Semileptonic $B_c^- \to D^{*0}\ell\nu$ transition in three–point QCD sum rules and HQET with gluon condensate corrections

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Abstract

Taking into account the gluon condensate contributions, the form factors of the semileptonic $B_c^- \to D^{*0}\ell\nu$ transition with $l = \tau, e$ are calculated in the framework of the three point QCD sum rules. The heavy quark effective theory limit of the form factors are also computed. The relevant total decay width as well as the branching ratio are evaluated and compared with the predictions of the other non-perturbative approaches.

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1 Introduction

In 1998, the CDF collaboration reported the first experimental observation of the B_c meson[1]. The heavy meson B_c with $\bar{b}c$ quark structure is made of two heavy quarks with different charge and flavors. It is located between two heavy meson families called charmonium $\bar{c}c$ and bottomonium $\bar{b}b$, so this meson is similar to the the charmonium and bottomonium in the spectroscopy. The predictions for the mass spectra of the $\bar{b}c$ levels were obtained in the potential models (PM) and lattice simulations [2–6]. In contrast to the charmonium and bottomonium, the B_c decays only via weak interaction and holds long life time. For this reason the B_c transitions are very interesting tool to calculate more precise values for the Cabibbo-Kabayashi-Maskawa(CKM) matrix elements and study the CP and T violations that occur in weak interactions. It is predicted that the LHC experiments may give interesting informations about this meson that could be used as a basis for future investigations[7, 8].

Some decay modes of this meson have been studied by different methods. The $B_c \to D_{s,d}\ell^+\ell^-/\nu\bar{\nu}$, $B_c \to D^*_{s,d}\ell^+\ell^-$, $B_c \to X(D^*, D^*_s, D_1, D_{s1})\nu\bar{\nu}$ transitions have been discussed via three point QCD sum rules (3PSR) in [9–11], the $B_c \to j/\psi\ell\nu$ has been analyzed by means of the 3PSR and non-relativistic QCD (NRQCD) [12] and the $B_c \to \ell\bar{\nu}\gamma$, $B_c \to \rho^+\gamma$, $B_c \to K^{*+}\gamma$, $B_c \to B^*_u\ell^+\ell^-$ channels have been investigated in the framework of the light-cone QCD sum rules [14–17]. A large set of the exclusive non-leptonic and semileptonic decays of the B_c meson have been studied within the potential model (PM) (see [18–27]), and also operator product expansion in inverse powers of the heavy quark masses [28]. In this work, considering the gluon corrections to the relevant form factors, the $B_c^- \to D^{*0}\ell\nu$ mode is investigated in the framework of the three-point QCD sum rules (3PSR) and also in the heavy quark effective theory (HQET). This decay mode has been discussed in different methods (for instance see[13, 22, 25, 29–31]). This transition has also been investigated in the QCD sum rule approach, for example in [13, 32] but without considering the gluon corrections. In [13], the coulomb like corrections were considered in the calculations to decrease the uncertainties. The main points in the present work are the calculation of the gluon corrections and check whether their contributions guarantee the convergence of the sum rules for the form factors or not and also the comparison between the form factors and their HQET limit. For this aim, we plot the dependence of both form factors and their HQET limit on the transferred momentum square (q^2) and compare them at high and low q^2 values.

This paper includes five sections. The calculation of the sum rules for the relevant form factors are presented in Section 2. In the sum rules expressions for the form factors, the light quark condensate do not have any contributions since applying the double Borel transformation with respect to the momentum of the initial and final states kills their contributions. Therefore, as a first correction on the non-perturbative part of the correlation function, the two gluon condensate contributions are taken into account, so in Section3, the gluon condensate contributions in the Borel transform scheme is presented. Section 4 is devoted to the explanation of the heavy quark effective theory. In this section HQET limit of the form factors are derived. Section 5 depicts our numerical analysis of the form factors and their comparison with the HQET limit. This section also contains the calculation of the total decay width as well as the branching ratio of the $B_c^- \to D^{*0}\ell\nu$ ($\ell = e, \tau$) via 3PSR and HQET and their comparison with the predictions of other approaches.

2 Sum rules for $B_c^- \to D^{*0}\ell\nu$transition form factors

At quark level, the tree level $b \to u$ transition is responsible for $B_c^- \to D^{*0}$ decay mode. The Hamiltonian of this decay is written as:

$$\mathcal{H}_q = \frac{G_F}{\sqrt{2}} V_{ub}\, \overline{\nu}\ \gamma_\mu(1-\gamma_5) l\ \overline{u}\ \gamma_\mu(1-\gamma_5) b, \tag{1}$$

where, G_F is the Fermi constant and V_{ub} is the CKM matrix element. The decay amplitude for $B_c^- \to D^{*0}\ell\nu$ is obtained by sandwiching the Eq. (1) between the initial and final meson states

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} V_{ub}\, \overline{\nu}\ \gamma_\mu(1-\gamma_5) l \langle D^*(p',\varepsilon) \mid\ \overline{u}\ \gamma_\mu(1-\gamma_5) b \mid B_c(p)\rangle. \tag{2}$$

Our main task is to calculate the matrix element appearing in the Eq.(2). Both axial and vector parts of the transition current are involved in this matrix element. Using the Lorentz invariance and parity conservation, it can be parameterized in terms of some form factors as:

$$< D^*(p',\varepsilon) \mid \overline{u}\gamma_\mu b \mid B_c(p) >= -f_V'(q^2)\varepsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}p^\alpha p'^\beta, \tag{3}$$

$$\begin{aligned} < D^*(p',\varepsilon) \mid \overline{u}\gamma_\mu\gamma_5 b \mid B_c(p) > &= -i\left[f_0'(q^2)\varepsilon^*_\mu\right. \\ +f_1'(q^2)(\varepsilon^* p)P_\mu &+ \left. f_2'(q^2)(\varepsilon^* p)q_\mu\right], \end{aligned} \tag{4}$$

where:

$$\begin{aligned} f_V'(q^2) &= \frac{2f_V(q^2)}{(m_{B_c}+m_{D^*})}, & f_0'(q^2) &= f_0(q^2)(m_{B_c}+m_{D^*}), \\ f_1'(q^2) &= -\frac{f_1(q^2)}{(m_{B_c}+m_{D^*})}, & f_2'(q^2) &= -\frac{f_2(q^2)}{(m_{B_c}+m_{D^*})}, \end{aligned} \tag{5}$$

and the $f_V(q^2)$, $f_0(q^2)$, $f_1(q^2)$ and $f_2(q^2)$ are the transition form factors, $P_\mu = (p+p')_\mu$, $q_\mu = (p-p')_\mu$ and ε is the four–polarization vector of the D^* meson.

To find the above form factors via sum rules, we use the following three-point correlation function:

$$\Pi_{\mu\nu}(p,p',q) = i^2 \int d^4x d^4y e^{+ip'x-ipy} \left\langle 0 \left| \mathrm{T}\left\{ J_{D^*\nu}(x) J_\mu(0) J^\dagger_{B_c}(y)\right\}\right| 0\right\rangle\ , \tag{6}$$

where $J_{D^*\nu}(x) = \overline{c}\gamma_\nu u$, $J_{B_c}(y) = \overline{c}\gamma_5 b$, $J_\mu^V = \overline{u}\gamma_\mu b$ and $J_\mu^A = \overline{u}\gamma_\mu\gamma_5 b$ are the interpolating currents of the D^*, B_c, vector and axial vector parts of the transition current, respectively.

The following relation hold for the Lorentz structures of the selected correlation functions:

$$\Pi^{(V-A)}_{\mu\nu} = \epsilon_{\mu\nu\alpha\beta}\, p^\alpha p'^\beta \Pi_{f_V} + \Pi_{f_0} g_{\mu\nu} + \Pi_{f_1}\mathcal{P}_\mu p_\nu + \Pi_{f_2} q_\mu p_\nu + ... \tag{7}$$

By inserting two complete sets of the intermediate states with the same quantum number as the currents J_{D^*} and J_{B_c}, we can calculate the phenomenological part of the correlators given in the Eq. (6) as follow:

$$\Pi_{\mu\nu}^{V-A}(p^2, p'^2, q^2) = \frac{< 0 \mid J_{D^*\nu} \mid D^*(p', \varepsilon) >< D^*(p', \varepsilon) \mid J_\mu^{V-A} \mid B_c(p) >< B_c(p) \mid J_{Bc} \mid 0 >}{(p'^2 - m_{D^*}^2)(p^2 - m_{Bc}^2)} + \cdots \tag{8}$$

where... denotes the contributions coming from the higher states and continuum. In the Eq. (8), the vacuum to the initial and final meson state matrix elements are defined as:

$$< 0 \mid J_{D^*}^\nu \mid D^*(p') >= f_{D^*} m_{D^*} \varepsilon^\nu \ , \quad < 0 \mid J_{B_c} \mid B_c(p) >= i \frac{f_{B_c} m_{B_c}^2}{m_b + m_c}, \tag{9}$$

where f_{D^*} and f_{B_c} are the leptonic decay constants of the D^* and B_c mesons, respectively. Using the Eqs. (3), (4) and (9) in the Eq. (8) and performing summation over the polarization of the D^* meson, we get the following result for the physical part:

$$\begin{aligned} \Pi_{\mu\nu}^V(p^2, p'^2, q^2) &= -\frac{f_{B_c} m_{B_c}^2}{(m_b + m_c)} \frac{f_{D^*} m_{D^*}}{(p'^2 - m_{D^*}^2)(p^2 - m_{B_c}^2)} \times [f_0' g_{\mu\nu} + f_1' P_\mu p_\nu \\ &+ f_2' q_\mu p_\nu] + \text{excited states.} \\ \Pi_{\mu\nu}^A(p^2, p'^2, q^2) &= -i\varepsilon_{\mu\nu\alpha\beta} p'^\alpha p^\beta \frac{f_{B_c} m_{B_c}^2}{(m_b + m_c)} \frac{f_{D^*} m_{D^*}}{(p'^2 - m_{D^*}^2)(p^2 - m_{B_c}^2)} f_V' + \\ & \text{excited states.} \end{aligned} \tag{10}$$

The coefficients of the Lorentz structures $\epsilon_{\mu\nu\alpha\beta} p^\alpha p'^\beta$, $ig_{\mu\nu}$, $P_\mu p_\nu$ and $q_\mu p_\nu$ in the correlation functions $\Pi_{\mu\nu}^V$and $\Pi_{\mu\nu}^A$ will be chosen in determination of the form factors $f_V(q^2)$, $f_0(q^2)$, $f_1(q^2)$ and $f_2(q^2)$, respectively.

The QCD side of the correlation function is calculated with the help of the operator product expansion (OPE) in the deep Euclidean region where $p^2 \ll (m_b + m_c)^2$, $p'^2 \ll m_c^2$. In the Eq. (6), using the expansion of the time ordered products of currents in terms of a series of local operators with increasing dimension, we will have [16]:

$$\begin{aligned} -\int d^4x d^4y e^{-i(px - p'y)} T\Big\{ J_{D^*\nu} J_\mu J_{B_c} \Big\} &= (C_0)_{\nu\mu} I + (C_3)_{\nu\mu} \bar{q}q + (C_4)_{\nu\mu} G_{\alpha\beta} G^{\alpha\beta} \\ &+ (C_5)_{\nu\mu} \bar{q} \sigma_{\alpha\beta} G^{\alpha\beta} q + (C_6)_{\nu\mu} \bar{q}\Gamma q \bar{q}\Gamma' q \ , \end{aligned} \tag{11}$$

where $(C_i)_{\mu\nu}$ are the Wilson coefficients, $G_{\alpha\beta}$ is the gluon field strength tensor, I is the unit operator, Γ and Γ' are the matrices appearing in the calculations. Taking into account the vacuum expectation value of the OPE, the expansion of the correlation function in terms of the local operators is written as follow:

$$\begin{aligned} \Pi_{\nu\mu}(p_1^2, p_2^2, q^2) = &(C_0)_{\nu\mu} + (C_3)_{\nu\mu} \langle \bar{q}q \rangle + (C_4)_{\nu\mu} \langle G^2 \rangle + (C_5)_{\nu\mu} \langle \bar{q} \sigma_{\alpha\beta} G^{\alpha\beta} q \rangle \\ &+ (C_6)_{\nu\mu} \langle \bar{q}\Gamma q \bar{q}\Gamma' q \rangle \ . \end{aligned} \tag{12}$$

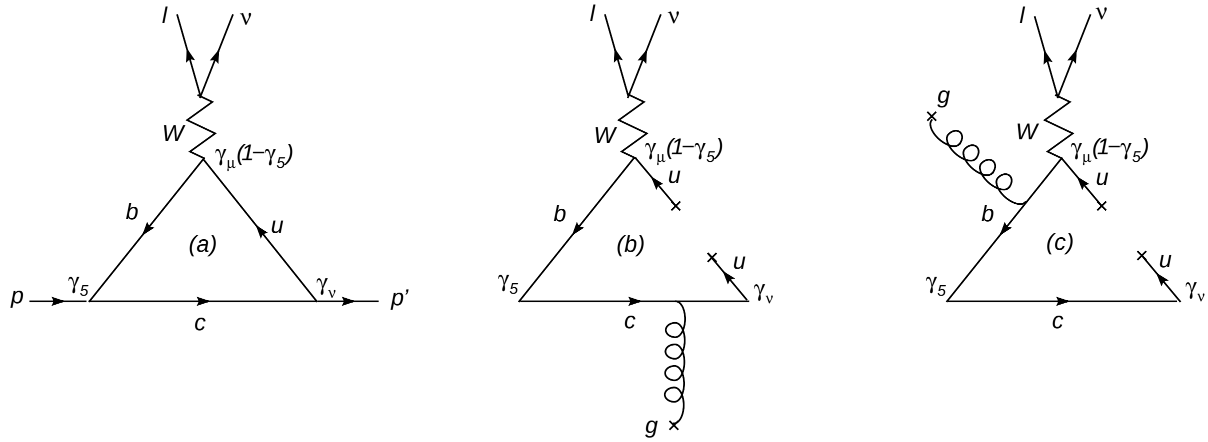

Figure 1: loop diagrams for $B_c \to D^* \ell \nu$ transitions, bare loop (diagram a) and light quark condensates with one gluon emission (diagrams b, c)

The heavy quark condensate contributions are suppressed by inverse of the heavy quark mass and can be safely omitted. The light u quark condensate contributions are zero after applying the double Borel transformation with respect to the momentum of the initial and final states because only one of them appears in the denominator (see in Fig. 1(b,c)).

