Semileptonic $B_q \to D_q^* l\nu$ $(q = s, d, u)$ Decays in QCD Sum Rules

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Abstract

The form factors relevant to $B_q \to D_q^*(J^P = 1^-)\ell\nu$ $(q = s, d, u)$ decays are calculated in the framework of the three point QCD sum rules approach. The heavy quark effective theory prediction of the form factors as well as $1/m_b$ corrections to those form factors are obtained. A comparison of the results for the ratio of form factors at zero recoil limit and other values of q^2 with the predictions of the subleading Isgur-Wise form factor application for $B \to D^*\ell\nu$ is presented. The total decay width and branching ratio for these decays are also evaluated using the q^2 dependencies of these form factors. The results are in good agreement with the constituent quark meson model and existing experimental data. The $q = s$ case can also be detected at LHC in the near future.

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1 Introduction

Semileptonic pseudoscalar B_q decays are crucial tools to restrict the Standard Model (SM) parameters and search for new physics beyond the SM. These decays provide possibility to calculate the elements of the Cabbibo-Kobayashi-Maskawa (CKM) matrix, leptonic decay constants as well as the origin of the CP violation.

When LHC begins to operate, a large number of B_q mesons will be produced. This will provide experimental framework to check the B_q decay channels. An important class of B_s and entire $B_{u,d}$ decays occur via the b quark decays. Among the b decays, $b \to c$ transition plays a significant role, because this transition is the most dominant transition among b decays. Some of the B decay channels could be $B_q \to D_q^* l\nu$ $(q = s, d, u)$ via $b \to c$ transition. These decays could give useful information about the structure of the vector D_s^* mesons. The observation of two narrow resonances with charm and strangeness, $D_{sJ}(2317)$ in the $D_s\pi^0$ invariant mass distribution [1]–[6], and $D_{sJ}(2460)$ in the $D_s^*\pi^0$ and $D_s\gamma$ mass distribution [2, 3, 4, 6, 7, 8], has raised discussions about the nature of these states and their quark contents [9, 10]. Analysis of the $D_{s_0}(2317) \to D_s^*\gamma$, $D_{sJ}(2460) \to D_s^*\gamma$ and $D_{sJ}(2460) \to D_{s_0}(2317)\gamma$ indicates that the quark content of these mesons is probably $\bar{c}s$ [11].

Form factors are central objects in studying of the semileptonic $B_q \to D_q^* l\nu$ decays. For the calculation of these form factors, we need reliable non-perturbative approaches. Among all non-perturbative models, the QCD sum rules has received especial attention since this model is based on the QCD Lagrangian. QCD sum rules is a framework which connects hadronic parameters with QCD parameters. In this method, hadrons are represented by their interpolating currents taken at large virtualities. The correlation function is calculated in hadrons and quark-gluon languages. The physical quantities are determined by matching these two representations of correlators . The application of sum rules has been extended remarkably during the past twenty years and applied for wide variety of problems (For review see for example [12]).

The aim of this paper is to analyze the semileptonic $B_q \to D_q^* l\nu$ decays using three point QCD sum rules method. Note that, this problem has been studied for $B_q \to D_q^* l\nu$ ($q = s, d, u$) in constituent quark meson (CQM) model in [13] and for $q = d, u(B^0, B^{\pm})$ cases in experiment [14]. The application of subleading Isgur-Wise form factor for $B \to D^* l\nu$ at heavy quark effective theory (HQET) is calculated in [15] (see also [16, 17]). Present work takes into account the SU(3) symmetry breaking and could be considered as an extension of the form factors of $D \to K^* e\nu$ presented in [18].

The paper is organized as fallow: In section II, sum rules expressions for form factors relevant to these decays and their HQET limit and $1/m_b$ corrections are obtained. The numerical analysis for form factors and their HQET limit at zero recoil and other values of y, conclusion and comparison of our results with the other approaches are presented in section III.

2 Sum rules for the $B_q \to D_q^* \ell\nu$ transition form factors

The $B_q \to D_q^*$ transitions occur via the $b \to c$ transition at the quark level. At this level, the matrix element for this transition is given by:

$$M_q = \frac{G_F}{\sqrt{2}} V_{cb}\ \overline{\nu}\ \gamma_\mu(1-\gamma_5) l\ \overline{c}\ \gamma_\mu(1-\gamma_5) b. \tag{1}$$

To derive the matrix elements for $B_q \to D_q^* l\nu$ decays, it is necessary to sandwich Eq. (1) between initial and final meson states. The amplitude of the $B_q \to D_q^* l\nu$ decays can be written as follows:

$$M = \frac{G_F}{\sqrt{2}} V_{cb}\ \overline{\nu}\ \gamma_\mu(1-\gamma_5) l < D_q^*(p', \varepsilon) \mid\ \overline{c}\ \gamma_\mu(1-\gamma_5) b \mid B_q(p) > . \quad (2)$$

The aim is to calculate the matrix element $< D_q^*(p', \varepsilon) \mid \overline{c}\gamma_\mu(1-\gamma_5)b \mid B_q(p) >$ appearing in Eq. (2). Both the vector and the axial vector part of $\overline{c}\ \gamma_\mu(1-\gamma_5)b$ contribute to the matrix element stated above. Considering Lorentz and parity invariances, this matrix element can be parameterized in terms of the form factors below:

$$< D_q^*(p', \varepsilon) \mid \overline{c}\gamma_\mu b \mid B_q(p) >= i \frac{f_V(q^2)}{(m_{B_q} + m_{D_q^*})} \varepsilon_{\mu\nu\alpha\beta} \varepsilon^{*\nu} p^\alpha p'^\beta, \quad (3)$$

$$\begin{aligned} < D_q^*(p', \varepsilon) \mid \overline{c}\gamma_\mu\gamma_5 b \mid B_q(p) > \ &= \ i \Big[f_0(q^2)(m_{B_q} + m_{D_q^*}) \varepsilon_\mu^* \\ - \frac{f_+(q^2)}{(m_{B_q} + m_{D_q^*})} (\varepsilon^* p) P_\mu \ &- \ \frac{f_-(q^2)}{(m_{B_q} + m_{D_q^*})} (\varepsilon^* p) q_\mu \Big], \end{aligned} \quad (4)$$

where $f_V(q^2)$, $f_0(q^2)$, $f_+(q^2)$ and $f_-(q^2)$ are the transition form factors and $P_\mu = (p + p')_\mu$, $q_\mu = (p - p')_\mu$. In order to calculate these form factors, the QCD sum rules method is applied. Initially the following correlator is considered:

$$\Pi_{\mu\nu}^{V;A}(p^2, p'^2, q^2) = i^2 \int d^4x d^4y e^{-ipx} e^{ip'y} < 0 \mid T[J_{\nu D_q^*}(y) J_\mu^{V;A}(0) J_{B_q}(x)] \mid 0 >, \quad (5)$$

where $J_{\nu D_q^*}(y) = \overline{q}\gamma_\nu c$ and $J_{B_q}(x) = \overline{b}\gamma_5 q$ are the interpolating currents of D_q^* and B_q mesons, respectively and $J_\mu^V = \overline{c}\gamma_\mu b$ and $J_\mu^A = \overline{c}\gamma_\mu\gamma_5 b$ are vector and axial vector transition currents . Two complete sets of intermediate states with the same quantum numbers as the currents $J_{D_q^*}$ and J_{B_q} are inserted to calculate the phenomenological part of the correlation function given in Eq. (5). After standard calculations, the following equation is obtained:

