Lepton flavor violating Z boson decays induced by scalar unparticle

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Abstract

We predict the branching ratios of the lepton flavor violating Z boson decays $Z \to e^{\pm}\mu^{\pm}$, $Z \to e^{\pm}\tau^{\pm}$ and $Z \to \mu^{\pm}\tau^{\pm}$ in the case that the lepton flavor violation is carried by the scalar unparticle mediation. We observe that their BRs are strongly sensitive to the unparticle scaling dimension and the branching ratios can reach to the values of the order of 10^{-8}, for the heavy lepton flavor case, for the small values of the scaling dimension.

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Lepton flavor violating (LFV) interactions reached great interest since they are sensitive the physics beyond the standard model (SM) and the related experimental measurements are improved at present. $Z \to l_1 \, l_2$ decays are among the LFV interactions and the theoretical predictions of their branching ratios (BRs) in the framework of the SM are extremely small [1, 2, 3]:

$$\begin{aligned} BR(Z \to e^{\pm}\mu^{\pm}) \sim BR(Z \to e^{\pm}\tau^{\pm}) &\sim 10^{-54} \, , \\ BR(Z \to \mu^{\pm}\tau^{\pm}) &< 4 \times 10^{-60} \end{aligned} \tag{1}$$

in the case of non-zero lepton mixing mechanism [4]. These results are far from the experimental limits obtained at LEP1 [5]:

$$\begin{aligned} BR(Z \to e^{\pm}\mu^{\pm}) &< 1.7 \times 10^{-6} \;\; [6] \, , \\ BR(Z \to e^{\pm}\tau^{\pm}) &< 9.8 \times 10^{-6} \;\; [6, \; 7] \, , \\ BR(Z \to \mu^{\pm}\tau^{\pm}) &< 1.2 \times 10^{-5} \, , [6, \; 8] \end{aligned} \tag{2}$$

and from the improved ones at Giga-Z [9]:

$$\begin{aligned} BR(Z \to e^{\pm}\mu^{\pm}) &< 2 \times 10^{-9} \, , \\ BR(Z \to e^{\pm}\tau^{\pm}) &< f \times 6.5 \times 10^{-8} \, , \\ BR(Z \to \mu^{\pm}\tau^{\pm}) &< f \times 2.2 \times 10^{-8} \end{aligned} \tag{3}$$

with $f = 0.2 - 1.0$[1]. On the other hand the Giga-Z option of the Tesla project aims to increase the production of Z bosons at resonance [10]. These numerical values and the forthcoming projects stimulate one to make theoretical works on the LFV Z decays and to enhance their BRs by considering new scenarios beyond the SM. There are various works related to these decays in the literature [1]-[3], [5]-[9], [11]-[18], namely the extension of νSM with one (two) heavy ordinary Dirac (right-handed singlet Majorana) neutrino(s) [3], the Zee model [11], the two Higgs doublet model (2HDM) [12], the 2HDM with extra dimensions [13, 14, 15], the supersymmetric models [16, 17], top-color assisted technicolor model [18].

In the present work, we consider that the lepton flavor (LF) violation is carried by the scalar unparticle (U)-lepton-lepton vertex and unparticles appear in the internal line, in the

[1] Notice that these numbers are obtained for the decays $Z \to \bar{l}_1 l_2 + \bar{l}_2 l_1$ where

$$BR(Z \to l_1^{\pm} l_2^{\pm}) = \frac{\Gamma(Z \to \bar{l}_1 l_2 + \bar{l}_2 l_1)}{\Gamma_Z} .$$

loop. The unparticle idea is introduced by Georgi [19, 20] and its effect in the processes, which are induced at least in one loop level, is studied in various works [21]-[29]. The starting point of this idea is the interaction of the SM and the ultraviolet sector, having non-trivial infrared fixed point, at high energy level. The ultraviolet sector comes out as new degrees of freedom, called unparticles, being massless and having non integral scaling dimension d_u around, $\Lambda_U \sim 1\,TeV$. The effective lagrangian which drives the interactions of unparticles with the SM fields in the low energy level reads

$$\mathcal{L}_{eff} \sim \frac{\eta}{\Lambda_U^{d_u+d_{SM}-n}}\, O_{SM}\, O_U \,, \tag{4}$$

where O_U is the unparticle operator, the parameter η is related to the energy scale of ultraviolet sector, the low energy one and the matching coefficient [19, 20, 30] and n is the space-time dimension.

At this stage, we choose the appropriate operators in order to drive the LFV decays[2]. The effective interaction lagrangian responsible for the LFV decays in the low energy effective theory is

$$\mathcal{L}_1 = \frac{1}{\Lambda_U^{du-1}} \Big(\lambda_{ij}^S\, \bar{l}_i\, l_j + \lambda_{ij}^P\, \bar{l}_i\, i\gamma_5\, l_j \Big)\, O_U \,, \tag{5}$$

where l is the lepton field and λ_{ij}^S (λ_{ij}^P) is the scalar (pseudoscalar) coupling. On the other hand, there is a possibility that tree level $U - Z - Z$ interaction exists[3] and it has a contribution to the LFV Z decays (see Fig 1 (b) and (c)). The corresponding effective Lagrangian reads

$$\mathcal{L}_2 = \frac{\lambda_0}{\Lambda_U^{du}}\, F_{\mu\nu}\, F^{\mu\nu}\, O_U \,, \tag{6}$$

where $F_{\mu\nu}$ is the field tensor for the Z_μ field and λ_0 is the effective coupling constant.

The one loop level $Z \to l_1\, l_2$ decay (see Fig.1) is carried with the help of the scalar unparticle propagator, which is obtained by using the scale invariance [20, 32]:

$$\int d^4x\, e^{ipx} \; < 0|T\Big(O_U(x)\, O_U(0)\Big) 0 >= i\frac{A_{d_u}}{2\,\pi} \int_0^\infty ds\, \frac{s^{d_u-2}}{p^2 - s + i\epsilon} = i\, \frac{A_{d_u}}{2\, sin\,(d_u \pi)}\, (-p^2 - i\epsilon)^{d_u-2}, \tag{7}$$

with the factor A_{d_u}

$$A_{d_u} = \frac{16\,\pi^{5/2}}{(2\,\pi)^{2\,d_u}}\, \frac{\Gamma(d_u + \frac{1}{2})}{\Gamma(d_u - 1)\,\Gamma(2\,d_u)} \,. \tag{8}$$

[2]Notice that the operators with the lowest possible dimension are chosen since they have the most powerful effect in the low energy effective theory (see for example [31]).

