# The Momentum 4-Vector in Bulk Viscous Bianchi Type-V Space-time

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Using the Einstein and Bergmann-Thomson prescriptions, the energy and momentum distributions for the Bianchi type-V bulk viscous space-time are evaluated in both general relativity and the teleparallel gravity (the tetrad theory of gravity). It is shown that for the Bianchi type-V bulk viscous solution, the energy and momentum due to matter and fields including gravity are the same in both the methods used. This paper indicates an important point that these energy-momentum definitions agree with each other not only in general relativity but also in teleparallel gravity and sustains the results obtained by some physicist who show that the energy-momentum definitions of Einstein, Landau-Lifshitz, Papapetrou, Weinberg, Penrose and Bergmann-Thomson complexes give the same energy expression in general relativity.

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## I. INTRODUCTION

The conserved quantities such as energy and momentum play a important role as they provide the first integrals of equations of motions, helping one to solve otherwise intractable problems [1]. Furthermore the energy content in a sphere of radius $R$ in a given space-time gives a taste of the effective gravitational mass that a test particle situated at the same distance from the gravitating object experiences. A large number of researchers have devoted considerable attention to the problem of finding the energy as well as momentum and angular momentum associated with various space-times.

The problem of calculating the energy is considered for general relativity and also the teleparallel theory of gravity. Since the advents of these different gravitation theories various calculation methods have been proposed to deduce the conservation laws that characterize the gravitational systems. The first of such attempts was made by Einstein [2] who proposed an expression for the energy-momentum distribution of the gravitational field. There are many attempts to resolve the energy-momentum problem [3, 4, 5, 6, 7, 8, 9, 10, 11]. There exists an opinion that the energy-momentum definitions are not useful to get finite and meaningful results in a given geometry. Virbhadra and his collaborators re-opened the problem of calculating energy-momentum by using the energy-momentum prescriptions. The Einstein energy-momentum formulation, used for calculating the energy in general relativistic systems, was followed by many definitions: e.g. Tolman, Papapetrou, Bergmann-Thomson, Møller, Landau-Liftshitz, Weinberg, Qadir-Sharif and the teleparallel gravity versions of the Einstein, Landau-Lifshitz, Bergmann-Thomson and Møller's. The energy-momentum formulations give meaningful results when we transform the line element in quasi-Cartesian coordinates. The energy and momentum complex of Møller gives the possibility to perform the calculations in any coordinate system [12]. To this end Virbhadra and his collaborators have considered many space-time models and have shown that several energy-momentum complexes give the same and acceptable results for a given space-time [13, 14, 15, 16, 17, 18, 19, 21]. Virbhadra [17], using the energy and momentum complexes of Einstein, Landau-Lifshitz, Papapetrou and Weinberg for a general non-static spherically symmetric metric of the Kerr-Schild class, showed that all of these energy-momentum formulations give the same energy distribution as in the Penrose energy-momentum formulation. Later, Xulu [22], Radinschi [23] and Saltı-Havare [24] considered the Bergmann-Thomson energy and/or momentum formulation for different space-time model and showed that the definition of Bergmann-Thomson agree with the other energy-momentum complexes. Xulu made the calculations using the Kerr-Schild cartesian coordinates and the Bergmann-Thomson definition provides for the given metric the same energy expression for the energy-momentum distributions as the Einstein, Landau-Lisfhitz, Papapetrou and Weinberg energy-momentum definitions.

Recently, this problem has also been studied in teleparallel gravity. Vargas [11] using the definitions of Einstein and Landau-Lifshitz in teleparallel gravity, found that the total energy is zero in Friedmann-Robertson-Walker space-times. This results agree with the previous works by Rosen [25] and Johri [26]. Saltı *et al.* [24, 27, 28, 29] considered different space-times for various definitions in teleparallel gravity and obtained the energy-momentum distributions in a given model. Firstly, Saltı and Havare [24] considered Bergmann-Thomson's complex in both general relativity and teleparallel gravity for the Viscous Kasner-type metric and in another work, Saltı [27] using the Einstein and Landau-Lifshitz's complexes associated with the same metric in teleparallel gravity, found that total energy and momentum are zero. At the last, We

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[28] used Møller's definition in teleparallel gravity for the Bianchi-I type metric and found that the total energy is zero, so the result is the same as obtained in general relativity.

