# A SPINOR MODEL FOR QUANTUM COSMOLOGY

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#### ABSTRACT

The question of the interpretation of Wheeler-DeWitt solutions in the context of cosmological models is addressed by implementing the Hamiltonian constraint as a spinor wave equation in minisuperspace. We offer a relative probability interpretation based on a non-closed vector current in this space and a prescription for a parametrisation of classical solutions in terms of classical time. Such a prescription can accommodate classically degenerate metrics describing manifolds with signature change. The relative probability density, defined in terms of a Killing vector of the Dewitt metric on minisuperspace, should permit one to identify classical loci corresponding to geometries for a classical manifold. This interpretation is illustrated in the context of a quantum cosmology model for two-dimensional dilaton gravity.

#### 1. Introduction

One of the many outstanding problems in trying to construct a quantum field theory of gravitation concerns the appropriate interpretation of quantum states for configurations that make no overt reference to "time". Thus it is difficult in general to endow the theory with any traditional Hilbert space structure based on a hermitian inner product and a unitary evolution. Although many alternative schemes have been suggested difficulties in interpretation remain. Some of the difficulties are intrinsic to the infinite dimensional aspect of field quantisation and in this respect one often seeks guidance by studying truncated field configurations corresponding to situations with high symmetry. We shall not rehearse here the many cogent arguments that urge caution in extending deductions from such models to the full quantum field theory [1]. However symmetric models are a useful theoretical laboratory for testing ideas that may have more general validity, and enable one to disentangle conceptual problems from technical ones. In the context of minisuperspace models a number of authors have noticed that it is possible to implement the Hamiltonian constraint for Bianchi-type models in general relativity, on a multicomponent wavefunction [2]. In an attempt to relate such states to modes of the gravitino field such models have been examined in the context of N=1 supergravity [3] although the relation with the original Wheeler-Dewitt equation is then lost [4]. In this letter we focus on a particular minisuperspace analysis that gives rise to a Hamiltonian constraint, classically describing the zero energy configuration of an oscillator ghost-oscillator pair. This gives rise to a Wheeler-DeWitt equation that has occurred in a number of different contexts. It appears in certain 4-dimensional spacetime cosmologies [5], [6], [7], [8], and we have discussed it in the context of a class of 2-dimensional dilaton-gravity models. These models have arisen either from string-inspired limits or from the suppression of inhomogeneous modes in Einstein's theory of general relativity, [9], [10], [11], [12], [13],

[14] [15]. Our interest with this class of models stems from the properties of coherent state solutions to the corresponding Wheeler-DeWitt equation and their relation to classical solutions to general relativity including those that change signature [16]. We show in this letter that it is possible to implement the Hamiltonian constraint in the cosmological sector as a first order wave equation for a multicomponent state vector and to endow the space of spinor solutions to such an equation with a Hilbert space structure. We offer an interpretation of such states in terms of relative probabilities defined by a non-conserved current. This is possible since such solutions enable one to construct such a current with a positive definite density defined by the Killing isometry of the minisuperspace metric. We suggest that such an interpretation is not unnatural in a quantum theory that attempts to accommodate states whose classical limits describe manifolds with degenerate metrics where the signature can change. Such limits correspond to more exotic spaces in which the global topology may be non-trivial and the geometry non-Riemannian. In such situations the emergence of an arrow of classical time may have its origin in an underlying quantum description of such spaces.