As a result, the correlation functions receive contribution from the bare-loop, Fig. 1(a) and gluon condensates, Fig. 2(a-f) i.e.,

$$\Pi_i(p_1^2, p_2^2, q^2) = \Pi_i^{per}(p_1^2, p_2^2, q^2) + \Pi_i^{\langle G^2 \rangle}(p_1^2, p_2^2, q^2) \frac{\alpha_s}{\pi} \langle G^2 \rangle \ . \tag{13}$$

Using the double dispersion representation, the bare-loop contribution is determined:

$$\Pi_i^{per} = -\frac{1}{(2\pi)^2} \int \int \frac{\rho_i(s, s', q^2)}{(s - p^2)(s' - p'^2)} ds ds' + \text{subtraction terms} \ . \tag{14}$$

The following non-equalities give the integration limits of the Eq. (14):

$$-1 \leq \frac{2ss' + (s + s' - q^2)(m_b^2 - m_c^2 - s) + 2s(m_c^2 - m_u^2)}{\lambda^{1/2}(s, s', q^2)\lambda^{1/2}(m_b^2, m_c^2, s)} \leq +1 \ , \tag{15}$$

where $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$.

By replacing the propagators with the Dirac-delta functions(Gutkovsky rule):

$$\frac{1}{k^2 - m^2} \to -2i\pi\delta(k^2 - m^2) \ , \tag{16}$$

the spectral densities $\rho_i(s, s^{'}, q^2)$ are found as:

$$\begin{aligned} \rho_V &= -4\, N_c\ I_0(s, s', q^2) \left\{ m_c + B_2\,(m_c - \ m_u) + B_1\,(m_c - m_b) \right\} \ , \\ \rho_0 &= -2\, N_c\ I_0(s, s', q^2) \left\{ 4\, A_1\,(m_b - m_c) + \Delta'\,(m_b - m_c) - \Delta\,(m_c + m_u) \right. \end{aligned}$$

$$
\begin{aligned}
& +2\, {m_c}^2 (m_c - m_b - m_u) + m_c (u + 2\, m_b m_u) \Bigg\} \ , \\
\rho_1 \;\; = \;\; & -2\, N_c \; I_0(s,s',q^2) \; \Bigg\{ B_1 (\, m_b - 3 m_c) + 2\, A_2\, (m_b - m_c) + 2\, A_3\, (m_b - m_c) \\
& + B_2\, (m_u - \; m_c) - m_c \Bigg\} \ , \\
\rho_2 \;\; = \;\; & -2\, N_c \; I_0(s,s',q^2) \; \Bigg\{ 2\, A_3 (m_c - \; m_b) - B_1\, (m_b + m_c) + 2\, A_2\, (m_b - m_c) \\
& + B_2 (m_c - m_u) + \; m_c \Bigg\} \ .
\end{aligned} \tag{17}
$$

where

$$
\begin{aligned}
I_0(s,s',q^2) &= \frac{1}{4\lambda^{1/2}(s,s',q^2)}, \\
\lambda(s,s',q^2) &= s^2 + {s'}^2 + q^4 - 2sq^2 - 2s'q^2 - 2ss', \\
B_1 &= \frac{1}{\lambda(s,s',q^2)} [2s'\Delta - \Delta' u], \\
B_2 &= \frac{1}{\lambda(s,s',q^2)} [2s\Delta' - \Delta u], \\
A_1 &= \frac{1}{2\lambda(s,s',q^2)} [{\Delta'}^2 s + \Delta^2 s' - 4m_u^2 ss' - \Delta\Delta' u + m_c^2 u^2], \\
A_2 &= \frac{1}{\lambda^2(s,s',q^2)} [2{\Delta'}^2 ss' + 6\Delta^2 {s'}^2 - 8m_c^2 s{s'}^2 - 6\Delta\Delta' s' u \\
&\quad + {\Delta'}^2 u^2 + 2m_c^2 s' u^2], \\
A_3 &= \frac{1}{\lambda^2(s,s',q^2)} [-3\Delta^2 us' - 3{\Delta'}^2 us + 4m_c^2 us's + 4\Delta\Delta' ss' \\
&\quad + 2\Delta\Delta' u^2 - m_c^2 u^3].
\end{aligned}
$$

The $N_c = 3$ is the color factor, $u = s + s' - q^2$, $\Delta = s + m_c^2 - m_b^2$ and $\Delta' = s' + m_c^2 - m_u^2$.

3 Gluon condensate contribution

In this section, the gluon condensate contributions related to the non-perturbative part of the QCD sum rules are discussed. The diagrams for contributions of the gluon condensates are depicted in Fig. 2.

To calculate these diagrams, the Fock-Schwinger fixed-point gauge, $x^\mu G^a_\mu = 0$, are used, where G^a_μ is the gluon field. In the evaluation of the diagrams in Fig. 2, integrations of the following type are encountered.

$$
I_{\mu\nu\ldots\tau}(a,b,c) = \int \frac{d^4k}{(2\pi)^4} \frac{k_\mu k_\nu \ldots k_\tau}{[k^2 - m_b^2]^a [(p+k)^2 - m_c^2]^b [(p'+k)^2 - m_u^2]^c}. \tag{18}
$$

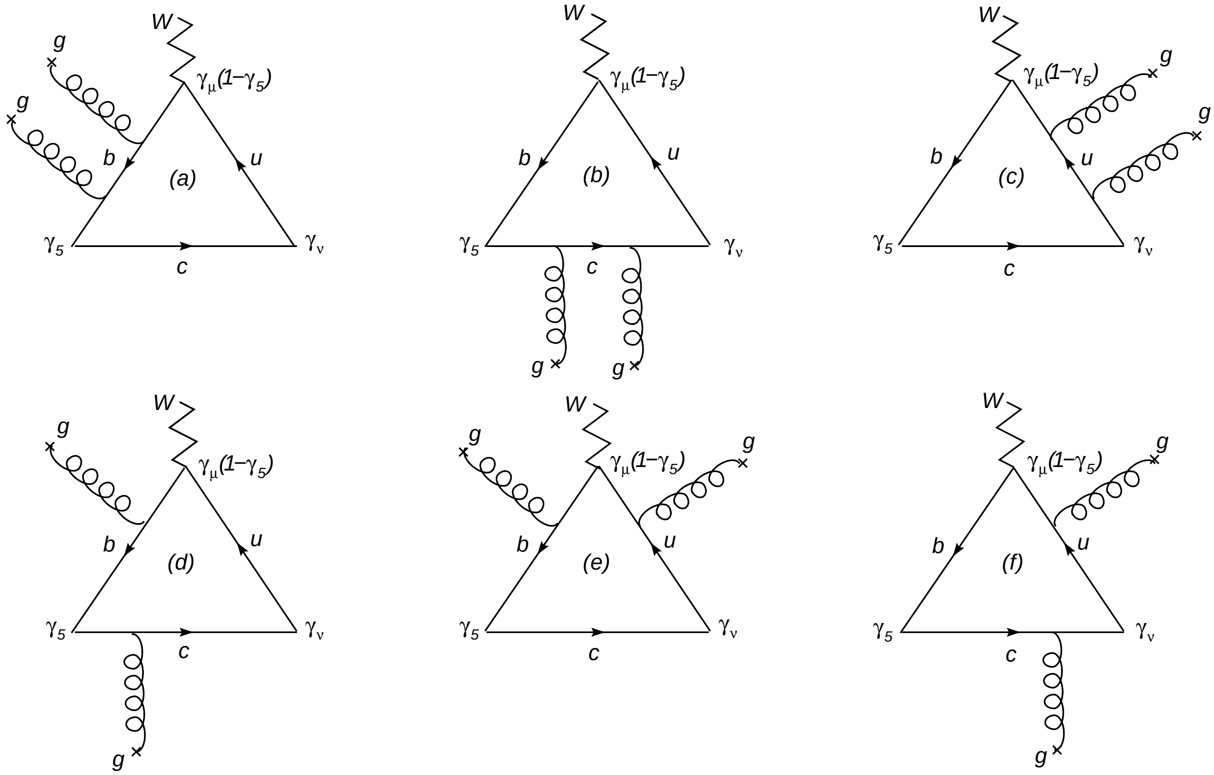

Figure 2: Gluon condensate contributions to $B_c \to D^* \ell \nu$ transitions

In our case, the following three types of integrals are appeared:

$$
\begin{aligned}
I_0(a,b,c) &= \int \frac{d^4k}{(2\pi)^4} \frac{1}{\left[k^2 - m_b^2\right]^a \left[(p+k)^2 - m_c^2\right]^b \left[(p'+k)^2 - m_u^2\right]^c} \, , \\
I_\mu(a,b,c) &= \int \frac{d^4k}{(2\pi)^4} \frac{k_\mu}{\left[k^2 - m_b^2\right]^a \left[(p+k)^2 - m_c^2\right]^b \left[(p'+k)^2 - m_u^2\right]^c} \, , \\
I_{\mu\nu}(a,b,c) &= \int \frac{d^4k}{(2\pi)^4} \frac{k_\mu k_\nu}{\left[k^2 - m_b^2\right]^a \left[(p+k)^2 - m_c^2\right]^b \left[(p'+k)^2 - m_u^2\right]^c} \, . \qquad (19)
\end{aligned}
$$

These integrals can be calculated by continuing to Euclidean space-time and using Schwinger representation for the Euclidean propagator

$$
\frac{1}{(k^2+m^2)^n} = \frac{1}{\Gamma(n)} \int_0^\infty d\alpha \, \alpha^{n-1} e^{-\alpha(k^2+m^2)} \, ,
$$

which fits the Borel transformation because:

$$
\mathcal{B}_{\hat{p}^2}(M^2) e^{-\alpha p^2} = \delta(1/M^2 - \alpha) \, .
$$

In order to obtain the Borel transformed form of the integrals in the Eq. (19), the integration is performed over loop momentum and over the two parameters used in the exponential representation of the propagators:

$$\hat{I}_0(a,b,c) = \frac{(-1)^{a+b+c}}{16\pi^2\,\Gamma(a)\Gamma(b)\Gamma(c)}(M_1^2)^{2-a-b}(M_2^2)^{2-a-c}\,\mathcal{U}_0(a+b+c-4,1-c-b)\ ,$$

$$\hat{I}_\mu(a,b,c) = \hat{I}_1(a,b,c)p_\mu + \hat{I}_2(a,b,c)p'_\mu\ ,$$

$$\hat{I}_{\mu\nu}(a,b,c) = \hat{I}_6(a,b,c)g_{\mu\nu} + \hat{I}_3 p_\mu p_\nu + \hat{I}_4 p_\mu p'_\nu + \hat{I}_4 p'_\mu p_\nu + \hat{I}_5 p'_\mu p'_\nu\ . \qquad (20)$$

$\hat{I}$ in the Eq.(20) stands for the double Borel transformed form of the Eq.(19), in the Schwinger representation.

where:

$$\hat{I}_{1(2)}(a,b,c) = i\frac{(-1)^{a+b+c+1}}{16\pi^2\,\Gamma(a)\Gamma(b)\Gamma(c)}(M_1^2)^{1-a-b+1(2)}(M_2^2)^{4-a-c-1(2)}\,\mathcal{U}_0(a+b+c-5,1-c-b)\ ,$$

$$\hat{I}_j(a,b,c) = i\frac{(-1)^{a+b+c+1}}{16\pi^2\,\Gamma(a)\Gamma(b)\Gamma(c)}(M_1^2)^{-a-b-1+j}(M_2^2)^{7-a-c-j}\,\mathcal{U}_0(a+b+c-5,1-c-b)\ ,$$

$$\hat{I}_6(a,b,c) = i\frac{(-1)^{a+b+c+1}}{32\pi^2\,\Gamma(a)\Gamma(b)\Gamma(c)}(M_1^2)^{3-a-b}(M_2^2)^{3-a-c}\,\mathcal{U}_0(a+b+c-6,2-c-b)\ , \qquad (21)$$

Here, $j = 3,4,5$, M_1^2 and M_2^2 are the Borel parameters in the s and s' channel, respectively, and the function $\mathcal{U}_0(\alpha,\beta)$ is defined as

$$\mathcal{U}_0(a,b) = \int_0^\infty dy(y+M_1^2+M_2^2)^a y^b\, exp\left[-\frac{B_{-1}}{y} - B_0 - (B_{+1})y\right]\ ,$$

where

$$B_{-1} = \frac{1}{M_1^2M_2^2}\left[m_u^2M_1^4 + m_b^2M_2^4 + M_2^2M_1^2(m_b^2+m_u^2-q^2)\right]\ ,$$

$$B_0 = \frac{1}{M_1^2M_2^2}\left[(m_u^2+m_c^2)M_1^2 + M_2^2(m_b^2+m_c^2)\right]\ ,$$

$$B_{+1} = \frac{m_c^2}{M_1^2M_2^2}\ .$$

Performing the double Borel transformations over the variables p^2 and $p^{'2}$ on the physical parts of the correlation functions and bare-loop diagrams and also equating two representations of the correlation functions, the sum rules for the form factors f_i are obtained:

$$f'_i = \frac{(m_b+m_c)}{f_{B_c}m_{B_c}^2 f_{D^*}m_{D^*}} e^{\frac{m_{B_c}^2}{M_1^2}} e^{\frac{m_{D^*}^2}{M_2^2}} \times\left\{-\frac{1}{4\pi^2}\int_{m_b^2}^{s'_0} ds'\int_{s_L}^{s_0}\rho_i(s,s',q^2)e^{\frac{-s}{M_1^2}}e^{\frac{-s'}{M_2^2}} - iM_1^2M_2^2\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\frac{C_4^i}{6}\right\}\ . \qquad (22)$$

where $i = V, 0, +1$ and 2, s_0 and $s_0^{'}$ are the continuum thresholds in the pseudoscalar B_c and vector D^* channels, respectively, and the lower bound integration limit of s_L is as follow:

$$s_L = \frac{(m_c^2 + q^2 - m_b^2 - s')(m_b^2 s' - m_c^2 q^2)}{(m_b^2 - q^2)(m_c^2 - s')} \ .$$

The explicit expressions for the C_4^i are presented in the Appendix–A.