[3]The vertex factor: $\frac{4\,i}{\Lambda_U^{d_u}}\, \lambda_0\, (k_{1\nu}\, k_{2\mu} - k_1.k_2\, g_{\mu\,\nu})$ where $k_{1(2)}$ is the four momentum of Z boson with polarization vector $\epsilon_{1\,\mu\,(2\,\nu)}$.

The function $\frac{1}{(-p^2-i\epsilon)^{2-d_u}}$ in eq. (7) becomes

$$\frac{1}{(-p^2-i\epsilon)^{2-d_u}} \rightarrow \frac{e^{-i\,d_u\,\pi}}{(p^2)^{2-d_u}}\,, \tag{9}$$

for $p^2>0$ and a non-trivial phase appears as a result of non-integral scaling dimension.

Now, we present the general effective vertex for the interaction of on-shell Z-boson with a fermionic current:

$$\Gamma_\mu=\gamma_\mu(f_V-f_A\,\gamma_5)+\frac{i}{m_W}\,(f_M+f_E\,\gamma_5)\,\sigma_{\mu\,\nu}\,q^\nu\,, \tag{10}$$

where q is the momentum transfer, $q^2=(p-p')^2$, f_V (f_A) is vector (axial-vector) coupling, f_M (f_E) magnetic (electric) transitions of unlike fermions. Here p ($-p'$) is the four momentum vector of lepton (anti-lepton). The form factors f_V, f_A, f_M and f_E in eq. (10) are obtained as

$$\begin{aligned}
f_V &= \int_0^1 dx\, f_{V\,self}+\int_0^1 dx\,\int_0^{1-x} dy\, f_{V\,vert}\,,\\
f_A &= \int_0^1 dx\, f_{A\,self}+\int_0^1 dx\,\int_0^{1-x} dy\, f_{A\,vert}\,,\\
f_M &= \int_0^1 dx\,\int_0^{1-x} dy\, f_{M\,vert}\,,\\
f_E &= \int_0^1 dx\,\int_0^{1-x} dy\, f_{E\,vert}\,.
\end{aligned} \tag{11}$$

Taking into account all the masses of internal leptons and external lepton (anti-lepton), the explicit expressions of $f_{V\,self}$, $f_{A\,self}$, $f_{V\,vert}$, $f_{A\,vert}$, $f_{M\,vert}$ and $f_{E\,vert}$ read as

$$\begin{aligned}
f_{V\,self} &= \frac{c_{self}\,(1-x)^{1-d_u}}{32\,s_W\,c_W\,\pi^2\left(m^2_{l_2^+}-m^2_{l_1^-}\right)(1-d_u)}\sum_{i=1}^{3}\Bigg\{(L^{d_u-1}_{self}-L'^{d_u-1}_{self})\Bigg(m_i\Big(\\
&\times(m_{l_1^-}\,c_2+m_{l_2^+}\,c_1)\,(\lambda^S_{il_1}+i\,\lambda^P_{il_1})\,(\lambda^P_{il_2}-i\,\lambda^S_{il_2})-(m_{l_1^-}\,c_1+m_{l_2^+}\,c_2)\,(\lambda^S_{il_1}-i\,\lambda^P_{il_1})\,(\lambda^P_{il_2}+i\,\lambda^S_{il_2})\Big)\\
&- m_{l_1^-}\,m_{l_2^+}\,(1-x)\Big(c_1\,(i\,\lambda^P_{il_1}-\lambda^S_{il_1})\,(\lambda^P_{il_2}-i\,\lambda^S_{il_2})+c_2\,(i\,\lambda^P_{il_1}+\lambda^S_{il_1})\,(\lambda^P_{il_2}+i\,\lambda^S_{il_2})\Big)\Bigg)\\
&+ (m^2_{l_2^+}\,L'^{d_u-1}_{self}-m^2_{l_1^-}\,L^{d_u-1}_{self})\,(1-x)\Big(c_1\,(i\,\lambda^P_{il_1}+\lambda^S_{il_1})\,(\lambda^P_{il_2}+i\,\lambda^S_{il_2})\\
&+ c_2\,(\lambda^P_{il_1}+i\,\lambda^S_{il_1})\,(\lambda^S_{il_2}+i\,\lambda^P_{il_2})\Big)\Bigg\}\,,
\end{aligned}$$

$$\begin{aligned}
f_{A\,self} &= \frac{c_{self}\,(1-x)^{1-d_u}}{32\,s_W\,c_W\,\pi^2\left(m_{l_2^+}-m_{l_1^-}\right)(1-d_u)}\sum_{i=1}^{3}\Bigg\{(L^{d_u-1}_{self}-L'^{d_u-1}_{self})\Bigg(m_i\Big(\\
&\times(m_{l_2^+}\,c_1-m_{l_1^-}\,c_2)\,(\lambda^S_{il_1}+i\,\lambda^P_{il_1})\,(\lambda^P_{il_2}-i\,\lambda^S_{il_2})-(m_{l_1^-}\,c_1-m_{l_2^+}\,c_2)(\lambda^S_{il_1}-i\,\lambda^P_{il_1})\,(\lambda^P_{il_2}+i\,\lambda^S_{il_2})\Big)\\
&- m_{l_1^-}\,m_{l_2^+}\,(1-x)\Big(c_1\,(\lambda^P_{il_1}+i\,\lambda^S_{il_1})\,(i\,\lambda^P_{il_2}+\lambda^S_{il_2})+c_2\,(\lambda^P_{il_1}-i\,\lambda^S_{il_1})\,(\lambda^S_{il_2}-i\,\lambda^P_{il_2})\Big)\Bigg)\\
&+ (m^2_{l_2^+}\,L'^{d_u-1}_{self}-m^2_{l_1^-}\,L^{d_u-1}_{self})\,(1-x)\Big(c_1\,(\lambda^S_{il_1}+i\,\lambda^P_{il_1})\,(\lambda^P_{il_2}+i\,\lambda^S_{il_2})\\
&+ c_2\,(\lambda^S_{il_1}-i\,\lambda^P_{il_1})\,(\lambda^P_{il_2}-i\,\lambda^S_{il_2})\Big)\Bigg\}\,,
\end{aligned}$$