#### 2. The Model

We have recently developed [16] a canonical quantisation of the 2-dimensional dilatongravity theory based on the classical action

$$S[g,\psi] = \int_{N} \left\{ \frac{1}{2} \psi \star \mathcal{R} + cd\psi \wedge \star d\psi + \star (\Lambda_{0} + \alpha e^{c\psi}) \right\}$$
(1)

where N is some domain of a two-dimensional manifold,  $\psi$  is a real scalar field and  $\mathcal{R}$  is the curvature scalar of the Levi-Civita connection associated with the metric tensor g. The operator  $\star$  denotes the Hodge map of g and c,  $\Lambda_0$  and  $\alpha$  are constants. The classical cosmological sector of this theory can be solved exactly and admits solutions with a degenerate metric where the signature changes from being Lorentzian to Euclidean. The standard approach for implementing the Hamiltonian constraint in the quantum version of such theories is to search for complex scalar valued functions on the appropriate manifold of matter and space geometry configurations. Thus in [16] we took  $\mathbf{R}^2$  as a minisuperspace with global coordinates  $\{X, Y\}$  labeling these configurations and sought Wheeler-DeWitt solutions  $\Psi : \mathbf{R}^2 \mapsto \mathbf{C}$  to the equation:

$$H\Psi = 0 \tag{2}$$

where

$$H = (\omega^2 X^2 - \frac{\partial_X^2}{4}) - (\omega^2 Y^2 - \frac{\partial_Y^2}{4})$$
(3)

The Wheeler-DeWitt equation (2) endows  $\mathbf{R}^2$  with a natural (Lorentzian signature) metric  $\mathcal{G}$ . In terms of the coordinates  $\{X, Y\}$ :

$$\mathcal{G} = \partial_X \otimes \partial_X - \partial_Y \otimes \partial_Y. \tag{4}$$

If # denotes the associated Hodge map then (2) may be written:

$$d\#\,d\Psi - W\#\,\Psi = 0\tag{5}$$

where  $W(X, Y) = 4\omega^2(X^2 - Y^2)$ . By multiplying (5) by  $\overline{\Psi}$  and subtracting from the corresponding equation obtained by complex conjugation we readily verify that

$$d\mathcal{J} = 0 \tag{6}$$

where the current 1-form

$$\mathcal{J} = Im(\overline{\Psi} \# d\Psi). \tag{7}$$

Although this current is conserved there is no preferred spacelike foliation of  $\mathbb{R}^2$  that defines a "density" component of  $\mathcal{J}$  that does not in general change sign. Furthermore, although the minisuperspace is flat there is no natural way to restrict solutions to have a positive definite norm. The Killing vectors of the metric (4) do not generate a symmetry of the

equation (5). Thus there appears no invariant way to normalise solutions of (5), construct a Hilbert subspace of normalisable solutions and endow the quantum theory with the standard probabilistic interpretation. By contrast the traditional Klein-Gordon quantisation of the relativistic free particle in Minkowski spacetime exists because the Killing isometry of the spacetime metric induces a classical symmetry of the Klein-Gordon equation. Furthermore one can then exploit translational symmetry to restrict solutions to either the positive or negative mass-hyperboloid in the space of spatial Fourier modes on which the above current induces a positive-definite inner-product. However if one considers the quantisation in a non-stationary spacetime (or in a stationary spacetime with a time dependent potential) again one may loose the timelike Killing symmetries that enable one to effect the above construction and the particle interpretation of the field system is at best an asymptotic notion in a second quantised formulation. Before the advent of second quantisation Dirac was motivated to implement the constraint arising from the reparametrisation of the action for a relativistic free particle by a first order equation for the space of quantum states. In a similar vein we are interested here in the possibility of implementing the constraint (2) by a first order equation for a complex multicomponent field  $\Phi : \mathbf{R}^2 \mapsto \mathbf{C}^n$  for some **n**, such that each component of  $\Phi$  satisfies (2). If this is possible it is natural to seek for a current constructed from  $\Phi$  that admits a positive-definite charge density for some class of foliations that are spacelike with reference to the metric (4). If this is possible then the choice of a probability interpretation is determined by the Lorentzian structure of (mini-)superspace. (This Lorentzian structure has its origins in the universal gravitational attraction between matter and must be clearly distinguished from the light-cone structure of classical spacetime). Such a choice seems natural if we wish to extend the interpretation of the theory to accommodate classical limits that include spacetime metrics with a signature transition. In a domain with Euclidean signature one has no natural means of defining the spacelike and timelike components of a current. Thus the definition of a probability current must transcend any definition of any preferred classical time for classical spacetimes. We shall reiterate our views on the latter problem in the last section.

