

Møller Energy-Momentum Prescription for a Locally Rotationally Symmetric Space-Time

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Abstract. The energy distribution in the Locally Rotationally Symmetric (LRS) Bianchi type II space-time is obtained by considering the Møller energy-momentum definition in both Einstein's theory of general relativity and teleparallel theory of relativity. The energy distribution which includes both the matter and gravitational field is found to be zero in both of these different gravitation theories. This result agrees with previous works of Cooperstock and Israelit, Rosen, Johri *et al.*, Banerjee and Sen, Vargas, and Aydogdu and Salti. Our result that the total energy of the universe is zero supports the view points of Albrow and Tryon.

Keywords: Energy-momentum; LRS Bianchi type II; General Relativity; Teleparallel Gravity.
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1. Introduction

Energy-momentum which is associated with a symmetry of space-time geometry is regarded as the most fundamental conserved quantity in physics. Furthermore, the definition of an energy-momentum density for a gravitational field is one of the oldest and most controversial problems of gravitation.

The problem of finding energy-momentum density associated with various spacetimes are considered in a large number of researches. Both Einstein's theory of general relativity and teleparallel theory of gravity consider the problem of obtaining the generally acceptable energy-momentum definition. Various methods have been proposed to deduce the conservation laws that characterize the gravitational systems since the advent of these different gravitation theories. The first of such attempts was made by Einstein who proposed an expression for the energy-momentum distribution of the gravitational field. Following the Einstein's energy-momentum complex[1] used for calculating the energy in general relativistic systems, various energy-momentum complexes have been introduced: e.g, Tolman[2], Papapetrou[3], Bergmann and Thomson[4], Møller[5], Landau and Lifshitz[6], Weinberg[7], Quadir and Sharif[8], and the teleparallel gravity analogs of the Einstein[9], Landau and Lifshitz[9], Bergmann and Thomson[9], and

Møller[10].

There exist an opinion that energy-momentum definitions are not useful to get finite and meaningful results for a given geometry. If we transform the line element into quasi-cartesian coordinates, the energy-momentum complexes give meaningful results. Specifically, the Møller energy-momentum complex allows us to compute the energy in any coordinate system[11]. Using energy-momentum complexes, Virbhadra and his colleagues re-opened the problem of the energy-momentum density. They have shown that several energy momentum complexes give the same and acceptable results for a given space-time[12, 13, 14, 15, 16, 17, 18]. For a general non-static spherical symmetric metric of the Kerr-Schild class, Virbhadra shown that Einstein, Landau and Lifshitz, Papapetrou, and Weinberg complexes give the same energy distribution as in the Penrose energy-momentum complex[17].

Albrow[19] and Tryon[20] claimed that all conserved quantities have to vanish in our universe. Tryon's big bang model predicted a homogenous, isotropic and closed universe including matter and anti-matter equally. Therefore, they argue that any closed universe has zero energy. Cooperstock and Israelit[21] found that the total energy of any homogeneous isotropic universe described by a Friedmann-Robertson-Walker(FRW) metric in the context of general relativity is zero. Following this interesting work, Rosen[22] who considered a closed homogeneous isotropic universe found that the total energy is vanishing, using Einstein's energy-momentum complex. Johri *et al.*[23], using the Landau and Lifshitz's energy-momentum complex, obtained that the total energy of a FRW spatially closed universe is zero at all times. The total energy of the Bianchi type I space-time is obtained by Banerjee and Sen[24], using the Einstein's definition. They found that the total energy is zero. This result agrees with the studies of Johri *et al.* as the line element of the Bianchi type I space-time reduce to spatially flat FRW line element in a special case. Using the definitions of Einstein and Landau and Lifshitz's energy-momentum complexes in teleparallel gravity, Vargas[9] calculated that the total energy is zero in FRW space-times. These results agree with the works of Rosen and Johri *et al.*. In recent papers, Salti and Havare[25] who considered Bergmann-Thomson's complex in both general relativity and teleparallel gravity for the viscous Kasner type metric and in other work, Salti[26], using Einstein and Landau and Lifshitz's complexes associated with the same metric in teleparallel gravity, found that the total energy is zero. At the last, Aydogdu and Salti[27] who used Møller's definition in teleparallel gravity for Bianchi type I metric and obtained that the total energy is zero everywhere.

