Asymmetry Parameter of the $K_1(1270, 1400)$ by Analyzing the $B \to K_1 \nu \overline{\nu}$ Transition Form Factors within QCD

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Abstract

Separating the mixture of the $K_1(1270)$ and $K_1(1400)$ states, the $B \to K_1(1270, 1400)\nu\overline{\nu}$ transition form factors are calculated in the three-point QCD sum rules approach. The longitudinal, transverse and total decay widths as well as the asymmetry parameter, characterizing the polarization of the axial $K_1(1270, 1400)$ and the branching ratio for these decays are evaluated.

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1 Introduction

The $B \to K_1(1270, 1400)\nu\overline{\nu}$ transitions are governed by the flavor changing neutral current (FCNC) decay of $b \to s\nu\overline{\nu}$ which is of fundamental interest because of the following reasons: Such transition occurs at loop level and is forbidden at tree level in the Standard Model (SM). This transition is a good candidate for searching new physics beyond the SM and constrains the parameters beyond it. A search for SUSY particles [1], light dark matter [2] and also fourth generation of the quarks is possible by analyzing such loop level transition. The $B \to K_1(1270, 1400)\nu\overline{\nu}$ decays also provide a new framework for precise calculation of the V_{tb} and V_{ts} as elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix.

Experimentally, the $K_1(1270)$ and $K_1(1400)$ are the mixtures of the strange members of two axial-vector SU(3) octets ${}^3P_1(K_1^A)$ and ${}^1P_1(K_1^B)$. The $K_1(1270, 1400)$ and $K_1^{A,B}$ states are related to each other as [3, 4]:

$$
\begin{aligned}
\mid K_1(1270) > &= \mid K_1^A > sin\theta + \mid K_1^B > cos\theta \\
\mid K_1(1400) > &= \mid K_1^A > cos\theta - \mid K_1^B > sin\theta,
\end{aligned}
\tag{1}
$$

the angle θ lies in the interval $37^\circ \leq \theta \leq 58^\circ$, $-58^\circ \leq \theta \leq -37^\circ$ [3, 4, 5, 6, 7]. The sign ambiguity for the mixing angle is related to the fact that one can add arbitrary phase to the $\mid K_1^A >$ and $\mid K_1^B >$. In the recent studies for $B \to K_1(1270)\gamma$ and $\tau \to K_1(1270)\nu_\tau$, the following values has been obtained for θ [8], which we are going to use in the present work:

$$\theta = -(34 \pm 13)^\circ \tag{2}$$

The $B \to K_1\gamma$ decay has been investigated in the next-to-leading order in the large energy effective theory (LEET) and in the framework of light cone sum rules in [9] and [3], respectively. In [10], the $B \to K_1(1270)l\bar{l}$ transition has also been investigated in the LEET model. In this work, separating the $K_1(1270)$ and $K_1(1400)$ states, we analyze the $B \to K_1(1270, 1400)\nu\overline{\nu}$ decay modes in the framework of the three-point QCD sum rules. First, we calculate the form factors of the B to axial $\mid K_1^A >$ and $\mid K_1^B >$ states. Then, using the relations among the form factors of the $K_1(1270)$, $K_1(1400)$, $\mid K_1^A >$ and $\mid K_1^B >$, we calculate the form factors of the $B \to K_1(1270)$ and $B \to K_1(1400)$ transitions.

The transition form factors play fundamental role in the evaluating of the longitudinal, transverse and total decay widths as well as the asymmetry parameter of $K_1(1270, 1400)$. For calculating these form factors, we use the well established QCD sum rules method as a non-perturbative method based on the fundamental QCD lagrangian.

The paper encompasses three sections. In section 2, the form factors of the $B \to K_1^{A(B)}\nu\overline{\nu}$ transition as well as the longitudinal and transverse component of the decay width and asymmetry parameter for $K_1(1270, 1400)$ are calculated. Section 3 is devoted to the numerical analysis and discussions.

2 Sum rules for the $B \to K_1^{A(B)}\nu\overline{\nu}$ transition form factors

At the quark level, the $B \to K_1^{A(B)}\nu\overline{\nu}$ decay proceeds by the loop $b \to s\nu\overline{\nu}$ transition. The Hamiltonian responsible for this transition is given by:

$$H_{eff} = \frac{G_F \alpha_{em}}{2\sqrt{2}\pi} C_{10} V_{tb} V_{ts}^* \; \overline{\nu} \; \gamma_\mu (1-\gamma_5) \nu \; \overline{s} \; \gamma_\mu (1-\gamma_5) b. \tag{3}$$

To obtain the transition amplitude for $B \to K_1^{A(B)} \nu\overline{\nu}$ decay, it is necessary to sandwich the Eq. (3) between the initial and final meson states.

$$M = \frac{G_F \alpha_{em}}{2\sqrt{2}\pi} C_{10} V_{tb} V_{ts}^* \ \overline{\nu}\ \gamma_\mu (1-\gamma_5) \nu < K_1^{A(B)}(p', \varepsilon) \mid \ \overline{s}\ \gamma_\mu (1-\gamma_5) b \mid B(p) >, \quad (4)$$

where G_F is the Fermi constant, α_{em} is the fine structure constant at Z mass scale and V_{ij} are the elements of the CKM matrix. Both vector and axial vector part of the transition current, $\overline{s}\ \gamma_\mu (1-\gamma_5) b$, contribute to the matrix element stated in the Eq. (4). Considering the Lorentz and parity invariances, this matrix element can be parameterized in terms of some form factors as follows:

$$\begin{aligned} < K_1^{A(B)}(p', \varepsilon) \mid \overline{s}\gamma_\mu b \mid B(p) > &= i \Big[f_0^{A(B)}(q^2)(m_B + m_{K_1^{A(B)}}) \varepsilon_\mu^* \\ &- \frac{f_+^{A(B)}(q^2)}{(m_B + m_{K_1^{A(B)}})} (\varepsilon^* p) P_\mu - \frac{f_-^{A(B)}(q^2)}{(m_B + m_{K_1^{A(B)}})} (\varepsilon^* p) q_\mu \Big], \end{aligned} \quad (5)$$

$$< K_1^{A(B)}(p', \varepsilon) \mid \overline{s}\gamma_\mu \gamma_5 b \mid B(p) > = -\frac{f_V^{A(B)}(q^2)}{(m_B + m_{K_1^{A(B)}})} \varepsilon_{\mu\nu\alpha\beta} \varepsilon^{*\nu} p^\alpha p'^\beta, \quad (6)$$

where $f_V^{A(B)}(q^2)$, $f_0^{A(B)}(q^2)$, $f_+^{A(B)}(q^2)$ and $f_-^{A(B)}(q^2)$ are the transition form factors and $P_\mu = (p+p')_\mu$, $q_\mu = (p-p')_\mu$. Here, we should mention that the $f_-^{A(B)}(q^2)$ form factor does not appear in the expressions of the decay widths, so we don't consider it in our calculations. Using the Eqs. (1, 5, 6) we obtain:

$$\begin{aligned} f_0^{B\to K_1(1270)} &= \frac{m_B + m_{K_1^A}}{m_B + m_{K_1(1270)}} f_0^{B\to K_1^A} sin\theta + \frac{m_B + m_{K_1^B}}{m_B + m_{K_1(1270)}} f_0^{B\to K_1^B} cos\theta, \\ f_0^{B\to K_1(1400)} &= \frac{m_B + m_{K_1^A}}{m_B + m_{K_1(1400)}} f_0^{B\to K_1^A} cos\theta - \frac{m_B + m_{K_1^B}}{m_B + m_{K_1(1400)}} f_0^{B\to K_1^B} sin\theta, \end{aligned} \quad (7)$$

$$f_{+,-,V}^{B\to K_1(1270)} = \frac{m_B + m_{K_1(1270)}}{m_B + m_{K_1^A}} f_{+,-,V}^{B\to K_1^A} sin\theta + \frac{m_B + m_{K_1(1270)}}{m_B + m_{K_1^B}} f_{+,-,V}^{B\to K_1^B} cos\theta,$$
$$f_{+,-,V}^{B\to K_1(1400)} = \frac{m_B + m_{K_1(1400)}}{m_B + m_{K_1^A}} f_{+,-,V}^{B\to K_1^A} cos\theta - \frac{m_B + m_{K_1(1400)}}{m_B + m_{K_1^B}} f_{+,-,V}^{B\to K_1^B} sin\theta. \quad (8)$$

For simplicity, we will set $f_i^{B\to K_1^{A(B)}} = f_i^{A(B)}$ in the future calculations.

