THE NEUTRALINO MASS: CORRELATION WITH THE CHARGINOS

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Abstract

As the fundamental SU(2) supersymmetric parameters can be determined in the chargino sector, and the remaining fundamental parameters of the minimal supersymmetric extensions of the standard model can be analyzed in the neutralino sector, the two sectors can be correlated via these parameters. We have shown that for the CP conserving case, the masses of all the neutralinos can be determined in terms of the chargino masses, and $\tan \beta$. In this case the neutralino masses are quite insensitive to the variations of $\tan \beta$; they change by about %15 when $\tan \beta$ varies in the range from 5 to 50. In the CP violating case, the neutralino masses are found to be quite sensitive to the variations of the CP violating phase. For the heavier neutralinos the dependence of the masses to the CP violating phase show complementary behaviour at CP violating points.

1. Introduction and Summary

The Lagrangian of the Minimal Supersymmetric Standard Model (MSSM) contains various mass parameters which are not necassarily real [1]. The phases of these parameters appear in several CP violating processes such as the electric dipole moments [2], the decays and mixings of mesons [3], the Higgs phenomenology [4,5,6], and the chargino/neutralino systems [7,8,9].

One of the simplest sectors in supersymmetric (SUSY) theories is that of the charginos. The 2×2 chargino mass matrix

$$M_\chi = \begin{pmatrix} M_2 & \sqrt{2} M_W \sin \beta \\ \sqrt{2} M_W \cos \beta & \mu \end{pmatrix} , \qquad (1)$$

is built up by the SU(2) gaugino, and the Higgsino mass parameters, M_2 and μ, respectively, and the ratio $\tan \beta = v_2 / v_1$ of the expectation values of the two neutral Higgs fields which break the electroweak symmetry.

In the CP violating theories M_2, and μ can be complex. However, by the reparametrization of the fields M_2 can be taken as real and positive, so that the remaining non-trivial phase can be attributed to the μ parameter. We define,

$$\mu = |\mu| e^{i\varphi_\mu} \qquad (2)$$

The chargino mass matrix M_χ can be diagonalized by the following transformation:

$$\mathcal{U}^* M_\chi \mathcal{V}^{-1} = \mathrm{Diag}(M_{\chi_1^+}, M_{\chi_2^+}) \ , \tag{3}$$

with the chargino mass eigenvalues $m^2_{\tilde{\chi}^+_{1,2}}$:

$$m^2_{\tilde{\chi}^+_{1,2}} = \frac{1}{2} \left[M_2^2 + |\mu|^2 + 2M_W^2 \mp \Delta_\chi \right] \tag{4}$$

where

$$
\begin{aligned}
\Delta_\chi &= \Big[(M_2^2 - |\mu|^2 - 2M_W^2 \cos 2\beta)^2 \\
&+ 8M_W^2 (M_2^2 \cos^2\beta + |\mu|^2 \sin^2\beta + M_2|\mu| \sin 2\beta \cos\varphi_\mu) \Big]^{1/2}
\end{aligned} \tag{5}
$$

gives the difference between the two chargino masses $(M^2_{\chi_2^+} - M^2_{\chi_1^+})$.

For given $\tan\beta$, the fundamental SUSY parameters M_2 and $|\mu|$ can be derived from these two masses [8,10]. The sum and the difference of the chargino masses lead to the following equations involving M_2 and $|\mu|$:

$$M_2^2 + |\mu|^2 = M^2_{\chi_1^+} + M^2_{\chi_2^+} - 2M_W^2 \ , \tag{6}$$

$$M_2^2 |\mu|^2 - 2M_W^2 \sin 2\beta \cos\varphi_\mu M_2|\mu| + (M_W^4 \sin^2 2\beta - M^2_{\chi_1^+} M^2_{\chi_2^+}) = 0 \ . \tag{7}$$

The solution of (7) is given as:

$$M_2|\mu| = M_W^2 \cos\varphi_\mu \sin 2\beta \pm \sqrt{M^2_{\chi_1^+} M^2_{\chi_2^+} - M_W^4 \sin^2 2\beta \sin^2\varphi_\mu} \ . \tag{8}$$

From (6) and (8) one obtains the following solutions for M_2 and $|\mu|$:

$$2M_2^2 = (M^2_{\chi_1^+} + M^2_{\chi_2^+} - 2M_W^2) \mp \sqrt{(M^2_{\chi_1^+} + M^2_{\chi_2^+} - 2M_W^2)^2 - Q_\pm} \ , \tag{9}$$

$$2|\mu|^2 = (M^2_{\chi_1^+} + M^2_{\chi_2^+} - 2M_W^2) \pm \sqrt{(M^2_{\chi_1^+} + M^2_{\chi_2^+} - 2M_W^2)^2 - Q_\pm} \ , \tag{10}$$

with

$$
\begin{aligned}
Q_\pm &= 4 \Big[M^2_{\chi_1^+} M^2_{\chi_2^+} + M_W^4 \cos 2\,\varphi_\mu \sin^2 2\beta \\
&\pm 2M_W^2 \cos\varphi_\mu \sin 2\beta \sqrt{M^2_{\chi_1^+} M^2_{\chi_2^+} - M_W^4 \sin^2\varphi_\mu \sin^2 2\beta} \Big] \ ,
\end{aligned} \tag{11}
$$

where the upper signs correspond to $M_2 < |\mu|$ regime, and the lower ones to $M_2 > |\mu|$.

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Therefore, for given $\tan\beta$, M_2 and $|\mu|$ can be determined in terms of the masses of the charginos ($M_{\chi_1^+}$ and $M_{\chi_2^+}$) by using (9), and (10) from which one gets four solutions corresponding to different physical scenarios. For $|\mu| < M_2$, the lightest chargino has a stronger higgsino-like component and therefore is referred as higgsino-like [9,10]. The solution $|\mu| > M_2$, corresponding to gaugino-like situation, can be readily obtained by the substitutions: $M_2 \to |\mu|$, and $\mu \to \text{sign}(\mu)\, M_2$ [9,11].

Let us now consider the mass matrix of the neutralino system:

$$M_{\chi^0} = \begin{pmatrix} M_1 & 0 & -M_Z s_W \cos\beta & M_Z s_W \sin\beta \\ 0 & M_2 & M_Z c_W \cos\beta & -M_Z c_W \sin\beta \\ -M_Z s_W \cos\beta & M_Z c_W \cos\beta & 0 & -\mu \\ M_Z s_W \sin\beta & -M_Z c_W \sin\beta & -\mu & 0 \end{pmatrix}, \quad (12)$$

The main difference of the mass spectra of the neutralino and chargino system is the appearence of the SU(2) gaugino mass M_1, in the former.

