

# The effects of non-universal extra dimensions on the $\tau \rightarrow \mu \bar{\nu}_i \nu_i$ decay

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## Abstract

We study  $\tau \rightarrow \mu \bar{\nu}_i \nu_i$ ,  $i = e, \mu, \tau$  decays in the two Higgs doublet model, with the inclusion of one and two spatial non-universal extra dimensions. We observe that the branching ratio is sensitive to two extra dimensions in contrary to a single extra dimension.

arXiv:hep-ph/0411091v1 5 Nov 2004

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# 1 Introduction

Lepton flavor violating (LFV) interactions are rich to analyze since they exist in the loop level and they are sensitive to the physics beyond the standard model (SM). On the other hand they are clean theoretically because they are free from the nonperturbative effects. The experimental work done and the numerical results obtained stimulate the theoretical studies on these decays. The  $\mu \rightarrow e\gamma$  and  $\tau \rightarrow \mu\gamma$  processes are among the LFV decays and the experimental current limits for their branching ratios (BR) have been predicted as  $1.2 \times 10^{-11}$  [1] and  $1.1 \times 10^{-6}$  [2], respectively.

There is an extensive theoretical work done on the LFV decays in the literature. Such interactions are studied in a model independent way in [3], in the seesaw model [4], in the framework of the two Higgs doublet model (2HDM) [5, 6], in supersymmetric models [7, 8, 9, 10, 11, 12, 13].

In the present work we study  $\tau \rightarrow \mu \bar{\nu}_i \nu_i$ ,  $i = e, \mu, \tau$  decay in the model version of the 2HDM with the inclusion of the non-universal extra dimensions. This process exists at least at one loop level in the model III. The lepton flavor violation is driven by the internal scalar bosons  $h^0$  and  $A^0$  and the transition  $\tau \rightarrow \mu$  is obtained. Furthermore the internal Z boson connects this transition to the  $\bar{\nu}\nu$  output (see Fig. 1). Notice that we respect the assumption of the non-existence of Cabibbo-Kobayashi-Maskawa (CKM) type matrix in the leptonic sector and vanishing charged Flavor Changing (FC) interactions.

With the inclusion of additional dimensions there may be an enhancement in the BR of the decay under consideration. The extra dimensions have been studied in the literature extensively [14]-[31]. These new dimensions could not be detected at present and the most favorable description is their compactification on a surface with small radii. From the 4D point of view this compactification results in Kaluza-Klein (KK) modes of the particles with masses regulated by the parameter  $R$ , which is a typical size of the extra dimension. If all the fields are accessible to the extra dimensions, the extra dimensional momentum, and therefore, the KK number at each vertex is conserved. Such extra dimensions are called universal extra dimensions (UED) and they are studied in various works [15, 16, 17, 18, 20, 21, 22, 23] in the literature. If the extra dimensions are accessible to some fields but not all in the theory, such type of extra dimensions are called non-universal extra dimensions (NUED) (see for example [25, 26, 30, 31]). In this case the KK number at each vertex is not conserved and tree level interaction of KK modes with the ordinary particles can exist.

In our work we take the extra dimensions as non-universal and assume that the first Higgs

doublet and the gauge fields are accessible to the extra dimensions, however the leptons, the quarks and the second Higgs doublet, which contains new Higgs particles, live in the 4D brane. In this case the contributions due to the additional dimensions comes from the internal Z boson KK modes after the compactification of the external dimensions on orbifold  $S^1/Z_2$  ( $(S^1 \times S^1)/Z_2$ ) for a single (two) extra dimensions. We observe that there is almost two orders larger enhancement on the BR of the process we study in the case of two NUED. This enhancement is obviously due to the abundance of Z boson KK modes.

The paper is organized as follows: In Section 2, we present the theoretical expression for the decay width of the LFV decay  $\tau \rightarrow \mu \bar{\nu}_i \nu_i$ ,  $i = e, \mu, \tau$ , in the framework of the model III, with the inclusion of one and two NUEDs. Section 3 is devoted to discussion and our conclusions. In the appendix section, we give explicit expressions of the functions appearing in the general effective vertex for the interaction of off-shell Z-boson with a fermionic current.

## 2 $\tau \rightarrow \mu \bar{\nu}_i \nu_i$ decay in the general two Higgs doublet model with the inclusion of non-universal extra dimensions

The LFV  $\tau \rightarrow \mu \bar{\nu}_i \nu_i$  decay exists at least in the one loop level and, therefore, the physical quantities like the BR contains rich information about the model used and the free parameters existing. The extension of the Higgs sector in the SM makes the flavor violation (FV) possible with the help of the new Yukawa interactions coming from the new Higgs scalars. In the multi Higgs doublet models, the flavor changing neutral current (FCNC) at tree level, which induces the FV interactions, can appear and the general 2HDM, so called model III, is one of the candidate. The additional Yukawa interactions arising from new Higgs doublet are responsible for the non vanishing theoretical values of the physical quantities like the BR. The inclusion of extra dimensions may bring further contributions to these physical quantities. In the present work we assume that the first Higgs doublet and the gauge fields feel the extra dimensions, however, the leptons, the quarks and the second Higgs doublet, which contains new Higgs particles, live in the 4D brane. With the addition of single (double) extra dimension, the Yukawa interaction, including the LFV part in the model III, reads