$$
\begin{aligned}
f_{V\,vert} &= \frac{-c_{ver}\,(1-x-y)^{1-d_u}}{32\,\pi^2}\,\sum_{i=1}^{3}\frac{1}{L_{vert}^{2-d_u}}\Big\{m_i\,(1-x-y\,)\\
&\times\Big((m_{l_2^+}\,c_1+m_{l_1^-}\,c_2)\,(\lambda^S_{il_1}+i\,\lambda^P_{il_1})\,(\lambda^P_{il_2}-i\,\lambda^S_{il_2})-(m_{l_1^-}\,c_1+m_{l_2^+}\,c_2)\ (\lambda^S_{il_1}-i\,\lambda^P_{il_1})\,(\lambda^P_{il_2}+i\,\lambda^S_{il_2})\Big)\\
&- \ m_i^2\,\Big(c_1\,(\lambda^S_{il_1}+i\,\lambda^P_{il_1})\,(\lambda^P_{il_2}+i\,\lambda^S_{il_2})+\ c_2\,(i\,\lambda^P_{il_1}-\lambda^S_{il_1})\,(\lambda^P_{il_2}-i\,\lambda^S_{il_2})\Big)\\
&- \ \Big(c_1\,(i\,\lambda^P_{il_1}-\lambda^S_{il_1})\,(\lambda^P_{il_2}-i\,\lambda^S_{il_2})+c_2\,(i\,\lambda^P_{il_1}+\lambda^S_{il_1})\,(\lambda^P_{il_2}+i\,\lambda^S_{il_2})\Big)\\
&\times \ \Big(m_Z^2\,x\,y+m_{l_1^-}\,m_{l_2^+}\,(1-x-y)^2-\frac{L_{vert}}{1-d_u}\Big)\Big\}\\
&- \ \frac{\lambda_0\,m_Z^2}{16\,\pi^2}\,\sum_{i=1}^{3}\Big\{\frac{b_{ver}\,y^{1-d_u}}{L_{1\,vert}^{2-d_u}}\,\big\{(\lambda^P_{il_2}+i\,\lambda^S_{il_2})\,\Big(c_1\,m_i\,(1-x+y)+c_2\,\Big(m_{l_1^-}\,x\,(x+y-1)\\
&+ \ m_{l_2^+}\,y\,(1+x+y)\Big)\Big)-(\lambda^P_{il_2}-i\,\lambda^S_{il_2})\,\Big(c_2\,m_i\,(1-x+y)+c_1\,\Big(m_{l_1^-}\,x\,(x+y-1)\\
&+m_{l_2^+}\,y\,(1+x+y)\Big)\Big)\big\}-\frac{b'_{ver}\,x^{1-d_u}}{L_{2\,vert}^{2-d_u}}\,\big\{(\lambda^P_{il_1}-i\,\lambda^S_{il_1})\,\Big(c_1\,m_i\,(1+x-y)+c_2\,\Big(m_{l_2^+}\,y\,(x+y-1)\\
&+ \ m_{l_1^-}\,x\,(1+x+y)\Big)\Big)-(\lambda^P_{il_1}+i\,\lambda^S_{il_1})\,\Big(c_2\,m_i\,(1+x-y)+c_1\,\Big(m_{l_2^+}\,y\,(x+y-1)\\
&+ \ m_{l_1^-}\,x\,(1+x+y)\Big)\Big)\big\}\Big\}\,,
\end{aligned}
$$

$$
\begin{aligned}
f_{A\,vert} &= \frac{-c_{ver}\,(1-x-y)^{1-d_u}}{32\,\pi^2}\,\sum_{i=1}^{3}\frac{1}{L_{vert}^{2-d_u}}\Big\{m_i\,(1-x-y\,)\\
&\times\Big((m_{l_2^+}\,c_1-m_{l_1^-}\,c_2)\,(\lambda^S_{il_1}+i\,\lambda^P_{il_1})\,(\lambda^P_{il_2}-i\,\lambda^S_{il_2})-(m_{l_2^+}\,c_2-m_{l_1^-}\,c_1)\ (i\,\lambda^P_{il_1}-\lambda^S_{il_1})\,(\lambda^P_{il_2}+i\,\lambda^S_{il_2})\Big)\\
&+ \ m_i^2\,\Big(c_1\,(\lambda^P_{il_1}-i\,\lambda^S_{il_1})\,(\lambda^S_{il_2}-i\,\lambda^P_{il_2})+\ c_2\,(\lambda^P_{il_1}+i\,\lambda^S_{il_1})\,(\lambda^S_{il_2}+i\,\lambda^P_{il_2})\Big)+\Big(c_1\,(\lambda^P_{il_1}+i\,\lambda^S_{il_1})\\
&\times \ (\lambda^S_{il_2}+i\,\lambda^P_{il_2})+\ c_2\,(\lambda^P_{il_1}-i\,\lambda^S_{il_1})\,(\lambda^S_{il_2}-i\,\lambda^P_{il_2})\Big)\,\Big(m_Z^2\,x\,y-m_{l_1^-}\,m_{l_2^+}\,(1-x-y)^2-\frac{L_{vert}}{1-d_u}\Big)\Big\}\\
&+ \ \frac{\lambda_0\,m_Z^2}{16\,\pi^2}\,\sum_{i=1}^{3}\Big\{\frac{b_{ver}\,y^{1-d_u}}{L_{1\,vert}^{2-d_u}}\,\big\{(\lambda^P_{il_2}+i\,\lambda^S_{il_2})\,\Big(c_1\,m_i\,(x-y-1)+c_2\,\Big(m_{l_1^-}\,x\,(1-x-y)\\
&+ \ m_{l_2^+}\,y\,(1+x+y)\Big)\Big)+(\lambda^P_{il_2}-i\,\lambda^S_{il_2})\,\Big(c_2\,m_i\,(x-y-1)+c_1\,\Big(m_{l_1^-}\,x\,(1-x-y)\\
&+m_{l_2^+}\,y\,(1+x+y)\Big)\Big)\big\}+\frac{b'_{ver}\,x^{1-d_u}}{L_{2\,vert}^{2-d_u}}\,\big\{(\lambda^P_{il_1}-i\,\lambda^S_{il_1})\,\Big(c_1\,m_i\,(1+x-y)+c_2\,\Big(m_{l_2^+}\,y\,(x+y-1)\\
&- \ m_{l_1^-}\,x\,(1+x+y)\Big)\Big)+(\lambda^P_{il_1}+i\,\lambda^S_{il_1})\,\Big(c_2\,m_i\,(1+x-y)+c_1\,\Big(m_{l_2^+}\,y\,(x+y-1)\\
&- \ m_{l_1^-}\,y\,(1+x+y)\Big)\Big)\big\}\Big\}\,,
\end{aligned}
$$