The neutralino mass matrix can be diagonalized as follows:

$$\mathcal{N}^T\, M_{\chi^0}\, \mathcal{N} = \text{Diag}\left(M_{\chi_4^0}, \cdots, M_{\chi_1^0}\right) , \quad (13)$$

with ordering $M_{\chi_4^0} > M_{\chi_3^0} > M_{\chi_2^0} > M_{\chi_1^0}$.

Assuming the two chargino masses are known, it is possible to express the neutralino masses in terms of these, for given $\tan\beta$. In this work, we have obtained the neutralino masses numerically, by using (13). In doing this, we use (9) and (10), for given $\tan\beta$.

Complete analytical solutions can be derived for the neutralino mass eigenvalues (13) as functions of the SUSY parameters for both CP conserving [12], and CP violating theories [9]. Admitedly, the diagonalization of the neutralino mass matrix is no easy job and the analytic expressions of the resulting eigenvalues are rather lengthy and complicated.

However, there are theoretically well motivated assumptions, like for instance the universality of the soft mass parameters, which could be easily implementable to the system. Typically, the gaugino mass parameter universality at the grand unification (GUT) scale, leads to the approximate relation [10]:

$$M_1(M_Z) = 5/3 \tan^2\theta_W M_2(M_Z) . \quad (14)$$

Furthermore, there are also very reasonable approximations to these mass eigenvalues in limiting cases which are sufficiently compact to allow a good understanding of the analytic dependencies. For instance, a particularly interesting limit is approached when the the supersymmetry mass parameters and their splittings are much larger than the electroweak scale $M_{SUSY}^2 >> M_Z^2$. In this limit the neutralino mass eigenvalues can be written in compact (approximate) form as [9]:

$$M_{\chi_1^0} = |M_1| + \mathcal{Z}_1\left[|M_1| + |\mu|\sin 2\beta \cos\varphi_\mu\right] , \quad (15)$$

$$M_{\chi_2^0} = |M_2| + \mathcal{Z}_2\left[|M_2| + |\mu|\sin 2\beta \cos\varphi_\mu\right] , \quad (16)$$

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$$M_{\chi_3^0} = |\mu| \left[1 - \frac{(1 - \sin 2\beta)}{2} \left(\mathcal{Z}_1 + \mathcal{Z}_2 \right) \right]$$

$$+ \quad \frac{(1 - \sin 2\beta)}{2} \left[\mathcal{Z}_1 |M_1| + \mathcal{Z}_2 |M_2| \right] \cos \varphi_\mu \ , \tag{17}$$

$$M_{\chi_4^0} = |\mu| \left[1 - \frac{(1 + \sin 2\beta)}{2} \left(\mathcal{Z}_1 + \mathcal{Z}_2 \right) \right]$$

$$- \quad \frac{(1 + \sin 2\beta)}{2} \left[\mathcal{Z}_1 |M_1| + \mathcal{Z}_2 |M_2| \right] \cos \varphi_\mu \ , \tag{18}$$

where

$$\mathcal{Z}_1 = \frac{m_Z^2 \, s_W^2}{|M_1|^2 - |\mu|^2} \qquad \text{and} \qquad \mathcal{Z}_2 = \frac{m_Z^2 \, c_W^2}{|M_2|^2 - |\mu|^2} \ . \tag{19}$$

The masses of the charginos and neutralinos are interesting observables which provide clues about the SUSY-breaking structure of the system [13]. Therefore, particle masses, SUSY parameters, the relations between the masses themselves, the relations between the basic gaugino parameters and the physical masses, are important for calculations. Previous works on the subject include the analysis at the lowest order processes [11], in the on-shell scheme [14], and aim to reconstruct the basic parameters based on chargino production [7,8].

In this work, our aim is to obtain the neutralino masses, from those of the charginos and investigate the effects of the CP violating phase on the masses of the neutralinos, taking the two chargino masses and $\tan \beta$ as input parameters.

In CP violating theories, the gaugino mass M_2, and the Higgsino-Dirac mass parameter μ can be complex. However, the gaugino mass M_2 can be taken to be real, and hence the phase of the μ parameter becomes the only non-trivial CP violating phase in the theory. In this work, we choose M_2 to be real. which means M_1 to be real also, due to the interrelation between them. Thus, the only non-trivial CP violating phase can be attributed to the μ parameter.

In the following, we first briefly consider the CP conserving case, where we calculate the neutralino masses numerically, and analyze their dependence of $\tan \beta$. Then we turn to the case for which there is CP violation in the theory, and study the sensitivity of the neutralino masses to the CP violating phase.

2. Numerical Analysis

2.1. *CP conserving case*

In the first part of the analysis, we set $\varphi_\mu = 0$ and we take the two chargino masses, and $\tan \beta$ as input parameters, and calculate the neutralino masses $M_{\chi_i^0}$.

In our analysis, we fix the heavy chargino mass as $M_{\chi_2^+} = 320$ GeV, and choose two different values for the light chargino mass $(M_{\chi_1^+})$, as $\tan \beta$ varies from 5 to 50.

