BLOCH IMPEDANCE ANALYSIS FOR A LEFT HANDED TRANSMISSION LINE

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In this study, the dispersion relation and the frequency dependence of Bloch impedance in a left handed transmission line (LH-TL) is carried out using the F-matrix formulation and Bloch-Floquet theorem. The artificial LH-TL formed by periodic lumped elements is described and the F-matrix, dispersion relation and the Bloch impedance are formulated according to this description. Numerical results for lossless and lossy LH-TL are presented and discussed.

K e y w o r d s: left/right handed, transmission line, Bloch impedance, Floquet theorem

1 INTRODUCTION

A medium in which both the permittivity ε and the permeability μ are simultaneously negative is called the left handed (LH) medium and it was first investigated by Veselago in 1968 [1]. The first experiment using LH materials at microwave frequencies goes back to 2000 [2]. Since then, due to the progresses of the fabrication technologies, LH materials are used also in integrated components, such as antennas, microwave filters and radomes, *etc.* The topic continues to be of great interest and practical importance due to a variety of potential applications.

In this paper, the dispersion relation of LH-TL is reviewed by using the F-matrix (ABCD-matrix) formulation. Then, the frequency dependence of the Bloch impedance in a LH-TL is presented. In addition, numerical investigation is considered for the application of the theoretical formulation. Although LH materials have been extensively analyzed in literature [3]-[5], in the author knowledge there is no work which directly relates to the Bloch impedance of LH-TL except [6] and [7]. These studies ([6] and [7]) investigate the frequency dependence and high pass filter characteristic of Bloch impedance in a LH-TL only for lossless cases.

The analysis is structured in two main steps; firstly the concept of artificial LH-TL formed from composite periodic structure is presented. Then, F-matrix parameters for the unit cell of artificial LH-TL are derived. A general dispersion relation is found by using F-matrix and applying the Bloch-Floquet theorem; the dispersion relation of LH-TL is also considered for both lossless and lossy cases. Furthermore, the formulation of the frequency dependence of the Bloch impedance is carried out. Numerical results for both the lossless and lossy cases are presented and discussed.

2 THEORETICAL ANALYSIS

It is well known that the properties of a periodic structure can be deduced from the properties of its unit cell. The characteristic impedance of the periodic artificial line is identical to the characteristic impedance of its unit cell, while the dispersion relation of a periodic artificial line is equivalent to the dispersion relation of its unit cell compounded by the number of unit cells in the line [8]. Design and implementation processes can be developed by finding the general properties of TL and then apply them to a LH-TL.

The artificial LH-TL considered here is a cascade of a basic unit cell (UC) circuit formed by lumped-elements that can be obtained by exchanging the inductance/capacitance and inverting the series/parallel arrangements in the equivalent circuit of the conventional right-handed transmission line (RH-TL). The corresponding lumped equivalent circuit is shown in Fig. 1.

2.1 F-Matrix formulation and dispersion relation

When a unit cell circuit has a ladder-network topology, the most convenient method of analysis is by F-matrix (ABCD matrix). If we consider the infinite TL as being composed of a cascade of identical two-port networks, we can relate the voltages and currents on either side of n-th unit cell using the F-matrix

$$\begin{bmatrix} V_n \\ I_n \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_{n+1} \\ I_{n+1} \end{bmatrix} \tag{1}$$

where A, B, C, and D are the matrix parameters for cascade of a TL section of length d_u. The F-matrix of the unit cell shown in Fig. 1 can be expressed as follows

$$\begin{bmatrix} V_n \\ I_n \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_{n+1} \\ I_{n+1} \end{bmatrix} \tag{2}$$

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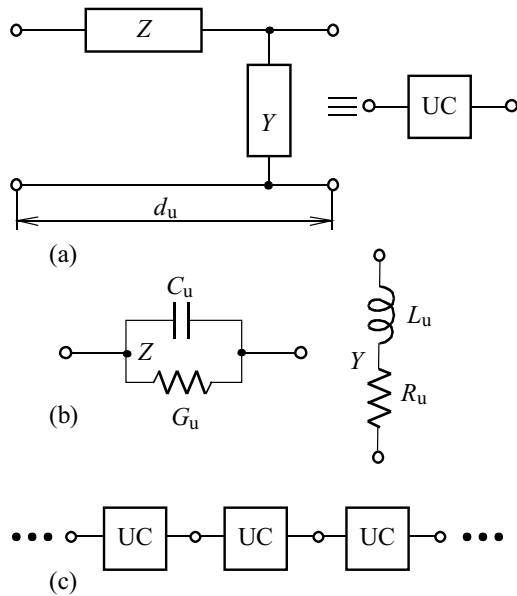
Fig. 1. Equivalent circuit model of LH-TL: (a) – unit cell of LH-TL, (b) – lumped elements of unit cell, (c) – a periodic artificial LH-TL composed of cascading the identical unit cells

