Investigation of the D_{s1} structure via B_c to $D_{s1}l^+l^-/\nu\bar{\nu}$ transitions in QCD.

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Abstract

We investigate the structure of the $D_{s1}(2460, 2536)(J^P = 1^+)$ mesons via analyzing the semileptonic $B_c \to D_{s1}l^+l^-$, $l = \tau, \mu, e$ and $B_c \to D_{s1}\nu\bar{\nu}$ transitions in the framework of the three–point QCD sum rules. We consider the D_{s1} meson in two ways, the pure $|c\bar{s}\rangle$ state and then as a mixture of two $|^3P_1\rangle$ and $|^1P_1\rangle$ states. Such type rare transitions take place at loop level by electroweak penguin and weak box diagrams in the standard model via the flavor changing neutral current transition of $b \to s$. The relevant form factors are calculated taking into account the gluon condensate contributions. These form factors are numerically obtained for $|c\bar{s}\rangle$ case and plotted in terms of the unknown mixing angle θ_s, when the D_{s1} meson are considered as mixture of two $|^3P_1\rangle$ and $|^1P_1\rangle$ states. The obtained results for the form factors are used to evaluate the decay rates and branching ratios. Any future experimental measurement on these form factors as well as decay rates and branching fractions and their comparison with the obtained results in the present work can give considerable information about the structure of this meson and the mixing angle θ_s.

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1 Introduction

The structure of the even–parity charmed D_{sJ} mesons has not known exactly yet and has been debated in the quark model. The observation of two narrow resonances with charm and strangeness, $D_{s0}(2317)$ in the invariant mass distribution of $D_s\pi^0$ [1–6] and $D_{s1}(2460)$ in the $D_s^*\pi^0$ and $D_s\gamma$ mass distributions [3–8], has raised discussions about the structure of these states and their quark contents [9, 10]. Analysis of the $D_{s_0}(2317) \to D_s^*\gamma$, $D_{s1}(2460) \to D_s^*\gamma$ and $D_{s_1}(2460) \to D_{s_0}(2317)\gamma$ shows that the quark content of these mesons are probably $c\bar{s}$ [11]. Among these mesons, the axial vector charm–strange meson D_{s1} is more attractive ones, because the discovery of the $D_{s1}(2460)(J^P = 1^+)$ meson [2–5] and its measured mass indicated a lower mass than expected in potential model (PM) [12] and quark model (QM) [13, 14] predictions. In other words, the $D_{s1}(2460)$ does not fit easily in to the $c\bar{s}$ spectroscopy [15]. However, some physicists presumed that this discovered state is conventional $c\bar{s}$ meson [16–26]. Many different theoretical efforts have been dedicated to the understanding of this unexpected and surprising disparity between theory and experiment [27–35]. As a result of the above discussion, we will consider the D_{s1} meson in two ways, the pure $|c\bar{s}\rangle$ state and also as a mixture of two $|^3P_1\rangle$ and $|^1P_1\rangle$ states.

Heavy–light mesons are not charge conjugation eigen states and so mixing can occur among states with the same J^P and different mass that are forbidden for neutral states. These occur between states with $J = L$ and $S = 1$ or 0 [15]. Hence, the mixing of the physical D_{s1} and D'_{s1} states can be parameterized in terms of a mixing angle θ_s, as follow:

$$\begin{aligned} |D_{s1}\rangle &= sin\theta_s|^3P_1\rangle + cos\theta_s|^1P_1\rangle, \\ |D'_{s1}\rangle &= cos\theta_s|^3P_1\rangle - sin\theta_s|^1P_1\rangle. \end{aligned} \quad (1)$$

where, the spectroscopic notation $^{2S+1}L_J$ has been used to introduce the mixing states. Considering $|^3P_1\rangle \equiv |D_{s1}1\rangle$ and $|^1P_1\rangle \equiv |D_{s1}2\rangle$ with different masses and decay constants [36], we can apply these relations for axial vectors $D_{s1}(2460)$ and $D_{s1}(2536)$ mesons with two different masses. i.e.,

$$\begin{aligned} |D_{s1}(2460)\ \rangle &= sin\theta_s|D_{s1}1\rangle + cos\theta_s|D_{s1}2\rangle, \\ |D_{s1}(2536)\ \rangle &= cos\theta_s|D_{s1}1\rangle - sin\theta_s|D_{s1}2\rangle. \end{aligned} \quad (2)$$

The masses of $D_{s1}1$ and $D_{s1}2$ states are presented in Table 1. These values have been obtained in QM approach.

Ref	[12]	[13]	[14]
$D_{s1}1(^3P_1)$	2.57	2.55	2.535
$D_{s1}2(^1P_1)$	2.53	2.55	2.605

Table 1: Masses of 1^1P_1 and 1^3P_1 heavy-light mesons in quark models.

Note that, in the heavy quark limit the physical eigen states D_{s1} and D'_{s1} can be identified with $P_1^{1/2}$ and $P_1^{3/2}$ with notation L_J^j, where j is the total angular momentum of the light quark [37], corresponding to $\theta_s = -54.7°$ [36].

In this work, taking into account the gluon condensate corrections, we analyze the rare semileptonic $B_c \to D_{s1}\ l^+l^-$, $l = \tau, \mu, e$ and $B_c \to D_{s1}\nu\bar{\nu}$ transitions in three–point QCD sum rules (3PSR) approach. Note that, the $B_c \to (D^*, D_s^*, D_{s1}(2460))\nu\bar{\nu}$ transitions have been studied in Ref. [38], but assuming the D_{s1} only as $c\bar{s}$. The $B_c \to D_q\ l^+l^-/\nu\bar{\nu}$ [39], $B_c \to D_q^*\ l^+l^-$, $(q = d, s)$ [40] transitions have also been analyzed in the same framework.

The heavy B_c meson contains two heavy quarks b and c with different charges. This meson is similar to the charmonium and bottomonium in the spectroscopy, but in contrast to the charmonium and bottomonium, the B_c decays only via weak interaction and has a long lifetime. The study of the B_c transitions are useful for more precise determination of the Cabibbo, Kabayashi, Maskawa (CKM) matrix elements in the weak decays.

The rare semileptonic $B_c \to D_{s1}l^+l^-/\nu\bar{\nu}$ decays occur at loop level by electroweak penguin and weak box diagrams in the standard model (SM) via the flavor changing neutral current (FCNC) transition of $b \to sl^+l^-$. The FCNC decays of B_c meson are sensitive to new physics (NP) contributions to penguin operators. Therefore, the study of such FCNC transitions can improve the information about:

- The CP violation, T violation and polarization asymmetries in $b \to s$ penguin channels, that occur in weak interactions,
- New operators or operators that are subdominant in the SM,
- Establishing NP and flavor physics beyond the SM.

To obtain the form factors of the semileptonic $B_c \to D_{s1}(2460[2536])$ transitions, first, we will suppose the $D_{s1}(2460)$ and $D_{s1}(2536)$ axial vector mesons as the pure $|c\bar{s}\rangle$ state and calculate the related form factors. Second, we will consider the D_{s1} meson as a mixture of two components $|D_{s1}1\rangle$ and $|D_{s1}2\rangle$ states and calculate the form factors of the $B_c \to D_{s1}1$ and $B_c \to D_{s1}2$ transitions. With the help of Eq. (2) and the definition of the form factors which will be presented in the next section, we will derive the transition form factors of $B_c \to D_{s1}(2460[2536])$ decays as a function of the mixing angle θ_s. The future experimental study of such rare decays and comparison of the results with the predictions of theoretical calculations can improve the information about the structure of D_{s1} meson and the mixing angle θ_s.

This paper is organized as follow. In section 2, we calculate the form factors for $B_c \to D_{s1}$ transition in the 3PSR. In section 3, the two-gluon condensate contributions as non-perturbative corrections are calculated. The calculation of the decay rates for $B_c \to D_{s1}l^+l^-$ and $B_c \to D_{s1}\nu\bar{\nu}$ transitions are presented in section 4. Finally, section 5 is devoted to the numeric results and discussions.

2 The form factors of $B_c \to D_{s1}$ transition in 3PSR

In the standard model, the effective Hamiltonian responsible for the rare semileptonic $B_c \to D_{s1}l^+l^-$ and $B_c \to D_{s1}\nu\bar{\nu}$ decays, which are described via $b \to s\ l^+l^-$ loop transitions (see Fig. 1) at quark-level, can be written as:

$$\mathcal{H}_{eff} = \frac{G_F\alpha}{2\pi\sqrt{2}}\, V_{tb}V_{ts}^*\Big[C_9^{eff}\,\overline{s}\,\gamma_\mu(1-\gamma_5)b\,\overline{\ell}\gamma_\mu\ell + C_{10}\,\overline{s}\gamma_\mu(1-\gamma_5)b\,\overline{\ell}\gamma_\mu\gamma_5\ell$$

$$- \quad 2C_7^{eff}\frac{m_b}{q^2}\,\bar{s}\,i\sigma_{\mu\nu}q^\nu(1+\gamma_5)b\,\bar{\ell}\gamma_\mu\ell\Big] \,. \tag{3}$$

where C_7^{eff}, C_9^{eff} and C_{10} are the Wilson coefficients, G_F is the Fermi constant, α is the fine structure constant at the Z mass scale and V_{ij} are the elements of the CKM matrix.

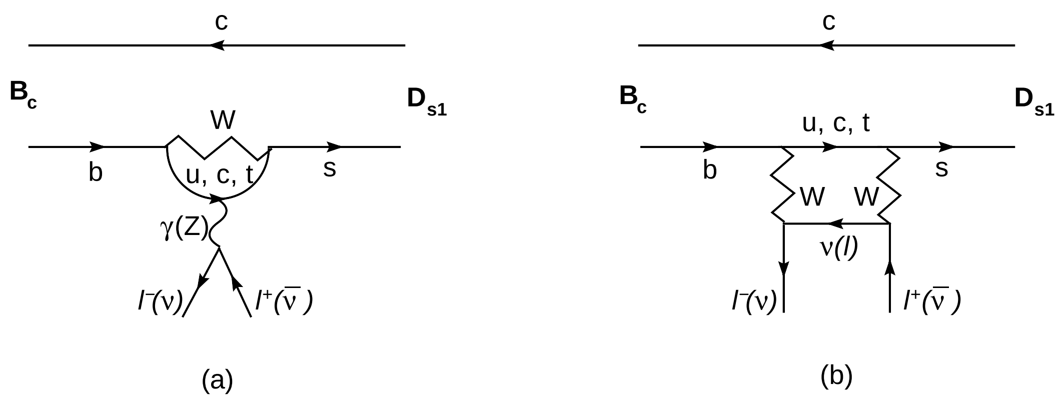

Figure 1: The loop diagrams of the semileptonic decay of B_c to D_{s1}. The electroweak penguin and box diagrams are shown in parts (a) and (b), respectively.

These loop transitions occur via the intermediate u, c, t quarks. In the SM, the measurement of the forward-backward asymmetry and invariant dilepton mass distribution in $b \to q' l^+ l^-$, $(q' = d, s)$ transitions provide information on the short distance contributions dominated by the top quark loops [41]. The electroweak penguin involving the contributions of photon and Z bosons is shown in Fig. 1(a) and Fig. 1(b) presents the contribution of the W box diagram. It is reminded that the $b \to s\ \nu\bar{\nu}$ transition receives contributions only from Z-penguin and box diagrams.