$$
\begin{aligned}
f_{M\,vert} &= \frac{-i\,(1-x-y)^{1-d_u}}{32\,\pi^2} \sum_{i=1}^{3} \frac{c_{ver}\, m_Z\, c_W}{L_{vert}^{2-d_u}} \Big\{ m_i \Big((x+y)\,(\lambda^S_{il_1}\,\lambda^S_{il_2} - \lambda^P_{il_1}\,\lambda^P_{il_2})(c_1+c_2) \\
&- i\,(x-y)\,(\lambda^S_{il_1}\,\lambda^P_{il_2} + \lambda^P_{il_1}\,\lambda^S_{il_2})(c_2-c_1)\Big) + (1-x-y)\,(m_{l_1^-}\,x + m_{l_2^+}\,y) \\
&\times \Big(c_1\,(\lambda^P_{il_1} + i\,\lambda^S_{il_1})\,(\lambda^P_{il_2} - i\,\lambda^S_{il_2}) + c_2\,(\lambda^P_{il_1} - i\,\lambda^S_{il_1})\,(\lambda^P_{il_2} + i\,\lambda^S_{il_2})\Big)\Big\} \\
&- \frac{i\,\lambda_0}{16\,\pi^2} \sum_{i=1}^{3} \Big\{ \frac{b_{ver}\, m_Z\, c_W\, y^{1-d_u}}{L_{1\,vert}^{2-d_u}} \Big(\Big(c_1\,(\lambda^S_{il_2} + i\,\lambda^P_{il_2}) + c_2\,(\lambda^S_{il_2} - i\,\lambda^P_{il_2})\Big) \\
&\times \Big(2\,m_Z^2\,x\,y + (1-x-y)\,\Big(m^2_{l_1^-}\,x + m^2_{l_2^+}\,y - m_{l_1^-}\,m_{l_2^+}\,(x+y)\Big) - 2\,\frac{L_{1\,vert}}{1-d_u}\Big) \\
&- \Big(c_1\,(\lambda^S_{il_2} - i\,\lambda^P_{il_2}) + c_2\,(\lambda^S_{il_2} + i\,\lambda^P_{il_2})\Big)\,(1-x-y)\,m_i\,(m_{l_1^-} - m_{l_2^+})\Big) \\
&+ \frac{b'_{ver}\, m_Z\, c_W\, x^{1-d_u}}{L_{2\,vert}^{2-d_u}} \Big(\Big(c_1\,(\lambda^S_{il_1} - i\,\lambda^P_{il_1}) + c_2\,(\lambda^S_{il_1} + i\,\lambda^P_{il_1})\Big) \\
&\times \Big(2\,m_Z^2\,x\,y + (1-x-y)\,\Big(m^2_{l_1^-}\,x + m^2_{l_2^+}\,y - m_{l_1^-}\,m_{l_2^+}\,(x+y)\Big) - 2\,\frac{L_{2\,vert}}{1-d_u}\Big) \\
&+ \Big(c_1\,(\lambda^S_{il_1} + i\,\lambda^P_{il_1}) + c_2\,(\lambda^S_{il_1} - i\,\lambda^P_{il_1})\Big)\,(1-x-y)\,m_i\,(m_{l_1^-} - m_{l_2^+})\Big)\Big\}\,,
\end{aligned}
$$

