DESY 97-076
hep-ph/9704385
April 1997

Leading logarithmic QCD corrections to the $B_s \to \gamma\gamma$ decay rate including long-distance effects through $B_s \to \phi\gamma \to \gamma\gamma$

G. Hiller*,
Deutsches Elektronen-Synchrotron DESY, Hamburg

E. O. Iltan †‡
Physics Department, Middle East Technical University
Ankara, Turkey

Abstract

We present leading logarithmic QCD corrections to the decay $B_s \to \gamma\gamma$ in the Standard Model. Further, the form factor $F_1(0)$ of $B_s \to \phi\gamma$ is calculated in the framework of QCD sum rules and found to be in agreement with the result existing in the literature. Using Vector Meson Dominance model, the amplitude for $B_s \to \phi\gamma \to \gamma\gamma$ is calculated as an estimate of the O_7-type contributions to the long-distance effects in the $B_s \to \gamma\gamma$ decay. The resulting branching ratio $\mathcal{B}(B_s \to \gamma\gamma)_{SD+LD_{O_7}}$ is analysed in view of its strong dependence on the non-perturbative parameter $\bar{\Lambda}_s$, describing bound state effects, and the renormalization scale μ.

*E-mail address: ghiller@x4u2.desy.de
†E-mail address: eiltan@heraklit.physics.metu.edu.tr
‡Supported by TUBITAK

1 Introduction

Rare B decays induced by flavor changing neutral currents (FCNC) are known to provide information about the Standard Model (SM) at quantum level and quantitative information on the SM parameters, such as the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements. The CLEO observation [1] of the radiative decay mode $B \to X_s\gamma$ has been analysed in the SM and the rate agrees with the SM-based theoretical calculations [2]. Another example is $B_s \to \phi\gamma$, which is CKM allowed due to the dominant CKM matrix element dependence of the decay rate. The calculational procedure of such decay rates is to use an effective Hamiltonian obtained by integrating out the top quark and the $W^{\pm}$ bosons [3]

$$\mathcal{H}_{eff} = -4\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{8} C_i(\mu) O_i(\mu) \ . \tag{1}$$

Here O_i are suitable operators and C_i are Wilson coefficients renormalized at the scale μ. The coefficients can be calculated perturbatively. Hadronic matrix elements $< V|O_i|B >$ can be calculated using some non-perturbative methods like QCD sum rules, which is one of the powerful methods to calculate matrix elements in a model independent way.

Among rare decays, $B_s \to \gamma\gamma$ is a potential candidate to test the SM and search for new physics. The final state contains CP-odd and CP-even states, allowing us to study CP violating effects. Measurement of these odd and even states is a powerful test of the underlying theory, in particular SM. In the literature, $B_s \to \gamma\gamma$ decay has been investigated earlier in the lowest order [4–6] and the branching ratio is found to be $4.5 \cdot 10^{-7}$ in the SM context for $m_s = 0.5$ GeV and other parameters given in Table 1, using the constituent quark model.

In the present work we give the leading logarithmic QCD-improved rates for $B_s \to \gamma\gamma$. This can be achieved through a matching of the full theory with the effective theory at a scale $\mu = m_W$, using the effective Hamiltonian in eq. (1), and performing an evolution of the Wilson coefficients from m_W down to $\mu \sim \mathcal{O}(m_b)$, thus resumming all large logarithms of the form $\alpha_s^n(m_b) log^m(\frac{m_b}{m_W})$, where $m \le n$ $(n = 0, 1, 2, \ldots)$. In the leading logarithmic approximation, which we use here, $m = n$. The effective Hamiltonian in eq. (1) is identical for $b \to s\gamma$ and for $b \to s\gamma\gamma$ to this order of $\frac{1}{m_W^2}$. Since there exists after applying the equations of motion no gauge-invariant FCNC-2-photon operator with field dimension ≤ 6, the set of operators (eq. (6)) is a basis for both decays [7]. The bound state effects of the B_s meson are modeled through an Heavy Quark Effective Theory (HQET) inspired approach following [8]. We estimate further the additional contribution in the decay $B_s \to \gamma\gamma$ through $B_s \to \phi\gamma$ followed by $\phi \to \gamma$ using Vector Meson Dominance (VMD) [9]. In the language of the operator basis in eq. (1), this contribution involves the operator O_7, (see eq. (6) below). The decay $B_s \to \phi\gamma$ was studied in the literature in the framework of Light-cone QCD sum rules [10]. We have repeated the calculation using the ordinary QCD sum rules including the contribution from the gluon condensate. The CP-odd and CP-even amplitudes in $B_s \to \gamma\gamma$ are then estimated by considering the $\phi \to \gamma$ process using a ϕ-photon conversion factor supplied by the VMD model. In this part an extrapolation from $p'^2 = m_\phi^2$ (needed for $B_s \to \phi\gamma$) to $p'^2 = 0$ (required for $B_s \to \gamma\gamma$) is necessary. We assume, that the form factor is dominated by a single pole, which is a good approximation for light mesons. The decay rate for $B_s \to \gamma\gamma$ depends sensitively on the model parameters $(m_b, \bar\Lambda_s)$ and μ. For typical values $(m_b, \bar\Lambda_s) = (5\,\mathrm{GeV}, 370\,\mathrm{MeV})$ and μ=5 GeV, we get (including long-distance effects through O_7) the branching ratio $\mathcal{B}(B_s \to \gamma\gamma))_{SD+LD_{O_7}} = 1.18 \cdot 10^{-6}$, which is a factor 1.9 larger compared to the lowest order estimate for the same values of the parameters. However, varying $(m_b, \bar\Lambda_s)$ and μ in the allowed range results in significant variation on the branching ratio, yielding $0.38 \cdot 10^{-6} \leq \mathcal{B}(B_s \to \gamma\gamma)_{SD+LD_{O_7}} \leq 1.43 \cdot 10^{-6}$.

The paper is organized as follows: In section 2 we display the amplitude for $B_s \to \gamma\gamma$ in a HQET inspired model and present the leading logarithmic QCD corrections. In section 3, we calculate the form factor F_1 in the decay $B_s \to \phi\gamma$ using QCD sum rules and compare our result with the previous result obtained in [10]. Section 4 is devoted to the estimate of the $B_s \to \phi\gamma \to \gamma\gamma$ amplitude in the framework of VMD. We discuss the resulting branching ratio $\mathcal{B}(B_s \to \gamma\gamma)_{SD+LD_{O_7}}$ and its parametric dependence on the model parameters $(m_b, \bar\Lambda_s)$ and the scale μ in section 5.