$$\Pi_{\mu\nu}^{V,A}(p^2, p'^2, q^2) =$$

$$\frac{< 0 \mid J^{\nu}_{D^*_q} \mid D^*_q(p',\varepsilon) >< D^*_q(p',\varepsilon) \mid J^{V,A}_{\mu} \mid B_q(p) >< B_q(p) \mid J_{B_q} \mid 0 >}{(p'^2 - m^2_{D^*_q})(p^2 - m^2_{Bq})} + \cdots \tag{6}$$

where $\cdots$ represents contributions coming from higher states and continuum. The matrix elements in Eq. (6) are defined as:

$$< 0 \mid J^{\nu}_{D^*_q} \mid D^*_q(p',\varepsilon) >= f_{D^*_q} m_{D^*_q} \varepsilon^{\nu} \ , \quad < B_q(p) \mid J_{B_q} \mid 0 >= -i \frac{f_{B_q} m^2_{B_q}}{m_b + m_q}, \tag{7}$$

where $f_{D^*_q}$ and f_{B_q} are the leptonic decay constants of D^*_q and B_q mesons, respectively. Using Eq. (3), Eq. (4) and Eq. (7) and performing summation over the polarization of the D^*_q meson in Eq. (6) the equation below are derived:

$$\begin{aligned}
\Pi^{A}_{\mu\nu}(p^2, p'^2, q^2) &= \frac{f_{B_q} m^2_{B_q}}{(m_b + m_q)} \frac{f_{D^*_q} m_{D^*_q}}{(p'^2 - m^2_{D^*_q})(p^2 - m^2_{B_q})} \\
&\times [-f_0 g_{\mu\nu}(m_{B_q} + m_{D^*_q}) + \frac{f_+ P_\mu p_\nu}{(m_{B_q} + m_{D^*_q})} + \frac{f_- q_\mu p_\nu}{(m_{B_q} + m_{D^*_q})}] \\
&+ \text{ excited states,} \\
\Pi^{V}_{\mu\nu}(p^2, p'^2, q^2) &= -\varepsilon_{\alpha\beta\mu\nu} p^\alpha p'^\beta \frac{f_{B_q} m^2_{B_q}}{(m_b + m_s)(m_{B_q} + m_{D^*_q})} \frac{f_{D^*_q} m_{D^*_q}}{(p'^2 - m^2_{D^*_q})(p^2 - m^2_{B_q})} f_V \\
&+ \text{ excited states.}
\end{aligned} \tag{8}$$

From the QCD (theoretical) sides, $\Pi_{\mu\nu}(p^2, p'^2, q^2)$ can also be calculated by the help of OPE in the deep space-like region where $p^2 \ll (m_b + m_q)^2$ and $p'^2 \ll (m_c + m_q)^2$. The theoretical part of the correlation function is calculated by means of OPE, and up to operators having dimension $d = 6$, it is determined by the bare-loop (Fig. 1 a) and the power corrections (Fig. 1 b, c, d) from the operators with $d = 3$, $< \overline{\psi}\psi >$, $d = 4$, $m_s < \overline{\psi}\psi >$, $d = 5$, $m_0^2 < \overline{\psi}\psi >$ and $d = 6$, $< \overline{\psi}\psi\bar{\psi}\psi >$. The $d = 6$ operator is ignored in the calculations. To calculate the bare-loop contribution, the

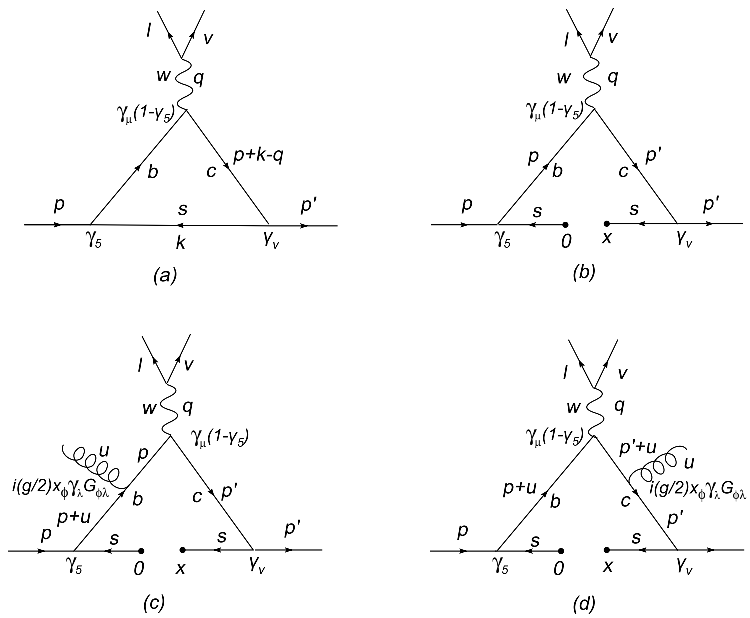

Figure 1: Feynman diagrams for $B_q \to D_q^* l\nu$ $(q = s, d, u)$ transitions.

double dispersion representation for the coefficients of corresponding Lorentz structures appearing in the correlation function are used:

$$\Pi_i^{per} = -\frac{1}{(2\pi)^2}\int ds' \int ds \frac{\rho_i(s,s',q^2)}{(s-p^2)(s'-p'^2)} + \text{ subtraction terms.} \qquad (9)$$

The spectral densities $\rho_i(s,s',q^2)$ can be calculated from the usual Feynman integral with the help of Cutkosky rules, i.e., by replacing the quark propagators with Dirac delta functions: $\frac{1}{p^2-m^2} \to -2\pi\delta(p^2-m^2)$, which implies that all quarks are real. After long and straightforward calculations for the corresponding spectral densities the following expressions are obtained:

$$\begin{aligned}
\rho_V(s,s',q^2) &= 4N_c I_0(s,s',q^2)\left[(m_b-m_q)A + (m_c-m_q)B - m_q\right], \\
\rho_0(s,s',q^2) &= -2N_c I_0(s,s',q^2)\Big[2m_q^3 - 2m_q^2(m_c+m_b) \\
&+ m_q(q^2+s+s'-2m_b m_c) + [q^2(m_b-m_q)
\end{aligned}$$

$$
\begin{aligned}
&+ \quad s(3m_q-2m_c-m_b)+s'(m_q-m_b)]A+[q^2(m_c-m_q) \\
&+ \quad s(m_q-m_c)+s'(3m_q-2m_b-m_c)]B+4(m_b-m_s)C\Big], \\
\rho_+(s,s',q^2) &= 2N_cI_0(s,s',q^2)\Big[m_q+(3m_q-m_b)A+(m_q-m_c)B \\
&+ \quad 2(m_q+m_b)D+2(m_q-m_b)E\Big], \\
\rho_-(s,s',q^2) &= 2N_cI_0(s,s',q^2)\Big[-m_q+(m_q+m_b)A-(m_q+m_c)B \\
&+ \quad 2(m_q-m_b)D+2(m_b-m_q)E\Big],
\end{aligned} \tag{10}
$$