We define the G-parity conserving decay constants of the axial vector mesons K_1^A and K_1^B as

$$< K_1^A(p', \varepsilon) \mid J_\nu = \bar{s}\gamma_\nu\gamma_5 u \mid 0 > = -i f_{K_1^A} m_{K_1^A} \varepsilon_\nu,$$
$$< K_1^B(p', \varepsilon) \mid J_{\nu\nu'} = \bar{s}\sigma_{\nu\nu'}\gamma_5 u \mid 0 > = f_{K_1^{B\perp}}(1 \ \ GeV)(\varepsilon_\nu p'_{\nu'} - \varepsilon_{\nu'} p'_\nu), \quad (9)$$

where $f_{K_1^A}$ is the scale-independent decay constant of the K_1^A meson, however $f_{K_1^{B\perp}}$ is the scale-dependent leptonic constant of the K_1^B meson. The $f_{K_1^{B\perp}}$ is calculated at scale 1 GeV. The decay constants $f_{K_1^A}$ and $f_{K_1^{B\perp}}$ are calculated in the framework of the light cone QCD sum rules with the help of the distribution amplitudes of the axial K_1^A and K_1^B states in [11, 12]. On the other hand, the G-party violating decay constants are defined as

$$< K_1^A(p', \varepsilon) \mid J_{\nu\nu'} = \bar{s}\sigma_{\nu\nu'}\gamma_5 u \mid 0 > = f_{K_1^A} \ a_0^{\perp, K_1^A}(\varepsilon_\nu p'_{\nu'} - \varepsilon_{\nu'} p'_\nu),$$
$$< K_1^B(p', \varepsilon) \mid J_\nu = \bar{s}\gamma_\nu\gamma_5 u \mid 0 > = i f_{K_1^{B\perp}}(1 \ \ GeV) \ a_0^{\parallel, K_1^B} m_{K_1^B} \ \varepsilon_\nu, \quad (10)$$

where $a_0^{\perp, K_1^A}$ and $a_0^{\parallel, K_1^B}$ are the zeroth order Gegenbauer moments. They are zero in the $SU(3)$ symmetry limit. The $a_0^{\perp, K_1^A}$ and $a_0^{\parallel, K_1^B}$ have been calculated in the framework of the QCD sum rules [3, 8, 11]. In these works, instead of the individual sum rules for the $a_0^{\perp, K_1^A}$ and $a_0^{\parallel, K_1^B}$, the sum rules for the combination of these moments have been obtained. These calculations led to the relation $a_0^{\perp, K_1^A} + (0.59 \pm 0.15) \ a_0^{\parallel, K_1^B} = 0.17 \pm 0.11$. Due to the data for the branching ratio of $B \to K_1(1270)\gamma$ which is very large than that of the $B \to K_1(1400)\gamma$ and also $\tau^- \to K_1^-(1270)\nu_\tau$, the mixing angle and $a_0^{\parallel,K_1^B}$ should be negative. Assuming that the G-party violation contribution is about 30 $^o/_o$, the values for $a_0^{\perp,K_1^A}$ and $a_0^{\parallel,K_1^B}$ are obtained as presented in the numerical analysis section. The Eqs. (9, 10) show that the main contributions of the axial $J_\nu = \bar{s}\gamma_\nu\gamma_5 u$ and pseudo tensor $J_{\nu\nu'} = \bar{s}\sigma_{\nu\nu'}\gamma_5 u$ currents come from their couplings to the $\mid K_1^A >$ and $\mid K_1^B >$, respectively.

To calculate the form factors, we start considering the following correlation functions:

$$
\begin{aligned}
\Pi^{V;A}_{\mu\nu}(p^2,p'^2,q^2) &= i^2\int d^4x d^4y e^{-ipx}e^{ip'y} < 0 \mid T[J_{\nu K_1^{A(B)}}(y) J_\mu^{V;A}(0) J_B(x)] \mid 0 >, \\
\Pi^{V;A}_{\mu\nu\nu'}(p^2,p'^2,q^2) &= i^2\int d^4x d^4y e^{-ipx}e^{ip'y} < 0 \mid T[J_{\nu\nu' K_1^{A(B)}}(y) J_\mu^{V;A}(0) J_B(x)] \mid 0 >,
\end{aligned}
\tag{11}
$$

where $J_{\nu K_1^{A(B)}}(y) = \bar{s}\gamma_\nu\gamma_5 u$ and $J_{\nu\nu' K_1^{A(B)}} = \bar{s}\sigma_{\nu\nu'}\gamma_5 u$ are the axial vector and pseudo tensor interpolating currents of the $K_1^{A(B)}$ mesons and $J_B(x) = \bar{b}\gamma_5 u$ is the interpolating current of B meson. The $J_\mu^V = \bar{s}\gamma_\mu b$ and $J_\mu^A = \bar{s}\gamma_\mu\gamma_5 b$ are the vector and axial vector part of the transition currents. From the general aspect of the QCD sum rules, the above mentioned correlators are calculated in two different approaches. First, they are saturated with towers of hadrons with the same quantum numbers as the interpolating currents called the physical or phenomenological part and on the other side they describe hadrons as quarks and gluons interacting with the QCD vacuum called the QCD or theoretical part. Considering the quark-hadron duality, equating these two representations of the correlation functions and applying double Borel transformation with respect to the momentum of the initial and final states, we get the sum rules for the form factors. To calculate the correlation functions in the phenomenological side, we insert complete sets of the intermediate states with the same quantum numbers as the interpolating currents and sum over the $\mid K_1^A>$ and $\mid K_1^B>$ states. As a result

$$\Pi^{V,A}_{\mu\nu}(p^2,p'^2,q^2) = \frac{<0\mid J^{\nu}_{K_1^A}\mid K_1^A(p',\varepsilon)><K_1^A(p',\varepsilon)\mid J^{V,A}_{\mu}\mid B(p)><B(p)\mid J_B\mid 0>}{(p'^2-m^2_{K_1^A})(p^2-m_B^2)} + \frac{<0\mid J^{\nu}_{K_1^B}\mid K_1^B(p',\varepsilon)><K_1^B(p',\varepsilon)\mid J^{V,A}_{\mu}\mid B(p)><B(p)\mid J_B\mid 0>}{(p'^2-m^2_{K_1^B})(p^2-m_B^2)} + \cdots \quad (12)$$

$$\Pi^{V,A}_{\mu\nu\nu'}(p^2,p'^2,q^2) = \frac{<0\mid J^{\nu\nu'}_{K_1^A}\mid K_1^A(p',\varepsilon)><K_1^A(p',\varepsilon)\mid J^{V,A}_{\mu}\mid B(p)><B(p)\mid J_B\mid 0>}{(p'^2-m^2_{K_1^A})(p^2-m_B^2)} + \frac{<0\mid J^{\nu\nu'}_{K_1^B}\mid K_1^B(p',\varepsilon)><K_1^B(p',\varepsilon)\mid J^{V,A}_{\mu}\mid B(p)><B(p)\mid J_B\mid 0>}{(p'^2-m^2_{K_1^B})(p^2-m_B^2)} + \cdots \quad (13)$$

are obtained. Here, the $\cdots$ represents contributions coming from the higher states and continuum. The vacuum to the hadronic state matrix element for B meson in the Eq. (12, 13) are defined in terms of the leptonic decay constant of this meson as:

$$<B(p)\mid J_B\mid 0> = -i\frac{f_B m_B^2}{m_b+m_u}. \quad (14)$$

This matrix element for K_1^A and K_1^B states are presented Eqs. (9, 10).