2 Leading logarithmic improved short-distance contributions in $B_s \to \gamma\gamma$ decay

The amplitude for the decay $B_s \to \gamma\gamma$ can be decomposed as [4–6]

$$\mathcal{A}(B_s \to \gamma\gamma) = \epsilon_1^\mu(k_1)\epsilon_2^\nu(k_2)(A^+ g_{\mu\nu} + iA^- \epsilon_{\mu\nu\alpha\beta}k_1^\alpha k_2^\beta) \ , \tag{2}$$

where the k_i and $\epsilon_i^\nu(k_i)$ denote the four-momenta and the polarization vectors of the outgoing photons, respectively [1]. Using the effective Hamiltonian in eq. (1), the CP-even (A^+) and CP-odd (A^-) parts in the SM can be written as (for diagrams see fig. 1 and fig. 2) in a HQET inspired approach [2] :

$$\begin{aligned} A^+ &= -\frac{\alpha_{em}G_F}{\sqrt{2}\pi} f_{B_s}\lambda_t \left(\frac{1}{3}\frac{m_{B_s}^4(m_b^{eff} - m_s^{eff})}{\bar\Lambda_s(m_{B_s} - \bar\Lambda_s)(m_b^{eff} + m_s^{eff})} C_7^{eff}(\mu) \right. \\ &- \left. \frac{4}{9}\frac{m_{B_s^2}}{m_b^{eff} + m_s^{eff}}(-m_b J(m_b) + m_s J(m_s))D(\mu) \right) , \\ A^- &= -\frac{\alpha_{em}G_F}{\sqrt{2}\pi} 2 f_{B_s}\lambda_t \left(\frac{1}{3}\frac{1}{m_{B_s}\bar\Lambda_s(m_{B_s} - \bar\Lambda_s)} g_- C_7^{eff}(\mu) \right. \\ &- \left. \sum_q Q_q^2 I(m_q) C_q(\mu) + \frac{1}{9(m_b^{eff} + m_s^{eff})}(m_b\triangle(m_b) + m_s\triangle(m_s))D(\mu) \right) , \end{aligned} \tag{3}$$

[1]We adopt the convention $Tr(\gamma^\mu\gamma^\nu\gamma^\alpha\gamma^\beta\gamma_5) = 4i\epsilon^{\mu\nu\alpha\beta}$, with $\epsilon^{0123} = +1$.

[2]In an earlier version of this paper the contributions of the operators $O_{1,3...6}$ in the irreducible part in A^+ and A^- were not completely taken into account. This is corrected here. As a second improvement we give the amplitudes in a formalism inspired by HQET to estimate the uncertainties coming from the bound state.

where we have used the unitarity of the CKM-matrix $\sum_{i=u,c,t} V_{is}^* V_{ib} = 0$ and have neglected the contribution due to $V_{us}^* V_{ub} \ll V_{ts}^* V_{tb} \equiv \lambda_t$. In eq. (3) N_c is the colour factor ($N_c = 3$ for QCD) and $Q_q = \frac{2}{3}$ for $q = u, c$ and $Q_q = -\frac{1}{3}$ for $q = d, s, b$. The QCD-corrected Wilson coefficients in leading logarithmic approximation [3], $C_{1\ldots6}(\mu)$ and $C_7^{eff}(\mu)$, enter the amplitudes in the combinations

$$\begin{aligned}
C_u(\mu) &= C_d(\mu) = (C_3(\mu) - C_5(\mu))N_c + C_4(\mu) - C_6(\mu) \ , \\
C_c(\mu) &= (C_1(\mu) + C_3(\mu) - C_5(\mu))N_c + C_2(\mu) + C_4(\mu) - C_6(\mu) \ , \\
C_s(\mu) &= C_b(\mu) = (C_3(\mu) + C_4(\mu))(N_c + 1) - N_c C_5(\mu) - C_6(\mu) \ , \\
D(\mu) &= C_5(\mu) + C_6(\mu) N_c \ . \qquad (4)
\end{aligned}$$

While $C_{1\ldots6}(\mu)$ are the coefficients of the operators $O_{1\ldots6}$, $C_7^{eff}(\mu)$ is the "effective" coefficient of O_7 and contains renormalization scheme dependent contributions from the four-quark operators $O_{1\ldots6}$ in $\mathcal{H}_{eff}$ to the effective vertex in $b \to s\gamma$. In the NDR scheme, which we use here, $C_7^{eff}(\mu) = C_7(\mu) - \frac{1}{3}C_5(\mu) - C_6(\mu)$, see [3] for details. The initial values of $C_{1\ldots6}(\mu)$ and $C_7^{eff}(\mu)$ in the SM are

$$\begin{aligned}
C_{1,3\ldots6}(m_W) &= 0 \ , \\
C_2(m_W) &= 1 \ , \\
C_7^{eff}(m_W) &= \frac{3x^3 - 2x^2}{4(x-1)^4} \ln x + \frac{-8x^3 - 5x^2 + 7x}{24(x-1)^3} \ , \qquad (5)
\end{aligned}$$

and $x = m_t^2/m_W^2$. For comparison, $C_1(m_b) = -0.246$, $C_2(m_b) = 1.106$, $C_3(m_b) = 0.011$, $C_4(m_b) = -0.025$, $C_5(m_b) = 0.007$, $C_6(m_b) = -0.031$ and $C_7^{eff}(m_b) = -0.313$ for the input values given in Table 1. The operator basis of $\mathcal{H}_{eff}$ is given as

$$\begin{aligned}
O_1 &= (\bar{s}_{L\alpha}\gamma_\mu b_{L\alpha})(\bar{c}_{L\beta}\gamma^\mu c_{L\beta}), \\
O_2 &= (\bar{s}_{L\alpha}\gamma_\mu b_{L\beta})(\bar{c}_{L\beta}\gamma^\mu c_{L\alpha}), \\
O_3 &= (\bar{s}_{L\alpha}\gamma_\mu b_{L\alpha}) \sum_{q=u,d,s,c,b} (\bar{q}_{L\beta}\gamma^\mu q_{L\beta}), \\
O_4 &= (\bar{s}_{L\alpha}\gamma_\mu b_{L\beta}) \sum_{q=u,d,s,c,b} (\bar{q}_{L\beta}\gamma^\mu q_{L\alpha}), \\
O_5 &= (\bar{s}_{L\alpha}\gamma_\mu b_{L\alpha}) \sum_{q=u,d,s,c,b} (\bar{q}_{R\beta}\gamma^\mu q_{R\beta}), \\
O_6 &= (\bar{s}_{L\alpha}\gamma_\mu b_{L\beta}) \sum_{q=u,d,s,c,b} (\bar{q}_{R\beta}\gamma^\mu q_{R\alpha}), \\
O_7 &= \frac{e}{16\pi^2} \bar{s}_\alpha \sigma_{\mu\nu}(m_b R + m_s L) b_\alpha F^{\mu\nu}, \\
O_8 &= \frac{g}{16\pi^2} \bar{s}_\alpha T^a_{\alpha\beta} \sigma_{\mu\nu}(m_b R + m_s L) b_\beta G^{a\mu\nu}, \qquad (6)
\end{aligned}$$