The transition amplitude of $B_c \to D_{s1} l^+ l^- / \nu\bar{\nu}$ decays is obtained sandwiching Eq. (3) between the initial and final states, i.e.,

$$\begin{aligned} \mathcal{M} \;=\; & \frac{G_F\alpha}{2\pi\sqrt{2}}\, V_{tb}V_{ts}^*\Big[C_9^{eff}\,\langle D_{s1}(p') \mid\ \bar{s}\ \gamma_\mu(1-\gamma_5)b \mid B_c(p)\rangle \bar{\ell}\gamma_\mu\ell \\ & + C_{10}\ \langle D_{s1}(p') \mid \bar{s}\ \gamma_\mu(1-\gamma_5)b \mid B_c(p)\rangle \bar{\ell}\gamma_\mu\gamma_5\ell \\ & -2\ C_7^{eff}\ \frac{m_b}{q^2}\langle D_{s1}(p') \mid \bar{s}\ i\sigma_{\mu\nu}q^\nu(1+\gamma_5)b \mid B_c(p)\rangle \bar{\ell}\gamma_\mu\ell\Big]\,, \end{aligned} \tag{4}$$

where, p and p' are the momentum of initial and final meson states, respectively, and ε is the polarization vector of the D_{s1} meson. Our aim is to parameterized the matrix elements appearing in Eq. (4) in terms of the transition form factors considering the Lorentz invariance and parity considerations.

$$\begin{aligned} \langle D_{s1}(p',\varepsilon) \mid \bar{s}\gamma_\mu\gamma_5 b \mid B_c(p)\rangle \;&=\; \frac{2A_V^{B_c\to D_{s1}}(q^2)}{m_{B_c}+m_{D_{s1}}}\ \varepsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}p^\alpha p'^\beta, \\ \langle D_{s1}(p',\varepsilon) \mid \bar{s}\gamma_\mu b \mid B_c(p)\rangle \;&=\; -\ iA_0^{B_c\to D_{s1}}(q^2)(m_{B_c}+m_{D_{s1}})\varepsilon_\mu^* + i\frac{A_1^{B_c\to D_{s1}}(q^2)}{m_{B_c}+m_{D_{s1}}}(\varepsilon^* p)P_\mu \end{aligned}$$

$$
\begin{aligned}
&+ \quad i\frac{A_2^{B_c\to D_{s1}}(q^2)}{m_{B_c}+m_{D_{s1}}}(\varepsilon^* p)q_\mu \ , \\
\langle D_{s1}(p',\varepsilon) \mid \bar{s}\sigma_{\mu\nu}q^\nu\gamma_5 b \mid B_c(p)\rangle &= 2\, T_V^{B_c\to D_{s1}}(q^2)\ i\varepsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}p^\alpha p'^\beta, \\
\langle D_{s1}(p',\varepsilon) \mid \bar{s}\sigma_{\mu\nu}q^\nu b \mid B_c(p)\rangle &= T_0^{B_c\to D_{s1}}(q^2)[\varepsilon^*_\mu(m_{B_c}^2-m_{D_{s1}}^2)-(\varepsilon^* p)P_\mu] \\
&+ \quad T_1^{B_c\to D_{s1}}(q^2)\ (\varepsilon^* p)[q_\mu - \frac{q^2}{m_{B_c}^2-m_{D_{s1}}^2}P_\mu], \qquad (5)
\end{aligned}
$$

where $A_i^{B_c\to D_{s1}}(q^2)$, $i=V,0,1,2$ and $T_j^{B_c\to D_{s1}}(q^2)$, $j=V,0,1$ are the transition form factors, $P_\mu=(p+p')_\mu$ and $q_\mu=(p-p')_\mu$. Here, q^2 is the momentum transfer squared of the Z boson (photon). In order to our calculations be simple, the following redefinitions of the transition form factors are considered :

$$
\begin{aligned}
A_V'^{B_c\to D_{s1}}(q^2) &= \frac{2A_V^{B_c\to D_{s1}}(q^2)}{m_{B_c}+m_{D_{s1}}}\ , & A_0'^{B_c\to D_{s1}}(q^2) &= A_0^{B_c\to D_{s1}}(q^2)(m_{B_c}+m_{D_{s1}}), \\
A_1'^{B_c\to D_{s1}}(q^2) &= -\frac{A_1^{B_c\to D_{s1}}(q^2)}{m_{B_c}+m_{D_{s1}}}\ , & A_2'^{B_c\to D_{s1}}(q^2) &= -\frac{A_2^{B_c\to D_{s1}}(q^2)}{m_{B_c}+m_{D_{s1}}}, \\
T_V'^{B_c\to D_{s1}}(q^2) &= -2T_V^{B_c\to D_{s1}}(q^2)\ , & T_0'^{B_c\to D_{s1}}(q^2) &= -\ T_0^{B_c\to D_{s1}}(q^2)(m_{B_c}^2-m_{D_{s1}}^2), \\
T_1'^{B_c\to D_{s1}}(q^2) &= -\ T_1^{B_c\to D_{s1}}(q^2). & & \qquad (6)
\end{aligned}
$$

To calculate the form factors within three-point QCD sum rules method, the following three-point correlation functions are used:

$$
\begin{aligned}
\Pi_{\mu\nu}^{V-A}(p^2,p'^2,q^2) &= i^2\int d^4x d^4y e^{-ipx}e^{ip'y}\langle 0 \mid T[J_\nu^{D_{s1}}(y)J_\mu^{V-A}(0)J^{B_c\dagger}(x)] \mid 0\rangle, \\
\Pi_{\mu\nu}^{T-PT}(p^2,p'^2,q^2) &= i^2\int d^4x d^4y e^{-ipx}e^{ip'y}\langle 0 \mid T[J_\nu^{D_{s1}}(y)J_\mu^{T-PT}(0)J^{B_c\dagger}(x)] \mid 0\rangle, \qquad (7)
\end{aligned}
$$

where $J_\nu^{D_{s1}}(y)=\bar{c}\gamma_\nu\gamma_5 s$ and $J^{B_c}(x)=\bar{c}\gamma_5 b$ are the interpolating currents of the initial and final meson states, respectively. $J_\mu^{V-A}=\bar{s}\gamma_\mu(1-\gamma_5)b$ and $J_\mu^{T-PT}=\bar{s}\sigma_{\mu\nu}q^\nu(1+\gamma_5)b$ are the vector-axial vector and tensor-pseudo tensor parts of the transition currents. In QCD sum rules approach, we can obtain the correlation function of Eq. (7) in two sides. The phenomenological or physical part is calculated saturating the correlator by a tower of hadrons with the same quantum numbers as interpolating currents. The QCD or theoretical part, on the other side, is obtained in terms of the quarks and gluons interacting in the QCD vacuum. To drive the phenomenological part of the correlators given in Eq. (7), two complete sets of intermediate states with the same quantum numbers as the currents $J_{D_{s1}}$ and J_{B_c} are inserted. This procedure leads to the following representations of the above-mentioned correlators:

$$
\begin{aligned}
\Pi_{\mu\nu}^{V-A}(p^2,p'^2,q^2) &= \frac{\langle 0 \mid J_\nu^{D_{s1}} \mid D_{s1}(p',\varepsilon)\rangle\langle D_{s1}(p',\varepsilon) \mid J_\mu^{V-A} \mid B_c(p)\rangle\langle B_c(p) \mid J^{B_c\dagger} \mid 0\rangle}{(p'^2-m_{D_{s1}}^2)(p^2-m_{B_c}^2)} \\
&+ \ \text{higher resonances and continuum states}\ ,
\end{aligned}
$$

$$\Pi_{\mu\nu}^{T-PT}(p^2,p'^2,q^2) = \frac{\langle 0 \mid J_\nu^{D_{s1}} \mid D_{s1}(p',\varepsilon)\rangle \langle D_{s1}(p',\varepsilon) \mid J_\mu^{T-PT} \mid B_c(p)\rangle \langle B_c(p) \mid J^{B_c\dagger} \mid 0\rangle}{(p'^2-m_{D_{s1}}^2)(p^2-m_{B_c}^2)} + \text{ higher resonances and continuum states .} \quad (8)$$

The following matrix elements are defined in the standard way in terms of the leptonic decay constants of the D_{s1} and B_c mesons as:

$$\langle 0 \mid J_{D_{s1}}^\nu \mid D_{s1}(p',\varepsilon)\rangle = f_{D_{s1}} m_{D_{s1}} \varepsilon^\nu \ , \quad \langle 0 \mid J_{B_c} \mid B_c(p)\rangle = i\frac{f_{B_c} m_{B_c}^2}{m_b+m_c}. \quad (9)$$

Using Eq. (5), Eq. (6) and Eq. (9) in Eq. (8) and performing summation over the polarization of D_{s1} meson we obtain:

$$\Pi_{\mu\nu}^{V-A}(p^2,p'^2,q^2) = -\frac{f_{B_c} m_{B_c}^2}{(m_b+m_c)} \frac{f_{D_{s1}} m_{D_{s1}}}{(p'^2-m_{D_{s1}}^2)(p^2-m_{B_c}^2)} \times \Big[iA_V'^{B_c\to D_{s1}}(q^2)\varepsilon_{\mu\nu\alpha\beta}p^\alpha p'^\beta + A_0'^{B_c\to D_{s1}}(q^2) g_{\mu\nu} + A_1'^{B_c\to D_{s1}}(q^2) P_\mu p_\nu + A_2'^{B_c\to D_{s1}}(q^2) q_\mu p_\nu \Big] + \text{excited states},$$

$$\Pi_{\mu\nu}^{T-PT}(p^2,p'^2,q^2) = -\frac{f_{B_c} m_{B_c}^2}{(m_b+m_c)} \frac{f_{D_{s1}} m_{D_{s1}}}{(p'^2-m_{D_{s1}}^2)(p^2-m_{B_c}^2)} \times \Big[T_V'^{B_c\to D_{s1}}(q^2)\varepsilon_{\mu\nu\alpha\beta}p^\alpha p'^\beta - i\, T_0'^{B_c\to D_{s1}}(q^2) g_{\mu\nu} - i\, T_1'^{B_c\to D_{s1}}(q^2) q_\mu p_\nu \Big] + \text{excited states.} \quad (10)$$

To calculate the form factors, A_V', A_0', A_1', A_2', T_V', T_0' and T_1', we will choose the structures, $i\varepsilon_{\mu\nu\alpha\beta}p^\alpha p'^\beta$, $g_{\mu\nu}$, $P_\mu p_\nu$, $q_\mu p_\nu$, from $\Pi_{\mu\nu}^{V-A}$ and $\varepsilon_{\mu\nu\alpha\beta}p^\alpha p'^\beta$, $ig_{\mu\nu}$ and $iq_\mu p_\nu$ from $\Pi_{\mu\nu}^{T-PT}$, respectively.

On the QCD side, using the operator product expansion (OPE), we can obtain the correlation function in quark-gluon language in the deep Euclidean region where $p^2 \ll (m_b+m_c)^2$ and $p'^2 \ll (m_c^2+m_s^2)$. For this aim, the correlators are written as:

$$\Pi_{\mu\nu}^{V-A}(p^2,p'^2,q^2) = i\,\Pi_V^{V-A}\varepsilon_{\mu\nu\alpha\beta}p^\alpha p'^\beta + \Pi_0^{V-A} g_{\mu\nu} + \Pi_1^{V-A} P_\mu p_\nu + \Pi_2^{V-A} q_\mu p_\nu,$$
$$\Pi_{\mu\nu}^{T-PT}(p^2,p'^2,q^2) = \Pi_V^{T-PT}\varepsilon_{\mu\nu\alpha\beta}p^\alpha p'^\beta - i\,\Pi_0^{T-PT} g_{\mu\nu} - i\,\Pi_1^{T-PT} q_\mu p_\nu, \quad (11)$$

where, each Π_i function is defined in terms of the perturbative and non-perturbative parts as:

$$\Pi_i(p^2,p'^2,q^2) = \Pi_i^{per}(p^2,p'^2,q^2) + \Pi_i^{nonper}(p^2,p'^2,q^2) \ . \quad (12)$$

To obtain the perturbative part of the correlation function, we should study the bare loop diagrams in Fig. 1. In calculating the bare loop contributions, we first write the double dispersion representation for the coefficients of the corresponding Lorentz structures appearing in each correlation function, as:

$$\Pi_i^{per} = -\frac{1}{(2\pi)^2}\int ds' \int ds \frac{\rho_i(s,s',q^2)}{(s-p^2)(s'-p'^2)} + \text{ subtraction terms.} \quad (13)$$

The spectral densities $\rho_i^{per}(s,s',q^2)$ are calculated by the help of the Gutkosky rules, i.e., the propagators are replaced by Dirac–delta functions

$$\frac{1}{p^2-m^2} \to -2i\pi\delta(p^2-m^2) \ , \quad (14)$$

expressing that all quarks are real. Note that, there are two main vertexes related to the bare loop diagrams that describe $b \to s\ l^+l^-$ transition in Fig. 1, i.e., $\gamma_\mu(1-\gamma_5)$ and $\sigma_{\mu\nu}q^\nu(1+\gamma_5)$. First, we calculate the spectral densities related to $\gamma_\mu(1-\gamma_5)$ vertex. Straightforward calculations end up in the following results:

$$\begin{aligned}
\rho_V^{V-A} &= 4N_c\ I_0(s,s',q^2)\ \{B_1(m_b-m_c)-B_2(m_s+m_c)-m_c\}\ ,\\
\rho_0^{V-A} &= -2N_c\ I_0(s,s',q^2)\ \{\Delta(m_c+m_s)-\Delta'(m_b-m_c)-4A_1(m_b-m_c)\\
&\quad +2m_c^2(m_b-m_c-m_s)+m_c(2m_bm_s-u)\}\ ,\\
\rho_1^{V-A} &= 2N_c\ I_0(s,s',q^2)\{B_1(m_b-3m_c)-B_2(m_c+m_s)+2A_2(m_b-m_c)\\
&\quad +2A_3(m_b-m_c)-m_c\}\ ,\\
\rho_2^{V-A} &= 2N_c\ I_0(s,s',q^2)\{2A_2(m_b-m_c)-2A_3(m_b-m_c)-B_1(m_b+m_c)\\
&\quad +B_2(m_c+m_s)+m_c\}\ . \qquad (15)
\end{aligned}$$