where $Z = (G_u + j\omega C_u)^{-1}$ and $Y = (R_u + j\omega L_u)^{-1}$. According to Floquet theorem, the current and voltage waves that are propagating along the periodic structure are modified after one period by a complex constant γd_u

$$[F] = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} YZ + 1 & Z \\ Y & 1 \end{bmatrix}. \qquad (3)$$

That is, the voltage and current at the $(n + 1)$-th terminals differ from the voltage and current at the n-th terminal by a multiplicative factor. Substituting this result back into (1) yields the following

$$\begin{bmatrix} V_n \\ I_n \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_{n+1} \\ I_{n+1} \end{bmatrix} = \begin{bmatrix} V_{n+1} \exp(\gamma d_u) \\ I_{n+1} \exp(\gamma d_u) \end{bmatrix}. \qquad (4)$$

After manipulating (4) a homogeneous matrix equation is obtained; for a non-trivial solution, the following condition must hold

$$AD + \exp(2\gamma d_u) - (A + D)\exp(\gamma d_u) - BC = 0. \qquad (5)$$

Since the network is reciprocal $AD - BC = 1$, and the expression can be simplified leading to the dispersion relation given by

$$\frac{1}{2}[ZY] =$$
$$1 + \frac{1}{2}\left[\frac{(G_u R_u - \omega^2 L_u C_u) - j\omega(L_u G_u + R_u C_u)}{(G_u R_u - \omega^2 L_u C_u)^2 + \omega^2(L_u G_u + R_u C_u)^2} \right]. \qquad (6)$$

Since $\gamma = \alpha + j\beta$ where α and β represent the attenuation and propagation constants (both real) along LH-TL we have

$$\cosh(\gamma d_u) =$$
$$\cosh(\alpha d_u)\cos(\beta d_u) + j\sinh(\alpha d_u)\sin(\beta d_u). \qquad (7)$$

For the case of $\alpha = 0$ which corresponds to a lossless case ($R_u = G_u = 0$) the dispersion relation yields

$$\cos(\beta d_u) = 1 - \frac{1}{2}\frac{1}{\omega^2 L_u C_u}. \qquad (8)$$

The dispersion relation is the keystone of LH-TL investigated here. The physical properties of LH-TL such as characteristic impedance, group and phase velocities, group delay, *etc* can be straightforwardly derived from the dispersion relation.

2.2 Bloch impedance

In this section, the frequency response of Bloch impedance is determined by the model discussed above. It is important to point out that the voltage and current waves defined in (3) are meaningful only when measured at the terminals of the unit cells. These waves are sometimes referred to as Bloch waves because of their similarity to the elastic waves that propagate through periodic crystal lattices [9]. The Bloch impedance defined as the characteristic impedance of waves on the structure and so it is given by

$$Z_B = Z_0 \frac{V_{n+1}}{I_{n+1}}. \qquad (9)$$

From (4) we have

$$(A - \exp(\gamma\, d_u)) V_{n+1} + B \cdot I_{n+1} = 0. \qquad (10)$$

Therefore

$$\bar{Z}_B = \frac{Z_B}{Z_0} = -\frac{B}{A - e^{\gamma d}} = -\frac{D - e^{\gamma d}}{C}. \qquad (11)$$

After some algebraic manipulations the expression for the Bloch impedance in terms of the transmission matrix elements of the unit cell yields

$$\bar{Z}_B^\pm = \frac{2B}{(D - A) \pm \sqrt{(A + D)^2 - 4}}. \qquad (12)$$

The $\pm$ solutions correspond to the characteristic impedance for positive and negative traveling waves, respectively. For symmetrical networks these impedances are the same except for the sign; the characteristic impedance for a negatively traveling wave turns out to be negative because we have defined current at the n-th unit cell as always being in the positive direction. In general, for a lossless structure [10],

