The $g_{V\sigma\gamma}$-coupling constants in hadron electrodynamics

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Abstract

Recent measurements of the branching ratios for the decays $\omega \to \pi^0\pi^0\gamma$ and $\rho^0 \to \pi^0\pi^0\gamma$ lead to coupling constants of the $V\sigma\gamma$-interaction (V is a vector meson) one order of magnitude smaller than previously assumed to describe the threshold cross section of $\gamma + p \to p + \rho^0$ and the HERMES effect. The new $g^2_{V\sigma\gamma}$ couplings are in contradiction with the predictions of the QCD sum rules, but are in good agreement with VDM-estimation of the $\sigma \to \gamma\gamma$-width.

1 Introduction

The coupling constants $g_{V\sigma\gamma}$, $V = \rho$ or ω are important ingredients for the theoretical analysis of many different hadronic electromagnetic processes. If $m_\sigma < m_V$, the decays $V \to \sigma + \gamma$ can occur directly-with radiation of electric dipole photons. In this respect, these decays are essentially different from the decays $V \to P + \gamma$, $P = \pi$, η, η' -with radiation of magnetic dipole photons. In the quark models, these last decays are induced by the quark magnetic moment, with transition $S = 1 \to S = 0$, where S is the total spin of the $q\overline{q}$-system (in the corresponding meson), whereas the decays $V \to \sigma + \gamma$ are induced by the internal motion of quarks, with transition $S = 1 \to S = 1$ [1].

Let us mention the main applications of the $g_{V\sigma\gamma}$-coupling constants:

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- The σ-exchange for the vector meson production [2–6] $\gamma + p \to p + \rho^0$, near threshold.
- The electromagnetic transition $\gamma^* \to \sigma + \omega$, in the space-like region of the virtual photon γ^*four-momenta, enters in the calculation of meson exchange currents, in particular in the analysis of the deuteron electromagnetic form factors at large momentum transfer [7–9].
- The branching ratio of rare radiative decays of vector mesons as $\rho^0 \to \pi^0\pi^0\gamma$ and $\omega \to \pi^0\pi^0\gamma$ are controlled by the $g_{V\sigma\gamma}$-coupling constants [10,11].
- The HERMES effect [12], concerning the inclusive electroproduction cross section on light nuclei can be explained under specific assumptions [13] about the absolute value of the $g_{\rho\sigma\gamma}$ and $g_{\omega\sigma\gamma}$ coupling constants and the corresponding electromagnetic form factors.
- The coupling constant of the $\sigma \to \gamma\gamma$-vertex, which can be calculated on the basis of the $g_{V\sigma\gamma}$ couplings, is important for the analysis of real and virtual Compton scattering on nucleons [14–16].
- The exact value of the $g_{V\sigma\gamma}$ coupling constant, in principle, allows one to constrain the $g_{\sigma NN}$-coupling, which describes the scalar exchange in the NN-potential [17].
- Finally, the $g_{V\sigma\gamma}$ coupling constants have an implicit theoretical interest, being fundamental coupling constants of hadron electrodynamics.

Attempts to estimate these coupling constants from QCD-sum rules, give relatively large absolute values [18]. Phenomenological methods based on the existing experimental data about different electromagnetic processes need serious additional assumptions. For example, in the framework of σ-exchange for $\gamma + p \to p + \rho^0$, it is possible to estimate the product $g_{\rho\sigma\gamma}g_{\sigma NN}$, assuming a definite form of the phenomenological form factors, which insure the correct behavior of the differential cross section $d\sigma/dt$, in the near-threshold region, and depend on ad-hoc cut-off parameters. Taking $g^2_{\sigma NN}/4\pi = 8$, one obtains $g_{\rho\sigma\gamma} = 2.71$ [2]. Considering only σ-exchange in $\gamma + p \to p + \rho^0$, the sign of this constant can not be determined, but some polarization observables are sensitive to this sign [19]. Other possible contributions to the matrix element for the process $\gamma + p \to p + \rho^0$, such as N^*-excitation, N-exchange in s- and u-channel are neglected in such oversimplified consideration [2].

Also the estimation of the discussed coupling constants done in framework of the HERMES effect [13] can not be considered as direct and model independent.

