# The $\tau \to \mu\, \bar{\nu}_i\, \nu_i$ decay in the Randall Sundrum background with localized $U(1)_Y$ gauge boson

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**Abstract**

We study the effects of localization of the $U(1)_Y$ gauge boson around the visible brane and the contributions of the KK modes of Z bosons on the BR of the LFV $\tau \to \mu\, \bar{\nu}_i\, \nu_i$ decay. We observe that the BR is sensitive to the amount of localization of Z boson in the bulk of the Randall Sundrum background.

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# 1 Introduction

Lepton flavor violating (LFV) interactions are rich from the theoretical point of view since they exist at the loop level and the measurable quantities of these decays carry considerable information about the free parameters of the model used. In addition to this, they are clean theoretically because they are free from the nonperturbative effects. The experimental work done stimulate the theoretical studies on LFV decays. The $\mu \to e\gamma$ and $\tau \to \mu\gamma$ processes are among the LFV decays and the experimental the current limits for their branching ratios (BR) have been predicted as $1.2 \times 10^{-11}$ [1] and $1.1 \times 10^{-6}$ [2], respectively.

In the framework of the standard model (SM) the theoretical values of the BRs of LFV interactions are extremely small. Therefore, one search new models beyond the SM to enhance these numerical values and the two Higgs doublet model (2HDM), with flavor changing neural currents (FCNC) at tree level, is one of the candidate. In this model, the LFV interactions are induced by the internal neutral Higgs bosons $h^0$ and $A^0$ with the help of the Yukawa couplings, appearing as free parameters, which can be determined by the experimental data.

The present work is devoted to the analysis of $\tau \to \mu\, \bar{\nu}_i\, \nu_i$, $i = e, \mu, \tau$ decay in the 2HDM with the inclusion of a single extra dimension, respecting the Randall Sundrum model (see [3] and [4] for the calculation of the BR of the same decay in the 2HDM without and with flat extra dimensions). Here, the LFV transition $\tau \to \mu$ is driven by the internal scalar bosons $h^0$ and $A^0$ and the internal Z boson connects this transition to the $\bar{\nu}\nu$ output (see Fig. 1)[1].

The hierarchy problem between weak and Planck scales could be explained by introducing the extra dimensions. The model introduced by Randall and Sundrum [5, 6] (the RS1 model) is related to the non-factorizable geometry where the gravity is localized in a 4D brane, so called Planck (hidden) brane, which is away from another 4D brane, which is the visible (TeV) brane where we live. In this scenario, the hierarchy is generated by the warped factor, which is an exponential function of the compactified radius in the extra dimension. There are other scenarios based on the RS1 background in the literature [7]-[16]. In [8, 9] the behavior of $U(1)$ gauge boson, accessible to the extra dimension in the RS1 background, has been studied and it was observed that the massless mode was not localized in the extra dimension. Furthermore, it was obtained that the KK excitations have large couplings to boundaries and this was not a phenomenologically favorable scenario since, for a perturbative theory, it would have been necessary to push the visible scale to energies greater than TeV. To have a zero mode localized

[1]Here, we respect the assumption that the Cabibbo-Kobayashi-Maskawa (CKM) type matrix in the leptonic sector doest not exist and the charged flavor changing (FC) interactions vanish.

in the bulk and to get small couplings of KK modes with the boundaries, the idea of brane localized mass terms has been considered for scalar fields [10]. These terms could change the boundary conditions to get a zero mode localized solution. In [10, 11], the bulk fermions were considered in the RS1 background. [12] was devoted to a extensive work on the bulk fields in various multi-brane models. In [16], the $U(1)$ gauge field with bulk and boundary mass terms were taken into account and only the $U(1)_Y$ gauge field was considered as localized in the bulk.

In this work, we study the effects of localization of the $U(1)_Y$ gauge boson around the visible brane and analyze the contributions of the KK modes of Z bosons on the BRs of the LFV $\tau \to \mu\, \bar{\nu}_i\, \nu_i$ decay, by following the idea that the $U(1)_Y$ gauge field is accessible to the extra dimension in the RS1 background and the other particles, including the new Higgs doublet lie on the visible brane.

The paper is organized as follows: In Section 2, we present the theoretical expression for the decay width of the LFV decay $\tau \to \mu\, \bar{\nu}_i\, \nu_i$, $i = e, \mu, \tau$, in the framework of the 2HDM by considering the non zero localization of the $U(1)_Y$ gauge in the RS1 background. Section 3 is devoted to discussion and our conclusions. In Appendix A, we present the model construction. Appendix B is devoted to the explicit expressions of the functions appearing in the general effective vertex for the interaction of off-shell Z-boson with a fermionic current.

# 2 The effect of the localized $U(1)_Y$ gauge boson on the $\tau \to \mu\, \bar{\nu}_i\, \nu_i$ decay in the Randall Sundrum background

LFV $\tau \to \mu\, \bar{\nu}_i\, \nu_i$ decay exists at least in the one loop level and, therefore, the physical quantities, like the BR, contain rich information about the model used and the free parameters existing. In the version of the 2HDM with the FCNCs at tree level, the LFV interactions exist with larger BRs, compared to ones obtained in the extended SM with massive neutrinos. These decays are driven by the Yukawa interaction lagrangian and the strength of the interaction is controlled by the new Yukawa couplings. The additional effects coming from the possible extra dimension(s) bring new contributions to the BRs of these decays and, in the present work, we study these effects by assuming that the $U(1)_Y$ gauge boson is accessible to the extra dimension and is localized on the visible brane, in the RS1 background.

The RS1 model is a higher dimensional scenario that is based on a non-factorizable geometry and the hierarchy is generated by the warped factor, which is an exponential function of the compactified radius in the extra dimension. This model is based on the idea that the gravity is localized on the so called hidden brane, which is one of the boundary of the $S^1/Z_2$ orbifold extra dimension, and extended into the bulk with varying strength, however, the SM fields live in another brane, the so called visible brane, which is the other boundary of the extra dimension. In this scenario, the 5D cosmological constant is non vanishing and both branes have equal and opposite tensions so that the low energy effective theory has flat 4D spacetime. The metric of such 5D world reads

$$ds^2 = e^{-2\,k\,y}\,\eta_{\mu\nu}\,dx^\mu\,dx^\nu + dy^2\,, \tag{1}$$

where $k$ is the bulk curvature constant, $R$ is the compactification radius, $y = R\,|\theta|$, $0 \leq |\theta| \leq \pi$ and $e^{-k\,R\,|\theta|}$ is the warp factor which causes that all the mass terms are rescaled on the visible brane for $\theta = \pi$. With the rough estimate, $kR \sim 11 - 12$, all mass terms are bring down to the TeV scale.