4 Heavy quark effective theory

In the present section, we analyze the infinite heavy quark mass limit of the form factors of the $B_c \to D^* \ell \nu$ calculated by 3PSR. To obtain the dependency of the form factors f_V, f_0, f_1 and f_2 on y the following parameterization is used: (see also[33])

$$y = \nu\nu' = \frac{m_{B_c}^2 + m_{D^*}^2 - q^2}{2 m_{B_c} m_{D^*}} \tag{23}$$

We also apply these definitions:

$$\begin{aligned} m_c &= \frac{m_b}{\sqrt{Z}}, \\ \sqrt{Z} &= y + \sqrt{y^2 - 1}, \\ T_1 &= \frac{M_1^2}{2m_b}, \\ T_2 &= \frac{M_2^2}{2m_c}. \end{aligned} \tag{24}$$

Where $m_b \to \infty$. In the above expressions, T_1 and T_2 are the new Borel parameters. The mass of light quark u is set to zero.

The continuum thresholds ν_0, $\nu_0^{'}$ and integration variables ν, $\nu^{'}$ are defined as:

$$\nu_0 = \frac{s_0 - m_b^2}{m_b}, \qquad \nu_0' = \frac{s_0' - m_c^2}{m_c}, \tag{25}$$

$$\nu = \frac{s - m_b^2}{m_b}, \qquad \nu' = \frac{s' - m_c^2}{m_c}. \tag{26}$$

The leptonic decay constants are also rescaled as:

$$\hat{f}_{B_c} = \sqrt{m_b} f_{B_c}, \qquad \hat{f}_{D^*} = \sqrt{m_c} f_{D^*}. \tag{27}$$

The correspond expressions for $\overline{I}_0(a,b,c)$, $\overline{I}_{1(2)}(a,b,c)$, $\overline{I}_j(a,b,c)$; $j = 3, 4, 5$ and $\overline{I}_6(a,b,c)$ in this limit are defined as:

$$\begin{aligned} \overline{I}_0(a,b,c) = & \frac{(-1)^{a+b+c}}{16\pi^2\,\Gamma(a)\Gamma(b)\Gamma(c)} (\frac{1}{\sqrt{Z}})^{2-a-c} (2m_b)^{4-2a-b-c}\ T_1^{2-a-b}\ T_2^{2-a-c} \\ & \mathcal{U}_0^{HQET}(a+b+c-4, 1-c-b)\ , \end{aligned}$$

$$\overline{I}_{1(2)}(a,b,c) = i\frac{(-1)^{a+b+c+1}}{16\pi^2\,\Gamma(a)\Gamma(b)\Gamma(c)}(\frac{1}{\sqrt{Z}})^{4-a-c-1(2)}(2m_b)^{5-2a-b-c}\ T_1^{1-a-b+1(2)}\ T_2^{4-a-c-1(2)}$$
$$\mathcal{U}_0^{HQET}(a+b+c-5,1-c-b)\ ,$$

$$\overline{I}_j(a,b,c) = i\frac{(-1)^{a+b+c}}{16\pi^2\,\Gamma(a)\Gamma(b)\Gamma(c)}(\frac{1}{\sqrt{Z}})^{7-a-c-j}(2m_b)^{6-2a-b-c}\ T_1^{-a-b-1+j}\ T_2^{7-a-c-j}$$
$$\mathcal{U}_0^{HQET}(a+b+c-6,1-c-b)\ ,$$

$$\overline{I}_6(a,b,c) = i\frac{(-1)^{a+b+c+1}}{32\pi^2\,\Gamma(a)\Gamma(b)\Gamma(c)}(\frac{1}{\sqrt{Z}})^{3-a-c}(2m_b)^{6-2a-b-c}\ T_1^{3-a-b}\ T_2^{3-a-c}$$
$$\mathcal{U}_0^{HQET}(a+b+c-6,2-c-b)\ . \tag{28}$$

The function $\mathcal{U}_0^{HQET}(m,n)$ takes the following form

$$\mathcal{U}_0^{HQET}(m,n) = \int_0^\infty (2m_b)^m\ (\frac{x}{2m_b}+T_1+\frac{T_2}{\sqrt{Z}})^m\ x^n[-\frac{\overline{B}_{-1}}{x}-\overline{B}_0-\overline{B}_1 x]dx, \tag{29}$$

with

$$\begin{aligned}
\overline{B}_{-1} &= \frac{\sqrt{Z}}{T_1T_2}\,[\frac{mb^2}{Z}\ T_2^2+\frac{1}{\sqrt{Z}}T_1T_2(m_b^2-q^2)],\\
\overline{B}_0 &= \frac{\sqrt{Z}}{2m_bT_1T_2}\ [m_c^2T_1+\frac{T_2}{\sqrt{Z}}(m_b^2+m_c^2)],\\
\overline{B}_1 &= \frac{1}{4\sqrt{Z}T_1T_2}.
\end{aligned} \tag{30}$$

After some calculations, we obtain the y-dependent expressions of the form factors as follows:

$$\begin{aligned}
f_V^{HQET}(y) &= \frac{1}{\hat{f}_{B_c}\hat{f}_{D^*}}e^{\frac{\Lambda}{T_1}}e^{\frac{\overline{\Lambda}}{T_2}}\Bigg\{\frac{1}{(1+\sqrt{Z})Z^{\frac{7}{4}}\sqrt{\frac{-1+y^2}{Z}}}\\
&\Bigg[-3+(3+9\,y)\sqrt{Z}-6\,y(1+y)Z\Bigg]\\
&\frac{1}{(2\pi)^2}\int_0^{\nu_0}d\nu\int_0^{\nu_0'}d\nu'\,e^{-\frac{\nu}{2T_1}}e^{-\frac{\nu'}{2T_2}}\theta(2y\nu\nu'-\nu^2-\nu'^2)\\
&+\lim_{m_b\to\infty}\ \left(i\ \frac{Z^{\frac{3}{4}}}{3m_b^3(1+\sqrt{Z})}\ \left\langle\frac{\alpha_s}{\pi}G^2\right\rangle C_V^{HQET}\right)\Bigg\}\ ,
\end{aligned} \tag{31}$$

$$\begin{aligned}
f_0^{HQET}(y) &= \frac{1}{\hat{f}_{B_c}\hat{f}_{D^*}}e^{\frac{\Lambda}{T_1}}e^{\frac{\overline{\Lambda}}{T_2}}\Bigg\{\frac{1}{2(1+\sqrt{Z})^3Z^{\frac{7}{4}}\sqrt{\frac{-1+y^2}{Z}}}\\
&\Bigg[3+3\,\sqrt{Z}\{-1+4\,y+2\,(-1+y)(1+3\,y)\sqrt{Z}-4\,y(-1+y+y^2)Z\}\Bigg]
\end{aligned}$$

$$
\frac{1}{(2\pi)^2}\int_0^{\nu_0} d\nu \int_0^{\nu_0^{'}} d\nu^{'} e^{-\frac{\nu}{2T_1}} e^{-\frac{\nu'}{2T_2}} \theta(2y\nu\nu^{'} - \nu^2 - \nu'^{\,2})
$$

$$
+ \lim_{m_b\rightarrow\infty} \left(i\ \frac{Z^{\frac{7}{4}}}{6m_b^5(1+\sqrt{Z})^3}\ \left\langle\frac{\alpha_s}{\pi}G^2\right\rangle C_0^{HQET}\right)\Bigg\}\ , \qquad (32)
$$

$$
\begin{aligned}
f_1^{HQET}(y) &= \frac{1}{\hat{f}_{B_c}\hat{f}_{D^*}} e^{\frac{\Lambda}{T_1}} e^{\frac{\bar{\Lambda}}{T_2}} \Bigg\{ \frac{1}{4(1+\sqrt{Z})(\frac{-1+y^2}{Z})^{\frac{3}{2}} Z^{\frac{15}{4}}} \\
&\Big[9 - 9\,(1+5\,y)\sqrt{Z} + (-6+45\,y+78\,y^2)Z \\
&-6\,(1+y[-3+y(11+9\,y)])Z^{\frac{3}{2}} + 12\,y(1+y[-1+y(2+y)])Z^2\Big] \\
&\frac{1}{(2\pi)^2}\int_0^{\nu_0} d\nu \int_0^{\nu_0^{'}} d\nu^{'} e^{-\frac{\nu}{2T_1}} e^{-\frac{\nu'}{2T_2}} \theta(2y\nu\nu^{'} - \nu^2 - \nu'^{\,2}) \\
&+ \lim_{m_b\rightarrow\infty} \left(i\ \frac{Z^{\frac{3}{4}}}{6m_b^3(1+\sqrt{Z})}\ \left\langle\frac{\alpha_s}{\pi}G^2\right\rangle C_1^{HQET}\right)\Bigg\}\ , \qquad (33)
\end{aligned}
$$

$$
\begin{aligned}
f_2^{HQET}(y) &= \frac{1}{\hat{f}_{B_c}\hat{f}_{D^*}} e^{\frac{\Lambda}{T_1}} e^{\frac{\bar{\Lambda}}{T_2}} \Bigg\{ \frac{1}{4(1+\sqrt{Z})(\frac{-1+y^2}{Z})^{\frac{3}{2}} Z^{\frac{15}{4}}} \\
&\Big[9 - 9\,(1+3\,y)\sqrt{Z} + 9\,(2+y)(-1+2\,y)Z \\
&+6\,(1+y[5+(-1+y)y])Z^{\frac{3}{2}} - 12\,y(1+y[-1+y(2+y)])Z^2\Big] \\
&\frac{1}{(2\pi)^2}\int_0^{\nu_0} d\nu \int_0^{\nu_0^{'}} d\nu^{'} e^{-\frac{\nu}{2T_1}} e^{-\frac{\nu'}{2T_2}} \theta(2y\nu\nu^{'} - \nu^2 - \nu'^{\,2}) \\
&+ \lim_{m_b\rightarrow\infty} \left(i\ \frac{Z^{\frac{3}{4}}}{6m_b^3(1+\sqrt{Z})}\ \left\langle\frac{\alpha_s}{\pi}G^2\right\rangle C_2^{HQET}\right)\Bigg\}\ . \qquad (34)
\end{aligned}
$$

In the heavy quark limit expressions of the form factors, the $\Lambda = m_{B_c} - m_b$ and $\bar{\Lambda} = m_{D_u^*} - m_c$ and the explicit expressions of the coefficients C_i^{HQET} are given in the Appendix–B.

At the end of this section, we would like to present the differential decay width $d\Gamma/dq^2$ for the process $B_c^- \rightarrow D^{*0}\ell\nu$ in terms of the form factors as follow:

$$
\begin{aligned}
\frac{d\Gamma_\pm(B_c\rightarrow D^*\ell\nu)}{dq^2} &= \frac{G^2\left|V_{ub}\right|^2}{192\pi^3 m_{B_c}^3}\ q^2\lambda^{1/2}(m_{B_c}^2, m_{D^*}^2, q^2)\left|H_\pm\right|^2\ , \\
\frac{d\Gamma_0(B_c\rightarrow D^*\ell\nu)}{dq^2} &= \frac{G^2\left|V_{ub}\right|^2}{192\pi^3 m_{B_c}^3}\ q^2\lambda^{1/2}(m_{B_c}^2, m_{D^*}^2, q^2)\left|H_0\right|^2\ , \qquad (35)
\end{aligned}
$$

$$
\begin{aligned}
H_\pm(q^2) &= (m_{B_c} + m_{D^*})f_0(q^2) \mp \frac{\lambda^{1/2}(m_{B_c}^2, m_{D^*}^2, q^2)}{m_{B_c}+m_{D^*}} f_V(q^2)\ , \\
H_0(q^2) &= \frac{1}{2m_{D^*}\sqrt{q^2}}\left[(m_{B_c}^2 - m_{D^*}^2 - q^2)(m_{B_c}+m_{D^*})f_0(q^2) - \frac{\lambda(m_{B_c}^2, m_{D^*}^2, q^2)}{m_{B_c}+m_{D^*}} f_1(q^2)\right]\ .
\end{aligned}
$$

where $\pm, 0$ refers to the D^* helicities.

5 Numerical analysis

The sum rules expressions of the form factors depict that the main input parameters entering the expressions are gluon condensate, elements of the CKM matrix V_{ub}, leptonic decay constants, f_{B_C} and f_{D^*}, Borel parameters M_1^2 and M_2^2, as well as the continuum thresholds s_0 and s_0'. We choose the values of the Gluon condensate, leptonic decay constants, CKM matrix elements, quark and meson masses as: $< \frac{\alpha_s}{\pi} G^2 >= 0.012\ GeV^4$ [34], $\mid V_{ub} \mid = 0.0037$ [31], $f_{B_c} = 0.35 \pm 0.025\ GeV$[32,35], $f_{D^*} = 0.22 \pm 0.016\ GeV$, $m_c(\mu = m_c) = 1.275 \pm 0.015\ GeV$, $m_u = (1.5 - 3)\ MeV$, $m_b = (4.7 \pm 0.01)\ GeV$, $m_{D^*} = 2.007\ GeV$, $m_{B_C} = 6.258\ GeV$[36], $\Lambda = 0.62\ GeV$[37] and $\overline{\Lambda} = 0.86\ GeV$[38].

The expressions for the form factors contain also four auxiliary parameters: Borel mass squares M_1^2 and M_2^2 and continuum threshold s_0 and s_0'. These are mathematical objects, so the physical quantities, form factors, should be independent of them. The parameters s_0 and s_0', which are the continuum thresholds of B_c and D^* mesons, respectively, are determined from the conditions that guarantee the sum rules to have the best stability in the allowed M_1^2 and M_2^2 region. The values of the continuum thresholds calculated from the two–point QCD sum rules are taken to be $s_0 = (45 - 50)\ GeV^2$ and $s_0' = (6 - 8)\ GeV^2$ [14, 34, 39]. The working regions for M_1^2 and M_2^2 are determined by requiring that not only contributions of the higher states and continuum are effectively suppressed, but the gluon condensate contributions are small, which guarantees that the contributions of higher dimensional operators are small. Both conditions are satisfied in the regions $10\ GeV^2 \leq M_1^2 \leq 20\ GeV^2$ and $4\ GeV^2 \leq M_2^2 \leq 10\ GeV^2$.

