

$\tau \rightarrow \mu \bar{\nu}_i \nu_i$ decay in the general two Higgs doublet model

E. O. Iltan ^{*} and H. Sundu [†]

Physics Department, Middle East Technical University
Ankara, Turkey

Abstract

We study $\tau \rightarrow \mu \bar{\nu}_i \nu_i$, $i = e, \mu, \tau$ decay in the model III version of the two Higgs doublet model. We calculated the BR at the order of the magnitude of $10^{-6} - 10^{-4}$ for the intermediate values of the Yukawa couplings. Furthermore, we predict the upper limit of the coupling for the $\tau - h^0(A^0) - \tau$ transition as ~ 0.3 in the case that the BR is $\sim 10^{-6}$. We observe that the experimental result of the process under consideration can give comprehensive information about the physics beyond the standard model and the free parameters existing.

^{*}E-mail address: eiltan@heraklit.physics.metu.edu.tr

[†]E-mail address: sundu@.metu.edu.tr

1 Introduction

Lepton flavor violating (LFV) interactions are interesting since they do not exist in the standard model (SM) and give strong signal about the new physics beyond. Such decays have reached great interest at present and the experimental search has been improved. Among LFV decays the ones existing in the leptonic sector are clean theoretically in the sense that they are free from the nonperturbative effects. The processes $\mu \rightarrow e\gamma$, $\tau \rightarrow e(\mu)\gamma$, $\tau \rightarrow e\bar{e}e$, $\tau \rightarrow e\bar{\mu}\mu$, are the examples of LFV interactions. There are on-going and planned experiments for $\mu \rightarrow e\gamma$ ($\tau \rightarrow \mu\gamma$) and the current limits for their branching ratios (BR) are 1.2×10^{-11} [1] (1.1×10^{-6} [2]). The numerical estimates predict that the BR of the processes $\tau \rightarrow e\bar{e}e$, $\tau \rightarrow e\bar{\mu}\mu$ are at the order of the magnitude of 10^{-6} [3], which is a measurable value in the LEP experiments and τ factories.

In such decays, the assumption of the non-existence of Cabibbo-Kobayashi-Maskawa (CKM) type matrix in the leptonic sector forbids the charged Flavor Changing (FC) interactions and therefore, the physics beyond the SM plays the main role, where the general two Higgs doublet model (2HDM), so called model III, is one of the candidate. This model predicts that the LFV interactions can exist at loop level and the internal neutral higgs bosons h_0 and A_0 play the main role. There are number of Yukawa couplings which describe the strength of the interactions lepton-lepton-neutral Higgs particle, appearing in the loops and their strength can be determined by the experimental data. In the literature, there are several studies on LFV interactions in different models. Such interactions are studied in a model independent way in [4], in the framework of model III 2HDM [5], in supersymmetric models [6, 7, 8, 9, 10, 11, 12].

Our work is devoted to the study of the $\tau \rightarrow \mu\bar{\nu}_i\nu_i$, $i = e, \mu, \tau$ decay in the model III version of 2HDM. This process exists with the help of internal scalar bosons h^0 and A^0 to obtain the flavor changing transition $\tau \rightarrow \mu$ and the internal Z boson to get the output $\bar{\nu}\nu$ (see Fig. 1). The $BR(\tau \rightarrow \mu\bar{\nu}_i\nu_i)$, ($i = e, \mu, \tau$) is predicted at the order of the magnitude of $10^{-6} - 10^{-4}$ for the intermediate values of the Yukawa couplings, which are the free parameters of the model used and is strongly sensitive to the couplings for $\tau - h^0(A^0) - \tau$ and $\tau - h^0(A^0) - \mu$ transitions. This can play an important role in the determination of the upper limits of these couplings, especially the one for the $\tau - h^0(A^0) - \tau$ transition. Notice that there have been some experimental studies on this process in the literature [13].

The paper is organized as follows: In Section 2, we present the theoretical expression for the decay width of the LFV decay $\tau \rightarrow \mu\bar{\nu}_i\nu_i$, $i = e, \mu, \tau$, in the framework of the model III. Section 3 is devoted to discussion and our conclusions.

2 $\tau \rightarrow \mu \bar{\nu}_i \nu_i$ decay in the general two Higgs doublet model

Type III 2HDM permits flavor changing neutral currents (FCNC) at tree level. The Yukawa interaction for the leptonic sector in the model III is

$$\mathcal{L}_Y = \eta_{ij}^E \bar{l}_{iL} \phi_1 E_{jR} + \xi_{ij}^E \bar{l}_{iL} \phi_2 E_{jR} + h.c. , \quad (1)$$

where i, j are family indices of leptons, L and R denote chiral projections $L(R) = 1/2(1 \mp \gamma_5)$, ϕ_i for $i = 1, 2$, are the two scalar doublets, l_{iL} and E_{jR} are lepton doublets and singlets respectively. Here ϕ_1 and ϕ_2 are chosen as

$$\phi_1 = \frac{1}{\sqrt{2}} \left[\begin{pmatrix} 0 \\ v + H^0 \end{pmatrix} + \begin{pmatrix} \sqrt{2}\chi^+ \\ i\chi^0 \end{pmatrix} \right] ; \phi_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}H^+ \\ H_1 + iH_2 \end{pmatrix} , \quad (2)$$

with the vacuum expectation values

$$\langle \phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} ; \langle \phi_2 \rangle = 0 . \quad (3)$$