$$\mathcal{L}_Y = \eta_{5(6)ij}^E \bar{l}_{iL} (\phi_1|_{y(z)=0}) E_{jR} + \xi_{ij}^{E\bar{}} \bar{l}_{iL} \phi_2 E_{jR} + h.c. \quad , \quad (1)$$

where  $i, j$  are family indices of leptons,  $L$  and  $R$  denote chiral projections  $L(R) = 1/2(1 \mp \gamma_5)$ ,  $\phi_i$  for  $i = 1, 2$ , are the two scalar doublets,  $l_{iL}$  and  $E_{jR}$  are lepton doublets and singlets respec-

tively. Here  $\eta_{5(6)ij}^E$  are 5(6)-dimensional dimensionful,  $\xi_{ij}^E$  are dimensionless Yukawa couplings and  $\eta_{5(6)ij}^E$  can be rescaled to the ones in 4-dimension as  $\eta_{5(6)ij}^E = \sqrt{2\pi R}(2\pi R)\eta_{ij}^E$ ,  $R$  is the compactification radius and  $y(z)$  is the coordinate represents the 5(6)'th dimension. The coupling  $\xi_{ij}^E$ , which has complex entries in general, is responsible for the FCNC at tree level<sup>1</sup>. At this stage we take two Higgs doublets  $\phi_1$  and  $\phi_2$  as

$$\phi_1 = \frac{1}{\sqrt{2}} \left[ \begin{pmatrix} 0 \\ v + H^0 \end{pmatrix} + \begin{pmatrix} \sqrt{2}\chi^+ \\ i\chi^0 \end{pmatrix} \right]; \phi_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}H^+ \\ H_1 + iH_2 \end{pmatrix}, \quad (2)$$

respecting that only the first one has a vacuum expectation value but not the second, namely,

$$\langle \phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}; \langle \phi_2 \rangle = 0. \quad (3)$$

Therefore the CP even neutral Higgs particles  $H_1$  and  $H^0$  do not mix in the tree level and they are obtained as the mass eigenstates  $h^0$  and  $H^0$  respectively. The neutral Higgs particle  $H_2$  is the well known CP odd  $A^0$ . In this case, the SM particles lie in the first doublet and the new particles in the second one.

The LFV  $\tau \rightarrow \mu \bar{\nu}_i \nu_i$  decay is induced by  $\tau \rightarrow \mu Z^*$  transition and  $Z^* \rightarrow \bar{\nu}_i \nu_i$  process (see Fig. 1). The  $\tau \rightarrow \mu Z^*$  transition, which needs the FCNC at tree level, is driven by the internal new neutral Higgs bosons  $h^0$  and  $A^0$ , which are living in the 4D brane. Since the gauge fields are accessible to the extra dimensions, the internal  $Z$  boson has KK modes, which have additional contribution to the physical quantities related to the decay studied. The KK modes of gauge fields appear after the compactification of the external dimensions on the orbifold  $S^1/Z_2$  ( $(S^1 \times S^1)/Z_2$ ) for a single (two) extra dimensions and, for two extra dimensions, the gauge fields can be expanded to the KK modes as:

$$\begin{aligned} A_\mu(x, y, z) &= \frac{1}{(2\pi R)^{d/2}} \left\{ A_\mu^{(0,0)}(x) + 2^{d/2} \sum_{n,r}^\infty A_\mu^{(n,r)}(x) \cos(ny/R + rz/R) \right\}, \\ A_i(x, y, z) &= \frac{1}{(2\pi R)^{d/2}} \left\{ 2^{d/2} \sum_{n,r}^\infty A_i^{(n,r)}(x) \sin(ny/R + rz/R) \right\}, \end{aligned} \quad (4)$$

where  $i = 5, 6$ ,  $d = 2$ , the indices  $n$  and  $r$  are positive integers including zero, but both are not zero at the same time. The KK modes of the gauge field  $Z$  have the masses  $\sqrt{m_Z^2 + m_n^2 + m_r^2}$ , where  $m_n = n/R$  and  $m_r = r/R$ . Notice that the gauge boson mass matrix is diagonal since the Higgs field, which has the non-zero vacuum expectation value, is in the bulk (see [30] for details). Here, we take the compactification radius  $R$  as the same for both new dimensions. In the case of a single extra dimension, one should set  $d = 1$ , take  $z = 0$ , and drop the summation over  $r$  in eq. (4).

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<sup>1</sup>Notice that, in the following, we replace  $\xi^E$  with  $\xi_N^E$  where "N" denotes the word "neutral".

Now, we present the general effective vertex for the interaction of off-shell Z-boson with a fermionic current

$$\Gamma_{\mu}^{(REN)}(\tau \rightarrow \mu Z^*) = f_1 \gamma_{\mu} + f_2 \gamma_{\mu} \gamma_5 + f_3 \sigma_{\mu\nu} k^{\nu} + f_4 \sigma_{\mu\nu} \gamma_5 k^{\nu}, \quad (5)$$

where  $k$  is the momentum transfer,  $k^2 = (p - p')^2$ ,  $p$  ( $p'$ ) is the four momentum vector of incoming (outgoing) lepton and the explicit expressions for the functions  $f_1$ ,  $f_2$ ,  $f_3$  and  $f_4$  are given in the appendix section. The matrix element  $M$  of the  $\tau \rightarrow \mu \bar{\nu}_i \nu_i$  process is obtained with the internal  $Z$  boson connection between the  $\tau \rightarrow \mu$  transition and the  $\bar{\nu}_i \nu_i$  pair. Since the gauge fields are accessible to the extra dimensions, the KK modes of the internal  $Z$  boson have additional contributions to the process (see Fig. 1). Notice that the KK number need not to be conserved in the vertices where  $Z$  boson appears since the the extra dimensions in this case are so called NUED. Using the matrix element  $M$  the decay width  $\Gamma$  of the decay under consideration can be obtained in the  $\tau$  lepton rest frame with the help of the well known expression