Another source of information about the $\rho\sigma\gamma$-vertex is the radiative decay $\rho^0 \to \pi^+\pi^-\gamma$-with relatively large branching ratio [20]. Namely this decay has been considered to favor large values of $g_{\rho\sigma\gamma}$ in comparison with $g_{\omega\sigma\gamma}$, because of the corresponding widths $\Gamma(\omega \to \pi^+\pi^-\gamma) \ll \Gamma(\rho \to \pi^+\pi^-\gamma)$ [2]. However it is necessary to point out that for the decay $\rho^0 \to \pi^+\pi^-\gamma$ the bremsstrahlung mechanism dominates, therefore the σ-contribution, being of the order of the error bars, can not give the $g_{\rho\sigma\gamma}$ with good accuracy. Note also that the precise determination of $g_{\rho\sigma\gamma}$ depends essentially on the mass and the width of the σ-meson. In [11] different values of $g_{\rho\sigma\gamma}$ could been found, with the general conclusion that the $g_{\rho\sigma\gamma}$ value from $\rho^0 \to \pi^+\pi^-\gamma$ is not in contradiction with the estimation done in [2].

Finally, having a definite value of $g_{\rho\sigma\gamma}$, it is possible to estimate the branching ratio for $\rho \to \pi^0\pi^0\gamma$, in framework of the effective Lagrangian approach. In this case, the absence of the bremsstrahlung mechanism results in the fact that the σ-exchange mechanism is important. But the predicted $BR(\rho^0 \to \pi^0\pi^0\gamma) \simeq (45 - 200) \cdot 10^{-5}$, with $g_{\rho\sigma\gamma} = 3 \div 5$, were too large in comparison with other theoretical expectations for this decay [21].

Moreover, and this will be the important background of this paper, large values of $g_{\rho\sigma\gamma}$ contradict essentially the recent experimental results from Novosibirsk, concerning the direct measurement of the decay $\rho^0 \to \pi^0\pi^0\gamma$ [22]:

$$Br(\rho \to \pi^0\pi^0\gamma) = (4.1^{+1.0}_{-0.9} \pm 0.3) \cdot 10^{-5},$$

$$Br(\rho \to \sigma\gamma \to \pi^0\pi^0\gamma) = (1.9^{+0.9}_{-0.8} \pm 0.4) \cdot 10^{-5}.$$

The main aim of this paper is to estimate the coupling constant $g_{\rho\sigma\gamma}$ on the basis of these new important experimental data, which is, in our opinion, the most straightforward way.

2 The σ-contribution to the $\rho \to \pi^0 + \pi^0 + \gamma$-decay

A crude estimation of the $g_{\rho\sigma\gamma}$ coupling constant can be easily obtained from the experimental data about $\rho^0 \to \sigma\gamma \to \pi^0\pi^0\gamma$ [22], under the assumption that the σ-mass is smaller than the ρ-meson mass, so that the decay $\rho \to \sigma+\gamma$ is allowed.

Neglecting for a moment the σ-width (which is an evident oversimplification of reality, as $\Gamma_\sigma = (0.6 \div 1)$ GeV [20]), we can find:

$$\Gamma(\rho^0 \to \sigma\gamma \to \pi^0\pi^0\gamma) = \frac{1}{3}\Gamma(\rho^0 \to \sigma\gamma), \tag{1}$$

taking into account the identity of the neutral π^0-mesons, produced in $\rho^0 \to \pi^0\pi^0\gamma$, and the isotopic relation: $g(\sigma \to \pi^0\pi^0) = g(\sigma \to \pi^+\pi^-)$.

The matrix element of the decay $V \to \sigma + \gamma$ can be written in the following form:

$$\mathcal{M} = \frac{e g_{V\sigma\gamma}}{M} \left(\epsilon^* \cdot U \; k \cdot p - \epsilon^* \cdot p \; U \cdot k \right), \tag{2}$$

where $\epsilon_\mu(k)$ and $U_\mu(p)$ are the four-vectors describing the vector polarization (four-momenta) of the photon and of the $V-$meson.

Averaging over the V-meson polarizations and summing over the photon polarizations, one can find for the width of the decay $V \to \sigma + \gamma$ (still considering the σ-meson as a stable particle):

$$\Gamma(V \to \sigma\gamma) = \alpha \frac{g_{V\sigma\gamma}^2}{24} M \left(1 - \frac{m_\sigma^2}{M^2}\right)^3 \simeq 231 g_{V\sigma\gamma}^2 \left(1 - \frac{m_\sigma^2}{M^2}\right)^3 \text{ keV}, \tag{3}$$

where $M(m_\sigma)$ is the mass of the $V(\sigma)$-meson. The numerical estimation (3) is done for M=0.77 GeV (ρ-meson). Evidently, at a given value of the $g_{V\sigma\gamma}$-coupling constant, the value of $\Gamma(V \to \sigma\gamma)$ depends essentially on the value taken for the σ-meson mass, which can be in a wide range: $m_\sigma = (0.4 \div 1.2)$ GeV [20]. In Table 1 we report the values of $\Gamma(\rho^0 \to \sigma\gamma)$ and $\Gamma(\rho^0 \to \sigma\gamma \to \pi^0\pi^0\gamma)$, calculated for the values of the $g_{\rho\sigma\gamma}$-constant and m_σ from Refs. [2] and [13]. Such estimations for $\Gamma(\rho^0 \to \sigma\gamma \to \pi^0\pi^0\gamma)$ are almost two orders of magnitude larger than the value given by the experiment [22]:

$$\Gamma(\rho^0 \to \sigma\gamma \to \pi^0\pi^0\gamma) = (2.85^{+1.4}_{-1.2} \pm 0.6) \text{ keV}. \tag{4}$$