The dependence of the form factors f_V, f_0, f_1 and f_2 on M_1^2 and M_2^2 for $B_c \to D^* \ell \nu$ are shown in Fig. 3. This figure shows a good stability of the form factors with respect to the Borel mass parameters in the working regions. Our numerical analysis shows that the contribution of the non-perturbative part (the gluon condensate diagrams) is about 5% of the total and the main contribution comes from the perturbative part of the form factors. This means that the contribution of the higher dimension operators is small and this guarantees the convergence of the sum rules expression of the form factors and those sum rules are reliable.

The values of the form factors at $q^2 = 0$ are shown in Table.1. In comparison, the predictions of the other approaches are also presented in this Table.

The sum rules for the form factors are truncated at about $10\ GeV^2$, so to extend our results to the full physical region, we look for a parameterization of the form factors in such a way that in the region $0 \leq q^2 \leq 10\ GeV^2$, this parameterization coincides with the sum rules predictions. Our numerical calculations shows that the sufficient parameterization of the form factors with respect to q^2 is as follows:

$$f_i(q^2) = \frac{f_i(0)}{1 + \alpha \hat{q} + \beta \hat{q}^2} \tag{36}$$

where $\hat{q} = q^2/m_{B_c}^2$. The values of the parameters $f_i(0)$, α and β are given in the Table 2.

The errors are estimated by the variation of the Borel parameters M_1^2 and M_2^2, the

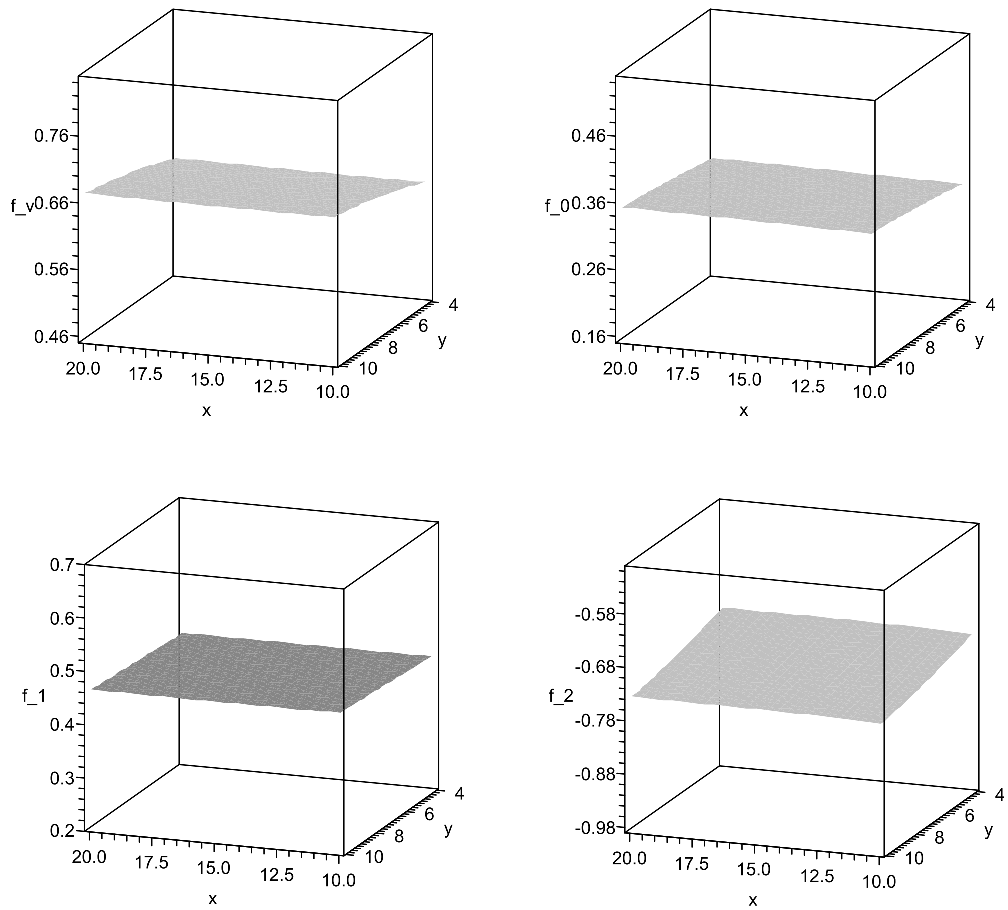

Figure 3: The dependence of the form factors on $x = M_1^2$ and $y = M_2^2$ for $B_c^- \to D^{*0}\ell\nu$.

variation of the continuum thresholds s_0 and s_0', the leptonic decay constants f_{B_c} and f_{D^*} and uncertainties in the values of the other input parameters. The main uncertainty comes from the continuum thresholds and the decay constants, which is about $\sim 19\%$ of the central value, while the other uncertainties are small, constituting a few percent.

Now, we compare the values for the form factors and their HQET values obtained from Eqs. (31-34) in Table 3 for $B_c^- \to D^{*0}\ell\nu$.

At $y = 1$ called the zero recoil limit, the HQET limit of the form factors are not finite and at this value, we can determine only the ratio of the form factors. For other values of y and corresponding q^2, the behavior of the form factors and their HQET values are the same, i.e., when y increases (q^2 decreases) both the form factors and their HQET values decrease. Moreover, at high q^2 values, the form factors and their HQET values are close to each other. For better comparison we prefer to plot the dependence of the relevant form factors and HQET limit of them on the momentum transfer square q^2 (Fig. 4). This figure shows a good agreement between both form factors and their HQET at high q^2 values. This figure also contains the fit function of the form factors (see Eq. (22)). The form factors and their fit functions coincide well in the interval $0 \leq q^2 \leq 10\ GeV^2$.

	Our					
Form factor	3PSR	HQET	LCSR[31]	3PSR[13]	PM[19]	QM[29]
$f_V^{B_c\to D^*}$	0.67 ± 0.16	0.36 ± 0.09	0.57	0.83	0.80	0.98
$f_0^{B_c\to D^*}$	0.35 ± 0.09	0.25 ± 0.06	0.32	0.43	0.43	0.56
$f_1^{B_c\to D^*}$	0.46 ± 0.11	0.20 ± 0.05	0.57	0.51	0.49	0.64
$f_2^{B_c\to D^*}$	-0.74 ± 0.18	-0.35 ± 0.09	-0.57	-0.83	-0.89	-1.17

Table 1: The form factors of the $B_c^- \to D^{*0}\ell\nu$ decay for $M_1^2 = 17\ GeV^2$, $M_2^2 = 8\ GeV^2$ at $q^2 = 0$ in different approaches: three–point sum rules (3PSR) with gluon condensate corrections, heavy quark effective theory (HQET), light cone sum rules (LCSR), three–point sum rules without gluon condensate corrections (3PSR), potential model(PM) and quark model(QM).

	f(0)	α	β
f_V	0.67	-0.53	-0.26
f_0	0.35	0.38	-3.08
f_1	0.46	1.92	-16.43
f_2	-0.74	1.04	-15.77

Table 2: Parameters appearing in the form factors of the $B_c \to D^*\ell\nu$ decay for $M_1^2 = 17\ GeV^2$, $M_2^2 = 8\ GeV^2$.

At the end of this section, we would like to present the values of the branching ratio for $B_c^- \to D^{*0}\ell\nu$. Integrating Eq. (35) over q^2 in the whole physical region and using the total mean life time $\tau \simeq 0.48 \pm 0.05\ ps$ of B_c meson [40], the branching ratio of the $B_c^- \to D^{*0}\ell\nu$ decay is obtained as presented in Table 4. The branching ratio of this decay obtained using the HQET limit of form factors Eqs. (31-34) is also shown in this Table. This Table also includes a comparison between our results via both SR and HQET and the predictions of the other approaches including the LCSR, 3PSR (without gluon condensate corrections), QM, BSE, PM and RM estimates.

Conclusion

Considering the gluon corrections, we investigated the $B_c^- \to D^{*0}\ell\nu$ channel in the frame work of three–point QCD sum rules. We found that the gluon correction contributions to the sum rules expression of the form factors are small. This implies the small contribution of the higher dimension operators and also it guarantees that the sum rules for the form factors are convergent and reliable. The HQET limit of the form factors with their corresponding

y	1	1.02	1.1	1.2	1.3	1.4	1.5	1.6	1.7
q^2	18.00	17.50	15.49	12.99	10.48	7.97	5.46	2.95	0.44
$f_V(q^2)$	5.81	5.11	2.87	1.86	1.38	1.10	0.91	0.79	0.68
$f_V^{HQET}(y)$	-	4.98	0.84	0.59	0.50	0.48	0.43	0.39	0.37
$f_0(q^2)$	0.83	0.72	0.57	0.48	0.42	0.39	0.37	0.36	0.35
$f_0^{HQET}(y)$	-	0.69	0.36	0.32	0.30	0.29	0.28	0.27	0.26
$f_1(q^2)$	2.18	1.89	1.19	0.84	0.68	0.59	0.52	0.49	0.47
$f_1^{HQET}(y)$	-	1.74	0.35	0.29	0.26	0.24	0.23	0.22	0.21
$f_2(q^2)$	-5.87	-5.01	-2.61	-1.69	-1.29	-1.07	-0.96	-0.85	-0.76
$f_2^{HQET}(y)$	-	-4.79	-0.62	-0.49	-0.44	-0.42	-0.39	-0.37	-0.36

Table 3: The comparison of the values for the form factors and their HQET limit for $B_c^- \to D^{*0}\ell\nu$ at $M_1^2 = 17\ GeV^2, M_2^2 = 8\ GeV^2$ and corresponding $T_1 = 1.80\ GeV, T_2 = 3.14\ GeV$.

	Our							
Mode	3PSR	HQET	LCSR[31]	3PSR[13]	QM[29]	PM[25]	BSE[30]	RM[22]
$D^*e\nu$	$(2.2 \pm 0.5)10^{-2}$	$(3 \pm 0.7)10^{-3}$	0.035	0.018	0.034	0.004	0.018	0.013
$D^*\tau\nu$	$(1.2 \pm 0.3)10^{-2}$	$(1 \pm 0.2)10^{-3}$	0.020	0.008	0.019	-	-	-

Table 4: The branching ratio of the $B_c^- \to D^{*0}\ell\nu$ decay in different approaches: 3PSR with gluon condensate corrections, HQET, LCSR [31], 3PSR without gluon corrections [13] , QM [29], PM [25], the Bethe-Salpeter equation(BSE) [30] and a relativistic model with factorization to obtain the nonleptonic decay widths (RM) [22].

gluon condensate corrections are also computed. A Comparison between the form factors and their HQET was made. Finally, we evaluated the total decay width and the branching fraction of this decay and compared with the predictions of the other approaches such as LCSR, 3PSR, QM, PM, BSE and RM.

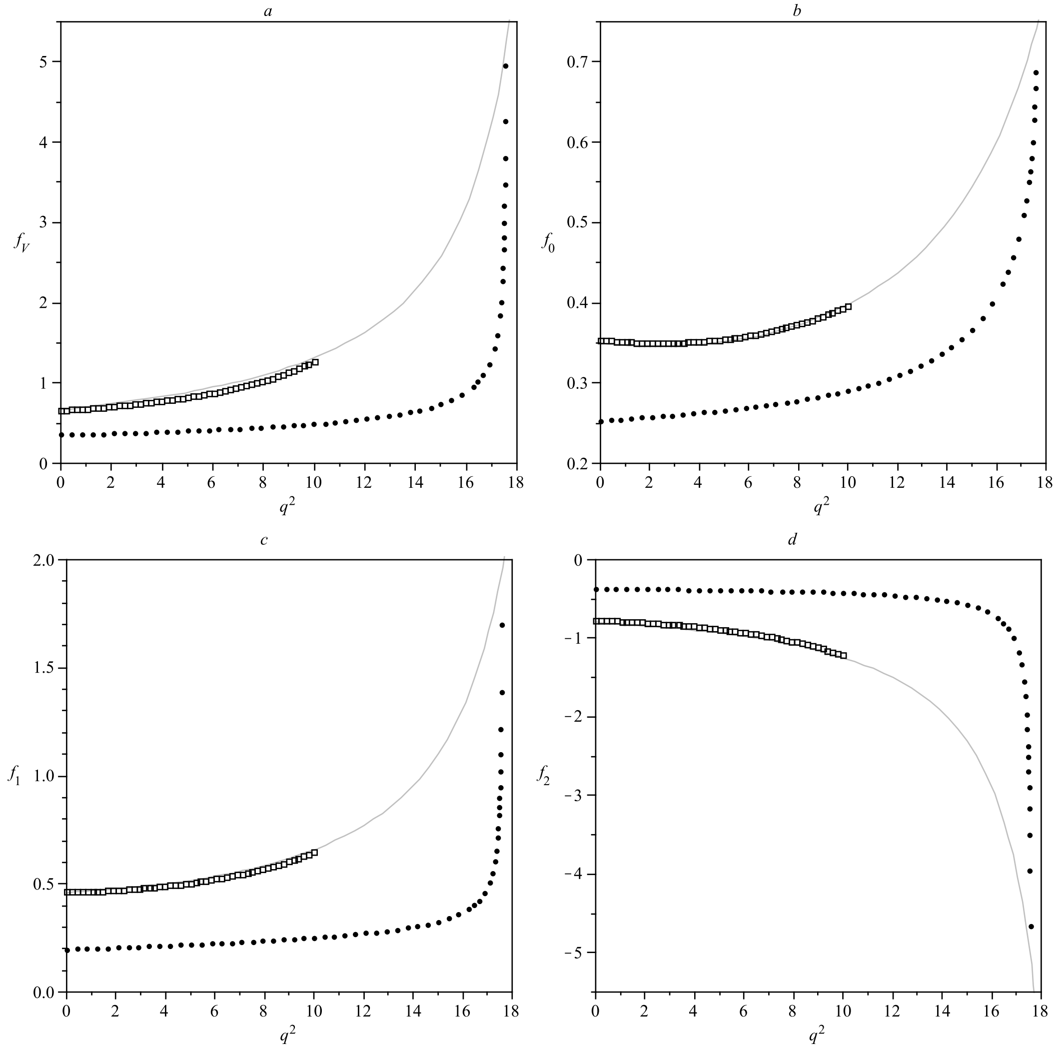

Figure 4: The dependence of the form factors and their HQET limit as well as the fit parameterization of the form factors on q^2. The small boxes correspond to the form factors, the solid lines are belong to the fit parameterization of form factors and dotted lines show the HQET limit of the form factors.