Then, the spectral densities related to $\sigma_{\mu\nu}q^\nu(1+\gamma_5)$ vertex are presented as:

$$\begin{aligned}
\rho_V^{T-PT}(s,s',q^2) &= 2N_c\ I_0(s,s',q^2)\ \{m_c(m_b-m_s)+B_1\ [\Delta-m_cm_s-m_c^2+m_b(m_c+m_s)-s]\\
&+\ B_2\ [\Delta'-m_cm_s-m_c^2+m_b(m_c+m_s)-s']\ \},\\
\rho_0^{T-PT}(s,s',q^2) &= 2N_c\ I_0(s,s',q^2)\ \{2A_1(2s-u)-\Delta\ [m_b(m_c+m_s)-m_c(m_c+m_s)+s']\\
&+\ \Delta'\ [m_b(m_c+m_s)-m_c(m_c+m_s)+s]-2m_c(m_b-m_c)s'\\
&-\ 2m_c(m_c+m_s)s+2m_c(m_b+m_s)u\},\\
\rho_1^{T-PT}(s,s',q^2) &= 2N_c\ I_0(s,s',q^2)\ \{m_c(m_b+m_s)+B_2\ [(m_b-m_c)(m_c+m_s)+s']\\
&+\ B_1\ [\Delta-\Delta'-m_b(m_c+m_s)+m_c(m_c+m_s)-s]\\
&+\ (A_2-A_3)(2s-u)\}, \qquad (16)
\end{aligned}$$

where

$$\begin{aligned}
I_0(s,s',q^2) &= \frac{1}{4\lambda^{1/2}(s,s',q^2)},\\
\lambda(a,b,c) &= a^2+b^2+c^2-2ac-2bc-2ab,\\
\Delta' &= (s'+m_c^2-m_s^2),\\
\Delta &= (s+m_c^2-m_b^2),\\
u &= s+s'-q^2,\\
B_1 &= \frac{1}{\lambda(s,s',q^2)}[2s'\Delta-\Delta'u],\\
B_2 &= \frac{1}{\lambda(s,s',q^2)}[2s\Delta'-\Delta u],\\
A_1 &= -\frac{1}{2\lambda(s,s',q^2)}[(4ss'm_c^2-s\Delta'^2-s'\Delta^2-u^2m_c^2+u\Delta\Delta')],
\end{aligned}$$

$$
\begin{aligned}
A_2 &= -\frac{1}{\lambda^2(s,s',q^2)}[8ss'^2m_c^2 - 2ss'\Delta'^2 - 6s'^2\Delta^2 - 2u^2s'm_c^2 \\
&+ 6s'u\Delta\Delta' - u^2\Delta'^2], \\
A_3 &= \frac{1}{\lambda^2(s,s',q^2)}[4ss'um_c^2 + 4ss'\Delta\Delta' - 3su\Delta'^2 - 3u\Delta^2 s' - u^3m_c^2 + 2u^2\Delta\Delta'].
\end{aligned}
$$

and $N_c = 3$ is the color factor.

The integration region in Eq. (13) is obtained requiring that the argument of three delta vanish, simultaneously. The physical region in the s and s' plane is described by the following inequalities:

$$
-1 \leq \frac{2ss' + (s+s'-q^2)(m_b^2 - s - m_c^2) + (m_c^2 - m_s^2)2s}{\lambda^{1/2}(m_b^2, s, m_c^2)\lambda^{1/2}(s,s',q^2)} \leq +1. \tag{17}
$$

From this inequality, to use in the lower limit of the integration over s in continuum subtractions, it is easy to express s in terms of s', i.e., s_L is as follow:

$$
s_L = \frac{(m_c^2 + q^2 - m_b^2 - s')(m_b^2 s' - q^2 m_c^2)}{(m_b^2 - q^2)(m_c^2 - s')} . \tag{18}
$$

3 Gluon condensate contribution

In this section, the non-perturbative part contributions to the correlation function are discussed. Here, we will follow the same procedure as stated in [38–40, 42]. The non-perturbative part contains the quark and gluon condensate diagrams. For this aim, we consider the condensate terms of dimension $3, 4$ and 5. It's found that the heavy quark condensate contributions are suppressed by inverse of the heavy quark mass and can be safely omitted. The light s quark condensate contribution is zero after applying the double Borel transformation with respect to the both variables p^2 and $p^{'2}$, because only one variable appears in the denominator. Therefore in this case, we consider the two gluon condensate diagrams with mass dimension 4 as non-perturbative corrections. The diagrams for contribution of the gluon condensates are depicted in Fig. 2. To obtain the contributions of these diagrams, the Fock-Schwinger fixed-point gauge, $x^\mu A_\mu^a = 0$, are used, where A_μ^a is the gluon field. In the evaluation of diagrams in Fig. 2, integrals of the following types are encountered.

$$
\begin{aligned}
I_0(a,b,c) &= \int \frac{d^4k}{(2\pi)^4} \frac{1}{[k^2 - m_c^2]^a\,[(p+k)^2 - m_b^2]^b\,[(p'+k)^2 - m_s^2]^c} , \\
I_\mu(a,b,c) &= \int \frac{d^4k}{(2\pi)^4} \frac{k_\mu}{[k^2 - m_c^2]^a\,[(p+k)^2 - m_b^2]^b\,[(p'+k)^2 - m_s^2]^c} , \\
I_{\mu\nu}(a,b,c) &= \int \frac{d^4k}{(2\pi)^4} \frac{k_\mu k_\nu}{[k^2 - m_c^2]^a\,[(p+k)^2 - m_b^2]^b\,[(p'+k)^2 - m_s^2]^c} .
\end{aligned} \tag{19}
$$

These integrals can be calculated using the Schwinger representation for the Euclidean propagator

$$
\frac{1}{(k^2+m^2)^n} = \frac{1}{\Gamma(n)} \int_0^\infty d\alpha\, \alpha^{n-1} e^{-\alpha(k^2+m^2)} , \tag{20}
$$

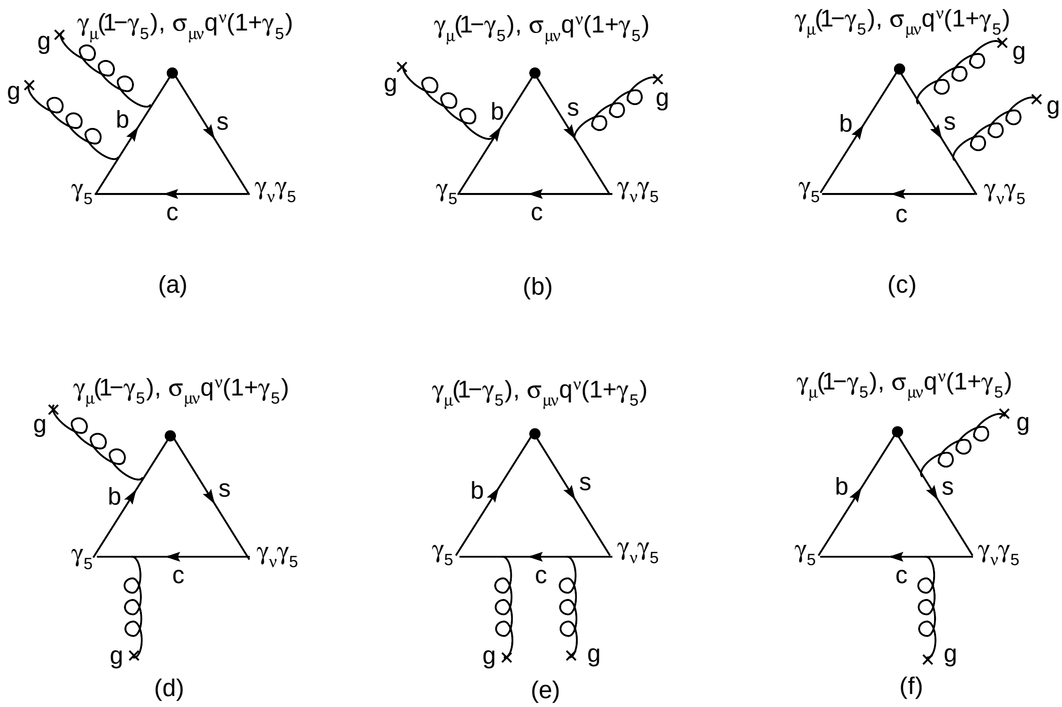

Figure 2: Contribution of gluon condensates for $B_c \to D_{s1}$ transition.

After Borel transformation using:

$$\mathcal{B}_{\hat{p}^2}(M^2)e^{-\alpha p^2} = \delta(1/M^2 - \alpha) \ , \tag{21}$$

we obtain

$$\hat{I}_0(a,b,c) = \frac{(-1)^{a+b+c}}{16\pi^2\,\Gamma(a)\Gamma(b)\Gamma(c)}(M_1^2)^{2-a-b}(M_2^2)^{2-a-c}\,\mathcal{U}_0(a+b+c-4, 1-c-b) \ ,$$

$$\hat{I}_\mu(a,b,c) \ = \ \hat{I}_1(a,b,c)p_\mu + \hat{I}_2(a,b,c)p'_\mu \ ,$$

$$\hat{I}_{\mu\nu}(a,b,c) \ = \ \hat{I}_6(a,b,c)g_{\mu\nu} + \hat{I}_3(a,b,c)p_\mu p_\nu + \hat{I}_4(a,b,c)p_\mu p'_\nu + \hat{I}_4(a,b,c)p'_\mu p_\nu + \hat{I}_5(a,b,c)p'_\mu p'_\nu \ . \tag{22}$$

$\hat{I}$ in Eq. (22) stands for double Borel transformed form of Eq. (19). In Schwinger representation:

$$\hat{I}_k(a,b,c) = i\frac{(-1)^{a+b+c+1}}{16\pi^2\,\Gamma(a)\Gamma(b)\Gamma(c)}(M_1^2)^{1-a-b+k}(M_2^2)^{4-a-c-k}\,\mathcal{U}_0(a+b+c-5, 1-c-b) \ ,$$

$$\hat{I}_m(a,b,c) = i\frac{(-1)^{a+b+c+1}}{16\pi^2\,\Gamma(a)\Gamma(b)\Gamma(c)}(M_1^2)^{-a-b-1+m}(M_2^2)^{7-a-c-m}\,\mathcal{U}_0(a+b+c-5, 1-c-b) \ ,$$

$$\hat{I}_6(a,b,c) = i\frac{(-1)^{a+b+c+1}}{32\pi^2\,\Gamma(a)\Gamma(b)\Gamma(c)}(M_1^2)^{3-a-b}(M_2^2)^{3-a-c}\,\mathcal{U}_0(a+b+c-6, 2-c-b) \ . \tag{23}$$

where $k=1,2$, $m=3,4,5$, M_1^2 and M_2^2 are the Borel parameters in the s and s' channel, respectively, and the function $\mathcal{U}_0(a,b)$ is defined as

$$\mathcal{U}_0(a,b) = \int_0^\infty dy (y+M_1^2+M_2^2)^a y^b\, exp\left[-\frac{B_{-1}}{y} - B_0 - B_1 y\right] ,$$

where

$$\begin{aligned} B_{-1} &= \frac{1}{M_1^2 M_2^2}\left[m_s^2 M_1^4 + m_b^2 M_2^4 + M_2^2 M_1^2 (m_b^2 + m_s^2 - q^2)\right] , \\ B_0 &= \frac{1}{M_1^2 M_2^2}\left[(m_s^2+m_c^2)M_1^2 + M_2^2(m_b^2+m_c^2)\right] , \\ B_1 &= \frac{m_c^2}{M_1^2 M_2^2} . \end{aligned} \tag{24}$$

Performing the double Borel transformation over the variables p^2 and $p^{'2}$ on the physical as well as perturbative parts of the correlation functions and equating the coefficients of the selected structures from both sides, the sum rules for the form factors $A_i^{'B_c\to D_{s1}}$ are obtained:

$$\begin{aligned} A_i^{'B_c\to D_{s1}} &= -\frac{(m_b+m_c)}{f_{B_c} m_{B_c}^2 f_{D_{s1}} m_{D_{s1}}} e^{\frac{m_{B_c}^2}{M_1^2}} e^{\frac{m_{D_{s1}}^2}{M_2^2}} \Bigg\{ -\frac{1}{4\pi^2}\int_{m_c^2}^{s_0'} ds' \int_{s_L}^{s_0} \rho_i^{V-A}(s,s',q^2) e^{\frac{-s}{M_1^2}} e^{\frac{-s'}{M_2^2}} \\ &- i M_1^2 M_2^2 \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \frac{C_i^{V-A}}{6} \Bigg\} , \end{aligned} \tag{25}$$

where $i=V,0,1,2$ and for the form factors $T_j^{'B_c\to D_{s1}}$, we get

$$\begin{aligned} T_j^{'B_c\to D_{s1}} &= -\frac{(m_b+m_c)}{f_{B_c} m_{B_c}^2 f_{D_{s1}} m_{D_{s1}}} e^{\frac{m_{B_c}^2}{M_1^2}} e^{\frac{m_{D_{s1}}^2}{M_2^2}} \Bigg\{ -\frac{1}{4\pi^2}\int_{m_c^2}^{s_0'} ds' \int_{s_L}^{s_0} \rho_j^{T-PT}(s,s',q^2) e^{\frac{-s}{M_1^2}} e^{\frac{-s'}{M_2^2}} \\ &- i M_1^2 M_2^2 \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \frac{C_j^{T-PT}}{6} \Bigg\} . \end{aligned} \tag{26}$$

where $j=V,0,1$. The s_0 and $s_0^{'}$ are the continuum thresholds in B_c and D_{s1} channels, respectively and lower bound s_L in the integrals is given in Eq. (18). We present the explicit expressions of the coefficients $C_{i(j)}^{V-A(T-PT)}$ correspond to gluon condensates in Appendix–A.