Note that the predictions of QCD sum rules [18] are also in serious disagreement with the direct experimental estimation (4).

m_σ [GeV]	$g_{\rho\sigma\gamma}$	$\Gamma(\rho^0 \to \sigma\gamma)$ [keV]	$\Gamma(\rho^0 \to \pi^0\pi^0\gamma)$ [keV]	Ref.
0.5	3	379	126	[2]
0.6	5-6	308-444	103-148	[13]

Table 1
Radiative widths for different masses of σ-meson, m_σ and coupling constant $g_{\rho\sigma\gamma}$.

This crude preliminary estimation leads to a conclusion which is inconsistent with the suggested interpretation of the HERMES effect [13] and the description of threshold behavior of the cross section for $\gamma + p \to p + \rho^0$ [2]. Therefore let us estimate more precisely the $g_{\rho\sigma\gamma}$ coupling constant on the basis of the decay $\rho^0 \to \sigma\gamma \to \pi^0\pi^0\gamma$, removing the hypothesis taken above, i.e. taking into account the finite value of the σ-width, and the possibility that in some cases it is possible to have $m_\sigma > M$.

Considering the sequence of decays $\rho^0 \to \sigma + \gamma \to \pi^0 + \pi^0 + \gamma$, one can find, for the effective $\pi^0 + \pi^0$-mass distribution, the following formula (see Appendix):

$$\frac{d\Gamma}{dw^2} = \frac{e^2 g^2_{\rho\sigma\gamma}}{288\pi^2} M \left(1 - \frac{w^2}{M^2}\right)^3 \frac{\beta_w}{\beta_\sigma} \frac{m_\sigma \Gamma_\sigma}{(w^2 - m_\sigma^2)^2 + \Gamma_\sigma^2 m_\sigma^2}, \tag{5}$$

where Γ_σ is the total width for the σ-meson, and:

$$\beta_\sigma = \sqrt{1 - \frac{4m_\pi^2}{m_\sigma^2}},\ \beta_w = \sqrt{1 - \frac{4m_\pi^2}{w^2}}.$$

For the estimation of the coupling constant $g^2_{\rho\sigma\gamma}$ on the basis of $\Gamma(\rho^0 \to \sigma\gamma \to \pi^0\pi^0\gamma)$ the following formula can be used:

$$g^2_{\rho\sigma\gamma} = \frac{\Gamma(\rho^0 \to \sigma\gamma \to \pi^0\pi^0\gamma) 72\pi}{M\alpha} \frac{1}{r_1 r_2 f(r_1, r_2)} \tag{6}$$

with

$$f(r_1, r_2) = \int_a^1 dx \frac{(1-x)^3}{(x - r_1^2)^2 + r_1^2 r_2^2} \left(\frac{1 - a/x}{1 - a/r_1^2}\right)^{1/2},\ r_1 = \frac{m_\sigma}{M},\ r_2 = \frac{\Gamma_\sigma}{M},\ a = \frac{4m_\pi^2}{M^2}.$$

Using the experimental information for the branching ratio for the radiative decay $\rho^0 \to \sigma\gamma \to \pi^0 + \pi^0 + \gamma$ [22], with the help of Eq. (6), one can deduce the constant $g_{\rho\sigma\gamma}$ as a function of the σ-meson parameters, its mass and its width (Fig. 1). One can see a strong dependence of the value of $g_{\rho\sigma\gamma}$ on the two coordinates r_1 and r_2, which vary in the wide interval suggested by [20]. Moreover, in the whole region of parameters r_1 and r_2 one can see that $g^2_{\rho\sigma\gamma} \leq 2.5$; for example, for $m_\sigma = 0.5$ GeV and $\Gamma_\sigma = 0.6$ GeV, we find $g^2_{\rho\sigma\gamma} = 0.196$.

