International Journal of Modern Physics A

AN OPEN INFLATIONARY MODEL FOR DIMENSIONAL REDUCTION AND ITS EFFECTS ON THE OBSERVABLE PARAMETERS OF THE UNIVERSE

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Assuming that higher dimensions existed in the early stages of the universe where the evolution was inflationary, we construct an open, singularity-free, spatially homogeneous and isotropic cosmological model to study the effects of dimensional reduction that may have taken place during the early stages of the universe. We consider dimensional reduction to take place in a stepwise manner and interpret each step as a phase transition. By imposing suitable boundary conditions we trace their effects on the present day parameters of the universe.

Keywords: Dimensional reduction; inflation; phase transition.

1. Introduction

During the past two decades, several closed cosmological models aiming the singularity-free description of the universe have been proposed.[1,2,3] In these works, the universe was modeled as a closed Friedmann-Lemaitre-Robertson-Walker spacetime, bouncing from a Planck mass and radius state with inflation, connecting to the standard model of radiation and then to the era of matter dominance. Although they correctly described the evolution of the universe, these models could not predicted the present value of the Hubble parameter within the observed range. The approach developed by Bayın, Cooperstock and Faraoni[3] was applied to open geometry by Karaca and Bayın[4] and it was shown that a singularity-free cosmological model which complies with the recent measurements is possible for open geometry. In this model, since the initially static condition ($\dot{a}(0) = 0$) used in the closed models can no longer be used in the open universe case, the universe starts with an initial expansion rate, i.e., $\dot{a}(0) \neq 0$. Hence, this model is a two-parameter universe model in which one of the parameters determines the strength of the initial vacuum dominance and the other corresponds to the initial expansion rate.

It is well known that some theories require the existence of higher dimensions in the early stages of the universe. In this paper, we investigate the possibility that higher dimensions might have existed during the inflationary prematter phase of the universe and explore its observable consequences. As in the previous papers[1,2,3,4] we adopt the Gliner[5]-Markov[6] picture of Planck limits to the physical quantities.

2. Description of the Model

In our model, universe starts its journey as a D ($D > 3$) dimensional open Friedmann model (D denotes the number of initial space dimensions). At $t = 0$ density of the universe is taken as the Planck density. Existence of higher dimensions and their reduction to three dimensions are assumed to take place during the inflationary prematter phase where the equation of state is given as

$$P_D = (\gamma_D - 1)\,\rho_D. \tag{1}$$

Here, γ_D is assumed to be a positive, small and dimensionless constant that characterizes the prematter in D dimensions, and P_D and ρ_D represent the pressure and energy density in D dimensions, respectively.

During this phase the universe expands isentropically while its temperature increases. This expansion continues until the maximum allowed temperature, i.e., the Planck temperature of that dimension ($T_{pl,D}$) is reached. At this point, we postulate that a phase transition to a lower dimension takes place, where the Planck temperature is higher. Hence, the universe finds more room for further inflation. This process continues until we reach $D = 3$. At this point, either the last reduction may carry the universe directly to the standard radiation era ($P = \frac{1}{3}\rho$) or prematter era may continue once more in the usual three-dimensional space and then comes the radiation era. In either case, the universe is eventually carried to the era of matter dominance ($P = 0$). In this work, we will consider the second alternative.

Naturally, neither the dimensional reduction takes place at an instant nor we could expect the universe to remain homogeneous and isotropic during dimensional reduction. However, we expect the duration of this process to be very short compared to the times that the universe spends in constant D eras where it is expanding as a D dimensional Friedmann model. Details of what happens during the dimensional reduction phase is beyond the scope of the present work. However, by imposing suitable boundary conditions at the transition points we could trace the effects of extra dimensions on the currently observable parameters of the universe, i.e., Hubble constant, age and density.

3. Boundary Conditions and Solutions for the Scale Factor

We assume open, homogeneous and isotropic $(1 + D)$ dimensional spacetimes where the metric is given as:

$$\begin{aligned} ds^2 = c^2dt^2 - a_D^2(t)[d\chi^2 + \sinh^2\chi d\theta_1^2 + \sinh^2\chi \sin^2\theta_1 d\theta_2^2 + ...\\ ..... + \sinh^2\chi \sin^2\theta_1 \sin^2\theta_2 ..... \sin^2\theta_{D-2} d\theta_{D-1}^2], \end{aligned} \tag{2}$$