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Appendix–A

In this appendix, the explicit expressions of the coefficients of the gluon condensate entering the sum rules of the form factors f_V, f_0, f_1 and f_2 are given.

$$
\begin{aligned}
C_4^V \;=\;& -10\,\hat{I}_1(3,2,2){m_c}^5-10\,\hat{I}_2(3,2,2){m_c}^5-10\,\hat{I}_0(3,2,2){m_c}^5+10\,\hat{I}_1(3,2,2){m_c}^4m_b \\
& +10\,\hat{I}_0(3,2,2){m_c}^3{m_b}^2+10\,\hat{I}_2(3,2,2){m_c}^3{m_b}^2+10\,\hat{I}_1(3,2,2){m_c}^3{m_b}^2-10\,\hat{I}_1(3,2,2){m_c}^2{m_b}^3 \\
& -10\,\hat{I}_0(3,1,2){m_c}^3+20\,\hat{I}_0^{[0,1]}(3,2,2){m_c}^3+20\,\hat{I}_2^{[0,1]}(3,2,2){m_c}^3-30\,\hat{I}_0(4,1,1){m_c}^3 \\
& -20\,\hat{I}_2(2,2,2){m_c}^3-10\,\hat{I}_2(3,1,2){m_c}^3+10\,\hat{I}_2(3,2,1){m_c}^3-20\,\hat{I}_0(2,2,2){m_c}^3 \\
& -20\,\hat{I}_1(2,2,2){m_c}^3-30\,\hat{I}_2(4,1,1){m_c}^3-30\,\hat{I}_1(4,1,1){m_c}^3+20\,\hat{I}_1^{[0,1]}(3,2,2){m_c}^3 \\
& +30\,\hat{I}_1(4,1,1){m_c}^2m_b+20\,\hat{I}_1(2,2,2){m_c}^2m_b-10\,\hat{I}_2(3,2,1){m_c}^2m_b-20\,\hat{I}_1^{[0,1]}(3,2,2){m_c}^2m_b \\
& +10\,\hat{I}_1(3,2,1){m_c}^2m_b+20\,\hat{I}_1(2,3,1){m_c}^2m_b+40\,\hat{I}_0(2,3,1){m_c}^2m_b+10\,\hat{I}_0(3,2,1){m_c}^2m_b \\
& +10\,\hat{I}_2^{[0,1]}(3,2,2)m_c{m_b}^2+10\,\hat{I}_1^{[0,1]}(3,2,2)m_c{m_b}^2-30\,\hat{I}_1(3,2,1)m_c{m_b}^2-20\,\hat{I}_2(3,2,1)m_c{m_b}^2 \\
& -20\,\hat{I}_0(3,2,1)m_c{m_b}^2+10\,\hat{I}_0^{[0,1]}(3,2,2)m_c{m_b}^2+60\,\hat{I}_1(1,4,1)m_c{m_b}^2+60\,\hat{I}_0(1,4,1)m_c{m_b}^2 \\
& -20\,\hat{I}_1(2,3,1){m_b}^3-60\,\hat{I}_1(1,4,1){m_b}^3-10\,\hat{I}_1^{[0,1]}(3,2,2){m_b}^3+10\,\hat{I}_1(2,2,2){m_b}^3 \\
& +20\,\hat{I}_1(3,2,1){m_b}^3+20\,\hat{I}_1(2,2,1)m_c+30\,\hat{I}_2^{[0,1]}(3,1,2)m_c-30\,\hat{I}_0(2,1,2)m_c \\
& -20\,\hat{I}_1(1,2,2)m_c+20\,\hat{I}_2^{[0,1]}(2,2,2)m_c-10\,\hat{I}_1^{[0,2]}(3,2,2)m_c-20\,\hat{I}_2(1,2,2)m_c \\
& +20\,\hat{I}_1^{[0,1]}(2,2,2)m_c+10\,\hat{I}_2(3,1,1)m_c-20\,\hat{I}_0(1,2,2)m_c-10\,\hat{I}_1(3,1,1)m_c \\
& +20\,\hat{I}_1^{[0,1]}(3,2,1)m_c+20\,\hat{I}_0^{[0,1]}(3,2,1)m_c+30\,\hat{I}_2(2,2,1)m_c-10\,\hat{I}_0^{[0,2]}(3,2,2)m_c \\
& +20\,\hat{I}_0(2,2,1)m_c+30\,\hat{I}_0^{[0,1]}(3,1,2)m_c-10\,\hat{I}_2^{[0,2]}(3,2,2)m_c+20\,\hat{I}_1^{[0,1]}(3,1,2)m_c \\
& +20\,\hat{I}_0^{[0,1]}(2,2,2)m_c-30\,\hat{I}_2(2,1,2)m_c-10\,\hat{I}_0(3,1,1)m_c+10\,\hat{I}_2^{[0,1]}(3,2,1)m_c \\
& -20\,\hat{I}_1(2,1,2)m_c+20\,\hat{I}_1^{[0,1]}(2,3,1)m_b+30\,\hat{I}_1(2,1,2)m_b+10\,\hat{I}_1^{[0,2]}(3,2,2)m_b \\
& +40\,\hat{I}_2^{[0,1]}(2,3,1)m_b+20\,\hat{I}_1(1,2,2)m_b-20\,\hat{I}_0(2,2,1)m_b+60\,\hat{I}_2(1,3,1)m_b \\
& -20\,\hat{I}_1^{[0,1]}(3,2,1)m_b-20\,\hat{I}_1^{[0,1]}(2,2,2)m_b+100\,\hat{I}_0(1,3,1)m_b-20\,\hat{I}_1^{[0,1]}(3,1,2)m_b \\
& -50\,\hat{I}_1(2,2,1)m_b-20\,\hat{I}_2(2,2,1)m_b+20\,\hat{I}_1(1,3,1)m_b
\end{aligned}
$$

$$
\begin{aligned}
C_4^0 \;=\;& 5\,\hat{I}_0(3,2,2){m_c}^6m_b-5\,\hat{I}_0(3,2,2){m_c}^5{m_b}^2-5\,\hat{I}_0(3,2,2){m_c}^4{m_b}^3+5\,\hat{I}_0(3,2,2){m_c}^3{m_b}^4 \\
& +20\,\hat{I}_6(3,2,2){m_c}^5-20\,\hat{I}_6(3,2,2){m_c}^4m_b+5\,\hat{I}_0(3,2,1){m_c}^4m_b+15\,\hat{I}_0(2,2,2){m_c}^4m_b \\
& -15\,\hat{I}_0^{[0,1]}(3,2,2){m_c}^4m_b+15\,\hat{I}_0(4,1,1){m_c}^4m_b+10\,\hat{I}_0(2,3,1){m_c}^4m_b-10\,\hat{I}_0(2,2,2){m_c}^3{m_b}^2 \\
& -5\,\hat{I}_0(3,2,1){m_c}^3{m_b}^2-20\,\hat{I}_6(3,2,2){m_c}^3{m_b}^2-15\,\hat{I}_0(4,1,1){m_c}^3{m_b}^2+10\,\hat{I}_0^{[0,1]}(3,2,2){m_c}^3{m_b}^2 \\
& +10\,\hat{I}_0(3,2,1){m_c}^2{m_b}^3-30\,\hat{I}_0(1,4,1){m_c}^2{m_b}^3-10\,\hat{I}_0(2,3,1){m_c}^2{m_b}^3+20\,\hat{I}_6(3,2,2){m_c}^2{m_b}^3 \\
& -10\,\hat{I}_0(3,2,1)m_c{m_b}^4+5\,\hat{I}_0^{[0,1]}(3,2,2)m_c{m_b}^4+30\,\hat{I}_0(1,4,1)m_c{m_b}^4+40\,\hat{I}_6(3,2,1){m_c}^3 \\
& -5\,\hat{I}_0(3,1,1){m_c}^3+20\,\hat{I}_6(3,1,2){m_c}^3-40\,\hat{I}_6^{[0,1]}(3,2,2){m_c}^3+40\,\hat{I}_6(2,2,2){m_c}^3 \\
& +60\,\hat{I}_6(4,1,1){m_c}^3-40\,\hat{I}_6(3,2,1){m_c}^2m_b-20\,\hat{I}_6(3,1,2){m_c}^2m_b-20\,\hat{I}_6(2,2,2){m_c}^2m_b \\
& -15\,\hat{I}_0^{[0,1]}(4,1,1){m_c}^2m_b-30\,\hat{I}_0^{[0,1]}(2,2,2){m_c}^2m_b-20\,\hat{I}_0^{[0,1]}(2,3,1){m_c}^2m_b-20\,\hat{I}_0^{[0,1]}(3,2,1){m_c}^2m_b \\
& +10\,\hat{I}_0(3,1,1){m_c}^2m_b-60\,\hat{I}_6(4,1,1){m_c}^2m_b+15\,\hat{I}_0(2,1,2){m_c}^2m_b+40\,\hat{I}_6^{[0,1]}(3,2,2){m_c}^2m_b
\end{aligned}
$$

$$
\begin{aligned}
&+40\,\hat{I}_6(2,3,1){m_c}^2 m_b + 15\,\hat{I}_0^{[0,2]}(3,2,2){m_c}^2 m_b - 10\,\hat{I}_0^{[0,1]}(3,1,2){m_c}^2 m_b - 10\,\hat{I}_0(1,3,1){m_c}^2 m_b\\
&-15\,\hat{I}_0(2,2,1){m_c}^2 m_b + 20\,\hat{I}_0(1,2,2){m_c}^2 m_b - 120\,\hat{I}_6(1,4,1)m_c\,{m_b}^2 + 30\,\hat{I}_0(1,3,1)m_c\,{m_b}^2\\
&+20\,\hat{I}_6(2,2,2)m_c\,{m_b}^2 + 20\,\hat{I}_0^{[0,1]}(3,1,2)m_c\,{m_b}^2 + 5\,\hat{I}_0^{[0,1]}(3,2,1)m_c\,{m_b}^2 + 10\,\hat{I}_0^{[0,1]}(2,2,2)m_c\,{m_b}^2\\
&-5\,\hat{I}_0^{[0,2]}(3,2,2)m_c\,{m_b}^2 - 20\,\hat{I}_6^{[0,1]}(3,2,2)m_c\,{m_b}^2 + 20\,\hat{I}_6(3,2,1)m_c\,{m_b}^2 - 15\,\hat{I}_0(2,1,2)m_c\,{m_b}^2\\
&+10\,\hat{I}_0(2,2,1)m_c\,{m_b}^2 - 10\,\hat{I}_0(1,2,2)m_c\,{m_b}^2 - 5\,\hat{I}_0(3,1,1)m_c\,{m_b}^2 + 20\,\hat{I}_0(1,3,1){m_b}^3\\
&-5\,\hat{I}_0(2,2,1){m_b}^3 - 20\,\hat{I}_6(3,2,1){m_b}^3 - 15\,\hat{I}_0^{[0,1]}(3,2,1){m_b}^3 + 30\,\hat{I}_0^{[0,1]}(1,4,1){m_b}^3\\
&+5\,\hat{I}_0^{[0,2]}(3,2,2){m_b}^3 + 5\,\hat{I}_0(1,2,2){m_b}^3 + 120\,\hat{I}_6(1,4,1){m_b}^3 - 10\,\hat{I}_0^{[0,1]}(2,3,1){m_b}^3\\
&-20\,\hat{I}_6(2,2,2){m_b}^3 + 20\,\hat{I}_6^{[0,1]}(3,2,2){m_b}^3 - 40\,\hat{I}_6(2,3,1){m_b}^3 - 10\,\hat{I}_0^{[0,1]}(2,2,2){m_b}^3\\
&-5\,\hat{I}_0(1,1,2)m_c + 20\,\hat{I}_6^{[0,2]}(3,2,2)m_c - 40\,\hat{I}_6(3,1,1)m_c - 20\,\hat{I}_6(2,2,1)m_c\\
&-40\,\hat{I}_6^{[0,1]}(2,2,2)m_c - 20\,\hat{I}_6^{[0,1]}(3,1,2)m_c - 5\,\hat{I}_0(1,2,1)m_c - 5\,\hat{I}_0^{[0,1]}(3,1,1)m_c\\
&-40\,\hat{I}_6^{[0,1]}(3,2,1)m_c + 5\,\hat{I}_0(2,1,1)m_c - 20\,\hat{I}_6(2,1,2)m_c + 40\,\hat{I}_6(1,3,1)m_b\\
&-15\,\hat{I}_0^{[0,1]}(2,2,1)m_b + 15\,\hat{I}_0^{[0,2]}(2,2,2)m_b + 15\,\hat{I}_0(1,1,2)m_b + 40\,\hat{I}_6^{[0,1]}(2,3,1)m_b\\
&+20\,\hat{I}_6^{[0,1]}(3,2,1)m_b + 20\,\hat{I}_6(2,1,2)m_b + 10\,\hat{I}_0^{[0,2]}(2,3,1)m_b + 15\,\hat{I}_0^{[0,2]}(3,2,1)m_b\\
&+40\,\hat{I}_6(2,2,1)m_b - 20\,\hat{I}_0^{[0,1]}(1,2,2)m_b + 10\,\hat{I}_0^{[0,2]}(3,1,2)m_b + 20\,\hat{I}_6(1,2,2)m_b\\
&+20\,\hat{I}_6^{[0,1]}(2,2,2)m_b + 40\,\hat{I}_6(3,1,1)m_b + 40\,\hat{I}_6^{[0,1]}(3,1,2)m_b + 10\,\hat{I}_0^{[0,1]}(1,3,1)m_b\\
&+15\,\hat{I}_0(1,2,1)m_b - 15\,\hat{I}_0(2,1,1)m_b - 10\,\hat{I}_0^{[0,1]}(3,1,1)m_b - 20\,\hat{I}_6^{[0,2]}(3,2,2)m_b
\end{aligned}
$$