The LFV $\tau \to \mu\,\bar{\nu}_i\,\nu_i$ decay is induced by $\tau \to \mu Z^*$ transition and $Z^* \to \bar{\nu}_i\,\nu_i$ process (see Fig. 1). The $\tau \to \mu Z^*$ transition, which needs the FCNC at tree level, is driven by the internal new neutral Higgs bosons $h^0$ and $A^0$, which are living on the visible brane. Now, we present the Yukawa interaction, which is responsible for the $\tau \to \mu$ transition of the decay,

$$\mathcal{S}_Y = \int d^5x\sqrt{-g}\left(\xi^E\,\bar{l}_{iL}\phi_2 E_{jR} + h.c.\right)\delta(y-\pi R)\ \ , \tag{2}$$

where $L$ and $R$ denote chiral projections $L(R) = 1/2(1 \mp \gamma_5)$, $\phi_2$ is the new scalar doublet, $l_{iL}$ ($E_{jR}$) are lepton doublets (singlets), $\xi^E_{ij}$ [2], with family indices $i, j$ , are the Yukawa couplings which induce the FV interactions in the lepton sector and $g$ is the determinant of the metric[3] (see eq. (1)). We assume that the Higgs doublet $\phi_1$ has non-zero vacuum expectation value to ensure the ordinary masses of the gauge fields and the fermions, however, the second doublet has no vacuum expectation value, namely, we choose the doublets $\phi_1$ and $\phi_2$ and their vacuum expectation values as

$$\phi_1 = \frac{1}{\sqrt{2}}\left[\left(\begin{array}{c} 0 \\ v + H^0 \end{array}\right) + \left(\begin{array}{c} \sqrt{2}\chi^+ \\ i\chi^0 \end{array}\right)\right] ; \phi_2 = \frac{1}{\sqrt{2}}\left(\begin{array}{c} \sqrt{2}H^+ \\ H_1 + iH_2 \end{array}\right)\ , \tag{3}$$

and

$$< \phi_1 > = \frac{1}{\sqrt{2}}\left(\begin{array}{c} 0 \\ v \end{array}\right)\ ; < \phi_2 > = 0\ . \tag{4}$$

[2] In the following, we replace $\xi^E$ with $\xi^E_N$ where "N" denotes the word "neutral".

[3] Notice that the term $\sqrt{-g} = e^{-4\,k\,y}$ is embedded into the redefinitions of the fields on the visible brane for $y = \pi\,R$, namely, they are warped as $\phi_2 \to e^{k\,\pi\,R}\,\phi_2^{warp}$, $l_i \to e^{3\,k\,\pi\,R/2}\,l_i^{warp}$ and in the following we use warped fields without the *warp* index.

This choice ensures that the mixing between neutral scalar Higgs bosons is switched off and it would be possible to separate the particle spectrum so that the SM particles are collected in the first doublet and the new particles in the second one[4].

In our analysis, we further assume that the $U(1)_Y$ gauge boson is accessible to the extra dimension and the other particles,including $SU(2)_L$ gauge bosons are confined on the visible brane. This leads to an effective localization of Z boson in the bulk of RS1 background. Therefore, the physical quantities related to the decay studied get additional contributions coming from the internal Z boson and its KK modes (see [16] for details and Appendix A for the summary of the model construction).

Now, we present the general effective vertex for the interaction of off-shell Z-boson with a fermionic current

$$\Gamma_{\mu}^{(REN)}(\tau \to \mu Z^*) = f_1\,\gamma_\mu + f_2\,\gamma_\mu\gamma_5 + f_3\,\sigma_{\mu\nu}k^\nu + f_4\,\sigma_{\mu\nu}\gamma_5 k^\nu\,, \tag{5}$$

where $k$ is the momentum transfer, $k^2 = (p-p')^2$, $p$ $(p')$ is the four momentum vector of incoming (outgoing) lepton and the explicit expressions for the functions $f_1$, $f_2$, $f_3$ and $f_4$ are given in Appendix B. The matrix element $M$ of the $\tau \to \mu\,\bar{\nu}_i\,\nu_i$ process is obtained by connecting the $\tau \to \mu$ transition to the $\bar{\nu}_i\nu_i$ pair with the internal Z boson and its KK modes (see Fig. 1). For the $n^{th}$ internal Z KK mode contribution, the coupling $g$ in $f_i, i = 1, ...4$, and at the $Z^{(n)}\bar{\nu}_i\nu_i$ vertex is replaced by $g^{(n)} \sim \frac{g}{\sqrt{\alpha}}$ and the $Z$ boson KK $n$ mode propagator is obtained by using the KK mode mass $m_n$ (see eq. (17)). Using the matrix element $M$, the decay width $\Gamma$ of the decay under consideration can be obtained in the $\tau$ lepton rest frame with the help of the well known expression

$$d\Gamma = \frac{(2\,\pi)^4}{2\,m_\tau}\,|M|^2\,\delta^4(p - \sum_{i=1}^{3} p_i)\,\prod_{i=1}^{3}\frac{d^3p_i}{(2\pi)^3 2E_i}\,, \tag{6}$$

where $p$ ($p_i$, i=1,2,3) is four momentum vector of $\tau$ lepton ($\mu$ lepton, incoming $\nu$, outgoing $\nu$).

# 3 Discussion

The BRs of the LFV decays are negligible in the SM, including non-zero neutrino masses. The extension of the Higgs sector brings new LFV vertices with the help of the new Higgs scalars. 2HDM with FCNC at tree level is the most primitive model in order to turn on the LFV interactions and the $\tau \to \mu\,\bar{\nu}_i\,\nu_i$ is induced by the LFV $\tau \to \mu$ transition which exists at least at one loop level in this model. The Yukawa couplings $\bar{\xi}^E_{N,ij}, i,j = e,\mu,\tau$ are the essential

[4] Here $H^1$ $(H^2)$ is the well known mass eigenstate $h^0$ $(A^0)$.

parameters driving the lepton flavor violation and they are free parameters which should be fixed by present and forthcoming experiments.[5] In our calculations, we assume that the Yukawa couplings $\bar{\xi}^{E}_{N,ij}$ are symmetric with respect to the indices $i$ and $j$ and take $\bar{\xi}^{E}_{N,ij}$, $i,j=e,\mu$, as small compared to $\bar{\xi}^{E}_{N,\tau\, i}\, i=e,\mu,\tau$ since we consider that the strengths of these couplings are related with the masses of leptons denoted by the indices of them. Therefore, we take only the $\tau$ lepton as an internal one and we choose the couplings $\bar{\xi}^{E}_{N,\tau\tau}$ and $\bar{\xi}^{E}_{N,\tau\mu}$ as non-zero. The upper limit of the coupling $\bar{\xi}^{E}_{N,\tau\mu}$ has been estimated as $30\, GeV$ (see [17] and references therein) by assuming that the new physics effects can not exceed experimental uncertainty $10^{-9}$ in the measurement of the muon anomalous magnetic moment. In our numerical calculation we choose $\bar{\xi}^{E}_{N,\tau\mu}=1\, GeV$ by respecting this upper limit. Since there is no restriction for the Yukawa coupling $\bar{\xi}^{E}_{N,\tau\tau}$, the numerical values we use are greater than $\bar{\xi}^{E}_{N,\tau\mu}$.