By considering the gauge and CP invariant Higgs potential which spontaneously breaks $SU(2) \times U(1)$ down to $U(1)$ as:

$$\begin{aligned} V(\phi_1, \phi_2) &= c_1(\phi_1^+ \phi_1 - v^2/2)^2 + c_2(\phi_2^+ \phi_2)^2 \\ &+ c_3[(\phi_1^+ \phi_1 - v^2/2) + \phi_2^+ \phi_2]^2 + c_4[(\phi_1^+ \phi_1)(\phi_2^+ \phi_2) - (\phi_1^+ \phi_2)(\phi_2^+ \phi_1)] \\ &+ c_5[Re(\phi_1^+ \phi_2)]^2 + c_6[Im(\phi_1^+ \phi_2)]^2 + c_7 , \end{aligned} \quad (4)$$

with constants c_i , $i = 1, \dots, 7$, H_1 and H_2 are obtained as the mass eigenstates h^0 and A^0 respectively, since no mixing occurs between two CP-even neutral bosons H^0 and h^0 in the tree level. Therefore it is possible to collect the SM particles in the first doublet and the new particles in the second one. The part which produce FCNC at tree level is

$$\mathcal{L}_{Y,FC} = \xi_{ij}^E \bar{l}_{iL} \phi_2 E_{jR} + h.c. . \quad (5)$$

Here the Yukawa matrices ξ_{ij}^E have complex entries in general. Notice that, in the following, we replace ξ^E with ξ_N^E where "N" denotes the word "neutral".

Now, we consider the lepton flavor changing process $\tau \rightarrow \mu \bar{\nu} \nu$ and we expect that the main contribution to this decay comes from the neutral Higgs bosons, namely, h_0 and A_0 in the loop level, in the leptonic sector of the model III, (see Fig. 1). The general effective vertex for the interaction of off-shell Z-boson with a fermionic current is obtained as

$$\Gamma_\mu^{(REN)}(\tau \rightarrow \mu Z^*) = f_1 \gamma_\mu + f_2 \gamma_\mu \gamma_5 + f_3 \sigma_{\mu\nu} k^\nu + f_4 \sigma_{\mu\nu} \gamma_5 k^\nu \quad (6)$$

where k is the momentum transfer, $k^2 = (p - p')^2$, p (p') is the four momentum vector of incoming (outgoing) lepton. Taking into account all the masses of internal (m_i) and external leptons (m_{l_1} , m_{l_2}), the explicit expressions for the functions f_1 , f_2 , f_3 and f_4 are

$$\begin{aligned}
f_1 &= \frac{g}{64 \pi^2 \cos \theta_W} \int_0^1 dx \frac{1}{m_{l_2}^2 - m_{l_1}^2} \left\{ c_V (m_{l_2} + m_{l_1}) \right. \\
&\left((-m_i \eta_i^+ + m_{l_1}(-1+x) \eta_i^V) \ln \frac{L_{1,h^0}^{self}}{\mu^2} + (m_i \eta_i^+ - m_{l_2}(-1+x) \eta_i^V) \ln \frac{L_{2,h^0}^{self}}{\mu^2} \right. \\
&+ (m_i \eta_i^+ + m_{l_1}(-1+x) \eta_i^V) \ln \frac{L_{1,A^0}^{self}}{\mu^2} - (m_i \eta_i^+ + m_{l_2}(-1+x) \eta_i^V) \ln \frac{L_{2,A^0}^{self}}{\mu^2} \Big) \\
&+ c_A (m_{l_2} - m_{l_1}) \\
&\left((-m_i \eta_i^- + m_{l_1}(-1+x) \eta_i^A) \ln \frac{L_{1,h^0}^{self}}{\mu^2} + (m_i \eta_i^- + m_{l_2}(-1+x) \eta_i^A) \ln \frac{L_{2,h^0}^{self}}{\mu^2} \right. \\
&+ (m_i \eta_i^- + m_{l_1}(-1+x) \eta_i^A) \ln \frac{L_{1,A^0}^{self}}{\mu^2} + (-m_i \eta_i^- + m_{l_2}(-1+x) \eta_i^A) \ln \frac{L_{2,A^0}^{self}}{\mu^2} \Big) \Big\} \\
&- \frac{g}{64 \pi^2 \cos \theta_W} \int_0^1 dx \int_0^{1-x} dy \left\{ m_i^2 (c_A \eta_i^A - c_V \eta_i^V) \left(\frac{1}{L_{A^0}^{ver}} + \frac{1}{L_{h^0}^{ver}} \right) \right. \\
&- (1-x-y) m_i \left(c_A (m_{l_2} - m_{l_1}) \eta_i^- \left(\frac{1}{L_{h^0}^{ver}} - \frac{1}{L_{A^0}^{ver}} \right) + c_V (m_{l_2} + m_{l_1}) \eta_i^+ \left(\frac{1}{L_{h^0}^{ver}} + \frac{1}{L_{A^0}^{ver}} \right) \right) \\
&- (c_A \eta_i^A + c_V \eta_i^V) \left(-2 + (k^2 x y + m_{l_1} m_{l_2} (-1+x+y)^2) \left(\frac{1}{L_{h^0}^{ver}} + \frac{1}{L_{A^0}^{ver}} \right) - \ln \frac{L_{h^0}^{ver}}{\mu^2} \frac{L_{A^0}^{ver}}{\mu^2} \right) \\
&- (m_{l_2} + m_{l_1}) (1-x-y) \left(\frac{\eta_i^A (x m_{l_1} + y m_{l_2}) + m_i \eta_i^-}{2 L_{A^0 h^0}^{ver}} + \frac{\eta_i^A (x m_{l_1} + y m_{l_2}) - m_i \eta_i^-}{2 L_{h^0 A^0}^{ver}} \right) \\
&+ \left. \frac{1}{2} \eta_i^A \ln \frac{L_{A^0 h^0}^{ver}}{\mu^2} \ln \frac{L_{h^0 A^0}^{ver}}{\mu^2} \right\}, \\
f_2 &= \frac{g}{64 \pi^2 \cos \theta_W} \int_0^1 dx \frac{1}{m_{l_2}^2 - m_{l_1}^2} \left\{ c_V (m_{l_2} - m_{l_1}) \right. \\
&\left((m_i \eta_i^- + m_{l_1}(-1+x) \eta_i^A) \ln \frac{L_{1,A^0}^{self}}{\mu^2} + (-m_i \eta_i^- + m_{l_2}(-1+x) \eta_i^A) \ln \frac{L_{2,A^0}^{self}}{\mu^2} \right. \\
&+ (-m_i \eta_i^- + m_{l_1}(-1+x) \eta_i^A) \ln \frac{L_{1,h^0}^{self}}{\mu^2} + (m_i \eta_i^- + m_{l_2}(-1+x) \eta_i^A) \ln \frac{L_{2,h^0}^{self}}{\mu^2} \Big) \\
&+ c_A (m_{l_2} + m_{l_1}) \\
&\left((m_i \eta_i^+ + m_{l_1}(-1+x) \eta_i^V) \ln \frac{L_{1,A^0}^{self}}{\mu^2} - (m_i \eta_i^+ + m_{l_2}(-1+x) \eta_i^V) \ln \frac{L_{2,A^0}^{self}}{\mu^2} \right. \\
&+ (-m_i \eta_i^+ + m_{l_1}(-1+x) \eta_i^V) \ln \frac{L_{1,h^0}^{self}}{\mu^2} + (m_i \eta_i^+ - m_{l_2}(-1+x) \eta_i^V) \ln \frac{L_{2,h^0}^{self}}{\mu^2} \Big) \Big\} \\
&+ \frac{g}{64 \pi^2 \cos \theta_W} \int_0^1 dx \int_0^{1-x} dy \left\{ m_i^2 (c_V \eta_i^A - c_A \eta_i^V) \left(\frac{1}{L_{A^0}^{ver}} + \frac{1}{L_{h^0}^{ver}} \right) \right.
\end{aligned}$$