#### 3. The Clifford Algebra of Minisuperspace

The natural Lorentzian null-cone structure in 2D minisuperspace endows the space with a (1,1) Clifford bundle structure [17]. Thus there is a matrix basis for the Clifford Algebra Cl(1,1) in which the 1-forms dX and dY are represented as matrices satisfying

$$dX \lor dX = 1 \tag{8}$$

$$dY \lor dY = -1 \tag{9}$$

$$dX \vee dY + dY \vee dX = 0 \tag{10}$$

where  $\vee$  denotes multiplication in the Clifford algebra. In conventional gamma matrix notation:  $(dX \mapsto \gamma^1, dY \mapsto \gamma^0)$ . The Clifford bundle has minisuperspace as base and Cl(1,1) as fibre. Let  $\Phi$  be a section of this bundle:

$$\Phi = \Phi_0 + \Phi_1 \, dX + \Phi_2 \, dY + \Phi_{12} \, dX \lor dY. \tag{11}$$

Since the bundle is trivial the components  $\Phi_j \equiv \{\Phi_0, \Phi_1, \Phi_2, \Phi_{12}\}$  may be regarded as complex functions on  $\mathbb{R}^2$ .

Introduce the Clifford potentials:

$$V_1 = 2i\omega(-Y + X\,dX \lor dY) \tag{12}$$

$$V_2 = 2i\omega(Y + X\,dX \lor dY). \tag{13}$$

We assert that if  $\Phi$  satisfies first order equation:

$$\mathcal{D}\Phi + V_1 \lor \Phi = 0 \tag{14}$$

where  $\mathcal{D} = (d - \delta)$  then each component of  $\Phi$  will satisfy (2). In this equation d denotes exterior differentiation and  $\delta$  is the coderivative:

$$\delta = \#^{-1}d \# \eta$$

where the involution  $\eta$  [17] is a linear operator on  $\Phi$  that preserves 0-forms, reverses the sign of 1-forms and

$$\eta(dX \lor dY) = dX \lor dY. \tag{15}$$

The above result follows from

$$(\mathcal{D} + V_2) \lor (\mathcal{D} + V_1) = (\mathcal{D}^2 - W) \tag{16}$$

and the recognition that  $\mathcal{D}^2 = -(d \,\delta + \delta \,d)$  is the Laplace-Beltrami operator. Since the basis forms in  $\Phi$  are all holonomic it follows that if  $\Phi$  satisfies (14) then its components satisfy

$$d\# \, d\Phi_j - W \# \, \Phi_j = 0. \tag{17}$$

#### 4. Spinor Solutions and Associated Currents

We observe that since our minisuperspace is flat with respect to (4) it is possible to find solutions  $\Psi$  of (14) that lie in a minimal (left) ideal of Cl(1,1) at each point. For example we may decompose

$$\Phi = \Phi \lor P_+ + \Phi \lor P_- \tag{18}$$

where  $P_{\pm} = \frac{1}{2}(1 \pm dX)$  and take  $\Psi = \Phi \vee P_{+}$ . Minimal ideals provide irreducible modules for the sub-group SPIN of the Clifford group of Cl(1,1) [17] and their elements are spinors. Since  $P_{+}$  is a parallel idempotent in minisuperspace, if  $\Phi$  is a solution of (14) then so is  $\Phi \vee P_{+}$  so  $\Psi$  may be regarded as a spinor solution of (14).