where L and R denote chiral projections, $L(R) = 1/2(1 \mp \gamma_5)$ and α and β are $SU(3)$ colour indices. Note that O_8 does not contribute here in this order of α_s. The functions $I(m_q)$, $J(m_q)$ and $\triangle(m_q)$

come from the irreducible diagrams with an internal q type quark propagating, see fig. 1, and are defined as

$$
\begin{aligned}
I(m_q) &= 1+\frac{m_q^2}{m_{B_s}^2}\triangle(m_q)\ , \\
J(m_q) &= 1-\frac{m_{B_s}^2-4m_q^2}{4m_{B_s}^2}\triangle(m_q)\ , \\
\triangle(m_q) &= \left(\ln(\frac{m_{B_s}+\sqrt{m_{B_s}^2-4m_q^2}}{m_{B_s}-\sqrt{m_{B_s}^2-4m_q^2}})-i\pi\right)^2 \quad \text{for } \frac{m_{B_s}^2}{4m_q^2}\geq 1, \\
\triangle(m_q) &= -\left(2\arctan(\frac{\sqrt{4m_q^2-m_{B_s}^2}}{m_{B_s}})-\pi\right)^2 \quad \text{for } \frac{m_{B_s}^2}{4m_q^2}< 1.
\end{aligned}
\tag{7}
$$

The parameter $\bar{\Lambda}_s$ enters eq. (3) through the bound state kinematics. For definiteness, we consider the decay $B_s \equiv (\bar{b}s) \to \gamma\gamma$. We write the momentum of the $\bar{b}$-quark inside the meson as $p = m_b v + k$, where k is a small residual momentum, v is the 4-velocity, which connects the quark with the meson kinematics through $P = m_{B_s} v$ and P is the momentum of the meson. In the B_s rest frame, $v = (1,0,0,0)$. For the reducible diagrams, see fig. 2, we need to evaluate $p.k_i$ and $p'.k_i$, $i = 1,2$, where k_i, p' are the momenta of the outgoing photon and s-quark, respectively. Now following [8], we average the residual momentum of the $\bar{b}$-quark through

$$
\begin{aligned}
< k_\alpha > &= -\frac{1}{2m_b}(\lambda_1+3\lambda_2)v_\alpha\ , \\
< k_\alpha k_\beta > &= \frac{\lambda_1}{3}(g_{\alpha\beta}-v_\alpha v_\beta)\ ,
\end{aligned}
\tag{8}
$$

where λ_1, λ_2 are matrix elements from the heavy quark expansion. Using $P = p - p'$, $P.k_i = \frac{m_{B_s}^2}{2}$, $v.k_i = \frac{m_{B_s}}{2}$ and the HQET relation [8]

$$
m_{B_s} = m_b + \bar{\Lambda}_s - \frac{1}{2m_b}(\lambda_1+3\lambda_2) \tag{9}
$$

one gets:

$$
\begin{aligned}
p.k_i &= \frac{m_{B_s}}{2}(m_{B_s}-\bar{\Lambda}_s)\ , \\
p'.k_i &= -\frac{m_{B_s}}{2}\bar{\Lambda}_s\ , \\
(m_b^{eff})^2 \equiv p^2 &= m_b^2-3\lambda_2\ , \\
(m_s^{eff})^2 \equiv p'^2 &= (m_b^{eff})^2-m_{B_s}^2+2m_{B_s}\bar{\Lambda}_s\ .
\end{aligned}
\tag{10}
$$

The non-perturbative parameter $\bar{\Lambda}_s$ can be related to $\bar{\Lambda}$, which has been extracted (together with λ_1) from data on semileptonic $B^\pm, B^0$ decays by [11], and the measured mass difference $\triangle m = m_{B_s} - m_B = 90$ MeV [12], defining $\bar{\Lambda}_s = \bar{\Lambda} + \triangle m$. The matrix element λ_2 is well determined from the $B^*_{(s)} - B_{(s)}$ mass splitting, $\lambda_2 = 0.12\,\mathrm{GeV}^2$. With the help of eq. (9), the correlated values of $\bar{\Lambda}$

and λ_1 can be transcribed into a correlation between $\bar{\Lambda}_{(s)}$ and m_b. We select 3 representative values [3] $(m_b, \bar{\Lambda}_s) = (5.03, 370), (4.91, 480), (4.79, 590)$ in (GeV, MeV) to study the hadronic uncertainties of our approach. Furthermore, we have used the definition

$$< 0|\bar{s}\gamma_\mu\gamma_5 b|B_s(P) > \;\; = \;\; i f_{B_s} P_\mu \;, \tag{11}$$

which leads together with the off-shellness of the quarks inside the meson to the matrix element of the pseudoscalar current

$$< 0|\bar{s}\gamma_5 b|B_s(P) > \;\; = \;\; -i f_{B_s} \frac{m_{B_s}^2}{m_b^{eff} + m_s^{eff}} \;. \tag{12}$$

The auxiliary function $g_- = g_-(m_b^{eff}, \bar{\Lambda}_s)$ is defined as

$$g_- = m_{B_s}(m_b^{eff} + m_s^{eff})^2 + \bar{\Lambda}_s(m_{B_s}^2 - (m_b^{eff} + m_s^{eff})^2) \;. \tag{13}$$

Note that in the limit $\bar{\Lambda}_s \to m_s$, $m_{b,s}^{eff} \to m_{b,s}$ and using $m_{B_s} = m_b + m_s$ we recover the result obtained by the constituent quark model [4–6], ignoring QCD corrections. Using the above expressions, the partial decay width is then given by :

$$\Gamma(B_s \to \gamma\gamma) = \frac{1}{32\pi m_{B_s}}(4|A^+|^2 + \frac{1}{2} m_{B_s}^4 |A^-|^2) \;. \tag{14}$$

Now, there are 2 new observations to be made:

First, the Wilson coefficients in eq. (3) depend on the scale μ. Therefore, since the behaviour of these short-distance (SD) coefficients under renormalization is known from the studies of $B \to X_s\gamma$ [2,3], one can give an improved width for $B_s \to \gamma\gamma$ by including the leading logarithmic QCD corrections, by renormalizing the coefficients $C_{1\ldots6}$ and C_7^{eff} from $\mu = m_W$ down to the relevant scale $\mu \approx \mathcal{O}(m_b)$. The explicit $\mathcal{O}(\alpha_s)$ improvement in the decay width $\Gamma(B_s \to \gamma\gamma)$ requires the calculation of a large number of virtual corrections, which we have not taken into account. Varying the scale μ in the range $\frac{m_b}{2} \leq \mu \leq 2m_b$, one introduces an uncertainty, which can be reduced only when the complete next-to-leading order (NLO)-analysis is available, similar to the recently completed calculation for the $B \to X_s\gamma$ decay [2].

