Lepton flavor violating $l_i \to l_j \gamma\gamma$ decays induced by scalar unparticle

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Abstract

We study the radiative lepton flavor violating $l_i \to l_j \gamma\gamma$ decays in the case that the lepton flavor violation is induced by the scalar unparticle mediation. We restrict the scaling dimension d_u and the scalar unparticle-photon-photon coupling by using the experimental upper limit of the branching ratio of the decay $\mu \to e\gamma\gamma$. Furthermore, we predict the BRs of the other radiative decays by using the restrictions we get. We observe that the measurements of upper limits of BRs of these decays ensure considerable information for testing the possible signals coming from unparticle physics.

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The radiative decays with two photon output are interesting to analyze since they are rich due to two different polarizations of photons, namely the parallel and the perpendicular ones. The measurement of the photon spin polarization provides comprehensive information about the free parameters existing in the model used. In the present work we study the radiative lepton decays with two photon output, i.e, $l_i \rightarrow l_j \gamma\gamma$, $i \neq j$. These processes are driven by a lepton flavor violating (LFV) mechanism and, in the framework of the SM, the lepton mixing matrix which arises with non-zero neutrino masses is responsible for this violation. In this case these decays exist at least in the loop level and their branching ratios (BRs) are highly suppressed. Here, for the lepton flavor violation, we consider the another mechanism based on the unparticle physics.

The unparticle stuff is proposed by Georgi [1, 2] and the so called unparticle, which looks like a number of d_u massless invisible particles, has non-integer scaling dimension d_u. The unparticle stuff is the low energy manifestation of a hypothetical non-trivial scale invariant ultraviolet sector, having a non-trivial infrared fixed point. The interactions of unparticles with the SM fields are driven by the effective lagrangian in the low energy level and the corresponding Lagrangian reads

$$\mathcal{L}_{eff} \sim \frac{\eta}{\Lambda_U^{d_u+d_{SM}-n}} \, O_{SM} \, O_U \, , \tag{1}$$

where O_U is the unparticle operator, the parameter η is related to the energy scale of ultraviolet sector, the low energy one and the matching coefficient [1, 2, 3] and n is the space-time dimension.

The search for unparticle(s) ensures a valuable information about the expected ultraviolet sector and the scale invariance. The missing energies at various processes which can be measured at LHC or e^+e^- colliders, the dipole moments of fundamental particles are among the possible candidates for searching the effects of unparticle(s). In the literature there is an extensive phenomenological work done on unparticles [2]-[7]. These studies are about the possible effects of unparticle stuff on the missing energy of many processes, the anomalous magnetic moments, the electric dipole moments, $D^0 - \bar{D}^0$ and $B^0 - \bar{B}^0$ mixing, lepton flavor violating interactions, direct CP violation in particle physics, cosmology and astrophysics.

In the present work, we consider that the LF violation is switched on with the scalar unparticle (U)-lepton-lepton vertex. Furthermore, we expect that U-photon (γ)-photon (γ) interaction exists and, finally, the radiative $l_i \rightarrow l_j \gamma\gamma$, $i \neq j$ decays appear in the tree level, with the scalar unparticle mediation. In our calculations, we respect the experimental upper limit of the BR of the $\mu \rightarrow e \gamma\gamma$ decay, BR$< 7.2\, 10^{-11}\, 90\%\, CL$ [8] and try to restrict the scaling dimension d_u and the $U-\gamma-\gamma$ coupling. In addition to this, we predict the BRs of the other radiative decays by using the restrictions we get.