We concentrate on those spinor solutions of (14) that are asymptotically well behaved as |X| or |Y| tends to infinity. Such solutions may be expressed in terms of a basis of Hermite

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functions. Thus if

$$\Psi = (u - vdY) \lor P_+ \tag{19}$$

(14) may be expressed as the coupled partial differential equations:

$$\partial_{\xi}F - 2i\omega\xi H = 0 \tag{20}$$

$$\partial_{\eta}H + 2i\omega\eta F = 0 \tag{21}$$

where  $\xi = (Y - X)/\sqrt{(2)} \eta = (Y + X)/\sqrt{(2)}$ , F = u - v, H = u + v. These have the solutions

$$F(X,Y) = \sum_{n=0}^{\infty} c_n e^{-(z_1^2 + z_2^2)/2} H_n(z_1) H_n(z_2)$$
(22)

$$H(X,Y) = \sum_{n=0}^{\infty} b_n e^{-(z_1^2 + z_2^2)/2} H_n(z_1) H_n(z_2)$$
(23)

where  $z_1 = \sqrt{(2\omega)}X$  and  $z_2 = \sqrt{(2\omega)}Y$ . The complex coefficients  $\{c_n\}$  and  $\{b_n\}$  are linearly correlated by the wave equation (14). A typical " coherent spinor state " solution takes the form:

$$F = Ce^{-(\alpha(X^2 + Y^2) - 2\beta XY)}$$
(24)

$$H = \frac{i}{\omega} (\alpha + \beta) F \tag{25}$$

where  $\alpha, \beta, C$  are arbitrary complex constants. Since our 2-dimensional minisuperspace is topologically trivial it is always possible to find closed 1-forms that are candidates for conserved currents. However it is non-trivial to construct a current that has a positive definite density for a class of solutions to (14). It is not difficult to verify that the (complex) current

$$\mathcal{J}' = H^2 \xi \, d\xi - F^2 \eta \, d\eta \tag{26}$$

is closed and hence gives rise to a conserved current in mini-superspace. However for general solutions of (14) there is no foliation of mini-superspace that enables one to construct a non-negative real density from such a conserved current.

We recall that in the Dirac theory of a relativistic particle described by a spinor  $\psi$  on spacetime, the Dirac vector current with components  $\overline{\psi}\gamma^{\mu}\psi$  is conserved and possesses a positive-definite density for any non-trivial spinor. In the language of Clifford bundles this current is the form:

$$j[\psi] = *ReS_1(\psi \lor \tilde{\psi}) \tag{27}$$

where  $S_1$  projects out the 1-form part of its argument and  $\tilde{\psi} = C^{-1} \vee \psi^{\mathcal{I}}$  for some involution  $\mathcal{I}$  in the (simple) Clifford algebra that is equivalent to hermitian conjugation:

$$A^{\dagger} = C^{-1} A^{\mathcal{I}} C \tag{28}$$

for all elements A in the Clifford algebra.

With this goal in mind we find from (14) and (27)

$$dj[\Psi] = Re \, Tr(\tilde{\Psi} \lor (V_2 - V_1) \lor \Psi) \lor \#1 \tag{29}$$

for all spinor solutions  $\Psi$  of (14) where

$$j[\Psi] = \#ReS_1(\Psi \lor \tilde{\Psi}). \tag{30}$$

Here transposition is induced by the involution  $\mathcal{I} \equiv \eta \xi$  where  $\xi$  is the main anti-involution [17] of the Clifford algebra and the adjoint spinor  $\tilde{\Psi} = C^{-1} \vee \overline{\Psi}^{\xi\eta}$ , where  $\overline{\Psi}$  denotes the complex conjugation of  $\Psi$ . In the spinor basis defined by the projectors  $P_{\pm}$  that we are using, the element C = dY. It follows from (29) that  $j[\Psi]$  defines a closed current for solutions that satisfy the condition  $\operatorname{Re} Tr(i\omega\tilde{\Psi} \vee \Psi) = 0$ . Although such solutions do exist we shall not impose this restrictive condition in the following discussion.