The second point concerns the strong dependence of the decay width $\Gamma(B_s \to \gamma\gamma)$ on $\bar{\Lambda}_s$, $\Gamma \sim \mathcal{O}(\frac{1}{\bar{\Lambda}_s^2})$ in eq. (14). It originates in the s-quark propagator in the diagram with an intermediate s-quark in fig. 2. In the earlier work the authors of e.g. [4] evaluated the decay width with $m_s \approx m_K$, assuming that the constituent quarks are to be treated as static quarks in the meson. This is a questionable assumption. In the HQET inspired approach, this gets replaced by $\bar{\Lambda}_s$, which is well-defined experimentally. This formalism implies, that the decay width $\Gamma(B_d \to \gamma\gamma)$ will involve the parameter $\bar{\Lambda}$, which avoids the unwanted uncertainty on m_d.

[3] We choose $(\lambda_1, \bar{\Lambda}) = (-0.09, 280), (-0.19, 390), (-0.29, 500)$ in (GeV^2, MeV) from fig. 1 in [11].

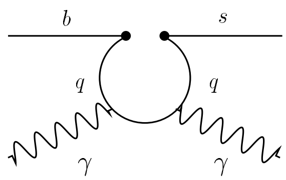

Figure 1: The generic diagram contributing to $b \to s\gamma\gamma$ in the effective theory due to the (Fierz ordered) four-quark operators. The diagram with interchanged photons is not shown.

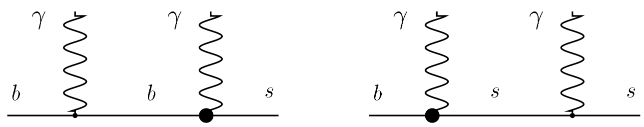

Figure 2: The reducible diagrams contributing to $b \to s\gamma\gamma$. The blob denotes the FCNC operator O_7. The diagrams with interchanged photons are not shown.

Lowering the scale μ from $\mu = m_W$ to $\mu \simeq \mathcal{O}(m_b)$ and $\bar{\Lambda}_s$ enhances the branching ratio $\mathcal{B}(B_s \to \gamma\gamma)$. The dependence of the branching ratio as a function of the scale μ for different values of $(m_b, \bar{\Lambda}_s)$ is discussed in the last section including the O_7-type long-distance (LD) estimate.

3 QCD sum rule for the $B_s \to \phi\gamma$ form factor

3.1 Calculation of the sum rule

The amplitude for the $B_s \to \phi\gamma$ transition $\mathcal{A}(B_s \to \phi\gamma) =< \phi\gamma|\mathcal{H}_{eff}|B_s >$ reduces to

$$\mathcal{A}(B_s \to \phi\gamma) = \epsilon^\mu C m_b < \phi(p')|\bar{s}\sigma_{\mu\nu} R q^\nu b|B_s(p) > \tag{15}$$

with the constant C

$$C = \frac{G_F}{\sqrt{2}} \frac{e}{2\pi^2} V_{ts}^* V_{tb} C_7^{eff}(\mu) \ , \tag{16}$$

where we just take the contribution due to the electromagnetic penguin operator O_7 into account and put $m_s = 0$, justified by $m_s \ll m_b$. Here ϵ and q are the photon polarization and the (outgoing) photon momentum, respectively. Lorentz decomposition gives further:

$$\begin{aligned} < \phi(p')|\bar{s}\sigma_{\mu\nu} R q^\nu b|B_s(p) > &= i\epsilon_{\mu\nu\rho\sigma}\epsilon^{\phi\nu} p^\rho p'^\sigma F_1(q^2) \\ &+ (\epsilon_\mu^\phi p.q - p_\mu q.\epsilon^\phi) G(q^2) \ , \end{aligned} \tag{17}$$

Parameter	Value
m_c	1.4 (GeV)
m_b	4.8 (GeV)
α_{em}^{-1}	129
λ_t	0.04
$\Gamma_{tot}(B_s)$	$4.09 \cdot 10^{-13}$ (GeV)
f_{B_s}	0.2 (GeV)
m_{B_s}	5.369 (GeV)
m_t	175 (GeV)
m_W	80.26 (GeV)
m_Z	91.19 (GeV)
$\Lambda_{QCD}^{(5)}$	0.214 (GeV)
$\alpha_s(m_Z)$	0.117
λ_2	0.12 (GeV^2)

Table 1: Values of the input parameters used in the numerical calculations unless otherwise specified.

where p, p' denote the four-momenta of the initial B_s-meson and the outgoing ϕ, respectively and ϵ_μ^ϕ is the polarization vector of the ϕ-meson. At the point $q^2 = 0$, it is enough to calculate $F_1(0)$, since both form factors coincide [13]. Note, that the form factors introduced above are in general functions of two variables q^2 and p'^2. Since ϕ is on-shell, we abbreviate here and in the following unless otherwise stated $F_1(q^2) \equiv F_1(q^2, p'^2 = m_\phi^2)$.

The starting point for the sum rule is the three-point function [14]

$$T_{\alpha\mu} = -\int d^4x e^{ipx - ip'y} < 0|T[J_\alpha(x) T_\mu(0) J_5(y)]|0 > , \quad (18)$$

where $J_\alpha = \bar{s}\gamma_\alpha s$, $J_5 = \bar{s} i\gamma_5 b$ and $T_\mu = \bar{s}\frac{1}{2}\sigma_{\mu\nu}q^\nu b$ correspond to the electromagnetic, pseudoscalar currents and the penguin operator, respectively. Performing now an operator product expansion (OPE) of $T_{\alpha\mu}$, we obtain a perturbative term, the so-called bare loop, and non-perturbative power corrections, diagrammatically shown in fig. 3. The bare loop diagram can be obtained using a double dispersion relation in p^2 and p'^2,

$$T_{bare} = \frac{1}{\pi^2} \int_{m_b^2}^\infty ds \int_0^\infty ds' \frac{\rho(s, s')}{(s - p^2)(s' - p'^2)} + \text{subtractions} . \quad (19)$$

Technically, the spectral density $\rho(s, s')$ can be calculated by using the Cutkowsky rule, namely, by replacing the usual propagator denominator by a delta function:
$\frac{1}{k^2 - m^2} \to -2\pi i \delta(k^2 - m^2)\theta(k_0)$. As a result we get

$$\rho(s, s') = \frac{N_c}{8} m_b^4 \frac{s'}{(s - s')^3} . \quad (20)$$

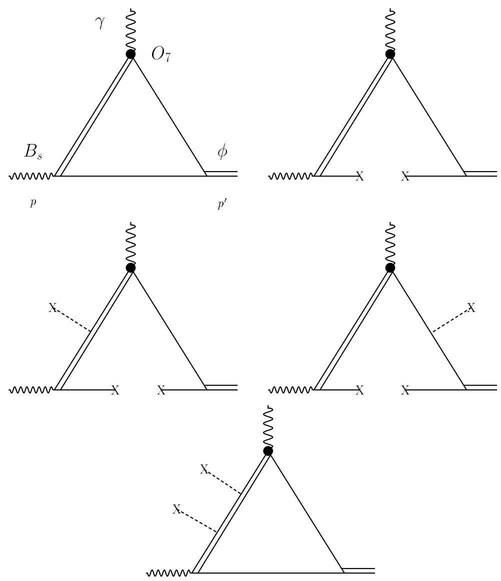

Figure 3: Contributions of perturbation theory and of vacuum condensates to the $B_s \to \phi\gamma$ decay. The dashed lines denote soft gluons.

