# The annihilation cross section of dark matter which is driven by scalar unparticle

**E. O. Iltan** *
Physics Department, Middle East Technical University
Ankara, Turkey

**Abstract**

We analyze the annihilation cross section of dark matter which interacts with the standard model sector over the scalar unparticle propagator. We observe that the annihilation cross section of dark matter pair is sensitive to the dark matter mass and the scaling dimension of scalar unparticle. We estimate a range for the dark matter mass and the scaling dimension of scalar unparticle by using the current dark matter abundance.

*E-mail address: eiltan@metu.edu.tr

The visible matter is considerably less than the amount of matter required in the universe and 23% of present Universe [1, 2, 3, 4, 5] is contributed by the dark matter (DM) that is not detectable by the radiation emitted. Although the nature of DM is not known at present, the weakly interacting massive particles (WIMPs) [1] are among the promising candidates of DM and they are expected in the mass range 10 GeV- a few TeV. WIMPs do not decay in to standard model (SM) particles since they are stable, however they disappear by pair annihilation (see for example [6, 7]). One needs a theoretical framework beyond the SM in order to explain the nature of DM and its stability that can be ensured by an appropriate discrete symmetry in various models (for details see for example [8] and references therein). From the experimental point of view the indirect detection of the DM candidate is based on the current relic density which can be explained by thermal freeze-out of their pair annihilation. By using the current DM abundance by the WMAP collaboration [5] one gets the appropriate range for the annihilation cross section and obtains a valuable information about the nature of DM. In the present work we take an additional scalar SM singlet DM field $\phi_S$ (see [9]-[15]) and assume that it interacts with the SM sector over the scalar unparticle propagator. Unparticles [16, 17] arise from the interaction of the SM and the ultraviolet sector with non-trivial infrared fixed point at high energy level. They are massless and they have non integral scaling dimension $d_U$. The unparticle sector weakly interacts with the SM one and the interactions of unparticles with the SM fields in the low energy level is defined by the effective lagrangian

$$\mathcal{L}_{eff} = \frac{\eta}{\Lambda_U^{d_U+d_{SM}-n}} \, O_{SM} \, O_U \, , \tag{1}$$

with the unparticle operator $O_U$, the energy scale $\Lambda_U$, the space-time dimension $n$ and the parameter $\eta$ which carries information about the energy scale of ultraviolet sector, the low energy sector and the matching coefficient [16, 17, 18]. In order to formulate the DM annihilation we start with the effective lagrangian which obeys the $Z_2$ symmetry $\phi_S \to -\phi_S$

$$\mathcal{L}_S = \frac{1}{2} \, \partial_\mu \, \phi_S \, \partial^\mu \, \phi_S - \frac{\lambda}{4} \, \phi_S^4 - \frac{1}{2} \, m_S^2 \, \phi_S^2 - \frac{\lambda_0}{\Lambda_U^{d_U-2}} \phi_S^2 \, O_U \, , \tag{2}$$

where $\lambda_0$[1] is the effective coupling which leads to tree level DM-DM-scalar unparticle interaction. Here the DM scalar $\phi_S$ has no vacuum expectation value and the $Z_2$ symmetry guarantees the stability of $\phi_S$ which appears as pairs and it can not decay into the SM particles. On the other hand they are expected to annihilate into SM particles with the annihilating cross section which obeys the observed DM abundance. The annihilation process is driven by the exchange

[1] Notice that we consider $\lambda_0$ as universal coupling (see for example [19]), i.e., we take $\eta = \lambda_0$ and $n = 4$ in eq.(1).