$$d\Gamma = \frac{(2\pi)^4}{2m_{\tau}} |M|^2 \delta^4(p - \sum_{i=1}^3 p_i) \prod_{i=1}^3 \frac{d^3 p_i}{(2\pi)^3 2E_i}, \quad (6)$$

where  $p$  ( $p_i$ ,  $i=1,2,3$ ) is four momentum vector of  $\tau$  lepton ( $\mu$  lepton, incoming  $\nu$ , outgoing  $\nu$ ).

### 3 Discussion

The  $\tau \rightarrow \mu \bar{\nu}_i \nu_i$  is induced by the LFV  $\tau \rightarrow \mu$  transition which depends on the various Yukawa couplings  $\bar{\xi}_{N,ij}^E$ <sup>2</sup>  $i = \mu, \tau$ ;  $j = e, \mu, \tau$  and they need to be restricted by using the present and forthcoming experiments. In our work, we take only the  $\tau$  lepton as an internal lepton and, therefore, we choose the couplings  $\bar{\xi}_{N,\tau\tau}^E$  and  $\bar{\xi}_{N,\tau\mu}^E$  as non-zero. Here we expect that the couplings which contain at least one  $\tau$  index are dominant similar to the Cheng-Sher scenario [32]. The upper limit of the coupling  $\bar{\xi}_{N,\tau\mu}^E$  has been estimated as  $30 GeV$  (see [33] and references therein) by assuming that the new physics effects can not exceed experimental uncertainty  $10^{-9}$  in the measurement of the muon anomalous magnetic moment. In our numerical calculation we choose  $\bar{\xi}_{N,\tau\mu}^E = 1 GeV$  by respecting this upper limit. Since there is no restriction for the Yukawa coupling  $\bar{\xi}_{N,\tau\tau}^E$ , the numerical values we use are greater than  $\bar{\xi}_{N,\tau\mu}^E$ .

This work is devoted to the a single and two NUED effects on the BR of the LFV processes  $\tau \rightarrow \mu \bar{\nu}_i \nu_i$ , in the type III 2HDM. In the case of two extra dimensions, we observe that the contribution of KK modes enhances the BR considerably, due to the crowd of Z boson KK

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<sup>2</sup>The dimensionfull Yukawa couplings  $\bar{\xi}_{N,ij}^E$  are defined as  $\xi_{N,ij}^E = \sqrt{\frac{4G_F}{\sqrt{2}}} \bar{\xi}_{N,ij}^E$ .

modes. Notice that we use the numerical values,  $m_Z = 91$  ( $GeV$ ),  $m_W = 80$  ( $GeV$ ),  $s_w = \sqrt{0.23}$ ,  $G_F = 1.6637 \times 10^{-5}$  ( $GeV^{-2}$ ),  $\Gamma_\tau = 2.27 \times 10^{-12}$  ( $GeV$ ), in our numerical calculations.

In Fig. 2, we present  $\bar{\xi}_{N,\tau\tau}^E$  dependence of the BR for  $\bar{\xi}_{N,\tau\mu}^E = 1$  ( $GeV$ ) and for different values of the compactification scale  $1/R$ , in the case of a single extra dimension. Here solid (dashed, small dashed, dotted, dash-dotted, three dashed-spaced) line represents the case without extra dimensions (with extra dimensions for  $1/R = 200, 400, 800, 1000, 2000$  ( $GeV$ )). It is observed that the BR lies in the range  $1.0 \times 10^{-6} - 5.0 \times 10^{-5}$  for the interval of the Yukawa coupling  $10$  ( $GeV$ )  $\leq \bar{\xi}_{N,\tau\tau}^E \leq 50$  ( $GeV$ ). The addition of a single extra dimension enhances the BR almost twice for the small values of the compactification scale. However this enhancement is not more than the order of 1% for its large values.

Fig. 3, represents the compactification scale  $1/R$  dependence of the BR for  $\bar{\xi}_{N,\tau\mu}^E = 1$  ( $GeV$ ) and for different values of the coupling  $\bar{\xi}_{N,\tau\tau}^E$ , in the case of a single extra dimension. Here solid-dashed-dotted straight lines (curved lines) represents the BR for  $\bar{\xi}_{N,\tau\tau}^E = 10 - 30 - 50$  ( $GeV$ ) without extra dimensions (with extra dimensions). This figure also shows that the contribution due to the extra dimensions is negligible especially for  $1/R \geq 600$  ( $GeV$ ).

Now we make the same analysis for two NUED. In this case the crowd of Z boson KK modes cause to enhance the BR of the decay analyzed and these effects can be observable. Notice that there is a possible divergence problem due to the abundance of KK modes in the summation of the KK mode contributions, however, the ratio  $\frac{1}{m_Z^2 + (n^2 + r^2)/R^2}$  appearing in the internal line converges to zero sharply with the increasing values of the integers  $n$  and  $r$  and the convergence of the KK sum is obtained for the region of the compactification scale that we study,  $1/R > 200$   $GeV$ .