The addition of a single extra spatial dimension brings new contributions to the BR of the decay under consideration and the source of these contributions are the KK excitations of the particles, which live in the bulk. Here, we study the BR of the LFV process $\tau\to\mu\,\bar{\nu}_i\,\nu_i$ in the framework of the 2HDM, including the extra dimension effects in the RS1 scenario. The RS1 model is an alternative scenario to solve the well known hierarchy problem. It is based on the assumption that the extra dimension is compactified into $S^1/Z_2$ orbifold with two 4D surfaces (branes) at the boundaries in 5D world and the extra dimensional bulk is populated by gravity, which is localized on the hidden brane. Furthermore, the SM particles live on the so called visible brane. However, in our case, we follow the idea [16] that the $U(1)_Y$ gauge field is accessible to the extra dimension in the RS1 background and the other particles, including the new Higgs doublet, lie on the visible brane[6]. With the help of the boundary mass term (see eq. (12)) it is possible to get zero the mode term and this mode is localized around the visible brane with the special choice of the parameters, $a$ and $\alpha$, related to the bulk and boundary mass terms (eq. (14)). If the parameter $\alpha$ vanishes, namely, there is no boundary mass term, KK mode coupling to fermions becomes almost one order larger compared to the zero mode one and, in order to obtain a perturbative theory, it would be necessary to push the visible scale to energies greater than TeV (see the discussion given in [8, 9]). This is not a phenomenologically favorable scenario. For $\alpha>0$ the zero mode is localized around the visible brane and the KK mode fermion coupling is small enough for $\alpha>1$ to obey the phenomenology. Notice that one gets the original RS1 model for infinitely large $\alpha$.

[5]The dimensionfull Yukawa couplings $\bar{\xi}^{E}_{N,ij}$ are defined as $\xi^{E}_{N,ij}=\sqrt{\frac{4\,G_F}{\sqrt{2}}}\,\bar{\xi}^{E}_{N,ij}$.

[6]Here we assume that the corresponding gauge field make small contribution to the bulk energy density not to disturb the solution of the Einstein's equations.

In the present work, we study the effects of localization of the $U(1)_Y$ gauge boson and the contributions of the KK modes of Z bosons on the BRs of the LFV $\tau \to \mu\,\bar{\nu}_i\,\nu_i$ decay. Throughout our calculations we use the input values given in Table (1).

| Parameter | Value |
|---|---|
| $m_\mu$ | 0.106 (GeV) |
| $m_\tau$ | 1.78 (GeV) |
| $m_{h^0}$ | 100 (GeV) |
| $m_{A^0}$ | 200 (GeV) |
| $G_F$ | $1.1663710^{-5}(GeV^{-2})$ |

Table 1: The values of the input parameters used in the numerical calculations.

In Fig. 2, we present the parameter $a$ dependence of the BR of the LFV $\tau \to \mu\,\bar{\nu}_i\,\nu_i$ decay for $\bar{\xi}^{E}_{N,\tau\tau} = 10\,GeV$ and $\bar{\xi}^{E}_{N,\tau\mu} = 1\,GeV$. The solid (dashed-small dashed-dotted) line represents the BR without extra dimension (with extra dimension for $kR = 12$, $k = 10^{18}\,GeV - kR = 11$, $k = 10^{18}\,GeV - kR = 11$, $k = 10^{17}\,GeV$). This figure shows that the BR is at the order of the magnitude of $10^{-6}$ for the free parameters used and it decreases with the increasing values of $a$. This is due to the fact that the interaction coupling of Z boson KK modes to the fermions becomes weak with the increasing values of the zero mode localization parameter $\alpha$ and the increase in KK mode masses results in a suppression in the KK mode contribution. For the limit $a \to \infty$, this coupling vanishes, the zero mode is confined in the visible brane and we reach the numerical values of BR for the original RS1 model, where all 2HDM particles are confined in the visible brane. On the other hand, we observe that for decreasing values of $kR$ the BR becomes smaller because of the weaker coupling $g^{(n)}$.

Fig. 3 is devoted to the parameter $k$ dependence of the BR($\tau \to \mu\,\bar{\nu}_i\,\nu_i$) for $\bar{\xi}^{E}_{N,\tau\tau} = 10\,GeV$ and $\bar{\xi}^{E}_{N,\tau\mu} = 1\,GeV$ and $kR = 11$. The solid (dashed-small dashed-dotted) line represents the BR without extra dimension (with extra dimension for $a = 0.1 - 0.5 - 1.0$). It is observed that the BR decreases with the increasing values of $k$. Here the KK mode masses increases with $k$ for fixed $kR$ and, as a result, the BR is suppressed and reaches to the one which is obtained without extra dimension.

Finally, for completeness, we study the Yukawa coupling dependence of the BR for different values of the parameter $a$. Fig. 4 represents $\bar{\xi}^{E}_{N,\tau\tau}$ dependence of the BR of the LFV $\tau \to \mu\,\bar{\nu}_i\,\nu_i$ decay, for $\bar{\xi}^{E}_{N,\tau\mu} = 1\,GeV$, $kR = 12$ and $k = 10^{18}\,GeV$. The solid (dashed-small dashed) line represents the BR without extra dimension (with extra dimension for $a = 0.5 - 1.0$). The BR is sensitive to the Yukawa couplings as it should be and the increase in the parameter $a$ pushes the numerical value of the BR to the one obtained without extra dimension because of strong localization of the Z field around the visible brane. This is case that the original RS1 model is reached, where the extra dimension effects are switched off for the process under consideration.