particle(s) and, here, we assume that the scalar unparticle propagator is responsible for this annihilation. The scalar unparticle propagator is obtained by using the scale invariance [17, 19]:

$$\int d^4x\, e^{ipx} \;<0|T\Big(O_U(x)\, O_U(0)\Big)0> = i\frac{A_{d_u}}{2\,\pi}\int_0^\infty ds\, \frac{s^{d_u-2}}{p^2-s+i\epsilon} = i\,\frac{A_{d_u}}{2\,sin\,(d_u\pi)}\,(-p^2-i\epsilon)^{d_u-2}\,, \quad (3)$$

where $A_{d_u} = \frac{16\,\pi^{5/2}}{(2\,\pi)^{2\,d_u}}\,\frac{\Gamma(d_u+\frac{1}{2})}{\Gamma(d_u-1)\,\Gamma(2\,d_u)}$ and the function $\frac{1}{(-p^2-i\epsilon)^{2-d_u}}$ becomes $\frac{1}{(-p^2-i\epsilon)^{2-d_u}} \to \frac{e^{-i\,d_u\,\pi}}{(p^2)^{2-d_u}}$ for $p^2>0$ with a non-trivial phase which appears as a result of non-integral scaling dimension.

The total averaging annihilation rate of DM can obtained by the process $\phi_S\,\phi_S \to U \to X_{SM}$,

$$< \sigma\, v_r > \;=\; \frac{4\,\lambda_0^2}{m_S\,\Lambda_U^{2\,(d_U-2)}}\left(\frac{A_{d_U}}{2\,sin\,d_U\,\pi}\Big(\frac{1}{4\,m_S^2}\Big)^{2-d_U}\right)^2\Gamma(\tilde{U}\to X_{SM})\,, \quad (4)$$

where $\Gamma(\tilde{U}\to X_{SM}) = \sum_i \Gamma(\tilde{U}\to X_{i\,SM})$ with virtual unparticle $\tilde{U}$ having mass $m_U = 2\,m_S$ (see [20, 21]) and $v_r = \frac{2\,p_{CM}}{m_S}$ is the average relative speed of two DM scalars (see for example [15]). At this stage we present the functions $\Gamma(\tilde{U}\to X_{i\,SM})$ which appear in the annihilation cross section arising from possible annihilations that are valid for the DM mass range we choose (see Discussion section): In this range the annihilations to the fermion-antifermion pairs[2], photon pair, gluon pair and $WW^*$, $ZZ^*$ can exist. For the fermion-antifermion output we have

$$\Gamma(\tilde{U}\to f\,\bar{f}) = \sum_f \frac{N_f\,(c_U^{ff})^2}{8\,\pi\,m_U^2}\,(m_U^2-4\,m_f^2)^{\frac{3}{2}}\,, \quad (5)$$

where $N_f = 1\,(3)$ for leptons (quarks) and $c_U^{ff} = \frac{\lambda_0}{\Lambda_U^{d_U-1}}$. The one for the photon-photon (gluon-gluon) pair reads

$$\Gamma(\tilde{U}\to V\,V) = \frac{\beta\,m_U^3}{64\,\pi}\,|c_U^{VV}|^2\,, \quad (6)$$

where $c_U^{VV} = \frac{4\,i\,\lambda_0}{\Lambda_U^{d_U}}$ and $\beta = 1\,(2)$ for $V = \gamma\,(g)$. Finally for $WW^*$ and $ZZ^*$ output we get[3]

$$\Gamma(\tilde{U}\to W(Z)W(Z)^*) = \sum_{ij}\Gamma_{ij}(\tilde{U}\to W(Z)W(Z)^*)\,, \quad (7)$$

with

$$\Gamma_{ij}(\tilde{U}\to W(Z)W(Z)^*) = \frac{(2\,\pi)^4}{2\,m_U}\int \delta[P-\sum_{i=1}^{3}p_i\,]\prod_{i=1}^{3}\frac{d^3p_i}{(2\,\pi)^3\,2\,E_i}\,N_f\,|M_{ij}^{W(Z)}|^2\,, \quad (8)$$

where $p_i\,(p_j,\,p_3)$ is the outgoing charged lepton or down quark (incoming neutrino or up quark, outgoing W boson) four momentum for $\Gamma_{ij}(\tilde{U}\to WW^*)$, the outgoing lepton or quark (antilepton or antiquark, outgoing Z boson) four momentum for $\Gamma_{ij}(\tilde{U}\to ZZ^*)$. In eq.(8) $|M_{ij}^W|^2$