OPE enables us further to parametrize the non-perturbative effects in terms of vacuum expectation values of gauge-invariant operators up to a certain dimension, the so-called condensates. We consider up to dimension-5 operators; i.e. the quark condensate, gluon condensate and the quark-gluon (mixed) condensate contributions (fig. 3). This calculation is carried out in the fixed point gauge, i.e. $A_\mu . x_\mu = 0$. We get

$$\begin{aligned}
T_{dim-3} &= \frac{-m_b}{2} < \bar{s}s > \frac{1}{(p^2 - m_b^2)p'^2} \ , \\
T_{dim-4} &= \frac{\alpha_s}{144\pi} < G^2 > \int_0^1 dx \int_0^{1-x} dy \int_0^\infty d\alpha \alpha^3 \\
&\cdot \ (c_1 + c_2 P^2 + c_3 P'^2) e^{-\alpha(d_1 + d_2 P^2 + d_3 P'^2)} \ , \\
T_{dim-5} &= \frac{m_b}{2} g < \bar{s}\sigma G s > [\frac{m_b^2}{2(p^2 - m_b^2)^3 p'^2} + \frac{m_b^2}{3(p^2 - m_b^2)^2 p'^4} \\
&+ \ \frac{1}{2(p^2 - m_b^2)^2 p'^2}] \ , \qquad (21)
\end{aligned}$$

where

$$\begin{aligned}
c_1 &= m_b^4 x^4 \ , \\
c_2 &= m_b^2 x^4 (1 - x - y) \ , \\
c_3 &= m_b^2 x^3 (3 + y)(1 - x - y) \ , \\
d_1 &= m_b^2 x \ , \\
d_2 &= x(1 - x - y) \ ,
\end{aligned}$$

$$d_3 = y(1-x-y) \ . \tag{22}$$

Here we used the exponential representation for the gluon condensate contribution:

$$\frac{1}{D^n} = \frac{1}{(n-1)!}\int_0^\infty d\alpha \ \alpha^{n-1} e^{-\alpha D} \ . \tag{23}$$

The momenta P, P' in eq. (21) are euclidean.

For the calculation of the physical part of the sum rules we insert a complete set of on-shell states with the same quantum numbers as B_s and ϕ in eq. (18) and get a double dispersion relation

$$T_{phys} = \frac{m_{B_s}^2 f_{B_s}}{m_b} f_\phi m_\phi \frac{1}{(p^2 - m_{B_s}^2)(p'^2 - m_\phi^2)} F_1(0) + \text{continuum} \ , \tag{24}$$

where f_ϕ and f_{B_s} are the leptonic decay constants of the ϕ and B_s mesons respectively, defined as usual by

$$\begin{aligned} <0|J_\alpha|\phi> &= m_\phi f_\phi \epsilon_\alpha^\phi \ , \\ <0|J_5|B_s(p)> &= f_{B_s} m_{B_s}^2 / m_b \ . \end{aligned} \tag{25}$$

We have absorbed all higher order states and resonances in the continuum.

Now, we equate the hadron-world with the quark-world by $T_{phys} = T_{bare} + T_3 + T_4 + T_5$. Using quark-hadron duality, we model the continuum contribution by purely perturbative QCD. To be definite, it is the part in eq. (19) above the so-called continuum thresholds s_0 and s_0'. To get rid of subtractions and to suppress the contribution of higher order states, we apply a Double Borel transformation $\hat{B}$ [17] with respect to p^2 and p'^2. We make use of the following properties of the Borel transform:

$$\hat{B}\left(\frac{1}{(p^2 - m^2)^n}\right) = \frac{(-1)^n}{(n-1)!}\frac{e^{-m^2/M^2}}{(M^2)^n} \ , \tag{26}$$

$$\hat{B}(e^{-\alpha p^2}) = \delta(1 - \alpha M^2) \ . \tag{27}$$

Finally, this yields the sum rule:

$$\begin{aligned} F_1(0) &= \exp\left(\frac{m_{B_s}^2}{M^2} + \frac{m_\phi^2}{M'^2}\right) \frac{m_b}{f_{B_s} f_\phi m_\phi m_{B_s}^2} \Big\{ \frac{1}{\pi^2}\int_{m_b^2}^{s_0} ds \int_0^{\bar{s}} ds' \rho(s,s') e^{-s/M^2 - s'/M'^2} \\ &- \frac{m_b}{2} <\bar{s}s> e^{(-m_b^2/M^2)} \left[1 - m_0^2\left(\frac{m_b^2}{4M^4} + \frac{m_b^2}{3M^2M'^2} - \frac{1}{2M^2}\right)\right] \\ &+ \frac{\alpha_s}{\pi} <G^2> \int_0^{x_{max}} N(x) dx \Big\} \ , \end{aligned} \tag{28}$$

where $\bar{s} = min(s - m_b^2, s_0')$ and $x_{max} = \frac{M'^2}{M^2 + M'^2}$. Here we used the parametrization

$$g <\bar{s}\sigma G s> = m_0^2 <\bar{s}s> \ . \tag{29}$$

The last term in eq. (28) is due to the gluon condensate contribution and the function $N(x)$ is defined by:

$$\begin{aligned} N(x) &= \frac{1}{48}\exp\left(-\frac{m_b^2}{M^2(1 - x - xM^2/M'^2)}\right) m_b^2 M'^6 x (m_b^2 M'^4 - 4M^2M'^4 + 5M^2M'^4 x \\ &+ 5M^4M'^2 x - M^2M'^4x^2 - 2M^4M'^2x^2 - M^6x^2)/(M^4(-M'^2 + M'^2x + M^2x)^5) \ . \end{aligned} \tag{30}$$