$$
\begin{aligned}
C_4^1 \;=\;& 5\,\hat{I}_1(3,2,2){m_c}^5 + 10\,\hat{I}_3(3,2,2){m_c}^5 + 10\,\hat{I}_4(3,2,2){m_c}^5 + 5\,\hat{I}_2(3,2,2){m_c}^5\\
&-10\,\hat{I}_4(3,2,2){m_c}^4 m_b - 10\,\hat{I}_3(3,2,2){m_c}^4 m_b - 10\,\hat{I}_3(3,2,2){m_c}^3{m_b}^2 - 10\,\hat{I}_4(3,2,2){m_c}^3{m_b}^2\\
&-5\,\hat{I}_1(3,2,2){m_c}^3{m_b}^2 - 5\,\hat{I}_2(3,2,2){m_c}^3{m_b}^2 + 10\,\hat{I}_4(3,2,2){m_c}^2{m_b}^3 + 10\,\hat{I}_3(3,2,2){m_c}^2{m_b}^3\\
&+15\,\hat{I}_2(4,1,1){m_c}^3 + 20\,\hat{I}_4(3,2,1){m_c}^3 + 15\,\hat{I}_1(4,1,1){m_c}^3 + 20\,\hat{I}_2(3,2,1){m_c}^3\\
&+20\,\hat{I}_3(2,2,2){m_c}^3 + 30\,\hat{I}_3(4,1,1){m_c}^3 + 20\,\hat{I}_1(3,2,1){m_c}^3 + 10\,\hat{I}_2(2,2,2){m_c}^3\\
&-10\,\hat{I}_6^{[0,1]}(3,2,2){m_c}^3 - 10\,\hat{I}_1^{[0,1]}(3,2,2){m_c}^3 + 20\,\hat{I}_4(2,2,2){m_c}^3 + 10\,\hat{I}_4(3,1,2){m_c}^3\\
&+10\,\hat{I}_3(3,1,2){m_c}^3 + 5\,\hat{I}_1(3,1,2){m_c}^3 + 20\,\hat{I}_3(3,2,1){m_c}^3 - 20\,\hat{I}_3^{[0,1]}(3,2,2){m_c}^3\\
&-20\,\hat{I}_4^{[0,1]}(3,2,2){m_c}^3 + 5\,\hat{I}_2(3,1,2){m_c}^3 + 10\,\hat{I}_1(2,2,2){m_c}^3 + 20\,\hat{I}_0(3,2,1){m_c}^3\\
&+30\,\hat{I}_4(4,1,1){m_c}^3 - 10\,\hat{I}_3(2,2,2){m_c}^2 m_b - 20\,\hat{I}_4(3,2,1){m_c}^2 m_b - 20\,\hat{I}_3(3,2,1){m_c}^2 m_b\\
&-15\,\hat{I}_2(3,2,1){m_c}^2 m_b + 20\,\hat{I}_1(2,3,1){m_c}^2 m_b + 20\,\hat{I}_4^{[0,1]}(3,2,2){m_c}^2 m_b + 20\,\hat{I}_3^{[0,1]}(3,2,2){m_c}^2 m_b\\
&-10\,\hat{I}_3(3,1,2){m_c}^2 m_b - 10\,\hat{I}_4(3,1,2){m_c}^2 m_b - 10\,\hat{I}_4(2,2,2){m_c}^2 m_b - 30\,\hat{I}_4(4,1,1){m_c}^2 m_b\\
&+20\,\hat{I}_2(2,3,1){m_c}^2 m_b - 30\,\hat{I}_3(4,1,1){m_c}^2 m_b + 20\,\hat{I}_3(2,3,1){m_c}^2 m_b - 15\,\hat{I}_1(3,2,1){m_c}^2 m_b\\
&+20\,\hat{I}_4(2,3,1){m_c}^2 m_b + 10\,\hat{I}_1(3,2,1)m_c\,{m_b}^2 - 60\,\hat{I}_4(1,4,1)m_c\,{m_b}^2 - 30\,\hat{I}_1(1,4,1)m_c\,{m_b}^2\\
&+10\,\hat{I}_4(3,2,1)m_c\,{m_b}^2 + 10\,\hat{I}_3(2,2,2)m_c\,{m_b}^2 - 10\,\hat{I}_3^{[0,1]}(3,2,2)m_c\,{m_b}^2 - 60\,\hat{I}_3(1,4,1)m_c\,{m_b}^2\\
&+10\,\hat{I}_3(3,2,1)m_c\,{m_b}^2 - 5\,\hat{I}_6^{[0,1]}(3,2,2)m_c\,{m_b}^2 - 10\,\hat{I}_4^{[0,1]}(3,2,2)m_c\,{m_b}^2 - 5\,\hat{I}_1^{[0,1]}(3,2,2)m_c\,{m_b}^2\\
&-30\,\hat{I}_2(1,4,1)m_c\,{m_b}^2 + 10\,\hat{I}_4(2,2,2)m_c\,{m_b}^2 + 10\,\hat{I}_2(3,2,1)m_c\,{m_b}^2 - 10\,\hat{I}_3(2,2,2){m_b}^3\\
&-10\,\hat{I}_3(3,2,1){m_b}^3 - 20\,\hat{I}_4(2,3,1){m_b}^3 + 10\,\hat{I}_4^{[0,1]}(3,2,2){m_b}^3 - 20\,\hat{I}_3(2,3,1){m_b}^3\\
&-10\,\hat{I}_4(2,2,2){m_b}^3 + 60\,\hat{I}_3(1,4,1){m_b}^3 + 10\,\hat{I}_3^{[0,1]}(3,2,2){m_b}^3 - 10\,\hat{I}_4(3,2,1){m_b}^3\\
&+60\,\hat{I}_4(1,4,1){m_b}^3 - 20\,\hat{I}_3(3,1,1)m_c - 10\,\hat{I}_1^{[0,1]}(3,2,1)m_c + 5\,\hat{I}_2^{[0,2]}(3,2,2)m_c
\end{aligned}
$$

$$
\begin{aligned}
& -10\,\hat{I}_4(2,1,2)m_c + 10\,\hat{I}_3^{[0,2]}(3,2,2)m_c - 5\,\hat{I}_2(3,1,1)m_c - 10\,\hat{I}_3^{[0,1]}(3,1,2)m_c \\
& -10\,\hat{I}_6^{[0,1]}(2,2,2)m_c - 10\,\hat{I}_3(2,1,2)m_c - 20\,\hat{I}_3^{[0,1]}(2,2,2)m_c - 10\,\hat{I}_3(2,2,1)m_c \\
& -10\,\hat{I}_1^{[0,1]}(2,2,2)m_c + 10\,\hat{I}_4^{[0,2]}(3,2,2)m_c - 20\,\hat{I}_3^{[0,1]}(3,2,1)m_c - 10\,\hat{I}_4(2,2,1)m_c \\
& +15\,\hat{I}_1(2,1,2)m_c - 10\,\hat{I}_6^{[0,1]}(3,2,1)m_c + 5\,\hat{I}_1^{[0,2]}(3,2,2)m_c - 20\,\hat{I}_4^{[0,1]}(2,2,2)m_c \\
& +10\,\hat{I}_0(2,2,1)m_c + 15\,\hat{I}_2(2,1,2)m_c - 20\,\hat{I}_4(3,1,1)m_c - 15\,\hat{I}_1^{[0,1]}(3,1,2)m_c \\
& -15\,\hat{I}_6^{[0,1]}(3,1,2)m_c - 20\,\hat{I}_4^{[0,1]}(3,2,1)m_c - 10\,\hat{I}_4^{[0,1]}(3,1,2)m_c - 5\,\hat{I}_1(3,1,1)m_c \\
& +20\,\hat{I}_4^{[0,1]}(2,3,1)m_b + 20\,\hat{I}_3(3,1,1)m_b + 10\,\hat{I}_4(2,1,2)m_b - 10\,\hat{I}_4^{[0,2]}(3,2,2)m_b \\
& +20\,\hat{I}_3(1,3,1)m_b + 10\,\hat{I}_3^{[0,1]}(3,2,1)m_b + 10\,\hat{I}_3(2,1,2)m_b + 10\,\hat{I}_4^{[0,1]}(3,2,1)m_b \\
& -10\,\hat{I}_3^{[0,2]}(3,2,2)m_b - 10\,\hat{I}_2(1,3,1)m_b + 20\,\hat{I}_3^{[0,1]}(2,3,1)m_b + 10\,\hat{I}_1(2,2,1)m_b \\
& +10\,\hat{I}_3^{[0,1]}(2,2,2)m_b + 20\,\hat{I}_4(3,1,1)m_b + 20\,\hat{I}_3(2,2,1)m_b + 20\,\hat{I}_4(2,2,1)m_b \\
& +10\,\hat{I}_2(2,2,1)m_b + 10\,\hat{I}_3(1,2,2)m_b + 10\,\hat{I}_4(1,2,2)m_b + 20\,\hat{I}_4(1,3,1)m_b
\end{aligned}
$$

$$
\begin{aligned}
C_4^2 \;=\; & 5\,\hat{I}_1(3,2,2){m_c}^5 + 10\,\hat{I}_3(3,2,2){m_c}^5 - 5\,\hat{I}_2(3,2,2){m_c}^5 - 10\,\hat{I}_4(3,2,2){m_c}^5 \\
& +10\,\hat{I}_4(3,2,2){m_c}^4m_b - 10\,\hat{I}_3(3,2,2){m_c}^4m_b - 5\,\hat{I}_1(3,2,2){m_c}^3{m_b}^2 + 10\,\hat{I}_4(3,2,2){m_c}^3{m_b}^2 \\
& -10\,\hat{I}_3(3,2,2){m_c}^3{m_b}^2 + 5\,\hat{I}_2(3,2,2){m_c}^3{m_b}^2 - 10\,\hat{I}_4(3,2,2){m_c}^2{m_b}^3 + 10\,\hat{I}_3(3,2,2){m_c}^2{m_b}^3 \\
& +15\,\hat{I}_1(4,1,1){m_c}^3 + 5\,\hat{I}_1(3,1,2){m_c}^3 - 10\,\hat{I}_1^{[0,1]}(3,2,2){m_c}^3 + 20\,\hat{I}_3(2,2,2){m_c}^3 \\
& -30\,\hat{I}_4(4,1,1){m_c}^3 + 20\,\hat{I}_0(3,2,1){m_c}^3 - 20\,\hat{I}_2(3,2,1){m_c}^3 + 20\,\hat{I}_3(3,2,1){m_c}^3 \\
& +20\,\hat{I}_1(3,2,1){m_c}^3 + 10\,\hat{I}_2^{[0,1]}(3,2,2){m_c}^3 - 20\,\hat{I}_4(3,2,1){m_c}^3 - 20\,\hat{I}_4(2,2,2){m_c}^3 \\
& +30\,\hat{I}_3(4,1,1){m_c}^3 + 10\,\hat{I}_1(2,2,2){m_c}^3 - 15\,\hat{I}_2(4,1,1){m_c}^3 + 20\,\hat{I}_4^{[0,1]}(3,2,2){m_c}^3 \\
& -5\,\hat{I}_2(3,1,2){m_c}^3 + 10\,\hat{I}_3(3,1,2){m_c}^3 - 10\,\hat{I}_2(2,2,2){m_c}^3 - 10\,\hat{I}_4(3,1,2){m_c}^3 \\
& -20\,\hat{I}_3^{[0,1]}(3,2,2){m_c}^3 - 15\,\hat{I}_1(3,2,1){m_c}^2m_b - 10\,\hat{I}_3(3,1,2){m_c}^2m_b + 20\,\hat{I}_4(3,2,1){m_c}^2m_b \\
& -20\,\hat{I}_3(3,2,1){m_c}^2m_b - 20\,\hat{I}_4(2,3,1){m_c}^2m_b + 20\,\hat{I}_3(2,3,1){m_c}^2m_b - 20\,\hat{I}_4^{[0,1]}(3,2,2){m_c}^2m_b \\
& -20\,\hat{I}_2(2,3,1){m_c}^2m_b - 10\,\hat{I}_3(2,2,2){m_c}^2m_b + 15\,\hat{I}_2(3,2,1){m_c}^2m_b + 20\,\hat{I}_3^{[0,1]}(3,2,2){m_c}^2m_b \\
& +10\,\hat{I}_4(2,2,2){m_c}^2m_b + 30\,\hat{I}_4(4,1,1){m_c}^2m_b + 20\,\hat{I}_1(2,3,1){m_c}^2m_b - 30\,\hat{I}_3(4,1,1){m_c}^2m_b \\
& +10\,\hat{I}_4(3,1,2){m_c}^2m_b + 30\,\hat{I}_2(1,4,1)m_c\,{m_b}^2 - 5\,\hat{I}_1^{[0,1]}(3,2,2)m_c\,{m_b}^2 + 10\,\hat{I}_1(3,2,1)m_c\,{m_b}^2 \\
& +60\,\hat{I}_4(1,4,1)m_c\,{m_b}^2 - 60\,\hat{I}_3(1,4,1)m_c\,{m_b}^2 + 5\,\hat{I}_2^{[0,1]}(3,2,2)m_c\,{m_b}^2 - 10\,\hat{I}_4(3,2,1)m_c\,{m_b}^2 \\
& -10\,\hat{I}_4(2,2,2)m_c\,{m_b}^2 - 30\,\hat{I}_1(1,4,1)m_c\,{m_b}^2 + 10\,\hat{I}_3(3,2,1)m_c\,{m_b}^2 + 10\,\hat{I}_3(2,2,2)m_c\,{m_b}^2 \\
& +10\,\hat{I}_4^{[0,1]}(3,2,2)m_c\,{m_b}^2 - 10\,\hat{I}_2(3,2,1)m_c\,{m_b}^2 - 10\,\hat{I}_3^{[0,1]}(3,2,2)m_c\,{m_b}^2 - 10\,\hat{I}_4^{[0,1]}(3,2,2){m_b}^3 \\
& -60\,\hat{I}_4(1,4,1){m_b}^3 - 10\,\hat{I}_3(3,2,1){m_b}^3 + 10\,\hat{I}_4(3,2,1){m_b}^3 - 20\,\hat{I}_3(2,3,1){m_b}^3 \\
& +60\,\hat{I}_3(1,4,1){m_b}^3 - 10\,\hat{I}_3(2,2,2){m_b}^3 + 10\,\hat{I}_4(2,2,2){m_b}^3 + 10\,\hat{I}_3^{[0,1]}(3,2,2){m_b}^3 \\
& +20\,\hat{I}_4(2,3,1){m_b}^3 + 20\,\hat{I}_4(3,1,1)m_c + 5\,\hat{I}_1^{[0,2]}(3,2,2)m_c - 10\,\hat{I}_1^{[0,1]}(2,2,2)m_c \\
& +10\,\hat{I}_2^{[0,1]}(2,2,2)m_c + 20\,\hat{I}_4^{[0,1]}(3,2,1)m_c + 10\,\hat{I}_4^{[0,1]}(3,1,2)m_c - 10\,\hat{I}_1^{[0,1]}(3,2,1)m_c \\
& +10\,\hat{I}_2^{[0,1]}(3,2,1)m_c - 10\,\hat{I}_4^{[0,2]}(3,2,2)m_c - 10\,\hat{I}_3(2,1,2)m_c + 10\,\hat{I}_4(2,2,1)m_c \\
& +15\,\hat{I}_2^{[0,1]}(3,1,2)m_c - 15\,\hat{I}_2(2,1,2)m_c + 10\,\hat{I}_4(2,1,2)m_c + 5\,\hat{I}_2(3,1,1)m_c \\
& +15\,\hat{I}_1(2,1,2)m_c - 5\,\hat{I}_2^{[0,2]}(3,2,2)m_c - 15\,\hat{I}_1^{[0,1]}(3,1,2)m_c + 10\,\hat{I}_0(2,2,1)m_c
\end{aligned}
$$