$$
\begin{aligned}
f_{E\,vert} &= \frac{-i\,(1-x-y)^{1-d_u}}{32\,\pi^2} \sum_{i=1}^{3} \frac{c_{ver}\, m_Z\, c_W}{L_{vert}^{2-d_u}} \Big\{ m_i \Big(i\,(x+y)\,(\lambda^S_{il_1}\,\lambda^P_{il_2} + \lambda^P_{il_1}\,\lambda^S_{il_2})\,(c_1+c_2) \\
&+ (x-y)\,(\lambda^P_{il_1}\,\lambda^P_{il_2} - \lambda^S_{il_1}\,\lambda^S_{il_2})\,(c_2-c_1)\Big) + (1-x-y)\,(m_{l_1^-}\,x - m_{l_2^+}\,y) \\
&\times \Big(c_1\,(\lambda^P_{il_1} + i\,\lambda^S_{il_1})\,(\lambda^P_{il_2} - i\,\lambda^S_{il_2}) - c_2\,(\lambda^P_{il_1} - i\,\lambda^S_{il_1})\,(\lambda^P_{il_2} + i\,\lambda^S_{il_2})\Big)\Big\} \\
&- \frac{i\,\lambda_0}{16\,\pi^2} \sum_{i=1}^{3} \Big\{ \frac{b_{ver}\, m_Z\, c_W\, y^{1-d_u}}{L_{1\,vert}^{2-d_u}} \Big(\Big(c_1\,(\lambda^S_{il_2} + i\,\lambda^P_{il_2}) - c_2\,(\lambda^S_{il_2} - i\,\lambda^P_{il_2})\Big) \\
&\times \Big(2\,m_Z^2\,x\,y + (1-x-y)\,\Big(m^2_{l_1^-}\,x + m^2_{l_2^+}\,y + m_{l_1^-}\,m_{l_2^+}\,(x+y)\Big) - 2\,\frac{L_{1\,vert}}{1-d_u}\Big) \\
&+ \Big(c_1\,(\lambda^S_{il_2} - i\,\lambda^P_{il_2}) - c_2\,(\lambda^S_{il_2} + i\,\lambda^P_{il_2})\Big)\,(1-x-y)\,m_i\,(m_{l_1^-} + m_{l_2^+})\Big) \\
&- \frac{b'_{ver}\, m_Z\, c_W\, x^{1-d_u}}{L_{1\,vert}^{2-d_u}} \Big(\Big(c_1\,(\lambda^S_{il_1} - i\,\lambda^P_{il_1}) - c_2\,(\lambda^S_{il_1} + i\,\lambda^P_{il_1})\Big) \\
&\times \Big(2\,m_Z^2\,x\,y + (1-x-y)\,\Big(m_{l_1^-}\,x + m^2_{l_2^+}\,y + m_{l_1^-}\,m_{l_2^+}\,(x+y)\Big) - 2\,\frac{L_{2\,vert}}{1-d_u}\Big) \\
&+ \Big(c_1\,(\lambda^S_{il_1} + i\,\lambda^P_{il_1}) - c_2\,(\lambda^S_{il_1} - i\,\lambda^P_{il_1})\Big)\,(1-x-y)\,m_i\,(m_{l_1^-} + m_{l_2^+})\Big)\Big\}\,, \qquad (12)
\end{aligned}
$$

with

$$
L_{self} = x\,\Big(m^2_{l_1^-}\,(1-x) - m_i^2\Big)\,,
$$

$$
\begin{aligned}
L'_{self} &= x\left(m^2_{l_2^+}\,(1-x) - m_i^2\right), \\
L_{vert} &= (m^2_{l_1^-}\,x + m^2_{l_2^+}\,y)\,(1-x-y) - m_i^2\,(x+y) + m_Z^2\,x\,y\,, \\
L_{1\,vert} &= \left(m^2_{l_1^-}\,x + m^2_{l_2^+}\,y - m_i^2\right)(1-x-y) + m_Z^2\,x\,(y-1)\,, \\
L_{2\,vert} &= \left(m^2_{l_1^-}\,x + m^2_{l_2^+}\,y - m_i^2\right)(1-x-y) + m_Z^2\,y\,(x-1)\,,
\end{aligned} \tag{13}
$$

and

$$
\begin{aligned}
c_{self} &= -\frac{e\,A_{d_u}}{2\,sin\,(d_u\pi)\,\Lambda_u^{2\,(d_u-1)}}\,, \\
c_{ver} &= -\frac{e\,A_{d_u}}{2\,s_W\,c_W\;sin\,(d_u\pi)\,\Lambda_u^{2\,(d_u-1)}}\,, \\
b_{ver} &= -\frac{e\,A_{d_u}}{2\,s_W\,c_W\;sin\,(d_u\pi)\,\Lambda_u^{2\,d_u-1}}\,, \\
b'_{ver} &= -b_{ver}\,.
\end{aligned} \tag{14}
$$

In eq. (12), the flavor changing scalar and pseudoscalar couplings $\lambda^{S,P}_{il_{1(2)}}$ represent the effective interaction between the internal lepton i, $(i = e, \mu, \tau)$ and the outgoing l_1^- (l_2^+) lepton (anti lepton). Finally, the BR for $Z \to l_1^-\,l_2^+$ can be obtained by using the form factors f_V, f_A, f_M and f_E as

$$
BR(Z \to l_1^-\,l_2^+) = \frac{1}{48\,\pi}\,\frac{m_Z}{\Gamma_Z}\,\{|f_V|^2 + |f_A|^2 + \frac{1}{2\,cos^2\,\theta_W}(|f_M|^2 + |f_E|^2)\}\,, \tag{15}
$$

where Γ_Z is the total decay width of Z boson. Note that, in general, the production of sum of charged states is considered with the corresponding BR

$$
BR(Z \to l_1^{\pm}\,l_2^{\pm}) = \frac{\Gamma(Z \to (\bar{l}_1\,l_2 + \bar{l}_2\,l_1)}{\Gamma_Z}\,, \tag{16}
$$

and in our numerical analysis we use this branching ratio.

Discussion

In this section, we estimate the BRs of LFV Z boson decays by considering that the flavor violation is carried by the scalar unparticle mediation. These decays exist at least in one loop level and, in the present case, we assume that the possible sources of LF violation are the U-lepton-lepton couplings in the framework of the effective theory. On the other hand, we take the $U - Z - Z$ coupling non-zero and we study the sensitivity of the BRs to this coupling. The couplings considered and the scaling dimension of unparticle(s) are free parameters and they should be restricted by respecting the current experimental measurements and some theoretical considerations. For the scaling dimension d_u we choose the range[4] $1 < d_u < 2$. For the LF violating couplings we consider the following restrictions:

- The (off) diagonal couplings are flavor (blind and universal) aware and $\lambda_{\tau\tau} > \lambda_{\mu\mu} > \lambda_{ee}$ ($\lambda_{ij}, i \neq j$). We take the greatest numerical value of diagonal coupling of the order of one and the off diagonal one as $\lambda_{ij} = \kappa \lambda_{ee}$ with $\kappa < 1$. In our numerical calculations, we choose $\kappa = 0.5$.

- As a second possibility, we consider that the (off) diagonal couplings are flavor blind and universal and of the order of one. Similar to the previous case, we take the off diagonal ones as $\lambda_{ij} = \kappa \lambda_{ii}$ with $\kappa = 0.5$.

Furthermore, we choose the coupling λ_0 for the tree level $U - Z - Z$ interaction (see eq. (6)) in the range $0.1 - 1.0$ and we take the energy scale of the order of TeV. Notice that throughout our calculations we use the input values given in Table (1).