The paper is organized as follows: In the next section, we introduce the Bianchi type-V cosmological model. In section III, using Einstein and Bergmann-Thomson's energy-momentum complexes in general relativity, we calculate the total energy due to matter plus fields including gravitation for a given space-time. Section IV gives us, the energy-momentum formulations of Einstein and Bergmann-Thomson, and the total energy distribution for the same metric in teleparallel gravity. At the last, we summarize and discuss our results. Throughout this paper, All indices run from 0 to 3 otherwise instead and we use the convention that $G = 1$, $c = 1$ units.

## II. THE BIANCHI TYPE-V BULK VISCOUS SPACE-TIME

The study of Bianchi type-V cosmological models create more interest as these models contain isotropic special cases and permit arbitrarily small anisotropy levels at some instant of cosmic time. This property make them suitable as model of our universe. Also Bianchi type-V models are more complicated than the simplest Bianchi type models e.g. the Einstein tensor has off-diagonal terms so that it is more natural to include tilt and heat conduction. Space-time model od Bianchi type I, V and IX universes are the generalizations of Friedmann-Robertson-Walker models and it will be interesting to construct cosmological models of the types which are of class one. Roy and Prasad [30] have investigated the Bianchi type-V universes which are locally rotationally symmetric and are of embedding class one filled with perfect fluid with heat conduction and radiation. Bianchi type-V cosmological models have considered by some other researchers [31, 32, 33, 34, 35, 36].

We consider the Bianchi type-V space-time in the form given below.

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 e^{2x}[dy^2 + dz^2] \tag{1}$$

where $A$ and $B$ are function of $t$ only.

The matrix form of the metric tensor $g_{\mu\nu}$ is defined by respectively by

$$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & A^2 & 0 & 0 \\ 0 & 0 & B^2 e^{2x} & 0 \\ 0 & 0 & 0 & B^2 e^{2x} \end{pmatrix} \tag{2}$$

and its inverse matrix $g^{\mu\nu}$ is

$$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & A^{-2} & 0 & 0 \\ 0 & 0 & B^{-2} e^{-2x} & 0 \\ 0 & 0 & 0 & B^{-2} e^{-2x} \end{pmatrix} \tag{3}$$

The non-trivial tetrad field induces a teleparallel structure on space-time which is directly related to the presence of the gravitational field, and the Riemannian metric arises as

$$g_{\mu\nu} = \eta_{ab} h^a{}_\mu h^b{}_\nu, \qquad \eta_{ab} = diag(-1, 1, 1, 1) \tag{4}$$

Using this relation, we obtain the tetrad components $h^a{}_\mu$ as follow:

$$h^a{}_\mu = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & A & 0 & 0 \\ 0 & 0 & Be^x & 0 \\ 0 & 0 & 0 & Be^x \end{pmatrix} \tag{5}$$

and its inverse

$$h_a{}^\mu = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & A^{-1} & 0 & 0 \\ 0 & 0 & B^{-1} e^{-x} & 0 \\ 0 & 0 & 0 & B^{-1} e^{-x} \end{pmatrix} \tag{6}$$

The Einstein field equations read as

$$R^\nu_\mu - \frac{1}{2} R g^\nu_\mu = -8\pi T^\nu_\mu \tag{7}$$

where $R^\nu_\mu$ is the Ricci tensor, $R = g^{\mu\nu} R_{\mu\nu}$ is the Ricci scalar, and $T^\nu_\mu$ is the stress energy tensor. For the Bianchi type-V bulk viscous metric, the non-vanishing components of the Einstein tensor are

$$G_{11} = \frac{B^2 - A^2(\dot{B}^2 + 2B\ddot{B})}{B^2} \tag{8}$$

$$G_{22} = G_{33} = -\frac{Be^{2x}}{A^2} \left( B(A\ddot{A} - 1) + A(\dot{A}\dot{B} + A\ddot{B}) \right) \tag{9}$$