$$\begin{aligned}
& - m_i (1 - x - y) \left(c_V (m_{l_2} - m_{l_1}) \eta_i^- + c_A (m_{l_2} + m_{l_1}) \eta_i^+ \right) \left(\frac{1}{L_{h^0}^{ver}} - \frac{1}{L_{A^0}^{ver}} \right) \\
& + (c_V \eta_i^A + c_A \eta_i^V) \left(-2 + (k^2 x y - m_{l_1} m_{l_2} (-1 + x + y))^2 \right) \left(\frac{1}{L_{h^0}^{ver}} + \frac{1}{L_{A^0}^{ver}} \right) - \ln \frac{L_{h^0}^{ver}}{\mu^2} \frac{L_{A^0}^{ver}}{\mu^2} \\
& - (m_{l_2} - m_{l_1}) (1 - x - y) \left(\frac{\eta_i^V (x m_{l_1} - y m_{l_2}) + m_i \eta_i^+}{2 L_{A^0 h^0}^{ver}} + \frac{\eta_i^V (x m_{l_1} - y m_{l_2}) - m_i \eta_i^+}{2 L_{h^0 A^0}^{ver}} \right) \\
& - \frac{1}{2} \eta_i^V \ln \frac{L_{A^0 h^0}^{ver}}{\mu^2} \ln \frac{L_{h^0 A^0}^{ver}}{\mu^2} \Bigg\}, \\
f_3 & = -i \frac{g}{64 \pi^2 \cos \theta_W} \int_0^1 dx \int_0^{1-x} dy \left\{ \left((1 - x - y) (c_V \eta_i^V + c_A \eta_i^A) (x m_{l_1} + y m_{l_2}) \right. \right. \\
& + \left. \left. m_i (c_A (x - y) \eta_i^- + c_V \eta_i^+ (x + y)) \right) \frac{1}{L_{h^0}^{ver}} \right. \\
& + \left. \left((1 - x - y) (c_V \eta_i^V + c_A \eta_i^A) (x m_{l_1} + y m_{l_2}) - m_i (c_A (x - y) \eta_i^- + c_V \eta_i^+ (x + y)) \right) \frac{1}{L_{A^0}^{ver}} \right. \\
& - \left. (1 - x - y) \left(\frac{\eta_i^A (x m_{l_1} + y m_{l_2})}{2} \left(\frac{1}{L_{A^0 h^0}^{ver}} + \frac{1}{L_{h^0 A^0}^{ver}} \right) + \frac{m_i \eta_i^-}{2} \left(\frac{1}{L_{h^0 A^0}^{ver}} - \frac{1}{L_{A^0 h^0}^{ver}} \right) \right) \right\}, \\
f_4 & = -i \frac{g}{64 \pi^2 \cos \theta_W} \int_0^1 dx \int_0^{1-x} dy \left\{ \left((1 - x - y) \left(- (c_V \eta_i^A + c_A \eta_i^V) (x m_{l_1} - y m_{l_2}) \right) \right. \right. \\
& - \left. \left. m_i (c_A (x - y) \eta_i^+ + c_V \eta_i^- (x + y)) \right) \frac{1}{L_{h^0}^{ver}} \right. \\
& + \left. \left((1 - x - y) \left(- (c_V \eta_i^A + c_A \eta_i^V) (x m_{l_1} - y m_{l_2}) \right) + m_i (c_A (x - y) \eta_i^+ + c_V \eta_i^- (x + y)) \right) \frac{1}{L_{A^0}^{ver}} \right. \\
& + \left. (1 - x - y) \left(\frac{\eta_i^V}{2} (m_{l_1} x - m_{l_2} y) \left(\frac{1}{L_{A^0 h^0}^{ver}} + \frac{1}{L_{h^0 A^0}^{ver}} \right) + \frac{m_i \eta_i^+}{2} \left(\frac{1}{L_{A^0 h^0}^{ver}} - \frac{1}{L_{h^0 A^0}^{ver}} \right) \right) \right\}, \quad (7)
\end{aligned}$$