where

$$
\begin{aligned}
I_0(s,s',q^2) &= \frac{1}{4\lambda^{1/2}(s,s',q^2)}, \\
\lambda(a,b,c) &= a^2+b^2+c^2-2ac-2bc-2ab, \\
A &= \frac{1}{(s'+s-q^2)^2-4ss'}\Big[(-2m_b^2+q^2+s-s')s' \\
&+ \quad m_q^2(q^2-s+s')+m_c^2(-q^2+s+s')\Big], \\
B &= \frac{1}{(s'+s-q^2)^2-4ss'}\Big[m_q^2(q^2+s-s') \\
&+ \quad (-2m_c^2+q^2-s+s')s+m_b^2(-q^2+s+s')\Big], \\
C &= \frac{1}{2[(s'+s-q^2)^2-4ss']}\Big[m_c^4s+m_b^4s' \\
&+ \quad q^2[m_q^4+m_q^2(q^2-s-s')+ss']+m_b^2m_c^2(q^2-s-s') \\
&- \quad (q^2+s-s')s'-m_q^2(q^2-s+s') \\
&- \quad m_c^2m_q^2(q^2+s-s')+s(q^2-s+s')\Big],
\end{aligned}
$$

$$
\begin{aligned}
D &= \frac{1}{[(s'+s-q^2)^2-4ss']^2}\Big[m_q^4[q^4-2q^2(s-2s')+(s-s')^2] \\
&+ [6m_b^4+q^4+q^2(4s-2s')+(s-s')^2-6m_b^2(q^2+s-s')]s'^2 \\
&+ m_c^4[q^4+s^2+4ss'+s'^2-2q^2(s+s')] \\
&- 2m_q^2s'[-2q^4+(s-s')^2+3m_b^2(q^2-s+s')+q^2(s+s')] \\
&- 2m_c^2m_q^2(q^2+s^2+ss'-2s'^2+q^2(-2s+s')) \\
&+ s'[q^4+q^2s-2s^2-2q^2s'+ss'+s'^2+3m_b^2(-q^2+s+s')]\Big], \\
E &= \frac{1}{[s'+s-q^2)^2-4ss']^2}\Big[2m_q^4q^4+m_q^2q^6-m_q^4q^2s-m_q^2q^4s-m_q^4s^2 \\
&- m_q^2q^2s^2+m_q^2s^3-m_q^4q^2s'-m_q^2q^4s'+2m_q^4ss' \\
&+ 6m_q^2q^2ss'+2q^2ss'-m_q^2s^2s'-q^2s^2s'-s^3s' \\
&+ 3m_b^4(q^2-s+s')s'-m_q^4s'^2-m_q^2q^2s'^2-m_q^2ss'^2 \\
&- q^2ss'^2+2s^2s'^2+m_q^2s'^3-ss'^3-3m_c^4s(-q^2+s+s') \\
&- 2m_c^2m_q^2[q^4-2s^2+q^2(s-2s')+ss'+s'^2)] \\
&+ s[q^2+s^2+ss'-2s'^2+q^2(-2s+s')] \\
&+ 2m_b^2\{-m_q^2(q^4-2q^2s+s^2+q^2s'+ss'-2s'^2) \\
&- s'(q^4+q^2s-2s^2-q^2s'+ss'+s'^2) \\
&+ m_c^2[q^4+s^2+4ss'+s'^2-2q^2(s+s')]\}\Big].
\end{aligned}
\tag{11}
$$

The subscripts V, 0 and $\pm$ correspond to the coefficients of the structures proportional to $i\varepsilon_{\mu\nu\alpha\beta}p'^{\alpha}p^{\beta}$, $g_{\mu\nu}$ and $\frac{1}{2}(p_\mu p_\nu \pm p'_\mu p_\nu)$, respectively. In Eq. (10) $N_c = 3$ is the number of colors.

The integration region for the perturbative contribution in Eq. (9) is determined from the condition that arguments of the three δ functions must vanish simultaneously. The physical region in s and s' plane is described by the following inequalities:

$$-1 \leq \frac{2ss' + (s + s' - q^2)(m_b^2 - s - m_q^2) + (m_q^2 - m_c^2)2s}{\lambda^{1/2}(m_b^2, s, m_q^2)\lambda^{1/2}(s, s', q^2)} \leq +1. \qquad (12)$$

From this inequalities, we calculate s in terms of s' in order to put to the lower limit of integration over s. For the contribution of power corrections, i.e., the contributions of operators with dimensions $d = 3$, 4 and 5, the following results were derived:

$$\begin{aligned}
f_V^{(3)} + f_V^{(4)} + f_V^{(5)} &= \frac{1}{2} < \bar{q}q > \Bigg[-\frac{1}{rr'^3} m_c^2(m_0^2 - 2m_q^2) \\
&- \frac{1}{3r^2r'^2}[-3m_q^2(m_b^2 + m_c^2 - q^2) \\
&+ m_0^2(m_b^2 + m_b m_c + m_c^2 - q^2)] \\
&- \frac{1}{rr'^2} m_c m_q - \frac{1}{r^3r'} m_b^2(m_0^2 - 2m_q^2) \\
&+ \frac{1}{3r^2r'}(2m_0^2 - 3m_b m_q) + \frac{2}{rr'} \Bigg], \\
f_0^{(3)} + f_0^{(4)} + f_0^{(5)} &= \frac{1}{4} < \bar{q}q > \Bigg[-\frac{1}{rr'^3} m_c^2(m_0^2 - 2m_q^2) \\
&\times (m_b^2 + 2m_b m_c + m_c^2 - q^2) \\
&- \frac{1}{3r^2r'^2}(m_b^2 + 2m_b m_c + m_c^2 - q^2) \\
&\times [-3m_q^2(m_b^2 + m_c^2 - q^2) + m_0^2(m_b^2 + m_b m_c + m_c^2 - q^2)] \\
&- \frac{1}{3rr'^2}[m_0^2(m_b^2 + 3m_b m_c - q^2) \\
&+ 3(m_c - m_q)m_q(m_b^2 + 2m_b m_c + m_c^2 - q^2)] \\
&- \frac{1}{r^3r'} m_b^2(m_0^2 - 2m_q^2)(m_b^2 + 2m_b m_c + m_c^2 - q^2) \\
&+ \frac{1}{3r^2r'}[-3(m_b - m_q)m_q(m_b^2 + 2m_b m_c + m_c^2 - q^2) \\
&+ m_0^2(m_c^2 + 3m_b m_c - q^2)] \\
&+ \frac{1}{3rr'}(4m_0^2 + 6m_b^2 + 12m_b m_c + 6m_c^2
\end{aligned}$$

$$
\begin{aligned}
&- && 3m_b m_q + 3m_c m_q - 6m_q^2 - 6q^2)\Bigg], \\
f_+^{(3)} + f_+^{(4)} + f_+^{(5)} &= && \frac{1}{4} < \overline{q}q > \Bigg[-\frac{1}{rr'^3} m_c^2 (m_0^2 - 2m_q^2) \\
&+ && \frac{1}{3r^2 r'^2} [-3m_q^2 (m_b^2 + m_c^2 - q^2) \\
&+ && m_0^2 (m_b^2 + m_b m_c + m_c^2 - q^2)] \\
&+ && \frac{1}{rr'^2} m_c m_q < \overline{q}q > + \frac{1}{4r^3 r'} m_b^2 (m_0^2 - 2m_q^2) \\
&+ && \frac{1}{3r^2 r'} [-4m_0^2 + 3m_q (m_b + 2m_q)] - \frac{1}{3rr'} \Bigg], \\
f_-^{(3)} + f_-^{(4)} + f_-^{(5)} &= && \frac{1}{4} < \overline{q}q > \Bigg[-\frac{1}{rr'^3} m_c^2 (m_0^2 - 2m_q^2) \\
&- && \frac{1}{3r^2 r'^2} [-3m_q^2 (m_b^2 + m_c^2 - q^2) \\
&+ && m_0^2 (m_b^2 + m_b m_c + m_c^2 - q^2)] \\
&- && \frac{1}{rr'^2} m_c m_q - \frac{1}{r^3 r'} m_b^2 (m_0^2 - 2m_q^2) \\
&+ && \frac{1}{r^2 r'} m_q (-m_b + 2m_q) + \frac{2}{rr'} \Bigg],
\end{aligned}
\tag{13}
$$