Now, the $A_i^{'B_c\to D_{s1}1(2)}$ and $T_j^{'B_c\to D_{s1}1(2)}$ form factors are obtained from the above equations replacing the $f_{D_{s1}}$ by decay constant $f_{D_{s1}1(2)}$, and $m_{D_{s1}}$ with $m_{D_{s1}1(2)}$, i.e.,

$$\begin{aligned} A_i^{'B_c\to D_{s1}1(2)} &= -\frac{(m_b+m_c)}{f_{B_c} m_{B_c}^2 f_{D_{s1}1(2)} m_{D_{s1}1(2)}} e^{\frac{m_{B_c}^2}{M_1^2}} e^{\frac{m_{D_{s1}1(2)}^2}{M_2^2}} \Bigg\{ -\frac{1}{4\pi^2}\int_{m_c^2}^{s_0'} ds' \int_{s_L}^{s_0} \rho_i^{V-A}(s,s',q^2) e^{\frac{-s}{M_1^2}} e^{\frac{-s'}{M_2^2}} \\ &- i M_1^2 M_2^2 \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \frac{C_i^{V-A}}{6} \Bigg\} , \end{aligned} \tag{27}$$

and

$$T_j^{\prime B_c\to D_{s1}1(2)} = -\frac{(m_b+m_c)}{f_{B_c}m_{B_c}^2 f_{D_{s1}1(2)}m_{D_{s1}1(2)}} e^{\frac{m_{B_c}^2}{M_1^2}} e^{\frac{m_{D_{s1}1(2)}^2}{M_2^2}} \Bigg\{ -\frac{1}{4\pi^2}\int_{m_c^2}^{s_0'} ds' \int_{s_L}^{s_0} \rho_i^{T-PT}(s,s',q^2) e^{\frac{-s}{M_1^2}} e^{\frac{-s'}{M_2^2}} - iM_1^2 M_2^2 \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \frac{C_i^{T-PT}}{6} \Bigg\} . \quad (28)$$

Using the straight forward calculations, the form factors of the $A_i^{B_c\to D_{s1}(2460)}$ and $T_j^{B_c\to D_{s1}(2460)}$ are found as follows:

$$\begin{aligned}
A_0^{B_c\to D_{s1}(2460)} &= \left(\frac{m_{B_c}+m_{D_{s1}1}}{m_{B_c}+m_{D_{s1}}}\right) A_0^{B_c\to D_{s1}1}\ sin\theta_s + \left(\frac{m_{B_c}+m_{D_{s1}2}}{m_{B_c}+m_{D_{s1}}}\right) A_0^{B_c\to D_{s1}2}\ cos\theta_s\ , \\
A_{i'}^{B_c\to D_{s1}(2460)} &= \left(\frac{m_{B_c}+m_{D_{s1}}}{m_{B_c}+m_{D_{s1}1}}\right) A_{i'}^{B_c\to D_{s1}1}\ sin\theta_s + \left(\frac{m_{B_c}+m_{D_{s1}}}{m_{B_c}+m_{D_{s1}2}}\right) A_{i'}^{B_c\to D_{s1}2}\ cos\theta_s\ , \\
T_0^{B_c\to D_{s1}(2460)} &= \left(\frac{m_{B_c}+m_{D_{s1}1}}{m_{B_c}+m_{D_{s1}}}\right) T_0^{B_c\to D_{s1}1}\ sin\theta_s + \left(\frac{m_{B_c}+m_{D_{s1}2}}{m_{B_c}+m_{D_{s1}}}\right) T_0^{B_c\to D_{s1}2}\ cos\theta_s\ , \\
T_{j'}^{B_c\to D_{s1}(2460)} &= T_{j'}^{B_c\to D_{s1}1}\ sin\theta_s + T_{j'}^{B_c\to D_{s1}2}\ cos\theta_s\ . \qquad (29)
\end{aligned}$$

where $i' = V, 1, 2$ and $j' = V, 1$. Note that, the $A_i^{B_c\to D_{s1}(2536)}$ and $T_j^{B_c\to D_{s1}(2536)}$ form factors are obtained from the above equations by replacing the $sin\theta_s \to cos\theta_s$ and $cos\theta_s \to -sin\theta_s$.

4 Decay widths

Now, we present the dilepton invariant mass distribution for the $B_c \to D_{s1}\nu\bar{\nu}$ and $B_c \to D_{s1}l\bar{l}$ decays. Using the parameterization of the $B_c \to D_{s1}$ transition in terms of form factors and also Eq. (3), the dilepton invariant mass distribution of the $B_c \to D_{s1}\nu\bar{\nu}$ decay can be written as [43]:

$$\frac{d\Gamma\left(B_c \to D_{s1}\nu\bar{\nu}\right)}{dq^2} = \frac{3G_F^2 m_{B_c}^3 |V_{tb}V_{ts}^*|^2 \alpha^2 |D\left(x_t\right)|^2}{2^8\pi^5 sin^4\theta_W} \phi_{D_{s1}}^{\frac{1}{2}} \left[s\alpha_1 + \frac{\phi_{D_{s1}}}{3}\beta_1\right], \quad (30)$$

where $s = q^2/m_{B_c}^2$, $x_t = m_t^2/m_W^2$. The parameters $D(x_t)$, $\phi_{D_{s1}}$, α_1 and β_1 are defined by

$$\begin{aligned}
D(x_t) &= \frac{x_t}{8}\left(\frac{2+x_t}{x_t-1} + \frac{3x_t-6}{(x_t-1)^2}\ \ln x_t\right), \\
\phi_{D_{s1}} &= \left(1-r_{D_{s1}}\right)^2 - 2s\left(1+r_{D_{s1}}\right) + s^2, \\
\alpha_1 &= (1-\sqrt{r_{D_{s1}}})^2 \left|A_0^{B_c\to D_{s1}}\right|^2 + \frac{\phi_{D_{s1}}}{(1+\sqrt{r_{D_{s1}}})^2}\left|A_V^{B_c\to D_{s1}}\right|^2\ , \\
\beta_1 &= \frac{(1-\sqrt{r_{D_{s1}}})^2}{4r_{D_{s1}}}\left|A_0^{B_c\to D_{s1}}\right|^2 - \frac{s}{(1+\sqrt{r_{D_{s1}}})^2}\left|A_V^{B_c\to D_{s1}}\right|^2 + \frac{\phi_{D_{s1}}\left|A_1^{B_c\to D_{s1}}\right|^2}{4r_{D_{s1}}(1+\sqrt{r_{D_{s1}}})^2} \\
&\quad + \frac{1}{2}\left(\frac{1-s}{r_{D_{s1}}} - 1\right)\frac{1-\sqrt{r_{D_{s1}}}}{1+\sqrt{r_{D_{s1}}}} Re(A_0^{B_c\to D_{s1}} A_1^{*B_c\to D_{s1}})\ , \qquad (31)
\end{aligned}$$

where $r_{D_{s1}} = m^2_{D_{s1}}/m^2_{B_c}$. The differential decay rates for $B_c \to D_{s1} l\bar{l}$ are found to be [44, 45]:

$$\frac{d\Gamma\left(B_c^+ \to D_{s1} l^+ l^-\right)}{dq^2} = \frac{G_F^2 m_{B_c}^3 |V_{tb}V_{ts}^*|^2 \alpha^2}{2^9 \pi^5} v\ \phi^{\frac{1}{2}}_{D_{s1}} \left[\left(1 + \frac{2m_l^2}{q^2}\right)\left(s\ \alpha_3 + \frac{\phi_{D_{s1}}}{3}\beta_3\right) + 4t\ \delta\right], \tag{32}$$

where $t = m_l^2/m^2_{B_c}$, $v = \sqrt{1 - 4m_l^2/q^2}$ and the expressions of α_3, β_3 and δ are given as:

$$\begin{aligned}\alpha_3 &= (1 - \sqrt{r_{D_{s1}}})^2 \left[\left|C_9^{eff} A_0^{B_c \to D_{s1}} - \frac{2\hat{m}_b C_7^{eff}(1 + \sqrt{r_{D_{s1}}}) T_0^{B_c \to D_{s1}}}{s}\right|^2 + \left|C_{10} A_0^{B_c \to D_{s1}}\right|^2\right] \\ &+ \frac{\phi_{D_{s1}}}{(1 + \sqrt{r_{D_{s1}}})^2} \left[\left|C_9^{eff} A_V^{B_c \to D_{s1}} - \frac{2\hat{m}_b C_7^{eff}(1 + \sqrt{r_{D_{s1}}}) T_V^{B_c \to D_{s1}}}{s}\right|^2 + \left|C_{10} A_V^{B_c \to D_{s1}}\right|^2\right],\end{aligned} \tag{33}$$

$$\begin{aligned}\beta_3 &= \frac{(1 - \sqrt{r_{D_{s1}}})^2}{4r_{D_{s1}}} \left[\left|C_9^{eff} A_0^{B_c \to D_{s1}} - \frac{2\hat{m}_b C_7^{eff}(1 + \sqrt{r_{D_{s1}}}) T_0^{B_c \to D_{s1}}}{s}\right|^2 + \left|C_{10} A_0^{B_c \to D_{s1}}\right|^2\right] \\ &- \frac{s}{(1 + \sqrt{r_{D_{s1}}})^2} \left[\left|C_9^{eff} A_V^{B_c \to D_{s1}} - \frac{2\hat{m}_b C_7^{eff}(1 + \sqrt{r_{D_{s1}}}) T_V^{B_c \to D_{s1}}}{s}\right|^2 + \left|C_{10} A_V^{B_c \to D_{s1}}\right|^2\right] \\ &+ \frac{\phi_{D_{s1}}}{4r_{D_{s1}}(1 + \sqrt{r_{D_{s1}}})^2} \left[\left|C_9^{eff} A_1^{B_c \to D_{s1}} - \frac{2\hat{m}_b C_7^{eff}(1 + \sqrt{r_{D_{s1}}}) T_1^{B_c \to D_{s1}}}{s}\right|^2 + \left|C_{10} A_1^{B_c \to D_{s1}}\right|^2\right] \\ &+ \frac{1}{2}\left(\frac{1 - s}{r_{D_{s1}}} - 1\right) \frac{1 - \sqrt{r_{D_{s1}}}}{1 + \sqrt{r_{D_{s1}}}} Re\left\{\left[C_9^{eff} A_0^{B_c \to D_{s1}} - \frac{2\hat{m}_b C_7^{eff}(1 + \sqrt{r_{D_{s1}}}) T_0^{B_c \to D_{s1}}}{s}\right]\right. \\ &\times \left.\left[C_9^{eff} A_1^{B_c \to D_{s1}} - \frac{2\hat{m}_b C_7^{eff}(1 + \sqrt{r_{D_{s1}}}) T_1^{B_c \to D_{s1}}}{s}\right] + |C_{10}|^2 Re(A_0^{B_c \to D_{s1}} A_1^{*B_c \to D_{s1}})\right\}.\end{aligned} \tag{34}$$

and

$$\begin{aligned}\delta &= \frac{|C_{10}|^2}{2(1 + \sqrt{r_{D_{s1}}})^2} \left\{ -2\phi_{D_{s1}} |A_V^{B_c \to D_{s1}}|^2 - 3(1 - r_{D_{s1}})^2 |A_0^{B_c \to D_{s1}}|^2 \right. \\ &+ \frac{\phi_{D_{s1}}}{4r_{D_{s1}}} \left[2(1 + r_{D_{s1}}) - s\right] |A_1^{B_c \to D_{s1}}|^2 + \frac{\phi_{D_{s1}} s}{4r_{D_{s1}}} |A_2^{B_c \to D_{s1}}|^2 \\ &+ \left. \frac{\phi_{D_{s1}}(1 - r_{D_{s1}})}{2r_{D_{s1}}} Re\left(A_0^{B_c \to D_{s1}} A_1^{*B_c \to D_{s1}} + A_0^{B_c \to D_{s1}} A_2^{*B_c \to D_{s1}} + A_1^{B_c \to D_{s1}} A_2^{*B_c \to D_{s1}}\right)\right\},\end{aligned} \tag{35}$$

where $\hat{m}_b = m_b/m_{B_c}$.