In Figure 1 and Figure 2, we plot the variation of the lightest neutralino mass $M_{\chi_1^0}$ with respect to $\tan \beta$, for $M_2 < |\mu|$ and for $M_2 > |\mu|$ regimes, respectively. In both Figures the left panels are for $M_{\chi_1^+} = 105$ GeV, and the right panels are for $M_{\chi_1^+} = 160$ GeV.
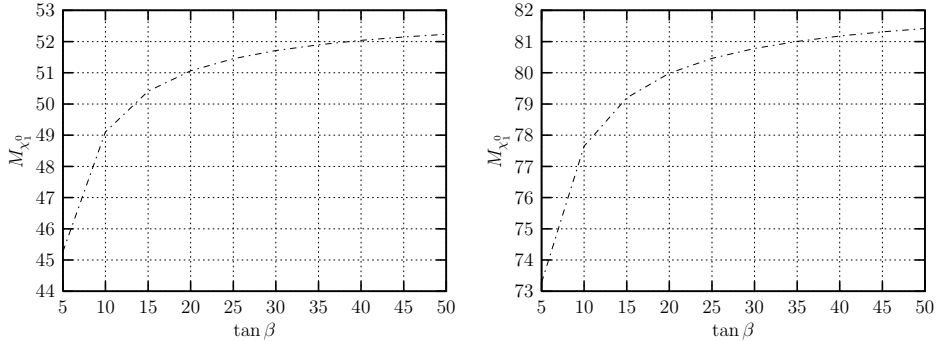
Fig. 1. The $\tan\beta$ dependence of $M_{\chi_1^0}$, when $M_{\chi_1^+} = 105$ GeV (left panel), and $M_{\chi_1^+} = 160$ GeV (right panel) for $M_2 < |\mu|$.
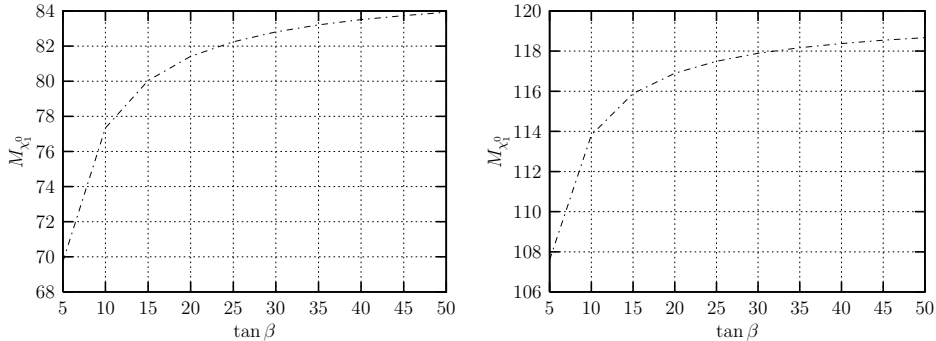

Fig. 2. The $\tan\beta$ dependence of $M_{\chi_1^0}$, when $M_{\chi_1^+} = 105$ GeV (left panel), and $M_{\chi_1^+} = 160$ GeV (right panel) for $M_2 > |\mu|$.

It can be seen from both Figures that $M_{\chi_1^0}$ increases with $\tan\beta$, at both values of the lightest chargino mass ($M_{\chi_1^+} = 105$ GeV and $M_{\chi_1^+} = 160$ GeV) for both $M_2 < |\mu|$ and $M_2 > |\mu|$ regimes.

One can deduce that when $M_{\chi_1^+} = 105$ GeV, the gaugino and Higgsino Dirac mass lie in the $M_2\,(|\mu|) \sim 104 - 113$ GeV and $|\mu|\,(M_2) \sim 299 - 296$ GeV intervals, respectively, for $M_2 < |\mu|$ $(M_2 > |\mu|)$. When $M_{\chi_1^+} = 160$ GeV, one can again deduce that $M_2\,(|\mu|)$ ranges from 164 to 175 GeV, whereas $|\mu|\,(M_2)$ changes from 297 to 290 GeV.

A comparative analysis of Figure 1 and Figure 2 suggest that the lightest neutralino mass $(M_{\chi_1^0})$ changes by at most %15 as $\tan\beta$ varies from 5 to 50, thus depicting a low sensitivity. For instance, the maximal and minimal values of $M_{\chi_1^0}$ can be read as 45 GeV, and 52 GeV, at $\tan\beta = 5$, and $\tan\beta = 50$, respectively, at $M_{\chi_1^+} = 105$ GeV, for $M_2 < |\mu|$. Similar observations can be made for the $M_2 > |\mu|$ regime.

In Figure 3 and in Figure 4, we plot the variation of the second light neutralino mass $M_{\chi_2^0}$ with respect to $\tan\beta$, for $M_2 < |\mu|$ and for $M_2 > |\mu|$ regimes, respectively. In both Figures the left panels are for $M_{\chi_1^+} = 105$ GeV, and the right panels are for $M_{\chi_1^+} = 160$ GeV.

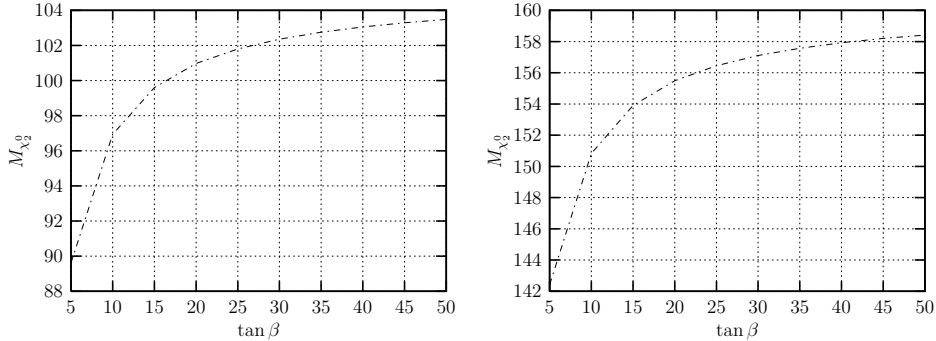It can be seen from Figure 3 and Figure 4 that similar to the variation of $M_{\chi_1^0}$

Fig. 3. The $\tan\beta$ dependence of $M_{\chi_2^0}$, when $M_{\chi_1^+} = 105$ GeV (left panel), and $M_{\chi_1^+} = 160$ GeV (right panel) $M_2 < |\mu|$.
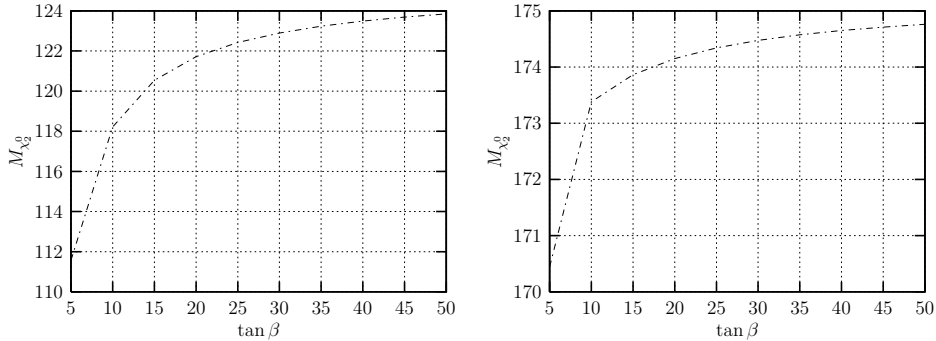

Fig. 4. The $\tan\beta$ dependence of $M_{\chi_2^0}$, when $M_{\chi_1^+} = 105$ GeV (left panel), and $M_{\chi_1^+} = 160$ GeV (right panel) $M_2 > |\mu|$.