where $r = p^2 - m_b^2$ and $r' = p'^2 - m_c^2$. Here we should mentioned that, considering the definition of double dispersion relation in Eq. (9) and parametrization of the form factors and the coefficient of selected structures, with the changes: 1) $b \to c$ and $c \to s$, 2) set the $m_q \to 0$ and 3) ignore the terms $\sim m_s^2$, the Eqs. (10, 13) reduce to the expressions for the spectral densities and quark condensate contributions up to 5 mass dimensions for the form factors f_V, f_0 and f_+ presented in the appendix A of [18] which describes the form factors of $D \to K^* e\nu$.

By equating the phenomenological expression given in Eq. (8) and the OPE expression given by Eqs. (10-13), and applying double Borel transformations with respect to the variables p^2 and p'^2 ($p^2 \to M_1^2$, $p'^2 \to M_2^2$) in order to suppress the contributions of higher states and continuum, the QCD sum rules for the form factors f_V, f_0, f_+ and f_- are obtained:

$$f_i(q^2) = \kappa \frac{(m_b + m_q)}{f_{B_q} m_{B_q}^2} \frac{\eta}{f_{D_q^*} m_{D_q^*}} e^{m_{B_q}^2/M_1^2 + m_{D_q^*}^2/M_2^2}$$
$$\times [\frac{1}{(2\pi)^2} \int_{(m_c+m_s)^2}^{s_0'} ds' \int_{f(s')}^{s_0} ds \rho_i(s, s', q^2) e^{-s/M_1^2 - s'/M_2^2}$$
$$+\hat{B}(f_i^{(3)} + f_i^{(4)} + f_i^{(5)})], \tag{14}$$

where $i = V, 0$ and $\pm$, and $\hat{B}$ denotes the double Borel transformation operator and $\eta = m_{B_q} + m_{D_q^*}$ for $i = V, \pm$ and $\eta = \frac{1}{m_{B_q} + m_{D_q^*}}$ for $i = 0$ are considered. Here $\kappa = +1$ for $i = \pm$ and $\kappa = -1$ for $i = 0$ and V. In Eq. (14), in order to subtract the contributions of the higher states and the continuum, the quark-hadron duality assumption is used, i.e., it is assumed that

$$\rho^{higherstates}(s, s') = \rho^{OPE}(s, s') \theta(s - s_0) \theta(s' - s_0'). \tag{15}$$

In calculations the following rule for the double Borel transformations is used:

$$\hat{B} \frac{1}{r^m} \frac{1}{r'^n} \to (-1)^{m+n} \frac{1}{\Gamma(m)} \frac{1}{\Gamma(n)} e^{-m_b^2/M_1^2} e^{-m_c^2/M_2^2} \frac{1}{(M_1^2)^{m-1} (M_2^2)^{n-1}}. \tag{16}$$

Here, we should mention that the contribution of higher dimensions are proportional to the powers of the inverse of the heavy quark masses, so this contributions are suppressed.

Next, we present the infinite heavy quark mass limit of the form factors for $B_q \to D_q^* l\nu$ transitions. In HQET, the following procedure are used (see [19, 20, 21]). First, we use the following parametrization:

$$y = \nu\nu' = \frac{m_{B_q}^2 + m_{D_q^*}^2 - q^2}{2 m_{B_q} m_{D_q^*}} \tag{17}$$

where ν and ν' are the four-velocities of the initial and final meson states, respectively and $y = 1$ is so called zero recoil limit. Next, we try to find the y dependent expressions of the form factors by taking $m_b \to \infty$, $m_c = \frac{m_b}{\sqrt{z}}$, where z is given by $\sqrt{z} = y + \sqrt{y^2-1}$ and setting the mass of light quarks to zero. In this limit the Borel parameters take the form $M_1^2 = 2T_1 m_b$ and $M_2^2 = 2T_2 m_c$ where T_1 and T_2 are the new Borel parameters.

The new continuum thresholds ν_0, and ν_0' take the following forms in this limit

$$\nu_0 = \frac{s_0 - m_b^2}{m_b}, \qquad \nu_0' = \frac{s_0' - m_c^2}{m_c}, \tag{18}$$

and the new integration variables are defined as:

$$\nu = \frac{s - m_b^2}{m_b}, \qquad \nu' = \frac{s' - m_c^2}{m_c}. \tag{19}$$

The leptonic decay constants are rescaled:

$$\hat{f}_{B_q} = \sqrt{m_b} f_{B_q}, \qquad \hat{f}_{D_q^*} = \sqrt{m_c} f_{D_q^*}. \tag{20}$$

After the standard calculations, we obtain the y-dependent expressions of the form factors as follows:

$$\begin{aligned} f_V &= \frac{(1+\sqrt{z})}{48 \hat{f}_{D_q^*} \hat{f}_{B_q} z^{1/4}} e^{(\frac{\Lambda}{T_1} + \frac{\overline{\Lambda}}{T_2})} \Bigg\{ \\ & \frac{3}{\pi^2 (y+1)\sqrt{y^2-1}} \int_0^{\nu_0} d\nu \int_0^{\nu_0'} d\nu' (\nu + \nu') e^{-\frac{\nu}{2T_1} - \frac{\nu'}{2T_2}} \theta(2y\nu\nu' - \nu^2 - \nu'^2) \\ &+ 16 < \overline{q} q > \left[1 - \frac{m_0^2}{8} \left(\frac{1}{2T_1^2} + \frac{1}{2T_2^2} + \frac{1}{3T_1 T_2} (1 + \frac{1}{\sqrt{z}} + \frac{1}{z}) \right) \right] \Bigg\}, \end{aligned} \tag{21}$$

$$\begin{aligned} f_0 &= \frac{z^{1/4}}{16 \hat{f}_{D_q^*} \hat{f}_{B_q} (1+\sqrt{z})} e^{(\frac{\Lambda}{T_1} + \frac{\overline{\Lambda}}{T_2})} \Bigg\{ \frac{3}{\pi^2 \sqrt{y^2-1}} \int_0^{\nu_0} d\nu \int_0^{\nu_0'} d\nu' (\nu + \nu') e^{-\frac{\nu}{2T_1} - \frac{\nu'}{2T_2}} \\ & \theta(2y\nu\nu' - \nu^2 - \nu'^2) + \frac{< \overline{q} q > \sqrt{z}}{3} \Bigg[\left(\frac{1}{2} + \frac{1}{2z} + \frac{1}{\sqrt{z}} \right) \\ & \left(16 - m_0^2 (\frac{1}{T_1^2} + \frac{1}{T_1^2}) \right) - \frac{m_0^2}{T_1 T_2} \left(1 + \frac{1}{3z^{\frac{3}{2}}} + \frac{4}{3\sqrt{z}} + \frac{1}{z} + \frac{\sqrt{z}}{3} \right) \Bigg] \Bigg\}, \end{aligned} \tag{22}$$

