# The exclusive $\bar{B} \rightarrow \pi e^{+} e^{-}$and $\bar{B} \rightarrow \rho e^{+} e^{-}$decays in the two Higgs doublet model with flavor changing neutral currents 

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#### Abstract

We calculate the leading logarithmic QCD corrections to the matrix element of the decay $b \rightarrow d e^{+} e^{-}$in the two Higgs doublet model with tree level flavor changing currents (model III). We continue studying the differential branching ratio and the CP violating asymmetry for the exclusive decays $B \rightarrow \pi e^{+} e^{-}$and $B \rightarrow \rho e^{+} e^{-}$and analysing the dependencies of these quantities on the selected model III parameters, $\xi^{U, D}$, including the leading logarithmic QCD corrections. Further, we present the forward-backward asymmetry of dileptons for the decay $B \rightarrow \rho e^{+} e^{-}$and discuss the dependencies to the model III parameters. We observe that there is a possibility to enhance the branching ratios and suppress the CP violating effects for both decays in the framework of the model III. Therefore, the measurements of these quantities will be an efficient tool to search the new physics beyond the SM.


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## 1 Introduction

Rare B meson decays, induced by flavor changing neutral current (FCNC) $b \rightarrow s(d)$ transitions are one of the interesting research area to test the Standard model (SM) at loop level. They are informative in the determination of the fundamental parameters, such as Cabbibo-KobayashiMaskawa (CKM) matrix elements, leptonic decay constants, etc. and useful for establishing the physics beyond the SM, such as two Higgs Doublet model (2HDM), Minimal Supersymmetric extension of the SM (MSSM) [1], etc.

Since the SM predicts the large Branching ratio ( $B r$ ), which is measurable in the near future, the exclusive decays induced by $b \rightarrow s l^{+} l^{-}$process become attractive. Such transitions has been investigated extensively in the SM, 2HDM and MSSM, in the literature [2]- [15]. For these transitions, the matrix element contains a term includes the virtual effects of the top quark proportional to $V_{t b} V_{t s}^{*}$ and additional terms describing the $c \bar{c}$ and $u \bar{u}$ loops, proportional to $V_{c b} V_{c s}^{*}$ and $V_{u b} V_{u s}^{*}$ respectively. Using the unitarity of CKM matrix, i.e. $V_{i b} V_{i s}^{*}=0, i=u, c, t$, and neglecting the factor $V_{u b} V_{u s}^{*}$ compared to $V_{t b} V_{t s}^{*}$ and $V_{c b} V_{c s}^{*}$, it is easy to see that the matrix element involves only one independent CKM factor, $V_{t b} V_{t s}^{*}$. This causes that the CP violating effects are suppressed within the SM [16, 17]. However, for $b \rightarrow d l^{+} l^{-}$decay, all the CKM factors $V_{t b} V_{t d}^{*}, V_{c b} V_{c d}^{*}$ and $V_{u b} V_{u d}^{*}$ are at the same order and this leads to a considerable CP violating asymmetry between the channels induced by the inclusive $b \rightarrow d l^{+} l^{-}$and $\bar{b} \rightarrow \bar{d} l^{+} l^{-}$ decays. These effects have been studied in the literature for the inclusive $b \rightarrow d e^{+} e^{-}$decay, in the framework of the SM [18]. The difficulties of the experimental investigation of the inclusive decays stimulate the study of the exclusive decays. However, the theoretical analysis of the exclusive decays is complicated due to the hadronic form factors which can be calculated using non-perturbative methods. The dispersion formulation of the light-cone constituent quark model is one of the method which can be used to calculate the hadronic matrix elements. In the literature, the form factors for $b \rightarrow d e^{+} e^{-}$induced exclusive $B \rightarrow(\pi, \rho) e^{+} e^{-}$decays have been calculated in the framework of this method [19, 20]. The CP violation effects for these exclusive decays have been studied in the framework of the SM 21].

In this work, we present the leading logarithmic (LLog) QCD corrected effective Hamiltonian in the 2HDM with flavor changing neutral currents (model III) for the inclusive $b \rightarrow d e^{+} e^{-}$ decay and calculate the differential $B r$ of the exclusive $\bar{B} \rightarrow(\pi, \rho) e^{+} e^{-}$process. Further, we study the CP-violation asymmetry $\left(A_{C P}\right)$ and forward-backward asymmetry $\left(A_{F B}\right)$ of dileptons for the decay $\bar{B} \rightarrow \rho e^{+} e^{-}$.

The paper is organized as follows: In Section 2, we give the LLog QCD corrected Hamilto-
nian responsible for the inclusive $b \rightarrow d e^{+} e^{-}$decay and calculate the matrix element. In section 3, we present the $B r$ and $A_{C P}$ of the exclusive $\bar{B} \rightarrow \pi e^{+} e^{-}$decay and analyse the dependencies of the $B r$ and $A_{C P}$ on the couplings $\bar{\xi}_{b b}^{D}, \bar{\xi}_{t t}^{U}$. In section 4, we study the $B r, A_{C P}$ and $A_{F B}$ of the exclusive $\bar{B} \rightarrow \rho e^{+} e^{-}$decay. Section 5 is devoted to our conclusions. In Appendix, we summarize the essential points of the model III and give the explicit forms of some functions we use in our calculations.

## 2 Leading logarithmic improved short-distance contributions in the model III for the decay $b \rightarrow d e^{+} e^{-}$with additional long-distance effects

In this section, we present the LLog QCD corrections to the inclusive $b \rightarrow d e^{+} e^{-}$decay amplitude in the 2HDM with tree level neutral currents (model III). The LLog QCD corrections to the $b \rightarrow d e^{+} e^{-}$decay amplitude can be calculated using the effective theory. In this method, the heavy degrees of freedom, $t$ quark, $W^{ \pm}, H^{ \pm}, H_{1}$, and $H_{2}$ bosons, in the present case, are integrated out. Here $H^{ \pm}$denote charged, $H_{1}$ and $H_{2}$ denote neutral Higgs bosons. The procedure is to match the full theory with the effective low energy theory at the high scale $\mu=m_{W}$ and evaluate the Wilson coefficients from $m_{W}$ down to the lower scale $\mu \sim O\left(m_{b}\right)$. In our calculations we choose the higher scale as $\mu=m_{W}$ since the current theoretical restrictions [22, 23] show that the charged Higgs mass is enough heavy to neglect the running from $m_{H^{ \pm}}$ to $m_{W}$.

The effective Hamiltonian relevant for the decay $b \rightarrow d e^{+} e^{-}$in the model III is

$$
\begin{align*}
\mathcal{H}_{e f f}=-4 \frac{G_{F}}{\sqrt{2}} V_{t b} V_{t d}^{*} & \left\{\sum_{i=1, \ldots, 12}\left(C_{i}(\mu) O_{i}(\mu)+C_{i}^{\prime}(\mu) O_{i}^{\prime}(\mu)\right)\right. \\
& +\lambda_{u} \sum_{i=1,2,11,12}\left(C_{i}(\mu)\left(O_{i}(\mu)-O_{i}^{\prime u}(\mu)\right)+C_{i}^{\prime}(\mu)\left(O_{i}^{\prime}(\mu)-O_{i}^{\prime u}(\mu)\right)\right\} \tag{1}
\end{align*}
$$

where $O_{i}^{\left({ }^{\prime}\right)}, O_{i}^{u\left({ }^{\prime}\right)}$, are the operators given in eqs. (2), (3) and $C_{i}^{\left({ }^{\prime}\right)}$ are the Wilson coefficients renormalized at the scale $\mu$. Here the unitarity of the Cobayashi-Maskawa matrix (CKM) is used, i.e. $V_{t b} V_{t d}^{*}+V_{u b} V_{u d}^{*}=-V_{c b} V_{c d}^{*}$ and the parameter $\lambda_{u}$ is defined as:

$$
\lambda_{u}=\frac{V_{u b} V_{u d}^{*}}{V_{t b} V_{t d}^{*}}
$$

Using Wolfenstein parametrization [24, $\lambda_{u}$ can be written as

$$
\lambda_{u}=\frac{\rho(1-\rho)-\eta^{2}}{(1-\rho)^{2}+\eta^{2}}-i \frac{\eta}{(1-\rho)^{2}+\eta^{2}}+O\left(\lambda^{2}\right)
$$

where $\rho, \eta$ and $\lambda \sim 0.221$ are Wolfenstein parameters. The parameter $\eta$ (and therefore $\lambda_{u}$ ) is the reason for the $C P$ violation in the SM .

The operator basis is similar to the one used for model III (25) and references therein), which is obtained by replacing $s$-quark by $d$-quark and adding new operators, i.e. $O_{i}^{u(\prime)}, i=1,2,11,12$ :

$$
\begin{align*}
O_{1} & =\left(\bar{d}_{L \alpha} \gamma_{\mu} c_{L \beta}\right)\left(\bar{c}_{L \beta} \gamma^{\mu} b_{L \alpha}\right) \\
O_{2} & =\left(\bar{d}_{L \alpha} \gamma_{\mu} c_{L \alpha}\right)\left(\bar{c}_{L \beta} \gamma^{\mu} b_{L \beta}\right) \\
O_{1}^{u} & =\left(\bar{d}_{L \alpha} \gamma_{\mu} u_{L \beta}\right)\left(\bar{u}_{L \beta} \gamma^{\mu} b_{L \alpha}\right), \\
O_{2}^{u} & =\left(\bar{d}_{L \alpha} \gamma_{\mu} u_{L \alpha}\right)\left(\bar{u}_{L \beta} \gamma^{\mu} b_{L \beta}\right), \\
O_{3} & =\left(\bar{d}_{L \alpha} \gamma_{\mu} b_{L \alpha}\right) \sum_{q=u, d, s, c, b}\left(\bar{q}_{L \beta} \gamma^{\mu} q_{L \beta}\right), \\
O_{4} & =\left(\bar{d}_{L \alpha} \gamma_{\mu} b_{L \beta}\right) \sum_{q=u, d, s, c, b}\left(\bar{q}_{L \beta} \gamma^{\mu} q_{L \alpha}\right), \\
O_{5} & =\left(\bar{d}_{L \alpha} \gamma_{\mu} b_{L \alpha}\right) \sum_{q=u, d, s, c, b}\left(\bar{q}_{R \beta} \gamma^{\mu} q_{R \beta}\right), \\
O_{6} & =\left(\bar{d}_{L \alpha} \gamma_{\mu} b_{L \beta}\right) \sum_{q=u, d, s, c, b}\left(\bar{q}_{R \beta} \gamma^{\mu} q_{R \alpha}\right), \\
O_{7} & =\frac{e}{16 \pi^{2}} \bar{d}_{\alpha} \sigma_{\mu \nu}\left(m_{b} R+m_{d} L\right) b_{\alpha} \mathcal{F}^{\mu \nu}, \\
O_{8} & =\frac{g}{16 \pi^{2}} \bar{d}_{\alpha} T_{\alpha \beta}^{a} \sigma_{\mu \nu}\left(m_{b} R+m_{d} L\right) b_{\beta} \mathcal{G}^{a \mu \nu} \\
O_{9} & =\frac{e}{16 \pi^{2}}\left(\bar{d}_{L \alpha} \gamma_{\mu} b_{L \alpha}\right)\left(\bar{l} \gamma_{\mu} l\right), \\
O_{10} & =\frac{e}{16 \pi^{2}}\left(\bar{d}_{L \alpha} \gamma_{\mu} b_{L \alpha}\right)\left(\bar{l} \gamma_{\mu} \gamma_{5} l\right), \\
O_{11} & =\left(\bar{d}_{L \alpha} \gamma_{\mu} c_{L \beta}\right)\left(\bar{c}_{R \beta} \gamma^{\mu} b_{R \alpha}\right), \\
O_{12} & =\left(\bar{d}_{L \alpha} \gamma_{\mu} c_{L \alpha}\right)\left(\bar{c}_{R \beta} \gamma^{\mu} b_{R \beta}\right), \\
O_{11}^{u} & =\left(\bar{d}_{L \alpha} \gamma_{\mu} u_{L \beta}\right)\left(\bar{u}_{R \beta} \gamma^{\mu} b_{R \alpha}\right), \\
O_{12}^{u} & =\left(\bar{d}_{L \alpha} \gamma_{\mu} u_{L \alpha}\right)\left(\bar{u}_{R \beta} \gamma^{\mu} b_{R \beta}\right), \tag{2}
\end{align*}
$$