In Fig. 4, we present  $\bar{\xi}_{N,\tau\tau}^E$  dependence of the BR for  $\bar{\xi}_{N,\tau\mu}^E = 1$  ( $GeV$ ) and for different values of the compactification scale  $1/R$  in the case of two NUEDs. Here solid (dashed, small dashed, dotted, dash-dotted, three dashed-spaced) line represents the case without extra dimensions (with extra dimensions for  $1/R = 200, 400, 800, 1000, 2000$  ( $GeV$ )). This figure shows that the BR is considerably enhanced for the small values of the compactification scale  $1/R$ . The BR is two orders (one order, three times, two times, 30 %) larger for  $1/R = 200$  ( $GeV$ ) ( $1/R = 400$  ( $GeV$ ),  $1/R = 800$  ( $GeV$ ),  $1/R = 1000$  ( $GeV$ ),  $1/R = 2000$  ( $GeV$ )) compared to the one without NUED. Even for large values of the compactification scale near  $1/R \sim 2000$  ( $GeV$ ) there is an enhancement in the BR.

Fig. 5 is devoted to the compactification scale  $1/R$  dependence of the BR for  $\bar{\xi}_{N,\tau\mu}^E = 1$  ( $GeV$ ) and for different values of the coupling  $\bar{\xi}_{N,\tau\tau}^E$ , for two NUEDs. Here solid-dashed-

dotted straight lines (curved lines) represents the BR for  $\bar{\xi}_{N,\tau\tau}^E = 10 - 30 - 50 (GeV)$  without extra dimensions (with extra dimensions). The enhancement in the BR is observed in this figure also. These contributions become negligible for the large values of the compactification scale, namely for  $1/R > 2000 (GeV)$ .

At this stage we would like to summarize our results:

- We predict the BR in the range  $1.0 \times 10^{-6} - 5.0 \times 10^{-5}$  for the interval of the Yukawa coupling  $10 (GeV) \leq \bar{\xi}_{N,\tau\tau}^E \leq 50 (GeV)$ . The inclusion of a single NUED brings contributions which increase the BR twice of the one without the extra dimension, for the small values of the compactification scale. However this enhancement is not more than the order of 1% for the large values of the compactification scale.
- With the inclusion of two extra dimensions the BR is considerably enhanced for the small values of the compactification scale  $1/R$ , even two orders. In the case of large values of the compactification scale, there is still an enhancement in the BR.

Therefore, the future theoretical and experimental investigations of the process  $\tau \rightarrow \mu \bar{\nu}_i \nu_i$  would ensure a valuable information about signals coming from the extra dimensions.

## 4 Appendix

The explicit expressions for the functions  $f_1$ ,  $f_2$ ,  $f_3$  and  $f_4$  appearing in eq. (5) read

