

THE LUKASH PLANE-WAVE ATTRACTOR AND RELATIVE ENERGY

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Abstract. We study energy distribution in the context of teleparallel theory of gravity, due to matter and fields including gravitation, of the universe based on the plane-wave Bianchi VII_δ spacetimes described by the Lukash metric. In order to make this calculation we consider the teleparallel gravity analogs of the energy-momentum formulations of Einstein, Bergmann-Thomson and Landau-Lifshitz. We find that Einstein and Bergmann-Thomson prescriptions agree with each other and give the same results for the energy distribution in a given spacetime, but the Landau-Lifshitz complex does not. Energy density turns out to be non-vanishing in all of these prescriptions. It is interesting to mention that the results can be reduced to the already available results for the Milne universe when we write $\omega = 1$ and $\Xi^2 = 1$ in the metric of the Lukash spacetime, and for this special case, we get the same relation among the energy-momentum formulations of Einstein, Bergmann-Thomson and Landau-Lifshitz as obtained for the Lukash spacetime. Furthermore, our results support the hypothesis by Cooperstock that the energy is confined to the region of non-vanishing energy-momentum tensor of matter and all non-gravitational fields, and also sustain the importance of the energy-momentum definitions in the evaluation of the energy distribution associated with a given spacetime.

Keywords: Lukash; plane-wave attractor; energy; teleparallel gravity.
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1. Introduction

There is and has long been an interest in the investigation of the spatially homogeneous Bianchi spacetimes and their cosmological applications to our understanding of singularities and of the observed level of isotropy in the universe. These discussions analyze the problems within the manageable domain of ordinary differential equations and provide only a finite number of alternative cosmologies [1]. The most general Bianchi universes which contain the open Friedmann spacetime as a special subcase are those of type VII_δ. The late-time asymptotes for the non-tilted type VII_δ spacetimes, with

$\delta \neq 0$ and a matter content that obeys the strong energy condition, evolve towards the vacuum plane-wave solution found by Doroshkevich et al. and Lukash [2, 3, 4] that is known as the Lukash spacetime. These metrics describe the most general effects of spatially homogeneous perturbations on open Friedmann universes [5, 6, 7, 8]. The Lukash spacetime plays a guiding role in the investigations mentioned above because of the subtle stability properties of isotropic expansion at late times in open universes. When the strong energy condition is obeyed, then isotropic expansion was found to be stable but not asymptotically stable at late times [6, 7, 8, 9].

Hence, it is very interesting to discuss the energy associated with this model of the universe. In this study to calculate energy in the expanding Lukash spacetime we focus on Einstein, Bergmann-Thomson and Landau-Lifshitz energy-momentum formulations in the teleparallel gravity. Since Einstein proposed the theory of general relativity, relativists have not been able to agree upon a definition of the energy-momentum distribution associated with the gravitational field [10, 11, 12]. Einstein [13] first obtained such an expression and many others such as Landau-Lifshitz, Papapetrou, Weinberg, Bergmann-Thomson, Tolman, Møller and Qadir-Sharif gave similar prescriptions [14, 15, 16, 17, 18, 19, 20]. The expressions they gave are called energy-momentum complexes because they can be expressed as a combination of the energy-momentum density which is usually defined by a second rank tensor T_i^k and a pseudo-tensor, which is interpreted to represent the energy and momentum of the gravitational field. These formulations have been heavily criticized because they are non-tensorial, i.e. they are coordinate dependent. Except for the Møller definition these formulations only give meaningful results if the calculations are performed in *Cartesian* coordinates. Møller proposed a new expression for the energy-momentum complex which could be utilized to any coordinate system. Virbhadra and collaborators revived the interest in this approach [21, 22, 23, 24, 25, 26, 27, 28, 29, 30] and since then numerous works on evaluating the energy and momentum distributions of several gravitational backgrounds have been completed [31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60]. Next, Lessner [61] argued that the Møller prescription is a powerful concept for the energy-momentum in general relativity. Recently, the problem of energy-momentum localization has also been considered in the teleparallel theory of gravity [62, 63, 64]. Møller showed that a tetrad description of a gravitational field equation allows a more satisfactory treatment of the energy-momentum complex than does general relativity. Vargas [63], using the definitions of Einstein and Landau-Lifshitz in the teleparallel gravity, found that the total energy is zero in Friedmann-Robertson-Walker space-times. After this work there are a few papers on the energy-momentum in the teleparallel gravity [65, 66, 67, 68, 69, 70, 71, 72].