5 Numerical analysis

In this section, we present our numerical analysis of the form factors A_i , $(i = V, 0, 1, 2)$ and T_j , $(j = V, 0, 1)$. From the sum rules expressions of the form factors, it is clear that the main input parameters entering the expressions are gluon condensates, elements of the CKM matrix V_{tb} and V_{ts}, leptonic decay constants f_{B_c}, $f_{D_{s1}}$, $f_{D_{s1}1}$ and $f_{D_{s1}2}$, Borel parameters M_1^2 and M_2^2 as well as the continuum thresholds s_0 and s_0'. We choose the values of the condensates (at a fixed renormalization scale of about 1 GeV), leptonic decay constants , CKM matrix elements, quark and meson masses as: $\langle \frac{\alpha_s}{\pi} G^2 \rangle = 0.012\ GeV^4$ [46], $\mid V_{tb} \mid = 0.77^{+0.18}_{-0.24}$, $\mid V_{ts} \mid = (40.6 \pm 2.7) \times 10^{-3}$ [47], $C_7^{eff} = -0.313$, $C_9^{eff} = 4.344$, $C_{10} = -4.669$ [48, 49], $f_{D_{s1}} = (225 \pm 25)\ MeV$, $f_{D_{s1}1} = (240 \pm 25)\ MeV$, $f_{D_{s1}2} = (63 \pm 7)\ MeV$ [36, 50], $f_{B_c} = (350 \pm 25)\ MeV$ [51–53], $m_s(1\ GeV) = (104^{+26}_{-34})\ MeV$, $m_c = \left(1.27^{+0.07}_{-0.11}\right)\ GeV$, $m_b = (4.7 \pm 0.07)\ GeV$, $m_{D_{s1}(2460)} = (2459.6 \pm 0.6)\ MeV$, $m_{D_{s1}(2536)} = (2535.35 \pm 0.34 \pm 0.5)\ MeV$ and $m_{B_c} = (6.276 \pm 0.004)\ GeV$ [54].

The sum rules for the form factors contain also four auxiliary parameters: Borel mass squares M_1^2 and M_2^2 and continuum thresholds s_0 and s_0'. These are not physical quantities, so the the form factors as physical quantities should be independent of them. The parameters s_0 and s_0', which are the continuum thresholds of B_c and D_{s1} mesons, respectively, are determined from the condition that guarantees the sum rules to practically be stable in the allowed regions for M_1^2 and M_2^2. The values of the continuum thresholds calculated from the two–point QCD sum rules are taken to be $s_0 = (45 - 50)\ GeV^2$ and $s_0' = (6 - 8)\ GeV^2$ [46, 55, 56]. The working regions for M_1^2 and M_2^2 are determined requiring that not only the contributions of the higher states and continuum are small, but the contributions of the operators with higher dimensions are also small. Both conditions are satisfied in the regions $10\ GeV^2 \le M_1^2 \le 25\ GeV^2$ and $4\ GeV^2 \le M_2^2 \le 10\ GeV^2$. First, we would like to consider the D_{s1} meson as the pure $|c\bar{s}\rangle$ state. The values of the form factors at $q^2 = 0$ are presented in Table 2. The sum rules for the form factors are truncated at about 10 GeV^2, so

$A_V^{B_c \to D_{s1}(2460)}(0)$	-0.23 ± 0.07	$A_V^{B_c \to D_{s1}(2536)}(0)$	-0.22 ± 0.06
$A_0^{B_c \to D_{s1}(2460)}(0)$	0.09 ± 0.02	$A_0^{B_c \to D_{s1}(2536)}(0)$	0.07 ± 0.02
$A_1^{B_c \to D_{s1}(2460)}(0)$	0.16 ± 0.05	$A_1^{B_c \to D_{s1}(2536)}(0)$	0.17 ± 0.05
$A_2^{B_c \to D_{s1}(2460)}(0)$	-0.26 ± 0.08	$A_2^{B_c \to D_{s1}(2536)}(0)$	-0.28 ± 0.09
$T_V^{B_c \to D_{s1}(2460)}(02)$	0.12 ± 0.03	$T_V^{B_c \to D_{s1}(2536)}(0)$	0.14 ± 0.04
$T_0^{B_c \to D_{s1}(2460)}(0)$	0.11 ± 0.03	$T_0^{B_c \to D_{s1}(2536)}(0)$	0.14 ± 0.04
$T_1^{B_c \to D_{s1}(2460)}(0)$	-0.14 ± 0.04	$T_1^{B_c \to D_{s1}(2536)}(0)$	-0.16 ± 0.05

Table 2: The value of the form factors of the $B_c \to D_{s1}(2460)$ and $B_c \to D_{s1}(2536)$ transitions at $q^2 = 0$, $M_1^2 = 15\ GeV^2$ and $M_2^2 = 8\ GeV^2$, when the D_{s1} meson are considered as the pure $|c\bar{s}\rangle$ state .

to extend our results to the full physical region, $0 \le q^2 \le (m_{B_c} - m_{D_{s1}})^2\ GeV^2$, we look for a parameterization of the form factors in such a way that in the region $0 \le q^2 \le 10\ GeV^2$, this parameterization coincides with the sum rules predictions. Our numerical calculations show that the sufficient parameterization of the form factors with respect to q^2 is as follows:

$$f_i(q^2) = \frac{a}{(1 - \frac{q^2}{m_{fit}^2})} + \frac{b}{(1 - \frac{q^2}{m_{fit}^2})^2}. \quad (36)$$

The values of the parameters a, b and m_{fit} are given in the Table 3. To calculate the

	a	b	m_{fit}		a	b	m_{fit}
$A_V^{B_c\to D_{s1}(2460)}(q^2)$	-0.13	-0.10	5.30	$A_V^{B_c\to D_{s1}(2536)}(q^2)$	-0.12	-0.10	5.22
$A_0^{B_c\to D_{s1}(2460)}(q^2)$	0.05	0.04	5.98	$A_0^{B_c\to D_{s1}(2536)}(q^2)$	0.04	0.03	5.99
$A_1^{B_c\to D_{s1}(2460)}(q^2)$	0.09	0.07	5.95	$A_1^{B_c\to D_{s1}(2536)}(q^2)$	0.09	0.08	5.98
$A_2^{B_c\to D_{s1}(2460)}(q^2)$	-0.16	-0.10	5.15	$A_2^{B_c\to D_{s1}(2536)}(q^2)$	-0.17	-0.11	5.17
$T_V^{B_c\to D_{s1}(2460)}(q^2)$	0.08	0.04	5.22	$T_V^{B_c\to D_{s1}(2536)}(q^2)$	0.09	0.05	5.05
$T_0^{B_c\to D_{s1}(2460)}(q^2)$	0.10	0.01	5.85	$T_0^{B_c\to D_{s1}(2536)}(q^2)$	0.11	0.03	5.96
$T_1^{B_c\to D_{s1}(2460)}(q^2)$	-0.09	-0.05	5.22	$T_1^{B_c\to D_{s1}(2536)}(q^2)$	-0.08	-0.07	5.14

Table 3: Parameters appearing in the fit function for the form factors of the $B_c \to D_{s1}(2460)$ and $B_c \to D_{s1}(2536)$ transitions at $M_1^2 = 15\ GeV^2$ and $M_2^2 = 8\ GeV^2$.

branching ratios of the $B_c \to D_{s1}(2460[2536])l^+l^-/\nu\bar{\nu}$ decays, we integrate Eqs. (30, 32) over q^2 in the whole physical region and use the total mean life time $\tau_{B_c} = (0.46 \pm 0.07)\ ps$ [54]. Our numerical analysis shows that the contribution of the non-perturbative part (the gluon condensate diagrams) is about 12% of the total and the main contribution comes from the perturbative part of the form factors. The values for the branching ratio of these decays are obtained as presented in Table 4, when only the short distance (SD) effects are considered. It should be noted that, the long distance (LD) effects for the charged lepton

MODS	BR	MODS	BR
$B_c \to D_{s1}(2460)\nu\bar{\nu}$	$(3.26 \pm 1.10) \times 10^{-7}$	$B_c \to D_{s1}(2536)\nu\bar{\nu}$	$(2.76 \pm 0.88) \times 10^{-7}$
$B_c \to D_{s1}(2460)e^+e^-$	$(5.40 \pm 1.70) \times 10^{-6}$	$B_c \to D_{s1}(2536)e^+e^-$	$(2.91 \pm 0.93) \times 10^{-6}$
$B_c \to D_{s1}(2460)\mu^+\mu^-$	$(2.27 \pm 0.95) \times 10^{-6}$	$B_c \to D_{s1}(2536)\mu^+\mu^-$	$(1.96 \pm 0.63) \times 10^{-6}$
$B_c \to D_{s1}(2460)\tau^+\tau^-$	$(1.42 \pm 0.45) \times 10^{-8}$	$B_c \to D_{s1}(2536)\tau^+\tau^-$	$(0.68 \pm 0.21) \times 10^{-8}$

Table 4: The branching ratios of the semileptonic $B_c \to D_{s1}(2460)l^+l^-/\nu\bar{\nu}$ and $B_c \to D_{s1}(2536)l^+l^-/\nu\bar{\nu}$ decays with SD effects.

modes are not included in the values of Table 4. With the LD effects, we introduce some cuts close to $q^2 = 0$ and around the resonances of J/ψ and ψ' and study the three regions as follows:

$$\begin{aligned} I: &\quad \sqrt{q^2_{min}} \le \sqrt{q^2} \le M_{J/\psi} - 0.20, \\ II: &\quad M_{J/\psi} + 0.04 \le \sqrt{q^2} \le M_{\psi'} - 0.10, \\ III: &\quad M_{\psi'} + 0.02 \le \sqrt{q^2} \le m_{B_c} - m_{D_{s1}}. \end{aligned} \tag{37}$$

where $\sqrt{q^2_{min}} = 2m_l$. In Table 5, we present the branching ratios in terms of the regions shown in Eq. (37). The errors are estimated by the variation of the Borel parameters M_1^2

MODS	I	II	III
$BR(B_c \to D_{s1}(2460)e^+e^-)$	$(4.40 \pm 1.35) \times 10^{-6}$	$(1.62 \pm 0.52) \times 10^{-7}$	$(1.01 \pm 0.35) \times 10^{-7}$
$BR(B_c \to D_{s1}(2536)e^+e^-)$	$(2.65 \pm 0.82) \times 10^{-6}$	$(0.90 \pm 0.28) \times 10^{-7}$	$(0.81 \pm 0.25) \times 10^{-7}$
$BR(B_c \to D_{s1}(2460)\mu^+\mu^-)$	$(1.76 \pm 0.58) \times 10^{-6}$	$(1.61 \pm 0.53) \times 10^{-7}$	$(1.01 \pm 0.35) \times 10^{-7}$
$BR(B_c \to D_{s1}(2536)\mu^+\mu^-)$	$(1.00 \pm 0.31) \times 10^{-6}$	$(0.96 \pm 0.29) \times 10^{-7}$	$(0.68 \pm 0.21) \times 10^{-7}$
$BR(B_c \to D_{s1}(2460)\tau^+\tau^-)$	undefined	$(5.20 \pm 1.61) \times 10^{-9}$	$(8.12 \pm 2.75) \times 10^{-9}$
$BR(B_c \to D_{s1}(2536)\tau^+\tau^-)$	undefined	$(4.16 \pm 1.29) \times 10^{-9}$	$(6.56 \pm 2.16) \times 10^{-9}$

Table 5: The branching ratios of the semileptonic $B_c \to D_{s1}(2460)l^+l^-/\nu\bar{\nu}$ and $B_c \to D_{s1}(2536)l^+l^-/\nu\bar{\nu}$ decays including LD effects.

and M_2^2, the variation of the continuum thresholds s_0 and s_0', the variation of b and c quark masses and leptonic decay constants f_{B_c} and $f_{D_{s1}}$.

Now, we would like to analyze the form factors obtained when we considered the D_{s1} meson as a mixture of two $|^3P_1\rangle$ and $|^1P_1\rangle$ states (see Eq. 29). The transition form factors of $B_c \to D_{s1}(2460[2536])l^+l^-/\nu\bar{\nu}$ at $q^2 = 0$ in the interval $-180^\circ \le \theta_s \le 180^\circ$ are shown in Figs. 3 and 4. From these figures, we see that all form factors have the following common behaviors: 1) they have extrema at the same mixing angles and 2) they come across at two points. The dependence of the form factors $B_c \to D_{s1}(2460)$ and $B_c \to D_{s1}(2536)$ transitions on the mixing angle, θ_s, and the transferred momentum square, q^2, are plotted in Figs. 5, 6 in the regions $0 \le q^2 \le (m_{B_c} - m_{D_{s1}})^2\ GeV^2$ and $-180^\circ \le \theta_s \le 180^\circ$ for q^2 and mixing angle, respectively. Using Eqs. 30 and 32 we analyze the decay widths and the branching ratios related to considered decays. For this aim, we denote the variations of the decay widths with respect to q^2 and θ_s in the regions $4m_l^2 \le q^2 \le (m_{B_c} - m_{D_{s1}})^2\ GeV^2$, $(l = \tau, \mu, e)$ and $-180^\circ \le \theta_s \le 180^\circ$ and branching ratios only in terms of mixing angle θ_s in Figs. 7-10. The results for electron and muon are approximately the same, so we consider

$l = \mu, \tau$. In Fig. 11, as an example, we only depict the variation of the branching ratio of $B_c \to D_{s1}(2460)\nu\bar{\nu}$ decay in terms of the mixing angle. The figures 9 and 10 depict a regular variation of the branching ratios for l^+l^- case with respect to the mixing angle, while we see an irregular variation of the branching ratio of the $B_c \to D_{s1}(2460)\nu\bar{\nu}$ transition with respect to the θ_s.