where $a_D(t)$ represents the scale factor of the universe, t is the comoving time, $0 \leq \theta_n \leq \pi$, $0 \leq \theta_{D-1} \leq 2\pi, 0 \leq \chi \leq \infty$ and $n = 1, 2, ..., D - 2$. We also assume that the universe is filled with a perfect fluid which is represented by a stress-energy tensor having the following nonvanishing components:

$$T_0^0 = \rho_D, \quad T_1^1 = T_2^2 = ... = T_D^D = -P_D. \tag{3}$$

Einstein's gravitational field equations in $(1+D)$ dimensional space-time are (see, e.g., Ref. 7, pp. 470-482)

$$\frac{D(D-1)}{2a_D^2}\left[\left(\frac{\dot{a}_D}{c}\right)^2-1\right]=2\frac{A_D G_D}{c_D^4}\rho_D, \tag{4}$$

$$(1-D)\left(\frac{\ddot{a}_D}{c_D^2 a_D}\right)-\frac{(D-1)(D-2)}{2a_D^2}\left[\left(\frac{\dot{a}_D}{c_D}\right)^2-1\right]=2\frac{A_D G_D}{c_D^4}P_D, \tag{5}$$

where the subscript D denotes the value of that quantity in D dimensions, A_D is the solid angle subtended by a sphere in a D dimensional space ($A_3=4\pi$, $A_4=2\pi^2$, $A_5=8\pi^2/3$,...) and a dot denotes differentiation with respect to the cosmic time t. We may combine Eqs. (1), (4) and (5) to obtain an equation involving only the scale factor $a(t)$. Furthermore, if we introduce a new dependent variable given by $u_D\equiv\frac{a'_D}{a_D}=\frac{dlna_D}{d\eta}$ and define conformal time as $\eta=c\int^t\frac{dt'}{a(t')}$ we get the Riccati equation

$$u'_D+c_D u_D^2-c_D=0, \tag{6}$$

where $c_D=\frac{D}{2}\gamma_D-1$ and a prime denotes differentiation with respect to η. In the following, we will consider values of γ_D such that $c_D\neq 0$. Eq. (6) can easily be solved by setting $u_D\equiv\frac{1}{c_D}\frac{w'_D}{w_D}=\left[\ln\left(w_D^{1/c_D}\right)\right]'$ which gives the solution

$$a_D\left(\eta\right)=a_{0,D}\sinh\left(c_D\eta+\delta_D\right)^{1/c_D}, \tag{7}$$

where we have introduced the subscript D to identify the era which it applies and $a_{0,D}$ and δ_D are the integration constants to be determined from the initial conditions.

We also assume that at the end of the prematter era the fluid pervading the universe attained thermal equilibrium with a thermal spectrum. At this point we take the density of the universe equal to that of a massless scalar field with a thermal spectrum at $T_{pl,D}$, i.e.,

$$\rho\left(\eta_D\right)=\frac{A_D}{\left(2\pi c\hbar\right)^D}\Gamma\left(D+1\right)\xi\left(D+1\right)\left(kT_{pl,D}\right)^{D+1}. \tag{8}$$

Then, it is possible to find the duration of the initial prematter era by making use of Eqs. (4) and (8).

According to our consideration, dimensional reduction takes place via a first order phase transition. But, its duration could be taken negligibly small when compared with the durations of the constant D eras in the history of the universe. Thus, dimensional reduction could be considered to be taking place at an instant of time. Denoting this time as η_D, we may write Eq. (4) in terms of conformal time η just before the dimensional reduction at $\eta=\eta_D-\mid\epsilon\mid$ as

$$\frac{D(D-1)}{2}\left[\left(\frac{a'_D}{a_D^2}\right)^2-\frac{1}{a_D^2}\right]=\frac{2A_D G_D}{c^4}\rho_D, \tag{9}$$

where ϵ is a small number. After the dimensional reduction at η_D, the number of space dimensions becomes $(D-1)$ and Eq. (9) takes the following form

$$\frac{(D-1)(D-2)}{2}\left[\left(\frac{a'_{D-1}}{a^2_{D-1}}\right)^2 - \frac{1}{a^2_{D-1}}\right] = \frac{2A_{D-1}G_{D-1}}{c^4}\rho_{D-1}, \tag{10}$$