[2]The annihilations into top- antitop quark pair and top quark- antineutrino do not exist.
[3]Notice that, in this expression, we ignore the mass of neutrinos.

reads

$$|M^W_{ij}|^2 = \frac{16\, g^2\, c^2_{UW}\, |V_{ij}|^2 \left((p_i+p_j).p_3\right)^2}{m_W^6 \left(m_W^2-(p_i+p_j)^2\right)^2} \Big\{ 2\, m_i^2\, m_j^2\, m_W^2\, (m_i^2+m_j^2-2\, m_W^2) + 2\, (m_i^2+m_j^2)\, m_W^2\, (p_i.p_j)^2$$
$$- \; 2 \left((m_i-m_W)\,(m_i+m_W)\, p_i.p_3 + m_i^2\, p_j.p_3\right) \left(m_j^2\, p_i.p_3 + (m_j-m_W)\,(m_j+m_W)\, p_j.p_3\right)$$
$$+ \; p_i.p_j \left( (m_i^4 + 6\, m_i^2\, m_j^2 + m_j^4)\, m_W^2 - (m_i^2+m_j^2) \left(2\, m_W^4 + \left((p_i+p_j).p_3\right)^2\right) - m_W^6 \right) \Big\} . \qquad (9)$$

Here $c_{UW} = \frac{\lambda_0}{\Lambda_U^{d_U}}$, $V_{ij}$ is the CKM matrix element for up-down quark pairs and $V_{ij} = 1$ for neutrino-charged lepton. Finally $|M^Z_{ij}|^2$ is

$$|M^Z_{ij}|^2 = \frac{32\, g^2\, c_{UZ} \left((p_i+p_j).p_3\right)^2}{c_W^2\, m_Z^6 \left(m_Z^2-(p_i+p_j)^2\right)^2} \Big\{ (c_L^2+c_R^2) \Big( 2\, m_i^2\, m_j^2 \Big( m_Z^2\, (m_i^2+m_j^2-2\, m_Z^2)$$
$$- \left((p_i+p_j).p_3\right)^2 \Big) + p_i.p_j \Big( m_Z^2 \Big( m_i^4 + m_j^4 - m_Z^4 - 2\, m_j^2\, (m_Z^2 - p_i.p_j)$$
$$+ \; 2\, m_i^2\, (3\, m_j^2 - m_Z^2 + p_i.p_j) \Big) - (m_i^2+m_j^2) \left((p_i+p_j).p_3\right)^2 \Big) + 2\, m_Z^2 \Big( m_j^2\, p_i.p_3\, (p_i+p_j).p_3$$
$$+ \; p_j.p_3\, (-m_Z^2\, p_i.p_3 + m_i^2\, (p_i+p_j).p_3) \Big) \Big) - 2\, c_L\, c_R\, m_i\, m_j \Big( m_Z^2 \Big( m_i^4 + m_j^4 + 3\, m_Z^4$$
$$- \; 2\, m_j^2\, (m_Z^2 - 2\, p_i.p_j) - 4\, m_Z^2\, p_i.p_j + 4\, (p_i.p_j)^2 + 2\, m_i^2\, (m_j^2 - m_Z^2 + 2\, p_i.p_j) \Big)$$
$$- \; (m_i^2 + m_j^2 - 2\, m_Z^2 + 2\, p_i.p_j) \left((p_i+p_j).p_3\right)^2 \Big) \Big\} , \qquad (10)$$

where $c_{UZ} = \frac{\lambda_0}{\Lambda_U^{d_U}}$, $c_L = \frac{-1}{2} + s_W^2\, (\frac{1}{2})$ for charged lepton (neutrino), $c_L = \frac{-1}{2} + \frac{s_W^2}{3}\, (\frac{1}{2} - \frac{2\, s_W^2}{3})$ for down quark (up quark), $c_R = s_W^2\, (0)$ for charged lepton (neutrino) and $c_R = \frac{s_W^2}{3}\, (-\frac{2\, s_W^2}{3})$ for down quark (up quark).-