Using the above equations and performing summation over the polarization of the $K_1^{A(B)}$ meson we obtain:

$$\Pi^V_{\mu\nu}(p^2,p'^2,q^2) = i\frac{f_B m_B^2}{(m_b+m_u)}\frac{f_{K_1^A}\, m_{K_1^A}}{(p'^2-m^2_{K_1^A})(p^2-m_B^2)} \times [f_0^A g_{\mu\nu}(m_B+m_{K_1^A}) - \frac{f_+^A P_\mu p_\nu}{(m_B+m_{K_1^A})} - \frac{f_-^A q_\mu p_\nu}{(m_B+m_{K_1^A})}]$$

$$
\begin{aligned}
&+ \quad i\frac{f_B m_B^2}{(m_b+m_u)}\frac{f_{K_1^{B\perp}}\ a_0^{\parallel,K_1^B}\ m_{K_1^B}}{(p'^2-m^2_{K_1^B})(p^2-m_B^2)} \\
&\times \quad [f_0^B g_{\mu\nu}(m_B+m_{K_1^B}) - \frac{f_+^B P_\mu p_\nu}{(m_B+m_{K_1^B})} - \frac{f_-^B q_\mu p_\nu}{(m_B+m_{K_1^B})}] \\
&+ \quad \text{excited states,} \\
\Pi^A_{\mu\nu}(p^2,p'^2,q^2) \quad &= \quad -\varepsilon_{\alpha\beta\mu\nu}p^\alpha p'^\beta \frac{f_B m_B^2}{(m_b+m_u)(m_B+m_{K_1^A})}\frac{f_{K_1^A} m_{K_1^A}}{(p'^2-m^2_{K_1^A})(p^2-m_B^2)}f_V^A \\
&- \quad \varepsilon_{\alpha\beta\mu\nu}p^\alpha p'^\beta \frac{f_B m_B^2}{(m_b+m_u)(m_B+m_{K_1^B})}\frac{f_{K_1^{B\perp}}\ a_0^{\parallel,K_1^B} m_{K_1^B}}{(p'^2-m^2_{K_1^B})(p^2-m_B^2)}f_V^B \\
&+ \quad \text{excited states.}
\end{aligned}
\tag{15}
$$

$$
\begin{aligned}
\Pi^V_{\mu\nu\nu'}(p^2,p'^2,q^2) \quad &= \quad i\frac{f_B m_B^2}{(m_b+m_u)}\frac{f_{K_1^{B\perp}}}{(p'^2-m^2_{K_1^B})(p^2-m_B^2)} \\
&\times \quad [f_0^B(m_B+m_{K_1^B})(g_{\nu\mu}p'_{\nu'}-g_{\nu'\mu}p'_\nu) + \frac{f_+^B P_\mu}{(m_B+m_{K_1^B})}(p_{\nu'}p'_\nu - p_\nu p'_{\nu'}) \\
&+ \quad \frac{f_-^B q_\mu}{(m_B+m_{K_1^B})}(p_{\nu'}p'_\nu - p_\nu p'_{\nu'})] \\
&+ \quad i\frac{f_B m_B^2}{(m_b+m_u)}\frac{f_{K_1^A}\ a_0^{\perp,K_1^A}}{(p'^2-m^2_{K_1^A})(p^2-m_B^2)} \\
&\times \quad [f_0^A(m_B+m_{K_1^A})(g_{\nu\mu}p'_{\nu'}-g_{\nu'\mu}p'_\nu) + \frac{f_+^A P_\mu}{(m_B+m_{K_1^A})}(p_{\nu'}p'_\nu - p_\nu p'_{\nu'}) \\
&+ \quad \frac{f_-^A q_\mu}{(m_B+m_{K_1^A})}(p_{\nu'}p'_\nu - p_\nu p'_{\nu'})] + \text{excited states,} \\
\Pi^A_{\mu\nu\nu'}(p^2,p'^2,q^2) \quad &= \quad (\varepsilon_{\alpha\beta\mu\nu}p'^{\nu'} - \varepsilon_{\alpha\beta\mu\nu'}p'^{\nu})\frac{p^\alpha p'^\beta\ f_B m_B^2 f_{K_1^{B\perp}} f_V^B}{(m_b+m_u)(m_B+m_{K_1^B})(p'^2-m^2_{K_1^B})(p^2-m_B^2)} \\
&+ \quad (\varepsilon_{\alpha\beta\mu\nu}p'^{\nu'} - \varepsilon_{\alpha\beta\mu\nu'}p'^{\nu})\frac{p^\alpha p'^\beta\ f_B m_B^2 f_{K_1^A}\ a_0^{\perp,K_1^A} f_V^A}{(m_b+m_u)(m_B+m_{K_1^A})(p'^2-m^2_{K_1^A})(p^2-m_B^2)} \\
&+ \quad \text{excited states.}
\end{aligned}
\tag{16}
$$

For extracting the expressions for the form factors $f_0^{A(B)}(q^2)$ and $f_+^{A(B)}(q^2)$, we choose the coefficients of the structures $g_{\mu\nu}$ and $\frac{1}{2}(p_\mu p_\nu + p'_\mu p_\nu)$ from $\Pi^V_{\mu\nu}(p^2, p'^2, q^2)$, respectively and the structure $i\varepsilon_{\mu\nu\alpha\beta}p'^\alpha p^\beta$ from $\Pi^A_{\mu\nu}(p^2, p'^2, q^2)$ is considered for the form factor $f_V^{A(B)}(q^2)$. On the other hand, from the $\Pi^V_{\mu\nu\nu'}(p^2, p'^2, q^2)$ and $\Pi^A_{\mu\nu\nu'}(p^2, p'^2, q^2)$ the structures $i\varepsilon_{\mu\nu'\alpha\beta}p'^\nu p^\alpha p'^\beta$, $g_{\nu'\mu}p'^\nu$, $\frac{1}{2}(p_{\nu'}P_\mu p'_\nu + p_{\nu'}p'_\nu q_\mu)$ are selected for the form factors $f_V^{A(B)}(q^2)$, $f_0^{A(B)}(q^2)$ and $f_+^{A(B)}(q^2)$, respectively. Here, we stress that there is no contribution of the K pole in the invariant structures chosen to evaluate the form factors.

Therefore, the correlation functions are written in terms of the selected structures as:

$$\begin{aligned}
\Pi^V_{\mu\nu}(p, p', q^2) &= g_{\mu\nu}\Pi_0 + \frac{1}{2}(p_\mu p_\nu + p'_\mu p_\nu)\Pi_+ + ..., \\
\Pi^A_{\mu\nu}(p, p', q^2) &= i\varepsilon_{\mu\nu\alpha\beta}p'^\alpha p^\beta \Pi_V + ... \\
\Pi^V_{\mu\nu\nu'}(p, p', q^2) &= g_{\nu'\mu}p'^\nu T_0 + \frac{1}{2}(p_{\nu'}P_\mu p'_\nu + p_{\nu'}p'_\nu q_\mu)T_+ + ..., \\
\Pi^A_{\mu\nu\nu'}(p, p', q^2) &= i\varepsilon_{\mu\nu'\alpha\beta}p'^\nu p^\alpha p'^\beta T_V + ... \;.
\end{aligned} \tag{17}$$

The QCD side of the correlation functions are calculated by the help of the operator product expansion (OPE) in the deep Euclidean region, where $p^2 \ll (m_b + m_u)^2$ and $p'^2 \ll (m_s + m_u)^2$. For this aim, we write each $\Pi_i[T_i]$ function in terms of the perturbative and non-perturbative parts as:

$$\Pi_i[T_i](p, p', q^2) = \Pi_i[T_i]^{pert}(p, p', q^2) + \Pi_i[T_i]^{non-pert}(p, p', q^2), \tag{18}$$

where i stands for 0, V and +. The non-perturbative parts contain the light quark ($< \bar{q}q >$) condensates.

