Energy Distribution in Reissner-Nordström anti-de Sitter black holes in Møller Prescription‡

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Abstract. The energy (due to matter plus fields including gravity) distribution of the Reissner-Nordström anti-de Sitter (RN AdS) black holes is studied by using the Møller energy-momentum definition in general relativity. This result is compered with the energy expression obtained by using the Einstein and Tolman complexes. Total energy depends on the black hole mass M and charge Q and cosmological constant Λ. Energy distribution of the RN AdS is also calculated by using the Møller prescription in teleparallel gravity. We get the same result for both of these different gravitation theories. The energy obtained is also independent of the teleparallel dimensionless coupling constant, which means that it is valid not only in the teleparallel equivalent of general relativity, but also in any teleparallel model. Under special cases of our model, we also discuss the energy distributions associated with the Schwarzschild AdS, RN and Schwarzschild black holes, respectively.

Keywords: Reissner-Nordström anti-de Sitter; black hole; energy; Møller; prescription; teleparallel gravity.
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1. Introduction

The localization of gravitational energy-momentum still remains one of the distinguished problems and this subject continuous to be one of the most active areas of research in both general relativity and teleparallel gravity (the tetrad theory of gravity). Many attempts have been performed to obtain local or quasi-local energy-momentum. However, there is no generally accepted definition. Meisner, Thorne and Wheler [1] claimed that the energy is localizable only for spherical systems. But, Cooperstock and Sarracino [2] argued that if the energy is localizable in spherical systems, it is localizable in all systems. To solve this problem, several researcher have proposed different energy-momentum definitions [3, 4, 5, 6, 7, 8, 9, 10]. The fundamental difficulty with these definitions is that they are coordinate dependent. Therefore, if the calculations are carried out in "Cartesian" coordinates, these complexes can give a reasonable and meaningful result. Several researcher supposed that energy-momentum complexes

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weren't well-defined measures because of variety of such once. Recently, however, the subject of the energy-momentum definition has been re-opened by Virbhadra and his collogues [11, 12, 13].

The Møller energy-momentum prescription does not necessitate carrying out calculation in "Cartesian" coordinates, while the others do. Therefore, we can calculate the energy density in any coordinate system. Lessner [14] argued that the Møller prescription is a powerful concept of energy-momentum in general relativity. Teleparallel version of this complex was obtained by Mikhail *et al* [15]. In his recent paper, Vargas [16] using the Einstein and Landau-Lifshitz complexes, calculated the energy-momentum density of the Friedman-Robertson-Walker space-time. Recently, Saltı, Aydogdu and their collaborators [17, 18, 19, 20] have calculated energy-momentum density using different complexes for a given space-time in the teleparallel gravity.

Since the RN AdS black holes is a standard example to study the AdS/CFT correspondence [21] and some striking resemblance of the RN AdS phase structure to that of the Van der Waals-Maxwell liquid-gas system has been observed and some classical critical phenomena has also been uncovered [22], the study of this black hole model is appealing.

The solution of the RN AdS black holes for free space with a negative cosmological constant $\Lambda = -3/l^2$ is defined by the line-element given below.

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = \chi dtr - \chi^{-1}dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2) \tag{1}$$

where

$$\chi = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} + \frac{r^2}{l^2}. \tag{2}$$

The asymptotic form of this line-element is AdS. There is an outer horizon located at $r = r_+$. The mass of the black hole is given by

$$2M = r_+ + \frac{r_+^3}{l^2} + \frac{Q^2}{r_+}. \tag{3}$$

The Hawking temperature is

$$T_H = \frac{1}{4\pi r_+}(1 - \frac{Q^2}{r_+^2} + \frac{3r_+^2}{l^2}) \tag{4}$$

and the potential is

$$\phi = \frac{Q}{r_+}. \tag{5}$$

In the extreme case r_+ and Q satisfy the following relation.