(Figures 1 and 2), $M_{\chi_2^0}$ increases as $\tan\beta$ varies from 5 to 50, for both $M_2 < |\mu|$ and $M_2 > |\mu|$ regimes. The lower-upper bounds of $M_{\chi_2^0}$ can be read as 90-104 GeV when $M_{\chi_1^+} = 105$ GeV (left panel of Fig. 3), and 142-160 GeV when $M_{\chi_1^+} = 160$ GeV (right panel of Fig. 3) for $M_2 < |\mu|$. In passing to $M_2 > |\mu|$ regime, it is seen that the lower-upper bounds of $M_{\chi_2^0}$ increase by an amount of $\sim \%10$ (Figure 4).

Up to now we have studied the $\tan\beta$ behaviour of the lighter neutralinos ($M_{\chi_1^0}$ and $M_{\chi_2^0}$). The assigned values for the fundamental parameters in our numerical analysis indeed satisfy the assumption which went into the expressions (15)-(16).

Next, we pass to the heavier neutralinos in which case we plot the variation of the next-to heaviest neutralino $M_{\chi_3^0}$ mass with respect to $\tan\beta$, for $M_2 < |\mu|$ and for $M_2 > |\mu|$ regimes in Figure 5 and in Figure 6, respectively. In both of the Figures the left panels are for $M_{\chi_1^+} = 105$ GeV, and the right panels are for $M_{\chi_1^+} = 160$ GeV.

We observe from Figure 5 that the behaviour of the χ_3^0 mass with respect to $\tan\beta$ is the same with the lighter neutralinos (χ_1^0 and χ_2^0), for the lighter chargino ($M_{\chi_1^+} = 105$ GeV), however it reverses for the heavier chargino ($M_{\chi_1^+} = 160$ GeV) for the $M_2 < |\mu|$ regime. To understand this interesting behaviour it may be useful
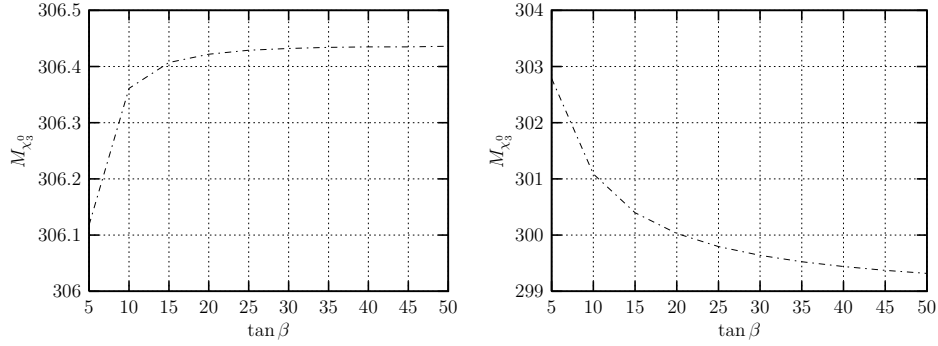
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Fig. 5. The $\tan\beta$ dependence of $M_{\chi_3^0}$, when $M_{\chi_1^+} = 105$ GeV (left panel), and $M_{\chi_1^+} = 160$ GeV (right panel) for $M_2 < |\mu|$.
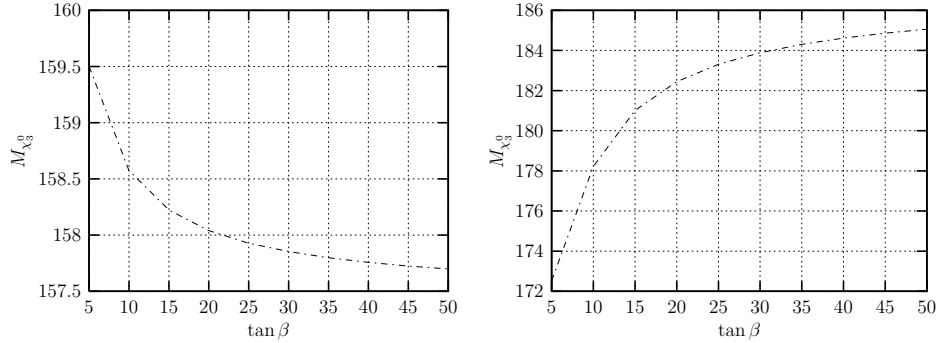

Fig. 6. The $\tan\beta$ dependence of $M_{\chi_3^0}$, when $M_{\chi_1^+} = 105$ GeV (left panel), and $M_{\chi_1^+} = 160$ GeV (right panel) for $M_2 > |\mu|$.

to look into the analytic expression (17), where $M_{\chi_3^0}$ is related to the gaugino masses of M_1 and M_2 by the combinations of Z_1 and Z_2. It can be observed that among the two contributions to the expression (17), the first term of (17) always dominates, as compared to the second term for both $M_{\chi_1^+} = 105$ GeV and $M_{\chi_1^+} = 160$ GeV, cases.

One notes that this term increases with increasing $\tan\beta$ for the lighter chargino mass ($M_{\chi_1^+} = 105$ GeV), whereas it decreases for the heavier chargino mass ($M_{\chi_1^+} = 160$ GeV). Since the contribution of this term is dominant, the neutralino mass gets heavier with the increase in $\tan\beta$ for the lighter chargino mass (the left panel of Figure 5), and the behaviour is reversed for the heavier chargino mass (the right panel of Figure 5).

As can be seen from (17), as the lighter chargino mass $M_{\chi_1^+}$ moves from a lower value ($M_{\chi_1^+} = 105$ GeV) to a higher one ($M_{\chi_1^+} = 160$ GeV), then the three different $\tan\beta$ contributions compete against each other, and their roles are changed at a certain critical value. This is the reason for the shift of the pattern of Figure 5 from one panel to the other. The critical value of the chargino mass is $M_{\chi_1^+} = 130$ GeV at which the $\tan\beta$-$M_{\chi_3^0}$ behaviour reverses.