The perturbative parts are written in terms of the double dispersion integrals as:

$$\Pi_i[T_i]^{pert} = -\frac{1}{(2\pi)^2}\int ds' \int ds \frac{\rho_i[\varrho_i](s, s', q^2)}{(s - p^2)(s' - p'^2)} + \text{ subtraction terms.} \tag{19}$$

The spectral densities $\rho_i(s,s',q^2)$ and $\varrho_i(s,s',q^2)$ can be calculated from the usual Feynman integrals with the help of the Cutkosky rules, i.e., by replacing the quark propagators with the Dirac-delta functions: $\frac{1}{p^2-m^2} \to -2\pi\delta(p^2-m^2)$, implying all quarks are real. Calculations lead to the following expressions for the spectral densities.

$$
\begin{aligned}
\rho_V(s,s',q^2) &= 4N_c I_0(s,s',q^2)\left[(m_u-m_b)A+(m_u+m_s)B+m_u\right], \\
\rho_0(s,s',q^2) &= -2N_c I_0(s,s',q^2)\Big[(m_b-m_u)(Au+2Bs'-4C) \\
&+ (m_s+m_u)(2As+Bu) \\
&+ m_u(2m_b m_s+m_b m_u-m_s m_u-m_u^2-u)\Big], \\
\rho_+(s,s',q^2) &= 2N_c I_0(s,s',q^2)\Big[A(3m_u-m_b)+B(m_u+m_s) \\
&+ 2(m_u-m_b)(D+E)+mu\Big],
\end{aligned}
$$

$$
\begin{aligned}
\varrho_V(s,s',q^2) &= -8N_c I_0(s,s',q^2)\left[B+E+F\right], \\
\varrho_0(s,s',q^2) &= 4N_c I_0(s,s',q^2)\Big[m_u(m_b-m_u)-As-2Es-Fu \\
&- B(m_s m_u+m_u^2-m_b(m_s+m_u)+u)\Big], \\
\varrho_+(s,s',q^2) &= 4N_c I_0(s,s',q^2)\left[B+E+F\right],
\end{aligned}
$$

where

$$
\begin{aligned}
I_0(s,s',q^2) &= \frac{1}{4\lambda^{1/2}(s,s',q^2)}, \\
\lambda(a,b,c) &= a^2+b^2+c^2-2ac-2bc-2ab,
\end{aligned}
$$

$$
\begin{aligned}
\Delta &= m_b^2 - m_u^2 - s, \\
\Delta' &= m_s^2 - m_u^2 - s', \\
u &= s + s' - q^2, \\
A &= \frac{1}{\lambda(s,s',q^2)}(\Delta' u - 2\Delta s), \\
B &= \frac{1}{\lambda(s,s',q^2)}(\Delta u - 2\Delta' s), \\
C &= \frac{1}{2\lambda(s,s',q^2)}(\Delta'^2 s + \Delta^2 s' - \Delta\Delta' u + m_u^2(-4ss' + u^2), \\
D &= \frac{1}{\lambda(s,s',q^2)^2}[-6\Delta\Delta' s' u + \Delta'^2(2ss' + u^2) + 2s'(3\Delta^2 s' + m_u^2(-4ss' + u^2))], \\
E &= \frac{1}{\lambda(s,s',q^2)^2}[-3\Delta^2 s' u + 2\Delta\Delta'(2ss' + u^2) - u(3\Delta'^2 s' + m_u^2(-4ss' + u^2))]. \\
F &= \frac{1}{\lambda(s,s',q^2)^2}[6\Delta^2 s^2 - 6\Delta\Delta' s u + 2m_u^2 s(-4ss' + u^2) + \Delta^2(2ss' + u^2)].
\end{aligned}
\tag{20}
$$

The subscripts V, 0 and + correspond to form factors f_V, f_0 and f_+, respectively. In the Eq. (20) $N_c = 3$ is the number of colors.

The integration region for the perturbative contribution in the Eq. (19) is determined from the condition that the arguments of the three δ functions must vanish simultaneously. The physical region in the s and s' plane is described by the following non-equality:

$$
-1 \leq f(s,s') = \frac{2ss' + (s + s' - q^2)(m_b^2 - s - m_u^2) + (m_u^2 - m_s^2)2s}{\lambda^{1/2}(m_b^2, s, m_u^2)\lambda^{1/2}(s,s',q^2)} \leq +1. \tag{21}
$$

For the contribution of the non-perturbative parts, i.e., the contributions of the operators with dimensions $d = 3$, 4 and 5, the following results are derived:

$$
\Pi_V^{non-pert} = \frac{1}{2} < \overline{q}q > \left[\frac{1}{rr'^3} m_s^2 (m_0^2 - 2m_u^2) \right.
$$

$$
\begin{aligned}
&+ \quad \frac{1}{3r^2r'^2}[-3m_u^2(m_b^2+m_s^2-q^2) \\
&+ \quad m_0^2(m_b^2-m_bm_s+m_s^2-q^2)] \\
&- \quad \frac{1}{rr'^2}m_sm_u+\frac{1}{r^3r'}m_b^2(m_0^2-2m_u^2) \\
&+ \quad \frac{1}{3r^2r'}(-2m_0^2-3m_bm_u)-\frac{2}{rr'}\Bigg],
\end{aligned}
$$

$$
\begin{aligned}
\Pi_0^{non-pert} &= \frac{1}{4}<\overline{q}q>\Bigg[\frac{1}{rr'^3}m_s^2(m_0^2-2m_u^2) \\
&\times \quad (m_b^2-2m_bm_s+m_s^2-q^2) \\
&+ \quad \frac{1}{3r^2r'^2}(m_b^2-2m_bm_s+m_s^2-q^2) \\
&\times \quad [-3m_u^2(m_b^2+m_s^2-q^2)+m_0^2(m_b^2-m_bm_s+m_s^2-q^2)] \\
&+ \quad \frac{1}{3rr'^2}[m_0^2(m_b^2+3m_bm_s-q^2) \\
&- \quad 3m_u(m_s+m_u)(m_b^2-2m_bm_s+m_s^2-q^2)] \\
&+ \quad \frac{1}{r^3r'}m_b^2(m_0^2-2m_u^2)(m_b^2-2m_bm_s+m_s^2-q^2) \\
&+ \quad \frac{1}{3r^2r'}[3m_u(m_b-m_u)(m_b^2-2m_bm_s+m_s^2-q^2) \\
&+ \quad m_0^2(3m_bm_s-m_s^2+q^2)] \\
&+ \quad \frac{1}{3rr'}(-4m_0^2-6m_b^2-6m_s^2+3m_sm_u \\
&+ \quad 6m_u^2+m_b(4m_s+m_u)+2q^2)\Bigg],
\end{aligned}
$$

$$
\begin{aligned}
\Pi_+^{non-pert} &= \frac{1}{4}<\overline{q}q>\Bigg[-\frac{1}{rr'^3}m_s^2(m_0^2-2m_u^2) \\
&+ \quad \frac{1}{3r^2r'^2}[3m_u^2(m_b^2+m_s^2-q^2) \\
&+ \quad m_0^2(-m_b^2+m_bm_s-m_s^2+q^2)] \\
&+ \quad \frac{1}{rr'^2}m_sm_u+\frac{1}{4r^3r'}m_b^2(m_0^2-2m_u^2) \\
&+ \quad \frac{1}{3r^2r'}[4m_0^2-3m_u(m_b+2m_u)]+\frac{2}{rr'}\Bigg],
\end{aligned}
\tag{22}
$$

$$
\begin{aligned}
T_V^{non-pert} &= 0, \\
T_0^{non-pert} &= \frac{1}{2} < \overline{q}q > \Bigg[\frac{1}{rr'^3} m_b m_s^2 (m_0^2 - 2m_u^2) \\
&+ \frac{m_b}{3r^2r'^2} [-3m_u^2(m_b^2 + m_s^2 - q^2) + m_0^2(2m_b^2 - m_b m_s + 2m_s^2 - 2q^2)] \\
&+ \frac{1}{3rr'^2} [-3m_u(-m_b^2 + m_b m_s - m_s^2 + m_s m_u - q^2) + m_0^2(m_b + 2m_s)] \\
&+ \frac{1}{r^3 r'} m_b^3 (m_0^2 - 2m_u^2) \\
&- \frac{m_b}{3r^2r'} [3m_u(m_b - m_u) + m_0^2] + \frac{2m_b}{rr'} \Bigg], \\
T_+^{non-pert} &= -\frac{1}{2} < \overline{q}q > [\frac{m_b m_0^2}{3r^2r'^2} + \frac{m_u}{rr'^2}], \qquad (23)
\end{aligned}
$$

where $r = p^2 - m_b^2$ and $r' = p'^2 - m_c^2$.