$$\begin{aligned}
f_1 &= \frac{g}{64 \pi^2 \cos \theta_W} \int_0^1 dx \frac{1}{m_{l_2}^2 - m_{l_1}^2} \left\{ c_V (m_{l_2} + m_{l_1}) \right. \\
&\left( (-m_i \eta_i^+ + m_{l_1}(-1+x) \eta_i^V) \ln \frac{L_{1,h^0}^{self}}{\mu^2} + (m_i \eta_i^+ - m_{l_2}(-1+x) \eta_i^V) \ln \frac{L_{2,h^0}^{self}}{\mu^2} \right. \\
&+ (m_i \eta_i^+ + m_{l_1}(-1+x) \eta_i^V) \ln \frac{L_{1,A^0}^{self}}{\mu^2} - (m_i \eta_i^+ + m_{l_2}(-1+x) \eta_i^V) \ln \frac{L_{2,A^0}^{self}}{\mu^2} \left. \right) \\
&+ c_A (m_{l_2} - m_{l_1}) \\
&\left( (-m_i \eta_i^- + m_{l_1}(-1+x) \eta_i^A) \ln \frac{L_{1,h^0}^{self}}{\mu^2} + (m_i \eta_i^- + m_{l_2}(-1+x) \eta_i^A) \ln \frac{L_{2,h^0}^{self}}{\mu^2} \right. \\
&+ (m_i \eta_i^- + m_{l_1}(-1+x) \eta_i^A) \ln \frac{L_{1,A^0}^{self}}{\mu^2} + (-m_i \eta_i^- + m_{l_2}(-1+x) \eta_i^A) \ln \frac{L_{2,A^0}^{self}}{\mu^2} \left. \right) \left. \right\} \\
&- \frac{g}{64 \pi^2 \cos \theta_W} \int_0^1 dx \int_0^{1-x} dy \left\{ m_i^2 (c_A \eta_i^A - c_V \eta_i^V) \left( \frac{1}{L_{A^0}^{ver}} + \frac{1}{L_{h^0}^{ver}} \right) \right. \\
&- (1-x-y) m_i \left( c_A (m_{l_2} - m_{l_1}) \eta_i^- \left( \frac{1}{L_{h^0}^{ver}} - \frac{1}{L_{A^0}^{ver}} \right) + c_V (m_{l_2} + m_{l_1}) \eta_i^+ \left( \frac{1}{L_{h^0}^{ver}} + \frac{1}{L_{A^0}^{ver}} \right) \right) \\
&- (c_A \eta_i^A + c_V \eta_i^V) \left( -2 + (k^2 x y + m_{l_1} m_{l_2} (-1+x+y)^2) \left( \frac{1}{L_{h^0}^{ver}} + \frac{1}{L_{A^0}^{ver}} \right) - \ln \frac{L_{h^0}^{ver}}{\mu^2} \frac{L_{A^0}^{ver}}{\mu^2} \right) \\
&- (m_{l_2} + m_{l_1}) (1-x-y) \left( \frac{\eta_i^A (x m_{l_1} + y m_{l_2}) + m_i \eta_i^-}{2 L_{A^0 h^0}^{ver}} + \frac{\eta_i^A (x m_{l_1} + y m_{l_2}) - m_i \eta_i^-}{2 L_{h^0 A^0}^{ver}} \right) \\
&+ \left. \frac{1}{2} \eta_i^A \ln \frac{L_{A^0 h^0}^{ver}}{\mu^2} \frac{L_{h^0 A^0}^{ver}}{\mu^2} \right\}, \\
f_2 &= \frac{g}{64 \pi^2 \cos \theta_W} \int_0^1 dx \frac{1}{m_{l_2}^2 - m_{l_1}^2} \left\{ c_V (m_{l_2} - m_{l_1}) \right. \\
&\left( (m_i \eta_i^- + m_{l_1}(-1+x) \eta_i^A) \ln \frac{L_{1,A^0}^{self}}{\mu^2} + (-m_i \eta_i^- + m_{l_2}(-1+x) \eta_i^A) \ln \frac{L_{2,A^0}^{self}}{\mu^2} \right. \\
&+ (-m_i \eta_i^- + m_{l_1}(-1+x) \eta_i^A) \ln \frac{L_{1,h^0}^{self}}{\mu^2} + (m_i \eta_i^- + m_{l_2}(-1+x) \eta_i^A) \ln \frac{L_{2,h^0}^{self}}{\mu^2} \left. \right) \\
&+ c_A (m_{l_2} + m_{l_1}) \\
&\left( (m_i \eta_i^+ + m_{l_1}(-1+x) \eta_i^V) \ln \frac{L_{1,A^0}^{self}}{\mu^2} - (m_i \eta_i^+ + m_{l_2}(-1+x) \eta_i^V) \ln \frac{L_{2,A^0}^{self}}{\mu^2} \right. \\
&+ (-m_i \eta_i^+ + m_{l_1}(-1+x) \eta_i^V) \ln \frac{L_{1,h^0}^{self}}{\mu^2} + (m_i \eta_i^+ - m_{l_2}(-1+x) \eta_i^V) \frac{\ln L_{2,h^0}^{self}}{\mu^2} \left. \right) \left. \right\} \\
&+ \frac{g}{64 \pi^2 \cos \theta_W} \int_0^1 dx \int_0^{1-x} dy \left\{ m_i^2 (c_V \eta_i^A - c_A \eta_i^V) \left( \frac{1}{L_{A^0}^{ver}} + \frac{1}{L_{h^0}^{ver}} \right) \right.
\end{aligned}$$



$$\begin{aligned}
& - m_i (1 - x - y) \left( c_V (m_{l_2} - m_{l_1}) \eta_i^- + c_A (m_{l_2} + m_{l_1}) \eta_i^+ \right) \left( \frac{1}{L_{h^0}^{ver}} - \frac{1}{L_{A^0}^{ver}} \right) \\
& + (c_V \eta_i^A + c_A \eta_i^V) \left( -2 + (k^2 x y - m_{l_1} m_{l_2} (-1 + x + y)^2) \left( \frac{1}{L_{h^0}^{ver}} + \frac{1}{L_{A^0}^{ver}} \right) - \ln \frac{L_{h^0}^{ver}}{\mu^2} \frac{L_{A^0}^{ver}}{\mu^2} \right) \\
& - (m_{l_2} - m_{l_1}) (1 - x - y) \left( \frac{\eta_i^V (x m_{l_1} - y m_{l_2}) + m_i \eta_i^+}{2 L_{A^0 h^0}^{ver}} + \frac{\eta_i^V (x m_{l_1} - y m_{l_2}) - m_i \eta_i^+}{2 L_{h^0 A^0}^{ver}} \right) \\
& - \left. \frac{1}{2} \eta_i^V \ln \frac{L_{A^0 h^0}^{ver}}{\mu^2} \frac{L_{h^0 A^0}^{ver}}{\mu^2} \right\}, \\
f_3 & = -i \frac{g}{64 \pi^2 \cos \theta_W} \int_0^1 dx \int_0^{1-x} dy \left\{ \left( (1 - x - y) (c_V \eta_i^V + c_A \eta_i^A) (x m_{l_1} + y m_{l_2}) \right. \right. \\
& + \left. \left. m_i (c_A (x - y) \eta_i^- + c_V \eta_i^+ (x + y)) \right) \frac{1}{L_{h^0}^{ver}} \right. \\
& + \left. \left( (1 - x - y) (c_V \eta_i^V + c_A \eta_i^A) (x m_{l_1} + y m_{l_2}) - m_i (c_A (x - y) \eta_i^- + c_V \eta_i^+ (x + y)) \right) \frac{1}{L_{A^0}^{ver}} \right. \\
& - \left. (1 - x - y) \left( \frac{\eta_i^A (x m_{l_1} + y m_{l_2})}{2} \left( \frac{1}{L_{A^0 h^0}^{ver}} + \frac{1}{L_{h^0 A^0}^{ver}} \right) + \frac{m_i \eta_i^-}{2} \left( \frac{1}{L_{h^0 A^0}^{ver}} - \frac{1}{L_{A^0 h^0}^{ver}} \right) \right) \right\}, \\
f_4 & = -i \frac{g}{64 \pi^2 \cos \theta_W} \int_0^1 dx \int_0^{1-x} dy \left\{ \left( (1 - x - y) \left( - (c_V \eta_i^A + c_A \eta_i^V) (x m_{l_1} - y m_{l_2}) \right) \right. \right. \\
& - \left. \left. m_i (c_A (x - y) \eta_i^+ + c_V \eta_i^- (x + y)) \right) \frac{1}{L_{h^0}^{ver}} \right. \\
& + \left. \left( (1 - x - y) \left( - (c_V \eta_i^A + c_A \eta_i^V) (x m_{l_1} - y m_{l_2}) \right) + m_i (c_A (x - y) \eta_i^+ + c_V \eta_i^- (x + y)) \right) \frac{1}{L_{A^0}^{ver}} \right. \\
& + \left. (1 - x - y) \left( \frac{\eta_i^V}{2} (m_{l_1} x - m_{l_2} y) \left( \frac{1}{L_{A^0 h^0}^{ver}} + \frac{1}{L_{h^0 A^0}^{ver}} \right) + \frac{m_i \eta_i^+}{2} \left( \frac{1}{L_{A^0 h^0}^{ver}} - \frac{1}{L_{h^0 A^0}^{ver}} \right) \right) \right\}, \quad (7)
\end{aligned}$$