Similar observations can be made for the $M_2 > |\mu|$ regime, by taking into account of the fact that the roles of M_2 and $|\mu|$ are interchanged for this case, under the substitution: $M_2 \to |\mu|$, and $\mu \to \text{sign}(\mu)\, M_2$ [9,11].

That this behaviour is observed for the mass of χ_3^0 particularly, is due to the fact that its mass lies in the transitional region from the lighter chargino masses to the heavier.
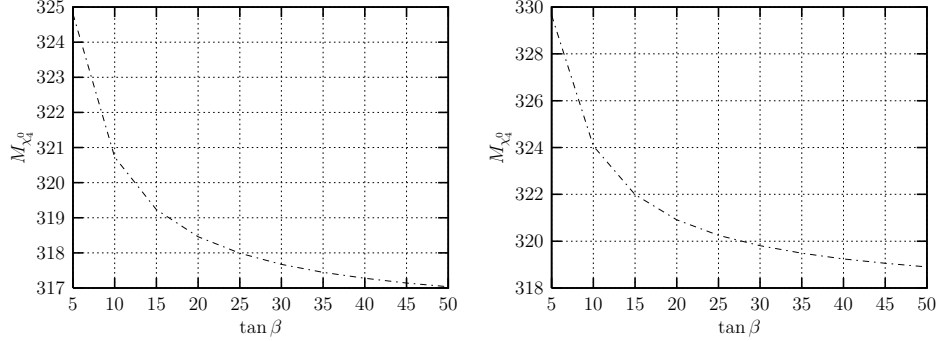

Fig. 7. The $\tan\beta$ dependence of $M_{\chi_4^0}$, when $M_{\chi_1^+} = 105$ GeV (left panel), and $M_{\chi_1^+} = 160$ GeV (right panel) for $M_2 < |\mu|$.
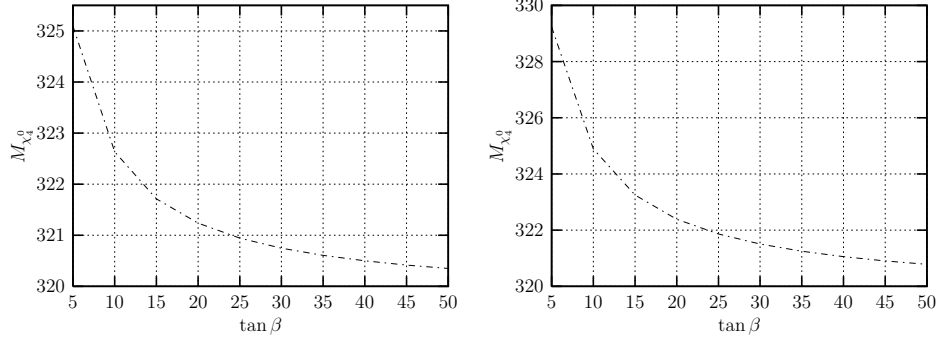

Fig. 8. The $\tan\beta$ dependence of $M_{\chi_4^0}$, when $M_{\chi_1^+} = 105$ GeV (left panel), and $M_{\chi_1^+} = 160$ GeV (right panel) for $M_2 > |\mu|$.

In Figure 7 and in Figure 8, we plot the variation of the mass of the heaviest neutralino $M_{\chi_4^0}$ with respect to $\tan\beta$, for $M_2 < |\mu|$ and for $M_2 > |\mu|$ regimes, respectively. In both Figures the left panels are for $M_{\chi_1^+} = 105$ GeV, and the right panels are for $M_{\chi_1^+} = 160$ GeV.

We see from Figure 7 and Figure 8 that $M_{\chi_4^0}$ decreases, as $\tan\beta$ increases. Like $M_{\chi_3^0}$, it can be observed that among the two contributions to (18), the first term of (18) always dominates, as compared to the second term for both $M_{\chi_1^+} = 105$ GeV and $M_{\chi_1^+} = 160$ GeV, cases. One notes that this term decreases with increasing $\tan\beta$ for both the lighter and the heavier chargino masses ($M_{\chi_1^+} = 105$ GeV and $M_{\chi_1^+} = 160$ GeV, respectively). Since the contribution of this term is dominant, $M_{\chi_4^0}$ gets lightened with the increase in $\tan\beta$.

A comparative analysis of Figure 7 and Figure 8 suggest that the mass of the heaviest neutralino remains around 325 GeV for the lighter chargino ($M_{\chi_1^+} = 105$ GeV), and does not exceed 330 GeV, for the heavier chargino ($M_{\chi_1^+} = 160$ GeV).

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2.2. *CP violating case*

In the second part of our analysis, we carry out the analysis when there is CP violation. In the following, we analyze the φ_μ dependence of $M_{\chi_i^0}$, as φ_μ ranges from 0 to 2π. In our analysis, we fix $M_{\chi_2^+} = 320$ GeV and we choose two values of the lightest chargino mass, like the CP conserving case. Here, we consider two specific values of $\tan\beta$; Namely, $\tan\beta = 5$, and $\tan\beta = 50$, representing the low and high $\tan\beta$ regimes.
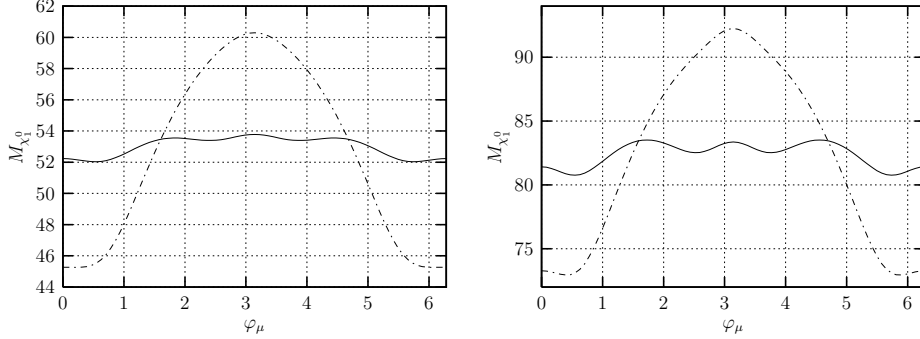
Fig. 9. The φ_μ dependence of $M_{\chi_1^0}$, when $M_{\chi_1^+} = 105$ GeV (left panel), and $M_{\chi_1^+} = 160$ GeV (right panel), for $M_2 < |\mu|$.