Equating the phenomenological expression given in the Eqs. (15,16) and the OPE expression given by Eqs. (18-23), and applying double Borel transformations with respect to the variables p^2 and p'^2 ($p^2 \to M_1^2,\ p'^2 \to M_2^2$) in order to suppress the contributions of the higher states and continuum, the QCD sum rules for the combinations of the form factors $f_V^{A(B)}$, $f_0^{A(B)}$ and $f_+^{A(B)}$ are obtained:

$$
\begin{aligned}
& \frac{f_{K_1^A} m_{K_1^A}}{(m_B + m_{K_1^A})} e^{-m_{K_1^A}^2/M_2^2} f_V^A(q^2) + \frac{f_{K_1^B\perp}\ a_0^{\parallel, K_1^B} m_{K_1^B}}{(m_B + m_{K_1^B})} e^{-m_{K_1^B}^2/M_2^2} f_V^B(q^2) \\
= & -\frac{(m_b + m_u)}{f_B m_B^2} e^{m_B^2/M_1^2} \Bigg\{ -\frac{1}{(2\pi)^2} \int_{(m_s+m_u)^2}^{s_0'} ds' \int_{(m_b+m_u)^2}^{s_0} ds\ \rho_V(s, s', q^2) \\
\times & \ \theta[1 - f^2(s, s')] e^{-s/M_1^2 - s'/M_2^2} + \hat{B}(\Pi_V^{non-pert}) \Bigg\},
\end{aligned}
$$

$$
\frac{f_{K_1^A}\ a_0^{\perp, K_1^A}}{(m_B + m_{K_1^A})} e^{-m_{K_1^A}^2/M_2^2} f_V^A(q^2) + \frac{f_{K_1^B\perp}}{(m_B + m_{K_1^B})} e^{-m_{K_1^B}^2/M_2^2} f_V^B(q^2)
$$

$$= -\frac{(m_b+m_u)}{f_B m_B^2} e^{m_B^2/M_1^2} \Bigg\{ -\frac{1}{(2\pi)^2} \int_{(m_s+m_u)^2}^{s_0'} ds' \int_{(m_b+m_u)^2}^{s_0} ds \; \varrho_V(s,s',q^2)$$
$$\times \; \theta[1-f^2(s,s')] e^{-s/M_1^2 - s'/M_2^2} + \hat{B}(T_V^{non-pert}) \Bigg\}, \tag{24}$$

$$f_{K_1^A} m_{K_1^A} (m_B + m_{K_1^A}) e^{-m_{K_1^A}^2/M_2^2} f_0^A(q^2) + f_{K_1^{B\perp}} \; a_0^{\parallel, K_1^B} m_{K_1^B} (m_B + m_{K_1^B}) e^{-m_{K_1^B}^2/M_2^2} f_0^B(q^2)$$
$$= \frac{(m_b+m_u)}{f_B m_B^2} e^{m_B^2/M_1^2} \Bigg\{ -\frac{1}{(2\pi)^2} \int_{(m_s+m_u)^2}^{s_0'} ds' \int_{(m_b+m_u)^2}^{s_0} ds \; \rho_0(s,s',q^2)$$
$$\times \; \theta[1-f^2(s,s')] e^{-s/M_1^2 - s'/M_2^2} + \hat{B}(\Pi_0^{non-pert}) \Bigg\},$$

$$-f_{K_1^A} \; a_0^{\perp, K_1^A} (m_B + m_{K_1^A}) e^{-m_{K_1^A}^2/M_2^2} f_0^A(q^2) - f_{K_1^{B\perp}} (m_B + m_{K_1^B}) e^{-m_{K_1^B}^2/M_2^2} f_0^B(q^2)$$
$$= \frac{(m_b+m_u)}{f_B m_B^2} e^{m_B^2/M_1^2} \Bigg\{ -\frac{1}{(2\pi)^2} \int_{(m_s+m_u)^2}^{s_0'} ds' \int_{(m_b+m_u)^2}^{s_0} ds \; \varrho_0(s,s',q^2)$$
$$\times \; \theta[1-f^2(s,s')] e^{-s/M_1^2 - s'/M_2^2} + \hat{B}(T_0^{non-pert}) \Bigg\}, \tag{25}$$

$$-\frac{f_{K_1^A} m_{K_1^A}}{(m_B + m_{K_1^A})} e^{-m_{K_1^A}^2/M_2^2} f_+^A(q^2) - \frac{f_{K_1^{B\perp}} \; a_0^{\parallel, K_1^B} m_{K_1^B}}{(m_B + m_{K_1^B})} e^{-m_{K_1^B}^2/M_2^2} f_+^B(q^2)$$
$$= \frac{(m_b+m_u)}{f_B m_B^2} e^{m_B^2/M_1^2} \Bigg\{ -\frac{1}{(2\pi)^2} \int_{(m_s+m_u)^2}^{s_0'} ds' \int_{(m_b+m_u)^2}^{s_0} ds \; \rho_+(s,s',q^2)$$
$$\times \; \theta[1-f^2(s,s')] e^{-s/M_1^2 - s'/M_2^2} + \hat{B}(\Pi_+^{non-pert}) \Bigg\},$$

$$\frac{f_{K_1^A} \; a_0^{\perp, K_1^A}}{(m_B + m_{K_1^A})} e^{-m_{K_1^A}^2/M_2^2} f_+^A(q^2) + \frac{f_{K_1^{B\perp}}}{(m_B + m_{K_1^B})} e^{-m_{K_1^B}^2/M_2^2} f_+^B(q^2)$$
$$= \frac{(m_b+m_u)}{f_B m_B^2} e^{m_B^2/M_1^2} \Bigg\{ -\frac{1}{(2\pi)^2} \int_{(m_s+m_u)^2}^{s_0'} ds' \int_{(m_b+m_u)^2}^{s_0} ds \; \varrho_+(s,s',q^2)$$
$$\times \; \theta[1-f^2(s,s')] e^{-s/M_1^2 - s'/M_2^2} + \hat{B}(T_+^{non-pert}) \Bigg\}. \tag{26}$$