where

$$\begin{aligned}
L_{1\{2\}, h^0(A^0)}^{self} & = m_{h^0(A^0)}^2 (1 - x) + (m_i^2 - m_{l_1\{2\}}^2) (1 - x) x, \\
L_{h^0(A^0)}^{ver} & = m_{h^0(A^0)}^2 (1 - x - y) + m_i^2 (x + y) - k^2 x y, \\
L_{h^0 A^0(A^0 h^0)}^{ver} & = m_{A^0(h^0)}^2 x + m_i^2 (1 - x - y) + (m_{h^0(A^0)}^2 - k^2 x) y, \quad (8)
\end{aligned}$$

and

$$\begin{aligned}
\eta_i^V & = \xi_{N, l_1 i}^E \xi_{N, i l_2}^{E*} + \xi_{N, i l_1}^{E*} \xi_{N, l_2 i}^E, \\
\eta_i^A & = \xi_{N, l_1 i}^E \xi_{N, i l_2}^{E*} - \xi_{N, i l_1}^{E*} \xi_{N, l_2 i}^E, \\
\eta_i^+ & = \xi_{N, i l_1}^{E*} \xi_{N, i l_2}^{E*} + \xi_{N, l_1 i}^E \xi_{N, l_2 i}^E, \\
\eta_i^- & = \xi_{N, i l_1}^{E*} \xi_{N, i l_2}^{E*} - \xi_{N, l_1 i}^E \xi_{N, l_2 i}^E. \quad (9)
\end{aligned}$$

The parameters c_V and c_A are $c_A = -\frac{1}{4}$ and $c_V = \frac{1}{4} - \sin^2 \theta_W$. In eq. (9) the flavor changing couplings $\xi_{N, l_j i}^E$ represent the effective interaction between the internal lepton i , ($i = e, \mu, \tau$)

and outgoing (incoming) $j = 1$ ($j = 2$) one. Here we take only the τ lepton in the internal line and we neglect all the Yukawa couplings except $\xi_{N,\tau\tau}^E$ and $\xi_{N,\tau\mu}^E$ in the loop contributions (see Discussion section). Notice that the parameter μ in eq. (7) is the renormalization scale, the functions f_1, f_2, f_3, f_4 are finite and independent of μ .

The matrix element M for the process $\tau \rightarrow \mu \bar{\nu}_i \nu_i$, $i = e, \mu, \tau$ is calculated in the framework of the model III, by connecting the $\tau \rightarrow \mu$ transition and the $\bar{\nu}_i \nu_i$ output with the help of the internal Z boson. Finally the decay width Γ is obtained in the τ lepton rest frame using the well known expression

$$d\Gamma = \frac{(2\pi)^4}{2m_\tau} |M|^2 \delta^4(p - \sum_{i=1}^3 p_i) \prod_{i=1}^3 \frac{d^3 p_i}{(2\pi)^3 2E_i}, \quad (10)$$

where p (p_i , $i=1,2,3$) is four momentum vector of τ lepton (μ lepton, incoming ν , outgoing ν).

3 Discussion

In the case of vanishing charged interactions, under the assumption that CKM type matrix in the leptonic sector does not exist, LFV interactions arise with the help the neutral Higgs bosons h^0 and A^0 , in the framework of model III. In this scenario the Yukawa couplings $\xi_{N,ij}^E$, $i, j = e, \mu, \tau$ play the main role in the determination of the BR of the processes under consideration. Since these couplings are free parameters of the theory, they should be restricted by using the present experimental limits of physical quantities, such as BR of various leptonic decays and electric dipole moments (EDM), anomalous magnetic moments (AMM) of leptons. In general, these Yukawa couplings are complex and they ensure non-zero lepton EDM.

Now, we briefly discuss the case that the known light neutrinos ν_i as massive particles and the lepton numbers L_i denote the leptons of i^{th} family are not conserved. If this is so, the lepton sector is an exact analogy to the quark sector and there exists a similar CKM type matrix, Maki-Nakagawa-Sakata (MNS) matrix $V_{l\nu}$ [14], that its elements are measured in neutrino oscillation experiments. It has been shown that the mixing between the muon neutrino and the heaviest mass eigenstate of the neutrino sector, the $V_{\mu 3}$ element, is nearly maximal [15, 16]. The experiments on solar neutrinos [15], [17] (the reactor experiments such as CHOOZ [18]) predicted the mixing between electron neutrino and the second heaviest mass eigenstate of the neutrino sector, the $V_{e 2}$ element (the heaviest mass eigenstate of the neutrino sector, the $V_{e 3}$ element). Notice that the corner element $V_{e 3}$ is much smaller than the others. On the other hand some off-diagonal elements of MNS matrix, such as $V_{\mu 3}$ are large and the MNS matrix is

far from diagonal in contrast to the CKM matrix (see for example [19] for more discussion on lepton mixing).