and the second operator set which are flipped chirality partners of the first:

$$
\begin{aligned}
O_{1}^{\prime} & =\left(\bar{d}_{R \alpha} \gamma_{\mu} c_{R \beta}\right)\left(\bar{c}_{R \beta} \gamma^{\mu} b_{R \alpha}\right), \\
O_{2}^{\prime} & =\left(\bar{d}_{R \alpha} \gamma_{\mu} c_{R \alpha}\right)\left(\bar{c}_{R \beta} \gamma^{\mu} b_{R \beta}\right), \\
O_{1}^{\prime u} & =\left(\bar{d}_{R \alpha} \gamma_{\mu} u_{R \beta}\right)\left(\bar{u}_{R \beta} \gamma^{\mu} b_{R \alpha}\right), \\
O_{2}^{\prime u} & =\left(\bar{d}_{R \alpha} \gamma_{\mu} u_{R \alpha}\right)\left(\bar{u}_{R \beta} \gamma^{\mu} b_{R \beta}\right), \\
O_{3}^{\prime} & =\left(\bar{d}_{R \alpha} \gamma_{\mu} b_{R \alpha}\right) \sum_{q=u, d, s, c, b}\left(\bar{q}_{R \beta} \gamma^{\mu} q_{R \beta}\right), \\
O_{4}^{\prime} & =\left(\bar{d}_{R \alpha} \gamma_{\mu} b_{R \beta}\right) \sum_{q=u, d, s, c, b}\left(\bar{q}_{R \beta} \gamma^{\mu} q_{R \alpha}\right),
\end{aligned}
$$

$$
\begin{align*}
O_{5}^{\prime} & =\left(\bar{d}_{R \alpha} \gamma_{\mu} b_{R \alpha}\right) \sum_{q=u, d, s, c, b}\left(\bar{q}_{L \beta} \gamma^{\mu} q_{L \beta}\right) \\
O_{6}^{\prime} & =\left(\bar{d}_{R \alpha} \gamma_{\mu} b_{R \beta}\right) \sum_{q=u, d, s, c, b}\left(\bar{q}_{L \beta} \gamma^{\mu} q_{L \alpha}\right), \\
O_{7}^{\prime} & =\frac{e}{16 \pi^{2}} \bar{d}_{\alpha} \sigma_{\mu \nu}\left(m_{b} L+m_{d} R\right) b_{\alpha} \mathcal{F}^{\mu \nu}, \\
O_{8}^{\prime} & =\frac{g}{16 \pi^{2}} \bar{d}_{\alpha} T_{\alpha \beta}^{a} \sigma_{\mu \nu}\left(m_{b} L+m_{d} R\right) b_{\beta} \mathcal{G}^{a \mu \nu}, \\
O_{9}^{\prime} & =\frac{e}{16 \pi^{2}}\left(\bar{d}_{R \alpha} \gamma_{\mu} b_{R \alpha}\right)\left(\bar{l} \gamma_{\mu} l\right), \\
O_{10}^{\prime} & =\frac{e}{16 \pi^{2}}\left(\bar{d}_{R \alpha} \gamma_{\mu} b_{R \alpha}\right)\left(\bar{l} \gamma_{\mu} \gamma_{5} l\right), \\
O_{11}^{\prime} & =\left(\bar{d}_{R \alpha} \gamma_{\mu} c_{R \beta}\right)\left(\bar{c}_{L \beta} \gamma^{\mu} b_{L \alpha}\right), \\
O_{12}^{\prime} & =\left(\bar{d}_{R \alpha} \gamma_{\mu} c_{R \alpha}\right)\left(\bar{c}_{L \beta} \gamma^{\mu} b_{L \beta}\right), \\
O_{11}^{\prime u} & =\left(\bar{d}_{R \alpha} \gamma_{\mu} u_{R \beta}\right)\left(\bar{u}_{L \beta} \gamma^{\mu} b_{L \alpha}\right), \\
O_{12}^{\prime \prime} & =\left(\bar{d}_{R \alpha} \gamma_{\mu} u_{R \alpha}\right)\left(\bar{u}_{L \beta} \gamma^{\mu} b_{L \beta}\right), \tag{3}
\end{align*}
$$

where $\alpha$ and $\beta$ are $S U(3)$ colour indices and $\mathcal{F}^{\mu \nu}$ and $\mathcal{G}^{\mu \nu}$ are the field strength tensors of the electromagnetic and strong interactions, respectively.

The initial values for the first set of operators (eq.(22)) [5, (25] are

$$
\begin{aligned}
C_{1,3, \ldots 6,11,12}^{S M}\left(m_{W}\right) & =0, \\
C_{2}^{S M}\left(m_{W}\right) & =1, \\
C_{7}^{S M}\left(m_{W}\right) & =\frac{3 x^{3}-2 x^{2}}{4(x-1)^{4}} \ln x+\frac{-8 x^{3}-5 x^{2}+7 x}{24(x-1)^{3}}, \\
C_{8}^{S M}\left(m_{W}\right) & =-\frac{3 x^{2}}{4(x-1)^{4}} \ln x+\frac{-x^{3}+5 x^{2}+2 x}{8(x-1)^{3}}, \\
C_{9}^{S M}\left(m_{W}\right) & =-\frac{1}{\sin ^{2} \theta_{W}} B(x)+\frac{1-4 \sin ^{2} \theta_{W}}{\sin ^{2} \theta_{W}} C(x)-D(x)+\frac{4}{9}, \\
C_{10}^{S M}\left(m_{W}\right) & =\frac{1}{\sin ^{2} \theta_{W}}(B(x)-C(x)), \\
C_{1, \ldots 6,11,12}^{H}\left(m_{W}\right) & =0, \\
C_{7}^{H}\left(m_{W}\right) & =\frac{1}{m_{t}^{2}}\left(\bar{\xi}_{N, t t}^{U}+\bar{\xi}_{N, t c}^{U} \frac{V_{c d}^{*}}{V_{t d}^{*}}\right)\left(\bar{\xi}_{N, t t}^{U}+\bar{\xi}_{N, t c}^{U} \frac{V_{c b}}{V_{t b}}\right) F_{1}(y), \\
& +\frac{1}{m_{t} m_{b}}\left(\bar{\xi}_{N, t t}^{U}+\bar{\xi}_{N, t c}^{U} \frac{V_{c d}^{*}}{V_{t d}^{*}}\right)\left(\bar{\xi}_{N, b b}^{D}+\bar{\xi}_{N, s b}^{D} \frac{V_{t s}}{V_{t b}}\right) F_{2}(y), \\
C_{8}^{H}\left(m_{W}\right) & =\frac{1}{m_{t}^{2}}\left(\bar{\xi}_{N, t t}^{U}+\bar{\xi}_{N, t c}^{U} \frac{V_{c d}^{*}}{V_{t d}^{*}}\right)\left(\bar{\xi}_{N, t t}^{U}+\bar{\xi}_{N, t c}^{U} \frac{V_{c b}}{V_{t b}}\right) G_{1}(y), \\
& +\frac{1}{m_{t} m_{b}}\left(\bar{\xi}_{N, t t}^{U}+\bar{\xi}_{N, t c}^{U} \frac{V_{c d}^{*}}{V_{t d}^{*}}\right)\left(\bar{\xi}_{N, b b}^{D}+\bar{\xi}_{N, s b}^{U} \frac{V_{t s}}{V_{t b}}\right) G_{2}(y), \\
C_{9}^{H}\left(m_{W}\right) & =\frac{1}{m_{t}^{2}}\left(\bar{\xi}_{N, t t}^{U}+\bar{\xi}_{N, t c}^{U} \frac{V_{c d}^{*}}{V_{t d}^{*}}\right)\left(\bar{\xi}_{N, t t}^{U}+\bar{\xi}_{N, t c}^{U} \frac{V_{c b}}{V_{t b}}\right) H_{1}(y),
\end{aligned}
$$

$$
\begin{equation*}
C_{10}^{H}\left(m_{W}\right)=\frac{1}{m_{t}^{2}}\left(\bar{\xi}_{N, t t}^{U}+\bar{\xi}_{N, t c}^{U} \frac{V_{c d}^{*}}{V_{t d}^{*}}\right)\left(\bar{\xi}_{N, t t}^{U}+\bar{\xi}_{N, t c}^{U} \frac{V_{c b}}{V_{t b}}\right) L_{1}(y), \tag{4}
\end{equation*}
$$