Fig. 10. The φ_μ dependence of $M_{\chi_1^0}$, when $M_{\chi_1^+} = 105$ GeV (left panel), and $M_{\chi_1^+} = 160$ GeV (right panel), for $M_2 > |\mu|$.

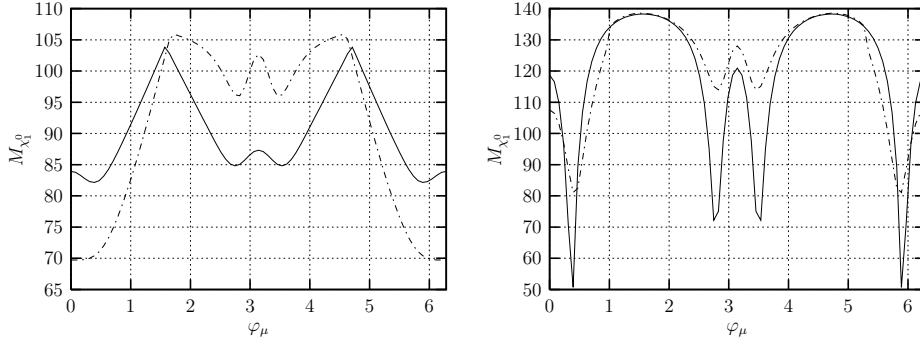In Figures 9, and 10, we show the φ_μ dependence of the mass of the lightest neutralino $M_{\chi_1^0}$, at $M_{\chi_1^+} = 105$ GeV (left panels), and $M_{\chi_1^+} = 160$ GeV (right panels), for $M_2 < |\mu|$, and $M_2 > |\mu|$ regimes, respectively. In each panel the dotted curves are for $\tan\beta = 5$, whereas the solid ones are for $\tan\beta = 50$.

When $M_{\chi_1^+} = 105$ GeV, $M_{\chi_1^0}$ changes from 45 to 60 GeV and from 70 to 105 GeV, for the $M_2 < |\mu|$ and $M_2 > |\mu|$ regimes, respectively, at $\tan\beta = 5$, as φ_μ varies in the $[0 , \pi]$ interval. For $M_{\chi_1^+} = 160$ GeV, it is seen that the lower and upper bounds of $M_{\chi_1^0}$ increases for both $M_2 < |\mu|$ and $M_2 > |\mu|$ regimes, as expected.

A comparative analysis of Figures 9 and Figure 10 suggest that $M_{\chi_1^0}$ is quite sensitive to the variations of φ_μ. It is interesting to note that when $M_{\chi_1^+} = 105$ GeV,

φ_μ dependence of $M_{\chi_1^0}$ is sharper at the CP violating points ($\varphi_\mu = \pi/2$ and $3\pi/2$), for the $M_2 > |\mu|$ regime, at both $\tan\beta = 5$ and $\tan\beta = 50$. When $M_{\chi_1^+} = 160$ GeV (right panel of Figure 10), one observes a slower variation at the CP violating points. On the other hand, for the $M_2 < |\mu|$ regime, it can be seen that the variation of φ_μ (at both $M_{\chi_1^+} = 105$ GeV and $M_{\chi_1^+} = 160$ GeV), is slower as compared to $M_2 > |\mu|$ regime (see, for instance the analytical expression of $M_{\chi_1^0}$ given by (15)).

One notes that, when $M_{\chi_2^+} = 320$ GeV, (i) $|\mu|$ changes from 299 GeV, to 292 GeV, whereas M_2 from 104 GeV to 124 GeV, for $M_{\chi_1^+} = 105$ GeV, (ii) $|\mu|$ changes from 297 GeV, to 280 GeV, whereas M_2 from 164 GeV to 192 GeV, for $M_{\chi_1^+} = 160$ GeV at $\tan\beta = 5$. On the other hand, (i) $|\mu|$ changes from 296 GeV, to 295 GeV, whereas M_2 from 113 GeV to 115 GeV, for $M_{\chi_1^+} = 105$ GeV, (ii) $|\mu|$ changes from 290 GeV, to 288 GeV, whereas M_2 from 175 GeV to 178 GeV, for $M_{\chi_1^+} = 160$ GeV at $\tan\beta = 50$.
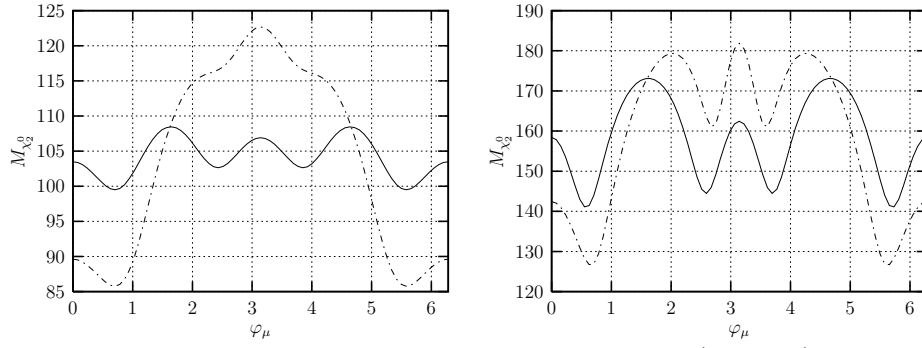

Fig. 11. The φ_μ dependence of $M_{\chi_2^0}$, when $M_{\chi_1^+} = 105$ GeV (left panel), and $M_{\chi_1^+} = 160$ GeV (right panel), for $M_2 < |\mu|$.

Fig. 12. The φ_μ dependence of $M_{\chi_2^0}$, when $M_{\chi_1^+} = 105$ GeV (left panel), and $M_{\chi_1^+} = 160$ GeV (right panel), for $M_2 > |\mu|$.