The aim of this paper is to calculate the total energy of a model of the universe based on the LRS Bianchi type II metric, using the energy-momentum complex of Møller. In the next section, we introduce LRS Bianchi type II cosmological models. In section III, we review Møller's energy-momentum complex in general relativity and calculate energy density. In section IV, we give Møller's definition in teleparallel gravity

and compute the energy density. The final section is devoted to the discussion and conclusion. Throughout this paper, Latin indices (i,j,k,...) represent the vector number and Greek indices (μ, ν, λ, \dots) represent the vector components. All indices run from 0 to 3 and we use the convention that $G = 1, c = 1$ units.

2. LRS Bianchi type II space-time

The FRW models have a significant role in cosmology. Whether these models correctly represent the universe or not isn't known, but it is believed that they are good global approximations of the present universe. Spatial homogeneity and isotropy characterize these models.

In last decade, theoretical interest in anisotropic cosmological models has been increased. In current modern cosmology, the spatial homogeneous and anisotropic Bianchi models which present a medium way between FRW models and completely inhomogeneous and anisotropic universes play an important role. Here, we consider the LRS model of Bianchi type II. The metric for Bianchi type II in the LRS case is given by[28]

$$ds^2 = dt^2 - A(t)^2 dx^2 - B(t)^2 dy^2 - [A(t)^2 + x^2 B(t)^2] dz^2 - 2xB(t)^2 dydz \quad (1)$$

$A(t)$ and $B(t)$ which are expansion factors could be determined via Einstein's field equations. The non-vanishing components of the Einstein tensor $G_{\mu\nu} (\equiv 8\pi T_{\mu\nu}$, where $T_{\mu\nu}$ is the energy-momentum tensor for the matter field described by a perfect fluid with density ρ , pressure p) are

$$G_{11} = A\ddot{A} + A^2 \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{B^2}{4A^2} \quad (2)$$

$$G_{22} = 2B^2 \frac{\ddot{A}}{A} + B^2 \frac{\dot{A}^2}{A^2} - \frac{3B^4}{4A^4} \quad (3)$$

$$G_{33} = (2x^2 B^2 + A^2) \frac{\ddot{A}}{A} + A^2 \frac{\ddot{B}}{B} + x^2 B^2 \frac{\dot{A}^2}{A^2} + \frac{A}{B} \dot{A}\dot{B} + \frac{B^2}{4A^2} - x^2 \frac{3B^2}{4A^4} \quad (4)$$

$$G_{00} = \frac{\dot{A}^2}{A^2} - 2 \frac{\dot{A}\dot{B}}{AB} + \frac{B^2}{4A^4} \quad (5)$$

$$G_{23} = B^2 \left(2 \frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} - \frac{3B^2}{4A^4} \right) \quad (6)$$

where dot represents derivation with respect to time.

For the line element given in Eq. (1), $g_{\mu\nu}$ and its inverse $g^{\mu\nu}$ are given as follow:

$$g_{\mu\nu} = \delta_\mu^0 \delta_\nu^0 - A^2 \delta_\mu^1 \delta_\nu^1 - B^2 \delta_\mu^2 \delta_\nu^2 - (A^2 + x^2 B^2) \delta_\mu^3 \delta_\nu^3 - xB^2 (\delta_\mu^3 \delta_\nu^2 + \delta_\mu^2 \delta_\nu^3) \quad (7)$$

$$g^{\mu\nu} = \delta_0^\mu \delta_0^\nu - A^{-2} \delta_1^\mu \delta_1^\nu - \frac{A^2 + x^2 B^2}{B^2 A^2} \delta_2^\mu \delta_2^\nu - A^{-2} \delta_3^\mu \delta_3^\nu + xA^{-2} (\delta_3^\mu \delta_2^\nu + \delta_2^\mu \delta_3^\nu) \quad (8)$$

The riemannian metric arises as

$$g_{\mu\nu} = \eta_{ij} h_\mu^i h_\nu^j \quad (9)$$

Using this relation, we obtain the tetrad components

$$h_\mu^i = \delta_0^i \delta_\mu^0 + A \delta_1^i \delta_\mu^1 + B \delta_2^i \delta_\mu^2 + A \delta_3^i \delta_\mu^3 + x B \delta_2^i \delta_\mu^3 \quad (10)$$

and its inverse is

$$h_i^\mu = \delta_i^0 \delta_0^\mu + A^{-1} \delta_i^1 \delta_1^\mu + B^{-1} \delta_i^2 \delta_2^\mu + A^{-1} \delta_i^3 \delta_3^\mu - \frac{x}{A} \delta_i^3 \delta_2^\mu \quad (11)$$