and for the second set of operators (eq. (3) ),

$$
\begin{align*}
C_{1, \ldots, 12}^{\prime S M}\left(m_{W}\right) & =0, \\
C_{1, \ldots, 6,11,12}^{\prime H}\left(m_{W}\right) & =0, \\
C_{7}^{\prime H}\left(m_{W}\right) & =\frac{1}{m_{t}^{2}}\left(\bar{\xi}_{N, b d}^{D} \frac{V_{t b}^{*}}{V_{t d}^{*}}+\bar{\xi}_{N, s d}^{D}\right)\left(\bar{\xi}_{N, b b}^{D}+\bar{\xi}_{N, s b}^{D} \frac{V_{t s}}{V_{t b}}\right) F_{1}(y), \\
& +\frac{1}{m_{t} m_{b}}\left(\bar{\xi}_{N, b d}^{D} \frac{V_{t b}^{*}}{V_{t d}^{*}}+\bar{\xi}_{N, s d}^{D} \frac{V_{t s}^{*}}{V_{t d}^{*}}\right)\left(\bar{\xi}_{N, t t}^{U}+\bar{\xi}_{N, t c}^{U} \frac{V_{c b}}{V_{t b}}\right) F_{2}(y), \\
C_{8}^{\prime H}\left(m_{W}\right) & =\frac{1}{m_{t}^{2}}\left(\bar{\xi}_{N, b d}^{D} \frac{V_{t b}^{*}}{V_{t d}^{*}}+\bar{\xi}_{N, s d}^{D}\right)\left(\bar{\xi}_{N, b b}^{D}+\bar{\xi}_{N, s b}^{D} \frac{V_{t s}}{V_{t b}}\right) G_{1}(y), \\
& +\frac{1}{m_{t} m_{b}}\left(\bar{\xi}_{N, b d}^{D} \frac{V_{t b}^{*}}{V_{t d}^{*}}+\bar{\xi}_{N, s d}^{D} \frac{V_{t s}^{*}}{V_{t d}^{*}}\right)\left(\bar{\xi}_{N, t t}^{U}+\bar{\xi}_{N, t c}^{U} \frac{V_{c b}}{V_{t b}}\right) G_{2}(y), \\
C_{9}^{\prime H}\left(m_{W}\right) & =\frac{1}{m_{t}^{2}}\left(\bar{\xi}_{N, b d}^{D} \frac{V_{t b}^{*}}{V_{t d}^{*}}+\bar{\xi}_{N, s d}^{D}\right)\left(\bar{\xi}_{N, b b}^{D}+\bar{\xi}_{N, s b}^{D} \frac{V_{t s}}{V_{t b}}\right) H_{1}(y), \\
C_{10}^{\prime H}\left(m_{W}\right) & =\frac{1}{m_{t}^{2}}\left(\bar{\xi}_{N, b d}^{D} \frac{V_{t b}^{*}}{V_{t d}^{*}}+\bar{\xi}_{N, s d}^{D}\right)\left(\bar{\xi}_{N, b b}^{D}+\bar{\xi}_{N, s b}^{D} \frac{V_{t s}}{V_{t b}}\right) L_{1}(y), \tag{5}
\end{align*}
$$

where $x=m_{t}^{2} / m_{W}^{2}$ and $y=m_{t}^{2} / m_{H^{ \pm}}^{2}$. In eqs. (4) and (5) we used the redefinition

$$
\begin{equation*}
\xi^{U, D}=\sqrt{\frac{4 G_{F}}{\sqrt{2}}} \bar{\xi}^{U, D} . \tag{6}
\end{equation*}
$$

Here the Wilson coefficients $C_{i}^{S M}\left(m_{W}\right)$ and $C_{i}^{H}\left(m_{W}\right)$ denote the SM and the additional charged Higgs contributions respectively. The functions $B(x), C(x), D(x), F_{1(2)}(y), G_{1(2)}(y), H_{1}(y)$ and $L_{1}(y)$ are given in appendix B. Note that in the calculations we neglect the contributions due to the neutral Higgs bosons since their interactions include negligible Yukawa couplings (see [26] for details).

Finally, the initial values of the Wilson coefficients in the model III (eqs. (4) and (5)) are

$$
\begin{aligned}
C_{1,3, \ldots 6,11,12}^{2 H D M}\left(m_{W}\right) & =0 \\
C_{2}^{2 H D M}\left(m_{W}\right) & =1 \\
C_{7}^{2 H D M}\left(m_{W}\right) & =C_{7}^{S M}\left(m_{W}\right)+C_{7}^{H}\left(m_{W}\right) \\
C_{8}^{2 H D M}\left(m_{W}\right) & =C_{8}^{S M}\left(m_{W}\right)+C_{8}^{H}\left(m_{W}\right), \\
C_{9}^{2 H D M}\left(m_{W}\right) & =C_{9}^{S M}\left(m_{W}\right)+C_{9}^{H}\left(m_{W}\right), \\
C_{10}^{2 H D M}\left(m_{W}\right) & =C_{10}^{S M}\left(m_{W}\right)+C_{10}^{H}\left(m_{W}\right), \\
C_{1,2,3, \ldots, 6,11,12}^{2 H D M}\left(m_{W}\right) & =0,
\end{aligned}
$$

$$
\begin{align*}
& C_{7}^{\prime 2 H D M}\left(m_{W}\right)=C_{7}^{\prime S M}\left(m_{W}\right)+C_{7}^{\prime H}\left(m_{W}\right) \\
& C_{8}^{2 H D M}\left(m_{W}\right)=C_{8}^{\prime S M}\left(m_{W}\right)+C_{8}^{\prime H}\left(m_{W}\right), \\
& C_{9}^{\prime 2 H D M}\left(m_{W}\right)=C_{9}^{\prime S M}\left(m_{W}\right)+C_{9}^{\prime H}\left(m_{W}\right), \\
& C_{10}^{\prime 2 H D M}\left(m_{W}\right)=C_{10}^{\prime S M}\left(m_{W}\right)+C_{10}^{\prime H}\left(m_{W}\right) . \tag{7}
\end{align*}
$$

These initial values help us calculate the coefficients $C_{i}^{2 H D M}$ and $C_{i}^{\prime 2 H D M}$ at any lower scale as in the SM ([27] references therein). The $\mu$ scale dependence of the coefficients in the LLog approximation can be found in the literature [13, 28, 29, 30]. The operators $O_{5}, O_{6}, O_{11}, O_{11}^{u}$, $O_{12}$ and $O_{12}^{u}\left(O_{5}^{\prime}, O_{6}^{\prime}, O_{11}^{\prime}, O_{11}^{\prime u}, O_{12}^{\prime}\right.$ and $\left.O_{12}^{\prime u}\right)$ give contribution to the leading order matrix element of $b \rightarrow s \gamma$ and the magnetic moment type coefficient $C_{7}^{e f f}(\mu)\left(C_{7}^{\text {leff }}(\mu)\right)$ is redefined in the NDR scheme as:

$$
\begin{align*}
C_{7}^{\text {eff }}(\mu) & =C_{7}^{2 H D M}(\mu)+Q_{d}\left(C_{5}^{2 H D M}(\mu)+N_{c} C_{6}^{2 H D M}(\mu)\right) \\
& +Q_{u}\left(\frac{m_{c}+m_{u}}{m_{b}} C_{12}^{2 H D M}(\mu)+N_{c} \frac{m_{c}+m_{u}}{m_{b}} C_{11}^{2 H D M}(\mu)\right) \\
C_{7}^{\prime e f f}(\mu) & =C_{7}^{\prime 2 H D M}(\mu)+Q_{d}\left(C_{5}^{\prime 2 H D M}(\mu)+N_{c} C_{6}^{2 H D M}(\mu)\right) \\
& +Q_{u}\left(\frac{m_{c}+m_{u}}{m_{b}} C_{12}^{\prime 2 H D M}(\mu)+N_{c} \frac{m_{c}+m_{u}}{m_{b}} C_{11}^{\prime 2 H D M}(\mu)\right) . \tag{8}
\end{align*}
$$

Since $O_{2}^{(u)}\left(O_{2}^{\prime(u)}\right)$ produce dilepton via virtual photon, their Wilson coefficient $C_{2}(\mu)\left(C_{2}^{\prime}(\mu)\right)$ and the coefficients $C_{1}(\mu), C_{3}(\mu), \ldots, C_{6}(\mu)\left(C_{2}^{\prime}(\mu), C_{3}^{\prime}(\mu), \ldots, C_{6}^{\prime}(\mu)\right)$ induced by the operator mixing, give contributions to $C_{9}^{e f f}(\mu)\left(C_{9}^{\prime e f f}(\mu)\right)$. In a more complete analysis, one has to take into account the long-distance (LD) contributions, produced by real $u \bar{u}, d \bar{d}$ and $c \bar{c}$ intermediate states, i.e. $\rho, \omega$ and $\psi^{(i)}, i=1, \ldots, 6$ (Table 1). These effects can be taken into account by introducing a Breit-Wigner form of the resonance propogator and it gives an additional contribution to $C_{9}^{e f f}(\mu)$ 8, 31] $\left(C_{9}^{\text {eff }}(\mu)\right)$. Finally the effective coefficients $C_{9}^{e f f}(\mu)$ [18, 30] and $C_{9}^{\text {leff }}(\mu)$ are defined in the NDR scheme as:

$$
\begin{align*}
C_{9}^{e f f}(\mu)= & C_{9}^{2 H D M}(\mu) \tilde{\eta}(\hat{s})+\left(h(z, \hat{s})-\frac{3}{\alpha_{e m}^{2}} \kappa \sum_{V_{i}=\psi_{i}} \frac{\pi \Gamma\left(V_{i} \rightarrow l l\right) m_{V_{i}}}{q^{2}-m_{V_{i}}^{2}+i m_{V_{i}} \Gamma_{V_{i}}}\right) \\
& \left(3 C_{1}(\mu)+C_{2}(\mu)+3 C_{3}(\mu)+C_{4}(\mu)+3 C_{5}(\mu)+C_{6}(\mu)\right) \\
+ & \lambda_{u}\left\{h(z, \hat{s})-\frac{3}{\alpha_{e m}^{2}} \kappa \sum_{V_{i}=\psi_{i}} \frac{\pi \Gamma\left(V_{i} \rightarrow l l\right) m_{V_{i}}}{q^{2}-m_{V_{i}}^{2}+i m_{V_{i}} \Gamma_{V_{i}}}\right. \\
- & \left.h(0, \hat{s})+\frac{16 \pi^{2}}{9} \sum_{V_{j}=\rho, \omega} \frac{f_{V_{j}}^{2}\left(q^{2}\right) / q^{2}}{q^{2}-m_{V_{j}}^{2}+i m_{V_{j}} \Gamma_{V_{j}}}\right\}\left(3 C_{1}(\mu)+C_{2}(\mu)\right) \\
- & \frac{1}{2} h(1, \hat{s})\left(4 C_{3}(\mu)+4 C_{4}(\mu)+3 C_{5}(\mu)+C_{6}(\mu)\right) \\
- & \frac{1}{2} h(0, \hat{s})\left(C_{3}(\mu)+3 C_{4}(\mu)\right)+\frac{2}{9}\left(3 C_{3}(\mu)+C_{4}(\mu)+3 C_{5}(\mu)+C_{6}(\mu)\right), \tag{9}
\end{align*}
$$

and

$$
\begin{align*}
C_{9}^{\prime e f f}(\mu)= & C_{9}^{\prime 2 H D M}(\mu) \tilde{\eta}(\hat{s})+\left(h(z, \hat{s})-\frac{3}{\alpha_{e m}^{2}} \kappa \sum_{V_{i}=\psi_{i}} \frac{\pi \Gamma\left(V_{i} \rightarrow l l\right) m_{V_{i}}}{q^{2}-m_{V_{i}}^{2}+i m_{V_{i}} \Gamma_{V_{i}}}\right) \\
& \left(3 C_{1}^{\prime}(\mu)+C_{2}^{\prime}(\mu)+3 C_{3}^{\prime}(\mu)+C_{4}^{\prime}(\mu)+3 C_{5}^{\prime}(\mu)+C_{6}^{\prime}(\mu)\right) \\
= & \lambda_{u}\left\{h(z, \hat{s})-\frac{3}{\alpha_{e m}^{2}} \kappa \sum_{V_{i}=\psi_{i}} \frac{\pi \Gamma\left(V_{i} \rightarrow l l\right) m_{V_{i}}}{q^{2}-m_{V_{i}}^{2}+i m_{V_{i}} \Gamma_{V_{i}}}\right. \\
= & \left.h(0, \hat{s})+\frac{16 \pi^{2}}{9} \sum_{V_{j}=\rho, \omega} \frac{f_{V_{j}}^{2}\left(q^{2}\right) / q^{2}}{q^{2}-m_{V_{j}}^{2}+i m_{V_{j}} \Gamma_{V_{j}}}\right\}\left(3 C_{1}^{\prime}(\mu)+C_{2}^{\prime}(\mu)\right) \\
- & \frac{1}{2} h(1, \hat{s})\left(4 C_{3}^{\prime}(\mu)+4 C_{4}^{\prime}(\mu)+3 C_{5}^{\prime}(\mu)+C_{6}^{\prime}(\mu)\right) \\
- & \frac{1}{2} h(0, \hat{s})\left(C_{3}^{\prime}(\mu)+3 C_{4}^{\prime}(\mu)\right)+\frac{2}{9}\left(3 C_{3}^{\prime}(\mu)+C_{4}^{\prime}(\mu)+3 C_{5}^{\prime}(\mu)+C_{6}^{\prime}(\mu)\right) . \tag{10}
\end{align*}
$$