$$1 - \frac{Q^2}{r_+^2} + \frac{3r_+^2}{l^2} = 0. \tag{6}$$

For the RN AdS black holes, the non-vanishing components of the Einstein tensor $G_{\mu\nu}$ ($\equiv 8\pi T_{\mu\nu}$, where $T_{\mu\nu}$ is the energy-momentum tensor for the matter field described by a perfect fluid of density ρ and pressure p) are

$$G_{11} = \frac{1}{r^2\chi}[\chi + r\chi' - 1], \tag{7}$$

$$G_{22} = \frac{r}{2}[r\chi'' + 2\chi'], \tag{8}$$

$$G_{33} = \frac{1}{2} r sin^2\theta [r\chi'' + 2\chi'], \tag{9}$$

$$G_{00} = \frac{-\chi}{r^2}[\chi + r\chi' - 1], \tag{10}$$

where the prime represents differentiation with respect to r.

Energy distributions of a charged dilaton black hole and Schwarzschild black hole in a magnetic universe are obtained by Xulu [13]. Radinschi [13], using Tolman's prescription, obtained the energy distribution of a dilaton dyonic black hole and her result is also the same as the result found by I-Ching Yang *et al.* [35]. It is of interest to investigate the energy distribution associated with RN AdS black hole model. We hope to find the same and an acceptable energy distribution in both general relativity and teleparallel gravity.

2. Gravitational Energy

The matrix form of the metric tensor $g_{\mu\nu}$ for the line-element (1) is defined by

$$\begin{pmatrix} (1 - \frac{2M}{r} + \frac{Q^2}{r^2} + \frac{r^2}{l^2}) & 0 & 0 & 0 \\ 0 & -\frac{1}{1-\frac{2M}{r}+\frac{Q^2}{r^2}+\frac{r^2}{l^2}} & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2\sin^2\theta \end{pmatrix} \tag{11}$$

and its inverse matrix $g^{\mu\nu}$ is

$$\begin{pmatrix} \frac{1}{1-\frac{2M}{r}+\frac{Q^2}{r^2}+\frac{r^2}{l^2}} & 0 & 0 & 0 \\ 0 & -(1 - \frac{2M}{r} + \frac{Q^2}{r^2} + \frac{r^2}{l^2}) & 0 & 0 \\ 0 & 0 & -\frac{1}{r^2} & 0 \\ 0 & 0 & 0 & -\frac{1}{r^2\sin^2\theta}. \end{pmatrix} \tag{12}$$

The general form of the tetrad, e_i^μ, having spherical symmetry, was given by Robertson [23]. In the Cartesian form it can be written as

$$e_0^0 = i\Upsilon, \qquad e_a^0 = \kappa x^a, \qquad e_0^\alpha = i\Pi x^\alpha, \qquad e_a^\alpha = \zeta\delta_a^\alpha + \Psi x^a x^\alpha + \epsilon_{a\alpha\beta}\Delta x^\beta \tag{13}$$

where $\Upsilon, \zeta, \kappa, \Pi, \Psi$, and Δ are functions of t and $r = \sqrt{x^a x^a}$, and the zeroth vector e_0^μ has the factor $i = \sqrt{-1}$ to preserve Lorentz signature. We impose the boundary condition that in the case of $r \to \infty$ the tetrad given above approaches the tetrad of Minkowski space-time, $e_a^\mu = \mathtt{diag}(i, \delta_a^\mu)$ (where $a = 1, 2, 3$). In the spherical, static and isotropic coordinate system $\mathbf{X}^1 = r\sin\theta\cos\phi$, $\mathbf{X}^2 = r\sin\theta\sin\phi$, $\mathbf{X}^3 = r\cos\theta$, the tetrad components of the RN AdS space-time can be obtained from the line-element given in the Eq. (1), using the general coordinate transformation $e_{a\mu} = \frac{\partial \mathbf{X}^{\nu}}{\partial \mathbf{X}^{\mu}} e_{a\nu}$.