$$
\begin{aligned}
&+20\,\hat{I}_4^{[0,1]}(2,2,2)m_c - 20\,\hat{I}_3^{[0,1]}(2,2,2)m_c - 10\,\hat{I}_3^{[0,1]}(3,1,2)m_c - 20\,\hat{I}_3^{[0,1]}(3,2,1)m_c \\
&+10\,\hat{I}_3^{[0,2]}(3,2,2)m_c - 5\,\hat{I}_1(3,1,1)m_c - 10\,\hat{I}_3(2,2,1)m_c - 20\,\hat{I}_3(3,1,1)m_c \\
&+10\,\hat{I}_1(2,2,1)m_b + 10\,\hat{I}_3(2,1,2)m_b + 10\,\hat{I}_4^{[0,2]}(3,2,2)m_b - 10\,\hat{I}_2(2,2,1)m_b \\
&-10\,\hat{I}_4^{[0,1]}(2,2,2)m_b + 10\,\hat{I}_2(1,3,1)m_b + 10\,\hat{I}_3^{[0,1]}(2,2,2)m_b - 20\,\hat{I}_4(3,1,1)m_b \\
&-10\,\hat{I}_1(1,3,1)m_b + 10\,\hat{I}_3(1,2,2)m_b - 10\,\hat{I}_4(1,2,2)m_b - 20\,\hat{I}_4^{[0,1]}(3,1,2)m_b \\
&-10\,\hat{I}_4^{[0,1]}(3,2,1)m_b - 10\,\hat{I}_3^{[0,2]}(3,2,2)m_b + 10\,\hat{I}_3^{[0,1]}(3,2,1)m_b - 20\,\hat{I}_4(2,2,1)m_b \\
&+20\,\hat{I}_3(1,3,1)m_b - 20\,\hat{I}_4(1,3,1)m_b - 10\,\hat{I}_4(2,1,2)m_b + 20\,\hat{I}_3(3,1,1)m_b
\end{aligned}
$$

where

$$
\hat{I}_n^{[i,j]}(a,b,c) = \left(M_1^2\right)^i \left(M_2^2\right)^j \frac{d^i}{d\left(M_1^2\right)^i}\frac{d^j}{d\left(M_2^2\right)^j}\left[\left(M_1^2\right)^i \left(M_2^2\right)^j \hat{I}_n(a,b,c)\right] \; .
$$

Appendix–B

In this appendix, the explicit expressions of the coefficients of the gluon condensate entering the HQET limit of the form factors f_V^{HQET}, f_0^{HQET}, f_1^{HQET} and f_2^{HQET} are presented.

$$
\begin{aligned}
C_V^{HQET} = \quad & 60\,\frac{\bar{I}_1(1,4,1){m_b}^3}{\sqrt{Z}} - 30\,\frac{\bar{I}_1(3,2,1){m_b}^3}{\sqrt{Z}} - 20\,\frac{\bar{I}_2(3,2,1){m_b}^3}{\sqrt{Z}} \\
& +10\,\frac{\bar{I}_0^{[0,1]}(3,2,2){m_b}^3}{\sqrt{Z}} + 60\,\frac{\bar{I}_0(1,4,1){m_b}^3}{\sqrt{Z}} + 10\,\frac{\bar{I}_2^{[0,1]}(3,2,2){m_b}^3}{\sqrt{Z}} \\
& +10\,\frac{\bar{I}_1^{[0,1]}(3,2,2){m_b}^3}{\sqrt{Z}} - 20\,\frac{\bar{I}_0(3,2,1){m_b}^3}{\sqrt{Z}} - 20\,\frac{\bar{I}_1^{[0,1]}(3,2,2){m_b}^3}{Z} \\
& +30\,\frac{\bar{I}_1(4,1,1){m_b}^3}{Z} - 10\,\frac{\bar{I}_1(3,2,2){m_b}^5}{Z} + 40\,\frac{\bar{I}_0(2,3,1){m_b}^3}{Z} \\
& +20\,\frac{\bar{I}_1(2,2,2){m_b}^3}{Z} + 10\,\frac{\bar{I}_0(3,2,1){m_b}^3}{Z} + 10\,\frac{\bar{I}_1(3,2,1){m_b}^3}{Z} \\
& -10\,\frac{\bar{I}_2(3,2,1){m_b}^3}{Z} + 20\,\frac{\bar{I}_1(2,3,1){m_b}^3}{Z} - 30\,\frac{\bar{I}_1(4,1,1){m_b}^3}{Z^{3/2}} \\
& +10\,\frac{\bar{I}_0(3,2,2){m_b}^5}{Z^{3/2}} + 10\,\frac{\bar{I}_2(3,2,2){m_b}^5}{Z^{3/2}} - 20\,\frac{\bar{I}_2(2,2,2){m_b}^3}{Z^{3/2}} \\
& -30\,\frac{\bar{I}_2(4,1,1){m_b}^3}{Z^{3/2}} + 10\,\frac{\bar{I}_1(3,2,2){m_b}^5}{Z^{3/2}} - 20\,\frac{\bar{I}_1(2,2,2){m_b}^3}{Z^{3/2}} \\
& -10\,\frac{\bar{I}_0(3,1,2){m_b}^3}{Z^{3/2}} + 20\,\frac{\bar{I}_2^{[0,1]}(3,2,2){m_b}^3}{Z^{3/2}} - 20\,\frac{\bar{I}_0(2,2,2){m_b}^3}{Z^{3/2}} \\
& -30\,\frac{\bar{I}_0(4,1,1){m_b}^3}{Z^{3/2}} + 20\,\frac{\bar{I}_0^{[0,1]}(3,2,2){m_b}^3}{Z^{3/2}} + 10\,\frac{\bar{I}_2(3,2,1){m_b}^3}{Z^{3/2}} \\
& -10\,\frac{\bar{I}_2(3,1,2){m_b}^3}{Z^{3/2}} + 20\,\frac{\bar{I}_1^{[0,1]}(3,2,2){m_b}^3}{Z^{3/2}} + 10\,\frac{\bar{I}_1(3,2,2){m_b}^5}{Z^2} \\
& -10\,\frac{\bar{I}_1(3,2,2){m_b}^5}{Z^{5/2}} - 10\,\frac{\bar{I}_0(3,2,2){m_b}^5}{Z^{5/2}} - 10\,\frac{\bar{I}_2(3,2,2){m_b}^5}{Z^{5/2}} \\
& +10\,\bar{I}_1(2,2,2){m_b}^3 + 20\,\bar{I}_1(3,2,1){m_b}^3 - 60\,\bar{I}_1(1,4,1){m_b}^3 \\
& -20\,\bar{I}_1(2,3,1){m_b}^3 - 10\,\bar{I}_1^{[0,1]}(3,2,2){m_b}^3
\end{aligned}
$$

$$
\begin{aligned}
C_0^{HQET} = \quad & -10\,\frac{\bar{I}_0(3,2,1){m_b}^5}{\sqrt{Z}} + 5\,\frac{\bar{I}_0^{[0,1]}(3,2,2){m_b}^5}{\sqrt{Z}} + 30\,\frac{\bar{I}_0(1,4,1){m_b}^5}{\sqrt{Z}} \\
& -10\,\frac{\bar{I}_0(2,3,1){m_b}^5}{Z} + 20\,\frac{\bar{I}_6(3,2,2){m_b}^5}{Z} - 30\,\frac{\bar{I}_0(1,4,1){m_b}^5}{Z} \\
& +10\,\frac{\bar{I}_0(3,2,1){m_b}^5}{Z} + 10\,\frac{\bar{I}_0^{[0,1]}(3,2,2){m_b}^5}{Z^{3/2}} - 5\,\frac{\bar{I}_0(3,2,1){m_b}^5}{Z^{3/2}} \\
& -20\,\frac{\bar{I}_6(3,2,2){m_b}^5}{Z^{3/2}} - 15\,\frac{\bar{I}_0(4,1,1){m_b}^5}{Z^{3/2}} - 10\,\frac{\bar{I}_0(2,2,2){m_b}^5}{Z^{3/2}} \\
& +5\,\frac{\bar{I}_0(3,2,2){m_b}^7}{Z^{3/2}} + 15\,\frac{\bar{I}_0(4,1,1){m_b}^5}{Z^2} + 15\,\frac{\bar{I}_0(2,2,2){m_b}^5}{Z^2}
\end{aligned}
$$

$$-15\,\frac{\bar{I}_0^{[0,1]}(3,2,2)m_b{}^5}{Z^2}+10\,\frac{\bar{I}_0(2,3,1)m_b{}^5}{Z^2}-5\,\frac{\bar{I}_0(3,2,2)m_b{}^7}{Z^2}$$
$$-20\,\frac{\bar{I}_6(3,2,2)m_b{}^5}{Z^2}+5\,\frac{\bar{I}_0(3,2,1)m_b{}^5}{Z^2}-5\,\frac{\bar{I}_0(3,2,2)m_b{}^7}{Z^{5/2}}$$
$$+20\,\frac{\bar{I}_6(3,2,2)m_b{}^5}{Z^{5/2}}+5\,\frac{\bar{I}_0(3,2,2)m_b{}^7}{Z^3}$$

$$C_1^{HQET} = \quad 40\,\frac{\bar{I}_1(3,2,1)m_b{}^3}{\sqrt{Z}}+10\,\frac{\bar{I}_1(2,2,2)m_b{}^3}{\sqrt{Z}}-5\,\frac{\bar{I}_2^{[0,1]}(3,2,2)m_b{}^3}{\sqrt{Z}}$$
$$-20\,\frac{\bar{I}_1^{[0,1]}(3,2,2)m_b{}^3}{\sqrt{Z}}+20\,\frac{\bar{I}_4(3,2,1)m_b{}^3}{\sqrt{Z}}-20\,\frac{\bar{I}_4^{[0,1]}(3,2,2)m_b{}^3}{\sqrt{Z}}$$
$$+15\,\frac{\bar{I}_0(3,2,1)m_b{}^3}{\sqrt{Z}}+20\,\frac{\bar{I}_4(2,2,2)m_b{}^3}{\sqrt{Z}}+20\,\frac{\bar{I}_3(2,2,2)m_b{}^3}{\sqrt{Z}}$$
$$-5\,\frac{\bar{I}_1(3,1,2)m_b{}^3}{\sqrt{Z}}+10\,\frac{\bar{I}_2(3,2,1)m_b{}^3}{\sqrt{Z}}-20\,\frac{\bar{I}_3^{[0,1]}(3,2,2)m_b{}^3}{\sqrt{Z}}$$
$$-120\,\frac{\bar{I}_1(1,4,1)m_b{}^3}{\sqrt{Z}}-120\,\frac{\bar{I}_3(1,4,1)m_b{}^3}{\sqrt{Z}}-30\,\frac{\bar{I}_0(1,4,1)m_b{}^3}{\sqrt{Z}}$$
$$+20\,\frac{\bar{I}_3(3,2,1)m_b{}^3}{\sqrt{Z}}-120\,\frac{\bar{I}_4(1,4,1)m_b{}^3}{\sqrt{Z}}-5\,\frac{\bar{I}_0^{[0,1]}(3,2,2)m_b{}^3}{\sqrt{Z}}$$
$$-30\,\frac{\bar{I}_2(1,4,1)m_b{}^3}{\sqrt{Z}}+40\,\frac{\bar{I}_4(2,3,1)m_b{}^3}{Z}+40\,\frac{\bar{I}_3(2,3,1)m_b{}^3}{Z}$$
$$+70\,\frac{\bar{I}_1(2,3,1)m_b{}^3}{Z}-50\,\frac{\bar{I}_1(3,2,1)m_b{}^3}{Z}-20\,\frac{\bar{I}_4(3,1,2)m_b{}^3}{Z}$$
$$-10\,\frac{\bar{I}_1(3,1,2)m_b{}^3}{Z}+40\,\frac{\bar{I}_4^{[0,1]}(3,2,2)m_b{}^3}{Z}+40\,\frac{\bar{I}_3^{[0,1]}(3,2,2)m_b{}^3}{Z}$$
$$+20\,\frac{\bar{I}_0(2,3,1)m_b{}^3}{Z}-60\,\frac{\bar{I}_3(4,1,1)m_b{}^3}{Z}-20\,\frac{\bar{I}_3(3,1,2)m_b{}^3}{Z}$$
$$-60\,\frac{\bar{I}_4(4,1,1)m_b{}^3}{Z}+10\,\frac{\bar{I}_1^{[0,1]}(3,2,2)m_b{}^3}{Z}-40\,\frac{\bar{I}_4(3,2,1)m_b{}^3}{Z}$$
$$-20\,\frac{\bar{I}_4(2,2,2)m_b{}^3}{Z}-15\,\frac{\bar{I}_1(4,1,1)m_b{}^3}{Z}+20\,\frac{\bar{I}_2(2,3,1)m_b{}^3}{Z}$$
$$+5\,\frac{\bar{I}_1(3,2,2)m_b{}^5}{Z}-40\,\frac{\bar{I}_3(3,2,1)m_b{}^3}{Z}-5\,\frac{\bar{I}_0(3,2,1)m_b{}^3}{Z}$$
$$-15\,\frac{\bar{I}_2(3,2,1)m_b{}^3}{Z}+20\,\frac{\bar{I}_4(3,2,2)m_b{}^5}{Z}+20\,\frac{\bar{I}_3(3,2,2)m_b{}^5}{Z}$$
$$-20\,\frac{\bar{I}_3(2,2,2)m_b{}^3}{Z}-20\,\frac{\bar{I}_3(3,2,2)m_b{}^5}{Z^{3/2}}-5\,\frac{\bar{I}_0(3,2,2)m_b{}^5}{Z^{3/2}}$$
$$+20\,\frac{\bar{I}_2(3,2,1)m_b{}^3}{Z^{3/2}}-20\,\frac{\bar{I}_4(3,2,2)m_b{}^5}{Z^{3/2}}-5\,\frac{\bar{I}_2(3,2,2)m_b{}^5}{Z^{3/2}}$$
$$+40\,\frac{\bar{I}_4(2,2,2)m_b{}^3}{Z^{3/2}}+20\,\frac{\bar{I}_4(3,1,2)m_b{}^3}{Z^{3/2}}+25\,\frac{\bar{I}_1(3,1,2)m_b{}^3}{Z^{3/2}}$$