The paper is organized as follow. In the next section, first we introduce Einstein, Bergmann-Thomson and Landau-Lifshitz's prescriptions of energy-momentum distribution in the teleparallel gravity, and then calculate the energy of the expanding Lukash metric of a plane-wave attractor. Finally, section 3 is devoted to the discussions. In this paper we use convention that $G = 1$ and $c = 1$. Except for the cases we give the

special values of the indices, all indices take the values from 0 to 3.

2. Teleparallel Energy

The Bianchi VII_δ type spacetimes belong to the non-exceptional family of the Behr class B spatially homogeneous metrics. The plane-wave Lukash solution is the late-time attractor of the Bianchi VII_δ models for a broad range of initial data and matter properties. These vacuum models correspond to equilibrium points of the associated autonomous dynamical system and are self-similar [73, 74, 75]. The metric of Lukash spacetime is defined as

$$ds^2 = -dt^2 + t^2 dx^2 + t^{2\omega} e^{2\omega x} \left[(A dy + B dz)^2 + (B dy + A dz)^2 \right], \quad (1)$$

where ω is an arbitrary constant parameter in the range $0 < \omega < 1$, and $A = \cos \Lambda$, $B = \Xi^{-1} \sin \Lambda$, $C = -\Xi \sin \Lambda$, $\Lambda = k(x + \ln t)$ [76, 77]. Note that Ξ and k are constants related to ω by

$$\frac{k^2}{\Xi^2} (1 - \Xi^2)^2 = 4\omega(1 - \omega), \quad (2)$$

and

$$w^2 = \delta k^2, \quad (3)$$

where δ is the associated group parameter. Constraints (2) and (3) are the Lukash analogue of the Friedmann equation. We also point out that when we take $\omega = 1$ and $\Xi^2 = 1$, the Lukash metric can be reduced to that of the empty Milne universe.

Now, let's calculate the energy associated with the metric (1) in the teleparallel gravity. Teleparallel gravity (the tetrad theory of gravitation), which corresponds to a gauge theory for the translation group based on the Weitzenböck geometry, [80] is an alternative approach to Einstein gravitation [81, 82]. In this theory, gravitation is attributed to torsion [83], which plays the role of a force [84], whereas the curvature tensor vanishes identically. The fundamental field is a nontrivial tetrad field, which gives rise to the metric as a by-product. The last translational gauge potentials appear as the nontrivial part of the tetrad field, and thus they induce on space-time a teleparallel structure which is directly related to the presence of the gravitational field. The interesting point of teleparallel gravity is that it can reveal a more appropriate approach to considering the same specific problem due to gauge structure. This is the case, for example, for the energy-momentum problem, which becomes more transparent when considered from the teleparallel point of view.

Teleparallel theories of gravity, whose basic entities are tetrad fields $\xi_{a\mu}$ (a and μ are SO(3,1) and spacetime indices, respectively) have been considered long time ago by Møller [78, 79] in connection with attempts to define the energy of the gravitational field. Teleparallel theories of gravity are defined on Weitzenböck spacetime [80], which is endowed with the affine connection

$$\Gamma_{\mu\nu}^\lambda = \xi^{a\lambda} \partial_\mu \xi_{a\nu}, \quad (4)$$

This connection defines a spacetime with an absolute parallelism or teleparallelism of vector fields [85]. In this geometrical framework the gravitational effects are due to the spacetime torsion corresponding to the above mentioned connection.

As remarked by Hehl [86], by considering Einstein's general relativity as the best available alternative theory of gravity, its teleparallel equivalent is the next best one. Therefore it is interesting to perform studies of the space-time structure as described by the teleparallel gravity.