$$
\begin{aligned}
f_+ &= \frac{(1+\sqrt{z})}{96\hat{f}_{D_q^*}\hat{f}_{B_q}z^{1/4}} e^{(\frac{\Lambda}{T_1}+\frac{\overline{\Lambda}}{T_2})}\Bigg\{ \\
&\frac{9}{\pi^2(y+1)\sqrt{y^2-1}}\int_0^{\nu_0} d\nu \int_0^{\nu_0'} d\nu'(\nu+\nu')e^{-\frac{\nu}{2T_1}-\frac{\nu'}{2T_2}}\theta(2y\nu\nu'-\nu^2-\nu'^2) \\
- &\; 16 < \overline{q}q > \left[1+\frac{m_0^2}{8}\left(\frac{1}{2T_1^2}+\frac{1}{2T_2^2}+\frac{1}{3T_1T_2}(1+\frac{1}{\sqrt{z}}+\frac{1}{z})\right)\right]\Bigg\}, \qquad (23)
\end{aligned}
$$

$$
\begin{aligned}
f_- &= -\frac{(1+\sqrt{z})}{96\hat{f}_{D_q^*}\hat{f}_{B_q}z^{1/4}} e^{(\frac{\Lambda}{T_1}+\frac{\overline{\Lambda}}{T_2})}\Bigg\{ \\
&\frac{9}{\pi^2(y+1)\sqrt{y^2-1}}\int_0^{\nu_0} d\nu \int_0^{\nu_0'} d\nu'(\nu+\nu')e^{-\frac{\nu}{2T_1}-\frac{\nu'}{2T_2}}\theta(2y\nu\nu'-\nu^2-\nu'^2) \\
+ &\; 16 < \overline{q}q > \left[1-\frac{m_0^2}{8}\left(\frac{1}{2T_1^2}+\frac{1}{2T_2^2}+\frac{1}{3T_1T_2}(1+\frac{1}{\sqrt{z}}+\frac{1}{z})\right)\right]\Bigg\}, \qquad (24)
\end{aligned}
$$

where $\Lambda = m_{B_q} - m_b$ and $\bar{\Lambda} = m_{D_q^*} - m_c$.

At the end of this section, we would like to present $\frac{1}{m_b}$ corrections for the form factors in Eqs. (21)-(24) using subleading Isgur-Wise form factors similar to [15] (see also [20, 22]). These corrections are given as:

$$
\begin{aligned}
f_V^{(1/m_b)} &= \frac{m_B+m_D^*}{\sqrt{m_B m_D^*}}\left\{\frac{\Lambda}{2m_b}+\frac{\Lambda}{m_b}[\rho_1(y)-\rho_4(y)]\right\}, \\
f_0^{(1/m_b)} &= \frac{(y+1)\sqrt{m_B m_D^*}}{m_B+m_D^*}\left\{\frac{\Lambda}{2m_b}\frac{y-1}{y+1}+\frac{\Lambda}{m_b}[\rho_1(y)-\frac{y-1}{y+1}\rho_4(y)]\right\}, \\
f_+^{(1/m_b)} &= \frac{1}{2}f_V^{(1/m_b)}, \\
f_-^{(1/m_b)} &= -f_+^{(1/m_b)}, \qquad (25)
\end{aligned}
$$

where the explicit expressions for $\rho_i(y)$ functions are given in [15]. The value of those functions at zero recoil limit ($y = 1$) are given as

$$
\rho_1(1) = \rho_2(1) = 0, \qquad \rho_3(1) \simeq 0, \qquad \rho_4(1) \simeq \frac{1}{3}. \qquad (26)
$$

3 Numerical analysis

This section is devoted by numerical analysis for the form factors $f_V(q^2)$, $f_0(q^2)$, $f_+(q^2)$ and $f_-(q^2)$. From sum rule expressions of these form factors it is clear that the condensates, leptonic decay constants of B_q and D_q^* mesons, continuum thresholds s_0 and s_0' and Borel parameters M_1^2 and M_2^2 are the main input parameters. In the numerical analysis the values of the condensates are chosen at a fixed renormalization scale of about 1 GeV. The values of the condensates are[23] : $<\overline{u}u>=<\overline{d}d>=-(240\pm10\ MeV)^3$, $<\overline{s}s>=(0.8\pm0.2)<\overline{u}u>$ and $m_0^2=0.8\ GeV^2$. The quark masses are taken to be $m_c(\mu=m_c)=1.275\pm0.015\ GeV$, $m_s=95\pm25\ MeV$, $m_u=(1.5-3)\ MeV$, $m_d\simeq(3-5)\ MeV$ [14] and $m_b=(4.7\pm0.1)\ GeV$ [23]. The mesons masses are chosen to be $m_{D_s^*}=2.112\ GeV$, $m_{D_u^*}=2.007\ GeV$, $m_{D_d^*}=2.010\ GeV$, $m_{B_s}=5.3\ GeV$, $m_{B_d}=5.2794\ GeV$and $m_{B_u}=5.2790\ GeV$[14]. For the values of the leptonic decay constants of B_q and D_q^* mesons the results obtained from two-point QCD analysis are used: $f_{B_s}=0.209\pm38\ GeV$ [12], $f_{D_s^*}=0.266\pm0.032\ GeV$[11]. For the others $f_{B_{d(u)}}=0.14\pm0.01\ GeV$ and $f_{D^*_{d(u)}}=0.23\pm0.02\ GeV$[14] are selected. The threshold parameters s_0 and s_0' are also determined from the two-point QCD sum rules: $s_0=(35\pm2)\ GeV^2$ [24] and $s_0'=(6-8)\ GeV^2$ [11]. The Borel parameters M_1^2 and M_2^2 are not physical quantities, hence form factors should not depend on them. The reliable regions for the Borel parameters M_1^2 and M_2^2 can be determined by requiring that both the continuum contribution and the contribution of the operator with the highest dimension be small. As a result of the above-mentioned requirements, the working regions are determined to be $10\ GeV^2<M_1^2<25\ GeV^2$ and $4\ GeV^2<M_2^2<10\ GeV^2$.

To determine the decay width of $B_q\to D_q^*l\nu$, the q^2 dependence of the form factors $f_V(q^2)$, $f_0(q^2)$, $f_+(q^2)$ and $f_-(q^2)$ in the whole physical region

$m_l^2 \leq q^2 \leq (m_{B_q} - m_{D_q^*})^2$ are needed. The value of the form factors at $q^2 = 0$ are given in Table 1.

$f_i(0)$	$B_s \to D_s^* \ell \nu$	$B_d \to D_d^* \ell \nu$	$B_u \to D_u^* \ell \nu$
$f_V(0)$	0.36 ± 0.08	0.47 ± 0.13	0.46 ± 0.13
$f_0(0)$	0.17 ± 0.03	0.24 ± 0.05	0.24 ± 0.05
$f_+(0)$	0.11 ± 0.02	0.14 ± 0.025	0.13 ± 0.025
$f_-(0)$	-0.13 ± 0.03	-0.16 ± 0.04	-0.15 ± 0.04

Table 1: The value of the form factors at $q^2 = 0$

.