where

$$\begin{aligned}
L_{1, h^0}^{self} & = m_{h^0}^2 (1 - x) + (m_i^2 - m_{l_1}^2 (1 - x)) x, \\
L_{1, A^0}^{self} & = L_{1, h^0}^{self} (m_{h^0} \rightarrow m_{A^0}), \\
L_{2, h^0}^{self} & = L_{1, h^0}^{self} (m_{l_1} \rightarrow m_{l_2}), \\
L_{2, A^0}^{self} & = L_{1, A^0}^{self} (m_{l_1} \rightarrow m_{l_2}), \\
L_{h^0}^{ver} & = m_{h^0}^2 (1 - x - y) + m_i^2 (x + y) - k^2 x y, \\
L_{h^0 A^0}^{ver} & = m_{A^0}^2 x + m_i^2 (1 - x - y) + (m_{h^0}^2 - k^2 x) y, \\
L_{A^0}^{ver} & = L_{h^0}^{ver} (m_{h^0} \rightarrow m_{A^0}), \\
L_{A^0 h^0}^{ver} & = L_{h^0 A^0}^{ver} (m_{h^0} \rightarrow m_{A^0}), \quad (8)
\end{aligned}$$

and

$$\eta_i^V = \xi_{N, l_1 i}^E \xi_{N, i l_2}^{E*} + \xi_{N, i l_1}^{E*} \xi_{N, l_2 i}^E,$$

$$\begin{aligned}
\eta_i^A &= \xi_{N,l_1 i}^E \xi_{N,i l_2}^{E*} - \xi_{N,i l_1}^{E*} \xi_{N,l_2 i}^E, \\
\eta_i^+ &= \xi_{N,i l_1}^{E*} \xi_{N,i l_2}^{E*} + \xi_{N,l_1 i}^E \xi_{N,l_2 i}^E, \\
\eta_i^- &= \xi_{N,i l_1}^{E*} \xi_{N,i l_2}^{E*} - \xi_{N,l_1 i}^E \xi_{N,l_2 i}^E.
\end{aligned} \tag{9}$$

The parameters  $c_V$  and  $c_A$  are  $c_A = -\frac{1}{4}$  and  $c_V = \frac{1}{4} - \sin^2 \theta_W$ . In eq. (9) the flavor changing couplings  $\xi_{N,j i}^E$  represent the effective interaction between the internal lepton  $i$ , ( $i = e, \mu, \tau$ ) and outgoing (incoming)  $j = l_1$  ( $j = l_2$ ) one. Here we take only the  $\tau$  lepton in the internal line and we neglect all the Yukawa couplings except  $\xi_{N,\tau\tau}^E$  and  $\xi_{N,\tau\mu}^E$  in the loop contributions (see Discussion section). The Yukawa couplings  $\xi_{N,j i}^E$  are complex in general, however, in the present work, we take them real.

## 5 Acknowledgement

This work has been supported by the Turkish Academy of Sciences in the framework of the Young Scientist Award Program. (EOI-TUBA-GEBIP/2001-1-8)