From the Christoffel symbols defined by

$$\Gamma_{\mu\nu}^\alpha = \frac{1}{2} g^{\alpha\tau} (\partial_\mu g_{\tau\nu} + \partial_\nu g_{\tau\mu} - \partial_\tau g_{\mu\nu}) \quad (12)$$

we obtain non-vanishing components:

$$\begin{aligned} \Gamma_{11}^0 &= A\dot{A}, & \Gamma_{22}^0 &= B\dot{B}, & \Gamma_{33}^0 &= A\dot{A} + x^2 B\dot{B}, & \Gamma_{01}^1 &= \frac{\dot{A}}{A} \\ \Gamma_{02}^2 &= \frac{\dot{B}}{B}, & \Gamma_{03}^3 &= \frac{\dot{A}}{A}, & \Gamma_{03}^2 &= \frac{x(A\dot{B} - B\dot{A})}{BA}, & \Gamma_{12}^2 &= -\frac{x^2 B^2}{2A^2} \\ \Gamma_{13}^2 &= \frac{A^2 - x^2 B^2}{2A^2}, & \Gamma_{12}^3 &= \frac{x^2 B^2}{2A^2}, & \Gamma_{13}^3 &= \frac{x^2 B^2}{2A^2} \end{aligned} \quad (13)$$

The notion of global rotation of the universe has become a notably significant physical aspect in the calculations of general relativity since Gamov[29] and Gödel's[30] works. For the LRS Bianchi type II metric, one can introduce the tetrad basis as follows:

$$\Theta^0 = dt, \quad \Theta^1 = Adx, \quad \Theta^2 = Bdy + xBdz, \quad \Theta^3 = Adz \quad (14)$$

Using the co-moving tetrad formalism, the kinematical variables of this model can be expressed only in terms of the structure coefficients of the tetrad basis defined as

$$d\Theta^\xi = \frac{1}{2} \Xi_{\mu\nu}^\xi \Theta^\mu \wedge \Theta^\nu \quad (15)$$

Four-acceleration vector	: $a_\gamma = \Delta_{\gamma 0}^0$
Vorticity tensor	: $w_{\gamma\beta} = \frac{1}{2} \Delta_{\gamma\beta}^0$
Expansion tensor	: $\chi_{\alpha\beta} = \frac{1}{2} (\Delta_{\alpha 0\beta} + \Delta_{\beta 0\alpha})$
Expansion scalar	: $\chi = \Delta_{01}^1 + \Delta_{02}^2 + \Delta_{03}^3$
Vorticity vector	: $w^1 = \frac{1}{2} \Delta_{23}^0, \quad w^2 = \frac{1}{2} \Delta_{31}^0, \quad w^3 = \frac{1}{2} \Delta_{12}^0$
Vorticity scalar	: $w^2 = \frac{1}{4} [(\Delta_{23}^0)^2 + (\Delta_{31}^0)^2 + (\Delta_{12}^0)^2]$
Shear tensor	: $\sigma_{\mu\nu} = \chi_{\mu\nu} - \frac{1}{3} \chi \delta_{\mu\nu}$

We calculated the following kinematical quantities for the line element (1), taking the exterior derivatives of the tetrad basis and using the kinematics[31] formulas which are given in Table ??:

$$a_\gamma = w^i = w_{\gamma\beta} = w = 0 \quad (16)$$

$$\chi_{11} = \frac{\dot{A}}{A}, \quad \chi_{22} = \frac{\dot{B}}{B}, \quad \chi_{33} = \frac{\dot{A}}{A}, \quad \chi = 2\frac{\dot{A}}{A} + \frac{\dot{B}}{B} \quad (17)$$

$$\sigma_{11} = \frac{1}{3}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right), \quad \sigma_{22} = \frac{2}{3}\left(\frac{\dot{B}}{B} - \frac{\dot{A}}{A}\right), \quad \sigma_{33} = \frac{1}{3}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) \quad (18)$$

From above result, we see that the model given in Eq. (1) has non-vanishing shear expansion. This model describes a cosmological model which has vanishing vorticity and four-acceleration. We also note that LRS Bianchi type II universe has vanishing shear expansion when we take $A = B$.