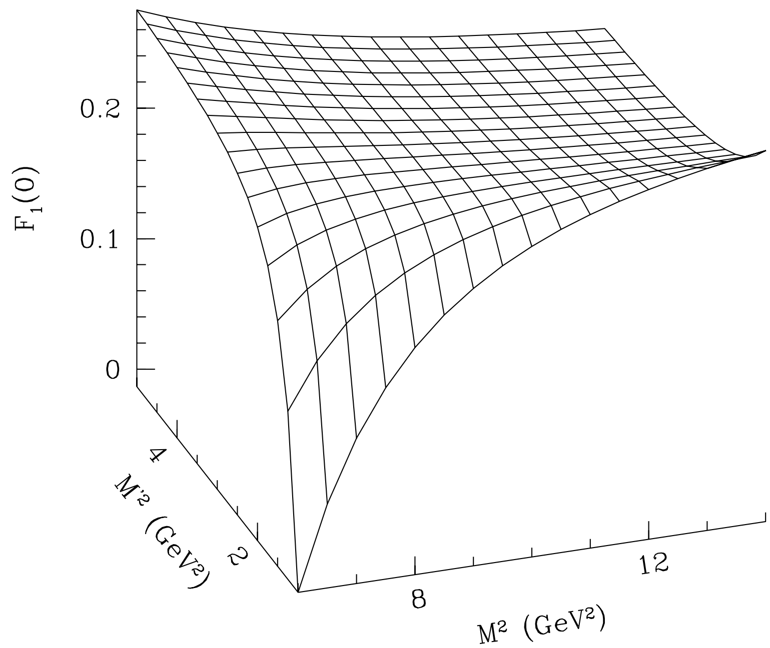

Figure 4: The dependence of the decay constant $F_1(0)$ on the Borel parameters M^2 and M'^2 for $s_0 = 33$ GeV^2.

3.2 Analysis of the sum rule

First we list the values of the input parameters entering the sum rules (eq. (28)), which are not included in Table 1: $m_0^2 = 0.8$ GeV^2 [15], $< \bar{s}s >= -0.011$ GeV^3 [16], $\frac{\alpha_s}{\pi} < G^2 >= 0.03$ GeV^4 [17], $m_\phi = 1.019$ GeV and $f_\phi = 0.23$ GeV [18].

We do the calculations for two different continuum threshold values $s_0 = 33$ GeV^2 and $s_0 = 35$ GeV^2 and take $s_0' = 1.8$ GeV^2. In fig. 4 we present the dependence of $F_1(0)$ on M^2 and M'^2 for $s_0 = 33$ GeV^2. According to the QCD sum rules method, it is necessary to find a range of M^2 and M'^2, where the dependence of $F_1(0)$ on these parameters is very weak and, at the same time, the power corrections and the continuum contribution remain under control. From fig. 4 and fig. 5 follows that the best stability region for $F_1(0)$ is 7 GeV$^2 \leq M^2 \leq 9$ GeV^2, 2 GeV$^2 \leq M'^2 \leq 3$ GeV^2 for $s_0 = 33, 35$ GeV^2. We get:

$$F_1(0) = 0.24 \pm 0.02 \ . \tag{31}$$

This agrees for our value of m_b within errors with the result given in the literature, based on Light-cone QCD sum rule calculations [10].

Numerical analysis shows, as also mentioned in [14], that the natural hierarchy of the bare loop, the power corrections and continuum contributions does not hold due to the smallness of the integration region, and the power corrections exceed the bare loop contribution. The gluon condensate contribution is $\leq 1\%$ of the dim-3 + dim-5 condensate contributions and can therefore be safely neglected in

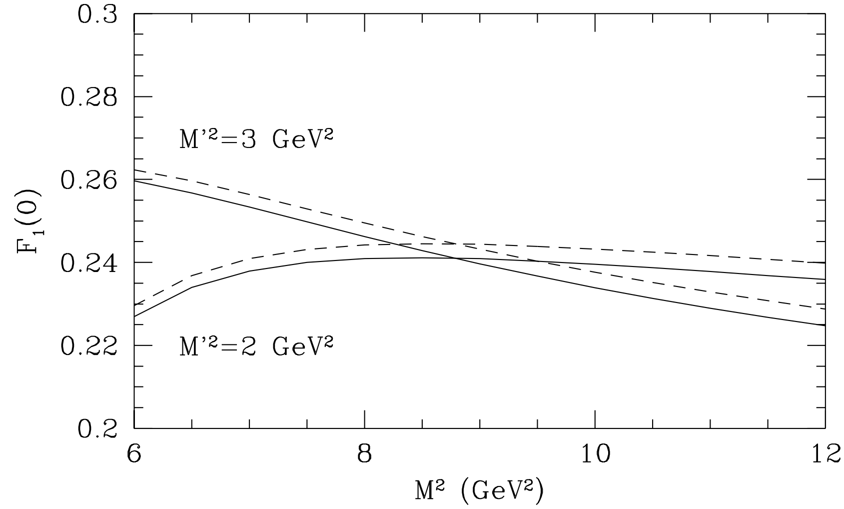

Figure 5: The dependence of the decay constant $F_1(0)$ on the Borel parameter M^2 for fixed M'^2 at $s_0 = 33$ GeV^2 (solid) and $s_0 = 35$ GeV^2 (dashed).

numerical calculations.

4 The $B_s \to \phi\gamma \to \gamma\gamma$ amplitude using VMD model

We consider the construction of a VMD amplitude using the amplitude for the decay $B_s \to \phi\gamma$ as an input. Our aim is to continue the $B_s \to \phi\gamma$ decay amplitude from $p'^2 = m_\phi^2$ to $p'^2 = 0$, such that the ϕ meson propagates as a massless virtual particle before converting into a photon. Note that we suppressed in our notation the dependence of the form factor $F_1(q^2) = F_1(q^2, p'^2 = m_\phi^2)$ on the second argument p'^2. We define here $\bar{F}_1(Q^2) \equiv F_1(q^2 = 0, Q^2)$ for virtual momenta $Q^2 = -p'^2$. Assuming pole-type behaviour of the form factor $\bar{F}_1(Q^2)$ we extrapolate using the single-pole form

$$\bar{F}_1(Q^2) = \frac{\bar{F}_1(0)}{1 - Q^2/m_{pole}^2} \ , \tag{32}$$

which works well for light mesons. Using an m_{pole} of order $1.7 - 1.9$ GeV, which corresponds to the mass of the higher resonances of ϕ, we estimate $\bar{F}_1(0) = 0.16 \pm 0.02$.