$$\bar{Z}_B^- = -(\bar{Z}_B^+)^*. \qquad (13)$$
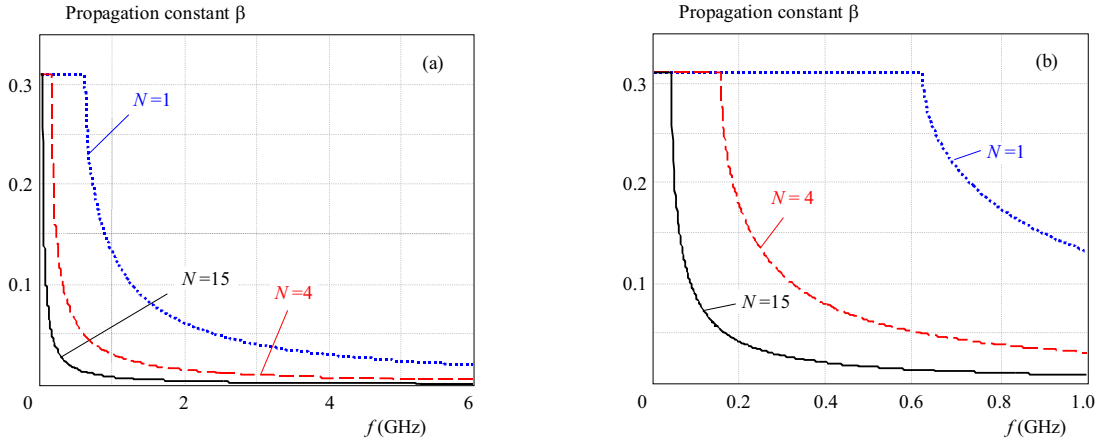
in the passband, since $|A + D| < 2$.

Fig. 2. Frequency response of propagation constant for lossless case; (a) Propagation constant versus frequency; (b) Zoom of Figure 2(a)

Table 1. Design parameters for the artificial lossless LH-TL

N	C_u(pF)	L_u(nH)	f_C(GHz)
1	2.56	6.4	0.6217
4	10.24	25.6	0.1554
15	38.4	96	0.0414

Table 2. Design parameters for the artificial lossy LH-TL

N	$R_u(\Omega)$	G_uS)	C_u(pF)	L_u(nH)
1	0.1	0.01	2.56	6.4
4	0.4	0.04	10.24	25.6
15	1.5	0.15	38.4	96

It is well known that the Bloch impedance Z_B represents the ratio of the voltage and current in the eigenmodes, $Z_B = V_B/I_B$. Hence, given that the power of the eigenmodes can be defined as

$$P_B = \frac{1}{2}\,\mathrm{Re}\left\{V_B I_B^*\right\} = \frac{1}{2}\,\mathrm{Re}\left\{Z_B\right\}|I_B|^2. \quad (14)$$

The sign of $\mathrm{Re}\{Z_B\}$ is related to the direction of the energy flow for the eigen-modes. On the other hand, the real part of the propagation constant represents the phase propagation; therefore, the LH frequency band can be identified by the sign of $\mathrm{Re}\{\gamma\}\,\mathrm{Re}\{Z_B\}$. In the frequency band where $\mathrm{Re}\{\gamma\}\,\mathrm{Re}\{Z_B\} < 0$ the direction of energy flow and wave propagation are anti-parallel. That is the eigenmodes are backward waves and the frequency bands are LH bands [11].

3 NUMERICAL RESULTS

In this section, the theoretical formulation is applied to numerically calculate the frequency responses of the dispersion relation and the Bloch impedance for a lossless and lossy LH-TL. For the artificial lossless LH-TL $R = G = 0$, $L = 64\,\mathrm{nH\,m}$, $C = 25.6\,\mathrm{pF\,m}$ and $d = 10\,\mathrm{m}$

are used and unit cell component values are directly calculated from

$$K_u = \frac{K}{d_u} = K\left(\frac{N}{d}\right) \quad (15)$$

where K is the generic variable for $\{R,\ G,\ L,\ C\}$, N is the number of unit cells and d is the length of fictitious LH-TL [5]. Table 1 shows the design parameters for artificial lossless LH-TL for different values of unit cells.

The artificial lossy LH-TL is considered with the same parameters as the lossless structure except for $R = 1\,\Omega\,\mathrm{m}$, $G = 0.1\,\mathrm{S\,m}$. The design parameters for artificial lossy LH-TL for different values of unit cells are shown in Table 2.