Now, we start by choosing the appropriate operators with the lowest possible dimension since they have the most powerful effect in the low energy effective theory (see for example [4]). The effective interaction lagrangian driving the LFV decays reads

$$\mathcal{L}_1 = \frac{1}{\Lambda_U^{du-1}} \Big(\lambda_{ij}^S \, \bar{l}_i \, l_j + \lambda_{ij}^P \, \bar{l}_i \, i\gamma_5 \, l_j \Big) \, O_U \, , \tag{2}$$

where l is the lepton field, O_U is the scalar unparticle (U) operator and λ_{ij}^S (λ_{ij}^P) is the scalar (pseudoscalar) coupling. On the other hand, the effective Lagrangian which is responsible for two photon radiation is

$$\mathcal{L}_2 = \frac{1}{\Lambda_U^{du}} \Big(\lambda_0 \, F_{\mu\nu} \, F^{\mu\nu} + \lambda'_0 \, \tilde{F}_{\mu\nu} \, F^{\mu\nu} \Big) \, O_U \, , \tag{3}$$

where $F_{\mu\nu}$ is the electromagnetic field tensor and $\tilde{F}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} \, F^{\alpha\beta}$. In radiative two photon decays the outgoing photons are in one of the possible states given by $F^{\mu\nu} \, F_{\mu\nu}$ and $\tilde{F}_{\mu\nu} \, F^{\mu\nu}$ and these states corresponds to the parallel $(\epsilon_1 . \epsilon_2)$ and perpendicular $(\epsilon_1 \times \epsilon_2)$ spin polarizations of photons which are regulated by the couplings λ_0 and λ'_0 in the present case. In our calculations we consider a parameter α such that $\lambda'_0 = \alpha \, \lambda_0$.

The tree level $l_i \to l_j \gamma\gamma$ decay (see Fig. 1) is carried by connecting the LFV vertex[1] and the $U-\gamma-\gamma$ vertex[2] by the scalar unparticle propagator which is obtained by using the scale invariance. The two point function of the unparticle reads [2, 5]

$$\int d^4x \, e^{ipx} \, < 0 | T \Big(O_U(x) \, O_U(0) \Big) 0 >= i \frac{A_{d_u}}{2\,\pi} \int_0^\infty ds \, \frac{s^{d_u-2}}{p^2 - s + i\epsilon} = i \, \frac{A_{d_u}}{2 \, sin \, (d_u \pi)} \, (-p^2 - i\epsilon)^{d_u-2} \, , \tag{4}$$

with the factor A_{d_u}

$$A_{d_u} = \frac{16 \, \pi^{5/2}}{(2\,\pi)^{2\,d_u}} \, \frac{\Gamma(d_u + \frac{1}{2})}{\Gamma(d_u - 1) \, \Gamma(2\,d_u)} \, . \tag{5}$$

The function $\frac{1}{(-p^2 - i\epsilon)^{2-d_u}}$ in eq. (4) becomes

$$\frac{1}{(-p^2 - i\epsilon)^{2-d_u}} \to \frac{e^{-i\,d_u\,\pi}}{(p^2)^{2-d_u}} \, , \tag{6}$$

for $p^2 > 0$ and a non-trivial phase appears as a result of non-integral scaling dimension.

[1]The vertex factor: $\frac{i}{\Lambda_U^{d_u-1}} (\lambda^S + i \, \gamma_5 \, \lambda^P)$.

[2]The vertex factor: $\frac{4\,i}{\Lambda_U^{d_u}} \left(\lambda_0 \, (k_{1\nu} \, k_{2\mu} - k_1 . k_2 \, g_{\mu\,\nu}) + \lambda'_0 \, \epsilon_{\alpha\beta\mu\nu} \, k_1^\alpha \, k_2^\beta \right)$ where $k_{1(2)}$ is the four momentum of photon with polarization vector $\epsilon_{1\,\mu\,(2\,\nu)}$.