As a result, the BR of the LFV $\tau \rightarrow \mu\,\bar{\nu}_i\,\nu_i$ decay is sensitive to the parameter $a$ and it decreases with the increasing values of $a$. In addition to this, the BR decreases with the increasing values of $k$ for fixed $kR$, since the masses of KK modes are proportional to $k$. The sensitivity of the BR($\tau \rightarrow \mu\,\bar{\nu}_i\,\nu_i$) to the extra dimension effects in RS1 model is informative and the more accurate future experimental measurement of this decay can ensure a possible test for the existence of the model used and the determination of the signals coming from the extra dimensions.

# Appendix

## A Model Construction

Now, we would like to summarize the model construction, following the work [16], by respecting the assumption that $U(1)_Y$ gauge boson is accessible to the extra dimension and the $SU(2)_L$ gauge bosons and the other particles, fermions, Higgs bosons, are living on the visible brane, in the RS1 background. The starting point is the 5D action

$$S = \int d^5x \sqrt{-g} \Big\{ -\frac{1}{4} F^{MN} F_{MN} + \Big( -\frac{1}{4} G^{a\,\mu\nu} G^a_{\mu\nu} + (D^\mu \phi)^\dagger D_\mu \phi - V(\phi) \Big) \delta(y - \pi R) \Big\} , \quad (7)$$

with field strength tensors $F^{MN}$ and $G^{a\,\mu\nu}$ for $U(1)_Y$ and $SU(2)_L$ gauge bosons, where $M, N = 0, ..., 4$; $\mu, \nu = 0, ..., 3$. The covariant derivative $D_\mu$ reads

$$D_\mu = \partial_\mu - \frac{i}{2} g\, A^a_\mu(x)\, \sigma^a - i\, g_5\, Y\, B_\mu(x, y) \ , \quad (8)$$

where $B_\mu(x,y)$, $A^a_\mu(x)$ are $U(1)_Y$, $SU(2)_L$ gauge bosons, respectively; $g$, $g_5$ are the corresponding couplings to the Higgs boson and $\sigma^a$ (Y=1/2) is the Pauli spin matrix (the hypercharge). Here the gauge field $B_\mu(x,y)$ is expanded to its Kaluza Klein (KK) modes as

$$B_\mu(x,y) = \sum_n B^{(n)}_\mu(x)\, \chi^{(n)}(y) \ , \quad (9)$$

and $\chi^{(n)}(y)$ satisfy the differential equation (see also [8, 9])

$$\Big( \frac{\partial^2}{\partial y^2} - 2\,k\, \frac{\partial}{\partial y} - m^2 + e^{2\,k\,y}\, m_n^2 \Big) \chi^{(n)}(y) = 0 \ , \quad (10)$$

with the normalization condition

$$\int_0^{\pi R} dy\, \chi^{(n)}(y)\, \chi^{(m)}(y) = \delta_{nm} \ . \quad (11)$$

Here $k$ is the curvature scale and $m$ is the bulk mass of the gauge field $B_\mu(x,y)$, which is at the order of the magnitude of $k$. To obtain zero mode, which appears in the construction of the photon and Z boson fields, the boundary mass term

$$S' = - \int d^5x \sqrt{-g}\, \alpha\, k\, B^\mu(x,y)\, B_\mu(x,y) \Big( \delta(y) - \delta(y - \pi R) \Big) \ , \quad (12)$$

is considered[7] and this term induces the boundary condition

$$\Big( \frac{\partial B_\mu(x,y)}{\partial y} - \alpha\, k\, B_\mu(x,y) \Big)\Big|_{y=0,\pi R} = 0 \ , \quad (13)$$

[7] The idea of brane localized mass terms has been considered for scalar fields in [7],[10].

which results in non-vanishing zero mode with the fine tuning of the parameters $\alpha$ and $a = m^2/k^2$,

$$\alpha_\pm = 1 \pm \sqrt{1+a}\ . \tag{14}$$

Finally, the normalized zero mode is obtained as

$$\chi^{(0)} = \sqrt{\frac{2\,\alpha k}{e^{2\,\alpha\,k\,\pi\,R} - 1}}\, e^{\alpha\,k\,y}\ , \tag{15}$$

and the $n$ mode reads

$$\chi^{(n)}(y) = N_n\, e^{k\,y} \left( J_\nu(\frac{m_n}{k}\, e^{k\,y}) + b_{n\nu}\, Y_\nu(\frac{m_n}{k}\, e^{k\,y}) \right)\ , \tag{16}$$

with the normalization constant $N_n$, and the parameter $\nu = \sqrt{1+a}$. Using the boundary condition at $y = 0$ and $y = \pi\,R$ (see eq.( 13)), the mass spectrum of KK modes ($n = 1, 2, ...$) reads

$$m_n \simeq (n \pm \frac{1}{2}\,\alpha_\pm - \frac{1}{4})\,\pi\,k\,e^{-k\,\pi\,R}\ , \tag{17}$$

for $k\,e^{-k\,\pi\,R} \ll m_n \ll k$.

Now, we present the mass Lagrangian for the photon and Z fields. After allowing the Higgs boson vacuum expectation value (see eq. (4)) and expanding the $U(1)_Y$ gauge field $B_\mu(x, y)$ to KK modes (see eq. (9), the mass Lagrangian becomes

$$L_m = \sum_{n=1}^{\infty} \frac{1}{2}\, m_n^2\, (B_\mu^{(n)}(x))^2 + \frac{1}{2}\,(\frac{v}{2})^2 \left( - g\, A_\mu^3(x) + g'\, B_\mu^0(x) + g' \sum_{n=1}^{\infty} \frac{\chi^{(n)}(\pi\,R)}{\chi^{(0)}(\pi\,R)}\, B_\mu^n(x) \right)^2\ , \tag{18}$$

where $g' = g_5\,\chi^{(0)}(\pi\,R)$. By considering the new basis and using the mixing angle $\theta_W$, we rewrite the above Lagrangian in terms of photon and Z fields as

$$L_m = \sum_{n=1}^{\infty} \frac{1}{2}\, m_n^2 (B_\mu^{(n)}(x))^2 + \frac{1}{2} \left( m_Z\, Z_\mu(x) - \frac{g'\,v}{2} \sum_{n=1}^{\infty} \frac{\chi^{(n)}(\pi\,R)}{\chi^{(0)}(\pi\,R)}\, B_\mu^n(x) \right)^2\ . \tag{19}$$

Here the photon field is massless as it should be and there exists mixing among Z boson and $B$ field KK modes. With the assumption that $m_Z \ll m_n$, the diagonalization of the mass matrix results in the shift in the mass of Z boson as

$$m_Z^{phys} = m_Z \sqrt{1 - (\frac{g'\,v}{2})^2 \sum_{n=1}^{\infty} \Big(\frac{1}{m_n} \frac{\chi^{(n)}(\pi\,R)}{\chi^{(0)}(\pi\,R)}\Big)^2}\ , \tag{20}$$

and the other physical masses coming from the mixing can be approximated to $m_n$ (see eq. (17)). On the other hand, the coupling of zero mode (KK mode) physical Z boson to the fermions reads $g' = g_5\,\chi^{(0)}$ ($g'^{(n)} \sim g_5\,\chi^{(n)}$). Here $g'^{(n)} \sim g' \sqrt{\frac{1-e^{-2\alpha\,\pi\,k\,R}}{\alpha}}$ and, for positive $\alpha$, $g'^{(n)} \sim g' \sqrt{\frac{1}{\alpha}}$. This is the case that the photon and Z bosons are effectively localized around the visible brane.