where $z=\frac{m_{c}}{m_{b}}$ and $\hat{s}=\frac{q^{2}}{m_{b}^{2}}$. In the above expression, $\tilde{\eta}(\hat{s})$ represents the one gluon correction to the matrix element $O_{9}$ with $m_{d}=0$ 29] The functions $\tilde{\eta}(\hat{s}), \omega(\hat{s}), h(z, \hat{s})$ and $h(0, \hat{s})$ are given in appendix C. In eqs. (9) and (10), the phenomenological parameter $\kappa=2.3$ is chosen to be able to reproduce the correct value of the branching ratio $\operatorname{Br}(B \rightarrow J / \psi X \rightarrow X l \bar{l})=B r(B \rightarrow$ $J / \psi X) \operatorname{Br}(J / \psi \rightarrow X l \bar{l})$ 10].

In the derivations of $\rho$ and $\omega$ meson resonance effects, we used the $q^{2}$ dependence of the coupling $f_{V_{j}}$ through the expression 32

$$
\begin{equation*}
f_{V_{j}}\left(q^{2}\right)=f_{V_{j}}(0)\left(1+\frac{q^{2}}{P_{V_{j}}(0)}\left(P_{V_{j}}^{\prime}(0)+\tilde{P}_{V_{j}}\left(q^{2}\right)\right)\right) \tag{11}
\end{equation*}
$$

where the coupling $f_{V_{j}}$ is defined as $<0\left|\bar{q} \gamma_{\mu} q\right| V_{j}\left(q^{2}\right) \mid 0>=f_{V_{j}}\left(q^{2}\right) \epsilon_{\mu}, P_{V_{j}}(0)$ and $P_{V_{j}}^{\prime}(0)$ are the subtraction constants (Table (22)). The function $\tilde{P}_{V_{j}}\left(q^{2}\right)$ is (32]

$$
\begin{equation*}
\tilde{P}_{V_{j}}\left(q^{2}\right)=\frac{1}{16 \pi^{2} r}\left(-4-\frac{20}{3} r+4(1+2 r)\left(\frac{1-r}{r}\right)^{1 / 2} \operatorname{Arctan}\left(\frac{r}{1-r}\right)^{1 / 2}\right), \tag{12}
\end{equation*}
$$

where $r=q^{2} / 4 m_{q}^{2}$ and $m_{q}$ is the mass of the quark which produces the meson. This expression is valid in the region $0 \leq q^{2} \leq 4 m_{q}^{2}$. For the $q^{2}$ values, $q^{2}>4 m_{q}^{2}$, we use the assumption [32] $f_{V_{j}}\left(q^{2}\right)=f_{V_{j}}\left(m_{V_{j}}^{2}\right)($ Table(2) $)$.

Finally, neglecting the down quark mass, the matrix element for $b \rightarrow d e^{+} e^{-}$decay is obtained as:

$$
\begin{align*}
\mathcal{M} & =-\frac{G_{F} \alpha_{e m}}{2 \sqrt{2} \pi} V_{t b} V_{t d}^{*}\left\{\left(C_{9}^{e f f}(\mu) \bar{d} \gamma_{\mu}\left(1-\gamma_{5}\right) b+C_{9}^{\prime e f f}(\mu) \bar{d} \gamma_{\mu}\left(1+\gamma_{5}\right) b\right) \bar{e} \gamma^{\mu} e\right. \\
& +\left(C_{10}(\mu) \bar{d} \gamma_{\mu}\left(1-\gamma_{5}\right) b+C_{10}^{\prime}(\mu) \bar{d} \gamma_{\mu}\left(1+\gamma_{5}\right) b\right) \bar{e} \gamma^{\mu} \gamma_{5} e  \tag{13}\\
& \left.-2\left(C_{7}^{e f f}(\mu) \frac{m_{b}}{q^{2}} \bar{d} i \sigma_{\mu \nu} q^{\nu}\left(1+\gamma_{5}\right) b+C_{7}^{\prime e f f}(\mu) \frac{m_{b}}{q^{2}} \bar{d} i \sigma_{\mu \nu} q^{\nu}\left(1-\gamma_{5}\right) b\right) \bar{e} \gamma^{\mu} e\right\}
\end{align*}
$$

| $\psi$ | $m_{\psi}(\mathrm{GeV})$ | $\Gamma\left(\psi \rightarrow l^{+} l^{-}\right)(\mathrm{GeV})$ |
| :---: | :---: | :---: |
| $J / \psi$ | 3.097 | $5.2810^{-6}$ |
| $\psi^{(2)}$ | 3.686 | $2.3510^{-6}$ |
| $\psi^{(3)}$ | 3.770 | $2.6410^{-7}$ |
| $\psi^{(4)}$ | 4.040 | $7.2810^{-7}$ |
| $\psi^{(5)}$ | 4.160 | $7.8010^{-7}$ |
| $\psi^{(6)}$ | 4.420 | $4.7310^{-7}$ |

Table 1: Masses of $\psi$ mesons and decay widths $\Gamma\left(\psi \rightarrow l^{+} l^{-}\right)$used in the calculations.

|  | $f_{V}(0)(\mathrm{GeV})$ | $f_{V}\left(m_{V}^{2}\right)(\mathrm{GeV})$ | $P_{V}(0)$ | $P_{V}^{\prime}(0)$ |
| :---: | :---: | :---: | :---: | :---: |
| $\rho$ | 0.162 | 0.17 | -0.7498 | -0.0430 |
| $\omega$ | 0.166 | 0.180 | -0.7744 | -0.0430 |

Table 2: The decay couplings and the substraction constants for $\rho$ and $\omega$ mesons.

## 3 The exclusive $\bar{B} \rightarrow \pi e^{+} e^{-}$decay

### 3.1 The formulation

Now, we continue to present the differential decay rate and CP violating asymmetry in the process $\bar{B} \rightarrow \pi e^{+} e^{-}$. To calculate the decay width, branching ratio, etc., for the exclusive $\bar{B} \rightarrow$ $\pi e^{+} e^{-}$decay, we need the matrix elements $\langle\pi| \bar{d} \gamma_{\mu}\left(1 \pm \gamma_{5}\right) b|\bar{B}\rangle$, and $\langle\pi| \bar{d} i \sigma_{\mu \nu} q^{\nu}\left(1 \pm \gamma_{5}\right) b|\bar{B}\rangle$. Using the parametrization

$$
\begin{align*}
<\pi\left(p_{\pi}\left|\bar{d} \gamma_{\mu}\left(1 \pm \gamma_{5}\right) b\right| \bar{B}\left(p_{B}\right)>\right. & =\left(2 p_{B}-q\right)_{\mu} f_{+}\left(q^{2}\right)+q_{\mu} f_{-}\left(q^{2}\right), \\
<\pi\left(p_{\pi}\left|\bar{d} i \sigma_{\mu \nu} q^{\nu}\left(1 \pm \gamma_{5}\right) b\right| \bar{B}\left(p_{B}\right)>\right. & =-\left\{\left(2 p_{B}-q\right)_{\mu} q^{2}-\left(m_{B}^{2}-m_{\pi}^{2}\right) q_{\mu}\right\} v\left(q^{2}\right), \tag{14}
\end{align*}
$$

where $p_{B}$ and $p_{\pi}$ are four momentum vectors of $B$ and $\pi$ mesons respectively and $q=p_{B}-p_{\pi}$, we get the double differential decay rate:

$$
\begin{equation*}
\frac{d \Gamma\left(\bar{B} \rightarrow \pi e^{+} e^{-}\right)}{d \sqrt{s} d z}=\frac{G_{F}^{2} \alpha_{e m}^{2} m_{B}^{5}\left|V_{t b} V_{t d}^{*}\right|^{2} \lambda^{1 / 2} \sqrt{s}}{2^{10} \pi^{5}} \Omega_{\pi} \tag{15}
\end{equation*}
$$

Here

$$
\begin{equation*}
\Omega_{\pi}=\left\{\left|\left(C_{9}^{e f f}+C_{9}^{\prime e f f}\right) f_{+}\left(q^{2}\right)+2\left(C_{7}^{e f f}+C_{7}^{\prime e f f}\right) v\left(q^{2}\right) m_{b}\right|^{2}+\left|\left(C_{10}+C_{10}^{\prime}\right) f_{+}\left(q^{2}\right)\right|^{2}\right\}\left(1-z^{2}\right) \tag{16}
\end{equation*}
$$

and $z=\cos \theta, \theta$ is the angle between the momentum of the electron and that of $B$ meson in the center of mass frame of the lepton pair,

$$
\begin{equation*}
\lambda=1+t^{2}+s^{2}-2 t-2 s-2 t s \tag{17}
\end{equation*}
$$

where $t=\frac{m_{\pi}^{2}}{m_{B}^{2}}$ and $s=\frac{q^{2}}{m_{B}^{2}}$.
For the form factors $f_{+}\left(q^{2}\right)$ and $v\left(q^{2}\right)$, we use the results due to the dispersion formulation of the light-cone constituent quark model 20

$$
\begin{align*}
f_{+}\left(q^{2}\right) & =\frac{f_{+}(0)}{\left(1-\frac{q^{2}}{m_{f_{+}}^{2}}\right)^{2.35}} \\
v\left(q^{2}\right) & =\frac{v(0)}{\left(1-\frac{q^{2}}{m_{v}^{2}}\right)^{2.31}} \tag{18}
\end{align*}
$$

where $f_{+}(0)=0.24, v(0)=0.05$ and $m_{f_{+}}=6.71 \mathrm{GeV}, m_{v}=6.68 \mathrm{GeV}$.
Let us now turn to the CP-violating asymmetry, which is defined as

$$
\begin{equation*}
A_{C P}=\frac{\frac{d \Gamma\left(\bar{B} \rightarrow \pi e^{+} e^{-}\right)}{d \sqrt{s}}-\frac{d \Gamma\left(B \rightarrow \bar{\pi} e^{+} e^{-}\right)}{d \sqrt{s}}}{\frac{d \Gamma\left(\bar{B} \rightarrow \pi e^{+} e^{-}\right)}{d \sqrt{s}}+\frac{d \Gamma\left(B \rightarrow \bar{\pi} e^{+} e^{-}\right)}{d \sqrt{s}}} . \tag{19}
\end{equation*}
$$