With the inclusion of MNS matrix, the existence of the lepton flavor violating (LFV) processes, $\tau \rightarrow \mu$ transition in the present work, in the SM, would be possible. However, we expect that the tiny masses of the internal neutrinos bring small contribution even with the choice of maximal mixing in the leptonic sector. (See for example [20], for the discussion of the existence of the MNS matrix and its effects on a special LFV process). In any case, the process $\tau \rightarrow \mu \bar{\nu}_i \nu_i$, $i = e, \mu, \tau$ should be examined theoretically if the lepton mixing is switched on.

In the framework of the model III the $\tau \rightarrow \mu Z^*$ transition can be switched on with the internal neutral Higgs bosons h^0 and A^0 , and internal leptons e, μ, τ . This brings unknown free parameters $\xi_{N,\mu j}^E$ and $\xi_{N,\tau j}^E$, $i, j = e, \mu, \tau$. By assuming that only the internal τ lepton contribution is considerable, the Yukawa couplings which does not contain τ index can be neglected. Such a choice respects the statement that the strength of the couplings are related with the masses of leptons denoted by the indices of them, similar to the Cheng-Sher scenario [21]. Furthermore, we take $\xi_{N,ij}^E$ symmetric with respect to the indices i and j . Finally, we are left with the couplings $\xi_{N,\tau\tau}^E$ and $\xi_{N,\tau\mu}^E$, which are complex in general. Notice that in the following we use the parametrization

$$\xi_{N,ij}^E = \sqrt{\frac{4G_F}{\sqrt{2}}} \bar{\xi}_{N,ij}^E, \quad (11)$$

and we present the numerical values of some parameters used in the calculations as a table:

m_τ	1.78 (GeV)
m_Z	91 (GeV)
m_W	80 (GeV)
s_w	$\sqrt{0.23}$
G_F	$1.16637 \times 10^{-5} (GeV^{-2})$
Γ_Z	2.49 (GeV)
Γ_τ	$2.27 \times 10^{-12} (GeV)$

Table 1: The numerical values of the physical parameters used in the numerical calculations.

The measurement of the BRs of $\tau \rightarrow \mu \bar{\nu}_i \nu_i$, $i = e, \mu, \tau$ decays [13] are based on counting the number of candidate jets and correcting for efficiency and event selection. In additions to this,

the backgrounds coming from taus decaying to hadrons or cosmic rays should be detected. In the process we study the output contains $\bar{\nu}_i \nu_i$, $i = e, \mu, \tau$ and the extraction of this output from the most probable one $\bar{\nu}_\mu \nu_\tau$ (BR $\sim 17.37 \pm 0.09\%$ [22]), which exist in the SM theoretically, is difficult from the experimental point of view.

In this work we studied the BR of the process $\tau \rightarrow \mu \bar{\nu}_i \nu_i$, $i = e, \mu, \tau$ and we used the numerical value $\bar{\xi}_{N,\tau\mu}^E$ in the interval $5 (GeV) < |\bar{\xi}_{N,\tau\mu}^E| < 25 (GeV)$. Here, the upper limit for the coupling $|\bar{\xi}_{N,\tau\mu}^E|$ has been estimated in [23] as $\sim 30 GeV$. In this work it is assumed that the new physics effects are of the order of the experimental uncertainty of muon AMM, namely 10^{-9} by respecting , the new experimental world average announced at BNL [24]

$$a_\mu = 11\,659\,203(8) \times 10^{-10} , \quad (12)$$

which has about half of the uncertainty of previous measurements. Here we have not used any restriction for the coupling $\bar{\xi}_{N,\tau\tau}^E$ except that we choose its numerical value larger compared to $\bar{\xi}_{N,\tau\mu}^E$. In addition to this, we expect the upper limit of $\bar{\xi}_{N,\tau\tau}^E$ by taking the upper limit of $\bar{\xi}_{N,\tau\mu}^E$ and the BR of the process as 10^{-4} - 10^{-6} .

In Fig. 2, we present $\bar{\xi}_{N,\tau\tau}^E$ dependence of the BR for real couplings. Here solid (dashed, small dashed, dotted, dash-dotted) line represents the case for $\bar{\xi}_{N,\tau\mu}^E = 5 GeV$ (10, 15, 20, 25 GeV). This figure shows that the BR enhances with the increasing values of both couplings and it reaches to values at order of magnitude of 10^{-4} . Fig. 3 shows the possible values of $\bar{\xi}_{N,\tau\tau}^E$ and the ratio $\frac{\bar{\xi}_{N,\tau\mu}^E}{\bar{\xi}_{N,\tau\tau}^E}$ for the fixed values of the BR , $BR = 10^{-4}$ (solid line) and $BR = 10^{-6}$ (dashed line). For $\frac{\bar{\xi}_{N,\tau\mu}^E}{\bar{\xi}_{N,\tau\tau}^E} = 0.1$, the $BR = 10^{-4}$ is obtained if the coupling $\bar{\xi}_{N,\tau\tau}^E \sim 150 GeV$ and the $BR = 10^{-6}$ is obtained if the coupling $\bar{\xi}_{N,\tau\tau}^E \sim 50 GeV$. The possible experimental search of the process $\tau \rightarrow \mu \bar{\nu}_i \nu_i$, $i = e, \mu, \tau$ would ensure a strong clue in the prediction of the upper limit of the coupling $\bar{\xi}_{N,\tau\tau}^E$.