3 Estimation of $g_{V\sigma\gamma}$ couplings from $\sigma \to 2\gamma$

Another source for an independent estimation of the $g_{\rho\sigma\gamma}$ coupling constant, which confirms our previous finding, is the radiative decay $\sigma \to 2\gamma$. In principle it is possible to estimate the width of this decay analyzing the σ-contribution in the amplitude of the elastic scattering of photons by nucleons, in the framework of the $t-$channel exchange, Fig. 2a, or in the two-photon production of a $\pi^+\pi^-$ or $\pi^0\pi^0$-pair in $\gamma\gamma$-collisions, Fig. 2b, in the corresponding region of the $\pi\pi$-effective mass [24].

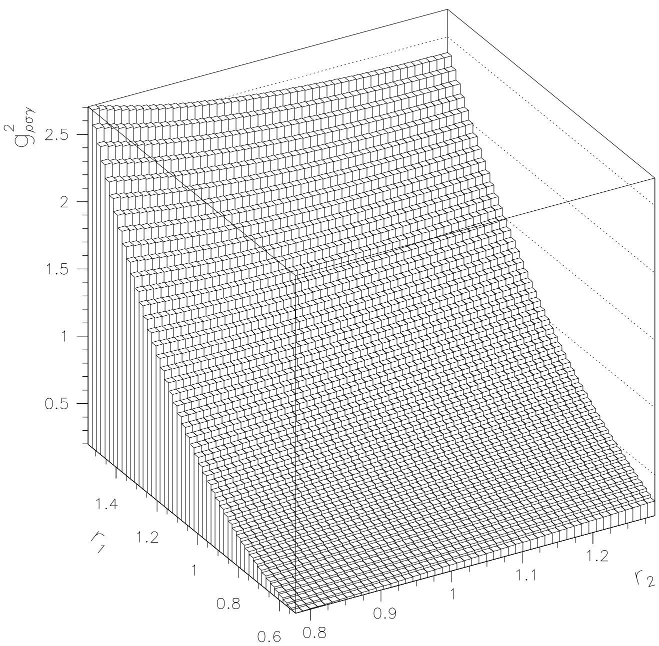

Fig. 1. Two-dimensional plot of the coupling constant $g^2_{\rho\sigma\gamma}$ as a function of $r_1 = \dfrac{m_\sigma}{M}$ and $r_2 = \dfrac{\Gamma_\sigma}{M}$.

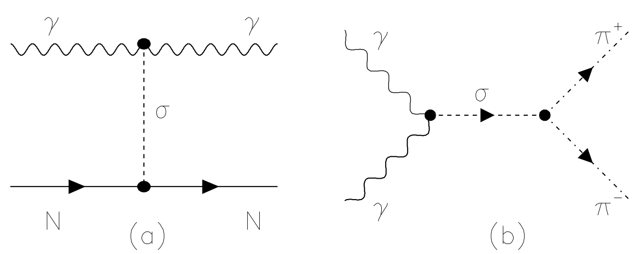

Fig. 2. Feynman diagrams for σ-meson contribution to nucleon Compton scattering (a) and to $\pi^+\pi^-$-production in $\gamma\gamma$-collisions (b).

Following the VDM approach, (Fig. 3), the matrix element for $\sigma \to 2\gamma$ can be written in terms of the $g_{V\sigma\gamma}$ coupling constants, as follows:

$$\mathcal{M}(\sigma \to 2\gamma) = e^2(g_\rho g_{\rho\sigma\gamma} + g_\omega g_{\omega\sigma\gamma})\vec{e}_1 \cdot \vec{e}_2 m_\sigma^2/M, \tag{7}$$

where $\vec{e}_1$ and $\vec{e}_2$ are the three-vectors of polarization for the two produced photons. The coupling constant g_V determines the width of the V-meson leptonic

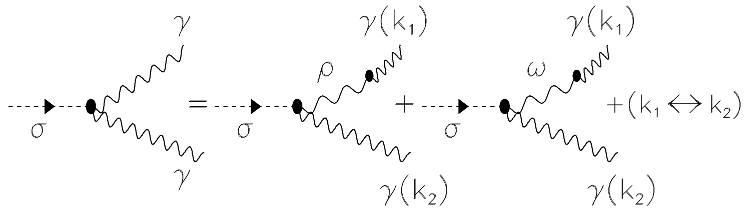

Fig. 3. Feynman diagrams for $\sigma \to 2\gamma$-decay in the VDM.

decay $V \to e^+e^-$ (through one-photon exchange): $\Gamma(V \to e^+e^-) = \alpha^2 g_V^2 M \frac{4\pi}{3}$. Note that this formula holds for zero lepton mass. The existing data about $\rho(\omega) \to e^+e^-$ decays [20] allow to estimate g_ρ and g_ω with good accuracy: $g_\rho^2 \simeq 0.04, \quad g_\omega^2 \simeq 0.09 g_\rho^2$.