Figure 2 shows the relationship between the propagation factor and frequency for the different values of N for lossless case. The propagation factor decreases when the frequency increases for all N. With increasing the number of unit cells (N), the cut-off frequency decreases, which means that stop band region becomes narrower. Furthermore, the variation in the group velocity decreases when the number of the unit cells increases. For example around the 0.8 GHz, the slope of the dispersion diagram increase when N decreases; that is the group velocity and the number of the unit cells N are inversely proportional.

Figure 3 shows the normalized Bloch impedance versus frequency for lossless case. The stop band region from Fig. 3(a) is wider when the number of unit cells is smaller. The real parts of the normalized $\overline{Z}_B^+$ and $\overline{Z}_B^-$ approach the value of $-50\,\Omega$ and $50\,\Omega$, respectively, when the frequency increases. Figure 3(c) shows that after the cut-off frequency the imaginary part of the normalized $\overline{Z}_B^+$ and $\overline{Z}_B^-$ assume the same value. Also they go to zero if N increases. Furthermore (19) is satisfied in the passband region for all N.

The propagation and attenuation constants as a function of frequency for lossy case are shown in Fig. 4. It is seen that propagation constant is negative in this case. The propagation and attenuation constants close to zero when the number of unit cells and frequency increase.
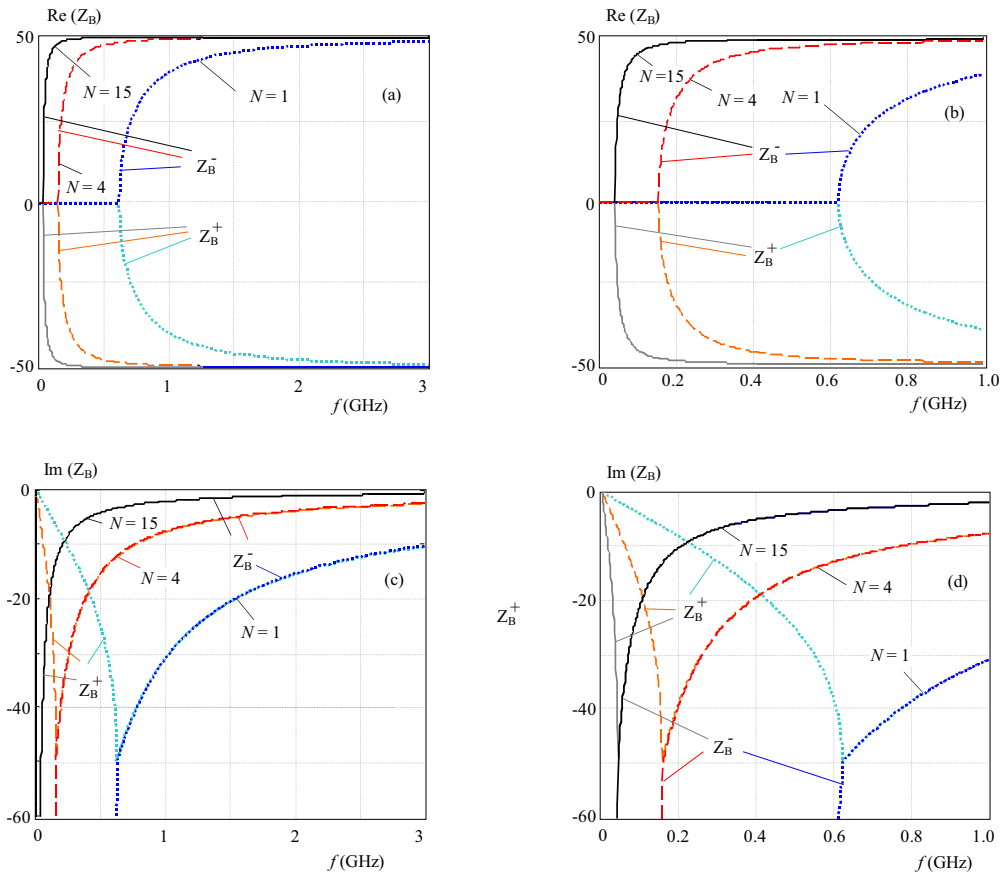
Fig. 3. Frequency response of Bloch impedance for lossless case: (a)– real part of normalized Bloch impedance versus frequency, (b) – zoom of Fig. 3(a),(c) – imaginary part of normalized Bloch impedance versus frequency, (d) – zoom of Fig. 3(c), note the different lines for $\overline{Z}_B^+$, and $\overline{Z}_B^-$
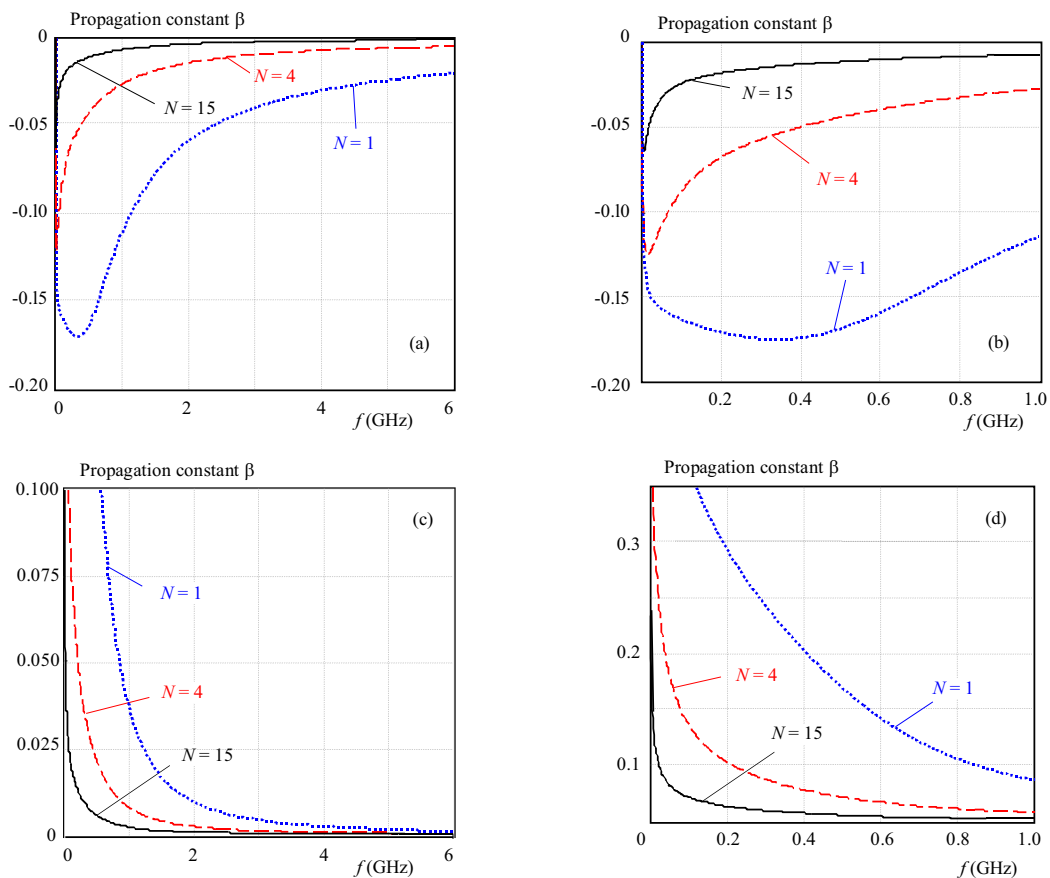