In each set of the above equations, we have two equations with two unknowns (form factors). To obtain the form factors $f_V^A(q^2)$, $f_V^B(q^2)$, $f_0^A(q^2)$, $f_0^B(q^2)$, $f_+^A(q^2)$ and $f_+^B(q^2)$, we solve each set simultaneously. Finally, we obtain the sum rules as following:

$$
\begin{aligned}
f_V^A(q^2) &= -\frac{1}{12 f_B f_{K_1^A} m_B^2 (m_{K_1^A} - a_0^{\perp,K_1^A} a_0^{\parallel,K_1^B} m_{K_1^B})\pi^2} \\
&\times \Bigg\{ e^{m_{K_1^A}^2/M_2^2 + m_B^2/M_1^2} (m_B + m_{K_1^A})(m_b + m_u) \Bigg[4\pi^2 \hat{B}(\Pi_V^{non-pert}) \\
&+ 3 a_0^{\parallel,K_1^B} m_{K_1^B} \int_{(m_s+m_u)^2}^{s_0'} ds' \int_{(m_b+m_u)^2}^{s_0} ds\ \varrho_V(s,s',q^2)\theta[1-f^2(s,s')] e^{-s/M_1^2 - s'/M_2^2} \\
&- 3 \int_{(m_s+m_u)^2}^{s_0'} ds' \int_{(m_b+m_u)^2}^{s_0} ds\ \rho_V(s,s',q^2)\theta[1-f^2(s,s')] e^{-s/M_1^2 - s'/M_2^2} \Bigg] \Bigg\},
\end{aligned} \tag{27}
$$

$$
\begin{aligned}
f_V^B(q^2) &= \frac{1}{12 f_B f_{K_1^{B\perp}} m_B^2 (m_{K_1^A} - a_0^{\perp,K_1^A} a_0^{\parallel,K_1^B} m_{K_1^B})\pi^2} \\
&\times \Bigg\{ e^{m_{K_1^A}^2/M_2^2 + m_B^2/M_1^2} (m_B + m_{K_1^B})(m_b + m_u) \Bigg[4\pi^2 a_0^{\perp,K_1^A} \hat{B}(\Pi_V^{non-pert}) \\
&+ 3 m_{K_1^A} \int_{(m_s+m_u)^2}^{s_0'} ds' \int_{(m_b+m_u)^2}^{s_0} ds\ \varrho_V(s,s',q^2)\theta[1-f^2(s,s')] e^{-s/M_1^2 - s'/M_2^2} \\
&- 3 a_0^{\perp,K_1^A} \int_{(m_s+m_u)^2}^{s_0'} ds' \int_{(m_b+m_u)^2}^{s_0} ds\ \rho_V(s,s',q^2)\theta[1-f^2(s,s')] e^{-s/M_1^2 - s'/M_2^2} \Bigg] \Bigg\},
\end{aligned} \tag{28}
$$

$$
\begin{aligned}
f_0^A(q^2) &= \frac{1}{12 f_B f_{K_1^A} m_B^2 (m_B + m_{K_1^A})(m_{K_1^A} - a_0^{\perp,K_1^A} a_0^{\parallel,K_1^B} m_{K_1^B})\pi^2} \\
&\times \Bigg\{ e^{m_{K_1^A}^2/M_2^2 + m_B^2/M_1^2} (m_b + m_u) \Bigg[4\pi^2 \Bigg(\hat{B}(\Pi_0^{non-pert}) - a_0^{\parallel,K_1^B} m_{K_1^B} \hat{B}(T_0^{non-pert}) \Bigg)
\end{aligned}
$$

$$
\begin{aligned}
&- \quad 3\int_{(m_s+m_u)^2}^{s'_0} ds' \int_{(m_b+m_u)^2}^{s_0} ds \;\; \rho_0(s,s',q^2)\theta[1-f^2(s,s')]e^{-s/M_1^2-s'/M_2^2} \\
&- \quad 3a_0^{\parallel,K_1^B} m_{K_1^B} \int_{(m_s+m_u)^2}^{s'_0} ds' \int_{(m_b+m_u)^2}^{s_0} ds \;\; \varrho_0(s,s',q^2)\theta[1-f^2(s,s')]e^{-s/M_1^2-s'/M_2^2}\Bigg]\Bigg\},
\end{aligned} \tag{29}
$$

$$
\begin{aligned}
f_0^B(q^2) \quad &= \quad -\frac{1}{12 f_B f_{K_1^{B\perp}} m_B^2 (m_B + m_{K_1^B})(-m_{K_1^A} + a_0^{\perp,K_1^A} a_0^{\parallel,K_1^B} m_{K_1^B})\pi^2} \\
&\times \quad \Bigg\{ e^{m_{K_1^A}^2/M_2^2 + m_B^2/M_1^2}(m_b+m_u)\Bigg[4\pi^2\Bigg(- a_0^{\perp,K_1^A}\hat{B}(\Pi_+^{non-pert}) + m_{K_1^A}\hat{B}(T_+^{non-pert})\Bigg) \\
&+ \quad 3a_0^{\perp,K_1^A}\int_{(m_s+m_u)^2}^{s'_0} ds' \int_{(m_b+m_u)^2}^{s_0} ds \;\; \rho_0(s,s',q^2)\theta[1-f^2(s,s')]e^{-s/M_1^2-s'/M_2^2} \\
&+ \quad 3m_{K_1^A}\int_{(m_s+m_u)^2}^{s'_0} ds' \int_{(m_b+m_u)^2}^{s_0} ds \;\; \varrho_0(s,s',q^2)\theta[1-f^2(s,s')]e^{-s/M_1^2-s'/M_2^2}\Bigg]\Bigg\},
\end{aligned} \tag{30}
$$

$$
\begin{aligned}
f_+^A(q^2) \quad &= \quad \frac{1}{12 f_B f_{K_1^A} m_B^2 (m_{K_1^A} - a_0^{\perp,K_1^A} a_0^{\parallel,K_1^B} m_{K_1^B})\pi^2} \\
&\times \quad \Bigg\{ e^{m_{K_1^A}^2/M_2^2 + m_B^2/M_1^2}(m_B + m_{K_1^A})(m_b+m_u) \\
&\times \quad \Bigg[- 4\pi^2\Bigg(\hat{B}(\Pi_+^{non-pert}) - a_0^{\parallel,K_1^B} m_{K_1^B}\hat{B}(T_+^{non-pert})\Bigg) \\
&+ \quad 3\int_{(m_s+m_u)^2}^{s'_0} ds' \int_{(m_b+m_u)^2}^{s_0} ds \;\; \rho_+(s,s',q^2)\theta[1-f^2(s,s')]e^{-s/M_1^2-s'/M_2^2} \\
&+ \quad 3a_0^{\parallel,K_1^B} m_{K_1^B}\int_{(m_s+m_u)^2}^{s'_0} ds' \int_{(m_b+m_u)^2}^{s_0} ds \;\; \varrho_+(s,s',q^2)\theta[1-f^2(s,s')]e^{-s/M_1^2-s'/M_2^2}\Bigg]\Bigg\},
\end{aligned} \tag{31}
$$

$$
f_+^B(q^2) \quad = \quad -\frac{1}{12 f_B f_{K_1^{B\perp}} m_B^2 (m_{K_1^A} - a_0^{\perp,K_1^A} a_0^{\parallel,K_1^B} m_{K_1^B})\pi^2}
$$

$$
\begin{aligned}
&\times \quad \Bigg\{ e^{m^2_{K_1^A}/M_2^2 + m_B^2/M_1^2} (m_B + m_{K_1^B})(m_b + m_u) \\
&\times \quad \Bigg[4\pi^2 \Bigg(- a_0^{\perp, K_1^A} \hat{B}(\Pi_+^{non-pert}) + m_{K_1^A} \hat{B}(T_+^{non-pert}) \Bigg) \\
&+ \quad 3a_0^{\perp, K_1^A} \int_{(m_s+m_u)^2}^{s'_0} ds' \int_{(m_b+m_u)^2}^{s_0} ds \ \rho_+(s,s',q^2)\theta[1 - f^2(s,s')] e^{-s/M_1^2 - s'/M_2^2} \\
&+ \quad 3m_{K_1^A} \int_{(m_s+m_u)^2}^{s'_0} ds' \int_{(m_b+m_u)^2}^{s_0} ds \ \varrho_+(s,s',q^2)\theta[1 - f^2(s,s')] e^{-s/M_1^2 - s'/M_2^2} \Bigg] \Bigg\}.
\end{aligned}
\tag{32}
$$

In the above equation, in order to subtract the contributions of the higher states and the continuum, the quark-hadron duality assumption is used, i.e.,

$$
\rho^{higherstates}(s,s') = \rho^{OPE}(s,s')\theta(s - s_0)\theta(s' - s'_0). \tag{33}
$$