As both couplings $g_{\rho\sigma\gamma}$ and $g_{\omega\sigma\gamma}$ enter in Eq. (7), we assume, for simplicity, the validity of the $SU(3)$ relation: $g_{\rho\sigma\gamma}/g_{\omega\sigma\gamma} \simeq 3$. The width $\Gamma(\sigma \to 2\gamma)$ can be written as:

$$\Gamma(\sigma \to 2\gamma) = \pi\alpha^2 \frac{m_\sigma^3}{M^2} g_\rho^2 g_{\rho\sigma\gamma}^2 \left(1 + \frac{g_\omega}{3g_\rho}\right)^2 \simeq 13.7 g_{\rho\sigma\gamma}^2 \left(\frac{m_\sigma}{1 \text{ GeV}}\right)^3 \text{ keV}. \quad (8)$$

Taking $g_{\rho\sigma\gamma} \simeq 3$ [2], one finds

$$\Gamma(\sigma \to 2\gamma) = 123 \text{ keV}, \text{ for } m_\sigma = 1 \text{ GeV}. \quad (9)$$

The same or even larger numbers for $\Gamma(\sigma \to 2\gamma)$ follow from the QCD-sum rule estimation for $g_{\rho\sigma\gamma}$ [18] or from the value deduced from the analysis of $\rho^0 \to \pi^+\pi^-\gamma$-decays [11]. These numbers contradict the corresponding experimental estimations:

$$\Gamma(\sigma \to 2\gamma) = 10.6 \text{ keV [20]}, \; \Gamma(\sigma \to 2\gamma) = (3.8 \pm 1.5) \text{ keV [24]}. \quad (10)$$

And, again, the discrepancy is large, up to one order magnitude. The situation is even worse, taking the value of $g_{\rho\sigma\gamma}$ from Ref. [13], which leads to

$$\Gamma(\sigma \to 2\gamma) = (340 \div 490) \text{ keV}. \quad (11)$$

On the other hand, taking the experimental data about the $\sigma \to 2\gamma$-decay, (10), one can deduce directly a value

$$0.28 \leq g^2_{\rho\sigma\gamma} \left(\frac{m_\sigma}{1 \text{ GeV}}\right)^3 \leq 0.78, \tag{12}$$

in evident contradiction with the previous large $g_{\rho\sigma\gamma}$ values [2,11,13,18], but in good agreement with the present estimation, based on the new data about the decay $\rho \to \pi^0\pi^0\gamma$ [22].

4 Discussion and conclusions

Let us discuss the possible consequences of a smaller value of the $g_{\rho\sigma\gamma}$ coupling constant; more precisely, let us consider the case $g^2_{\rho\sigma\gamma} \leq 1$. Here this range is derived from two independent sources: the radiative decay of the ρ^0-meson, $\rho^0 \to \sigma\gamma \to \pi^0 + \pi^0 + \gamma$, on one side, and the radiative decay of the σ-meson: $\sigma \to 2\gamma$, from another side. Both these decays can be described in a relative simple and transparent theoretical framework: the σ-dominance for the first one and the VDM approach for the second one. Note that the VDM model gives a good description of the different numerous radiative decays involving vector and pseudoscalar mesons. The coupling constants g_ρ and g_ω (for the $\gamma \to V^0$-transition), which enter in our consideration of the decay $\sigma \to 2\gamma$, are well known from the experimental data about the decays $V \to \ell^+\ell^-$, which have quite good accuracy. Therefore, the main uncertainty in the determination of $g_{\rho\sigma\gamma}$ on the basis of the existing data on $\sigma \to 2\gamma$, derives from the relatively large interval for $\Gamma(\sigma \to 2\gamma)$. The parameters of the σ-meson also do not affect very much this estimation: there is no dependence on the σ-width, and only a cubic dependence on the $\sigma-$meson mass, so that:

$$g^2_{\rho\sigma\gamma} = (g^2_{\rho\sigma\gamma})_0 \left(\frac{m_\sigma}{1 \text{ GeV}}\right)^{-3}, \tag{13}$$

where$(g^2_{\rho\sigma\gamma})_0$ is the value of the considered coupling constant for m_σ=1 GeV. From this scaling law it follows the $g_{\rho\sigma\gamma}$ constant gets smaller for higher σ-masses.