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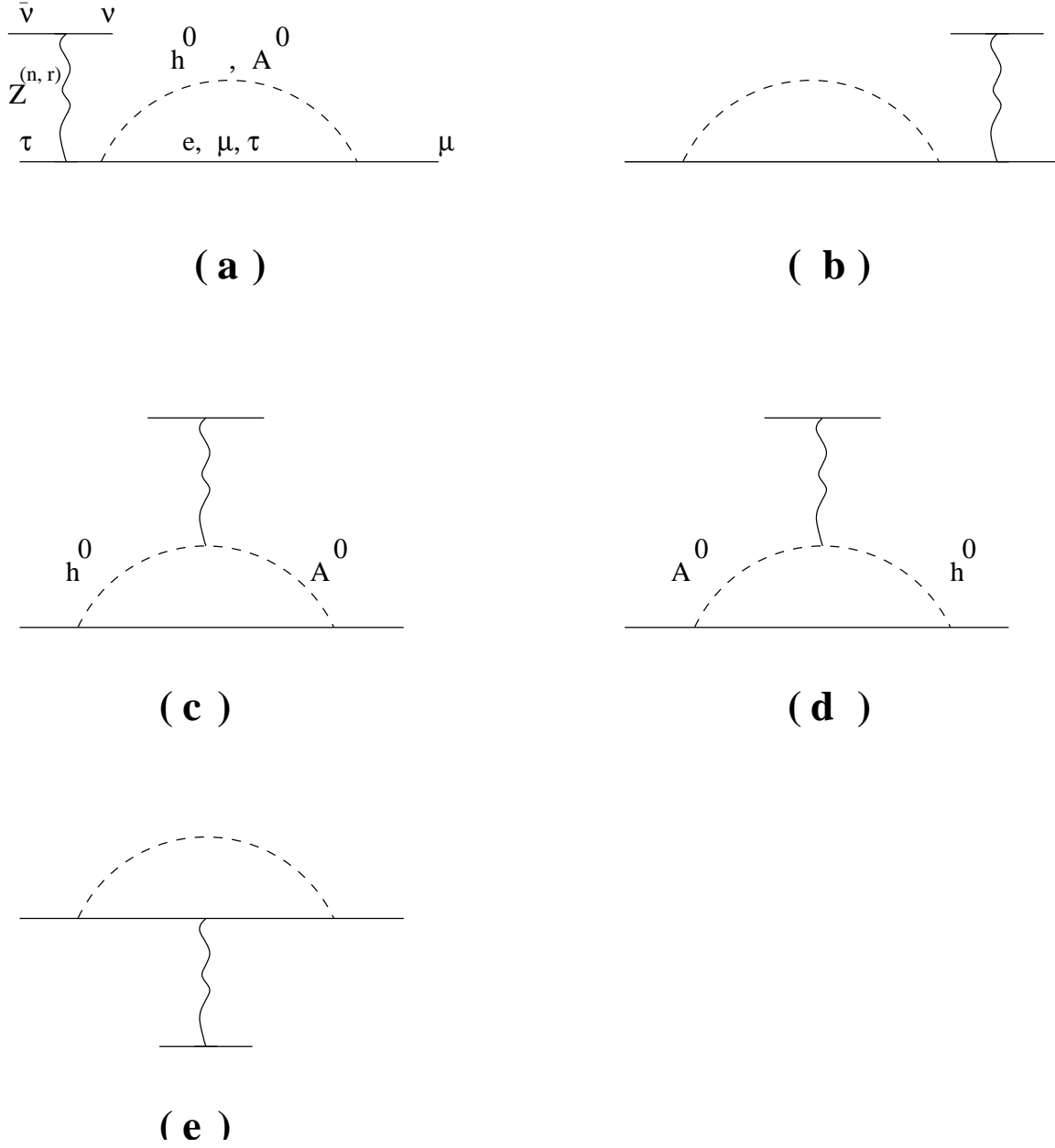


Figure 1: One loop diagrams contribute to  $\tau \rightarrow \mu \bar{\nu}_i \nu_i$ ,  $i = e, \mu, \tau$  decay due to the neutral Higgs bosons  $h_0$  and  $A_0$  in the model III version of 2HDM. Solid lines represent leptons and neutrinos, curly (dashed) lines represent the virtual  $Z$  boson and its KK modes in 6 dimensions ( $h_0$  and  $A_0$  fields).

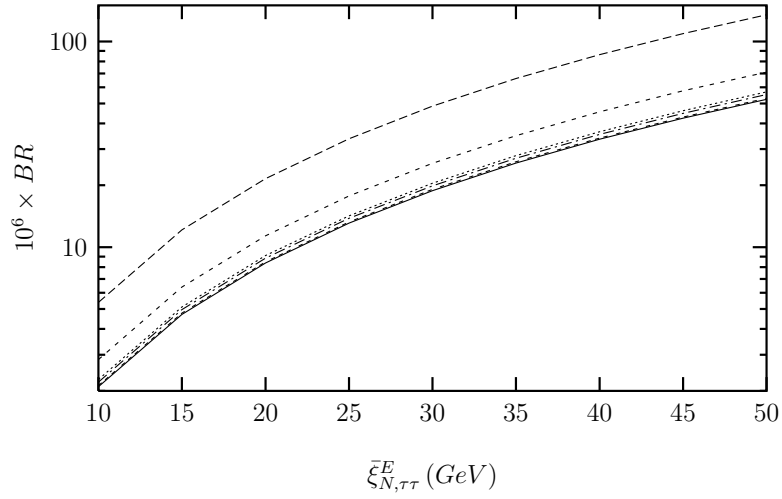


Figure 2:  $\bar{\xi}_{N,\tau\tau}^E$  dependence of the BR for  $\bar{\xi}_{N,\tau\mu}^E = 1 (GeV)$  and for different values of the compactification scale  $1/R$ , in the case of a single extra dimension. Here solid (dashed, small dashed, dotted, dash-dotted, three dashed-spaced) line represents the BR without extra dimensions (with extra dimensions for  $1/R = 200, 400, 800, 1000, 2000 (GeV)$ ).