Now, we present the decay amplitude for the radiative decay $l_i \to l_j \gamma\gamma$:

$$M(l_i \to l_j \gamma\gamma) = \epsilon_1^\mu(k_1)\, \epsilon_2^\nu(k_2)\, \bar{l}_j(p')\, T_{\mu\nu}\, l_i(p)\,, \tag{7}$$

where the structure $T_{\mu\nu}$ reads

$$T_{\mu\nu} = \frac{-A_{d_u}\,(-q^2)^{d_u-2}}{2\,sin\,(d_u\,\pi)\,\lambda_U^{2d_u-1}} \left(\lambda_{ij}^S + \gamma_5\,\lambda_{ij}^P\right) A_{\mu\nu}\,, \tag{8}$$

with

$$A_{\mu\nu} = 4\left((k_{1\nu}\,k_{2\mu} - k_1.k_2\,g_{\mu\nu})\,\lambda_0 + \epsilon_{\alpha\beta\mu\nu}\,k_1^\alpha\,k_2^\beta\,\lambda_0'\right). \tag{9}$$

Finally, the partial decay with $d\Gamma$ for $l_i \to l_j\,\gamma\gamma$ decay can be obtained by using the matrix element square $|M|^2$ as

$$d\Gamma = \frac{1}{128\,\pi^3\,m_{l_i}}\,|M|^2\,dE_1\,dE_j\,, \tag{10}$$

where $E_1(E_j)$ is the energy of the photon with polarization four vector $\epsilon_1^\mu(k_1)$ (the outgoing lepton). In our numerical calculations we analyze the BRs of the decays under consideration by using the total decay widths of incoming leptons [8] (see Table I).

Discussion

This section is devoted to analysis of the radiative LFV $l_i \to l_j\,\gamma\gamma$ decays. Here, we consider that the LF violation is switched on with the U-lepton-lepton coupling, in the framework of the effective theory. On the other hand, the possible $U-\gamma-\gamma$ vertex results in that the decays under consideration exist even in the tree level, with the unparticle mediation. Therefore, the physical quantities like the BRs of these decays can be informative in the determination of the free parameters, the scaling dimension of the unparticle, the couplings and the energy scale in the scenario studied. Here, we choose the scaling dimension d_u in the range[3] $1 < d_u < 2$. For off diagonal U-lepton-lepton couplings λ_{ij}^S and λ_{ij}^P[4] we take the numerical values of the order of $10^{-3} - 10^{-1}$ and we predict the $U-\gamma-\gamma$ coupling λ_0 by restricting the BR($\mu \to e\gamma\gamma$) to its current experimental upper limit, by taking the energy scale at the order of $\Lambda_U = 10\,(TeV)$. This analysis restricts the pair of parameters, the scaling dimension d_u and the coupling λ_0. Furthermore, we estimate the BRs of the LFV radiative decays $\tau \to e\gamma\gamma$ and $\tau \to \mu\gamma\gamma$, by using the restriction obtained for the pair d_u and λ_0. Notice that throughout our calculations we use the input values given in Table (1).

[3] $d_u > 1$ is chosen since one is free from the non-integrable singularity problem in the decay rate [2]. Furthermore, the momentum integrals converge for $d_u < 2$ [6]

[4] In the following we drop the indices i and j.

Parameter	Value
m_e	0.0005 (GeV)
m_μ	0.106 (GeV)
m_τ	1.780 (GeV)
Γ_μ^{Tot}	2.99×10^{-19} (GeV)
Γ_τ^{Tot}	2.26×10^{-10} (GeV)

Table 1: The values of the input parameters used in the numerical calculations.