## B Explicit Expresions

The explicit expressions for the functions $f_1$, $f_2$, $f_3$ and $f_4$ appearing in eq. (5) read

$$
\begin{aligned}
f_1 &= \frac{g}{64\,\pi^2\, cos\,\theta_W}\int_0^1 dx\,\frac{1}{m_{l_2}^2-m_{l_1}^2}\Bigg\{c_V\,(m_{l_2}+m_{l_1})\\
&\Bigg( (-m_i\,\eta_i^+ + m_{l_1}(-1+x)\,\eta_i^V)\,ln\,\frac{L^{self}_{1,\,h^0}}{\mu^2} + (m_i\,\eta_i^+ - m_{l_2}(-1+x)\,\eta_i^V)\,ln\,\frac{L^{self}_{2,\,h^0}}{\mu^2}\\
&+ (m_i\,\eta_i^+ + m_{l_1}(-1+x)\,\eta_i^V)\,ln\,\frac{L^{self}_{1,\,A^0}}{\mu^2} - (m_i\,\eta_i^+ + m_{l_2}(-1+x)\,\eta_i^V)\,ln\,\frac{L^{self}_{2,\,A^0}}{\mu^2}\Bigg)\\
&+ c_A\,(m_{l_2}-m_{l_1})\\
&\Bigg( (-m_i\,\eta_i^- + m_{l_1}(-1+x)\,\eta_i^A)\,ln\,\frac{L^{self}_{1,\,h^0}}{\mu^2} + (m_i\,\eta_i^- + m_{l_2}(-1+x)\,\eta_i^A)\,ln\,\frac{L^{self}_{2,\,h^0}}{\mu^2}\\
&+ (m_i\,\eta_i^- + m_{l_1}(-1+x)\,\eta_i^A)\,ln\,\frac{L^{self}_{1,\,A^0}}{\mu^2} + (-m_i\,\eta_i^- + m_{l_2}(-1+x)\,\eta_i^A)\,ln\,\frac{L^{self}_{2,\,A^0}}{\mu^2}\Bigg)\Bigg\}\\
&- \frac{g}{64\,\pi^2\, cos\,\theta_W}\int_0^1 dx\int_0^{1-x} dy\,\Bigg\{m_i^2\,(c_A\,\eta_i^A - c_V\,\eta_i^V)\,(\frac{1}{L^{ver}_{A^0}}+\frac{1}{L^{ver}_{h^0}})\\
&- (1-x-y)\,m_i\Bigg(c_A\,(m_{l_2}-m_{l_1})\,\eta_i^-\,(\frac{1}{L^{ver}_{h^0}}-\frac{1}{L^{ver}_{A^0}}) + c_V\,(m_{l_2}+m_{l_1})\,\eta_i^+\,(\frac{1}{L^{ver}_{h^0}}+\frac{1}{L^{ver}_{A^0}})\Bigg)\\
&- (c_A\,\eta_i^A + c_V\,\eta_i^V)\Bigg(-2 + (k^2\,x\,y + m_{l_1}\,m_{l_2}\,(-1+x+y)^2)\,(\frac{1}{L^{ver}_{h^0}}+\frac{1}{L^{ver}_{A^0}}) - ln\,\frac{L^{ver}_{h^0}}{\mu^2}\,\frac{L^{ver}_{A^0}}{\mu^2}\Bigg)\\
&- (m_{l_2}+m_{l_1})\,(1-x-y)\,\Bigg(\frac{\eta_i^A\,(x\,m_{l_1}+y\,m_{l_2}) + m_i\,\eta_i^-}{2\,L^{ver}_{A^0\,h^0}} + \frac{\eta_i^A\,(x\,m_{l_1}+y\,m_{l_2}) - m_i\,\eta_i^-}{2\,L^{ver}_{h^0\,A^0}}\Bigg)\\
&+ \frac{1}{2}\eta_i^A\,ln\,\frac{L^{ver}_{A^0\,h^0}}{\mu^2}\,\frac{L^{ver}_{h^0\,A^0}}{\mu^2}\Bigg\}\,,\\
f_2 &= \frac{g}{64\,\pi^2\, cos\,\theta_W}\int_0^1 dx\,\frac{1}{m_{l_2}^2-m_{l_1}^2}\Bigg\{c_V\,(m_{l_2}-m_{l_1})\\
&\Bigg( (m_i\,\eta_i^- + m_{l_1}(-1+x)\,\eta_i^A)\,ln\,\frac{L^{self}_{1,\,A^0}}{\mu^2} + (-m_i\,\eta_i^- + m_{l_2}(-1+x)\,\eta_i^A)\,ln\,\frac{L^{self}_{2,\,A^0}}{\mu^2}\\
&+ (-m_i\,\eta_i^- + m_{l_1}(-1+x)\,\eta_i^A)\,ln\,\frac{L^{self}_{1,\,h^0}}{\mu^2} + (m_i\,\eta_i^- + m_{l_2}(-1+x)\,\eta_i^A)\,ln\,\frac{L^{self}_{2,\,h^0}}{\mu^2}\Bigg)\\
&+ c_A\,(m_{l_2}+m_{l_1})\\
&\Bigg( (m_i\,\eta_i^+ + m_{l_1}(-1+x)\,\eta_i^V)\,ln\,\frac{L^{self}_{1,\,A^0}}{\mu^2} - (m_i\,\eta_i^+ + m_{l_2}(-1+x)\,\eta_i^V)\,ln\,\frac{L^{self}_{2,\,A^0}}{\mu^2}\\
&+ (-m_i\,\eta_i^+ + m_{l_1}(-1+x)\,\eta_i^V)\,ln\,\frac{L^{self}_{1,\,h^0}}{\mu^2} + (m_i\,\eta_i^+ - m_{l_2}(-1+x)\,\eta_i^V)\,\frac{ln\,L^{self}_{2,\,h^0}}{\mu^2}\Bigg)\Bigg\}\\
&+ \frac{g}{64\,\pi^2\, cos\,\theta_W}\int_0^1 dx\int_0^{1-x} dy\,\Bigg\{m_i^2\,(c_V\,\eta_i^A - c_A\,\eta_i^V)\,(\frac{1}{L^{ver}_{A^0}}+\frac{1}{L^{ver}_{h^0}})
\end{aligned}
$$