In summary, We analyzed the semileptonic $B_c \to D_{s1}(2460[2536])l^+l^-$, $l = e, \mu, \tau$ and $B_c \to D_{s1}(2460[2536])\nu\bar{\nu}$ decays in the framework of the three–point QCD sum rules. First, we assumed the $D_{s1}(2460)$ and $D_{s1}(2536)$ axial vector mesons as the pure $|c\bar{s}\rangle$ states. In this case, the related form factors were computed. The branching ratios of these decays were also estimated with both the short distance (SD) and long distance (LD) effects, for the charged lepton modes. Second, $D_{s1}(2460[2536])$ mesons were considered as a combinations of two states $|{}^3P_1\rangle \equiv |D_{s1}1\rangle$ and $|{}^1P_1\rangle \equiv |D_{s1}2\rangle$ with different masses and decay constants. We evaluated the transitions form factors and the decay widths of these decays with respect to the mixing angle θ_s and the transferred momentum square q^2. The dependence of the branching ratios on θ_s was also presented. Detection of these channels and their comparison with the phenomenological models like QCD sum rules could give useful information about the structure of the D_{s1} meson and the mixing angle θ_s.

Acknowledgments

Partial support of Shiraz university research council is appreciated. K. A. would like to thank T. M. Aliev, M. T. Zeyrek and A. Ozpineci for their useful discussions and also TUBITAK, Turkish Scientific and Technical Research Council, for their partial financial support provided under the project 108T502.

References

[1] A. Drutskoy et al., Belle Collaboration, Phys. Rev. Lett. 96, 061802 (2005).

[2] B. Aubert et al., BaBar, Collaboration, Phys. Rev. Lett. 90, 242001 (2003).

[3] D. Besson et al., CLEO Collaboration, Phys. Rev. D 68, 032002 (2003).

[4] Y. Nikami et al., Belle Collaboration, Phys. Rev. Lett. 92, 012002 (2004).

[5] P. Krokovny et al., Belle Collaboration, Phys. Rev. Lett. 91, 262002 (2003).

[6] B. Aubert et al., Babar Collaboration, Phys. Rev. Lett. 93, 181807 (2004).

[7] B. Aubert et al., Babar Collaboration, Phys. Rev. D 69, 031101 (2004).

[8] B. Aubert et al., Babar Collaboration, hep-ex/0408067

[9] P. Colangelo, F. De Fazio, R. Ferrandes, Mod. Phys. Lett. A 19, 2083 (2004).

[10] E. S. Swanson, Phys. Rept. 429, 243 (2006).

[11] P. Colangelo, F. De Fazio, A. Ozpineci, Phys. Rev. D 72, 074004 (2005).

[12] S. Godfrey, N. Isgur, Phys. Rev. D 32, 189 (1985).

[13] S. Godfrey, R. Kokoski, Phys. Rev. D 43, 1679 (1991).

[14] M. Di Pierro, E. Eichten, Phys. Rev. D 64, 114004 (2001).

[15] F. E. Close, E. S. Swanson, Phys. Rev. D 72, 094004 (2005).

[16] S. Godfrey, Phys. Lett. B 568, 254 (2003).

[17] W. A. Bardeen, E. J. Eichten, C. T. Hill, Phys. Rev. D 68, 054024 (2003).

[18] M. A. Nowak, M. Rho, I. Zahed, Acta Phys. Polon. B 35, 2377 (2004).

[19] A. Deandrea, G. Nardulli, A. D. Polosa, Phys. Rev. D 68, 097501 (2003).

[20] R. N. Cahn, J. D. Jackson, Phys. Rev. D 68, 037502 (2003).

[21] Y. B. Dai, C. S. Huang, C. Liu, S. L. Zhu, Phys. Rev. D 68, 114011 (2003).

[22] W. Lucha, F. Schobert, Mod. Phys. Lett. A 18, 2837 (2003).

[23] J. Hofmann, M. F. M. Lutz, Nucl. Phys. A 733, 142 (2004).

[24] M. Sadzikowski, Phys. Lett. B 579, 39 (2004).

[25] D. Becirevic et al., Phys. Lett B 599, 59 (2004).

[26] T. Lee, I. W. Lee, D. P. Min, B. Y. Park, Eur. Phys. J. C 49, 737 (2007).

[27] E. Kolomeitsev, M. Lutz, Phys. Lett. B 582, 39 (2004).

[28] T. Barnes, F. E. Close, H. J. Lipkin, Phys. Rev. D 68, 054006 (2003).

[29] H. Y. Cheng, W. S. Hou, Phys. Lett. B 566, 193 (2003).

[30] K. Terasaki, Phys. Rev. D 68, 011501 (2003).

[31] T. E. Browder, S. Pakvasa, A. A. Petrov, Phys. Lett. B 578, 365 (2004).

[32] U. Dmitrasinovic, Phys. Rev. D 70, 096011 (2004); Phys. Rev. Lett. 94, 162002 (2005).

[33] M. E. Bracco, A. Lozea, R. D. Matheus, F. S. Navarra, M. Nielsen, Phys. Lett. B 624, 217 (2005).

[34] H. Kim, Y. Oh, Phys. Rev. D 72, 074012 (2005).

[35] A.P. Szczepaniak, Phys. Lett. B 567, 23 (2003).

[36] C. E. Thomas, Phys. Rev. D 73, 054016 (2006).

[37] H. Y. Cheng, Phys. Rev. D 68, 094005 (2003).

[38] K. Azizi, R. Khosravi, V. Bashiry, Eur. Phys. J. C 56, 357, (2008).

[39] K. Azizi, R. Khosravi, Phys. Rev. D 78, 036005 (2008).

[40] K. Azizi, F. Falahati, V. Bashiry, S. M. Zebarjad, Phys. Rev. D 77, 114024 (2008).

[41] N. G. Deshpande, J. Trampetic, K. Panose, Phys. Rev. D 39, 1461 (1989); C. S. Lim, T. Morozumi, A. I. Sanda, Phys. Lett. B 218, 343 (1989).

[42] N. Ghahramany, R. Khosravi, K. Azizi, Phys. Rev. D 78, 116009 (2008).

[43] C. Q. Geng, C. W. Hwang, C. C. Liu, Phys. Rev. D 65, 094037 (2002).

[44] C. Breub, A. Ioannissian, D. Wyler, Phys. Lett. B 346, 149 (1995).

[45] C.Q. Geng, C. P. Kao, Phys. Rev. D 54, 5636 (1996).

[46] M. A. Shifman, A. I. Vainshtein, V. I. Zakharov, Nucl. Phys. B 147, 385 (1979).

[47] A. Ceccucci, Z. Ligeti,Y. Sakai, PDG, J. Phys. G 33, 139 (2006).

[48] A. J. Buras, M. Muenz, Phys. Rev. D 52, 186 (1995).

[49] V. Bashiry, K. Azizi, JHEP 0707, 064 (2007).

[50] S. Veseli, I. Dunietz, Phys. Rev. D 54, 6803 (1996).

[51] P. Colangelo, G. Nardulli, N. Paver, Z. Phys. C 57, 43 (1993).

[52] V. V. Kiselev, A. V. Tkabladze, Phys. Rev. D 48, 5208 (1993).

[53] T. M. Aliev, O. Yilmaz, Nuovo Cimento. A 105, 827 (1992).

[54] C. Amsler et al., Particle Data Group, Phys. Lett. B 667, 1 (2008).

[55] P. Colangelo, F. De Fazio, A. Ozpineci, Phys. Rev. D 72, 074004 (2005).

[56] T. M. Aliev, M. Savci, Phys. Lett. B 434, 358 (1998).

Appendix–A

In this appendix, the explicit expressions of the coefficients of the gluon condensate entering the sum rules of the form factors $A_i^{'B_c\to D_{s1}}$, $(i=V,0,1,2)$ and $T_j^{'B_c\to D_{s1}}$, $(j=V,0,1)$ are given.

$$
\begin{aligned}
C_V^{V-A} \;=\;& -10\,\hat{I}_1(3,2,2)m_b{}^3m_c{}^2+10\,\hat{I}_1(3,2,2)m_b{}^2m_c{}^3+10\,\hat{I}_2(3,2,2)m_b{}^2m_c{}^3+10\,\hat{I}_0(3,2,2)m_b{}^2m_c{}^3\\
&+10\,\hat{I}_1(3,2,2)m_bm_c{}^4+10\,\hat{I}_0^{[0,1]}(3,2,2)m_b{}^2m_c-30\,\hat{I}_1(3,2,1)m_b{}^2m_c+60\,\hat{I}_1(1,4,1)m_b{}^2m_c\\
&+60\,\hat{I}_2(1,4,1)m_b{}^2m_c-20\,\hat{I}_2(3,2,1)m_b{}^2m_c+10\,\hat{I}_2^{[0,1]}(3,2,2)m_b{}^2m_c-20\,\hat{I}_0(3,2,1)m_b{}^2m_c\\
&+60\,\hat{I}_0(1,4,1)m_b{}^2m_c+10\hat{I}_1^{[0,1]}(3,2,2)m_b{}^2m_c+20\,\hat{I}_1(2,2,2)m_bm_c{}^2+10\,\hat{I}_2(3,2,1)m_bm_c{}^2\\
&+10\,\hat{I}_1(3,2,1)m_bm_c{}^2+40\,\hat{I}_2(2,3,1)m_bm_c{}^2-10\,\hat{I}_0(3,2,1)m_bm_c{}^2+20\,\hat{I}_1(2,3,1)m_bm_c{}^2\\
&-20\,\hat{I}_1^{[0,1]}(3,2,2)m_bm_c{}^2+30\,\hat{I}_1(4,1,1)m_bm_c{}^2-10\,\hat{I}_2(3,2,2)m_c{}^5-10\,\hat{I}_1(3,2,2)m_c{}^5\\
&-10\,\hat{I}_0(3,2,2)m_c{}^5+20\,\hat{I}_1(3,2,1)m_b{}^3+10\,\hat{I}_1(2,2,2)m_b{}^3-20\,\hat{I}_1(2,3,1)m_b{}^3\\
&-60\,\hat{I}_1(1,4,1)m_b{}^3-10\,\hat{I}_1^{[0,1]}(3,2,2)m_b{}^3-30\,\hat{I}_2(4,1,1)m_c{}^3+20\,\hat{I}_2^{[0,1]}(3,2,2)m_c{}^3\\
&+10\,\hat{I}_0(3,2,1)m_c{}^3-10\,\hat{I}_2(3,1,2)m_c{}^3-20\,\hat{I}_0(2,2,2)m_c{}^3-20\,\hat{I}_2(2,2,2)m_c{}^3\\
&-20\,\hat{I}_1(2,2,2)m_c{}^3-30\,\hat{I}_0(4,1,1)m_c{}^3-30\,\hat{I}_1(4,1,1)m_c{}^3+20\,\hat{I}_1^{[0,1]}(3,2,2)m_c{}^3\\
&-10\,\hat{I}_0(3,1,2)m_c{}^3+20\,\hat{I}_0^{[0,1]}(3,2,2)m_c{}^3-50\,\hat{I}_1(2,2,1)m_b+20\,\hat{I}_1^{[0,1]}(2,3,1)m_b\\
&-20\,\hat{I}_1^{[0,1]}(3,2,1)m_b+20\,\hat{I}_1(1,2,2)m_b+60\,\hat{I}_0(1,3,1)m_b-20\,\hat{I}_2(2,2,1)m_b\\
&-20\,\hat{I}_1^{[0,1]}(3,1,2)m_b-20\,\hat{I}_0(2,2,1)m_b+30\,\hat{I}_1(2,1,2)m_b+100\,\hat{I}_2(1,3,1)m_b\\
&+10\,\hat{I}_1^{[0,2]}(3,2,2)m_b-20\,\hat{I}_1^{[0,1]}(2,2,2)m_b+40\,\hat{I}_0^{[0,1]}(2,3,1)m_b+20\,\hat{I}_1(1,3,1)m_b\\
&+30\,\hat{I}_0(2,2,1)m_c+30\,\hat{I}_2^{[0,1]}(3,1,2)m_c+20\,\hat{I}_2^{[0,1]}(3,2,1)m_c+10\,\hat{I}_0^{[0,1]}(3,2,1)m_c\\
&+20\,\hat{I}_1^{[0,1]}(3,2,1)m_c-10\,\hat{I}_0^{[0,2]}(3,2,2)m_c+20\,\hat{I}_1^{[0,1]}(3,1,2)m_c+20\,\hat{I}_1^{[0,1]}(2,2,2)m_c\\
&+20\,\hat{I}_2(2,2,1)m_c-30\,\hat{I}_2(2,1,2)m_c+10\,\hat{I}_0(3,1,1)m_c+20\,\hat{I}_0^{[0,1]}(2,2,2)m_c\\
&-20\,\hat{I}_0(1,2,2)m_c-20\,\hat{I}_2(1,2,2)m_c+30\,\hat{I}_0^{[0,1]}(3,1,2)m_c-10\,\hat{I}_1(3,1,1)m_c\\
&+20\,\hat{I}_2^{[0,1]}(2,2,2)m_c-10\,\hat{I}_2(3,1,1)m_c-20\,\hat{I}_1(2,1,2)m_c-30\,\hat{I}_0(2,1,2)m_c\\
&-10\,\hat{I}_2^{[0,2]}(3,2,2)m_c+20\,\hat{I}_1(2,2,1)m_c-20\,\hat{I}_1(1,2,2)m_c-10\,\hat{I}_1^{[0,2]}(3,2,2)m_c
\end{aligned}
$$