With the help of VMD [19] and factorization we can now present the amplitude for $B_s \to \gamma\gamma$. Using the intermediate propagator $\frac{-1}{Q^2+m_\phi^2}$ at $Q^2 = 0$, the $\phi \to \gamma$ conversion vertex from the VMD mechanism

$$< 0|J_{\mu\ em}|\phi(p', \epsilon) >= eQ_s f_\phi(0) m_\phi \epsilon_\mu \ , \tag{33}$$

and the $\mathcal{A}(B_s \to \phi\gamma)$ amplitude, see eq. (15), we get:

$$\mathcal{A}(B_s \to \phi\gamma \to \gamma\gamma) \ = \ \epsilon_1^\mu(k_1)\epsilon_2^\nu(k_2)(A^+_{LD_{O_7}} g_{\mu\nu} + iA^-_{LD_{O_7}} \epsilon_{\mu\nu\alpha\beta} k_1^\alpha k_2^\beta) \ , \tag{34}$$

with the CP-even ($A^+_{LD_{O_7}}$) and CP-odd ($A^-_{LD_{O_7}}$) parts:

$$A^+_{LD_{O_7}} \ = \ 2\chi C m_b \frac{m_{B_s}^2 - m_\phi^2}{2} \bar{F}_1(0)$$

$$= \sqrt{2}\frac{\alpha_{em}G_F}{\pi}\bar{F}_1(0)f_\phi(0)\lambda_t\frac{m_b(m_{B_s}^2-m_\phi^2)}{3m_\phi}C_7^{eff}(\mu) ,$$

$$A_{LD_{O_7}}^- = 2\chi C m_b\bar{F}_1(0)$$

$$= 2\sqrt{2}\frac{\alpha_{em}G_F}{\pi}\bar{F}_1(0)f_\phi(0)\lambda_t\frac{m_b}{3m_\phi}C_7^{eff}(\mu) , \quad (35)$$

where $f_\phi(0) = 0.18$ GeV [9], $Q_s = -1/3$ and C is defined in eq. (16). The factor 2 comes from the addition of the diagrams with interchanged photons. Note, that while for the analysis of the sum rule for $B_s \to \phi\gamma$ we have used $f_\phi \equiv f_\phi(m_\phi^2)$, here we take into account the suppression in $f_\phi(Q^2)$ going from $Q^2 = m_\phi^2$ to $Q^2 = 0$. We treated the polarization vector ϵ^ϕ as transversal and replaced $\epsilon \to \epsilon_1$, $\epsilon^\phi \to \epsilon_2$, $q \to k_1$, $p' \to k_2$. The conversion factor χ is defined as

$$\chi = -eQ_s\frac{f_\phi(0)}{m_\phi} . \quad (36)$$

Adding this to the short-distance amplitudes (eq. (3)), we obtain the $B_s \to \gamma\gamma$ width including the O_7-type long-distance effects:

$$\Gamma(B_s \to \gamma\gamma)_{SD+LD_{O_7}} = \frac{1}{32\pi m_{B_s}}(4|A^+ + A_{LD_{O_7}}^+|^2 + \frac{1}{2}m_{B_s}^4|A^- + A_{LD_{O_7}}^-|^2) . \quad (37)$$

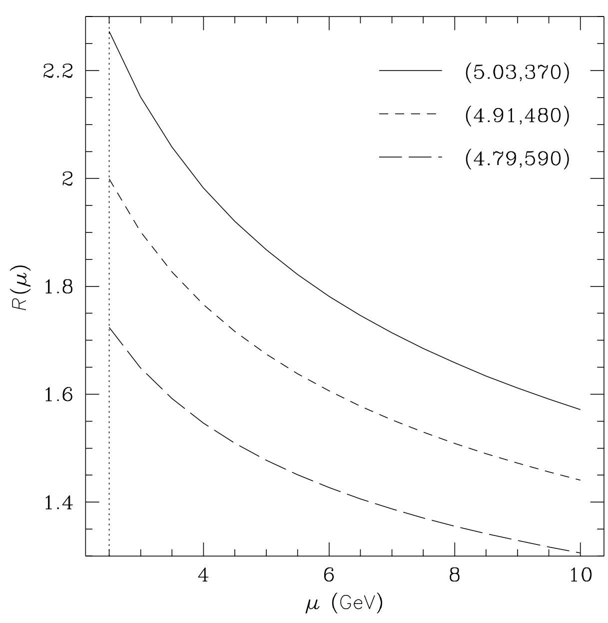

Figure 6: Scale dependence of the ratio $R(\mu)$ defined in eq. (38). The solid, short-dashed and long-dashed lines correspond to the values $(m_b, \bar{\Lambda}_s)$ in (GeV, MeV) as indicated in the figure. The dotted line depicts the suggested choice of the scale μ from $B \to X_s\gamma$ studies in NLO [2,3]. The parameters used are given in Table 1.

5 Numerical estimates

First we study the leading logarithmic μ-dependence of the ratio

$$R(\mu) = \frac{\Gamma(B_s \to \gamma\gamma)(\mu)_{SD+LD_{O_7}}}{\Gamma(B_s \to \gamma\gamma)(m_W)_{SD+LD_{O_7}}} \ . \tag{38}$$

In the numerical analysis we neglect the masses of the light quarks. From fig. 6 we find an enhancement factor of $1.3 - 2.3$ relative to the lowest order result obtained by setting $\mu = m_W$, depending on the model parameter $(m_b, \bar{\Lambda}_s)$. Varying μ in the range 2.5 GeV $\leq \mu \leq$ 10.0 GeV, gives an uncertainty $\triangle R/R(\mu = 5 \text{ GeV}) \approx \pm(17, 19, 22)\%$ for $\bar{\Lambda}_s = (590, 480, 370)$ MeV, respectively. Here one can argue, that the choice $\mu = \frac{m_b}{2}$ takes into account effectively the bulk of the NLO correction as suggested by the NLO calculation for $B \to X_s\gamma$ [2].

Table 2 shows the combined μ and model parameter dependence of the branching ratio

$$\mathcal{B}(B_s \to \gamma\gamma)_{SD+LD_{O_7}} = \frac{\Gamma(B_s \to \gamma\gamma)_{SD+LD_{O_7}}}{\Gamma_{tot}(B_s)} \ . \tag{39}$$

The dependence of the form factor $\bar{F}_1(m_\phi^2)$ on the b quark mass has been extrapolated from fig. 3 [10]. Here $\bar{F}_1(0) = 0.14, 0.15, 0.16$ has been used for $m_b = (5.03, 4.91, 4.79)$ GeV, respectively. Qualitatively,

μ (GeV)	$\bar{\Lambda}_s = 370$ MeV $m_b = 5.03$ GeV	$\bar{\Lambda}_s = 480$ MeV $m_b = 4.91$ GeV	$\bar{\Lambda}_s = 590$ MeV $m_b = 4.79$ GeV
2.5	$1.43 \cdot 10^{-6}$	$8.1 \cdot 10^{-7}$	$5.0 \cdot 10^{-7}$
5.0	$1.18 \cdot 10^{-6}$	$6.8 \cdot 10^{-7}$	$4.3 \cdot 10^{-7}$
10.0	$0.99 \cdot 10^{-6}$	$5.9 \cdot 10^{-7}$	$3.8 \cdot 10^{-7}$