$$G_{00} = \frac{\dot{B}^2}{B^2} - \frac{3}{A^2} + \frac{2\dot{A}\dot{B}}{AB} \tag{10}$$

$$G_{01} = 2\left( \frac{\dot{A}}{A} - \frac{\dot{B}}{B} \right) \tag{11}$$

## III. THE MOMENTUM 4-VECTOR IN GENERAL RELATIVITY

### A. Formulations

The energy-momentum prescription of Einstein [2] is given by

$$\Theta^\nu_\mu = \frac{1}{16\pi} H^{\nu\alpha}_{\mu,\alpha} \tag{12}$$

where

$$H^{\nu\alpha}_\mu = \frac{g_{\mu\beta}}{\sqrt{-g}} \left[ -g(g^{\nu\beta} g^{\alpha\xi} - g^{\alpha\beta} g^{\nu\xi}) \right]_{,\xi} \tag{13}$$

$\Theta^0_0$ is the energy density, $\Theta^0_\alpha$ are the momentum density components, and $\Theta^\alpha_0$ are the components of energy current density. The Einstein energy and momentum density satisfies the local conservation laws

$$\frac{\partial \Theta^\nu_\mu}{\partial x^\nu} = 0. \tag{14}$$

and energy and momentum components are given by

$$P_\mu = \int\int\int \Theta^0_\mu dxdydz. \tag{15}$$

Further Gauss's theorem furnishes

$$P_\mu = \frac{1}{16\pi}\int\int H^{0\lambda}_\mu \eta_\lambda dS. \tag{16}$$

$\eta_\lambda$ (where $\lambda = 1, 2, 3$) stands for the 3-components of unit vector over an infinitesimal surface element $dS = r^2 \sin\theta d\theta d\phi$. $P_i$ give momentum(energy current) components $P_1$, $P_2$, $P_3$ and $P_0$ gives the energy.

The energy-momentum prescription of Bergmann-Thomson [5, 6] is given by

$$\Lambda^{\mu\nu} = \frac{1}{16\pi}\Pi^{\mu\nu\alpha}_{,\alpha} \tag{17}$$

where

$$\Pi^{\mu\nu\alpha} = g^{\mu\beta}V^{\nu\alpha}_\beta \tag{18}$$

with

$$V^{\nu\alpha}_\beta = -V^{\alpha\nu}_\beta = \frac{g_{\beta\xi}}{\sqrt{-g}}\left[-g\left(g^{\nu\xi}g^{\alpha\rho} - g^{\alpha\xi}g^{\nu\rho}\right)\right]_{,\rho}. \tag{19}$$

The Bergmann-Thomson energy-momentum prescription satisfies the following local conservation laws

$$\frac{\partial \Lambda^{\mu\nu}}{\partial x^\nu} = 0 \tag{20}$$

in any coordinate system. The energy and momentum(energy current) density components are respectively represented by $\Lambda^{00}$ and $\Lambda^{a0}$. The energy and momentum components are given by

$$P^\mu = \int\int\int \Lambda^{\mu 0} dxdydz \tag{21}$$

For the time-independent metric one has

$$P^\mu = \frac{1}{16\pi}\int\int \Pi^{\mu 0 a}\kappa_a dS. \tag{22}$$

here $\kappa_\beta$ is the outward unit normal vector over the infinitesimal surface element $dS$. $P^i$ give momentum components $P^1$, $P^2$, $P^3$ and $P^0$ gives the energy.

### B. Calculations

To calculate Einstein's energy and momentum, using equation (13), the non-vanishing components of $H^{\nu\alpha}_\mu$ are found as

$$H^{01}_0 = \frac{4B^2}{A}e^{2x}, \qquad H^{01}_1 = 4A\dot{B}e^{2x} \tag{23}$$

Next, considering these results with equation (12), the energy and momentum densities of Einstein are found as

$$\Theta^0_0 = \frac{1}{2\pi}\frac{B^2}{A}e^{2x} \tag{24}$$

$$\Theta^0_1 = \frac{1}{2\pi}AB\dot{B}e^{2x}, \qquad \Theta^0_2 = 0 \qquad \Theta^0_3 = 0 \tag{25}$$