The energy-momentum complex of Einstein in the teleparallel gravity [63] is given by

$$4\pi\xi\mathfrak{R}^\mu{}_\nu = \frac{\partial\Delta_\nu{}^{\mu\lambda}}{\partial x^\lambda}. \quad (5)$$

Next, the Bergmann-Thomson formulation is defined as

$$4\pi\xi\Pi^{\mu\nu} = \frac{\partial(g^{\mu\beta}\Delta_\beta{}^{\nu\lambda})}{\partial x^\lambda}, \quad (6)$$

and the Landau-Lifshitz formulation is

$$4\pi\xi\Sigma^{\mu\nu} = \frac{\partial(\xi g^{\mu\beta}\Delta_\beta{}^{\nu\lambda})}{\partial x^\lambda}, \quad (7)$$

where $\xi = \mathbf{det}(\xi^a{}_\mu)$ and $\Delta_\beta{}^{\nu\lambda}$ is the Freud's super-potential, which is defined by:

$$\Delta_\beta{}^{\nu\lambda} = \xi\mathfrak{S}_\beta{}^{\nu\lambda}. \quad (8)$$

Here $\mathfrak{S}^{\mu\nu\lambda}$ is the tensor

$$\mathfrak{S}^{\mu\nu\lambda} = \aleph_1 T^{\mu\nu\lambda} + \frac{\aleph_2}{2}(T^{\nu\mu\lambda} - T^{\lambda\mu\nu}) + \frac{\aleph_3}{2}(g^{\mu\lambda}T^{\beta\nu}{}_\beta - g^{\nu\mu}T^{\beta\lambda}{}_\beta) \quad (9)$$

with \aleph_1 , \aleph_2 and \aleph_3 the three dimensionless coupling constants of teleparallel gravity [83]. For the teleparallel equivalent of general relativity the specific choice of these three constants are $\aleph_1 = \frac{1}{4}$, $\aleph_2 = \frac{1}{2}$ and $\aleph_3 = -1$. To calculate this tensor, first we must calculate the Weitzenböck connection:

$$\Gamma^\alpha{}_{\mu\nu} = \xi_a{}^\alpha \partial_\nu \xi^a{}_\mu, \quad (10)$$

and after this calculation, one gets the torsion of the Weitzenböck connection:

$$T^\mu{}_{\nu\lambda} = \Gamma^\mu{}_{\lambda\nu} - \Gamma^\mu{}_{\nu\lambda}. \quad (11)$$

In the Einstein, Bergmann-Thomson and Landau-Lifshitz complexes, for $P_\mu = (E, \vec{P})$, we have the following expressions:

$$E^E = \int_\varphi \xi \mathfrak{R}^0{}_0 dx dy dz, \quad P_i^E = \int_\varphi \xi \mathfrak{R}^0{}_i dx dy dz, \quad (12)$$

$$E^{BT} = \int_\varphi \xi \Pi^0{}_0 dx dy dz, \quad P_i^{BT} = \int_\varphi \xi \Pi^0{}_i dx dy dz, \quad (13)$$

$$E^{LL} = \int_\varphi \xi \Sigma^0{}_0 dx dy dz, \quad P_i^{LL} = \int_\varphi \xi \Sigma^0{}_i dx dy dz \quad (14)$$

where P_i give momentum components P_1, P_2, P_3 while $P_0 (E)$ gives the energy and the integration hyper-surface φ is described by $x^0 = t = \text{constant}$.

The components of the metric tensor $g_{\mu\nu}$ for the line-element (1) are

$$g_{\mu\nu} = -\delta_\mu^0\delta_\nu^0 + t^2\delta_\mu^1\delta_\nu^1 + t^{2\omega}e^{2\omega x}(A^2 + B^2)\delta_\mu^2\delta_\nu^2 + t^{2\omega}e^{2\omega x}A(B + C)(\delta_\mu^2\delta_\nu^3 + \delta_\mu^3\delta_\nu^2) + t^{2\omega}e^{2\omega x}(A^2 + C^2)\delta_\mu^3\delta_\nu^3, \quad (15)$$