3. Energy distribution in LRS Bianchi type II space-times

In this section, we compute energy-momentum distributions associated with LRS Bianchi type II universe models, using Bergmann-Thomson's definition in both general relativity and teleparallel gravity.

4. The Møller Energy in General Relativity

In general relativity, the energy-momentum complex of Møller[5] is given by

$$M_{\mu}^{\nu} = \frac{1}{8\pi}\Xi_{\mu,\alpha}^{\nu\alpha} \quad (19)$$

Also, it satisfies the local conservation laws:

$$\frac{\partial M_{\mu}^{\nu}}{\partial x^{\nu}} = 0 \quad (20)$$

where the antisymmetric super-potential $\Xi_{\mu}^{\nu\alpha}$ is

$$\Xi_{\mu}^{\nu\alpha} = \sqrt{-g}[g_{\mu\beta,\gamma} - g_{\mu\gamma,\beta}]g^{\nu\gamma}g^{\alpha\beta} \quad (21)$$

The locally conserved energy-momentum complex M_{μ}^{ν} contains contributions from the matter, non-gravitational and gravitational fields. M_0^0 is the energy density and M_i^0 are the momentum density components. The momentum four-vector of Møller is given by

$$P_{\mu} = \int \int \int M_{\mu}^0 dx dy dz \quad (22)$$

Using Gauss's theorem, we obtain

$$P_{\mu} = \frac{1}{8\pi} \int \int \Xi_{\mu}^{0\alpha} \eta_{\alpha} dS \quad (23)$$

where η_{α} (where $\alpha = 1, 2, 3$) is the outward unit normal vector over the infinitesimal surface element dS . P_i give momentum components P_1, P_2, P_3 and P_0 (or E_{GR}) gives the energy.

We want to find the total energy in the LRS Bianchi type II space-time which is described by the line element (1). From Eq. (21) with Eqs. (8) and (9), we obtain that the required non-vanishing $\Xi_{\mu}^{\nu\alpha}$ component is

$$\Xi_1^{01} = 2BA\dot{A} \quad (24)$$

Using above result, we find

$$M_0^0 = M_i^0 = 0 \quad (25)$$

As a result of this, we easily see that Møller's energy in the LRS Bianchi type II space-time is

$$E_{GR} = 0 \quad (26)$$

5. The Møller Energy in Teleparallel Gravity

The teleparallel theory of gravity (tetrad theory of gravitation) is an alternative approach to gravitation and corresponds to a gauge theory for the translation group based on Weitzenböck geometry[32]. The theory of teleparallel gravity is attributed to torsion[33], which plays the role of a force[34], and the curvature tensor vanishes identically. The essential field is acted by a nontrivial tetrad field, which gives rise to the metric as a by-product. The translational gauge potentials appear as the nontrivial item of the tetrad field, so induces on space-time a teleparallel structure which is directly related to the presence of the gravitational field. The interesting place of teleparallel theory is that it can reveal a more appropriate approach to consider some specific problem due to its gauge structure. This is the situation, for instance, in the energy and momentum problem, which becomes more transparent.

Møller modified general relativity by constructing a new field theory in teleparallel space. The aim of this theory was to overcome the problem of the energy-momentum complex that appears in Riemannian Space[35]. The field equations in this new theory were derived from a Lagrangian which is not invariant under local tetrad rotation. Saez[36] generalized Møller theory into a scalar tetrad theory of gravitation. Meyer[37] showed that Møller theory is a special case of Poincare gauge theory[38, 39].

In teleparallel gravity, the super-potential of Møller is given by Mikhail *et al.*[10] as

$$U_{\mu}^{\nu\alpha} = \frac{(-g)^{1/2}}{2\kappa} P_{\chi\rho\sigma}^{\tau\nu\alpha} [\Phi^{\rho} g^{\sigma\chi} g_{\mu\tau} - \lambda g_{\tau\mu} \Upsilon^{\chi\rho\sigma} - (1 - 2\lambda) g_{\tau\mu} \Upsilon^{\sigma\rho\chi}] \quad (27)$$

where $\Upsilon_{\alpha\beta\mu} = h_{i\alpha} h^i{}_{\beta;\mu}$ is the con-torsion tensor. κ is the Einstein constant and λ is free-dimensionless coupling parameter of teleparallel gravity. For the teleparallel equivalent of general relativity, there is a specific choice of this constant.