at $\eta = \eta_D + \mid \epsilon \mid$. In Eqs. (9) and (10), G_D and G_{D-1} denote the gravitational constant in D and $D-1$ dimensions, respectively. At this point, as the most natural and simple boundary conditions we take $a_D(\eta)$ and $a'_D(\eta)$ as continuous physical quantities at the transition points. Of course, strictly speaking neither of them could be continuous. However, since the phase transitions take place in a time interval whose duration is expected to be significantly smaller than those of the constant D eras and the evolution of the universe during these eras is inflationary, we could argue that during the transition period the change in $a_D(\eta)$ is negligible. On the other hand, in the limit as $\epsilon \to 0$ a discontinuity in $a'_D(\eta)$ would imply infinite forces acting on the universe, which in turn implies infinite values for the combinations $G_D\rho_D$ and $G_D P_D$ during the dimensional reduction. Thus, for the time being we argue that taking $a'_D(\eta)$ as continuous at the transition points is a good working assumption. We now consider Eqs. (9) and (10) in the limit as $\epsilon \to 0$ and take their ratios to obtain

$$\frac{G_D}{G_{D-1}} = \frac{D}{D-2}\frac{A_{D-1}}{A_D}\left[\frac{\rho_{D-1}}{\rho_D}\right]_{\eta=\eta_D}. \tag{11}$$

With the help of Eq. (11) we relate the change in G_D to that in ρ_D. Thus, in order to find how G_D changes as D changes, we have to know the discontinuity in density when the universe experiences a dimensional reduction. We may express this discontinuity by defining

$$l_{D,D-1} = \left[\frac{\rho_{D-1}}{\rho_D}\right]_{\eta=\eta_D}, \tag{12}$$

which gives us a critical length parameter with $[l_{D,D-1}] = cm$. Now, we could write the following expression for the change in G_D:

$$\frac{G_D}{G_{D-1}} = \frac{D}{D-2}\frac{A_{D-1}}{A_D}l_{D,D-1}, \tag{13}$$

where $l_{D,D-1}$ defines a characteristic length scale which marks the point at which dimensional reduction occurs.

4. Numerical Results and Conclusion

In order to demonstrate the basic features of our model we now consider a numerical model which starts from a $D = 4$ prematter era and evolves into a $D = 3$ prematter. In this model, to produce numerical results for the cosmological parameters of the universe one has to assign numerical values to the characteristic parameters of the model, i.e., c_3, c_4, $l_{4,3}$ and the initial value of the Hubble constant $H(0)$. It is apparent from Eqs. (1) and (7) that c_3 and c_4 represent the vacuum dominance of the universe in $D = 3$ and $D = 4$ prematter eras, respectively. In determining the critical parameters of the model we first consider the present value of the density:

$$\rho(\eta_{now}) = 6.2269 \cdot 10^{-35} T_m \left(\frac{4l_{4,3}}{\pi l_{pl}}\right)^{\frac{2(2-c_3)}{3(2c_3-1)}} gr/cm^3, \tag{14}$$

where T_m, which stands for the recombination temperature, is roughly at the order of $10^3 K$ (see, e.g., Ref. 7, pp. 70-82). Furthermore, since the evolution of the universe is inflationary during the prematter eras, we have

$$c_4 \ and \ c_3 \in (-1, 0)\,. \tag{15}$$

It is evident from Eqs. (14) and (15) that as the numerical value of the term $(4l_{4,3}/\pi l_{pl})$ gets far from unity, the present value of the density predicted in Eq. (14) falls rapidly below $10^{-31} gr/cm^3$ which represents the order of the present value of the total density. Hence, we may conclude that a phase transition is signalled when the critical length parameter approaches the Planck length of the lower dimension.

We also note that while our model is insensitive to the choice of c_3, its predictions strongly depend on the choice of c_4 and $H(0)$. We get numerical results that comply with recent observations[8] only when c_4 gets numerical values close to -1 and $H(0)$ remains at the order of 10^{63} $km/sec.mpc$. As a specific example, we took $T_m = 3000\,K$ and considered the following choices of parameters: $H(0) = 1.6 \cdot 10^{63}$ $km/sec.mpc$ and $l_{4,3} = 1.4 \cdot 10^{-33}$ cm. For this particular case, it is to be noted that only for a narrow range of initial conditions corresponding to the values of c_4 between -0.99145 and -0.99141, outcomes of this specific model are within the observed ranges for the cosmological parameters, i.e., Hubble constant H_0 ($60\ km/sec.mpc - 80\ km/sec.mpc$), age t_0 ($1.2 \cdot 10^{10}\ yrs - 1.6 \cdot 10^{10}\ yrs$) and density ρ_0 ($\sim 10^{-31} gr/cm^3$).

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