The wilson coefficient $C_{9}^{e f f}$ is the origin of the CP violating asymmetry since it is a function of $\lambda_{u}=\frac{V_{u b} V_{u d}^{*}}{V_{t b} V_{t d}^{*}}$. With the parametrization

$$
\begin{align*}
C_{9}^{e f f} & =\xi_{1}+\lambda_{u} \xi_{2} \\
C_{9}^{\prime e f f} & =\xi_{1}^{\prime}+\lambda_{u} \xi_{2}^{\prime} \tag{20}
\end{align*}
$$

and using eq. (19) we get

$$
\begin{equation*}
A_{C P}=-2 \operatorname{Im}\left(\lambda_{u}\right) \frac{\Delta_{\pi}}{\Omega_{\pi}} \lambda \tag{21}
\end{equation*}
$$

where

$$
\begin{equation*}
\Delta_{\pi}=\left\{\operatorname{Im}\left(\xi_{1}^{t *} \xi_{2}^{t}\right) f_{+}\left(q^{2}\right)+2 m_{b} \operatorname{Im}\left(\xi_{2}^{t}\right)\left(C_{7}^{e f f}+C_{7}^{\text {eff }}\right)\right\} v\left(q^{2}\right)\left|f_{+}\left(q^{2}\right)\right| \tag{22}
\end{equation*}
$$

and

$$
\begin{align*}
& \xi_{1}^{t}=\xi_{1}+\xi_{1}^{\prime} \\
& \xi_{2}^{t}=\xi_{2}+\xi_{2}^{\prime} \tag{23}
\end{align*}
$$

In our numerical analysis we used the input values given in Table (3).

### 3.2 Discussion

In this section, we would like to study the $q^{2}$ dependencies of the differential $B r$, and $A_{C P}$ of the decay $\bar{B} \rightarrow \pi e^{+} e^{-}$, for the selected parameters of the model III ( $\bar{\xi}_{N t t}^{U}, \bar{\xi}_{N b b}^{D}$ ), using the

| Parameter | Value |
| :--- | :--- |
| $m_{c}$ | $1.4(\mathrm{GeV})$ |
| $m_{b}$ | $4.8(\mathrm{GeV})$ |
| $\alpha_{\text {em }}^{-1}$ | 129 |
| $\lambda_{t}$ | 0.04 |
| $\Gamma_{\text {tot }}\left(B_{d}\right)$ | $3.96 \cdot 10^{-13}(\mathrm{GeV})$ |
| $m_{B_{d}}$ | $5.28(\mathrm{GeV})$ |
| $m_{\rho}$ | $0.768(\mathrm{GeV})$ |
| $m_{\pi}$ | $0.139(\mathrm{GeV})$ |
| $m_{t}$ | $175(\mathrm{GeV})$ |
| $m_{W}$ | $80.26(\mathrm{GeV})$ |
| $m_{Z}$ | $91.19(\mathrm{GeV})$ |
| $\Lambda_{Q C D}$ | $0.214(\mathrm{GeV})$ |
| $\alpha_{s}\left(m_{Z}\right)$ | 0.117 |
| $\sin \theta_{W}$ | $\sqrt{0.2325}$ |

Table 3: The values of the input parameters used in the numerical calculations.
constraints [27] coming from the $\Delta F=2(F=K, D, B)$ mixing ,the $\rho$ parameter [33] and the measurement by CLEO collaboration 34,

$$
\begin{equation*}
\operatorname{Br}\left(B \rightarrow X_{s} \gamma\right)=(2.32 \pm 0.07 \pm 0.35) 10^{-4} \tag{24}
\end{equation*}
$$

In the calculations, we take $\bar{\xi}_{N t c} \ll \bar{\xi}_{N t t}^{U}, \bar{\xi}_{N b b}^{D}$ and $\bar{\xi}_{N i j}^{D} \sim 0$ where $i$ or $j$ are first or second generation indices (see [26] for details). Under this assumption the Wilson coefficients $C_{7}^{\prime}, C_{9}^{\prime}$ and $C_{10}^{\prime}$ can be neglected compared to unprimed ones and the neutral Higgs contributions are suppressed.

In figs. 1 and 2 we plot the differential $B r$ of the decay $\bar{B} \rightarrow \pi e^{+} e^{-}$with respect to the dilepton mass $q^{2}$ for the fixed values of $\bar{\xi}_{N, b b}^{D}=40 m_{b}$ and charged Higgs mass $m_{H^{ \pm}}=400 \mathrm{GeV}$ at the scale $\mu=m_{b}$. Fig. [1] represents the case where the ratio $\left|r_{t b}\right|=\left|\frac{\xi_{N, t t}^{U}}{\xi_{N, b b}^{D}}\right| \ll 1$. It is shown that the differential $B r$ obtained in the model III is smaller compared to the one calculated in the SM. Fig. 2 (3) devoted to the case where $r_{t b} \gg 1$ for the fixed value of $\bar{\xi}_{N, b b}^{D}, \bar{\xi}_{N, b b}^{D}=40 m_{b}$ $\left(\bar{\xi}_{N, b b}^{D}=90 m_{b}\right)$. The differential $B r$ in the model III increases at this region $\left(r_{t b} \gg 1\right)$ and it enhances strongly compared to the SM with the increasing $\bar{\xi}_{N, b b}^{D}$ (Fig. 3).

Now we present the values of $B r$ for the $\bar{B} \rightarrow \pi e^{+} e^{-}$decay in the SM and model III, without LD effects. After integrating over $q^{2}$, we get

$$
\begin{equation*}
\operatorname{Br}\left(B \rightarrow \pi e^{+} e^{-}\right)=0.62 \times 10^{-7}(S M) \tag{25}
\end{equation*}
$$

and for the model III

$$
\operatorname{Br}\left(B \rightarrow K^{*} l^{+} l^{-}\right)= \begin{cases}0.27 \times 10^{-7} & \left(\left|r_{t b}\right| \ll 1, \bar{\xi}_{N, b b}^{D}=40 m_{b}\right)  \tag{26}\\ 0.54 \times 10^{-7} & \left(r_{t b} \gg 1, \bar{\xi}_{N, b b}^{D}=40 m_{b}\right) \\ 2.65 \times 10^{-7} & \left(r_{t b} \gg 1, \bar{\xi}_{N, b b}^{D}=90 m_{b}\right)\end{cases}
$$

Here, the strong enhancement of the $B r$ can be observed for $r_{t b} \gg 1$, especially with increasing $\bar{\xi}_{N, b b}^{D}$. Note that, in the calculations of $B r$ and the differential $B r$, we used the Wolfenstein parameters, $\rho=-0.07, \eta=0.34$.

Figs. Hand $^{2}$ show the $q^{2}$ dependence of $A_{C P}$ for the Wolfenstein parameters $\rho=-0.07, \eta=$ 0.34 , fixed values of $\bar{\xi}_{N, b b}^{D}=40 m_{b}$ and charged Higgs mass $m_{H^{ \pm}}=400 \mathrm{GeV}$ at the scale $\mu=m_{b}$, for $\left|r_{t b}\right| \ll 1$ and $r_{t b} \gg 1$ respectively. The CP violation in the model III for $\left|r_{t b}\right| \ll 1$ and $\bar{\xi}_{N, b b}^{D}=40 m_{b}$ is slightly greater than the one in the SM. However, it decreases for $r_{t b} \gg 1$ and becomes extremely smaller compared to the one calculated in the SM with increasing $\bar{\xi}_{N, b b}^{D}$ (Fig. [6).

We also present $<A_{C P}>$ for two different Wolfenstein parameters in two different dilepton mass regions

| ( $\rho, \eta$ ) | SM | $\begin{gathered} \hline \hline \text { model III } \\ \xi_{b b}^{D}=40 m_{b} \\ \left\|r_{t b}\right\| \ll 1 \\ \hline \end{gathered}$ | $\begin{gathered} \hline \hline \text { model III } \\ \xi_{b b}^{D}=40 m_{b} \\ r_{t b} \gg 1 \\ \hline \end{gathered}$ | $\begin{gathered} \text { model III } \\ \xi_{b b}^{D}=90, m_{b} \\ r_{t b} \gg 1 \end{gathered}$ | $q^{2}$ regions |
| :---: | :---: | :---: | :---: | :---: | :---: |
| (0.3, 0.34) | $2.2010^{-2}$ | $2.2110^{-2}$ | $1.5810^{-2}$ | $0.7210^{-2}$ | I |
|  | $0.6310^{-2}$ | $0.6310^{-2}$ | $0.4810^{-2}$ | $0.2410^{-2}$ | II |
| (-0.07, 0.34) | $0.9910^{-2}$ | $1.1810^{-2}$ | $0.8210^{-2}$ | $0.3610^{-2}$ | I |
|  | $0.3210^{-2}$ | $0.3210^{-2}$ | $0.2410^{-2}$ | $0.1110^{-2}$ | II |

Table 4: The average asymmetry $<A_{C P}>$ for regions I ( $\left.1 G e V \leq \sqrt{q}^{2} \leq m_{J / \psi}-20 \mathrm{MeV}\right)$ and II $\left(m_{J / \psi}+20 \mathrm{MeV} \leq \sqrt{q}^{2} \leq m_{\psi^{\prime}}-20 \mathrm{MeV}\right)$

In conclusion, we analyse the dependencies of the differential $B r, B r, A_{C P}$ and the average CP-asymmetry $<A_{C P}>$ on the selected model III parameters ( $\bar{\xi}_{N, b b}^{D}, \bar{\xi}_{N, t t}^{U}$ ) for the decay $\bar{B} \rightarrow \pi e^{+} e^{-}$. We obtain that the strong enhancement of the differential $\operatorname{Br}(B r)$ is possible in the framework of the model III and observe that $A_{C P}$ is sensitive to the model III parameters $\left(\bar{\xi}_{N, b b}^{D}, \bar{\xi}_{N, t t}^{U}\right)$.