#### 5. Discussion

For a spinor solution (19) (18) with components  $\{u, v\}$  we find in the chart  $\{X, Y\}$ :

$$j[\Psi] = uv \, dY - \left(\frac{1}{2}|u|^2 + \frac{1}{2}|v|^2\right) dX \tag{31}$$

which clearly displays the non-negativity of the density

$$\rho_K(X,Y) = -j[\Psi](K) \tag{32}$$

where K denotes the Killing vector field  $\partial_X$ . If  $\Psi$  carries a representation of SPIN, then such a density, defined by a spacelike Killing vector field will remain positive for all proper Lorentz transformations that preserve the metric. Thus it may be adopted as a probability measure for the interpretation of the theory. However as the notation indicates a choice of spacelike Killing vector K is implied. Since the current  $j[\Psi]$  is not conserved for all  $\Psi$  there exists no choice of spacelike foliation such that the integral of the probability density over a particular leaf of the foliation is independent of the leaf chosen. The existence of such a leaf dependent "charge" means that one cannot identify such a leaf as an "instant of time" and interpret such a charge as a normalisation factor for a state in the traditional manner. If we adopt as the Hilbert space norm of a state  $\Psi$ 

$$(\Psi,\Psi)_K = \int_{\mathbf{R}^2} \rho_K \# 1 \tag{33}$$

then this also will depend on the choice of Killing vector K. Inasmuch as any convenient norm can be used to define a Hilbert space this is not necessarily a drawback. However the probabilistic interpretation of the theory must be restricted to describing relative probabilities between configurations. Thus for  $K = \partial_X$  the relative probability densities for configuration  $Y_1$  and  $Y_2$  irrespective of X may be defined as  $\mu(Y_1)/\mu(Y_2)$  where:

$$\mu(Y) = \int_{-\infty}^{\infty} \rho_K(X, Y) \, dX \tag{34}$$

and in general  $\mu(Y)$  will depend on the configuration variable Y.

Up to this point no mention has been made of classical time. Indeed until this issue is resolved there is little to recommend any particular probabilistic interpretation of the theory since the obvious questions that need to be addressed involve classical observers in a classical spacetime. Thus we assert that in order to give substance to the above one should concentrate on particular quantum states that can be related to classical cosmologies. As we have stressed we do not wish to exclude from such classical cosmologies those that admit degenerate spacetime metrics. Thus we focus on those solutions that for a given choice of Killing vector K enable one to constructs functions  $\rho_K$  that have maxima in the vicinity of those loci in (mini-)superspace corresponding to parametrised solutions to the classical field equations. For a classical manifold with a proscribed topology and a proscribed signature structure we concentrate on particular classical solutions with degenerate metrics. Furthermore the manifold should enable one to perform a Hamiltonian description of the field equations so that the classical and quantum degrees of freedom can be put into correspondence [16]. In the cosmological context of the model in this discussion such a correspondence is given by a parametrised curve in mini-superspace:

$$\tau \mapsto \{X = X(\tau), Y = Y(\tau)\} \qquad \tau_0 < \tau < \tau_1 \tag{35}$$

Such a parametrisation of a classical solution may describe a Euclidean signature metric for part of the manifold and a Lorentzian signature metric elsewhere. Thus it is natural to use  $\tau$  as a choice of classical evolution parameter which is a classical time in the Lorentzian domain. We may now transfer the relative probability interpretation to the class of classical observers that inhabit the classical cosmology defined by the locus of the maxima of  $\rho_K$ . The density  $\rho_K(X(\tau), Y(\tau))$  now offers a means of predicting the relative probabilities for finding the classical configurations  $\{X, Y\}$  at "times"  $\tau_1$  and  $\tau_2$ . The freedom in choosing different parametrisations to describe the same classical solutions corresponds to the freedom in choosing different coordinate systems on the classical manifolds.