The estimation of $g^2_{\rho\sigma\gamma}$ on the basis of the decay $\rho^0 \to \sigma + \gamma \to \pi^0 + \pi^0 + \gamma$ depends essentially on the mass and width of the σ-meson, only, without additional unknown coupling constants. We can build a two-dimensional representation on $g^2_{\rho\sigma\gamma}$, with non trivial dependence on m_σ and Γ_σ. It turns out that, for the 'standard' value m_σ =0.5 GeV, in the interval $0.6 \leq \Gamma_\sigma \leq 1$ GeV, one has the following limits:

$$0.19 \leq g^2_{\rho\sigma\gamma} \leq 0.30, \tag{14}$$

definitely smaller than the values used in [2] and [13,18]. On the other hand, the interval (14) is in agreement with the interval (12). We must stress that the value $g^2_{\rho\sigma\gamma} = 9$ in the consideration of the process $\gamma + p \to p + \rho^0$ [2], has been found for m_σ=0.5 GeV, which has been chosen quite arbitrarily. The σ-exchange amplitude and the corresponding estimation of $g_{\rho\sigma\gamma}$ depend essentially on m_σ. For example, in the near-threshold conditions for $\gamma + p \to p + \rho^0$, another scaling law holds for $g_{\rho\sigma\gamma}$:

$$\frac{g^2_{\rho\sigma\gamma}}{(m^2_\sigma + a)^2} = const, \tag{15}$$

where $a = M^2 m_N/(m_N + M) \simeq 0.32$ GeV^2 (m_N is the nucleon mass). This relation shows a large correlation between $g_{\rho\sigma\gamma}$ deduced from the data about $\gamma+p \to p+\rho^0$ and the σ-mass, in such a way that larger value of m_σ correspond to larger value of the coupling constant.

Moreover, the differential cross section for $\gamma+p \to p+\rho^0$, calculated in framework of σ-exchange, contains a strong dependence on the σ-mass, through the phenomenological form factor, which has to be introduced here to improve the $t-$behavior of the differential cross section $d\sigma(\gamma p \to p\rho^0)/dt$ at large $|t|$. To deduce a definite value of the $g_{\rho\sigma\gamma}$ coupling constant from a fit of the differential cross section data, it is also necessary to assume a definite value for the $g_{\sigma NN}$-coupling. Typically this value is determined from the NN-potential.

The estimation of $g_{\rho\sigma\gamma}$ from the data about $\rho^0 \to \pi^+ + \pi^- + \gamma$, which suggests the large value: $g^2_{\rho\sigma\gamma} > 10$, can not be considered very precise, as the possible σ-exchange is hidden by the large contribution of bremsstrahlung (photon radiation by charged pions). Therefore the coupling $g_{\rho\sigma\gamma}$ derived by this decay, results in a very large width for $\Gamma(\rho^0 \to \pi^0 + \pi^0 + \gamma)$, in strong contradiction with the experiment and and with the other theoretical estimations of this width.

The HERMES-effect can not be considered a reliable source of information about the $g_{\rho\sigma\gamma}$-coupling constant. It is more correct to say, that, in order to explain this effect in the framework of the model [13], one needs large values, which are difficult to justify on the basis of what is known from the radiative decays $V \to \pi\pi\gamma$. For example, the large value of $g_{\omega\sigma\gamma}$ was justified in [13] by taking the lower limit for $\omega \to \pi^+\pi^-\gamma$ from [20], instead of a much smaller and more precise value from $\omega \to \pi^0\pi^0\gamma$. Following selection rules, the relation $\Gamma(\omega \to \pi^+\pi^-\gamma) = 2\Gamma(\omega \to \pi^0\pi^0\gamma)$ [23] is correct independently on the decay mechanism. One deduces, for $g_{\omega\sigma\gamma}$, a value which is two orders of magnitude lower then in Ref. [13].

If the small values for the $g_{\rho\sigma\gamma}$-coupling constant, as we suggest in the present paper are correct, the interpretation of the following different electromagnetic processes has to be revised:

- The explanation of the near-threshold cross section for $\gamma + p \to p + \rho^0$ in frame of the σ-model can not work, because the σ-contribution has to be small. The σ-contribution, taking $g^2_{\rho\sigma\gamma} \leq 1$, turns out to be one order of magnitude smaller than necessary for the explanation of the existing data. In principle, one can recover by increasing correspondingly the $g_{\sigma NN}$-coupling constant, but this would seriously modify the NN-potential. Moreover, such increase would change the scale of the σ-contribution to the differential cross section of $\gamma + p \to p + \omega$, in the near threshold region, making the σ and π-exchanges of the same order. So other contributions, such as Pomeron and (or) f_2-exchanges, which have been extrapolated up to threshold, have to be taken into account. This allows to decrease $g_{\rho\sigma\gamma}$ up to unity [5].
- The explanation of the HERMES effect, in terms of coherent contribution of mesons, does not hold anymore.
- The predictions of these constants in the framework of the QCD-sum rules do not hold.