Fig. 4. Propagation and attenuation constants as a function of frequency for lossy case: (a) – propagation constant versus frequency, (b) – zoom of Figure 4(a), (c) – attenuation constant versus frequency;, (d) – zoom of Fig. 4(c)
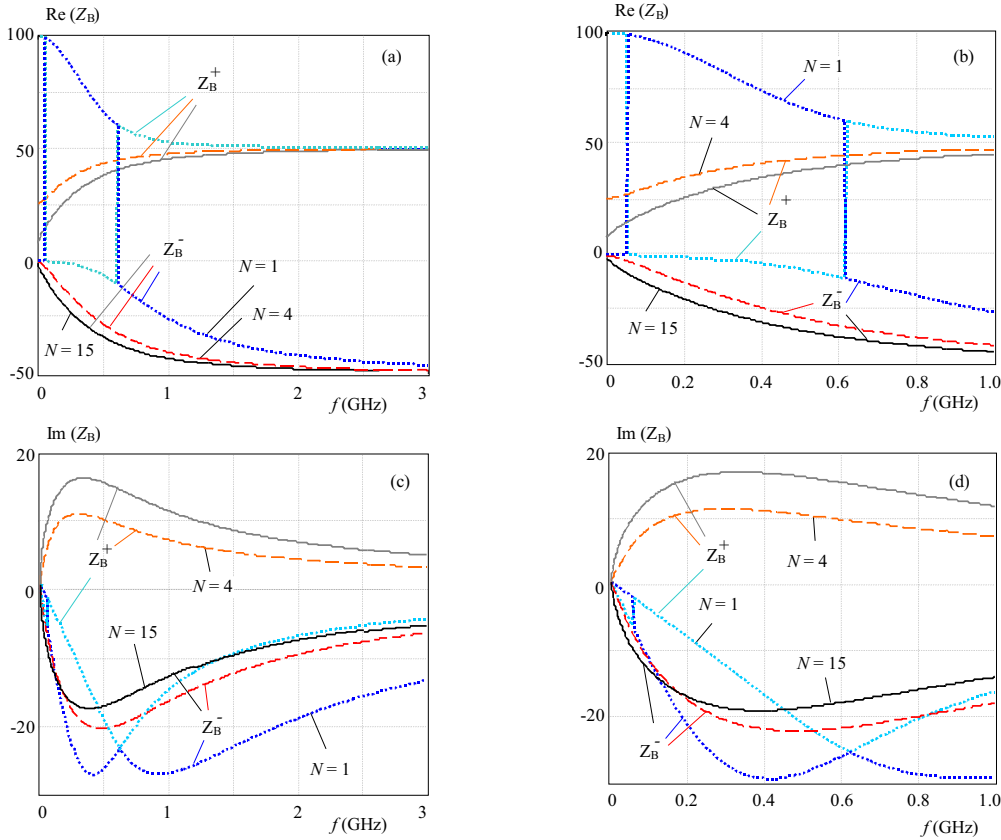
Fig. 5. Frequency response of Bloch impedance for lossy case: (a) – real part of normalized Bloch impedance versus frequency, (b) – Zoom of Fig. 5(a), (c) – imaginary part of normalized Bloch impedance versus frequency, (d) – zoom of Fig. 5(c), note the different lines for $\overline{Z}_B^+$, and $\overline{Z}_B^-$)