$$+15\,\frac{\bar{I}_0(4,1,1)m_b{}^3}{Z^{3/2}} - 40\,\frac{\bar{I}_4^{[0,1]}(3,2,2)m_b{}^3}{Z^{3/2}} + 65\,\frac{\bar{I}_1(3,2,1)m_b{}^3}{Z^{3/2}}$$
$$+5\,\frac{\bar{I}_2(3,1,2)m_b{}^3}{Z^{3/2}} - 40\,\frac{\bar{I}_1^{[0,1]}(3,2,2)m_b{}^3}{Z^{3/2}} + 25\,\frac{\bar{I}_0(3,2,1)m_b{}^3}{Z^{3/2}}$$
$$+60\,\frac{\bar{I}_1(4,1,1)m_b{}^3}{Z^{3/2}} + 60\,\frac{\bar{I}_3(4,1,1)m_b{}^3}{Z^{3/2}} + 10\,\frac{\bar{I}_0(2,2,2)m_b{}^3}{Z^{3/2}}$$
$$+60\,\frac{\bar{I}_4(4,1,1)m_b{}^3}{Z^{3/2}} + 10\,\frac{\bar{I}_2(2,2,2)m_b{}^3}{Z^{3/2}} + 5\,\frac{\bar{I}_0(3,1,2)m_b{}^3}{Z^{3/2}}$$
$$+20\,\frac{\bar{I}_3(3,1,2)m_b{}^3}{Z^{3/2}} + 40\,\frac{\bar{I}_1(2,2,2)m_b{}^3}{Z^{3/2}} + 40\,\frac{\bar{I}_3(3,2,1)m_b{}^3}{Z^{3/2}}$$
$$-10\,\frac{\bar{I}_0^{[0,1]}(3,2,2)m_b{}^3}{Z^{3/2}} - 40\,\frac{\bar{I}_3^{[0,1]}(3,2,2)m_b{}^3}{Z^{3/2}} - 10\,\frac{\bar{I}_2^{[0,1]}(3,2,2)m_b{}^3}{Z^{3/2}}$$
$$+40\,\frac{\bar{I}_3(2,2,2)m_b{}^3}{Z^{3/2}} + 40\,\frac{\bar{I}_4(3,2,1)m_b{}^3}{Z^{3/2}} - 20\,\frac{\bar{I}_1(3,2,2)m_b{}^5}{Z^{3/2}}$$
$$+15\,\frac{\bar{I}_2(4,1,1)m_b{}^3}{Z^{3/2}} - 20\,\frac{\bar{I}_4(3,2,2)m_b{}^5}{Z^2} - 5\,\frac{\bar{I}_1(3,2,2)m_b{}^5}{Z^2}$$
$$-20\,\frac{\bar{I}_3(3,2,2)m_b{}^5}{Z^2} + 20\,\frac{\bar{I}_3(3,2,2)m_b{}^5}{Z^{5/2}} + 20\,\frac{\bar{I}_4(3,2,2)m_b{}^5}{Z^{5/2}}$$
$$+20\,\frac{\bar{I}_1(3,2,2)m_b{}^5}{Z^{5/2}} + 5\,\frac{\bar{I}_0(3,2,2)m_b{}^5}{Z^{5/2}} + 5\,\frac{\bar{I}_2(3,2,2)m_b{}^5}{Z^{5/2}}$$
$$-10\,\bar{I}_1(2,3,1)m_b{}^3 + 120\,\bar{I}_4(1,4,1)m_b{}^3 + 120\,\bar{I}_3(1,4,1)m_b{}^3$$
$$-20\,\bar{I}_4(3,2,1)m_b{}^3 + 30\,\bar{I}_1(1,4,1)m_b{}^3 - 10\,\bar{I}_1(3,2,1)m_b{}^3$$
$$+20\,\bar{I}_3^{[0,1]}(3,2,2)m_b{}^3 - 20\,\bar{I}_3(3,2,1)m_b{}^3 - 20\,\bar{I}_4(2,2,2)m_b{}^3$$

$$C_2^{HQET} = \quad 20\,\frac{\bar{I}_4^{[0,1]}(3,2,2)m_b{}^3}{\sqrt{Z}} - 20\,\frac{\bar{I}_4(3,2,1)m_b{}^3}{\sqrt{Z}} - 120\,\frac{\bar{I}_3(1,4,1)m_b{}^3}{\sqrt{Z}}$$
$$+30\,\frac{\bar{I}_0(1,5,1)m_b{}^3}{\sqrt{Z}} + 10\,\frac{\bar{I}_1(3,2,1)m_b{}^3}{\sqrt{Z}} + 20\,\frac{\bar{I}_3(2,2,2)m_b{}^3}{\sqrt{Z}}$$
$$+60\,\frac{\bar{I}_2(1,4,1)m_b{}^3}{\sqrt{Z}} + 120\,\frac{\bar{I}_4(1,4,1)m_b{}^3}{\sqrt{Z}} + 20\,\frac{\bar{I}_3(3,2,1)m_b{}^3}{\sqrt{Z}}$$
$$+10\,\frac{\bar{I}_2^{[0,1]}(3,2,2)m_b{}^3}{\sqrt{Z}} - 20\,\frac{\bar{I}_2(3,2,1)m_b{}^3}{\sqrt{Z}} + 15\,\frac{\bar{I}_0(3,2,1)m_b{}^3}{\sqrt{Z}}$$
$$-30\,\frac{\bar{I}_1(1,4,1)m_b{}^3}{\sqrt{Z}} - 20\,\frac{\bar{I}_3^{[0,1]}(3,2,2)m_b{}^3}{\sqrt{Z}} - 5\,\frac{\bar{I}_0^{[0,1]}(3,2,2)m_b{}^3}{\sqrt{Z}}$$
$$-5\,\frac{\bar{I}_1^{[0,1]}(3,2,2)\ m_b{}^3}{\sqrt{Z}} - 20\,\frac{\bar{I}_4(2,2,2)\ m_b{}^3}{\sqrt{Z}} - 5\,\frac{\bar{I}_1(2,2,2)\ m_b{}^3}{Z}$$
$$-60\,\frac{\bar{I}_3(4,1,1)m_b{}^3}{Z} + 20\,\frac{\bar{I}_4(2,2,2)m_b{}^3}{Z} - 20\,\frac{\bar{I}_3(2,2,2)m_b{}^3}{Z}$$
$$+20\,\frac{\bar{I}_4(3,1,2)m_b{}^3}{Z} - 5\,\frac{\bar{I}_0(3,2,1)m_b{}^3}{Z} + 40\,\frac{\bar{I}_4(3,2,1)m_b{}^3}{Z}$$

$$
\begin{aligned}
&-40\,\frac{\bar{I}_4(2,3,1){m_b}^3}{Z} + 40\,\frac{\bar{I}_3^{[0,1]}(3,2,2){m_b}^3}{Z} + 60\,\frac{\bar{I}_4(4,1,1){m_b}^3}{Z} \\
&-20\,\frac{\bar{I}_4(3,2,2){m_b}^5}{Z} - 40\,\frac{\bar{I}_2(2,3,1){m_b}^3}{Z} + 20\,\frac{\bar{I}_1(2,3,1){m_b}^3}{Z} \\
&-15\,\frac{\bar{I}_1(3,2,1){m_b}^3}{Z} - 20\,\frac{\bar{I}_3(3,1,2){m_b}^3}{Z} - 40\,\frac{\bar{I}_3(3,2,1){m_b}^3}{Z} \\
&+30\,\frac{\bar{I}_2(3,2,1){m_b}^3}{Z} + 20\,\frac{\bar{I}_0(2,3,1){m_b}^3}{Z} - 40\,\frac{\bar{I}_4^{[0,1]}(3,2,2){m_b}^3}{Z} \\
&+20\,\frac{\bar{I}_3(3,2,2){m_b}^5}{Z} + 40\,\frac{\bar{I}_3(2,3,1){m_b}^3}{Z} - 10\,\frac{\bar{I}_0^{[0,1]}(3,2,2){m_b}^3}{Z^{3/2}} \\
&+20\,\frac{\bar{I}_1(3,2,1){m_b}^3}{Z^{3/2}} + 15\,\frac{\bar{I}_1(4,1,1){m_b}^3}{Z^{3/2}} + 20\,\frac{\bar{I}_3(3,1,2){m_b}^3}{Z^{3/2}} \\
&+40\,\frac{\bar{I}_3(2,2,2){m_b}^3}{Z^{3/2}} - 20\,\frac{\bar{I}_3(3,2,2){m_b}^5}{Z^{3/2}} - 10\,\frac{\bar{I}_1^{[0,1]}(3,2,2){m_b}^3}{Z^{3/2}} \\
&-20\,\frac{\bar{I}_4(3,1,2){m_b}^3}{Z^{3/2}} - 40\,\frac{\bar{I}_3^{[0,1]}(3,2,2){m_b}^3}{Z^{3/2}} - 30\,\frac{\bar{I}_2(4,1,1){m_b}^3}{Z^{3/2}} \\
&+10\,\frac{\bar{I}_0(2,2,2){m_b}^3}{Z^{3/2}} - 10\,\frac{\bar{I}_2(3,1,2){m_b}^3}{Z^{3/2}} - 40\,\frac{\bar{I}_4(2,2,2){m_b}^3}{Z^{3/2}} \\
&+25\,\frac{\bar{I}_0(3,2,1){m_b}^3}{Z^{3/2}} + 40\,\frac{\bar{I}_4^{[0,1]}(3,2,2){m_b}^3}{Z^{3/2}} - 40\,\frac{\bar{I}_4(3,2,1){m_b}^3}{Z^{3/2}} \\
&-60\,\frac{\bar{I}_4(4,1,1){m_b}^3}{Z^{3/2}} - 20\,\frac{\bar{I}_2(2,2,2){m_b}^3}{Z^{3/2}} + 60\,\frac{\bar{I}_3(4,1,1){m_b}^3}{Z^{3/2}} \\
&+15\,\frac{\bar{I}_0(4,1,1){m_b}^3}{Z^{3/2}} + 20\,\frac{\bar{I}_2^{[0,1]}(3,2,2){m_b}^3}{Z^{3/2}} + 5\,\frac{\bar{I}_0(3,1,2){m_b}^3}{Z^{3/2}} \\
&+10\,\frac{\bar{I}_1(3,1,2){m_b}^3}{Z^{3/2}} - 5\,\frac{\bar{I}_1(3,2,2){m_b}^5}{Z^{3/2}} + 10\,\frac{\bar{I}_1(2,2,2){m_b}^3}{Z^{3/2}} \\
&-40\,\frac{\bar{I}_2(3,2,1){m_b}^3}{Z^{3/2}} + 40\,\frac{\bar{I}_3(3,2,1){m_b}^3}{Z^{3/2}} + 10\,\frac{\bar{I}_2(3,2,2){m_b}^5}{Z^{3/2}} \\
&-5\,\frac{\bar{I}_0(3,2,2){m_b}^5}{Z^{3/2}} + 20\,\frac{\bar{I}_4(3,2,2){m_b}^5}{Z^{3/2}} + 20\,\frac{\bar{I}_4(3,2,2){m_b}^5}{Z^2} \\
&-20\,\frac{\bar{I}_3(3,2,2){m_b}^5}{Z^2} - 10\,\frac{\bar{I}_2(3,2,2){m_b}^5}{Z^{5/2}} + 20\,\frac{\bar{I}_3(3,2,2){m_b}^5}{Z^{5/2}} \\
&-20\,\frac{\bar{I}_4(3,2,2){m_b}^5}{Z^{5/2}} + 5\,\frac{\bar{I}_0(3,2,2){m_b}^5}{Z^{5/2}} + 5\,\frac{\bar{I}_1(3,2,2){m_b}^5}{Z^{5/2}} \\
&+20\,\bar{I}_3^{[0,1]}(3,2,2){m_b}^3 + 20\,\bar{I}_4(2,2,2){m_b}^3 - 40\,\bar{I}_3(2,3,1){m_b}^3 \\
&+120\,\bar{I}_3(1,4,1){m_b}^3 + 40\,\bar{I}_4(2,3,1){m_b}^3 - 20\,\bar{I}_3(2,2,2){m_b}^3 \\
&+20\,\bar{I}_4(3,2,1){m_b}^3 - 20\,\bar{I}_4^{[0,1]}(3,2,2){m_b}^3 - 20\,\bar{I}_3(3,2,1){m_b}^3
\end{aligned}
$$

where

$$
\bar{I}_n^{[i,j]}(a,b,c) = \frac{(2m_b)^{i+j}}{(\sqrt{z})^j} \left(T_1^2\right)^i \left(T_2^2\right)^j \frac{d^i}{d\left(T_1^2\right)^i} \frac{d^j}{d\left(T_2^2\right)^j} \left[\left(T_1^2\right)^i \left(T_2^2\right)^j \bar{I}_n(a,b,c)\right] \ .
$$

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