5 Acknowledgment

We thank J. M. Laget for useful discussions on the problem of the $g_{\rho\sigma\gamma}$-coupling constant in different electromagnetic processes. One of us (M.P.R.) acknowledges the kind hospitality of Saclay, where this work was completed.

6 Appendix

In this Appendix we discuss the effect of the σ-width on the decay $\rho \to \pi + \pi + \gamma$.

The matrix element, corresponding to this diagram (Fig. 4) can be written as:

$$\mathcal{M} = \mathcal{M}(V\sigma^*\gamma)\frac{1}{w^2 - m_\sigma^2} m_\sigma g_{\sigma\pi\pi}, \tag{16}$$

where $\mathcal{M}(V\sigma^*\gamma)$ is the matrix element for the decay $V \to \sigma^* + \gamma$, with production of a virtual σ-meson, which mass w (different from the mass of the σ-meson, m_σ), coincides here with the effective mass of the produced $\pi^+\pi^-$-system, $m_\sigma \to m_\sigma - i\Gamma_\sigma/2$, Γ_σ is the total width of the σ-meson.

Taking the expression for $\mathcal{M}(V\sigma^*\gamma)$, Eq.(2), (where the $g_{V\sigma^*\gamma}$ coupling constant does not depend, following our assumption, on the virtuality of the

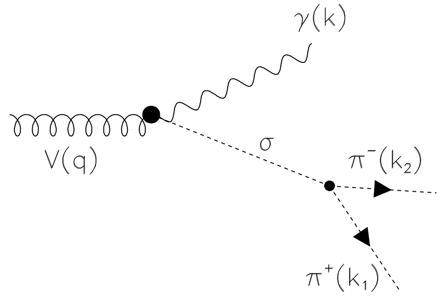

Fig. 4. Feynman diagrams for $V \to \pi^+ + \pi^- + \gamma$ decay through σ-meson production.

σ-meson), one can find, after summing over the photon polarizations and summing over the polarization of the $V-$meson:

$$|\overline{\mathcal{M}(V\sigma^*\gamma)}|^2 = e^2 \frac{g^2_{V\sigma^*\gamma}}{6} \frac{(M^2 - w^2)^2}{M^2}. \tag{17}$$

We are interested here in the $d\Gamma/dw^2$-distribution (integrated over the effective $\pi^+\pi^-$-mass) for the decay $V^0 \to \pi^+ + \pi^- + \gamma$ (we are considering here $\pi^+\pi^-$-production to have different final pions):

$$d\Gamma = \frac{|\overline{\mathcal{M}}|^2}{2M} (2\pi)^4 \frac{d^3k}{(2\pi)^3 2E_\gamma} \int \frac{d^3k_1}{(2\pi)^3 2E_1} \frac{d^3k_2}{(2\pi)^3 2E_2} \delta(q - k - k_1 - k_2).$$

As $|\overline{\mathcal{M}}|^2$ depends only on the variable w^2, it can be taken outside the integration, so that:

$$d\Gamma = \frac{|\overline{\mathcal{M}}|^2}{2^9 \pi^5 M} \frac{d^3k}{E_\gamma} \mathcal{I}, \tag{18}$$

where $\mathcal{I}$ is the result of an invariant integration (which has to be done in the CMS of the produced $\pi^+\pi^-$-system):

$$\mathcal{I} = \int \frac{d^3k_1}{E_1} \frac{d^3k_2}{E_2} \delta(q - k_1 - k_2) = 2\pi \frac{p^*}{E^*},\ E^* = \frac{w}{2},\ p^* = \sqrt{\frac{w^2}{4} - m_\pi^2} \tag{19}$$

So $\mathcal{I} = 2\pi\beta_w$, where $\beta_w = \dfrac{p^*}{E^*} = \sqrt{1 - 4m_\pi^2/w^2}$ is the velocity of the pion produced in the decay of the virtual σ-meson, $\sigma^* \to \pi^+ + \pi^-$, with mass w. It is the energy of the photon, produced in the decay $V \to \pi^+ + \pi^- + \gamma$, which determines the effective mass of the produced $\pi^+ + \pi^-$-system, through the following relation:

$$M^2 = w^2 + 2ME_\gamma \tag{20}$$

which holds in the rest system of the decaying $V-$meson. Therefore we can write:

$$\frac{d^3k}{E_\gamma} \to 4\pi E_\gamma dE_\gamma = -\pi\left(1-\frac{w^2}{M^2}\right)dw^2. \tag{21}$$