The q^2 dependence of the form factors can be calculated from QCD sum rules (for details, see [18, 25]). To obtain the q^2 dependent expressions of the form factors from QCD sum rules, q^2 should be stay approximately 1 GeV^2 below the perturbative cut, i.e., up to 10 GeV^2. Our sum rules, also, are truncated at $\simeq$ 10 GeV^2, but in the interval $0 \leq q^2 \leq 10\ GeV^2$ we can trust the sum rules. For the reliability of the sum rules in the full physical region, the parametrization of the form factors were identified such that in the region $0 \leq q^2 \leq 10\ GeV^2$, these parameterizations coincide with the sum rules prediction. Figs. 2, 3, 4 and 5 show the dependence of the form factors $f_V(q^2)$, $f_0(q^2)$, $f_+(q^2)$ and $f_-(q^2)$ on q^2. To find the extrapolation of the form factors, we choose the following two fit functions.

i)

$$f_i(q^2) = \frac{f_i(0)}{1 + \alpha\hat{q} + \beta\hat{q}^2 + \gamma\hat{q}^3 + \lambda\hat{q}^4}, \tag{27}$$

where $\hat{q} = q^2/m_{B_q}^2$. The values of the parameters $f_i(0), \alpha, \beta, \gamma$, and λ are given in Tables 2, 3 and 4.

ii)

$$f_i(q^2) = \frac{a}{(q^2 - m_{B^*}^2)} + \frac{b}{(q^2 - m_{fit}^2)}. \tag{28}$$

The values for a, b and m_{fit}^2 are given in Tables 5, 6 and 7. For details about the fit parametrization (ii) which is theoretically more reliable and some other fit functions see [26, 27]. These two parameterizations coincide well with the sum rules predictions in the whole physical region $0 \le q^2 \le 10\ GeV^2$ and also for $q^2 < 0$ region. For higher q^2, starting from the upper limit of the physical region the two fit functions deviate from each other and this behavior is almost the same for all form factors. As an example, we present the deviation of above mentioned fit functions in Fig. 6. From this figure, we see that in the outside of the physical region the fit (i) growthes more rapidly than fit (ii). The fit parametrization (ii) depicts that the m_{B^*} pole exists outside the allowed physical region and related to that one could calculate the hadronic parameters such as $g_{BB^*D^*}$ (see [26, 28]).

	f(0)	α	β	γ	λ
f_V	0.38	-2.53	2.77	-2.41	0.03
f_0	0.18	-1.77	0.98	-0.23	-3.50
f_+	0.12	-2.90	3.66	-3.72	-1.69
f_-	-0.15	-2.63	2.72	-0.99	-6.48

Table 2: Parameters appearing in the fit function (i) for form factors of the $B_s \to D_s^*(2112)\ell\nu$ at $M_1^2 = 19\ GeV^2$, $M_2^2 = 5\ GeV^2$.

	f(0)	α	β	γ	λ
f_V	0.46	-2.90	2.99	0.67	-5.04
f_0	0.24	-0.21	2.19	-1.68	-2.15
f_+	0.13	-4.21	9.52	-16.86	12.97
f_-	-0.15	-3.93	-8.03	-13.48	9.15

Table 3: Parameters appearing in the fit function (i) for form factors of the $B_u \to D_u^*(2007)\ell\nu$ at $M_1^2 = 19\ GeV^2$, $M_2^2 = 5\ GeV^2$.

	f(0)	α	β	γ	λ
f_V	0.47	-3.08	4.83	-5.95	2.95
f_0	0.24	-2.20	2.18	-1.83	-1.90
f_+	0.14	-4.13	8.99	-15.10	10.65
f_-	-0.16	-3.87	7.73	-12.71	8.26

Table 4: Parameters appearing in the fit function (i) for form factors of the $B_d \to D_d^*(2010)\ell\nu$ at $M_1^2 = 19\ GeV^2$, $M_2^2 = 5\ GeV^2$.

	a	b	m_{fit}^2
f_V	55.03	-54.30	23.18
f_0	1.43	-4.32	18.80
f_+	1.14	-2.57	14.88
f_-	-2.80	3.43	14.60

Table 5: Parameters appearing in the fit function (ii) for form factors of the $B_s \to D_s^*(2112)\ell\nu$ at $M_1^2 = 19\ GeV^2$, $M_2^2 = 5\ GeV^2$.

In deriving the numerical values for the ratio of the form factors at HQET limit, we take the value of the Λ and $\overline{\Lambda}$ obtained from two point sum rules, $\Lambda = 0.62 GeV$ [29] and $\overline{\Lambda} = 0.86 GeV$[30]. The following relations are defined for the ratio of the form factors,

$$
\begin{aligned}
R_{1(2)[3]} &= \left[1 - \frac{q^2}{(m_B + m_{D^*})^2}\right] \frac{f_{V(+)[-]}(y)}{f_0(y)}, \\
R_{4(5)} &= \left[1 - \frac{q^2}{(m_B + m_{D^*})^2}\right] \frac{f_{+(-)}(y)}{f_V(y)}, \\
R_6 &= \left[1 - \frac{q^2}{(m_B + m_{D^*})^2}\right] \frac{f_-(y)}{f_+(y)},
\end{aligned}
\tag{29}
$$

The numerical values of the above mentioned ratios and a comparison of our results with the predictions of [15] which presents the application of

	a	b	m^2_{fit}
f_V	118.69	-108.48	23.43
f_0	4.54	-5.12	20.74
f_+	7.79	-5.84	14.57
f_-	-6.72	5.46	14.02

Table 6: Parameters appearing in the fit function (ii) for form factors of the $B_u \to D^*_u(2007)\ell\nu$ at $M_1^2 = 19\ GeV^2$, $M_2^2 = 5\ GeV^2$.

	a	b	m^2_{fit}
f_V	115.74	-106.73	23.41
f_0	10.43	-12.85	20.66
f_+	5.50	-5.07	14.58
f_-	-5.36	4.90	14.03

Table 7: Parameters appearing in the fit function (ii) for form factors of the $B_d \to D^*_d(2010)\ell\nu$ at $M_1^2 = 19\ GeV^2$, $M_2^2 = 5\ GeV^2$.

the subleading Isgur-Wise form factors for $B \to D^*\ell\nu$ are shown in Table 8. Note that the values in this Table are obtained with $T_1 = T_2 = 2\ GeV$ correspond to $M_1^2 = 19\ GeV^2$ and $M_2^2 = 5\ GeV^2$ which are used in Tables [2-7].

Table 8 shows a good consistency between our results and the prediction of [15] for R_1 at zero recoil limit, $y = 1.1$ and 1.2, but for the other values of y, the changes in present work results are little greater. The values for R_2 shows an approximate agreement between two predictions, however the changes in the value of R_2 in our work is also a bit more then [15]. For both R_1 and R_2, our study and [15] predictions have the same behavior, i.e., R_1 decreases when the value of y is increased and increasing in the value of y causes the increasing in the value of R_2. From this Table, we also see that the

y	1 (zero recoil)	1.1	1.2	1.3	1.4	1.5
$q^2(GeV^2)$	10.69	8.57	6.45	4.33	2.20	0.08
R_1	1.34	1.31	1.25	1.19	1.10	0.95
R_2	0.80	0.99	1.10	1.22	1.30	1.41
R_3	-0.80	-0.79	-0.80	-0.81	-0.80	-0.80
R_4	0.50	0.64	0.77	0.94	1.20	1.46
R_5	-0.50	-0.51	-0.56	-0.62	-0.71	-0.89
R_6	-0.80	-0.67	-0.64	-0.61	-0.55	-0.53
R_1 [15]	1.31	1.30	1.29	1.28	1.27	1.26
R_2 [15]	0.90	0.90	0.91	0.92	0.92	0.93

Table 8: The values for the R_i and comparison of $R_{1,2}$ values with the predictions of [15].