In Fig.2, we present the magnitude of the coupling λ_0 with respect to d_u for the fixed value BR $(\mu \to e\gamma\,\gamma) = 7.2 \times 10^{-11}$ and the energy scale $\Lambda_U = 10\,(TeV)$. Here the solid-dashed-small dashed (upper dotted-intermediate dotted-lower dotted) line represents λ_0 for $\lambda^S = \lambda^P = 0.001 - 0.01 - 0.1$ and $\alpha = 1$ $(\alpha = 0)$. The coupling λ_0, which is sufficient to get the experimental upper limit of the BR, becomes stronger for the larger values of d_u. This is due to the fact that the BR is strongly sensitive to the scaling dimension d_u and it is suppressed with its increasing values. For $\lambda^S = \lambda^P = 0.001$ the coupling λ_0 should be in the range $0.001 - 1.0$ for the scaling dimension d_u, $1.1 < d_u < 1.4$. In the case of strong couplings $\lambda^S = \lambda^P = 0.01\,(0.1)$ this range is obtained for larger scaling dimension, $d_u < 1.5\,(1.6)$. Furthermore, it is observed that the amount of coupling λ_0 is weakly sensitive to the parameter α for its values that is less than one. Fig.3 represents the magnitude of the parameter α^2 with respect to d_u for the fixed value BR $(\mu \to e\gamma\,\gamma) = 7.2 \times 10^{-11}$ and the energy scale $\Lambda_U = 10\,(TeV)$. Here the solid (dashed, small dashed) line represents α^2 for $\lambda^S = \lambda^P = 0.001\,(0.01, 0.1)$ and $\lambda_0 = 0.01$. For the selected numerical value of the coupling λ_0, $\lambda_0 = 0.01$, the parameter α should be greater than one for $\lambda^S = \lambda^P = 0.001$ and 0.01. This is the case that the perpendicular spin polarization exceeds the parallel spin polarization for two photon system. Furthermore, the scaling parameter d_u should be > 1.18 and > 1.27 in order to get a solution. For $\lambda^S = \lambda^P = 0.1$, α can be less than one where the parallel spin polarization exceeds the perpendicular spin polarization for two photon system. This is the case that d_u should be > 1.34. These observations are interesting since the more accurate forthcoming measurement of the BR of the decay under consideration would ensure valuable information about the possible $U - \gamma - \gamma$ coupling and the scaling dimension d_u. In addition to this, the precise determination of the photon polarization in the experiments would be informative in the determination of these parameters.

Fig.4 is devoted to the BRs of the decays $\tau \to e\gamma\,\gamma$ and $\tau \to \mu\gamma\,\gamma$ with respect to d_u for $\lambda^S = \lambda^P = 0.01$, $\alpha = 1$ and $\Lambda_U = 10\,(TeV)$. Here the solid (dashed) line represents the BR for the pair d_u and λ_0 which is obtained by using the restriction BR $(\mu \to e\gamma\,\gamma) = 7.2 \times 10^{-11}$

(10^{-12}). The BRs for both decays almost coincides and they are of the order of 10^{-13} (10^{-15}) for the pair $\lambda_0 \sim 1.0$-$d_u \sim 1.5$. These numerical values are extremely small, however, there is a chance to increase them by considering that the off diagonal couplings of scalar U-lepton-lepton couplings are not flavor blind and sensitive to the lepton flavor.

For completeness, in Fig.5 we plot the BRs of $\tau \to e\gamma\,\gamma$ and $\tau \to \mu\gamma\,\gamma$ decays with respect to d_u for $\lambda^S = \lambda^P = 0.01$, $\alpha = 1$ in the case that there is no restriction for the pair d_u and λ_0. Here the solid (dashed, small dashed) line represents the BRs for $\lambda_0 = 0.01$ and $\Lambda_U = 1.0\,(5.0,\,10)\,(TeV)$. It is observed that the BRs of both decays are almost coincides and they are in the range $10^{-12} - 10^{-6}$ ($10^{-15} - 10^{-8}$, $10^{-16} - 10^{-9}$) for the energy scale $\Lambda_U = 1.0\,(5.0,\,10)\,(TeV)$ and the interval $1.1 < d_u < 1.5$.

As a summary, the radiative LFV decays $l_i \to l_j\gamma\gamma$ can exist in the tree level with the help of the unparticle idea. The measurements of upper limits of BRs of these decays (the more accurate one for the $\mu \to e\gamma\,\gamma$ decay) would be instructive for testing the possible signals coming from the new physics which drives the flavor violation and they would ensure considerable information on the restriction of free parameters.