$$\begin{pmatrix} \frac{i}{\sqrt{1-\frac{2M}{r}+\frac{Q^2}{r^2}+\frac{r^2}{l^2}}} & 0 & 0 & 0 \\ 0 & \sqrt{1-\frac{2M}{r}+\frac{Q^2}{r^2}+\frac{r^2}{l^2}}\ \sin\theta\cos\phi & \frac{1}{r}\ \cos\theta\cos\phi & -\frac{\sin\phi}{r\sin\theta} \\ 0 & \sqrt{1-\frac{2M}{r}+\frac{Q^2}{r^2}+\frac{r^2}{l^2}}\ \sin\theta\sin\phi & \frac{1}{r}\ \cos\theta\sin\phi & \frac{\cos\phi}{r\sin\theta} \\ 0 & \sqrt{1-\frac{2M}{r}+\frac{Q^2}{r^2}+\frac{r^2}{l^2}}\ \cos\theta & -\frac{1}{r}\ \sin\theta & 0 \end{pmatrix}. \tag{14}$$

2.1. The Møller Energy in General Relativity

In general relativity, the energy-momentum complex of Møller [9] is given by

$$M_{\mu}^{\nu} = \frac{1}{8\pi}\Sigma_{\mu,\alpha}^{\nu\alpha} \tag{15}$$

satisfying the local conservation laws

$$\frac{\partial M_{\mu}^{\nu}}{\partial x^{\nu}} = 0 \tag{16}$$

where the antisymmetric super-potential $\Sigma_{\mu}^{\nu\alpha}$ is

$$\Sigma_{\mu}^{\nu\alpha} = \sqrt{-g}[g_{\mu\beta,\gamma} - g_{\mu\gamma,\beta}]g^{\nu\gamma}g^{\alpha\beta}. \tag{17}$$

The locally conserved energy-momentum complex M_{μ}^{ν} contains contributions from the matter, non-gravitational and gravitational fields. M_0^0 is the energy density and M_a^0 are the momentum density components. The momentum four-vector of Møller is given by

$$P_{\mu} = \int\int\int M_{\mu}^{0} dxdydz. \tag{18}$$

Using Gauss's theorem, this definition transforms into

$$P_{\mu} = \frac{1}{8\pi}\int\int \Sigma_{\mu}^{0a}\mu_a dS. \tag{19}$$

where μ_a (where $a = 1, 2, 3$) is the outward unit normal vector over the infinitesimal surface element dS. P_i give momentum components P_1, P_2, P_3 and P_0 gives the energy.

Using the matrices given Eqs. (11) and (12), the required non-vanishing component of $\Sigma_{\mu}^{\nu\alpha}$ is

$$\Sigma_0^{01} = 2\sin\theta\left[M - \frac{Q^2}{r} + \frac{r^3}{l^2}\right]. \tag{20}$$

From this point of view, the energy of the RN AdS black holes in general relativity is found as given below.

$$E(r) = M - \frac{Q^2}{r} + \frac{r^3}{l^2}. \tag{21}$$

2.2. The Møller Energy in Teleparallel Gravity

The teleparallel theory of gravity (the tetrad theory of gravitation) is an alternative approach to gravitation and corresponds to a gauge theory for the translation group based on Weitzenböck geometry [24]. In the theory of teleparallel gravity, gravitation is attributed to torsion [25], which plays the role of a force [26], and the curvature tensor vanishes identically. The essential field is acted by a nontrivial tetrad field, which gives rise to the metric as a by-product. The translational gauge potentials appear as the nontrivial item of the tetrad field, so induces on space-time a teleparallel structure which is directly related to the presence of the gravitational field. The interesting place of teleparallel theory is that, due to its gauge structure, it can reveal a more appropriate approach to consider some specific problem. This is the situation, for example, in the energy and momentum problem, which becomes more transparent.