$$
\begin{aligned}
C_0^{V-A} \;=\;& -20\,\hat{I}_6(3,2,2)m_c{}^5-40\,\hat{I}_6(3,2,1)m_c{}^3-20\,\hat{I}_6(3,1,2)m_c{}^3+40\,\hat{I}_6^{[0,6]}(3,2,2)m_c{}^3\\
&-40\,\hat{I}_6(2,2,2)m_c{}^3-60\,\hat{I}_6(4,1,1)m_c{}^3+5\,\hat{I}_0(3,1,1)m_c{}^3-30\,\hat{I}_0^{[0,1]}(1,4,1)m_b{}^3\\
&+20\,\hat{I}_6(2,2,2)m_b{}^3+5\,\hat{I}_0(2,2,1)m_b{}^3-120\,\hat{I}_6(1,4,1)m_b{}^3+40\,\hat{I}_6(2,3,1)m_b{}^3\\
&+15\,\hat{I}_0^{[0,1]}(3,2,1)m_b{}^3-20\,\hat{I}_0(1,3,1)m_b{}^3+10\,\hat{I}_0^{[0,1]}(2,3,1)m_b{}^3-5\,\hat{I}_0^{[0,2]}(3,2,2)m_b{}^3\\
&+10\,\hat{I}_0^{[0,1]}(2,2,2)m_b{}^3-5\,\hat{I}_0(1,2,2)m_b{}^3-20\,\hat{I}_6^{[0,1]}(3,2,2)m_b{}^3+20\,\hat{I}_6^{[0,1]}(3,1,2)m_c\\
&+20\,\hat{I}_6(2,2,1)m_c+40\;\hat{I}_6^{[0,1]}(2,2,2)m_c-20\,\hat{I}_6^{[0,2]}(3,2,2)m_c+20\,\hat{I}_6(3,2,1)m_b{}^3\\
&+5\,\hat{I}_0^{[0,1]}(3,1,1)m_c+5\,\hat{I}_0(1,1,2)m_c+20\,\hat{I}_6(2,1,2)m_c+40\,\hat{I}_6(3,1,1)m_c\\
&+5\,\hat{I}_0(1,2,1)m_c-5\,\hat{I}_0(2,1,1)m_c-10\,\hat{I}_0^{[0,2]}(2,3,1)m_b-10\,\hat{I}_0^{[0,2]}(3,1,2)m_b\\
&-10\,\hat{I}_0^{[0,1]}(1,3,1)m_b-15\,\hat{I}_0(1,2,1)m_b-40\,\hat{I}_6(2,2,1)m_b+15\,\hat{I}_0^{[0,1]}(2,2,1)m_b
\end{aligned}
$$

$$
\begin{aligned}
&+20\,\hat{I}_0^{[0,1]}(1,2,2)m_b - 40\,\hat{I}_6(1,3,1)m_b - 40\,\hat{I}_6(3,1,1)m_b + 40\,\hat{I}_6^{[0,1]}(3,2,1)m_c \\
&-20\,\hat{I}_6^{[0,1]}(2,2,2)m_b + 20\,\hat{I}_6^{[0,2]}(3,2,2)m_b - 40\,\hat{I}_6^{[0,1]}(3,1,2)m_b - 15\,\hat{I}_0(1,1,2)m_b \\
&-15\,\hat{I}_0^{[0,2]}(2,2,2)m_b + 15\,\hat{I}_0(2,1,1)m_b - 20\,\hat{I}_6(2,1,2)m_b + 25\,\hat{I}_0^{[0,1]}(2,1,2)m_b \\
&+10\,\hat{I}_0^{[0,1]}(3,1,1)m_b - 15\,\hat{I}_0^{[0,2]}(3,2,1)m_b - 20\,\hat{I}_6(1,2,2)m_b - 40\,\hat{I}_6^{[0,1]}(2,3,1)m_b \\
&-20\,\hat{I}_6^{[0,1]}(3,2,1)m_b - 5\,\hat{I}_0(3,2,2){m_c}^6 m_b + 10\,\hat{I}_0(2,2,2){m_c}^3{m_b}^2 + 20\,\hat{I}_6(3,2,2){m_c}^3{m_b}^2 \\
&-10\,\hat{I}_0(2,3,1){m_c}^4 m_b + 15\,\hat{I}_0^{[0,1]}(3,2,2){m_c}^4 m_b + 20\,\hat{I}_6(3,2,2){m_c}^4 m_b - 15\,\hat{I}_0(2,2,2){m_c}^4 m_b \\
&-5\,\hat{I}_0(3,2,1){m_c}^4 m_b - 15\,\hat{I}_0(4,1,1){m_c}^4 m_b - 5\,\hat{I}_0(3,2,2){m_c}^3{m_b}^4 + 5\,\hat{I}_0(3,2,2){m_c}^4{m_b}^3 \\
&+5\,\hat{I}_0(3,2,2){m_c}^5{m_b}^2 - 30\,\hat{I}_0(1,4,1)m_c{m_b}^4 - 5\,\hat{I}_0^{[0,1]}(3,2,2)m_c{m_b}^4 + 10\,\hat{I}_0(3,2,1)m_c{m_b}^4 \\
&+10\,\hat{I}_0(2,3,1){m_c}^2{m_b}^3 - 10\,\hat{I}_0(3,2,1){m_c}^2{m_b}^3 - 20\,\hat{I}_6(3,2,2){m_c}^2{m_b}^3 + 30\,\hat{I}_0(1,4,1){m_c}^2{m_b}^3 \\
&-10\,\hat{I}_0^{[0,1]}(3,2,2){m_c}^3{m_b}^2 + 5\,\hat{I}_0(3,2,1){m_c}^3{m_b}^2 + 15\,\hat{I}_0(4,1,1){m_c}^3{m_b}^2 + 20\,\hat{I}_6(2,2,2){m_c}^2 m_b \\
&+10\,\hat{I}_0^{[0,1]}(3,1,2){m_c}^2 m_b - 40\,\hat{I}_6^{[0,1]}(3,2,2){m_c}^2 m_b - 15\,\hat{I}_0^{[0,2]}(3,2,2){m_c}^2 m_b \\
&+10\,\hat{I}_0(1,3,1){m_c}^2 m_b + 20\,\hat{I}_0^{[0,1]}(3,2,1){m_c}^2 m_b - 20\,\hat{I}_0(1,2,2){m_c}^2 m_b - 15\,\hat{I}_0(2,1,2){m_c}^2 m_b \\
&-40\,\hat{I}_6(2,3,1){m_c}^2 m_b + 15\,\hat{I}_0^{[0,1]}(4,1,1){m_c}^2 m_b + 30\,\hat{I}_0^{[0,1]}(2,2,2){m_c}^2 m_b + 60\,\hat{I}_6(4,1,1){m_c}^2 m_b \\
&-10\,\hat{I}_0(3,1,1){m_c}^2 m_b + 20\,\hat{I}_6(3,1,2){m_c}^2 m_b + 15\,\hat{I}_0(2,2,1){m_c}^2 m_b + 20\,\hat{I}_0^{[0,1]}(2,3,1){m_c}^2 m_b \\
&+40\,\hat{I}_6(3,2,1){m_c}^2 m_b - 10\,\hat{I}_0^{[0,1]}(2,2,2)m_c{m_b}^2 - 20\,\hat{I}_6(3,2,1)m_c{m_b}^2 + 20\,\hat{I}_6^{[0,1]}(3,2,2)m_c{m_b}^2 \\
&+15\,\hat{I}_0(2,1,2)m_c{m_b}^2 + 5\,\hat{I}_0(3,1,1)m_c{m_b}^2 - 20\,\hat{I}_0^{[0,1]}(3,1,2)m_c{m_b}^2 - 20\,\hat{I}_6(2,2,2)m_c{m_b}^2 \\
&-30\,\hat{I}_0(1,3,1)m_c{m_b}^2 + 120\,\hat{I}_6(1,4,1)m_c{m_b}^2 + 10\,\hat{I}_0(1,2,2)m_c{m_b}^2 - 5\,\hat{I}_0^{[0,1]}(3,2,1)m_c{m_b}^2 \\
&-10\,\hat{I}_0(2,2,1)m_c{m_b}^2 + 5\,\hat{I}_0^{[0,2]}(3,2,2)m_c{m_b}^2
\end{aligned}
$$

$$
\begin{aligned}
C_1^{V-A} \;=\; &-40\,\hat{I}_4^{[0,1]}(2,3,1)m_b + 20\;\hat{I}_4^{[0,2]}(3,2,2)m_b - 40\,\hat{I}_3(2,2,1)m_b - 20\,\hat{I}_1(1,2,2)m_b \\
&-20\,\hat{I}_1(2,1,2)m_b - 40\,\hat{I}_3^{[0,1]}(2,3,1)m_b - 20\;\hat{I}_3^{[0,1]}(3,2,1)m_b - 20\,\hat{I}_3(2,1,2)m_b \\
&-20\,\hat{I}_3^{[0,1]}(2,2,2)m_b - 20\,\hat{I}_3(1,2,2)m_b - 20\,\hat{I}_4(1,2,2)m_b - 10\,\hat{I}_1^{[0,1]}(2,3,1)m_b \\
&+5\;\hat{I}_1^{[0,2]}(3,2,2)m_b - 5\,\hat{I}_2(3,2,2){m_3}^5 - 5\,\hat{I}_0(3,2,2){m_c}^5 - 20\,\hat{I}_3(3,2,2){m_c}^5 \\
&-15\,\hat{I}_1(3,2,2){m_c}^5 - 45\,\hat{I}_1(3,2,1){m_c}^3 - 20\,\hat{I}_4(3,1,2){m_c}^3 - 20\,\hat{I}_2(3,2,1){m_c}^3 \\
&-15\,\hat{I}_2(4,1,1){m_c}^3 - 25\,\hat{I}_0(3,2,1){m_c}^3 - 40\,\hat{I}_4(3,2,1){m_c}^3 - 10\,\hat{I}_2(2,2,2){m_c}^3 \\
&-45\,\hat{I}_1(4,1,1){m_c}^3 - 20\,\hat{I}_4(3,2,2){m_c}^5 - 5\,\hat{I}_0(3,1,2){m_c}^3 - 40\,\hat{I}_3(2,2,2){m_c}^3 \\
&+10\,\hat{I}_0^{[0,1]}(3,2,2){m_c}^3 - 30\,\hat{I}_1(2,2,2){m_c}^3 + 40\,\hat{I}_3^{[0,1]}(3,2,2){m_c}^3 - 60\,\hat{I}_3(4,1,1){m_c}^3 \\
&-15\,\hat{I}_0(4,1,1){m_c}^3 - 5\,\hat{I}_2(3,1,2){m_c}^3 - 10\,\hat{I}_0(2,2,2){m_c}^3 - 20\,\hat{I}_1(3,1,2){m_c}^3 \\
&-40\,\hat{I}_3(3,2,1){m_c}^3 + 30\,\hat{I}_1^{[0,1]}(3,2,2){m_c}^3 + 40\,\hat{I}_4^{[0,1]}(3,2,2){m_c}^3 - 60\,\hat{I}_4(4,1,1){m_c}^3 \\
&-20\,\hat{I}_3(3,2,2){m_c}^2{m_b}^3 + 20\,\hat{I}_4(2,2,2){m_c}^2 m_b - 20\,\hat{I}_0(2,3,1){m_c}^2 m_b + 40\,\hat{I}_4(3,2,1){m_c}^2 m_b \\
&-40\,\hat{I}_4^{[0,1]}(3,2,2){m_c}^2 m_b - 20\,\hat{I}_2(2,3,1){m_c}^2 m_b + 60\,\hat{I}_4(4,1,1){m_c}^2 m_b + 60\,\hat{I}_3(4,1,1){m_c}^2 m_b \\
&+20\,\hat{I}_3(3,1,2){m_c}^2 m_b + 20\,\hat{I}_3(2,2,2){m_c}^2 m_b + 5\,\hat{I}_1(3,2,2){m_c}^4 m_b + 20\,\hat{I}_3(3,2,2){m_c}^4 m_b \\
&+20\,\hat{I}_4(3,2,2){m_c}^4 m_b + 5\,\hat{I}_0(3,2,2){m_c}^3{m_b}^2 + 20\,\hat{I}_4(3,2,2){m_c}^3{m_b}^2 + 5\,\hat{I}_2(3,2,2){m_c}^3{m_b}^2 \\
&+15\,\hat{I}_1(3,2,2){m_c}^3{m_b}^2 + 20\,\hat{I}_3(3,2,2){m_c}^3{m_b}^2 - 20\,\hat{I}_4(3,2,2){m_c}^2{m_b}^3 - 5\,\hat{I}_1(3,2,2){m_c}^2{m_b}^3 \\
&+15\,\hat{I}_2(3,2,1){m_c}^2 m_b + 15\,\hat{I}_1(4,1,1){m_c}^2 m_b + 5\,\hat{I}_0(3,2,1){m_c}^2 m_b + 10\,\hat{I}_1(3,1,2){m_c}^2 m_b \\
&-50\,\hat{I}_1(2,3,1){m_c}^2 m_b - 10\,\hat{I}_1^{[0,1]}(3,2,2){m_c}^2 m_b + 35\,\hat{I}_1(3,2,1){m_c}^2 m_b + 20\,\hat{I}_4(3,1,2){m_3}^2 m_b
\end{aligned}
$$