Fig. 4 represents the h^0 mass m_{h^0} dependence of the BR for the fixed values of $\bar{\xi}_{N,\tau\mu}^E$ and $\bar{\xi}_{N,\tau\tau}^E$, $\bar{\xi}_{N,\tau\mu}^E = 10 GeV$, $\bar{\xi}_{N,\tau\tau}^E = 100 GeV$. This figure shows that the BR is sensitive to m_{h^0} and it decreases with the increasing values of m_{h^0} .

Now, we take the coupling $\bar{\xi}_{N,\tau\tau}^E$ complex

$$\bar{\xi}_{N,\tau\tau}^E = |\bar{\xi}_{N,\tau\tau}^E| e^{i\theta_{\tau\tau}} , \quad (13)$$

and present the $\sin \theta_{\tau\tau}$ dependence of the BR for $\bar{\xi}_{N,\tau\tau}^E = 100 GeV$ for four different values of $\bar{\xi}_{N,\tau\mu}^E$, namely $\bar{\xi}_{N,\tau\mu}^E = 5, 10, 15, 20 GeV$ (solid, dashed, small dashed, dotted lines) in Fig. 5. From this figure it can be shown that the BR is not sensitive to the complexity of the coupling $\bar{\xi}_{N,\tau\tau}^E$.

At this stage we would like to summarize our results:

- We predict the BR at the order of the magnitude of $10^{-6} - 10^{-4}$ for the range of the couplings, $\bar{\xi}_{N,\tau\tau}^E \sim 30 - 100 \text{ GeV}$ and $\bar{\xi}_{N,\tau\mu}^E \sim 10 - 25 \text{ GeV}$. We predict the upper limit of the coupling for the $\tau - h^0(A^0) - \tau$ transition as ~ 0.3 in the case that the BR is $\sim 10^{-6}$. With the possible experimental measurement of the process $\tau \rightarrow \mu \bar{\nu}_i \nu_i$, $i = e, \mu, \tau$, a strong clue in the prediction of the upper limit of the coupling $\bar{\xi}_{N,\tau\tau}^E$ would be obtained. Notice that the upper limit of the coupling $\bar{\xi}_{N,\tau\mu}^E$ could be predicted previously (see for example [23]). This analysis also ensures a hint for the physics beyond the SM.
- We observe that the BR is sensitive to the neutral Higgs masses m_{h^0} and m_{A^0}
- We observe that the BR is not sensitive to the possible complexity of the Yukawa couplings.

Therefore, the future theoretical and experimental investigations of the process $\tau \rightarrow \mu \bar{\nu}_i \nu_i$ would be informative in the determination of the physics beyond the SM and the free parameters existing in this model.

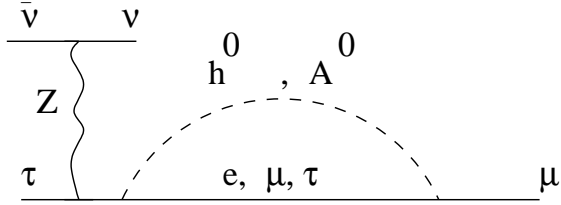
4 Acknowledgement

This work has been supported by the Turkish Academy of Sciences in the framework of the Young Scientist Award Program. (EOI-TUBA-GEBIP/2001-1-8)

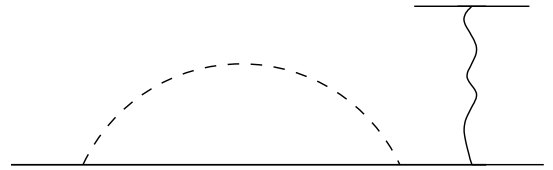
References

- [1] M. L. Brooks et. al., MEGA Collaboration, *Phys. Rev. Lett.* **83**, 1521 (1999).
- [2] S. Ahmed et.al., CLEO Collaboration, *Phys. Rev.* **D61**, 071101 (2000).
- [3] A. Ilakovac, A.Pilaftsis, *Nucl. Phys.* **B437**, 491 (1995).
- [4] D. Chang, W. S. Hou and W. Y. Keung, *Phys. Rev.* **D48**, 217 (1993).
- [5] E. Iltan, *Phys. Rev.* **D64**, 013013 (2001); *Phys. Rev.* **D64**, 115005 (2001); R. Diaz, R. Martinez and J-Alexis Rodriguez, hep-ph/0010149 (2000).
- [6] R. Barbieri and L. J. Hall, *Phys. Lett.* **B338**, 212 (1994).