Møller modified general relativity by constructing a new field theory in teleparallel space. The aim of this theory was to overcome the problem of the energy-momentum complex that appears in Riemannian Space [27]. The field equations in this new theory were derived from a Lagrangian which is not invariant under local tetrad rotation. Saez [28] generalized Møller theory into a scalar tetrad theory of gravitation. Meyer [29] showed that Møller theory is a special case of Poincare gauge theory [30, 31].

In teleparallel gravity, the super-potential of Møller is given by Mikhail *et al.* [15] as

$$U_\mu^{\nu\beta} = \frac{(-g)^{1/2}}{2\kappa} P_{\chi\rho\sigma}^{\tau\nu\beta} [\Phi^\rho g^{\sigma\chi} g_{\mu\tau} - \lambda g_{\tau\mu} \xi^{\chi\rho\sigma} - (1-2\lambda) g_{\tau\mu} \xi^{\sigma\rho\chi}] \tag{22}$$

where $\xi_{\alpha\beta\mu} = e_{i\alpha} e^i{}_{\beta;\mu}$ is the con-torsion tensor and $e_i{}^\mu$ is the tetrad field and defined uniquely by $g^{\alpha\beta} = e_i^\alpha e_j^\beta \eta^{ij}$ (here η^{ij} is the Minkowski space-time). κ is the Einstein constant and λ is free-dimensionless coupling parameter of teleparallel gravity. For the teleparallel equivalent of general relativity, there is a specific choice of this constant.

Φ_μ is the basic vector field given by

$$\Phi_\mu = \xi^\rho{}_{\mu\rho} \tag{23}$$

and $P_{\chi\rho\sigma}^{\tau\nu\beta}$ can be found by

$$P_{\chi\rho\sigma}^{\tau\nu\beta} = \delta_\chi^\tau g_{\rho\sigma}^{\nu\beta} + \delta_\rho^\tau g_{\sigma\chi}^{\nu\beta} - \delta_\sigma^\tau g_{\chi\rho}^{\nu\beta} \tag{24}$$

with $g_{\rho\sigma}^{\nu\beta}$ being a tensor defined by

$$g_{\rho\sigma}^{\nu\beta} = \delta_\rho^\nu \delta_\sigma^\beta - \delta_\sigma^\nu \delta_\rho^\beta. \tag{25}$$

The energy-momentum density is defined by

$$\Xi_\alpha^\beta = U_{\alpha,\lambda}^{\beta\lambda} \tag{26}$$

where comma denotes ordinary differentiation. The energy is expressed by the surface integral;

$$E = \lim_{r\to\infty} \int_{r=constant} U_0^{0\zeta} \eta_\zeta dS \tag{27}$$

where η_ζ is the unit three-vector normal to surface element dS. Now, we are interested in to find the total energy distribution. Since the intermediary mathematical exposition are length, we give only the final result. To find the super-potential of Møller, first we can calculate the required non-vanishing the basic vector field Φ_μ and the con-torsion tensor $\xi_{\alpha\beta\mu}$. After making the some calculations [32, 33], the required non-vanishing components of $\xi_{\alpha\beta\mu}$ and Φ_μ are obtained as following

$$\xi^0_{01} = -\xi^1_{11} = \left[ln\sqrt{1 - \frac{2M}{r} + \frac{Q^2}{r^2} + \frac{r^2}{l^2}} \right]_r , \tag{28}$$

$$\xi^1_{22} = -r \left[\sqrt{1 - \frac{2M}{r} + \frac{Q^2}{r^2} + \frac{r^2}{l^2}} \right], \tag{29}$$