At the end of this section, we would like to calculate the longitudinal and transverse component of the differential decay width in terms of the form factors of $K_1(1270)$ and $K_1(1400)$ (see Eq. (7) for the relation between the $K_1^{A(B)}$ and $K_1(1270, 1400)$ form factors). After some calculations, we obtain the longitudinal and transverse components of the differential decay width as

$$
\begin{aligned}
\frac{d\Gamma_L}{dq^2} &= \frac{3G_F^2 \alpha_{em}^2 \mid \overrightarrow{p'} \mid}{192\sqrt{2}\pi^5} \mid V_{tb}V_{ts}^* \mid^2 \mid C_{10} \mid^2 \Bigg\{ \frac{\mid \overrightarrow{p'} \mid^2 (m_{K_1}^2 - m_B^2 + q^2)}{m_{K_1}^2} Re[f_0 f_+] \\
&+ \quad \frac{(m_B + m_{K_1})^2 (m_B^2 \mid \overrightarrow{p'} \mid^2 + m_{K_1}^2 q^2)}{2m_B^2 m_{K_1}^2} \mid f_0 \mid^2 \\
&+ \quad \frac{1}{8m_B^2 m_{K_1}^2 (m_B + m_{K_1})^2} \Bigg[m_{K_1}^2 q^6 \mid f_V \mid^2 \\
&+ \quad 4m_B^2((m_B - m_{K_1})^2 - q^2)((m_B + m_{K_1})^2 - q^2) \mid \overrightarrow{p'} \mid^2 \mid f_+ \mid^2 \Bigg] \\
&+ \quad \frac{1}{8m_B^2 (m_B + m_{K_1})^2} \Bigg[m_B^4 + m_{K_1}^4 - 2m_{K_1}^2 q^2
\end{aligned}
$$

$$- \quad 2m_B^2(m_{K_1}^2 + 2 \mid \overrightarrow{p'} \mid^2 + q^2)\Big] \mid f_V \mid^2 q^2 \Big\}, \tag{34}$$

$$\begin{aligned} \frac{d\Gamma_T}{dq^2} &= \frac{3G_F^2 \alpha_{em}^2 \mid \overrightarrow{p'} \mid}{192\sqrt{2}\pi^5} \mid V_{tb}V_{ts}^* \mid^2 \mid C_{10} \mid^2 q^2 \Big\{ \frac{(m_B + m_{K_1})^2}{2m_B^2} \mid f_0 \mid^2 \\ &+ \frac{((m_B - m_{K_1})^2 - q^2)((m_B + m_{K_1})^2 - q^2)}{8m_B^2(m_B + m_{K_1})^2} \mid f_V \mid^2 \Big\}. \end{aligned} \tag{35}$$

The total decay width and the asymmetry parameter α, characterizing the polarization of the K_1 meson are define as:

$$\begin{aligned} \frac{d\Gamma_{tot}}{dq^2} &= \frac{d\Gamma_L}{dq^2} + 2\frac{d\Gamma_T}{dq^2}, \\ \alpha &= 2\frac{d\Gamma_L}{dq^2} / \frac{d\Gamma_T}{dq^2} - 1. \end{aligned} \tag{36}$$

3 Numerical analysis

In this section, we present our numerical analysis of the form factors f_V , f_0 and f_+, longitudinal, transverse and total decay width, branching ratio and the asymmetry parameter α, characterizing the polarization of the K_1 meson. The sum rules expressions for the form factors and also the expression for the decay widths depict that the main input parameters entering the expressions are the Wilson coefficient C_{10}, elements of the CKM matrix V_{tb} and V_{ts}^*, the leptonic decay constants; f_B and $f_{K_1^{A(B\perp)}}$, the Borel parameters M_1^2 and M_2^2, as well as the continuum thresholds s_0 and s_0'. In further numerical analysis, we choose the values of the leptonic decay constants, the CKM matrix elements, the Wilson coefficient C_{10}, the quark and meson masses as: $C_{10} = -4.669$ [13, 14], $\mid V_{tb} \mid = 0.77^{+0.18}_{-0.24}$, $\mid V_{ts} \mid = (40.6 \pm 2.7) \times 10^{-3}$ [15], $f_{K_1^A} = (250 \pm 13)\ MeV$, $f_{K_1^{B\perp}} = (190 \pm 10)\ MeV$ [3, 8], $f_B = 0.14 \pm 0.01\ GeV$,

[16], $m_c = 1.25 \pm 0.09\ GeV$, $m_s = 95 \pm 25\ MeV$, $m_b = (4.7 \pm 0.07)\ GeV$, $m_d = (3-7)\ MeV$, $m_B = 5.279\ GeV$, $m_{K_1}(1270) = 1.27\ GeV$, $m_{K_1}(1400) = 1.40\ GeV$[17], $m_{K_1^A} = (1.31 \pm 0.06)\ GeV$, $m_{K_1^B} = (1.34 \pm 0.08)\ MeV$ [3, 8, 18]. The zeroth order Gegenbaurer moments are taken to be $a_0^{\parallel, K_1^B}(1\ GeV) = -0.19 \pm 0.07$ and $a_0^{\perp, K_1^A}(1\ GeV) = 0.27^{+0.03}_{-0.17}$ [18].

The expressions for the form factors contain also four auxiliary parameters: Continuum thresholds s_0 and s'_0 and Borel mass squares M_1^2 and M_2^2. These are not physical quantities, hence the physical quantities, form factors, should be independent of them. The continuum thresholds s_0 and s'_0 in the B and K_1 channels are determined from the conditions that guarantee the sum rules to have the best stability in the allowed M_1^2 and M_2^2 region. The values of continuum thresholds calculated from the two–point QCD sum rules are taken to be $s_0 = (35 \pm 5)\ GeV^2$ and $s'_0 = (4 \pm 1)\ GeV^2$. The working regions for M_1^2 and M_2^2 are determined by requiring that not only contributions of the higher states and continuum are effectively suppressed, but the contributions of the higher dimensional operators are small. Both conditions are satisfied in the regions $10\ GeV^2 \leq M_1^2 \leq 22\ GeV^2$ and $3\ GeV^2 \leq M_2^2 \leq 8\ GeV^2$. The value of the form factors at $q^2 = 0$ are given in Table 1.

	$f_V(0)$	$f_0(0)$	$f_+(0)$
$B \to K_1(1270)\nu\overline{\nu}$	0.57 ± 0.21	0.24 ± 0.10	0.39 ± 0.14
$B \to K_1(1400)\nu\overline{\nu}$	0.40 ± 0.15	0.17 ± 0.07	0.29 ± 0.10

Table 1: The value of the form factors at $q^2 = 0$, $M_1^2 = 15\ GeV^2$, $M_2^2 = 4\ GeV^2$.

The sum rules expressions for the form factors are truncated at $12\ GeV^2$, about $4\ GeV^2$ below the upper limit of the q^2 which is about 16 (15) GeV^2 for $K_1(1270)$ ($K_1(1400)$). In order to extend our results to the whole physical

	$f_V^{K_1(1270)}$	$f_0^{K_1(1270)}$	$f_+^{K_1(1270)}$	$f_V^{K_1(1400)}$	$f_0^{K_1(1400)}$	$f_+^{K_1(1400)}$
a	-5.70	-3.05	-4.05	-4.05	-0.18	-0.48
b	6.28	3.29	4.43	4.43	0.35	0.75
m_{fit}	6.99	9.41	6.64	6.63	6.47	6.16

Table 2: Parameters appearing in the fit function for form factors of the $B \to K_1(1270, 1400)\overline{\nu}\nu$ at $M_1^2 = 15\ GeV^2$, $M_2^2 = 4\ GeV^2$.

region, i.e., $0 \leq q^2 < (m_B - m_{K_1})^2$ and for the reliability of the sum rules in the full physical region, we look for a fit parametrization such that in the region $0 \leq q^2 \leq 7\ GeV^2$, these parameterizations coincide with the sum rules predictions. To find the extrapolation of the form factors, we choose the following fit function

$$f_i(q^2) = \frac{a}{(1 - \frac{q^2}{m_{fit}^2})} + \frac{b}{(1 - \frac{q^2}{m_{fit}^2})^2}. \tag{37}$$

The values for a, b and m_{fit} are given in Table 2.