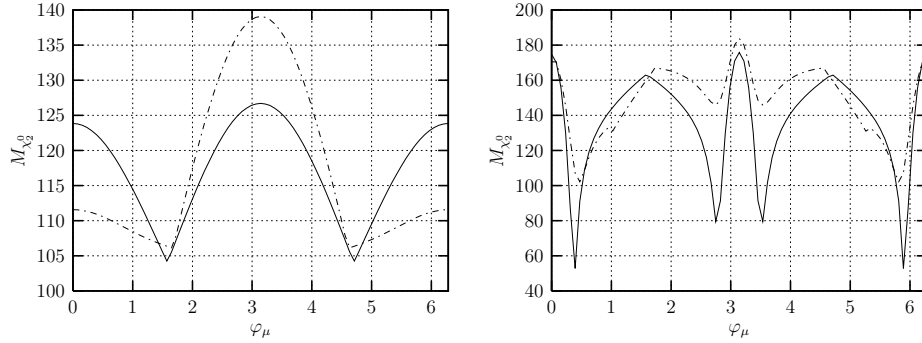In Figures 11, and 12, we show the φ_μ dependence of $M_{\chi_2^0}$, for $M_2 < |\mu|$, and $M_2 > |\mu|$ regimes, respectively, when $M_{\chi_1^+} = 105$ GeV (left panels), and $M_{\chi_1^+} =$

160 GeV (right panels). In each Figure, the dotted curves correspond to $\tan\beta = 5$, whereas the solid curves to $\tan\beta = 50$.

One notes from the left panel of Figure 11 that the variation is $M_{\chi_2^0}$, is more faster as compared to $M_{\chi_1^0}$ (see Figure 9). Such behaviour can be explained by referring into the analytic expression of $M_{\chi_2^0}$ (16).

On the other hand, as can be seen from the left panel of Figure 12, starting from $\varphi_\mu = 0$ at 112 GeV, $M_{\chi_2^0}$ decreases to 105 GeV at $\varphi_\mu = \pi/2$, then it increases to 140 GeV at $\varphi_\mu = \pi$, at $\tan\beta = 5$. For the heavier chargino (right panel) one observes a faster and sharper variation of φ_μ, as compared to the lighter chargino (left panel). However, it is seen that $M_{\chi_2^0}$ gets heavier together with the heavier chargino mass ($M_{\chi_1^+} = 160$ GeV), without causing too big splitting among the low and high $\tan\beta$ regimes. For instance, the maximal values at $\varphi_\mu = \pi$, for $\tan\beta = 5$ and $\tan\beta = 50$ cases, are 178 GeV and 180 GeV, respectively.
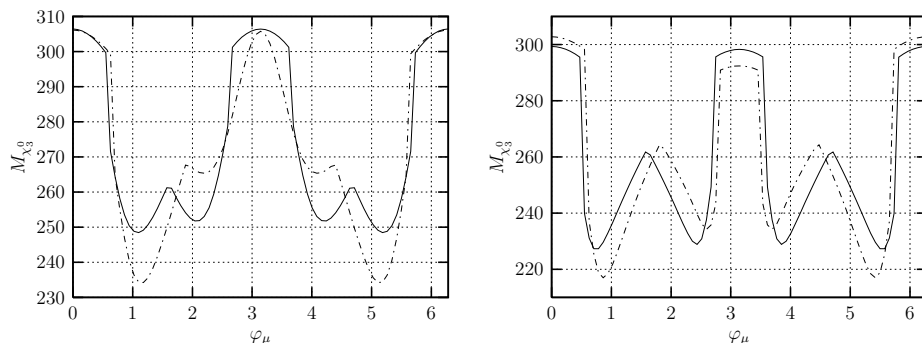

Fig. 13. The φ_μ dependence of $M_{\chi_3^0}$, when $M_{\chi_1^+} = 105$ GeV (left panel), and $M_{\chi_1^+} = 160$ GeV (right panel), for $M_2 < |\mu|$.

Fig. 14. The φ_μ dependence of $M_{\chi_3^0}$, when $M_{\chi_1^+} = 105$ GeV (left panel), and $M_{\chi_1^+} = 160$ GeV (right panel), for $M_2 > |\mu|$.

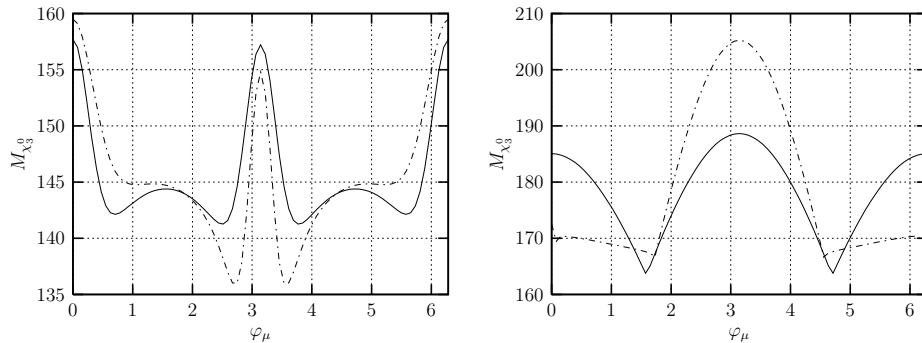In Figures 13, and 14, we show the φ_μ dependence of $M_{\chi_3^0}$, at $M_{\chi_1^+} = 105$ GeV (left panels), and $M_{\chi_1^+} = 160$ GeV (right panels), for $M_2 < |\mu|$, and $M_2 > |\mu|$ regimes, respectively. In the each panel, the dotted curves correspond to $\tan\beta = 5$, whereas the solid curves to $\tan\beta = 50$.

It is seen from the left panel of Figure 13 that the variation of $M_{\chi_3^0}$ with respect to φ_μ is much faster as compared to the lighter neutralinos $M_{\chi_1^0}$, and $M_{\chi_2^0}$, as expected (see for instance the analytical expression of $M_{\chi_3^0}$ given by the expression (17)).

It is interesting to note that $M_{\chi_3^0}$ at lighter $M_{\chi_1^+}$ ($M_{\chi_1^+} = 105$ GeV, the left panel of Figure 12) has similar φ_μ dependence with that of $M_{\chi_3^0}$ at heavier $M_{\chi_1^+}$ ($M_{\chi_1^+} = 160$ GeV, the right panel of Figure 14).