$$\xi^1_{33} = \xi^1_{22} \sin^2\theta, \tag{30}$$

$$\xi^2_{21} = \xi^3_{31} = r^{-1}, \tag{31}$$

$$\xi^3_{32} = \xi^3_{23} = \cot\theta, \tag{32}$$

$$\xi^2_{33} = -\sin\theta\cos\theta, \tag{33}$$

$$\xi^2_{12} = \xi^3_{13} = \left[r \left(\sqrt{1 - \frac{2M}{r} + \frac{Q^2}{r^2} + \frac{r^2}{l^2}} \right) \right]^{-1} , \tag{34}$$

$$\Phi_1 = -\left[ln\sqrt{1 - \frac{2M}{r} + \frac{Q^2}{r^2} + \frac{r^2}{l^2}} \right]_r + 2\left[r \left(\sqrt{1 - \frac{2M}{r} + \frac{Q^2}{r^2} + \frac{r^2}{l^2}} \right) \right]^{-1} \tag{35}$$

$$\Phi_2 = \cot\theta. \tag{36}$$

Substituting this results into Eq. (22), we obtain the non-vanishing required Møller's super-potential $U^{\nu\beta}_\mu$ as following

$$U^{01}_0 = \frac{2\sin\theta}{\kappa} \left[M - \frac{Q^2}{r} + \frac{r^3}{l^2} \right]. \tag{37}$$

Using above result in energy integral, we find the following energy for the RN Ads black hole

$$E(r) = M - \frac{Q^2}{r} + \frac{r^3}{l^2}. \tag{38}$$

We can easily see that the energy depends on the mass M and charge Q of the RN AdS black hole and cosmological constant Λ.

3. Discussion

The localization of energy-momentum in general relativity has been debated since the beginning of relativity. The energy-momentum pseudotensors are not tensorial object and one is forced to use "Cartesian" coordinates. Because of these reasons, this topic was not considered exactly for a long time. However, after Virbhadra, Rosen, Chamorro and Aguirregabiria's works [11], this subject was re-opened. In addition to this, Virbhadra [12] underlined that although the energy-momentum prescriptions are not tensorial objects, they do not disturb the principle of general covariance as the equations defining the local conservation laws with these objects are covariant. In another study, Chang, Nester and Chen [36] obtained that there exists a direct relationship between quasilocal and pseudotensor expressions; since every energy-momentum pseudotensor is associated with a legitimate Hamiltonian boundary term.

In general relativity, several studies have been devoted to calculate the energy(due to matter plus fields) distribution for a given space-time. For example; Chamorro-Virbhadra [11] and Xulu [13] showed, considering the general relativity analogs of Einstein and Møller's definitions, that the energy of a charged dilation black hole depends on the value h which controls the coupling between the dilation and the Maxwell fields.

$$E_{Einstein} = M - \frac{Q^2}{2r}(1-h^2), \qquad E_{Moller} = M - \frac{Q^2}{r}(1-h^2). \tag{39}$$

Also, Virbhadra[12] and Xulu[13] obtained that the energy distribution in the sense of Einstein and Møller disagree in general. Next, Lessner[14] showed that the Møller energy-momentum complex is a powerful concept of energy and momentum.

In this paper, to calculate the energy distribution(due to matter plus fields) associated with the RN AdS black holes, we investigated the Møller energy-momentum definition in both general relativity and teleparallel gravity. We obtained that the energy is the same in both of these different gravitation theories and also found that the energy depends on the mass M and charge Q of the RN AdS black hole and cosmological constant Λ. According to the Cooperstock hypothesis [2], the energy is confined to the region of non-vanishing energy-momentum tensor of matter and all non-gravitational fields. Radinschi [13] found that Einstein and Tolman prescriptions give the same energy for the RN AdS black hole which is given by

$$E_T(r) = E_E(r) = M - \frac{Q^2}{2r} + \frac{r^3}{2l^2}. \tag{40}$$

Using Møller complex, we found the energy of the RN AdS black hole in both general relativity and teleparallel gravity and showed that both of them give same result which is given by

$$E_M(r) = M - \frac{Q^2}{r} + \frac{r^3}{l^2} \tag{41}$$

The result support that the energy distribution in the sense of Einstein and Møller disagree in general. Difference between these two definitions is given by

$$E_E(r) - E_M(r) = \Delta(E) = \frac{Q^2}{2r} - \frac{r^3}{2l^2}. \tag{42}$$

The limiting of $\Lambda \to 0$ and $Q \to 0$, Einstein and Møller definitions give the same energy which is obtained as $E_E(r) = E_M(r) = M$.