To obtain the energy and momentum of Bergmann-Thomson, using equations (18) and (19), the required non-vanishing components of $\Pi^{\mu\nu\alpha}$ are found as

$$\Pi^{001} = -\frac{4B^2}{A}e^{2x}, \qquad \Pi^{101} = -\frac{4B}{A}\dot{B}e^{2x} \tag{26}$$

taking these results into equation (17), the energy and momentum distributions of Bergmann-Thomson are calculated as

$$\Lambda^0_0 = \frac{1}{2\pi}\frac{B^2}{A}e^{2x} \tag{27}$$

$$\Lambda^0_1 = \frac{1}{2\pi}AB\dot{B}e^{2x}, \qquad \Lambda^0_2 = 0 \qquad \Lambda^0_3 = 0 \tag{28}$$

## IV. THE MOMENTUM 4-VECTOR IN TELEPARALLEL GRAVITY

Teleparallel gravity is an alternative approach to gravitation [37] which corresponds to a gauge theory for the translation group based on the Weitzenböck geometry [38]. In this theory, gravitation is attributed to torsion [45], which plays the role of a force [40], whereas the curvature tensor vanishes identically. The fundamental field is represented by a nontrivial tetrad field, which gives rise to the metric as a by-product. The last translational gauge potentials appear as the nontrivial part of the tetrad field, thus induces on space-time a teleparallel structure which is directly related to the presence of the gravitational field. The interesting point of teleparallel gravity is that, due to gauge structure, it can reveal a more appropriate approach to consider the same specific problem. This is the case, for example, of the energy-momentum problem, which becomes more transparent when considered from the teleparallel point of view.

M$\phi$ller modified general relativity by constructing a new field theory in teleparallel space [41]. The aim of this theory was to overcome the problem of the energy-momentum complex that appears in Riemannian Space

[42]. The field equations in this new theory were derived from a Lagrangian which is not invariant under local tetrad rotation. Saez [43] generalized M$\phi$ller theory into a scalar tetrad theory of gravitation. Meyer [44] showed that M$\phi$ller theory is a special case of Poincare gauge theory [45, 46].

### A. Formulations

The teleparallel gravity analog of Einstein energy-momentum complex [11] is given by

$$hE^{\mu}{}_{\nu} = \frac{1}{4\pi}\partial_{\lambda}(U_{\nu}{}^{\mu\lambda}) \tag{29}$$

$$hB^{\mu\nu} = \frac{1}{4\pi}\partial_{\lambda}(g^{\mu\beta}U_{\beta}{}^{\nu\lambda}) \tag{30}$$

where $h = \det(h^{a}{}_{\mu})$ and $U_{\beta}{}^{\nu\lambda}$ is the Freud's super-potential, which is given by:

$$U_{\beta}{}^{\nu\lambda} = hS_{\beta}{}^{\nu\lambda}. \tag{31}$$

Here $S^{\mu\nu\lambda}$ is the tensor

$$\begin{aligned} S^{\mu\nu\lambda} &= k_1 T^{\mu\nu\lambda} + \frac{k_2}{2}(T^{\nu\mu\lambda} - T^{\lambda\mu\nu}) \\ &+ \frac{k_3}{2}(g^{\mu\lambda}T^{\beta\nu}{}_{\beta} - g^{\nu\mu}T^{\beta\lambda}{}_{\beta}) \end{aligned} \tag{32}$$

with $k_1$, $k_2$ and $k_3$ the three dimensionless coupling constants of teleparallel gravity [45]. For the teleparallel equivalent of general relativity the specific choice of these three constants are:

$$k_1 = \frac{1}{4}, \qquad k_2 = \frac{1}{2}, \qquad k_3 = -1 \tag{33}$$

To calculate this tensor firstly we must calculate Weitzenböck connection:

$$\Gamma^{\alpha}{}_{\mu\nu} = h_{a}{}^{\alpha}\partial_{\nu}h^{a}{}_{\mu} \tag{34}$$

and after this calculation we get torsion of the Weitzenböck connection:

$$T^{\mu}{}_{\nu\lambda} = \Gamma^{\mu}{}_{\lambda\nu} - \Gamma^{\mu}{}_{\nu\lambda} \tag{35}$$

For the Einstein complex, we have the relation,

$$P_{\mu} = \int_{\Sigma} hE^{0}{}_{\mu}dxdydz \tag{36}$$

where $P_i$ give momentum components $P_1$, $P_2$, $P_3$ while $P_0$ gives the energy and the integration hyper-surface $\Sigma$ is described by $x^0 = t$ =constant.