Now we are ready to analyze annihilation cross section $< \sigma\, v_r >$ and, by using the expression for the relic abundance,

$$\Omega\, h^2 = \frac{x_f\, 10^{-11}\, GeV^{-2}}{< \sigma\, v_r >} , \qquad (11)$$

with $x_f \sim 25$ [15], [22]-[24], we get the range $2.21 \times 10^{-9}\, GeV^{-2} \leq < \sigma\, v_r > \leq 2.44 \times 10^{-9}\, GeV^{-2}$. Here we respect the upper and the lower bounds of the present relic abundance [5]

$$\Omega\, h^2 = 0.1109 \pm 0.0056 . \qquad (12)$$

## Discussion

In the present work we analyze the annihilation cross section of DM which interacts with the SM sector over the scalar unparticle propagator. The DM-DM-unparticle coupling $\lambda_0$ plays an important role in the annihilation process and we study its numerical value by respecting the estimated upper and lower bounds of the annihilation cross section of the DM, namely, $2.21 \times 10^{-9}\, GeV^{-2} \leq <\sigma\, v_r> \leq 2.44 \times 10^{-9}\, GeV^{-2}$. Furthermore, the scaling dimension of scalar unparticle, the energy scale $\Lambda_U$ and the DM mass $m_S$ are among the free parameters of this scenario. For the the scaling dimension $d_U$ we choose the well known range $1 < d_u < 2$ (see [17, 25]). We consider the DM mass $m_S$ in the interval $10\, GeV \leq m_S \leq 70\, GeV$ and we take the the energy scale $\Lambda_U = 10\, TeV$.

In Fig.1 we plot the DM mass $m_S$ dependence of the coupling $\lambda_0$ for the annihilation cross section $<\sigma\, v_r>$ and different values of $d_U$. Here the lower-intermediate-upper solid (long dashed; dashed) line represents $\lambda_0$ for $d_U = 1.1 - 1.3 - 1.5$ and $<\sigma\, v_r>_{AV}$ ($<\sigma\, v_r>_{Max}$; $<\sigma\, v_r>_{Min}$). We observe that the coupling $\lambda_0$ is sensitive to $m_S$ and this sensitivity increases with the increasing values of the scaling dimension $d_U$. For small values of $d_U$ and $m_S$ $\lambda_0$ is more restricted and their increasing values result in that $\lambda_0$ lies in a broader range. In order to get the present experimental result of $<\sigma\, v_r>$, $\lambda_0$ must be at the order of the magnitude of 0.01 for $1.1 < d_u < 1.3$ and it must reach to 0.1 for $d_U = 1.5$ for the DM mass values $m_S > 40\, GeV$. For completeness we present the scaling dimension $d_U$ dependence

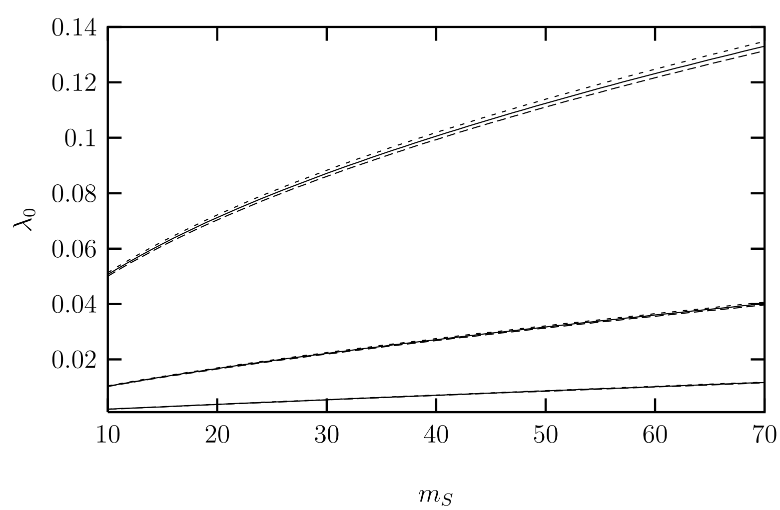


Figure 1: $\lambda_0$ as a function of $m_S$. Here the lower-intermediate-upper solid (long dashed; dashed) line represents $\lambda_0$ for $d_U = 1.1 - 1.3 - 1.5$ and $<\sigma\, v_r>_{AV}$ ($<\sigma\, v_r>_{Max}$; $<\sigma\, v_r>_{Min}$).