R_4 is sensitive to the changes in the value of y. However, the results of R_3, R_5 and R_6 vary slowly with respect to y. Our numerical analysis for $1/m_b$ corrections of form factors in Eq. (25) shows that this correction increase the HQET limit of the form factors f_V and f_+ about $7.1^0/_0$ and $6^0/_0$, respectively, however it doesn't change the f_0 and decrease the f_- about $6.5^0/_0$.

The next step is to calculate the differential decay width in terms of the form factors. After some calculations for differential decay rate

$$\begin{aligned}
\frac{d\Gamma}{dq^2} &= \frac{1}{8\pi^4 m_{B_q}^2} \mid \overrightarrow{p'} \mid G_F^2 \mid V_{cb} \mid^2 \{(2A_1 + A_2 q^2)[\mid f'_V \mid^2 (4m_{B_q}^2 \mid \overrightarrow{p'} \mid^2) + \mid f'_0 \mid^2]\} \\
&+ \frac{1}{16\pi^4 m_{B_q}^2} |\overrightarrow{p'}| G_F^2 |V_{cb}|^2 \Bigg\{ (2A_1 + A_2 q^2) \Bigg[\mid f'_V \mid^2 (4m_{B_q}^2 \mid \overrightarrow{p'} \mid^2 \\
&+ m_{B_q}^2 \frac{\mid \overrightarrow{p'} \mid^2}{m_{D_q^*}^2} (m_{B_q}^2 - m_{D_q^*}^2 - q^2)) + \mid f'_0 \mid^2 \\
&- \mid f'_+ \mid^2 \frac{m_{B_q}^2 \mid \overrightarrow{p'} \mid^2}{m_{D_q^*}^2} (2m_{B_q}^2 + 2m_{D_q^*}^2 - q^2) - \mid f'_- \mid^2 \frac{m_{B_q}^2 \mid \overrightarrow{p'} \mid^2}{m_{D_q^*}^2} q^2 \\
&- 2 \frac{m_{B_q}^2 \mid \overrightarrow{p'} \mid^2}{m_{D_q^*}^2} (Re(f'_0 f'^*_+ + f'_0 f'^*_- + (m_{B_q}^2 - m_{D_q^*}^2) f'_+ f'^*_-)) \Bigg]
\end{aligned}$$

$$
\begin{aligned}
&- \quad 2A_2 \frac{m_{B_q}^2 \mid \overrightarrow{p'} \mid^2}{m_{D_q^*}^2} \Big[\mid f_0' \mid^2 + (m_{B_q}^2 - m_{D_q^*}^2)^2 \mid f_+' \mid^2 + q^4 \mid f_-' \mid^2 \\
&+ \quad 2(m_{B_q}^2 - m_{D_q^*}^2) Re(f_0' f_+^{'*}) + 2q^2 f_0' f_-^{'*} + 2q^2 (m_{B_q}^2 - m_{D_q^*}^2) Re(f_+' f_-^{'*}) \Big] \Big\}, \qquad (30)
\end{aligned}
$$

is obtained, where

$$
\begin{aligned}
\mid \overrightarrow{p'} \mid &= \frac{\lambda^{1/2}(m_{B_q}^2, m_{D_q^*}^2, q^2)}{2m_{B_q}}, \\
A_1 &= \frac{1}{12q^2}(q^2 - m_l^2)^2 I_0, \\
A_2 &= \frac{1}{6q^4}(q^2 - m_l^2)(q^2 + 2m_l^2) I_0, \\
I_0 &= \frac{\pi}{2}(1 - \frac{m_l^2}{q^2}), \\
f_0' &= f_0(m_{D_q^*} + m_{B_q}), \\
f_V' &= \frac{f_V}{(m_{D_q^*} + m_{B_q})}, \\
f_+' &= \frac{f_+}{(m_{D_q^*} + m_{B_q})}, \\
f_-' &= \frac{f_-}{(m_{D_q^*} + m_{B_q})}. \qquad (31)
\end{aligned}
$$

The following part presents evaluation of the value of the branching ratio of these decays. Taking into account the q^2 dependence of the form factors and performing integration over q^2 in the interval $m_l^2 \leq q^2 \leq (m_{B_q} - m_{D_q^*})^2$ and using the total life-times $\tau_{B_u} = 1.638 \times 10^{-12}s$, $\tau_{B_d} = 1.53 \times 10^{-12}s$ [14] and $\tau_{B_s} = 1.46 \times 10^{-12}s$ [31], the branching ratios which are the same for both fit functions are obtained as:

$$
\boldsymbol{B}(B_s \to D_s^*(2112)\ell\nu) \quad = \quad (1.89 - 6.61) \times 10^{-2},
$$

$$\begin{aligned}
\boldsymbol{B}(B_d \to D_d^*(2010)\ell\nu) &= (4.36 - 8.94) \times 10^{-2}, \\
\boldsymbol{B}(B_u \to D_u^*(2007)\ell\nu) &= (4.57 - 9.12) \times 10^{-2}. \qquad (32)
\end{aligned}$$

The ranges appearing in the above equations are related to the different lepton masses (m_e, m_μ, m_τ) as well as the errors in the value of input parameters. Finally, we would like to compare our results of the branching ratios with the predictions of CQM model [13] and existing experimental data in Table 9. From this Table, we see a good agreement among the phenomenological models and the experiment for u and d cases. However for s case our results are about 1.7 times smaller than that of the CQM model. Also, there is a same behavior between present work results and the experiment. In the experiment, the value for branching ratios decreases from u to d. In our results also, this value decreases from u to s cases. The order of the branching fraction in present work for $B_s \to D_s^*\ell\nu$ decay shows that this transition could also be detected at LHC in the near future. For the present and future experiments about the semileptonic $b \to cl\nu$ based decays see [32]–[37] and references therein. The comparison of results from the experiments and phenomenological models like QCD sum rules could give useful information about the strong interaction inside the D_s^* and its structure.

In conclusion, the form factors related to the $B_q \to D_q^*\ell\nu$ decays were calculated using QCD sum rules approach. The HQET limit of the form factors as well as $1/m_b$ corrections to those limits were also obtained. A comparison of the results of form factors in HQET limit with the application of the subleading Isgur-Wise form factors at zero recoil limit and others values of y was presented. Taking into account the q^2 dependencies of the form factors, the total decay width and branching ratio for these decays were evaluated. Our results are in good agreement with that of the CQM model and existing experimental data. The result of $B_s \to D_s^*\ell\nu$ case shows a possibility to detect this decay channel at LHC in the near future.

	$B_s \to D_s^* \ell \nu$	$B_d \to D_d^* \ell \nu$	$B_u \to D_u^* \ell \nu$
Present study	$(1.89 - 6.61) \times 10^{-2}$	$(4.36 - 8.94) \times 10^{-2}$	$(4.57 - 9.12) \times 10^{-2}$
CQM model	$(7.49 - 7.66) \times 10^{-2}$	$(5.9 - 7.6) \times 10^{-2}$	$(5.9 - 7.6) \times 10^{-2}$
Experiment	-	$(5.35 \pm 0.20) \times 10^{-2}$	$(6.5 \pm 0.5) \times 10^{-2}$

Table 9: Comparison of the branching ratio of the $B_q \to D_q^* \ell \nu$ decays in present study, the CQM model [13] and the experiment [14].