Substituting Eqs. (19) and (21) in (17) one can find:

$$\frac{d\Gamma}{dw} = \frac{|\overline{\mathcal{M}}|^2}{2^8\pi^3 M}\left(1-\frac{w^2}{M^2}\right)\sqrt{1-4\frac{m_\pi^2}{w^2}}$$

$$= \frac{e^2 g_{V\sigma^*\gamma}}{3\pi^3 2^9}M\left(1-\frac{w^2}{M^2}\right)^3\sqrt{1-4\frac{m_\pi^2}{w^2}}g^2_{\sigma^*\pi^+\pi^-}\frac{m_\sigma^2}{(w^2-m_\sigma^2)^2} \tag{22}$$

The coupling constant $g_{\sigma\pi^+\pi^-}$ can be related to the total width Γ_σ of the σ-meson, assuming that $\sigma\to 2\pi$ is its main decay: $\Gamma_\sigma = \Gamma(\sigma\to\pi^+\pi^-)+\Gamma(\sigma\to\pi^0\pi^0)$. In terms of the coupling constant $g = g_{\sigma\pi^+\pi^-} = g_{\sigma\pi^0\pi^0}$ (on the basis of isotopic invariance), one can write:

$$\Gamma(\sigma\to\pi^+\pi^-) = \frac{|\overline{\mathcal{M}(\sigma\to\pi^+\pi^-)}|^2}{8\pi m_\sigma}q = \frac{g^2}{16\pi}m_\sigma\beta_\sigma, \tag{23}$$

where $\beta_\sigma = \sqrt{1-4m_\pi^2/m_\sigma^2}$ is the velocity of the pion produced in the decay of the real σ-meson, with mass m_σ. Taking into account the relation: $\Gamma(\sigma\to\pi^+\pi^-) = 2\Gamma(\sigma\to\pi^0\pi^0)$, due to the identity of the two neutral pions in the decay $\sigma\to\pi^0\pi^0$, one can write:

$$\Gamma_\sigma = \frac{3}{2}\Gamma(\sigma\to\pi^+\pi^-) = \frac{3}{32}\frac{g^2}{\pi}m_\sigma\beta_\sigma, \tag{24}$$

Substituting Eq. (24) in (22) one obtains:

$$\frac{d\Gamma}{dw^2} = \frac{e^2 g_{V\sigma^*\gamma}}{144\pi^2}M\left(1-\frac{w^2}{M^2}\right)^3\frac{\beta_w}{\beta_\sigma}\frac{m_\pi\Gamma_\sigma}{(w^2-m_\sigma^2)^2+\Gamma_\sigma^2 m_\sigma^2} \tag{25}$$

So, for the full width of the decay $V\to\pi^+ + \pi^- + \gamma$, the following formula holds:

$$\Gamma(V\to\pi^+\pi^-\gamma) = \int_{4m_\pi^2}^{M^2}\frac{d\Gamma}{dw^2}dw^2. \tag{26}$$

For the decay $V \to \pi^0 + \pi^0 + \gamma$, Eq. (26) holds, divided by a coefficient 2 - due to the identity of pions in the final state.

Let us test the validity of these formulas, considering the limit of zero width, $\Gamma_\sigma \to 0$, i.e. considering the σ-meson as a stable particle. We use the following symbolic relation:

$$\left|\frac{1}{(w^2 - m_\sigma^2) - i\Gamma_\sigma m_\sigma}\right|^2 = \frac{1}{(w^2 - m_\sigma^2) + i\Gamma_\sigma m_\sigma}\ \frac{1}{(w^2 - m_\sigma^2) - i\Gamma_\sigma m_\sigma} \overset{\Gamma_\sigma \to 0}{\to}$$

$$= \frac{1}{(w^2 - m_\sigma^2) + i\Gamma_\sigma m_\sigma}\left[\frac{1}{w^2 - m_\sigma^2} + i\pi\delta(w^2 - m_\sigma^2)\right] \to \frac{\pi}{m_\sigma \Gamma_\sigma}\delta(w^2 - m_\sigma^2)$$

This δ-function allows one to integrated easily Eq. (26), getting the following result:

$$\Gamma(V \to \sigma\gamma \to \pi^+\pi^-\gamma) = \frac{\alpha}{36} g^2_{V\sigma\gamma} M \left(1 - \frac{m_\sigma^2}{M^2}\right)^3. \tag{27}$$

To find the radiative width for the radiative decay $V \to \sigma\gamma$, taking into account both possibilities, $\sigma \to \pi^+ + \pi^-$ and $\sigma \to \pi^0 + \pi^0$, we have to introduce a factor 3/2, so that:

$$\Gamma(V \to \sigma\gamma) = \frac{\alpha}{24} g^2_{V\sigma\gamma} M \left(1 - \frac{m_\sigma^2}{M^2}\right)^3$$

in agreement with the direct calculation, see Eq. (3).

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