of the coupling $\lambda_0$ for the annihilation cross section $<\sigma\, v_r>$ and different values of $m_S$ in Fig.2. Here the lower-intermediate-upper solid (long dashed; dashed) line represents $\lambda_0$ for

$m_S = 30 - 50 - 70\, GeV$ and $< \sigma\, v_r >_{AV}$ ($< \sigma\, v_r >_{Max}$; $< \sigma\, v_r >_{Min}$). This figure also show the strong sensitivity of the coupling $\lambda_0$ to the scaling dimension $d_U$. The coupling reaches to the numerical values of the order of 1.0 for the upper bounds of $d_U$, namely for $d_U \sim 1.9$.

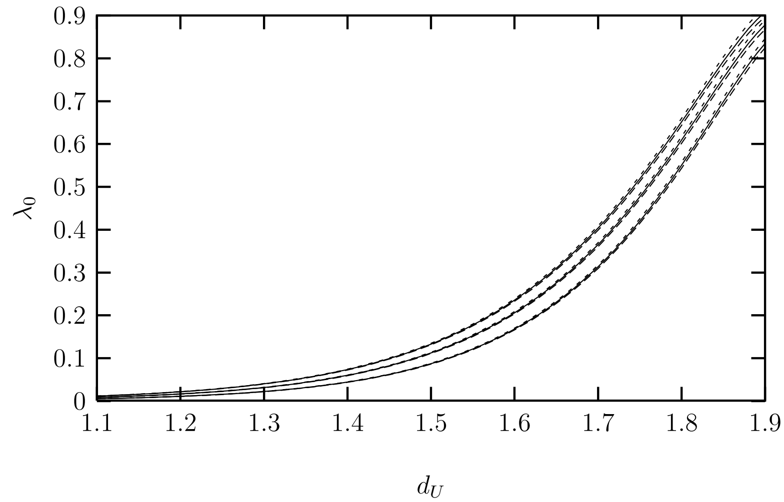


Figure 2: $\lambda_0$ as a function of $d_U$. Here the lower-intermediate-upper solid (long dashed; dashed) line represents $\lambda_0$ for $m_S = 30 - 50 - 70\, GeV$ and $< \sigma\, v_r >_{AV}$ ($< \sigma\, v_r >_{Max}$; $< \sigma\, v_r >_{Min}$).

Figs.3 and 4 represent $m_S$ and $d_U$ dependence of the annihilation cross section $< \sigma\, v_r >$ and, in both figures, the straight solid lines show the estimated upper and lower bounds.