### B. Calculations

For the Bianchi type-V bulk viscous space-time, the non-vanishing components of the Weitzenböck connection are obtained as:

$$\Gamma^{1}{}_{10} = \frac{\dot{A}}{A}, \qquad \Gamma^{2}{}_{20} = \Gamma^{3}{}_{30} = \frac{\dot{B}}{B} \tag{37}$$

$$\Gamma^{2}{}_{21} = \Gamma^{3}{}_{31} = 1 \tag{38}$$

The corresponding non-vanishing torsion components are found:

$$T^{110} = -T^{101} = \frac{\dot{A}}{A^3}, \tag{39}$$

$$T^{220} = -T^{202} = T^{330} = -T^{303} = \frac{\dot{B}}{B^3}e^{-2x} \tag{40}$$

$$T^{212} = -T^{221} = T^{313} = -T^{331} = \frac{1}{A^2B^2}e^{-2x} \tag{41}$$

Taking these results into equation (32), the required non-vanishing components of the tensor $S_{\mu}{}^{\nu\lambda}$ are calculated as:

$$S^{010} = -S^{001} = \frac{1}{A^2} \tag{42}$$

$$S^{101} = \frac{\dot{B}}{BA^2} \tag{43}$$

$$S^{202} = S^{303} = \frac{\dot{A}}{2AB^2}e^{-2x} \tag{44}$$

$$S^{212} = S^{313} = -\frac{1}{2A^2B^2}e^{-2x} \tag{45}$$

Now, using equation (31) the non-vanishing components of Freud's super-potential are found as

$$U_{0}^{01} = \frac{B^2}{A}e^{2x}, \qquad U_{1}^{01} = \frac{B}{A}\dot{B}e^{2x} \tag{46}$$

Using equations (29) and (30) with these results respectively, Einstein and Bergmann-Thomson's energy and momentum distributions due to matter plus fields including gravitation in teleparallel gravity are calculated as given below.

$$hE^{0}{}_{0} = hB^{0}{}_{0} = \frac{1}{2\pi}\frac{B^2}{A}e^{2x} \tag{47}$$

$$hE^{0}{}_{1} = hB^{0}{}_{1} = \frac{1}{2\pi}AB\dot{B}e^{2x} \tag{48}$$

$$hE^{0}{}_{2} = hB^{0}{}_{2} = 0 \tag{49}$$

$$hE^{0}{}_{3} = hB^{0}{}_{3} = 0 \tag{50}$$

From this point of view this result are the same as obtained in general relativity.

## V. CONCLUSIONS

The main object of this paper is to show that it is possible to evaluate the energy and momentum (due to matter and fields including gravitation) distributions by using the energy-momentum formulations not only in general relativity but also in the teleparallel gravity (the tetrad theory of gravity). To compute the energy and momentum densities (due to matter and fields including gravitation), we considered two different approaches of the Einstein and Bergmann-Thomson energy and/or momentum definitions.

We found that the energy and momentum distributions associated with the Bianchi type-V bulk viscous spacetime are the same in both general relativity and teleparallel gravity. Next, our results advocate the importance of energy-momentum complexes (opposes the against that different complexes could give different meaningless results for a given metric).

The components of Einstein energy-momentum tensor are different from our result, because the energy momentum densities obtained in this paper involve the matter and field including gravity inside arbitrary two surfaces.

Virbhadra [14], Xulu [22] and Radinschi [23] show that the energy momentum definitions of Einstein, Landau-Lifshitz, Papapetrou, Weinberg, Penrose and Bergmann-Thomson complexes give the same energy expression in general relativity. This paper indicates an important point that these energy momentum definitions agree with each other not only in general relativity but also in teleparallel gravity and sustains the results of the authors.

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