Φ_{μ} is the basic vector field given by

$$\Phi_{\mu} \doteq \Upsilon^{\tau}{}_{\mu\tau} \quad (28)$$

and $P_{\chi\rho\sigma}^{\tau\nu\beta}$ can be found by

$$P_{\chi\rho\sigma}^{\tau\nu\beta} = \delta_{\chi}^{\tau} g_{\rho\sigma}^{\nu\beta} + \delta_{\rho}^{\tau} g_{\sigma\chi}^{\nu\beta} - \delta_{\sigma}^{\tau} g_{\chi\rho}^{\nu\beta} \quad (29)$$

with $g_{\rho\sigma}^{\nu\beta}$ being a tensor defined by

$$g_{\rho\sigma}^{\nu\beta} = \delta_{\rho}^{\nu} \delta_{\sigma}^{\beta} - \delta_{\sigma}^{\nu} \delta_{\rho}^{\beta} \quad (30)$$

The energy-momentum density is defined by

$$\Sigma_{\alpha}^{\beta} = U_{\alpha}^{\beta\nu}{}_{,\nu} \quad (31)$$

where comma denotes ordinary differentiation. The energy is expressed by the surface integral;

$$E = \lim_{r \rightarrow \infty} \int_{r=\text{constant}} U_0^{0\gamma} \mu_\gamma dS \quad (32)$$

where μ_γ is the unit three-vector normal to surface element dS . Taking the results given by Eqs. (10) and (11) in $\Upsilon_{\alpha\beta\mu} = h_{i\alpha} h^i{}_{\beta;\mu}$, we get the non-vanishing components given as

$$\begin{aligned} \Upsilon_{101} &= -\Upsilon_{011} = A\dot{A}, \\ \Upsilon_{202} &= -\Upsilon_{022} = B\dot{B}, \\ \Upsilon_{303} &= -\Upsilon_{033} = A\dot{A}. \end{aligned} \quad (33)$$

From above result, the non-vanishing basic vector field component is given by

$$\Phi^0 = 2A\dot{A} + B\dot{B} \quad (34)$$

Using Eq. (27) with the results given in Eqs. (33) and (34), we obtain that the required components of Møller super-potential are zero. From this result, we easily see that Møller's energy which is given Eq. (32) in the LRS Bianchi type II space-time is

$$E_{TP} = 0 \quad (35)$$

at all times.

6. Discussions

The localization of energy-momentum in both general relativity and teleparallel gravity has been very exciting and interesting and it has been associated with some debate. Recently, some researches have shown interest in studying the energy content of the universe in various model. The total energy of a homogeneous isotropic universe described by FRW metric calculated by Rosen using Einstein's energy-momentum complex. He found that the total energy is zero everywhere. Using Landau and Lifshitz complex, Johri *et al.* shown that the total energy of the same universe is zero. Furthermore, Banerjee and Sen and Xulu, using Landau and Lifshitz and Einstein complexes, obtained that the total energy of the Bianchi type I universes is vanishing.

In this paper, using Møller's energy-momentum complex, we obtained the energy distribution in the LRS Bianchi type II universes in both general relativity and teleparallel gravity. These different gravitation theories give same result:

$$E_{GR} = E_{TP} = 0 \quad (36)$$

which agree with the result obtained by Cooperstock and Israelit, Rosen, Johri *et al.*, Banerjee and Sen, Vargas, Salti *et al.*. Although Einstein's energy-momentum tensor has non-vanishing components which are given in Eqs. (2)-(6), the total energy of the LRS Bianchi type II space-time is zero; because the energy-momentum contributions from the matter and field inside arbitrary two surfaces, in the case of the anisotropic model

based on the LRS Bianchi type II metric, cancel each other. Because of the independence of the dimensionless coupling constant of the teleparallel gravity, our result is valid for any teleparallel model. Moreover, this result supports the view points of Albrow and Tryon. Finally, our result that total energy density is vanishing everywhere maintains the importance of the energy-momentum complexes.