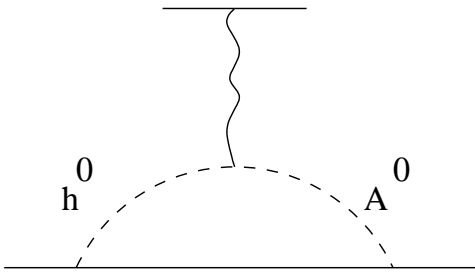
- [7] R. Barbieri, L. J. Hall and A. Strumia, *Nucl. Phys.* **B445**, 219 (1995).
- [8] R. Barbieri, L. J. Hall and A. Strumia, *Nucl. Phys.* **B449**, 437 (1995).
- [9] P. Ciafaloni, A. Romanino and A. Strumia, IFUP-YH-42-95.
- [10] T. V. Duong, B. Dutta and E. Keith, *Phys. Lett.* **B378**, 128 (1996).
- [11] G. Couture, et. al., *Eur. Phys. J.* **C7**, 135 (1999).
- [12] Y. Okada, K. Okumara and Y. Shimizu, *Phys. Rev.* **D61**, 094001 (2000).
- [13] M. Sasaki et.al., Opal Collaboration, UT-ICEPP-91-04, Sep 1991. 9pp.;
Alexander et al., Opal Collaboration, *Phys. Lett.* **B266**, 201 (1991), D. M. Edsall et al.,
Emes Hep Group, *Nucl. Phys. B* **55C**, 95 (1997), W. Bartel et. al, JADE Collaboration,
DESY 86-091 (1986), W. Bacino, et. al. , SLAC pub. 2223, October 1978.
- [14] Z. Maki, M. Nakagawa, S. Sakata, *Prog. Theor. Phys.* **28**, 870 (1962).
- [15] Superkamiokande data, see talk by T. Toshito at ICHEP'00, Osaka, (2000);
- [16] Soudan II data, see talk by G. Pearce at ICHEP'00, Osaka, (2000).
- [17] Q. R. Ahmad et. al. , *Phys. Rev. Lett.* **D87**, 071307 (2001); B. T. Cleveland et. al.,
Astropart Phys. **496**, 505 (1998); J. Abdurashitov et. al., *Phys. Rev.* **C60**, 055801 (1999);
J. W. Hampel et. al., *Phys. Lett.* **B447**, 127 (1999);
- [18] CHOOZ collaboration, M. Apollonio et. al. , *Phys. Lett.* **B420**, 397 (1998).
- [19] H. M. Chan, S. T. Tsou, *Acta Phys. Polon.* **B33**, 4041 (2002); R. N. Mohapatra, M. K.
Parida, G. Rajasekarar, hep-ph/0301234, (2003); S. A. Abel, C. Munoz, *JHEP* **0302**, 010
(2003);
- [20] J. I. Illiana, M. Jack, T. Riemann, hep-ph/0001273, (2000).
- [21] T. P. Cheng and M. Sher, *Phy. Rev.* **D35**, 3383 (1987).
- [22] Particle Data Group, *Euro. Phys. Jour.* **C3**, 1 (1998).
- [23] E. Iltan, H. Sundu, *Acta Phys. Slov.* **53**, 17 (2003).
- [24] M. Deile, et. al., Muon g-2 Collaboration, hep-ex/0211034 (2002).



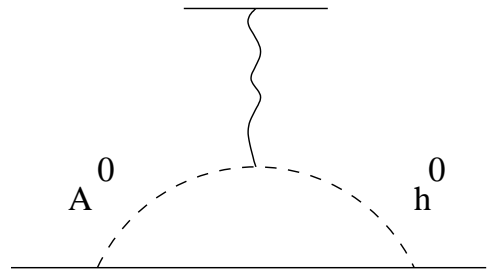
(a)



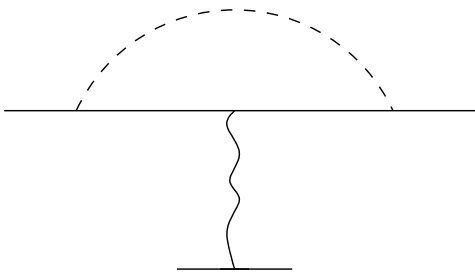
(b)



(c)



(d)



(e)

Figure 1: One loop diagrams contribute to $\tau \rightarrow \mu \bar{\nu}_i \nu_i$, $i = e, \mu, \tau$ decay due to the neutral Higgs bosons h_0 and A_0 in the model III version of 2HDM. Solid lines represent leptons and neutrinos, curly (dashed) lines represent the virtual Z boson (h_0 and A_0 fields).

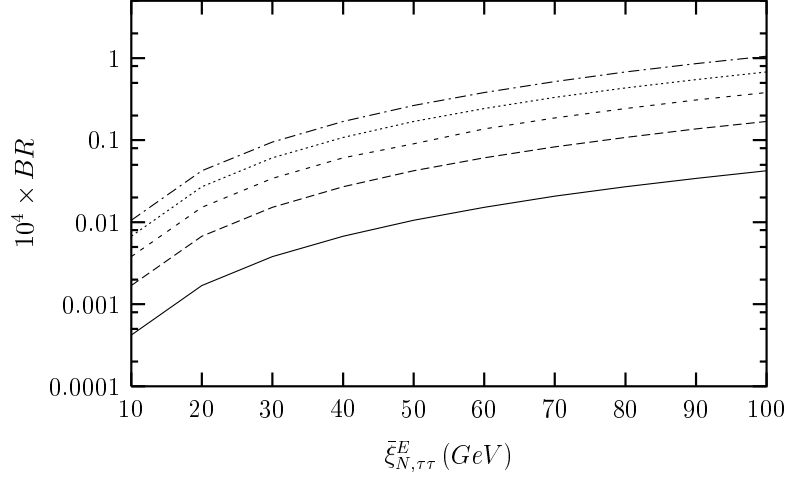


Figure 2: $\bar{\xi}_{N,\tau\tau}^E$ dependence of the BR for real couplings and $m_{h^0} = 85 \text{ GeV}$, $m_{A^0} = 90 \text{ GeV}$. Here solid (dashed, small dashed, dotted, dash-dotted) line represents the case for $\bar{\xi}_{N,\tau\mu}^E = 5 \text{ GeV}$ (10, 15, 20, 25 GeV).