$$
\begin{aligned}
&- && m_i\,(1-x-y)\left(c_V\,(m_{l_2}-m_{l_1})\,\eta_i^- + c_A\,(m_{l_2}+m_{l_1})\,\eta_i^+\right)\big(\frac{1}{L^{ver}_{h^0}}-\frac{1}{L^{ver}_{A^0}}\big) \\
&+ && (c_V\,\eta_i^A + c_A\,\eta_i^V)\bigg(-2+(k^2\,x\,y - m_{l_1}\,m_{l_2}\,(-1+x+y)^2)\big(\frac{1}{L^{ver}_{h^0}}+\frac{1}{L^{ver}_{A^0}}\big) - ln\,\frac{L^{ver}_{h^0}}{\mu^2}\,\frac{L^{ver}_{A^0}}{\mu^2}\bigg) \\
&- && (m_{l_2}-m_{l_1})\,(1-x-y)\left(\frac{\eta_i^V\,(x\,m_{l_1}-y\,m_{l_2})+m_i\,\eta_i^+}{2\,L^{ver}_{A^0\,h^0}} + \frac{\eta_i^V\,(x\,m_{l_1}-y\,m_{l_2})-m_i\,\eta_i^+}{2\,L^{ver}_{h^0\,A^0}}\right) \\
&- && \frac{1}{2}\eta_i^V\,ln\,\frac{L^{ver}_{A^0\,h^0}}{\mu^2}\,\frac{L^{ver}_{h^0\,A^0}}{\mu^2}\bigg\}\,, \\
f_3 &= && -i\frac{g}{64\,\pi^2\,cos\,\theta_W}\int_0^1 dx\,\int_0^{1-x} dy\bigg\{\bigg((1-x-y)\,(c_V\,\eta_i^V + c_A\,\eta_i^A)\,(x\,m_{l_1}+y\,m_{l_2}) \\
&+ && m_i\,(c_A\,(x-y)\,\eta_i^- + c_V\,\eta_i^+\,(x+y))\bigg)\,\frac{1}{L^{ver}_{h^0}} \\
&+ && \bigg((1-x-y)\,(c_V\,\eta_i^V + c_A\,\eta_i^A)\,(x\,m_{l_1}+y\,m_{l_2}) - m_i\,(c_A\,(x-y)\,\eta_i^- + c_V\,\eta_i^+\,(x+y))\bigg)\,\frac{1}{L^{ver}_{A^0}} \\
&- && (1-x-y)\bigg(\frac{\eta_i^A\,(x\,m_{l_1}+y\,m_{l_2})}{2}\,\big(\frac{1}{L^{ver}_{A^0\,h^0}}+\frac{1}{L^{ver}_{h^0\,A^0}}\big) + \frac{m_i\,\eta_i^-}{2}\,\big(\frac{1}{L^{ver}_{h^0\,A^0}}-\frac{1}{L^{ver}_{A^0\,h^0}}\big)\bigg)\bigg\}\,, \\
f_4 &= && -i\frac{g}{64\,\pi^2\,cos\,\theta_W}\int_0^1 dx\,\int_0^{1-x} dy\bigg\{\bigg((1-x-y)\,\Big(-(c_V\,\eta_i^A + c_A\,\eta_i^V)\,(x\,m_{l_1}-y\,m_{l_2})\Big) \\
&- && m_i\,(c_A\,(x-y)\,\eta_i^+ + c_V\,\eta_i^-\,(x+y))\bigg)\,\frac{1}{L^{ver}_{h^0}} \\
&+ && \bigg((1-x-y)\,\Big(-(c_V\,\eta_i^A + c_A\,\eta_i^V)\,(x\,m_{l_1}-y\,m_{l_2})\Big) + m_i\,(c_A\,(x-y)\,\eta_i^+ + c_V\,\eta_i^-\,(x+y))\bigg)\,\frac{1}{L^{ver}_{A^0}} \\
&+ && (1-x-y)\left(\frac{\eta_i^V}{2}\,(m_{l_1}\,x - m_{l_2}\,y)\,\big(\frac{1}{L^{ver}_{A^0\,h^0}}+\frac{1}{L^{ver}_{h^0\,A^0}}\big) + \frac{m_i\,\eta_i^+}{2}\,\big(\frac{1}{L^{ver}_{A^0\,h^0}}-\frac{1}{L^{ver}_{h^0\,A^0}}\big)\right)\bigg\}\,, \qquad (21)
\end{aligned}
$$

where

$$
\begin{aligned}
L^{self}_{1,\,h^0} &= m^2_{h^0}\,(1-x) + (m_i^2 - m^2_{l_1}\,(1-x))\,x\,, \\
L^{self}_{1,\,A^0} &= L^{self}_{1,\,h^0}(m_{h^0}\to m_{A^0})\,, \\
L^{self}_{2,\,h^0} &= L^{self}_{1,\,h^0}(m_{l_1}\to m_{l_2})\,, \\
L^{self}_{2,\,A^0} &= L^{self}_{1,\,A^0}(m_{l_1}\to m_{l_2})\,, \\
L^{ver}_{h^0} &= m^2_{h^0}\,(1-x-y) + m_i^2\,(x+y) - k^2\,x\,y\,, \\
L^{ver}_{h^0\,A^0} &= m^2_{A^0}\,x + m_i^2\,(1-x-y) + (m^2_{h^0} - k^2\,x)\,y\,, \\
L^{ver}_{A^0} &= L^{ver}_{h^0}(m_{h^0}\to m_{A^0})\,, \\
L^{ver}_{A^0\,h^0} &= L^{ver}_{h^0\,A^0}(m_{h^0}\to m_{A^0})\,, \qquad (22)
\end{aligned}
$$

and

$$
\eta_i^V = \xi^E_{N,l_1 i}\xi^{E\,*}_{N,il_2} + \xi^{E\,*}_{N,il_1}\xi^E_{N,l_2 i}\,,
$$

$$\begin{aligned}
\eta_i^A &= \xi^E_{N,l_1 i}\xi^{E*}_{N,il_2} - \xi^{E*}_{N,il_1}\xi^E_{N,l_2 i}\,, \\
\eta_i^+ &= \xi^{E*}_{N,il_1}\xi^{E*}_{N,il_2} + \xi^E_{N,l_1 i}\xi^E_{N,l_2 i}\,, \\
\eta_i^- &= \xi^{E*}_{N,il_1}\xi^{E*}_{N,il_2} - \xi^E_{N,l_1 i}\xi^E_{N,l_2 i}\,.
\end{aligned} \qquad (23)$$