Having the q^2 dependent expressions for the form factors, in the following part, we present the evaluation of the numerical values for the longitudinal and transverse components of the differential decay width, branching ratio and the asymmetry parameter of K_1 meson. Figures 1 and 2 depict the dependency of the $d\Gamma_L/dq^2$ and $d\Gamma_T/dq^2$ on $x = q^2/m_B^2$ in the allowed physical region for x, i.e., $0 < x < 0.57$ for $K_1(1270, 1400)$. From these figures, we see that the longitudinal and transverse components of the decay rate for $K_1(1270)$ is about four times greater than that of the $K_1(1400)$. The maximum values for the $d\Gamma_L/dq^2$ is at $x = 0.48$ and $x = 0.52$ for $K_1(1400)$ and $K_1(1270)$, respectively. However, the peak values for these states are at $x = 0.43$ and $x = 0.46$ for the $d\Gamma_T/dq^2$ case. These figures also show that the transverse component of the differential decay rates have nearly the same behavior at low values of the q^2, but in this region, the $d\Gamma_L/dq^2$ for $K_1(1270)$ is zero and has almost nonzero value for $K_1(1400)$. Finally, we denote the dependency of the asymmetry parameter α, characterizing the polarization of the K_1 meson in terms of the x. This figure depict that for both $K_1(1270)$ and $K_1(1400)$ cases, the α varies in the interval $-1 < \alpha < 1$ in the allowed region of the q^2. For $K_1(1270)$, the asymmetry parameter is unity with negative sign in the interval $0 < x < 0.3$. This implies that, in this region, the longitudinal component of the differential decay rate is zero and this is in agreement with Fig. 1. After $x = 0.3$, the $d\Gamma_L/dq^2$ approaches to zero and changes sign at $x = 0.5$. In this point, the longitudinal component of the decay rate is near to its maximum and half of the transverse component. The α for $K_1(1270)$ has positive sign in the region $0.50 < x < 0.57$. The asymmetry parameter for $K_1(1400)$ changes sign at $x = 0.4$ and has a minimum at about $x = 0.1$. Any measurement on the asymmetry parameter and determination of its sign can give valuable information about the structures of the $\mid K_1^A >$ and $\mid K_1^B >$ states.

$\boldsymbol{B}(B \to K_1(1270)\nu\overline{\nu})$	$\boldsymbol{B}(B \to K_1(1400)\nu\overline{\nu})$
$(3.54 \pm 1.27) \times 10^{-6}$	$(8.92 \pm 3.21) \times 10^{-7}$

Table 3: Values for the branching ratio of $B \to K_1\overline{\nu}\nu$.

Taking into account the q^2 dependency of the form factors and performing integration over q^2 for $d\Gamma_{tot}/dq^2$ (in Eq. 36) in the interval $0 \leq q^2 \leq (m_B - m_{K_1})^2$ and using the total life-time $\tau_B = 1.638 \times 10^{-12}s$ [17], the branching ratio for $B \to K_1(1270, 1400)\nu\overline{\nu}$ is obtained as Table 3. This Table shows that the branching fraction for $K_1(1270)$ about four times greater than that of the $K_1(1400)$ states.

In conclusion, separating the mixture of the $K_1(1270)$ and $K_1(1400)$

states, the form factors related to the $B \to K_1(1270, 1400)\overline{\nu}\nu$ decay were calculated using three-point QCD sum rules approach. Taking into account the q^2 dependencies of the form factors, the longitudinal and transverse component of the differential decay width as well as the asymmetry parameter α, characterizing the polarization of the K_1 meson and the branching ratio of these transitions were evaluated.

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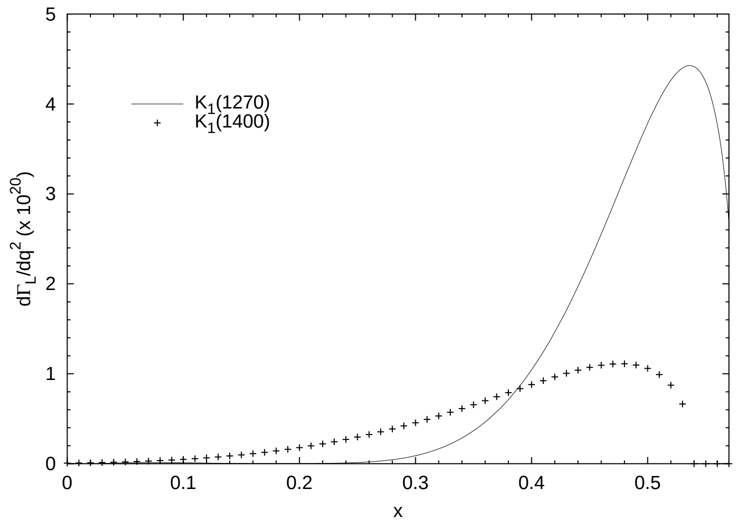

Figure 1: The dependence of the $d\Gamma_L/dq^2$ on $x = q^2/m_B^2$

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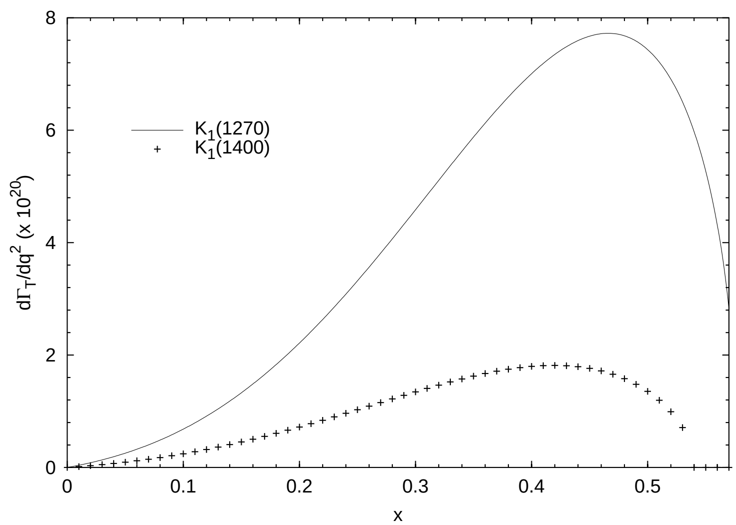

Figure 2: The dependence of the $d\Gamma_T/dq^2$ on $x = q^2/m_B^2$

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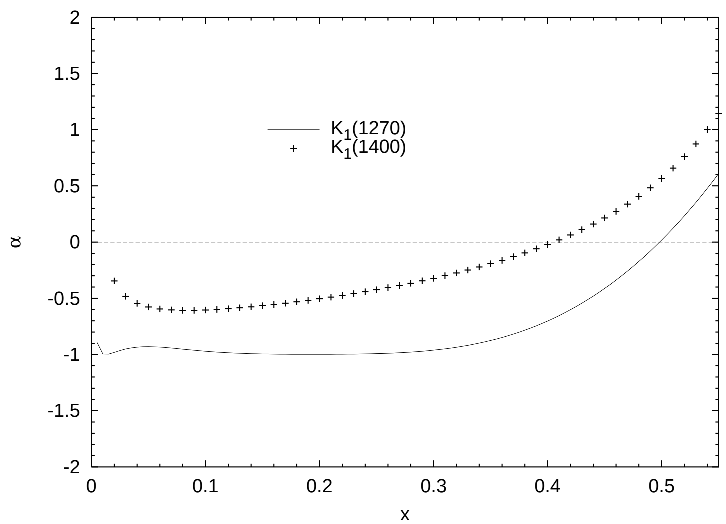

Figure 3: The dependence of the asymmetry parameter α on $x = q^2/m_B^2$.