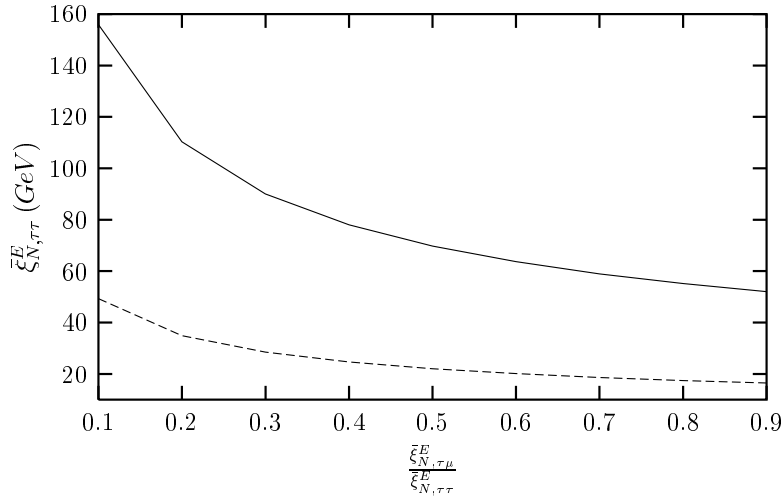


Figure 3: The possible values of $\bar{\xi}_{N,\tau\tau}^E$ and the ratio $\frac{\bar{\xi}_{N,\tau\mu}^E}{\bar{\xi}_{N,\tau\tau}^E}$ for $m_{h^0} = 85 \text{ GeV}$, $m_{A^0} = 90 \text{ GeV}$ and the fixed values of the BR , $BR = 10^{-4}$ (solid line) and $BR = 10^{-6}$ (dashed line)

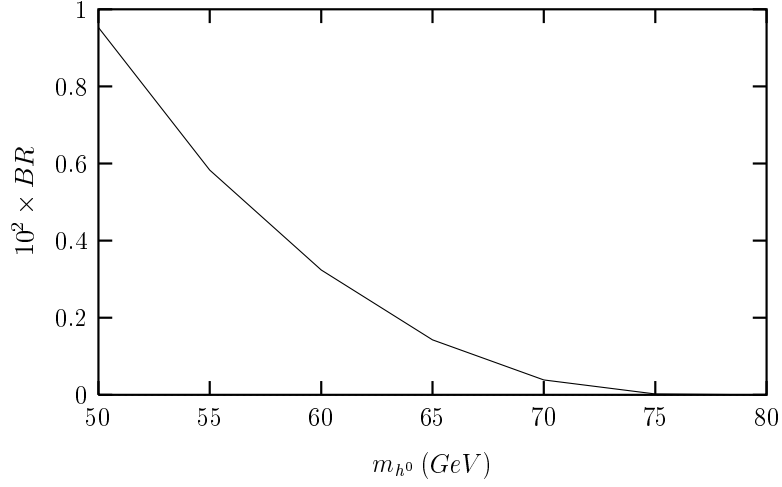


Figure 4: m_{h^0} dependence of the BR for the fixed values of $\bar{\xi}_{N,\tau\mu}^E = 10 \text{ GeV}$, $\bar{\xi}_{N,\tau\tau}^E = 100 \text{ GeV}$ and $m_{A^0} = 90 \text{ GeV}$

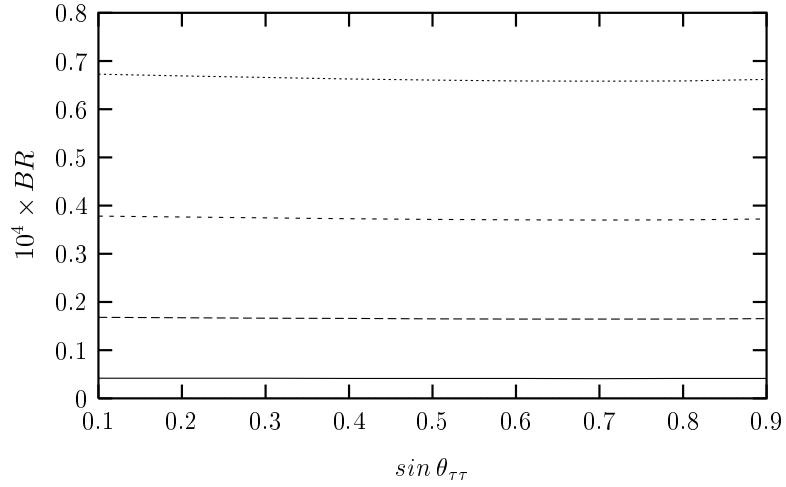


Figure 5: The $\sin \theta_{\tau\tau}$ dependence of the BR for $m_{h^0} = 85 \text{ GeV}$, $m_{A^0} = 90 \text{ GeV}$, $\bar{\xi}_{N,\tau\tau}^E = 100 \text{ GeV}$ and three different values of $\bar{\xi}_{N,\tau\mu}^E$, $\bar{\xi}_{N,\tau\mu}^E = 5, 10, 15, 20 \text{ GeV}$ (dashed, small dashed, dotted lines)