The parameters $c_V$ and $c_A$ are $c_A = -\frac{1}{4}$ and $c_V = \frac{1}{4} - sin^2\,\theta_W$. In eq. (23) the flavor changing couplings $\xi^E_{N,ji}$ represent the effective interaction between the internal lepton $i$, $(i = e, \mu, \tau)$ and outgoing (incoming) $j = l_1\,(j = l_2)$ one. Here we take only the $\tau$ lepton in the internal line and we neglect all the Yukawa couplings except $\xi^E_{N,\tau\tau}$ and $\xi^E_{N,\tau\mu}$ in the loop contributions (see Discussion section). The Yukawa couplings $\xi^E_{N,ji}$ are complex in general, however, in the present work, we take them real.

# References


[1] M. L. Brooks et. al., MEGA Collaboration, *Phys. Rev. Lett.* **83**, 1521 (1999).

[2] S. Ahmed et.al., CLEO Collaboration, *Phys. Rev.* **D61**, 071101 (2000).

[3] E. Iltan, H. Sundu, *J. Phys.* **G29** 1373 (2003)

[4] E. Iltan, *Mod. Phys.* **A20** 1845 (2005).

[5] L. Randall, R.Sundrum, *Phys. Rev. Lett.* **83** 3370 (1999);

[6] L. Randall, R.Sundrum, *Phys. Rev. Lett.* **83** 4690 (1999);

[7] W. D. Goldberger, M. B. Wise, *Phys. Rev. Lett.* **D 83** (1999) 4922.

[8] A. Pomarol, *Phys. Lett.* **B486** 153 (2000).

[9] H. Davoudias, J. L. Hewett, T. G. Rizzo, *Phys. Lett.* **B473** 43 (2000).

[10] T. Gherghetta, A. Pomarol, *Nucl. Phys.* **B586** 141 (2000).

[11] Y. Grossman, M. Neubert, *Phys. Lett.* **B474** 361 (2000).

[12] I. I. Kogan, S. Mouslopolous, A. Papazoglou, G. G. Ross, *Nucl. Phys.* **B615** 191 (2001).

[13] T. Gherghetta, A. Pomarol, *Nucl. Phys.* **B602** 3 (2001).

[14] S. J. Huber, Q. Shafi, *Phys. Rev.* **D63**, 045010 (2001).

[15] K. Ghoroku, A. Nakamura, *Phys. Rev.* **D65**, 084017 (2002).

[16] B. Batell, T. Gherghetta, *Phys. Rev.* **D73**, 045016 (2006).

[17] E. Iltan, H. Sundu, *Acta Phys.Slov.* **53** 17, (2003)