The normalized Bloch impedance versus frequency for the lossy case is illustrated in Fig. 5. Here the real part of $\overline{Z}_B^+$ ($\overline{Z}_B^-$) is positive (negative) resulting in an opposite behavior with respect to the lossless case. Finally, the imaginary part of $\overline{Z}_B^+$ ($\overline{Z}_B^-$) is positive (negative) for increasing values of N except for $N = 1$.

4 DISCUSSION AND CONCLUSION

The dispersion relation of LH-TL is carried out and the frequency dependence of the Bloch impedance for a LH-TL is analyzed by using the F-matrix (ABCD matrix) formulation and Bloch-Floquet theorem. The dispersion relation and the frequency dependence of Bloch impedance in a LH-TL is also calculated numerically to show their behaviors for the LH-TL. Numerical results for lossless and lossy cases are shown and compared. As it is seen that from the numerical results, the propagation constant is positive for the lossless case whereas it is negative for the lossy case for the given examples. Also $\overline{Z}_B^+$ is negative and $\overline{Z}_B^-$ is positive for the lossless case and vice versa for the lossy case. The physical properties, the cut-off frequency and the stop band region of a LH-TL can be achieved by using the results obtained in this study. It can be easily observed that if the number of unit cells decreases (increases), the cut-off frequency becomes

higher (lower) and the stop band region becomes wider (narrower). It is known that the unit cell components (R_u, G_u, L_u, and C_u) are directly proportional with the number of unit cells N. Also, the cut-off frequency and the stop band region are inversely proportional with the unit cell components. Thus, the cut-off frequency and the stop band region are also inversely proportional with the number of unit cells N. If it is desired to obtain lower cut-off frequency and narrower stop band region, larger values of the unit cell components are required due to the larger number of unit cells to keep the characteristics of the TL unchanged. In addition, the LH-TL and RH-TL structures have high-pass and low-pass filter properties, respectively. The Bloch impedance for the LH-TL shows the high-pass characteristics which are meaningful in its pass-band, as in the lossless case. On the other hand, the RH-TL exhibits the low-pass characteristics whose numerical results are skipped here. So that, these properties can be used in the periodic structures to obtain more efficient LH structures and devices. Furthermore, the results obtained here can be helpful to design new type of filters using the LH-TL. These results also open a way to think how the lossless and lossy LH materials will change the functionality of a device with LH-TL. Moreover, this study provides some insight into the potential applications of LH materials and LH-TL.

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