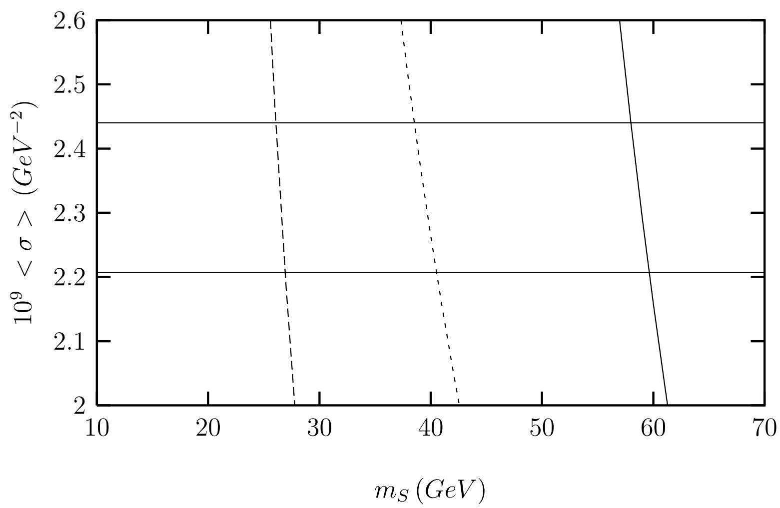


Figure 3: The annihilation cross section $< \sigma\, v_r >$ as a function of $m_S$. Here the solid (long dashed; dashed) line represents $< \sigma\, v_r >$ for $d_U = 1.1$ and $\lambda_0 = 0.01$ ($d_U = 1.2$ and $\lambda_0 = 0.01$; $d_U = 1.5$ and $\lambda_0 = 0.1$).

Fig.3 is devoted to the annihilation cross section $< \sigma\, v_r >$ with respect to $m_S$ for different values of $d_U$ and $\lambda_0$. Here the solid (long dashed; dashed) line represents $< \sigma\, v_r >$ for $d_U = 1.1$

and $\lambda_0 = 0.01$ ($d_U = 1.2$ and $\lambda_0 = 0.01$; $d_U = 1.5$ and $\lambda_0 = 0.1$). We observe that the $< \sigma\, v_r >$ is obtained in the estimated range for $d_U = 1.1$ and $\lambda_0 = 0.01$ in the case of $m_S \sim 60\, GeV$. For $d_U = 1.2$ and $\lambda_0 = 0.01$ the DM mass should be light, namely $m_S \sim 25\, GeV$, to get $< \sigma\, v_r >$ in the estimated range. For $d_U = 1.5$ and $\lambda_0 = 0.1$, $< \sigma\, v_r >$ lies in the estimated range for $m_S \sim 40\, GeV$. We see that, for a fixed coupling $\lambda_0$ (for $\lambda_0 = 0.01$ see this figure), the increase in the scaling dimension $d_U$ results in the decrease in the mass $m_S$ so that $< \sigma\, v_r >$ lies in the estimated range.

Fig.4 represents the annihilation cross section $< \sigma\, v_r >$ with respect to $d_U$ for $\lambda_0 = 0.1$ and different values of $m_S$. Here the solid (long dashed; dashed; dotted; dot-dashed) line represents $< \sigma\, v_r >$ for $m_S = 30\, (40; 50; 60; 70)\, GeV$. This figure shows that $< \sigma\, v_r >$ lies in the estimated range when $m_S$ respects $30\, GeV < m_S < 70\, GeV$ and $d_U \sim 1.5$.

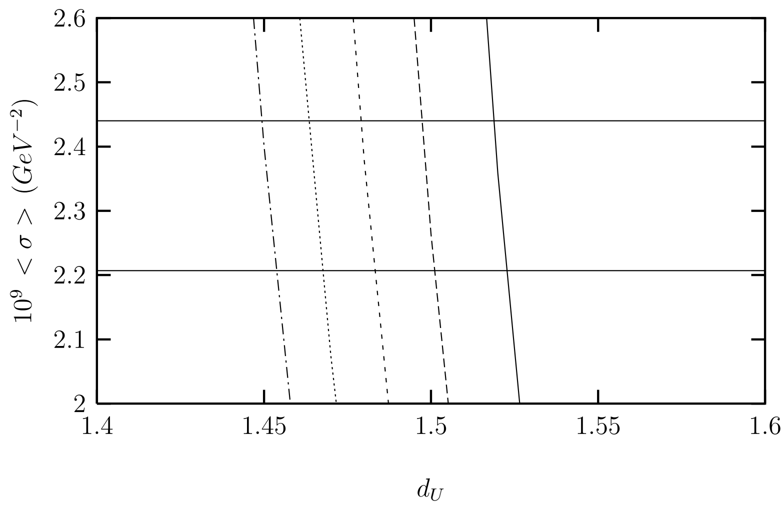


Figure 4: The annihilation cross section $< \sigma\, v_r >$ as a function of $d_U$. Here the solid (long dashed; dashed; dotted; dot-dashed) line represents $< \sigma\, v_r >$ for $m_S = 30\, (40; 50; 60; 70)\, GeV$.