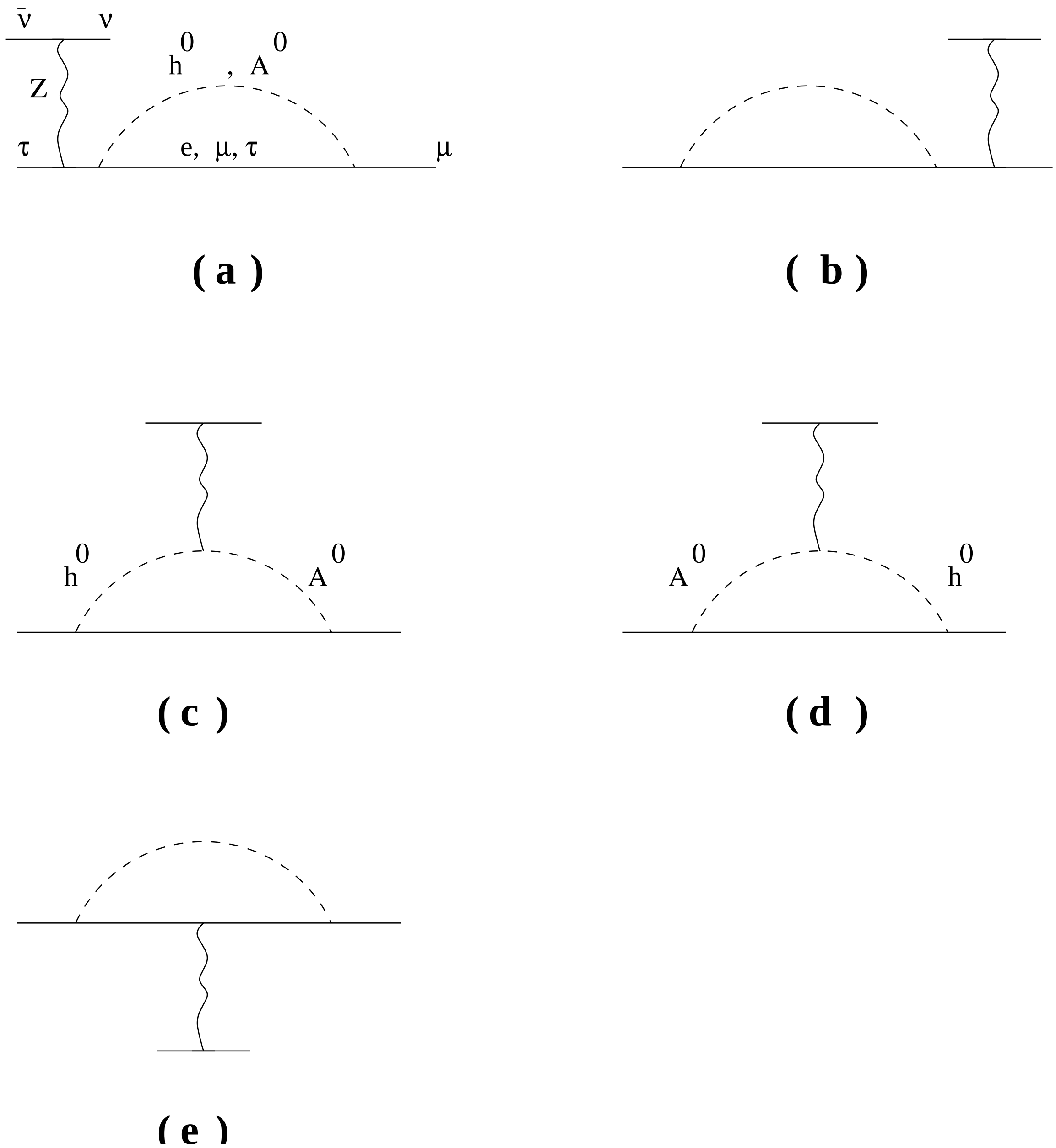


Figure 1: One loop diagrams contribute to $\tau \to \mu\,\bar{\nu}_i\,\nu_i$, $i = e, \mu, \tau$ decay due to the neutral Higgs bosons $h_0$ and $A_0$ in the 2HDM. Solid lines represent leptons and neutrinos, curly (dashed) lines represent the virtual $Z$ boson and its KK modes ($h_0$ and $A_0$ fields).

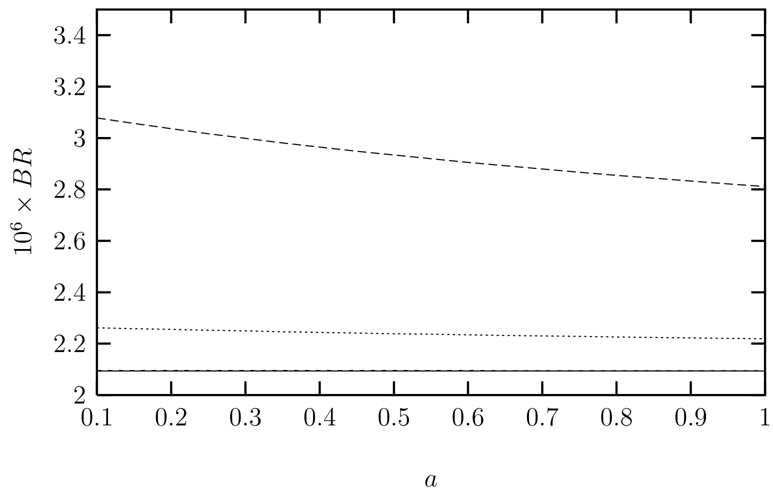


Figure 2: The parameter $a$ dependence of the BR of the LFV $\tau \to \mu \, \bar{\nu}_i \, \nu_i$ decay for $\bar{\xi}^{E}_{N,\tau\tau} = 10 \, GeV$ and $\bar{\xi}^{E}_{N,\tau\mu} = 1 \, GeV$. The solid (dashed-small dashed-dotted) line represents the BR without extra dimension (with extra dimension for $kR = 12$, $k = 10^{18} \, GeV - kR = 11$, $k = 10^{18} \, GeV - kR = 11$, $k = 10^{17} \, GeV$).

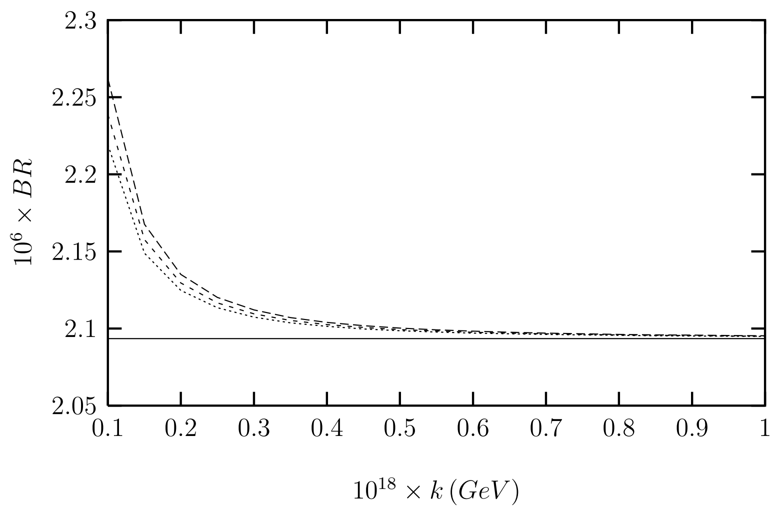


Figure 3: The parameter $k$ dependence of the BR of the LFV $\tau \to \mu \, \bar{\nu}_i \, \nu_i$ decay for $\bar{\xi}^{E}_{N,\tau\tau} = 10 \, GeV$ and $\bar{\xi}^{E}_{N,\tau\mu} = 1 \, GeV$ and $kR = 11$. The solid (dashed-small dashed-dotted) line represents the BR without extra dimension (with extra dimension for $a = 0.1 - 0.5 - 1.0$).

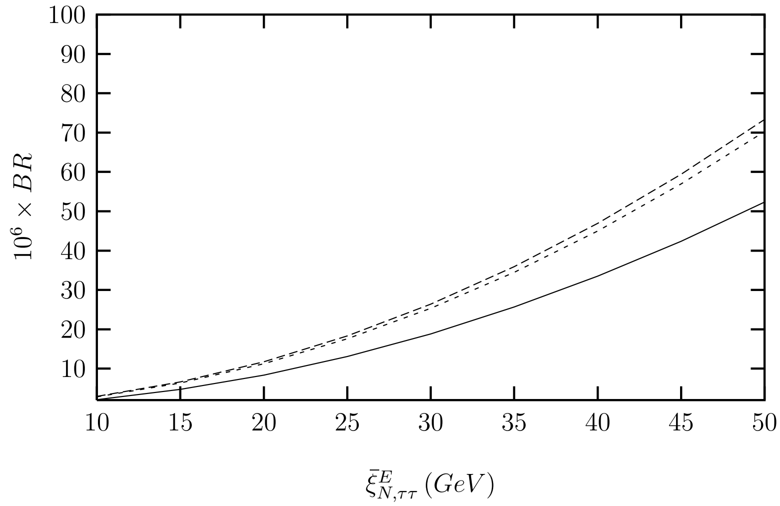


Figure 4: $\bar{\xi}^{E}_{N,\tau\tau}$ dependence of the BR of the LFV $\tau \to \mu\,\bar{\nu}_i\,\nu_i$ decay for $\bar{\xi}^{E}_{N,\tau\mu} = 1\,GeV$, $kR = 12$ and $k = 10^{18}\,GeV$. The solid (dashed-small dashed) line represents the BR without extra dimension (with extra dimension for $a = 0.5 - 1.0$).