Finally, in Figures 15, and 16, we show the φ_μ dependence of $M_{\chi_4^0}$, at $M_{\chi_1^+} = 105$ GeV (left panels), and $M_{\chi_1^+} = 160$ GeV (right panels), for $M_2 < |\mu|$, and $M_2 > |\mu|$ regimes, respectively. In the each panel, the dotted curves correspond to $\tan\beta = 5$, whereas the solid curves to $\tan\beta = 50$.
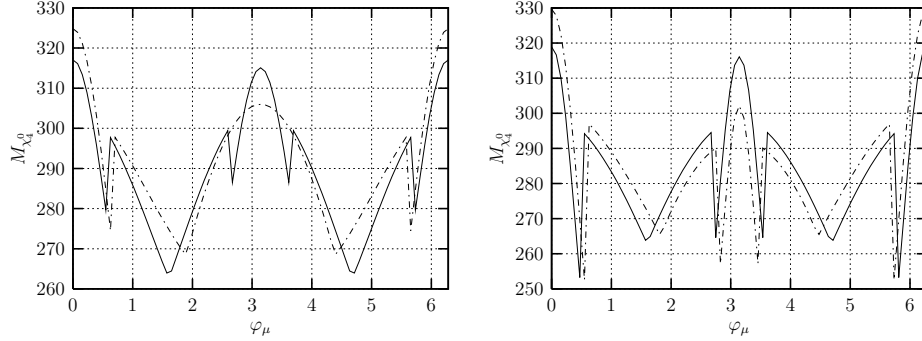

Fig. 15. The φ_μ dependence of $M_{\chi_4^0}$, when $M_{\chi_1^+} = 105$ GeV (left panel), and $M_{\chi_1^+} = 160$ GeV (right panel), for $M_2 < |\mu|$.

Fig. 16. The φ_μ dependence of $M_{\chi_4^0}$, when $M_{\chi_1^+} = 105$ GeV (left panel), and $M_{\chi_1^+} = 160$ GeV (right panel), for $M_2 > |\mu|$.

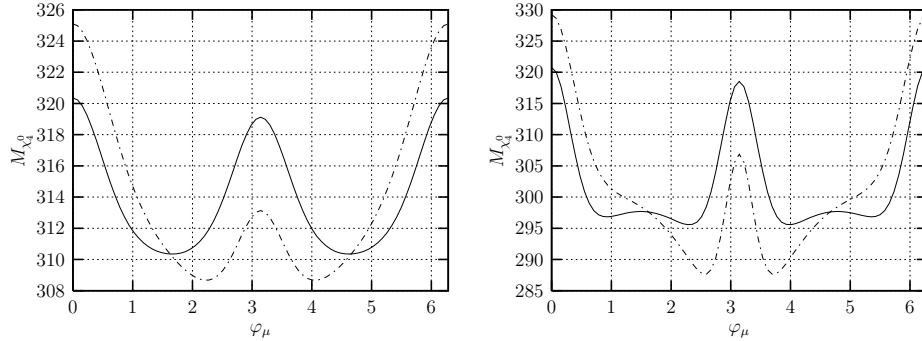A comparative analysis of Figures 13-16 suggest that for the heavier neutralinos (i=3,4), the dependence of the masses to φ_μ show complementary behaviour. For instance the CP violating points ($\varphi_\mu = \pi/2, 3\pi/2$) appear as local maxima for the χ_3^0 mass, as opposed to χ_4^0 case where those values are local minima, for both values of $M_{\chi_1^+} = 105$ GeV and $M_{\chi_1^+} = 160$ GeV, in the $M_2 < |\mu|$ regime. This complementarity at the CP violating points for the heavier sector can easily be seen from the expressions (17)-(18). A similar complementarity holds for the lighter

neutralinos (i=1,2), at the lighter chargino mass in the $M_2 > |\mu|$ regime (the left panels of Figures 10 and 12). On the other hand, the CP violating points appear as local maxima for both values of the lightest chargino masses in the $M_2 < |\mu|$ regime (Figures 9 and 11).

It can also be seen from Figures 9-16, that the variations of the lighter neutralino masses (i=1,2) with $\tan\beta$, is about %15 in the range from 5 to 50 at the CP violating points as in the CP conserving case. However, for the heavier neutralino case, the difference of the masses between high and low $\tan\beta$ regimes becomes very small.

3. Conclusions

We have analyzed the neutralino system, whose parameters are extracted from the chargino system, for both CP conserving and CP violating cases. Here is a brief summary of our main results:

When $\varphi_\mu = 0$, given $M_{\chi_1^+}$, $M_{\chi_2^+}$ and $\tan\beta$, the masses of all the neutralinos can be determined. The variation of $\tan\beta$ from 5 to 50 leads to at most %15 change in the neutralino masses.

The $\tan\beta$ behaviour of the lighter neutralinos $M_{\chi_i^0}$ (i=1,2) are opposite to the that of the heaviest neutralino (i=4) for the lower and the higher values of the lighter chargino mass ($M_{\chi_1^+} = 105$ GeV, and $M_{\chi_1^+} = 160$ GeV), for both $M_2 < |\mu|$ and $M_2 > |\mu|$ cases. The assigned values for the fundamental parameters in our numerical analysis indeed satisfy the assumption which went into the expressions (15)-(18). On the other hand, the switched behaviour of $M_{\chi_3^0}$ stems from the fact that the different $\tan\beta$ contributions given by (17) compete against each other, and their roles are changed at a certain critical value. Namely, there is a critical value of the lighter chargino mass $M_{\chi_1^+} = 130$ GeV at which the $\tan\beta$-$M_{\chi_3^0}$ behaviour reverses.

Our analysis shows that for the lower value of the lighter chargino mass ($M_{\chi_1^+} = 105$ GeV), M_2 ($|\mu|$) ranges in the $\sim 104 - 113$ GeV interval, as $|\mu|$ (M_2) changes from $\sim 299 - 296$ GeV, for $M_2 < |\mu|$ ($M_2 > |\mu|$), as $\tan\beta$ ranging from 5 to 50. On the other hand, for the higher value of the lighest chargino mass ($M_{\chi_1^+} = 160$ GeV), the corresponding values are: M_2 ($|\mu|$): 164-175 GeV, and $|\mu|$ (M_2): 297-290 GeV.

In the CP violating case, the complementary behaviour among the heavier neutralinos (i=3,4) can be observed in the sense that while the CP violating points appearing as local maxima for the χ_3^0 mass, they turn out to be local minima for χ_4^0, for values of the lighter chargino mass in the $M_2 < |\mu|$ regime. This complementarity at the CP violating points for the heavier sector can easily be seen from the expressions (17)-(18). Such a behaviour can also be observed for the lighter neutralinos (i=1,2), but only for the lighter chargino case in the $M_2 > |\mu|$ regime.

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