## 4 The exclusive $\bar{B} \rightarrow \rho e^{+} e^{-}$decay

### 4.1 The formulation

In this section, we analyse the differential decay rate, $A_{C P}$ and $A_{F B}$ in the process $\bar{B} \rightarrow \rho e^{+} e^{-}$. At this stage, we need the matrix elements $\langle\rho| \bar{d} \gamma_{\mu}\left(1 \pm \gamma_{5}\right) b|\bar{B}\rangle$, and $\langle\rho| \bar{d} \dot{i} \sigma_{\mu \nu} q^{\nu}\left(1 \pm \gamma_{5}\right) b|\bar{B}\rangle$. Using the parametrization of the form factors as in [35], the matrix element of the $\bar{B} \rightarrow \rho e^{+} e^{-}$ decay is obtained as [36]:

$$
\begin{align*}
\mathcal{M} & =-\frac{G \alpha_{e m}}{2 \sqrt{2} \pi} V_{t b} V_{t d}^{*}\left\{\bar{\ell} \gamma^{\mu} \ell\left[2 A_{t o t} \epsilon_{\mu \nu \rho \sigma} \epsilon^{* \nu} p_{\rho}^{\rho} q^{\sigma}+i B_{1 t o t} \epsilon_{\mu}^{*}-i B_{2 t o t}\left(\epsilon^{*} q\right)\left(p_{B}+p_{\rho}\right)_{\mu}-i B_{3 t o t}\left(\epsilon^{*} q\right) q_{\mu}\right]\right. \\
& \left.+\bar{\ell} \gamma^{\mu} \gamma_{5} \ell\left[2 C_{t o t} \epsilon_{\mu \nu \rho \sigma} \epsilon^{* \nu} p_{\rho}^{\rho} q^{\sigma}+i D_{1 t o t} \epsilon_{\mu}^{*}-i D_{2 t o t}\left(\epsilon^{*} q\right)\left(p_{B}+p_{\rho}\right)_{\mu}-i D_{3 t o t}\left(\epsilon^{*} q\right) q_{\mu}\right]\right\} \tag{27}
\end{align*}
$$

where $\epsilon^{* \mu}$ is the polarization vector of $\rho$ meson, $p_{B}$ and $p_{\rho}$ are four momentum vectors of $B$ and $\rho$ mesons, $q=p_{B}-p_{\rho}$ and

$$
\begin{align*}
A_{t o t} & =A+A^{\prime}, \\
B_{1 t o t} & =B_{1}+B_{1}^{\prime}, \\
B_{2 \text { tot }} & =B_{2}+B_{2}^{\prime}, \\
B_{3 t o t} & =B_{3}+B_{3}^{\prime}, \\
C_{t o t} & =C+C^{\prime}, \\
D_{1 \text { tot }} & =D_{1}+D_{1}^{\prime}, \\
D_{2 t o t} & =D_{2}+D_{2}^{\prime}, \\
D_{3 t o t} & =D_{3}+D_{3}^{\prime} \tag{28}
\end{align*}
$$

Here

$$
\begin{aligned}
A & =-C_{9}^{e f f} g\left(q^{2}\right)+2 C_{7}^{e f f} \frac{m_{b}}{q^{2}} g_{+}\left(q^{2}\right) \\
B_{1} & =-C_{9}^{e f f} f\left(q^{2}\right)+2 C_{7}^{e f f} \frac{m_{b}}{q^{2}}\left(\left(m_{B}^{2}-m_{\rho}^{2}\right) g_{+}\left(q^{2}\right)+q^{2} g_{-}\left(q^{2}\right)\right) \\
B_{2} & =C_{9}^{e f f} a_{+}\left(q^{2}\right)+2 C_{7}^{e f f} \frac{m_{b}}{q^{2}}\left(g_{+}\left(q^{2}\right)+\frac{q^{2} h\left(q^{2}\right)}{2}\right) \\
B_{3} & =C_{9}^{e f f} a_{-}\left(q^{2}\right)+2 C_{7}^{e f f} \frac{m_{b}}{q^{2}}\left(g_{-}\left(q^{2}\right)-\frac{\left(m_{B}^{2}-m_{\rho}^{2}\right) h\left(q^{2}\right)}{2}\right) \\
C & =-C_{10} g\left(q^{2}\right) \\
D_{1} & =-C_{10} f\left(q^{2}\right) \\
D_{2} & =C_{10} a_{+}\left(q^{2}\right)
\end{aligned}
$$

$$
\begin{equation*}
D_{3}=C_{10} a_{-}\left(q^{2}\right), \tag{29}
\end{equation*}
$$

and

$$
\begin{align*}
A^{\prime} & =-C_{9}^{\prime e f f} g\left(q^{2}\right)+2 C_{7}^{\prime e f f} \frac{m_{b}}{q^{2}} g_{+}\left(q^{2}\right) \\
B_{1}^{\prime} & =C_{9}^{\prime e f f} f\left(q^{2}\right)-2 C_{7}^{\prime e f f} \frac{m_{b}}{q^{2}}\left(\left(m_{B}^{2}-m_{\rho}^{2}\right) g_{+}\left(q^{2}\right)+q^{2} g_{-}\left(q^{2}\right)\right) \\
B_{2}^{\prime} & =-C_{9}^{\prime e f f} a_{+}\left(q^{2}\right)-2 C_{7}^{\prime e f f} \frac{m_{b}}{q^{2}}\left(g\left(q^{2}\right)+\frac{q^{2} h\left(q^{2}\right)}{2}\right) \\
B_{3}^{\prime} & =-C_{9}^{\prime e f f} a_{-}\left(q^{2}\right)-2 C_{7}^{\prime e f f} \frac{m_{b}}{q^{2}}\left(g_{-}\left(q^{2}\right)-\frac{\left(m_{B}^{2}-m_{\rho}^{2}\right) h\left(q^{2}\right)}{2}\right) \\
C^{\prime} & =-C_{10}^{\prime} g\left(q^{2}\right) \\
D_{1}^{\prime} & =C_{10}^{\prime} f\left(q^{2}\right) \\
D_{2}^{\prime} & =-C_{10}^{\prime} a_{+}\left(q^{2}\right) \\
D_{3}^{\prime} & =-C_{10}^{\prime} a_{-}\left(q^{2}\right) \tag{30}
\end{align*}
$$

For the formfactors $g\left(q^{2}\right), a_{-}\left(q^{2}\right), a_{+}\left(q^{2}\right), g_{+}\left(q^{2}\right), g_{-}\left(q^{2}\right), h\left(q^{2}\right)$, and $f\left(q^{2}\right)$ we use the dispersion formulation of the light-cone constituent quark model [20] in the following pole form

$$
\begin{align*}
g\left(q^{2}\right) & =\frac{0.036}{\left(1-\frac{q^{2}}{(6.55)^{2}}\right)^{2.75}}, \quad a_{+}\left(q^{2}\right)=\frac{-0.026}{\left(1-\frac{q^{2}}{(7.29)^{2}}\right)^{3.04}}, \\
a_{-}\left(q^{2}\right) & =\frac{0.03}{\left(1-\frac{q^{2}}{(6.88)^{2}}\right)^{2.85}}, \quad g_{+}\left(q^{2}\right)=\frac{-0.20}{\left(1-\frac{q^{2}}{(6.57)^{2}}\right)^{2.76}}, \\
g_{-}\left(q^{2}\right) & =\frac{0.18}{\left(1-\frac{q^{2}}{(6.50)^{2}}\right)^{2.73}}, \quad h\left(q^{2}\right)=\frac{0.003}{\left(1-\frac{q^{2}}{(6.43)^{2}}\right)^{3.42}} \\
f\left(q^{2}\right) & =\frac{1.10}{\left(1-\frac{q^{2}}{(5.59)^{2}}+\left(\frac{q^{2}}{(7.10)^{2}}\right)^{2}\right)} \tag{31}
\end{align*}
$$

Using eq.(27), we get the double differential decay rate:

$$
\begin{aligned}
\frac{d \Gamma}{d q^{2} d z} & =\frac{G^{2} \alpha_{e m}^{2}\left|V_{t b} V_{t s}^{*}\right|^{2} \lambda^{1 / 2}}{2^{12} \pi^{5} m_{B}}\left\{2 \lambda m_{B}^{4}\left[m_{B}^{2} s\left(1+z^{2}\right)\left(\left|A_{t o t}\right|^{2}+\left|C_{t o t}\right|^{2}\right)\right]\right. \\
& +\frac{\lambda m_{B}^{4}}{2 r}\left[\lambda m_{B}^{2}\left(1-z^{2}\right)\left(\left|B_{2 t o t}\right|^{2}+\left|D_{2 t o t}\right|^{2}\right)\right]
\end{aligned}
$$

$$
\begin{align*}
& +\frac{1}{2 r}\left[m_{B}^{2}\left\{\lambda\left(1-z^{2}\right)+8 r s\right\}\left(\left|B_{1 t o t}\right|^{2}+\left|D_{1 t o t}\right|^{2}\right)\right. \\
& \left.-2 \lambda m_{B}^{4}(1-r-s)\left(1-z^{2}\right)\left\{\operatorname{Re}\left(B_{1 t o t} B_{2 t o t}^{*}\right)+\operatorname{Re}\left(D_{1 t o t} D_{2 t o t}^{*}\right)\right\}\right] \\
& \left.-8 m_{B}^{4} s \lambda^{1 / 2} z\left[\left\{\operatorname{Re}\left(B_{1 t o t} C_{t o t}^{*}\right)+\operatorname{Re}\left(A_{t o t} D_{1 t o t}^{*}\right)\right\}\right]\right\} \tag{32}
\end{align*}
$$

where $z=\cos \theta, \theta$ is the angle between the momentum of electron $e$ and that of $B$ meson in the center of mass frame of the lepton pair, $\lambda=1+t^{2}+s^{2}-2 t-2 s-2 t s, t=\frac{m_{\rho}^{2}}{m_{B}^{2}}$ and $s=\frac{q^{2}}{m_{B}^{2}}$.

We continue to present the CP-violating asymmetry, which is defined as in eq. (19) with the replacement of $\pi \rightarrow \rho$. Using the same parametrization as in eq. (20) we get

$$
\begin{equation*}
A_{C P}=-2 \operatorname{Im}\left(\lambda_{u}\right) \frac{\Delta_{\rho}}{\Omega_{\rho}} \lambda \tag{33}
\end{equation*}
$$

where

$$
\begin{align*}
\Delta_{\rho}= & \operatorname{Im}\left(\xi_{1}^{t *} \xi_{2}^{t}\right)\left\{4 s m_{B}^{2} g^{2}\left(q^{2}\right)+\frac{f^{2}\left(q^{2}\right)}{\lambda m_{B}^{2}}\left(6 s+\frac{\lambda}{2 t}\right)+\frac{m_{B}^{2} \lambda}{2 t} a_{+}^{2}\left(q^{2}\right)+\frac{(1-s-t)}{t} f\left(q^{2}\right) a_{+}\left(q^{2}\right)\right\} \\
+ & \frac{2 C_{7}^{e f f}}{s} \operatorname{Im}\left(\xi_{2}\right)\left\{-4 \frac{(1+\sqrt{t})}{\sqrt{t}} m_{b} s g\left(q^{2}\right) g_{+}\left(q^{2}\right)-\frac{m_{b}}{2 m_{B}}\left((1-t) g_{+}\left(q^{2}\right)+s g_{-}\left(q^{2}\right)\right)(1+\sqrt{t})\right. \\
& \left(\frac{2 f\left(q^{2}\right)}{\lambda m_{B}}\left(6 s+\frac{\lambda}{2 t}\right)+m_{B} a_{+}\left(q^{2}\right) \frac{1-t-s}{t}\right)+\frac{m_{b}}{2 m_{B} t}\left(m_{B} \lambda a_{+}\left(q^{2}\right)+\frac{f\left(q^{2}\right)}{m_{B}}(1-t-s)\right) \\
& \left.\left(g_{+}\left(q^{2}\right)+\frac{s m_{B}^{2}}{2} h\left(q^{2}\right)\right)\right\} \tag{34}
\end{align*}
$$

and

$$
\begin{align*}
\Omega_{\rho} & =\lambda\left\{4 m_{B}^{2} s\left(\left|A_{t o t}\right|^{2}+|C|_{t o t}^{2}\right)+\frac{1}{m_{B}^{2} \lambda}\left(6 s+\frac{\lambda}{2 t}\right)\left(\left|B_{1 t o t}\right|^{2}+\left|D_{1 t o t}\right|^{2}\right)\right. \\
& \left.+\frac{\lambda}{2 t} m_{B}^{2}\left(\left|B_{2 t o t}\right|^{2}+\left|D_{2 t o t}\right|^{2}\right)-\lambda \frac{1-t-s}{t} \operatorname{Re}\left(B_{1 t o t} B_{2 t o t}^{*}+D_{1 t o t} D_{2 t o t}^{*}\right)\right\} \tag{35}
\end{align*}
$$