and of its inverse matrix $g^{\mu\nu}$ are

$$g^{\mu\nu} = -\delta_0^\mu\delta_0^\nu + \frac{1}{t^2}\delta_1^\mu\delta_1^\nu + \frac{t^{-2\omega}e^{-2\omega x}}{(A^2 + BC)^2}(A^2 + C^2)\delta_2^\mu\delta_2^\nu - \frac{t^{-2\omega}e^{-2\omega x}}{(A^2 - BC)^2}(A + BC)(\delta_2^\mu\delta_3^\nu + \delta_3^\mu\delta_2^\nu) + \frac{t^{-2\omega}e^{-2\omega x}}{(A^2 - BC)^2}(A^2 + B^2)\delta_3^\mu\delta_3^\nu \quad (16)$$

where δ_ν^μ is the four-index *Kronecker Delta* function.

The non-trivial tetrad field induces a teleparallel structure on space-time which is directly related to the presence of the gravitational field, and the Riemannian metric arises as

$$g_{\mu\nu} = \eta_{ab}\xi^a_\mu\xi^b_\nu, \quad \eta_{ab} = \text{diag}(-1, 1, 1, 1). \quad (17)$$

Using this definition, one can easily obtain the tetrad components ξ^a_μ as:

$$\xi^a_\mu = \delta_\mu^0\delta_0^a + t\delta_\mu^1\delta_1^a + t^\omega e^{\omega x}\sqrt{(A^2 + B^2)}\delta_\mu^2\delta_2^a + \frac{t^\omega e^{\omega x}(A + BC)}{\sqrt{A^2 + B^2}}\delta_\mu^2\delta_3^a + \frac{t^\omega e^{\omega x}(A^2 - BC)}{\sqrt{A^2 + B^2}}\delta_\mu^3\delta_3^a, \quad (18)$$

and the components of ξ_a^μ are

$$\xi_a^\mu = \delta_0^\mu\delta_a^0 + \frac{1}{t}\delta_1^\mu\delta_a^1 + \frac{t^{-\omega}e^{-\omega x}}{(A^2 + B^2)}\delta_2^\mu\delta_a^2 - \frac{t^{-\omega}e^{-\omega x}}{\sqrt{(A^2 + B^2)}}\delta_3^\mu\delta_a^2 + \frac{t^{-\omega}e^{-\omega x}\sqrt{(A^2 + B^2)}}{(A^2 - BC)}\delta_5^\mu\delta_a^3. \quad (19)$$

Hence, we obtain the following non-vanishing components of the Weitzenböck connection

$$\Gamma^1_{10} = \frac{1}{t}, \quad (20)$$

$$\Gamma^2_{21} = \Gamma^3_{30} = \frac{1}{t} \left\{ \omega - \frac{k(\Xi^2 - 1) \sin[2k(x + \ln t)]}{1 + \Xi^2 + (\Xi^2 - 1) \cos[2k(x + \ln t)]} \right\}, \quad (21)$$

$$\Gamma^2_{21} = \Gamma^3_{31} = \omega - \frac{k(\Xi^2 - 1) \sin[2k(x + \ln t)]}{1 + \Xi^2 + (\Xi^2 - 1) \cos[2k(x + \ln t)]}, \quad (22)$$

The corresponding non-vanishing torsion components are found:

$$T^1_{01} = -T^1_{10} = \frac{1}{t}, \quad (23)$$

$$T^2_{02} = -T^2_{20} = T^3_{03} = -T^3_{30} = \frac{1}{t} \left\{ \omega - \frac{k(\Xi^2 - 1) \sin[2k(x + \ln t)]}{1 + \Xi^2 + (\Xi^2 - 1) \cos[2k(x + \ln t)]} \right\}, \quad (24)$$

$$T^2_{12} = -T^2_{21} = T^3_{13} = -T^3_{31} = \omega - \frac{k(\Xi^2 - 1) \sin[2k(x + \ln t)]}{1 + \Xi^2 + (\Xi^2 - 1) \cos[2k(x + \ln t)]}. \quad (25)$$