Parameter	Value
m_e	0.0005 (GeV)
m_μ	0.106 (GeV)
m_τ	1.780 (GeV)
Γ_Z^{Tot}	2.49 (GeV)
s_W^2	0.23

Table 1: The values of the input parameters used in the numerical calculations.

In Fig.2, we present the BR ($Z \to \mu^\pm e^\pm$) with respect to the scale parameter d_u, for the energy scale $\Lambda_u = 10\,TeV$, the couplings $\lambda_{ee} = 0.01$, $\lambda_{\mu\mu} = 0.1$, $\lambda_{\tau\tau} = 1$ and $\lambda_{ij} = 0.005$, $i \neq j$. Here the solid (dashed) line represents the BR for total contribution and $\lambda_0 = 0.1$ (the contribution due to the $U - Z - Z$ vertex and $\lambda_0 = 1$). The BR is strongly sensitive to the scale d_u and, reaches to the numerical values 10^{-10}, for $d_u < 1.1$. The contribution of $U - Z - Z$ vertex is almost two order smaller than the total one, even for $\lambda_0 = 1$. With the increasing values of the scaling dimension d_u, the BR sharply decreases and becomes negligible. Fig. 3 represents the BR ($Z \to \mu^\pm e^\pm$) with respect to the couplings λ, for $d_u = 1.2$. Here the solid (dashed-small dashed) line represents the BR with respect to λ, for $\lambda = \lambda_{ee} = \lambda_{\mu\mu} = \lambda_{\tau\tau}$, $\lambda_{ij} = 0.5\,\lambda$, $\lambda_0 = 0.1$ and $\Lambda_u = 10\,TeV$ (with respect to λ_0 for $\lambda_{ee} = 0.01$, $\lambda_{\mu\mu} = 0.1$, $\lambda_{\tau\tau} = 1$, $\lambda_{ij} = 0.005$, $\Lambda_u = 1.0\,TeV$- with respect to λ_0 for $\lambda_{ee} = 0.01$, $\lambda_{\mu\mu} = 0.1$, $\lambda_{\tau\tau} = 1$, $\lambda_{ij} = 0.005$,

[4]Here, $d_u > 1$ is due to the non-integrable singularities in the decay rate [20] and $d_u < 2$ is due to the convergence of the integrals [22].

$\Lambda_u = 10\,TeV$). In the case that the diagonal (off diagonal) couplings are flavor blind, the BR can reach to the values of the order of 10^{-7} for $\lambda = 1$. This can ensure a valuable information about the unparticle physics and the LFV couplings with more accurate measurements of the decays under consideration. Furthermore, this figure shows that the BR is not sensitive to the coupling λ_0 and it enhances almost one order in the range $0.1 \leq \lambda_0 \leq 1.0$, for the energy $\Lambda_u = 1.0\,TeV$.

Fig.4 devotes the BR ($Z \to \tau^{\pm}\,\mu^{\pm}$)[5] with respect to the scale parameter d_u, for the energy scale $\Lambda_u = 10\,TeV$, the couplings $\lambda_{ee} = 0.01$, $\lambda_{\mu\mu} = 0.1$, $\lambda_{\tau\tau} = 1$ and $\lambda_{ij} = 0.005$, $i \neq j$. Here the solid (dashed) line represents the BR for total contribution and $\lambda_0 = 0.1$ (the contribution due to the $U\!-\!Z\!-\!Z$ vertex and $\lambda_0 = 1$). The BR enhances up to the values of the order of 10^{-8}, for $d_u < 1.1$ and the increasing values of the scaling dimension d_u results in the considerable suppression in the BR. The contribution due to the $U - Z - Z$ vertex is more than two orders smaller than the total one for $\lambda_0 = 1$ and it shows that the effect of the $U - Z - Z$ vertex becomes weaker for heavy flavor outputs. In Fig.5, we present the BR ($Z \to \tau^{\pm}\,\mu^{\pm}$) with respect to the couplings λ, for $d_u = 1.2$. Here the solid (dashed-small dashed) line represents the BR with respect to λ, for $\lambda = \lambda_{ee} = \lambda_{\mu\mu} = \lambda_{\tau\tau}$, $\lambda_{ij} = 0.5\,\lambda$, $\lambda_0 = 0.1$ and $\Lambda_u = 10\,TeV$ (with respect to λ_0 for $\lambda_{ee} = 0.01$, $\lambda_{\mu\mu} = 0.1$, $\lambda_{\tau\tau} = 1$, $\lambda_{ij} = 0.005$, $\Lambda_u = 1.0\,TeV$- with respect to λ_0 for $\lambda_{ee} = 0.01$, $\lambda_{\mu\mu} = 0.1$, $\lambda_{\tau\tau} = 1$, $\lambda_{ij} = 0.005$, $\Lambda_u = 10\,TeV$). For the flavor blind diagonal (off diagonal) couplings, the BR can reach to the values of the order of 10^{-6} for $\lambda = 1$. On the other hand the BR is not sensitive to the coupling λ_0 and, for the energy $\Lambda_u = 1.0\,TeV$, its numerical value is almost one order greater compared to the one for $\Lambda_u = 10\,TeV$.

As a summary, the LFV Z boson decays are strongly sensitive to the unparticle scaling dimension d_u and, for its small values $d_u < 1.1$, there is a considerable enhancement in the BR. In the case that the diagonal (off diagonal) couplings are flavor blind and of the order of one, the BR can reach to the values of the order of 10^{-6} (10^{-7}) for the decay $Z \to \tau^{\pm}\,l^{\pm}$, $l = \mu$ or e ($Z \to \mu^{\pm}\,e^{\pm}$). With the forthcoming more accurate measurements of the decays under consideration it would be possible to test the possible signals coming from the new physics which drives the flavor violation, here is the unparticle physics.

[5]For the BR ($Z \to \tau^{\pm}\,e^{\pm}$) decay we get almost the same results and we do not present the corresponding figures.