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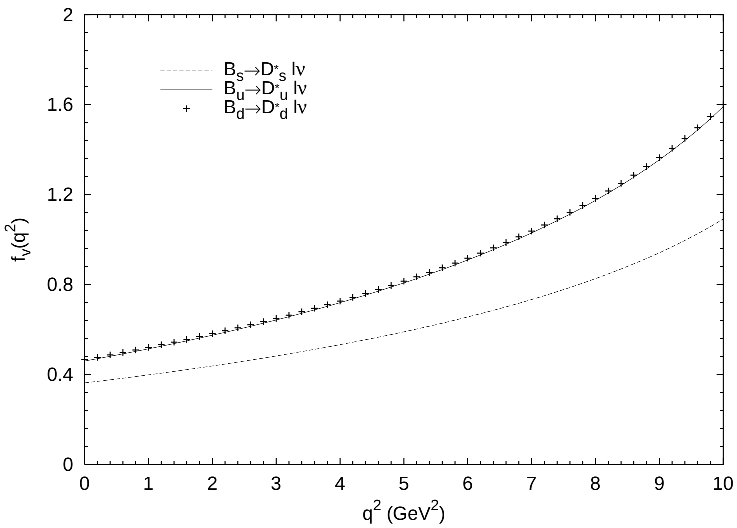

Figure 2: The dependence of f_V on q^2 at $M_1^2 = 19\ GeV^2$, $M_2^2 = 5\ GeV^2$, $s_0 = 35\ GeV^2$ and $s_0' = 6\ GeV^2$.

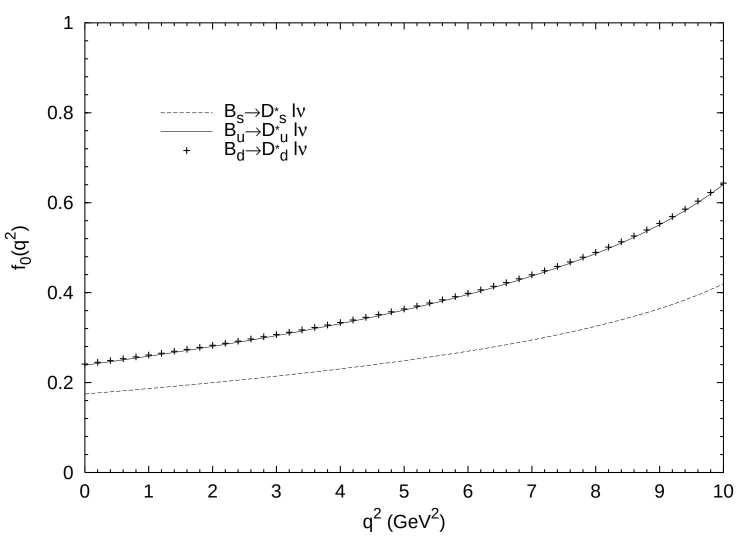

Figure 3: The dependence of f_0 on q^2 at $M_1^2 = 19\ GeV^2$, $M_2^2 = 5\ GeV^2$, $s_0 = 35\ GeV^2$ and $s_0' = 6\ GeV^2$.

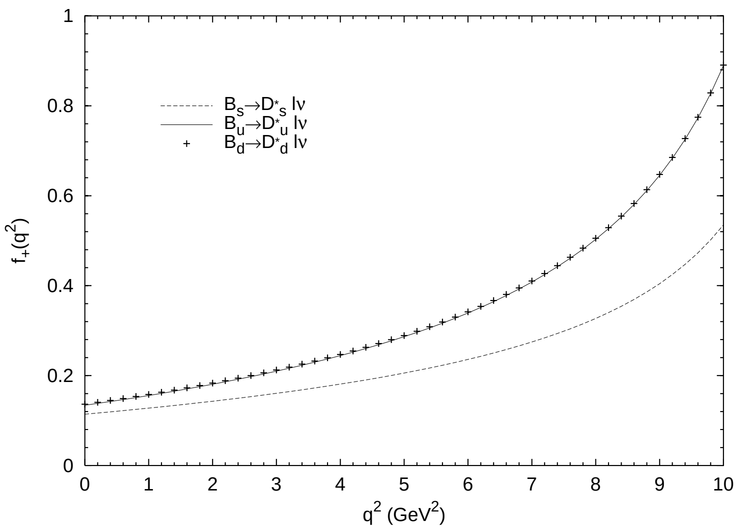

Figure 4: The dependence of f_+ on q^2 at $M_1^2 = 19\ GeV^2$, $M_2^2 = 5\ GeV^2$, $s_0 = 35\ GeV^2$ and $s_0' = 6\ GeV^2$.

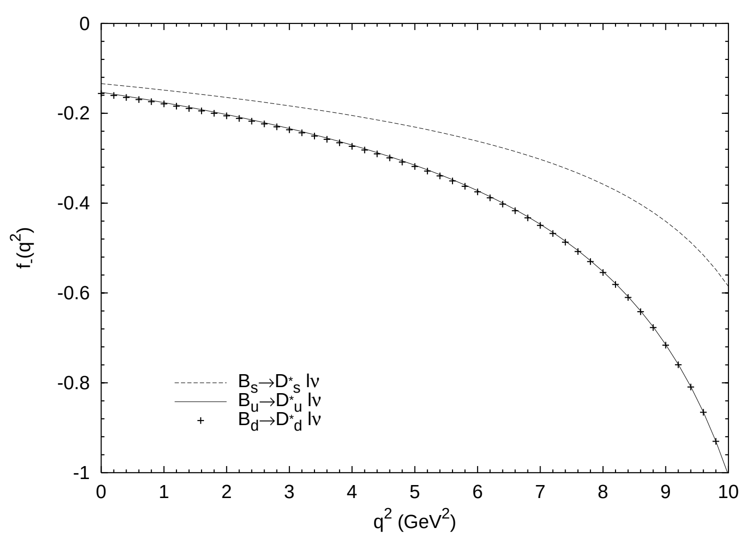

Figure 5: The dependence of f_- on q^2 at $M_1^2 = 19\ GeV^2$, $M_2^2 = 5\ GeV^2$, $s_0 = 35\ GeV^2$ and $s_0' = 6\ GeV^2$.

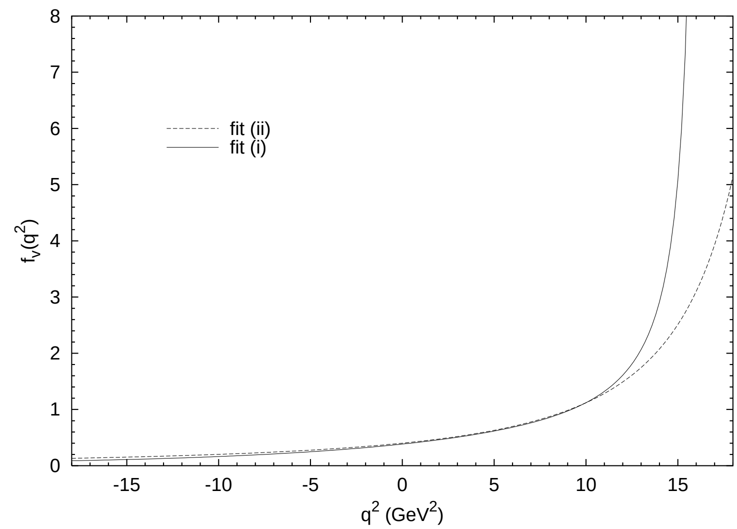

Figure 6: Comparison of fit functions (i) and (ii) for form factor f_V for $q = s$.