To illustrate the viability of this approach within the context of the model defined by

(2) we have sought a Killing vector K that enables one to construct a "coherent state" that can be used to construct a classical spacetime in the vicinity of its peak. Define  $\Psi_{s_1,s_2} = (u - vdY) \lor P_+$  with

$$u = s_1 e^{c_1 (X^2 + Y^2) + 2c_2 XY}$$
(36)

$$v = s_2 e^{c_1 (X^2 + Y^2) + 2c_2 XY} \tag{37}$$

for some complex constants  $c_1, c_2, s_1, s_2$ . This is a solution to (14) provided

$$c_1 = \frac{2i\omega s_1 s_2}{s_1^2 - s_2^2} \tag{38}$$

$$c_2 = \frac{i\omega(s_2^2 + s_1^2)}{s_2^2 - s_2^2}.$$
(39)

Then for suitable  $\{s_1, s_2, s_3, s_4\}$  the superposition

$$\Phi = \Psi_{s_1, s_2} - \Psi_{s_3, s_4} \tag{40}$$

enables one to construct a density  $\rho_{\partial_X}$  that peaks along classical loci for the theory defined by the action (1). This is illustrated in Fig 1 for the choice  $\{s_1 = (3+0.1i), s_2 = (1.3+0.1i), s_3 = (3.1+0.1i), s_4 = (1.4+0.1i)\}$ . The classical solutions correspond to the elliptical contours defined by this density profile and have been discussed in [16]. A notable feature of this state is the existence of a particular locus among the classical solutions for which the divergence of the vector current  $j[\Phi]$  does in fact vanish. In this sense one may say that there is approximate conservation in the vicinity of this particular classical configuration

Equation (29) is reminiscent of the equation that follows from the non-relativistic Schrödinger equation in the presence of a complex potential. Indeed the lack of hermiticity of the hamiltonian there is analogous to the property  $V_1 \neq \pm V_2^{\mathcal{I}}$ . In the Schrödinger situation the use of a complex potential models the absorptive properties of an open system. For a closed system a non-hermitian hamiltonian is usually regarded as pathological. However in the context of gravitation such a reaction requires caution [15]. For example if the non-unitary evolution of a pure state of matter to a mixed state via the Hawking process can be maintained when gravitational back reaction is taken into account then probability conservation in a gravitational context may not be tenable. It is clear from the behaviour of the state in Figure 1 why the conservation of our current is impossible. Since the state vanishes asymptotically in all directions in the configuration space there is no way that a flux of positive density from the peaks of the state can flow smoothly to zero. In such a scenario it is tempting to conjecture that it is the existence of degenerate classical geometries that are mandatory to accommodate the absorption of probability flux in the Euclidean domains. Just as the creation (and annihilation) of a classical cosmology may correspond to such domains where a classical spacetime description breaks down, the same may be true at the end points of localised gravitational collapse. Of course a cosmological model is insensitive to the subtleties required to accomodate a full quantisation of such a system. However it would be a novel approach to implement the untruncated canonical constraints in terms of a first order set of functional differential equations for a multicomponent state vector such that each component satisfies the traditional Wheeler-DeWitt equation.

#### 6. Conclusion

For our particular model we have chosen a symmetry vector of the DeWitt metric on superspace that enables one to construct, from a particular quantum state, a density that has maxima in the vicinity of classical cosmological loci. An internally consistent interpretation for such a density is provided in terms of relative probabilities of the occurrence of classical matter and a cosmological metric. In general the associated current is not closed on superspace although the divergence is zero in the vicinity of certain classical cosmologies. Whether the use of a non-conserved probability current has other implications for quantum cosmology will be pursued elsewhere.

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- 17 -Figure 1

 $|\Psi_{s_1,s_2} - \Psi_{s_3,s_4}|^2$  for the choice  $\{s_1 = (3+0.1i), s_2 = (1.3+0.1i), s_3 = (3.1+0.1i), s_4 = (1.4+0.1i)\}$ . The peak accentuates a classical solution for the theory defined by the action (1).

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