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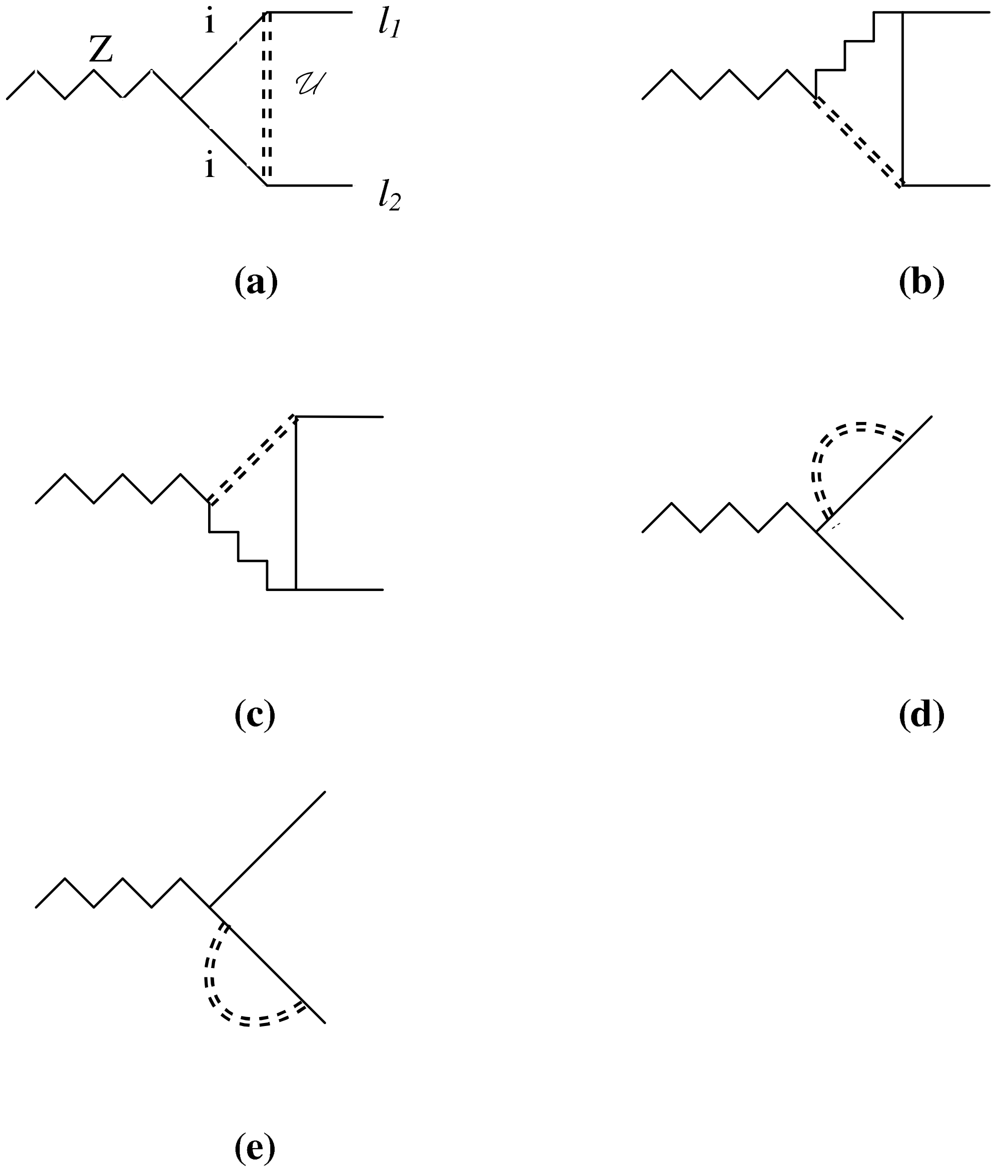

Figure 1: One loop diagrams contribute to $Z \to l_1^- \, l_2^+$ decay with scalar unparticle mediator. Solid line represents the lepton field: i represents the internal lepton, l_1^- (l_2^+) outgoing lepton (anti lepton), wavy line the Z boson field, double dashed line the unparticle field.

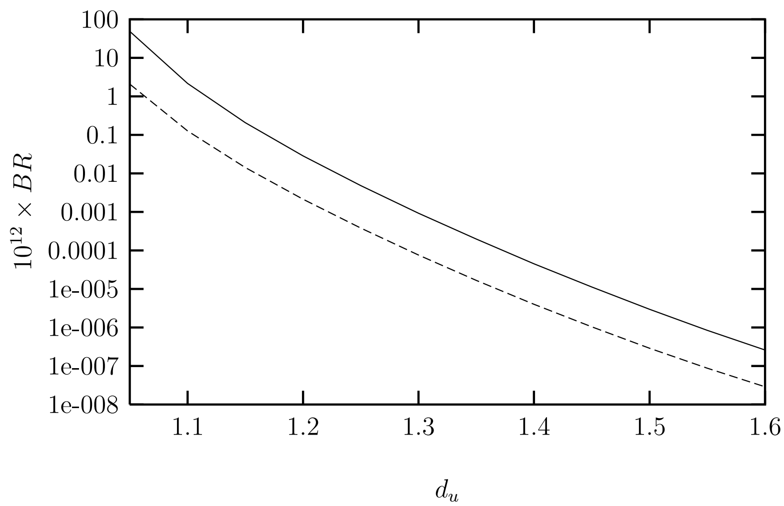

Figure 2: The scale parameter d_u dependence of the BR ($Z \to \mu^{\pm}\, e^{\pm}$) for $\Lambda_u = 10\, TeV$, the couplings $\lambda_{ee} = 0.01$, $\lambda_{\mu\mu} = 0.1$, $\lambda_{\tau\tau} = 1$ and $\lambda_{ij} = 0.005$, $i \neq j$. Here the solid (dashed) line represents the BR for total contribution and $\lambda_0 = 0.1$ (the contribution due to the $U - Z - Z$ vertex and $\lambda_0 = 1$).