Substituting these results into the equation (9), the non-zero energy component of the tensor $\mathfrak{S}_\mu^{\nu\lambda}$ is found as:

$$\mathfrak{S}^{001} = -\omega t^{-2}, \quad (26)$$

from this point of view, the only non-vanishing component of Freud's super-potential is

$$\Delta_0^{01} = e^{2\omega x} t^{2\omega-1} \omega. \quad (27)$$

Using equations (5), (6), (7), the relative Einstein, Bergmann-Thomson and Landau-Lifshitz's energy densities are found as:

$$\xi \mathfrak{R}^0_0 = \frac{1}{2\pi} e^{2\omega x} t^{2\omega-1} \omega^2, \quad (28)$$

$$\xi \Pi^{00} = -\frac{1}{2\pi} (e^{2\omega x} t^{2\omega-1} \omega^2), \quad (29)$$

$$\xi \Sigma^{00} = -\frac{1}{\pi} (e^{4\omega x} t^{4\omega} \omega^2) \quad (30)$$

It is evident that for the energy densities we have

$$\left(\xi \mathfrak{R}^0_0\right)_{Lukash} = \left(\xi \Pi^0_0\right)_{Lukash} \neq \left(\xi \Sigma^0_0\right)_{Lukash}, \quad (31)$$

which means that although Einstein and Bergmann-Thomson formulations agree with each other, the Landau-Lifshitz prescription gives different energy distribution in this universe. Furthermore, for the Milne universe, we have

$$\xi \mathfrak{R}^0_0 = \frac{t}{2\pi} e^{2x}, \quad (32)$$

$$\xi \Pi^{00} = -\frac{t}{2\pi} e^{2x}, \quad (33)$$

$$\xi \Sigma^{00} = -\frac{t^4}{\pi} e^{4x}, \quad (34)$$

and still we have the same relation among the energy-momentum formulations of Einstein, Bergmann-Thomson and Landau-Lifshitz.

$$\left(\xi \mathfrak{R}^0_0\right)_{Milne} = \left(\xi \Pi^0_0\right)_{Milne} \neq \left(\xi \Sigma^0_0\right)_{Milne}. \quad (35)$$

3. Discussions

The main object of the presented paper is to show that it is possible to evaluate the energy distribution by using the energy-momentum formulations in not only general relativity but also teleparallel gravity. In the context of teleparallel theory, we showed that the Einstein and Bergmann-Thomson formulations give the same results both in the Lukash plane-wave attractor spacetime and Milne universe, but the Landau-Lifshitz formulation does not.

$$hE^0_0 = hB^0_0 \neq hL^0_0. \quad (36)$$

It is interesting to mention that the results reduce to the already available results for the Milne universe when we write $\omega = 1$ and $\Xi^2 = 1$ in the metric of the Lukash spacetime. We find that the energy distribution (due to matter and fields including gravitation) turns out to be non-vanishing in three of the prescriptions used.

$${}_{Lukash}\xi\mathfrak{R}^0_0 = \frac{1}{2\pi}e^{2\omega x}t^{2\omega-1}\omega^2, \quad {}_{Milne}\xi\mathfrak{R}^0_0 = \frac{t}{2\pi}e^{2x}, \quad (37)$$

$${}_{Lukash}\xi\Pi^{00} = -\frac{1}{2\pi}(e^{2\omega x}t^{2\omega-1}\omega^2), \quad {}_{Milne}\xi\Pi^{00} = -\frac{t}{2\pi}e^{2x}, \quad (38)$$

$${}_{Lukash}\xi\Sigma^{00} = -\frac{1}{\pi}(e^{4\omega x}t^{4\omega}\omega^2), \quad {}_{Milne}\xi\Sigma^{00} = -\frac{t^4}{\pi}e^{4x}. \quad (39)$$

The energy distributions are also dependent of the teleparallel dimensionless coupling constants, which means that it is valid only in the teleparallel equivalent of general relativity, it is not valid any teleparallel model. Hence, one can also perform the calculations and get the same energy distributions in the general relativity.

Our results also (a) support the hypothesis by Cooperstock that the energy is confined to the region of non-vanishing energy-momentum tensor of matter and all non-gravitational fields, and (b) sustain the importance of the energy-momentum definitions in the evaluation of the energy distribution of a given spacetime.

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