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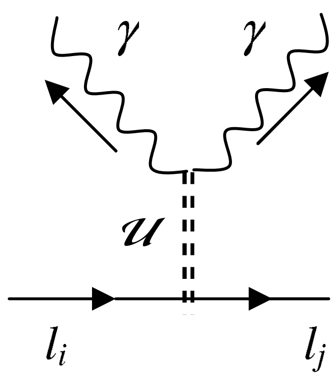

Figure 1: Tree level diagram contribute to $l_i \rightarrow l_j \gamma\gamma$ decay with scalar unparticle mediator. Solid line represents the lepton field, wavy line the photon field, double dashed line the scalar unparticle field.

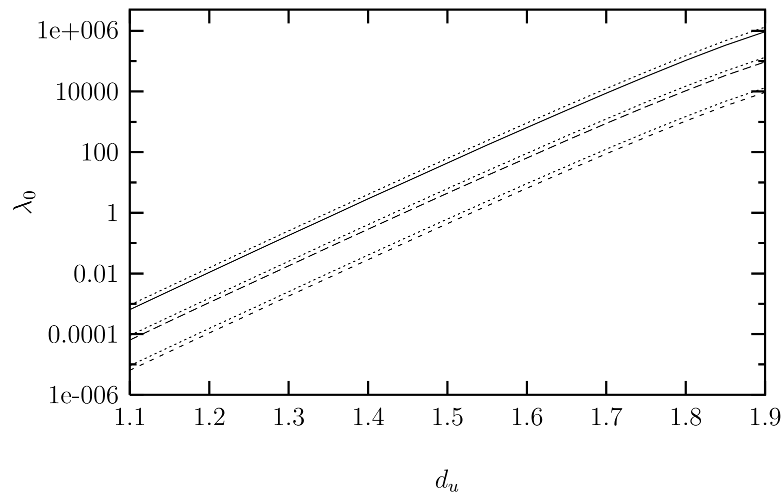

Figure 2: λ_0 with respect to d_u for the fixed value BR $(\mu \to e\gamma\,\gamma) = 7.2 \times 10^{-11}$ and the energy scale $\Lambda_U = 10\,(TeV)$. Here the solid-dashed-small dashed (upper dotted-intermediate dotted-lower dotted) line represents λ_0 for $\lambda^S = \lambda^P = 0.001 - 0.01 - 0.1$ and $\alpha = 1$ ($\alpha = 0$).

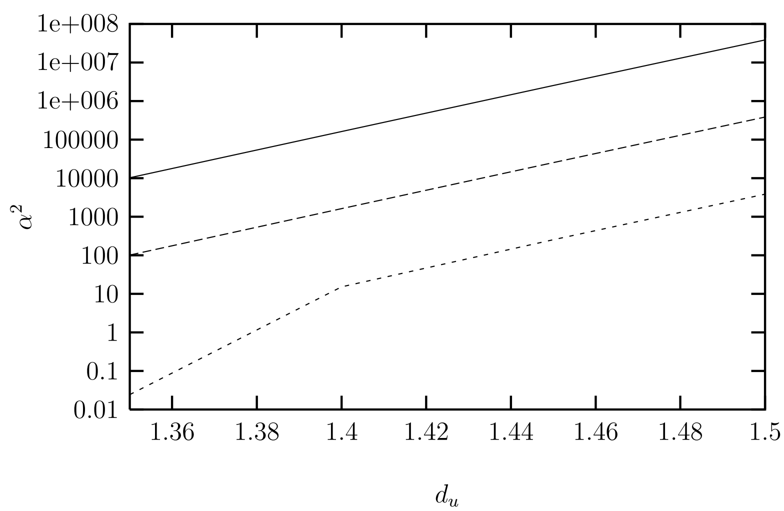

Figure 3: α^2 with respect to d_u for the fixed value BR $(\mu \to e\gamma\,\gamma) = 7.2 \times 10^{-11}$ and the energy scale $\Lambda_U = 10\,(TeV)$. Here the solid (dashed, small dashed) line represents α^2 for $\lambda^S = \lambda^P = 0.001\,(0.01, 0.1)$ and $\lambda_0 = 0.01$.