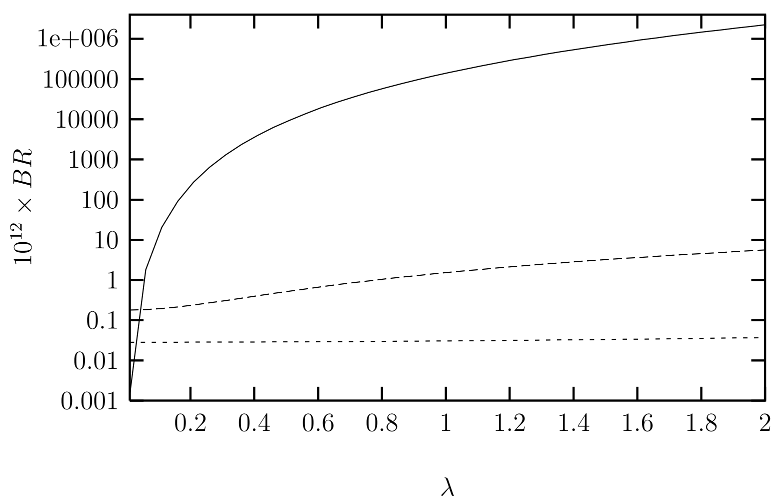

Figure 3: The BR ($Z \to \mu^{\pm} e^{\pm}$) with respect to the couplings λ, for $d_u = 1.2$. Here the solid (dashed-small dashed) line represents the BR with respect to λ, for $\lambda = \lambda_{ee} = \lambda_{\mu\mu} = \lambda_{\tau\tau}$, $\lambda_{ij} = 0.5\,\lambda$, $\lambda_0 = 0.1$ and $\Lambda_u = 10\,TeV$ (with respect to λ_0 for $\lambda_{ee} = 0.01$, $\lambda_{\mu\mu} = 0.1$, $\lambda_{\tau\tau} = 1$, $\lambda_{ij} = 0.005$, $\Lambda_u = 1.0\,TeV$- with respect to λ_0 for $\lambda_{ee} = 0.01$, $\lambda_{\mu\mu} = 0.1$, $\lambda_{\tau\tau} = 1$, $\lambda_{ij} = 0.005$, $\Lambda_u = 10\,TeV$).

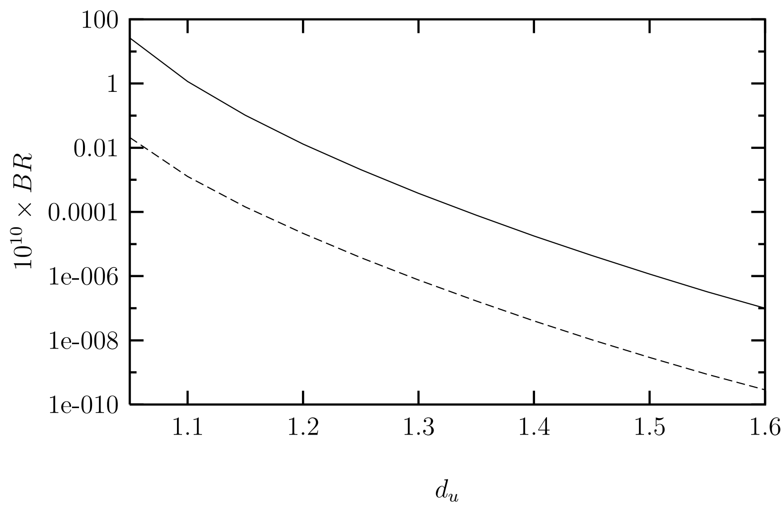

Figure 4: The scale parameter d_u dependence of the BR ($Z \to \tau^{\pm}\,\mu^{\pm}$) for $\Lambda_u = 10\,TeV$, the couplings $\lambda_{ee} = 0.01$, $\lambda_{\mu\mu} = 0.1$, $\lambda_{\tau\tau} = 1$ and $\lambda_{ij} = 0.005$, $i \neq j$. Here the solid (dashed) line represents the BR for total contribution and $\lambda_0 = 0.1$ (the contribution due to the $U - Z - Z$ vertex and $\lambda_0 = 1$).

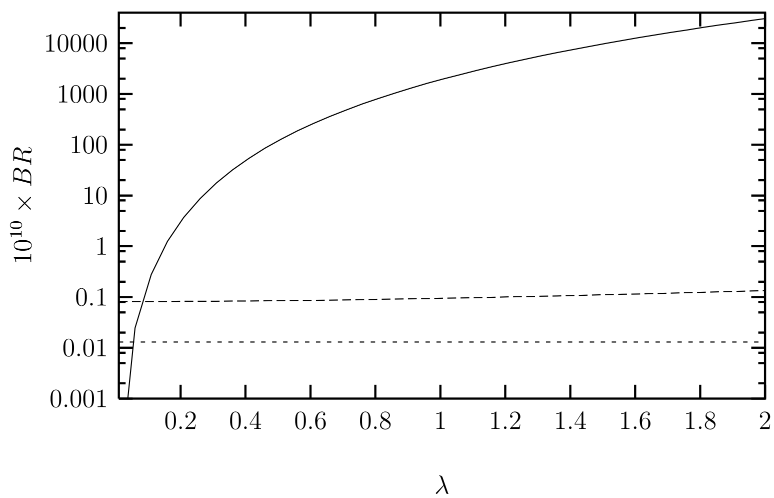

Figure 5: The BR ($Z \to \tau^{\pm}\,\mu^{\pm}$) with respect to the couplings λ, for $d_u = 1.2$. Here the solid (dashed-small dashed) line represents the BR with respect to λ for $\lambda = \lambda_{ee} = \lambda_{\mu\mu} = \lambda_{\tau\tau}$, $\lambda_{ij} = 0.5\,\lambda$, $\lambda_0 = 0.1$ and $\Lambda_u = 10\,TeV$ (with respect to λ_0 for $\lambda_{ee} = 0.01$, $\lambda_{\mu\mu} = 0.1$, $\lambda_{\tau\tau} = 1$, $\lambda_{ij} = 0.005$, $\Lambda_u = 1.0\,TeV$- with respect to λ_0 for $\lambda_{ee} = 0.01$, $\lambda_{\mu\mu} = 0.1$, $\lambda_{\tau\tau} = 1$, $\lambda_{ij} = 0.005$, $\Lambda_u = 10\,TeV$).