Finally, we present $A_{F B}$ which can give more precise information about the Wilson coefficients $C_{7}^{e f f}, C_{9}^{e f f}$ and $C_{10}$. It is defined as:

$$
\begin{equation*}
A_{F B}\left(q^{2}\right)=\frac{\int_{0}^{1} d z \frac{d \Gamma}{d q^{2} d z}-\int_{-1}^{0} d z \frac{d \Gamma}{d q^{2} d z}}{\int_{0}^{1} d z \frac{d \Gamma}{d q^{2} d z}+\int_{-1}^{0} d z \frac{d \Gamma}{d q^{2} d z}} \tag{36}
\end{equation*}
$$

After the standard calculation, we get

$$
\begin{align*}
A_{F B}= & 12 \lambda^{1 / 2} \frac{\operatorname{Re}\left(C_{10}+C_{10}^{\prime}\right)}{\Omega_{\rho}}\left\{s f\left(q^{2}\right) g\left(q^{2}\right) \operatorname{Re}\left(C_{9}^{e f f}+C_{9}^{\prime e f f}\right)-\frac{m_{b}}{m_{B}}\left(C_{7}^{e f f}+C_{7}^{\prime e f f}\right)\right. \\
& \left.\left(m_{B}(1+\sqrt{t}) g\left(q^{2}\right)\left((1-t) g_{+}\left(q^{2}\right)+s g_{-}\left(q^{2}\right)\right)+g_{+}\left(q^{2}\right) f\left(q^{2}\right) \frac{1+t}{m_{B}(1+\sqrt{t})}\right)\right\}( \tag{37}
\end{align*}
$$

### 4.2 Discussion

In this section, we study the $q^{2}$ dependencies of the differential $B r, A_{C P}$ and $A_{F B}$ of the decay $\bar{B} \rightarrow \rho e^{+} e^{-}$for the selected parameters of the model III $\left(\bar{\xi}_{N t t}^{U}, \bar{\xi}_{N b b}^{D}\right)$. In the calculations, we use the same restrictions for the model III parameters. (see section 3)

In figs. (8) (8) we plot the differential $B r$ of the decay $\bar{B} \rightarrow \rho e^{+} e^{-}$with respect to the dilepton mass $q^{2}$ for the fixed values of $\bar{\xi}_{N, b b}^{D}=40 m_{b}$ and charged Higgs mass $m_{H^{ \pm}}=400 \mathrm{GeV}$ at the scale $\mu=m_{b}$, for the ratio $\left|r_{t b}\right|=\left|\frac{\xi_{N, t t}^{U}}{\xi_{N, b b}^{N}}\right| \ll 1\left(r_{t b}=\frac{\bar{\xi}_{N, t t}^{U}}{\xi_{N, b b}^{D}} \gg 1\right)$. The differential $B r$, obtained in the model III, is smaller compared to the one calculated in the SM, for $\left|r_{t b}\right| \ll 1$. However, it increases at the region $r_{t b} \gg 1$ and enhances strongly compared to the SM with the increasing $\bar{\xi}_{N, b b}^{D}$ (Fig. $\bar{G}$ ), similar to the decay $\bar{B} \rightarrow \pi e^{+} e^{-}$. To be complete, we present the values of $B r$ for the $\bar{B} \rightarrow \rho e^{+} e^{-}$decay in the SM and model III, without the LD effects. After integrating over $q^{2}$, we get

$$
\begin{equation*}
\operatorname{Br}\left(\bar{B} \rightarrow \rho e^{+} e^{-}\right)=0.91 \times 10^{-7}(S M) \tag{38}
\end{equation*}
$$

and for the model III

$$
\operatorname{Br}\left(\bar{B} \rightarrow \rho e^{+} e^{-}\right)= \begin{cases}0.44 \times 10^{-7} & \left(\left|r_{t b}\right| \ll 1, \bar{\xi}_{N, b b}^{D}=40 m_{b}\right)  \tag{39}\\ 1.5 \times 10^{-7} & \left(r_{t b} \gg 1, \bar{\xi}_{N, b b}^{D}=40 m_{b}\right) \\ 3.2 \times 10^{-7} & \left(r_{t b} \gg 1, \bar{\xi}_{N, b b}^{D}=90 m_{b}\right)\end{cases}
$$

The strong enhancement of the $B r$ is observed for $r_{t b} \gg 1$, especially with increasing $\bar{\xi}_{N, b b}^{D}$. Note that we used the Wolfenstein parameters, $\rho=-0.07, \eta=0.34$, in the calculation of $B r$ and differential $B r$.

Figs. 10 and 11 show the $q^{2}$ dependence of $A_{C P}$ for the Wolfenstein parameters $\rho=$ $-0.07, \eta=0.34$, the fixed values of $\bar{\xi}_{N, b b}^{D}=40 m_{b}$ and charged Higgs mass $m_{H^{ \pm}}=400 \mathrm{GeV}$ at the scale $\mu=m_{b}$, for $\left|r_{t b}\right| \ll 1$ and $r_{t b} \gg 1$ respectively. The CP violation decreases in the region $r_{t b} \gg 1$, especially with increasing $\bar{\xi}_{N, b b}^{D}$ (Fig. 12). Now, we give $<A_{C P}>$ for two different Wolfenstein parameters in two different dilepton mass regions

Finally, we discuss $A_{F B}$ of the process under consideration. Figs. 13 and 14 show the $q^{2}$ dependence of $A_{F B}$ for the Wolfenstein parameters $\rho=-0.07, \eta=0.34$, the fixed values of $\bar{\xi}_{N, b b}^{D}=40 m_{b}$ and charged Higgs mass $m_{H^{ \pm}}=400 \mathrm{GeV}$ at the scale $\mu=m_{b}$, for $\left|r_{t b}\right| \ll 1$ and $r_{t b} \gg 1$ respectively. For $r_{t b} \gg 1$ (Fig. (13)) $A_{F B}$ changes its sign almost at $s=0.34$, however for $r_{t b} \gg 1$ (Fig. (14) it is positive without LD effects. Therefore the determination of the sign of $A_{F B}$ in the region $0 \leq s \leq 0.25$ (here the upper limit corresponds to the value where $A_{F B}$ change sign in the SM) can give a unique information about the existence of the model III.

| $(\rho, \eta)$ | $S M$ | model III <br> $\xi_{b b}^{D}=40 m_{b}$ <br> $r_{t b} \ll 1$ | model III <br> $\xi_{b b}^{D}=40 m_{b}$ <br> $\left\|r_{t b}\right\|>1$ | model III <br> $\xi_{b b}^{D}=90, m_{b}$ <br> $\left\|r_{t b}\right\|>1$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $(0.3,0.34)$ | $2.0010^{-2}$ | $1.9010^{-2}$ | $1.5010^{-2}$ | $0.5110^{-2}$ | regions |
|  | $0.6010^{-2}$ | $0.5710^{-2}$ | $0.5310^{-2}$ | $0.2510^{-2}$ | II |
| $(-0.07,0.34)$ | $0.9710^{-2}$ | $1.0010^{-2}$ | $0.7710^{-2}$ | $0.3410^{-2}$ | I |
|  | $0.3210^{-2}$ | $0.2910^{-2}$ | $0.2710^{-2}$ | $0.1410^{-2}$ | II |

Table 5: The average asymmetry $<A_{C P}>$ for regions I ( $\left.1 G e V \leq \sqrt{q}^{2} \leq m_{J / \psi}-20 \mathrm{MeV}\right)$ and II $\left(m_{J / \psi}+20 \mathrm{MeV} \leq \sqrt{q}^{2} \leq m_{\psi^{\prime}}-20 \mathrm{MeV}\right)$.

In conclusion, we analyse the selected model III parameters ( $\left.\bar{\xi}_{N, b b}^{D}, \bar{\xi}_{N, t t}^{U}\right)$ dependencies of the differential $B r, A_{C P}$ and $A_{F B}$ of the decay $\bar{B} \rightarrow \rho e^{+} e^{-}$. We obtain that the strong enhancement of the differential $B r$ is possible in the framework of the model III and observe that $A_{C P}$ and $A_{F B}$ are very sensitive to the model III parameters $\left(\bar{\xi}_{N, b b}^{D}, \bar{\xi}_{N, t t}^{U}\right)$.

## 5 Conclusion

We study the exclusive processes $\bar{B} \rightarrow \pi e^{+} e^{-}$and $\bar{B} \rightarrow \rho e^{+} e^{-}$which are induced by the inclusive $b \rightarrow d e^{+} e^{-}$decay. In such type of decays, it is informative to analyse the CP violating effects, in addition to the quantities like $B r, A_{F B}$. The origin of the CP violation in the SM is the parameter $\lambda_{u}=\frac{V_{u b} V_{u d}^{*}}{V_{t b} V_{t d}^{*}}$. In the model III, the couplings $\xi_{i j}^{U}$ and $\xi_{k l}^{D} \square$ can also create the CP violation in case they are not real. However, in our work disregard this possibility not to enlarge the number of free parameters and we assume that the only CP violating effect comes from the CKM matrix elements, similar to the SM.

Now, we would like to summarize the main results of our analysis:

- The $B r$ of the exclusive decays $\bar{B} \rightarrow \pi e^{+} e^{-}$and $\bar{B} \rightarrow \rho e^{+} e^{-}$are sensitive to the model III parameters. In the region $r_{t b} \gg 1$, a strong enhancement of the $B r$ is observed with increasing $\bar{\xi}_{b b}^{D}$ in both decays eqs. As an example, $B r_{n o L D}\left(\right.$ Model III) $\sim 3 B r_{n o L D}(S M)$ for $\bar{\xi}_{b b}^{D}=90 m_{b}$ (25,26) and (38,39). Therefore their experimental investigations are a crucial test for the physics beyond the SM.
- We calculated $<A_{C P}>$ for two different invariant mass region (see Table (4) and (5)). We observe that $<A_{C P}>$ decreases with increasing $\bar{\xi}_{b b}^{D}$ for $r_{t b} \gg 1$ in both regions. For

[^1]$r_{t b} \gg 1$ and $\bar{\xi}_{b b}^{D}=90 m_{b},<A_{C P}>$ is rather smaller compared the one in the SM, for both regions (region I and II), i.e. $<A_{C P}>_{\text {model III }} \sim \% 30<A_{C P}>_{S M}$.

- We calculated $A_{F B}$ for the $\bar{B} \rightarrow \rho e^{+} e^{-}$decay and observe that it does not change sign in the model III if the LD effects are excluded. This shows that the determination of the sign of $A_{F B}$ in the region $0 \leq s \leq 0.25$ will be informative to see the effects of the model III, if it exists.