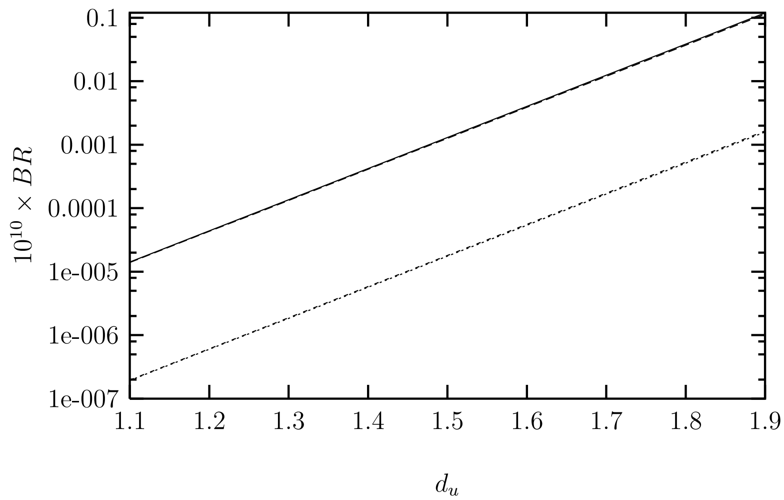

Figure 4: BR($\tau \to e(\mu)\gamma\,\gamma$) with respect to d_u for $\lambda^S = \lambda^P = 0.01$, $\alpha = 1$ and $\Lambda_U = 10\,(TeV)$. Here the solid (dashed) line represents the BR for the pair d_u and λ_0 which is obtained by using the restriction BR ($\mu \to e\gamma\,\gamma$) $= 7.2 \times 10^{-11}$ (10^{-12}).

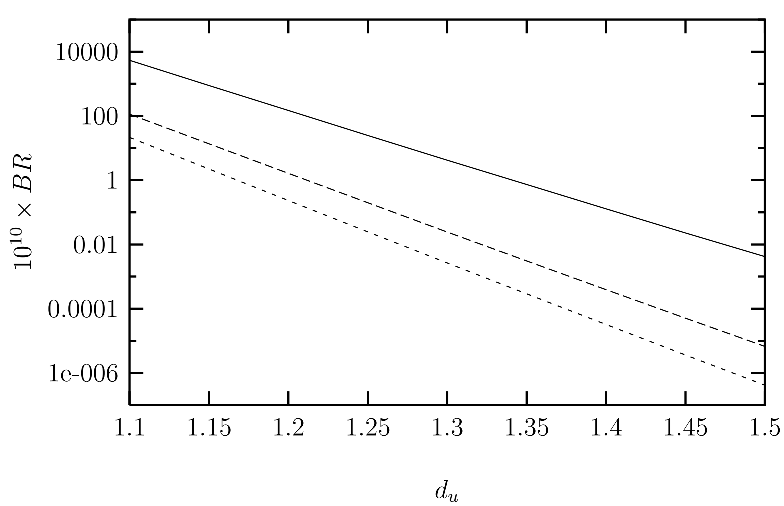

Figure 5: The BR ($\tau \to e(\mu)\gamma\,\gamma$) with respect to d_u for $\lambda^S = \lambda^P = 0.01$, $\alpha = 1$. Here the solid (dashed, small dashed) line represents the BR for $\lambda_0 = 0.01$ and $\Lambda_U = 1.0\,(5.0,\,10)\,(TeV)$.