As a summary, the annihilation cross section $< \sigma\, v_r >$ is sensitive to the DM-DM-unparticle coupling $\lambda_0$, the DM mass $m_S$ and the scaling dimension $d_U$. We observe that the coupling $\lambda_0$ is strongly restricted for the small values of $d_U$ and $m_S$. The experimental result of $< \sigma\, v_r >$ is obtained if $\lambda_0$ is at the order of the magnitude of 0.01 (0.1) for $1.1 < d_u < 1.3$ ($d_U \sim 1.5$) in the case of $m_S > 40\, GeV$. For $d_U \sim 1.9$, $\lambda_0$ reaches to the numerical values of the order of 1.0.

With the forthcoming more accurate experimental measurements one will provide a considerable information about the mechanism driving the possible annihilation process of DM and the role of unparticle physics on this process.

# References


[1] G. Jungman, M. Kamionkowski and K. Griest, *Phys. Rept.* **267**, 195 (1996).

[2] G. Bertone, D. Hooper and J. Silk, *Phys. Rept.* **405**, 279 (2005).

[3] D. Clowe et al.,*Astrophys. J.* **648**,L109 (2006).

[4] E. Komatsu et al., *Astrophys. J. Suppl. Ser.* **180**, 330 (2009).

[5] E. Komatsu et al. WMAP Collaboration, *Astrophys. J. Suppl.* **192** ,18 (2011).

[6] F. D. Eramo *Phys. Rev.* **D76**, 083522 (2007).

[7] W. L. Guo, X. Zhang, *Phys. Rev.* **D79**, 115023 (2009).

[8] C.-R. Chen, M. M. Nojiri, S. C. Park, J. Shu, IPMU09-0101, Aug 2009. 19pp., hep-ph/0908.4317 (2009).

[9] V. Silveira and A. Zee, *Phys. Lett.* **B161**, 136 (1985).

[10] D. E. Holz and A. Zee, *Phys. Lett.* **B517**, 239 (2001).

[11] J. McDonald, *Phys. Rev.* **D50**, 3637 (1994).

[12] B. Patt and F. Wilczek (2006), hep-ph/0605188.

[13] O. Bertolami, R. Rosenfeld, *Int. J. Mod. Phys.* **A23**, 4817 (2008).

[14] H. Davoudiasl, R. Kitano, T. Li, and H. Murayama, *Phys. Lett.* **B609**, 117 (2005).

[15] X.-G. He, T. Li, X.-Q. Li, and H.-C. Tsai, *Mod. Phys. Lett.* **A22**, 2121 (2007).

[16] H. Georgi, *Phys. Rev. Lett.* **98**, 221601 (2007).

[17] H. Georgi, *Phys. Lett.* **B650**, 275 (2007).

[18] R. Zwicky, *Phys. Rev.* **D77**, 036004 (2008).

[19] K. Cheung, W. Y. Keung, T. C. Yuan, *Phys. Rev. Lett.* **99**, 051803 (2007).

[20] C. Bird, P. Jackson, R. Kowalewski and M. Pospelov, *Phys. Rev. Lett.* **93**, 201803 (2004).

[21] C. Bird, R. Kowalewski and M. Pospelov, *Mod. Phys. Lett.* **A21**, 457 (2006).

[22] G. Servant and T. M. P. Tait, *Nucl. Phys.* **B650**, 391 (2003).

[23] S. Gopalakrishna, S. Gopalakrishna, A. de Gouvea, W. Porod, *JCAP* **0605**, 005 (2006).

[24] S. Gopalakrishna, S. J. Lee, J. D. Wells, *Phys. Lett.* **B680**, 88 (2009).

[25] Y. Liao, *Phys. Rev.* **D76**, 056006 (2007).