References

- [1] A. Einstein, *Sitzungsber. Prevs. Akad. Wiss. Berlin Math.Phys.* 778 (1915).
- [2] R. C. Tolman, *Oxford Univ. Press Loandon*, 227 (1934).
- [3] A. Papapetrou, *Proc. R. Irish. Acad.* **A52**, 11 (1948).
- [4] P. G. Bergmann and R. Thomson, *Phys. Rev.* **89**, 400 (1953).
- [5] C. Møller, *Ann. Phys.* **4**, 347 (1958); *Ann. Phys.* **12**, 118 (1961).
- [6] L. D. Landau and E. M. Lifshitz, *Pergamon Press, Oxford* (1987).
- [7] S. Weinberg, *John Wilet and Sons, Inc., New York* (1972).
- [8] A. Quadir and M. Sharif, *Phys. Lett.A*, **167**, 331 (1992).
- [9] T. Vargas, *Gen.Rel.Grav.* **36**, 1255 (2004).
- [10] F. I. Mikhail, M. I. Wanas, A. Hindawi and E. I. Lashin, *Int.J.Theor.Phys.* **32**, 1627 (1993).
- [11] I. Radinschi, *Mod.Phys.Lett.A*, **15**, 2171 (2000).
- [12] K. S. Vibhadra, *Phys. Rev.D* **41**, 1086 (1990).
- [13] K. S. Vibhadra, *Phys. Rev.D* **42**, 2919 (1990).
- [14] N. Rosen and K. S. Vibhadra, *Gen.Rel.Grav.*, **25**, 429 (1993).
- [15] A. Chamorro and K. S. Vibhadra, *Pramana J. Phys.*, **45**, 181 (1995).
- [16] J. M. Aguirregabiria, A. Chamorro and K. S. Vibhadra, *Gen.Rel.Grav.*, **28**, 1393 (1996).
- [17] K. S. Vibhadra, *Phys. Rev.D* **60**, 104041 (1999).
- [18] S. S. Xulu, *Int. J. Mod. Phys. A*, **15**, 4849 (2000).
- [19] M. G. Albrow, *Nature*, **241**, 56 (1973).
- [20] E. P. Tryon, *Nature*, **246**, 396 (1973).
- [21] F. I. Cooperstock *Gen.Rel.Grav.*, **26**, 323 (1994). F. I. Cooperstock and M. Israelit, *Found of Phys.*, **25**, 631 (1995).
- [22] N. Rosen, *Gen.Rel.Grav.*, **26**, 319 (1994).
- [23] V. B. Johri, D. Kalligas, G. P. Singh and C. W. F. Everitt, *Gen.Rel.Grav.*, **27**, 323 (1995).
- [24] N. Banerjee and S. Sen, *Pramana J. Phys.*, **49**, 609 (1997).
- [25] M. Salti and A. Havare, *Int.J.Mod.Phys.D*, **20**, 2169, (2005).
- [26] M. Salti, *Astrophys. Space Sci.* **299**, (2005) 159
- [27] O. Aydogdu and M. Salti, *Astrophys. Space Sci.* **299**, (2005) 227.
- [28] D. Lorenz, *Phys. Lett.A* **79**, 19 (1980).
- [29] G. Gamov, *Nature*, **178**, (1976), 549.
- [30] K. Gödel, *Rev. Mod. Phys.*, **21**, (1949), 447.
- [31] O. Gron and H. H. Soleng, *Acta Phys. Pol. B*, **20**, (1989), 557.
- [32] R. Weitzenböck, *Invarianttent theorie* (Groningen: Noordhoff, 1923).
- [33] K. Hayashi and T. Shirafuji, *Phys. Rev. D*, **19**, 3524 (1978).
- [34] V.V. de Andrade and J.G. Pereira, *Phys. Rev. D*, **56**, 4689 (1997).
- [35] C. Møller, *Mat. Fys. Medd. K. Vidensk. Selsk.*, **39**, 13 (1978); **1**, 10 (1961).
- [36] D. Saez, *Phys. Rev. D*, **27**, 2839 (1983).
- [37] H. Meyer, *Gen. Rel. Gravit.*, **14**, 531 (1982).
- [38] K. Hayashi and T. Shirafuji, *Prog. Theor. Phys.*, **64**, 866 (1980); **65**, 525 (1980).
- [39] F.W. Hehl, J. Nitsch and P. von der Heyde, *General Relativity and Gravitation*, edited by A. Held (Plenum, New York, 1980).