Table 2: Branching ratio $\mathcal{B}(B_s \to \gamma\gamma)_{SD+LD_{O_7}}$ for selected values $(m_b, \bar{\Lambda}_s)$ and the renormalization scale μ.

the influence of the LD contribution through $B_s \to \phi\gamma \to \gamma\gamma$ reduces the width because of the destructive interference of the LD + SD contributions. To quantify this, we define

$$\kappa \equiv \frac{\mathcal{B}(B_s \to \gamma\gamma)_{SD+LD_{O_7}} - \mathcal{B}(B_s \to \gamma\gamma)_{SD}}{\mathcal{B}(B_s \to \gamma\gamma)_{SD}} \ , \tag{40}$$

with $\Gamma(B_s \to \gamma\gamma)_{SD}$ given in eq. (14). We find, that κ lies in the range:

$$-15\% \leq \kappa \leq -27\% \ , \tag{41}$$

depending mainly on $(m_b, \bar{\Lambda}_s)$.

In conclusion, we have reanalysed the decay rate $B_s \to \gamma\gamma$ in the SM. We included the leading logarithmic QCD corrections and investigated the influence of the LD-contributions due to the chain $B_s \to \phi\gamma \to \gamma\gamma$. Depending on $\bar{\Lambda}_s$, the LD-contributions become sizeable. Other possible LD contributions may also arise from the O_2-type transitions.

The decay rate of $B_s \to \gamma\gamma$ depends sensitively on $(m_b, \bar{\Lambda}_s)$ and μ. Fixing μ to $\mu = \frac{m_b}{2}$ as suggested by the NLO calculation of $B \to X_s\gamma$ and varying the model parameter $(m_b, \bar{\Lambda}_s)$ (see Table 2), we find that the branching ratio $\mathcal{B}(B_s \to \gamma\gamma)_{SD+LD_{O_7}}$ is uncertain by a large factor

$$0.5 \cdot 10^{-6} \leq \mathcal{B}(B_s \to \gamma\gamma)_{SD+LD_{O_7}} \leq 1.4 \cdot 10^{-6} \ . \tag{42}$$

Improving this requires NLO calculation in the decay rate $B_s \to \gamma\gamma$. With the choice of $(m_b, \bar{\Lambda}_s) = (5.03\,\mathrm{GeV}, 370\,\mathrm{MeV})$, the resulting branching ratio $(1.4 \cdot 10^{-6})$ is substantially larger than what has been stated in the literature. The present best limit on the decay $B_s \to \gamma\gamma$ is [20]

$$\mathcal{B}(B_s \to \gamma\gamma) < 1.48 \cdot 10^{-4} \ , \tag{43}$$

which is still a factor $\approx 100 - 300$ away from the estimates given here.

Note added: Recently, the leading logarithmic QCD corrections for the short-distance part of the decay $B_s \to \gamma\gamma$ have also been calculated by Chang et al. [21]. They derived the decay rate with the full set of operators $O_{1\ldots8}$ and we agree with their analytical expression. Our model to incorporate the bound state effects in the B_s meson is inspired by HQET, resulting in the parameters $(m_b, \bar{\Lambda}_s)$. The strong parametric dependence of the decay rate $\Gamma(B_s \to \gamma\gamma)$ on $(m_b, \bar{\Lambda}_s)$ and on μ has been studied by us; Chang et al. [21] fix $\Lambda = m_{B_s} - m_b$, using the naive constituent quark model, and set $\mu = m_b$. We emphasize here that the decay rate is sensitive to both of these parameters and requires further theoretical investigation.

Acknowledgements

We thank Ahmed Ali for helpful discussions and input and Christoph Greub for discussions, especially on QCD corrections. E. O. I. would also like to thank the DESY theory group for the warm hospitality during his stay in Hamburg. The work of E. O. I. was supported by the Turkish Scientific and Technical Research Council (TUBITAK).

References

[1] M. S. Alam et al., (CLEO Collaboration), Phys. Rev. Lett. **74** (1995) 2885.

[2] C. Greub, T. Hurth and D. Wyler, Phys. Lett.B **380** (1996) 385; Phys. Rev. D **54** (1996) 3350; K. Chetyrkin, M. Misiak and M. Münz, hep-ph/9612313.

[3] A. Ali and C. Greub, Z. Phys. C **49** (1991) 431;
A. J. Buras et al., Nucl. Phys. B **424** (1994) 374.

[4] G.-L. Lin, J. Liu and Y.-P. Yao, Phys. Rev. Lett. **64** (1990) 1498;
G.-L. Lin, J. Liu and Y.-P. Yao, Phys. Rev. D **42** (1990) 2314.

[5] H. Simma and D. Wyler, Nucl. Phys. B **344** (1990) 283.

[6] T. M. Aliev and G. Turan, Phys. Rev. D **48** (1993) 1176.

[7] B. Grinstein, R. Springer and M. Wise, Phys. Lett. B **202** (1988) 138.

[8] A. Manohar and M. B. Wise, Phys. Rev. D **49** (1994) 1310.

[9] K. Terasaki, Nuovo Cim. Vol. **66** A, No. **4** (1981) 475.

[10] A. Ali, V. M. Braun and H. Simma, Z. Phys. C **63** (1994) 437.

[11] M. Gremm, A. Kapustin, Z. Ligeti and M.B. Wise, Phys. Rev. Lett. **77** (1996) 20.

[12] R. M. Barnett et al., Review of Particle Properties, Phys Rev D **54** (1996) 1.

[13] A. Ali and V. M. Braun, Phys. Lett. B **359** (1995) 223.

[14] P. Colangelo et al., Phys. Lett. B **317** (1993) 183.

[15] V.M.Belyaev, B.L.Ioffe, Sov. JETP **83** (1982) 876.

[16] B.L.Ioffe, A.V.Smilga, Phys.Let. B **133** (1985) 436.

[17] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B **147** (1979) 385.

[18] V. L. Chernyak and A. R. Zhitnitsky, Phys. Rep. **112** (1984) 173.

[19] J. J. Sakurai, Currents and Mesons (University of Chicago Press, Chicago, 1969);
E. Golowich and S. Pakvasa, Phys. Rev. D **51** (1995) 1215;
N. G. Deshpande, Xiao-Gang He and J. Trampetic, Phys. Lett. B **367** (1996) 362.

[20] M. Acciarri et al. (L3 Collaboration), Phys. Lett. B **363** (1995) 137.

[21] Chia-Hung V. Chang, Guey-Lin Lin and York-Peng Yao, hep-ph/9705345.