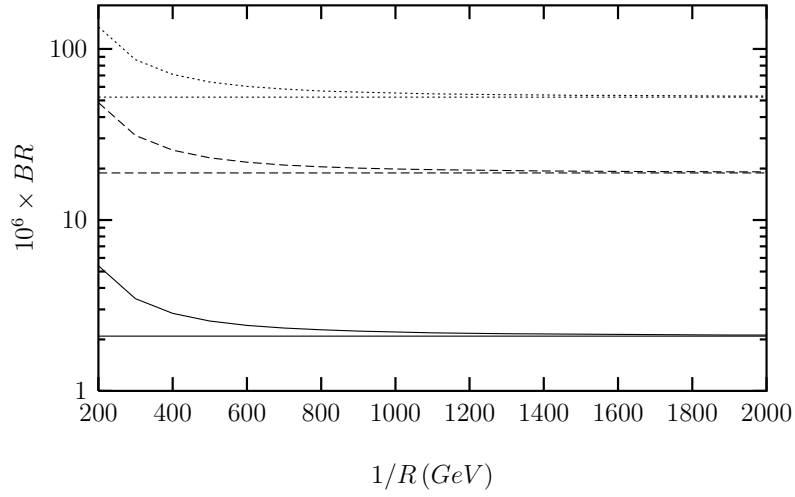


Figure 3: The compactification scale  $1/R$  dependence of the BR for  $\bar{\xi}_{N,\tau\mu}^E = 1 (GeV)$  and for different values of the coupling  $\bar{\xi}_{N,\tau\tau}^E$ , in the case of a single extra dimension. Here solid-dashed-dotted, straight lines (curved lines) represents the BR for  $\bar{\xi}_{N,\tau\tau}^E = 10 - 30 - 50 (GeV)$  without extra dimensions (with extra dimensions).

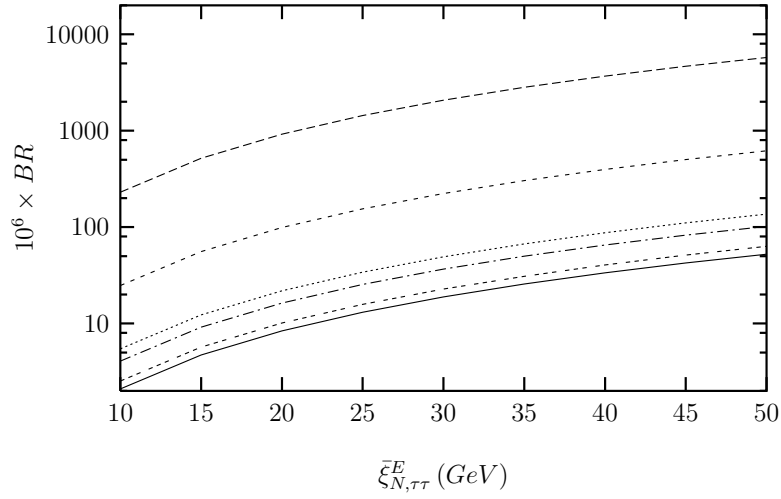


Figure 4:  $\bar{\xi}_{N,\tau\tau}^E$  dependence of the BR for  $\bar{\xi}_{N,\tau\mu}^E = 1 (GeV)$  and for different values of the compactification scale  $1/R$ , in the case of two extra dimensions. Here solid (dashed, small dashed, dotted, dash-dotted, three dashed-spaced) line represents the BR without extra dimensions (with extra dimensions for  $1/R = 200, 400, 800, 1000, 2000 GeV$ )



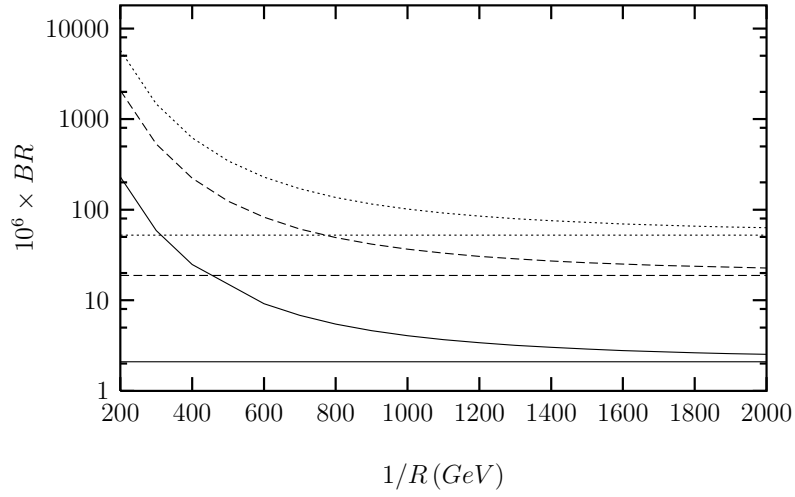


Figure 5: The compactification scale  $1/R$  dependence of the  $BR$  for  $\bar{\xi}_{N,\tau\mu}^E = 1 (GeV)$  and for different values of the coupling  $\bar{\xi}_{N,\tau\tau}^E$ , in the case of two extra dimensions. Here solid-dashed-dotted, straight lines (curved lines) represents the BR for  $\bar{\xi}_{N,\tau\tau}^E = 10 - 30 - 50 (GeV)$  without extra dimensions (with extra dimensions).