In some special cases, RN AdS black hole is reduced to the black holes known whose energies have been already calculated.

1. *Schwarzschild AdS limit*

We first consider the Schwarzschild AdS case. In this case, the RN AdS black hole is easily reduced to the Schwarzschild AdS black hole in the limiting of $Q \to 0$ (or without charge). From Eq. (21), the total energy becomes

$$E(r) = M + \frac{r^3}{l^2}. \tag{43}$$

The result is the same as that obtained by Salti and Aydogdu [34] for the Schwarzschild AdS black hole.

2. *RN limit*

The other limit is $\Lambda \to 0$ (or without cosmological constant). In this limit, the line-element (1) describes spherical symmetric solutions. From Eq. (21), the total energy becomes

$$E(r) = M - \frac{Q^2}{r}. \tag{44}$$

This result is also calculated by Chamorro and Virbhadra [11] for a charged dilaton black hole.

3. *Schwarzschild limit*

When $\Lambda \to 0$ and $Q \to 0$, the line-element (1) describes the Schwarzschild space. In this limit, the total energy is found as

$$E(r) = M. \tag{45}$$

Moreover, this paper sustains (a) the importance of the energy-momentum definitions in the evaluation of the energy distribution of a given space-time, (b) the viewpoint of Lessner [14], (c) the energy distribution in the sense of Einstein and Møller disagree in general and d) the Møller energy-momentum definition allows to make calculations in any coordinate system. Finally, the energy obtained is also independent of the teleparallel dimensionless coupling constant, which means that it is valid not only in the teleparallel equivalent of general relativity, but also in any teleparallel model..

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References

[1] C. W. Misner, K. S. Thorne and J. A. Wheeler, *Gravitation* (W. H. Freeman and N. Y. Co, N. Y., 1973).

[2] F. I. Cooperstock and R. S. Sarracino, J. of Phys. A **11** (1918), 877.

[3] A. Einstein, Sitzungsber. Preus. Akad. Wiss. Berlin (Math. Phys.) 778 (1915), Addendum-ibid. 799 (1915).

[4] A Trautman, in *Gravitation: and Introduction to Current Research*, edited by L. Witten (Wiley, New York, 1962), p.169-198.

[5] L.D. Landau and E.M. Lifshitz, *The Classical theory of fields* (Pergamon Press, 4th Edition, Oxford, re-printed in 2002).

[6] S. Weinberg, *Gravitation and Cosmology: Principle and Applications of General Theory of Relativity* (John Wiley and Sons, Inc., New York, 1972).

[7] A. Papapetrou, Proc. R. Irish. Acad. **A52**, 11 (1948).

[8] P.G. Bergmann and R. Thomson, Phys. Rev. **89**, 400 (1953).

[9] C. Mϕller, Ann. Phys. (NY) **4**, 347 (1958); Ann. Phys. (NY) **12**, 118 (1961)

[10] R.C. Tolman, *Relativity, Thermodinamics and Cosmology* (Oxford Univ. Pres. London, P.227) (1934).

[11] K. S. Virbhadra and N. Rosen, Gen. Rel. Grav. **25** (1993), 429; K. S. Virbhadra and A. Chamorro, Pramana-J Phys. **45** (1995), 181.

[12] K. S. Virbhadra, Phys. Rev. D **41** (1990), 1086; **42** (1990), 2919; **60** (1999), 104041; Pramana-J Phys. **45** (1995), 215.