As a conclusion, the experimental investigation of the quantities we present here, will be an efficient tool to search for new physics beyond the SM.

## Appendix

## A The essential points of the model III.

The Yukawa interaction in the general case of 2HDM (Model III) is

$$
\begin{equation*}
\mathcal{L}_{Y}=\eta_{i j}^{U} \bar{Q}_{i L} \tilde{\phi}_{1} U_{j R}+\eta_{i j}^{D} \bar{Q}_{i L} \phi_{1} D_{j R}+\xi_{i j}^{U} \bar{Q}_{i L} \tilde{\phi}_{2} U_{j R}+\xi_{i j}^{D} \bar{Q}_{i L} \phi_{2} D_{j R}+h . c . \tag{40}
\end{equation*}
$$

where $L$ and $R$ denote chiral projections $L(R)=1 / 2\left(1 \mp \gamma_{5}\right), \phi_{i}$ for $i=1,2$, are the two scalar doublets, $\eta_{i j}^{U, D}$ and $\xi_{i j}^{U, D}$ are the matrices of the Yukawa couplings. The Flavor Changing (FC) part of the interaction can be written as

$$
\begin{equation*}
\mathcal{L}_{Y, F C}=\xi_{i j}^{U} \bar{Q}_{i L} \tilde{\phi}_{2} U_{j R}+\xi_{i j}^{D} \bar{Q}_{i L} \phi_{2} D_{j R}+\text { h.c. } \tag{41}
\end{equation*}
$$

with the choice of $\phi_{1}$ and $\phi_{2}$

$$
\begin{equation*}
\phi_{1}=\frac{1}{\sqrt{2}}\left[\binom{0}{v+H^{0}}+\binom{\sqrt{2} \chi^{+}}{i \chi^{0}}\right] ; \phi_{2}=\frac{1}{\sqrt{2}}\binom{\sqrt{2} H^{+}}{H_{1}+i H_{2}} . \tag{42}
\end{equation*}
$$

Here the vacuum expectation values are,

$$
\begin{equation*}
<\phi_{1}>=\frac{1}{\sqrt{2}}\binom{0}{v} ;<\phi_{2}>=0 \tag{43}
\end{equation*}
$$

and the couplings $\xi^{U, D}$ for the FC charged interactions are

$$
\begin{align*}
& \xi_{c h}^{U}=\xi_{\text {neutral }} V_{C K M} \\
& \xi_{c h}^{D}=V_{C K M} \xi_{\text {neutral }} \tag{44}
\end{align*}
$$

where $\xi_{\text {neutral }}^{U, D} \|$ is defined by the expression

$$
\begin{equation*}
\xi_{N}^{U, D}=\left(V_{L}^{U, D}\right)^{-1} \xi^{U, D} V_{R}^{U, D} . \tag{45}
\end{equation*}
$$

Note that the charged couplings appear as a linear combinations of neutral couplings multiplied by $V_{C K M}$ matrix elements (more details see (37]).

[^2]
## B The necessary functions appear in the calculation of the Wilson coefficients

The functions $B(x), C(x), D(x), F_{1(2)}(y), G_{1(2)}(y), H_{1}(y)$ and $L_{1}(y)$ are given as

$$
\begin{align*}
B(x) & =\frac{1}{4}\left[\frac{-x}{x-1}+\frac{x}{(x-1)^{2}} \ln x\right] \\
C(x) & =\frac{x}{4}\left[\frac{x / 2-3}{x-1}+\frac{3 x / 2+1}{(x-1)^{2}} \ln x\right] \\
D(x) & =\frac{-19 x^{3} / 36+25 x^{2} / 36}{(x-1)^{3}}+\frac{-x^{4} / 6+5 x^{3} / 3-3 x^{2}+16 x / 9-4 / 9}{(x-1)^{4}} \ln x, \\
F_{1}(y) & =\frac{y\left(7-5 y-8 y^{2}\right)}{72(y-1)^{3}}+\frac{y^{2}(3 y-2)}{12(y-1)^{4}} \ln y, \\
F_{2}(y) & =\frac{y(5 y-3)}{12(y-1)^{2}}+\frac{y(-3 y+2)}{6(y-1)^{3}} \ln y, \\
G_{1}(y) & =\frac{y\left(-y^{2}+5 y+2\right)}{24(y-1)^{3}}+\frac{-y^{2}}{4(y-1)^{4}} \ln y, \\
G_{2}(y) & =\frac{y(y-3)}{4(y-1)^{2}}+\frac{y}{2(y-1)^{3}} \ln y, \\
H_{1}(y) & =\frac{1-4 \sin ^{2} \theta_{W}}{\sin ^{2} \theta_{W}} \frac{x y}{8}\left[\frac{1}{y-1}-\frac{1}{(y-1)^{2}} \ln y\right] \\
& -y\left[\frac{47 y^{2}-79 y+38}{108(y-1)^{3}}-\frac{3 y^{3}-6 y+4}{18(y-1)^{4}} \ln y\right] \\
L_{1}(y) & =\frac{1}{\sin ^{2} \theta_{W}} \frac{x y}{8}\left[-\frac{1}{y-1}+\frac{1}{(y-1)^{2}} \ln y\right] . \tag{46}
\end{align*}
$$

## C The functions which appear in the Wilson coefficients $C_{9}^{e f f}$ and $C_{9}^{l e f f}$

The function which represents the one gluon correction to the matrix element $O_{9}$ is [29]

$$
\begin{equation*}
\tilde{\eta}(\hat{s})=1+\frac{\alpha_{s}(\mu)}{\pi} \omega(\hat{s}), \tag{47}
\end{equation*}
$$

and

$$
\begin{align*}
\omega(\hat{s})= & -\frac{2}{9} \pi^{2}-\frac{4}{3} \operatorname{Li}_{2}(\hat{s})-\frac{2}{3} \ln \hat{s} \ln (1-\hat{s})-\frac{5+4 \hat{s}}{3(1+2 \hat{s})} \ln (1-\hat{s})- \\
& \frac{2 \hat{s}(1+\hat{s})(1-2 \hat{s})}{3(1-\hat{s})^{2}(1+2 \hat{s})} \ln \hat{s}+\frac{5+9 \hat{s}-6 \hat{s}^{2}}{6(1-\hat{s})(1+2 \hat{s})}, \tag{48}
\end{align*}
$$

$h(z, \hat{s})$ arises from the one loop contributions of the four quark operators $O_{1}, \ldots, O_{6}\left(O_{1}^{\prime}, \ldots, O_{6}^{\prime}\right)$

$$
\begin{align*}
h(z, \hat{s})= & -\frac{8}{9} \ln \frac{m_{b}}{\mu}-\frac{8}{9} \ln z+\frac{8}{27}+\frac{4}{9} x  \tag{49}\\
& -\frac{2}{9}(2+x)|1-x|^{1 / 2} \begin{cases}\left(\ln \left|\frac{\sqrt{1-x}+1}{\sqrt{1-x}-1}\right|-i \pi\right), & \text { for } x \equiv \frac{4 z^{2}}{\hat{s}}<1 \\
2 \arctan \frac{1}{\sqrt{x-1}}, & \text { for } x \equiv \frac{4 z^{2}}{\hat{s}}>1,\end{cases} \\
h(0, \hat{s})= & \frac{8}{27}-\frac{8}{9} \ln \frac{m_{b}}{\mu}-\frac{4}{9} \ln \hat{s}+\frac{4}{9} i \pi, \tag{50}
\end{align*}
$$

where $z=\frac{m_{c}}{m_{b}}$ and $\hat{s}=\frac{q^{2}}{m_{b}^{2}}$.

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Figure 1: Differential $B r$ as a function of $q^{2}$ for fixed $\bar{\xi}_{N, b b}^{D}=40 m_{b}$ in the region $\left|r_{t b}\right| \ll 1$, at the scale $\mu=m_{b}$ for the process $\bar{B} \rightarrow \pi e^{+} e^{-}$. Here solid line and corresponds to the model III with LD effects, dashed line to the model III withouth LD effects and dotted dashed line to the SM withouth LD effects.


Figure 2: The same as Fig 1, but at the region $r_{t b} \gg 1$.


Figure 3: The same as Fig 2, but for fixed $\bar{\xi}_{N, b b}^{D}=90 m_{b}$ value.


Figure 4: $A_{C P}$ as a function of $q^{2}$ for fixed $\bar{\xi}_{N, b b}^{D}=40 m_{b}$ in the region $\left|r_{t b}\right| \ll 1$, at the scale $\mu=m_{b}$, for the process $\bar{B} \rightarrow \pi e^{+} e^{-}$. Here solid line corresponds to the model III with LD effects, dashed line to the SM withouth LD effects and dotted dashed line to the SM with LD effects.


Figure 5: The same as Fig. ©, but at the region $r_{t b} \gg 1$.


Figure 6: The same as Fig 5 , but for fixed $\bar{\xi}_{N, b b}^{D}=90 m_{b}$ value. .


Figure 7: Differential $B r$ as a function of $q^{2}$ for fixed $\bar{\xi}_{N, b b}^{D}=40 m_{b}$ in the region $\left|r_{t b}\right| \ll 1$, at the scale $\mu=m_{b}$ for the process $\bar{B} \rightarrow \rho e^{+} e^{-}$. Here solid line and corresponds to the model III with LD effects, dashed line to the model III withouth LD effects and dotted dashed line to the SM withouth LD effects.


Figure 8: The same as Fig. 7, but at the region $r_{t b} \gg 1$.


Figure 9: The same as Fig. 8, but for fixed $\bar{\xi}_{N, b b}^{D}=90 m_{b}$ value.


Figure 10: $A_{C P}$ as a function of $q^{2}$ for fixed $\bar{\xi}_{N, b b}^{D}=40 m_{b}$ in the region $\left|r_{t b}\right| \ll 1$, at the scale $\mu=m_{b}$, for the process $\bar{B} \rightarrow \rho e^{+} e^{-}$. Here solid line corresponds to the model III with LD effects, dashed line to the SM withouth LD effects and dotted dashed line to the SM with LD effects.


Figure 11: The same as Fig 10, but at the region $r_{t b} \gg 1$.


Figure 12: The same as Fig 11, but for fixed $\bar{\xi}_{N, b b}^{D}=90 m_{b}$ value. .


Figure 13: $A_{F B}$ as a function of $q^{2}$ for fixed $\bar{\xi}_{N, b b}^{D}=40 m_{b}$ in the region $\left|r_{t b}\right| \ll 1$, at the scale $\mu=m_{b}$ for the process $\bar{B} \rightarrow \rho e^{+} e^{-}$. Here solid line and corresponds to the model III with LD effects, dashed line to the SM withouth LD effects and dotted dashed line to the SM with LD effects.


Figure 14: The same as Fig. 13, but at the region $r_{t b} \gg 1$.


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[^1]:    ${ }^{1}$ Here $i, j$ and $k, l$ denote up and down quarks respectively.

[^2]:    ${ }^{2}$ In all next discussion we denote $\xi_{\text {neutral }}^{U, D}$ as $\xi_{N}^{U, D}$.