[13] E. Vagenas, Int. J. Mod. Phys. A **18** (2003), 5781; Int. J. Mod. Phys. A **18** (2003), 5949; Mod. Phys. Lett. A **19** (2004) 213; Int. J. Mod. Phys. D **14** (2005), 573; S. S. Xulu, hep-th/0308077, Int. J. Mod. Phys. D **7** (1998), 773, Int.J.Mod.Phys. A **15** (2000), 2979; I. Radinschi, Fizika B **9** (2000), 43, Acta Physica Slovaca, **49** (1999), 1.

[14] G. Lessner, *Gen. Relativ. Gravit.* **28**, 527 (1996).

[15] F.I. Mikhail, M.I. Wanas, A. Hindawi and E.I. Lashin, Int. J. Theor. Phys. **32**, 1627 (1993).

[16] T. Vargas, Gen. Rel. Gravit. 36, 1255 (2004).

[17] M. Saltı and A. Havare, Int. J Mod. Phys. **A**20, 2169 (2005).

[18] M. Saltı, Astrophys. Space Sci. 299, 159 (2005).

[19] O. Aydogdu and M. Saltı: Astrophys. Space Sci. 299, 227 (2005).

[20] M. Saltı, Nuovo Cimento B**120**, 53 (2005); Mod. Phys. Lett. A**20**, 2175 (2005); Acta Phys. Slov., **55**, 563 (2005); O. Aydogdu, Int. J. Mod. Phys. A, to appear [gr-qc/0601070]; Int. J. Mod. Phys. D, to appear [gr-qc/0509047]; Fortschritte der Physik, to appear; O. Aydogdu, M. Saltı and M. Korunur, Acta Phys. Slov., **55**, 537 (2005); M. Saltı and O. Aydogdu, gr-qc/0511030, gr-qc/0509061; Astrophys. Space Sci., to appear [gr-qc/0509022]; Prog. Theor. Phys.**115**, 63 (2006).

[21] E. Witten, *Adv. Theor. Math. Phys.* **2**, 505 (1998).

[22] A. Chamblin, R. Emparan, C.V. Johnson and R.C. Myers; *Phys. Rev. D***60**, 064018 (1999).

[23] H.P. Robertson, *Ann. of Math. (Princeton)* **33**, 496 (1932).

[24] R. Weitzenböck, *Invarianttent theorie* (Gronningen: Noordhoff, 1923)

[25] K. Hayashi and T. Shirafuji, *Phys. Rev. D* **19**, 3524 (1978).

[26] V.V. de Andrade and J.G. Pereira, *Phys. Rev. D***56**, 4689 (1997).

[27] C. Møller, *Mat. Fys. Medd. K. Vidensk. Selsk.* **39**, 13 (1978); **1**, 10 (1961).

[28] D. Saez, *Phys. Rev. D* **27**, 2839 (1983).

[29] H. Meyer, *Gen. Rel. Gravit.* **14**, 531 (1982).

[30] K. Hayashi and T. Shirafuji, *Prog. Theor. Phys.* **64**, 866 (1980); **65**, 525 (1980).

[31] F.W. Hehl, J. Nitsch and P. von der Heyde, *General Relativity and Gravitation*, edited by A. Held (Plenum, New York, 1980)

[32] Wolfram Research, *Mathematica 5.0* (2003).

[33] TCI Software Research, *Scientific Workplace 3.0* (1998).

[34] M. Saltı and O. Aydogdu Found. Phys. Lett., to appear [gr-qc/0512080].

[35] Y-Ching Yang, Ching-Tzung Yeh, Rue-Ron Hsu and Chin-Rong Lee, Int. J. Mod. Phys. D **6** (1997), 349.

[36] Chia-Chen Chang, J. M. Nester and Chiang-Mei Chen, Phys.Rev.Lett. **83** (1999), 1897.