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\begin{aligned}
&-40\,\hat{I}_3^{[0,1]}(3,2,2){m_c}^2m_b-40\,\hat{I}_3(2,3,1){m_c}^2m_b-20\,\hat{I}_3(3,1,2){m_c}^3+10\,\hat{I}_2^{[0,1]}(3,2,2){m_c}^3\\
&+40\,\hat{I}_3(2,3,1){m_b}^3+20\,\hat{I}_3(2,2,2){m_b}^3-5\,\hat{I}_1^{[0,1]}(3,2,2){m_b}^3+40\,\hat{I}_4(2,3,1){m_b}^3\\
&-20\,\hat{I}_4^{[0,1]}(3,2,2){m_b}^3-120\,\hat{I}_3(1,4,1){m_b}^3+20\,\hat{I}_3(3,2,1){m_b}^3-120\,\hat{I}_4(1,4,1){m_b}^3\\
&+20\,\hat{I}_4(3,2,1){m_b}^3+10\,\hat{I}_1(2,3,1){m_b}^3-30\,\hat{I}_1(1,4,1){m_b}^3-40\,\hat{I}_4(2,2,2){m_c}^3\\
&+40\,\hat{I}_3(3,2,1){m_c}^2m_b-40\,\hat{I}_4(2,3,1){m_c}^2m_b+20\,\hat{I}_4^{[0,1]}(3,2,2)m_c{m_b}^2+5\,\hat{I}_0^{[0,1]}(3,2,2)m_c{m_b}^2\\
&-20\,\hat{I}_4(2,2,2)m_c{m_b}^2-30\,\hat{I}_1(3,2,1)m_c{m_b}^2+90\,\hat{I}_1(1,4,1)m_c{m_b}^2+120\,\hat{I}_3(1,4,1)m_3{m_b}^2\\
&+20\,\hat{I}_3^{[0,1]}(3,2,2)m_c{m_b}^2+120\,\hat{I}_4(1,4,1)m_c{m_b}^2+10\,\hat{I}_1(3,2,1){m_b}^3-20\,\hat{I}_3^{[0,1]}(3,2,2){m_b}^3\\
&+40\,\hat{I}_3(3,1,1)m_c-5\,\hat{I}_0(2,2,1)m_c+10\,\hat{I}_0^{[0,1]}(2,2,2)m_c+20\,\hat{I}_4(2,1,2)m_c+40\,\hat{I}_3^{[0,1]}(3,2,1)m_c\\
&+20\,\hat{I}_4^{[0,1]}(3,1,2)m_c+10\,\hat{I}_2^{[0,1]}(2,2,2)m_c+20\,\hat{I}_3(2,2,1)m_c-20\,\hat{I}_4^{[0,2]}(3,2,2)m_c\\
&+40\,\hat{I}_4^{[0,1]}(2,2,2)m_c+5\,\hat{I}_2(3,1,1)m_c+20\,\hat{I}_4(2,2,2){m_b}^3+30\,\hat{I}_0(1,4,1)m_c{m_b}^2\\
&+5\,\hat{I}_1(3,1,2)m_c{m_b}^2-15\,\hat{I}_0(3,2,1)m_c{m_b}^2+5\,\hat{I}_2^{[0,1]}(3,2,2)m_c{m_b}^2-10\,\hat{I}_2(3,2,1)m_c{m_b}^2\\
&+30\,\hat{I}_2(1,4,1)m_c{m_b}^2-20\,\hat{I}_4(3,2,1)m_c{m_b}^2+15\,\hat{I}_1^{[0,1]}(3,2,2)m_c{m_b}^2-10\,\hat{I}_1(2,2,2)m_c{m_b}^2\\
&-20\,\hat{I}_3(3,2,1)m_c{m_b}^2-20\,\hat{I}_3(2,2,2)m_c{m_b}^2-15\,\hat{I}_1^{[0,2]}(3,2,2)m_c-10\,\hat{I}_1(2,1,2)m_c\\
&-5\,\hat{I}_2^{[0,2]}(3,2,2)m_c+5\,\hat{I}_1(2,2,1)m_c+40\,\hat{I}_4^{[0,1]}(3,2,1)m_c+10\,\hat{I}_2^{[0,1]}(3,2,1)m_c\\
&+15\,\hat{I}_0^{[0,1]}(3,1,2)m_c+20\,\hat{I}_3^{[0,1]}(3,1,2)m_c+35\,\hat{I}_1^{[0,1]}(3,2,1)m_c+30\,\hat{I}_1^{[0,1]}(3,1,2)m_c\\
&-5\,\hat{I}_0^{[0,2]}(3,2,2)m_c+40\,\hat{I}_3^{[0,1]}(2,2,2)m_c+20\,\hat{I}_3(2,1,2)m_c-15\,\hat{I}_0(2,1,2)m_c\\
&-40\,\hat{I}_3^{[0,1]}(3,1,2)m_b-40\,\hat{I}_4(3,1,1)m_b-10\,\hat{I}_1^{[0,1]}(3,2,1)m_b-40\,\hat{I}_4(2,2,1)m_b\\
&+20\,\hat{I}_3^{[0,2]}(3,2,2)m_b-40\,\hat{I}_3(1,3,1)m_b-40\,\hat{I}_4^{[0,1]}(3,1,2)m_b+10\,\hat{I}_1(1,3,1)m_b\\
&-10\,\hat{I}_0(2,2,1)m_b-20\,\hat{I}_4^{[0,1]}(2,2,2)m_b-10\,\hat{I}_2(2,2,1)m_b-10\,\hat{I}_1^{[0,1]}(3,1,2)m_b\\
&+10\,\hat{I}_0(1,3,1)m_b-20\,\hat{I}_4^{[0,1]}(3,2,1)m_b-20\,\hat{I}_3^{[0,2]}(3,2,2)m_c-10\,\hat{I}_0(3,1,1)m_c\\
&+40\,\hat{I}_4(3,1,1)m_c-15\,\hat{I}_2(2,1,2)m_c+20\,\hat{I}_4(2,2,1)m_c+15\,\hat{I}_1(3,1,1)m_c+15\,\hat{I}_2^{[0,1]}(3,1,2)m_c\\
&+15\,\hat{I}_0^{[0,1]}(3,2,1)m_c-40\,\hat{I}_3(3,1,1)m_b-20\,\hat{I}_4(2,1,2)m_b+10\,\hat{I}_2(1,3,1)m_b-30\,\hat{I}_1(2,2,1)m_b\\
&-40\,\hat{I}_4(1,3,1)m_b+30\,\hat{I}_1^{[0,1]}(2,2,2)m_c
\end{aligned}
$$

$$
\begin{aligned}
C_2^{V-A} \;=\;& 15\,\hat{I}_2(4,1,1){m_c}^2m_b-40\,\hat{I}_3^{[0,1]}(3,2,2){m_c}^2m_b-40\,\hat{I}_4(3,2,1){m_c}^2m_b-10\,\hat{I}_2(2,3,1){m_3}^2m_b\\
&+20\,\hat{I}_3(3,1,2){m_c}^2m_b+60\,\hat{I}_3(4,1,1){m_c}^2m_b-20\,\hat{I}_4(2,2,2){m_c}^2m_b+40\,\hat{I}_3(3,2,1){m_c}^2m_b\\
&+40\,\hat{I}_4^{[0,1]}(3,2,2){m_c}^2m_b-60\,\hat{I}_4(4,1,1){m_c}^2m_b+40\,\hat{I}_4(2,3,1){m_c}^2m_b+20\,\hat{I}_3(2,2,2){m_c}^2m_b\\
&-5\,\hat{I}_0(3,2,1){m_c}^2m_b+15\,\hat{I}_1(3,2,1){m_c}^2m_b-20\,\hat{I}_1(2,3,1){m_c}^2m_b-40\,\hat{I}_3(2,3,1){m_c}^2m_b\\
&-20\,\hat{I}_4(3,1,2){m_c}^2m_b+10\,\hat{I}_2^{[0,1]}(3,2,2){m_c}^3+60\,\hat{I}_4(4,1,1){m_c}^3-20\,\hat{I}_3(3,1,2){m_c}^3\\
&+40\,\hat{I}_4(2,2,2){m_c}^3-15\,\hat{I}_1(4,1,1){m_c}^3-40\,\hat{I}_3(3,2,1){m_c}^3-40\,\hat{I}_3(2,2,2){m_c}^3\\
&-15\,\hat{I}_2(4,1,1){m_c}^3-5\,\hat{I}_2(3,2,1){m_c}^3+10\,\hat{I}_1^{[0,1]}(3,2,2){m_c}^3+5\,\hat{I}_0(3,1,2){m_c}^3\\
&-10\,\hat{I}_0^{[0,1]}(3,2,2){m_c}^3-10\,\hat{I}_1(2,2,2){m_c}^3+10\,\hat{I}_0(2,2,2){m_c}^3-10\,\hat{I}_2(3,1,2){m_c}^3\\
&-5\,\hat{I}_1(3,1,2){m_c}^3+15\,\hat{I}_0(4,1,1){m_c}^3-20\,\hat{I}_1(3,2,1){m_c}^3+20\,\hat{I}_4^{[0,1]}(3,2,2){m_b}^3\\
&-30\,\hat{I}_2(1,4,1){m_b}^3-20\,\hat{I}_3^{[0,1]}(3,2,2){m_b}^3+40\,\hat{I}_3(2,3,1){m_b}^3+5\,\hat{I}_2(3,2,1){m_c}^2m_b\\
&-20\,\hat{I}_4^{[0,1]}(3,2,2)m_c{m_b}^2+5\,\hat{I}_2(3,1,2)m_c{m_b}^2-20\,\hat{I}_3(3,2,1)m_c{m_b}^2+20\,\hat{I}_4(2,2,2)m_c{m_b}^2
\end{aligned}
$$

$$-5\,\hat{I}_0^{[0,1]}(3,2,2)m_c{m_b}^2-10\,\hat{I}_2(3,2,1)m_c{m_b}^2+15\,\hat{I}_0(3,2,1)m_c{m_b}^2+20\,\hat{I}_3^{[0,1]}(3,2,2)m_c{m_b}^2$$
$$-10\,\hat{I}_2(2,2,2)m_c{m_b}^2-30\,\hat{I}_0(1,4,1)m_c{m_b}^2+120\,\hat{I}_3(1,4,1)m_c{m_b}^2+5\,\hat{I}_1^{[0,1]}(3,2,2)m_c{m_b}^2$$
$$+30\,\hat{I}_1(1,4,1)m_c{m_b}^2-120\,\hat{I}_4(1,4,1)m_c{m_b}^2+5\,\hat{I}_2^{[0,1]}(3,2,2)m_c{m_b}^2-20\,\hat{I}_3(2,2,2)m_c{m_b}^2$$
$$+20\,\hat{I}_4(3,2,1)m_c{m_b}^2-10\,\hat{I}_1(3,2,1)m_c{m_b}^2+30\,\hat{I}_2(1,4,1)m_c{m_b}^2+20\,\hat{I}_3(2,2,2){m_b}^3$$
$$-20\,\hat{I}_4(2,2,2){m_b}^3-5\,\hat{I}_2^{[0,1]}(3,2,2){m_b}^3-40\,\hat{I}_4(2,3,1){m_b}^3+120\,\hat{I}_4(1,4,1){m_b}^3$$
$$-20\,\hat{I}_4(3,2,1){m_b}^3+10\,\hat{I}_2(2,3,1){m_b}^3+10\,\hat{I}_2(3,2,1){m_b}^3-120\,\hat{I}_3(1,4,1){m_b}^3$$
$$+20\,\hat{I}_3(3,2,1){m_b}^3+20\,\hat{I}_4^{[0,2]}(3,2,2)m_c+10\,\hat{I}_1^{[0,1]}(3,2,1)m_c-40\,\hat{I}_4^{[0,1]}(2,2,2)m_c$$
$$+5\,\hat{I}_0(2,2,1)m_c+15\,\hat{I}_1^{[0,1]}(3,1,2)m_c-5\,C1_2(3,2,2)m_c-15\,\hat{I}_0^{[0,1]}(3,2,1)m_c$$
$$+5\,\hat{I}_0^{[0,2]}(3,2,2)m_c-40\,\hat{I}_4(3,1,1)m_c+15\,\hat{I}_0(2,1,2)m_c+5\,\hat{I}_2(3,1,1)m_c$$
$$+40\,\hat{I}_3(3,1,1)m_c+5\,\hat{I}_1(3,1,1)m_c-20\,\hat{I}_4^{[0,1]}(3,1,2)m_c+10\,\hat{I}_0(3,1,1)m_c$$
$$+20\,\hat{I}_3(2,2,1)m_c+15\,\hat{I}_2^{[0,1]}(3,2,1)m_c-20\,\hat{I}_4(2,2,1)m_c+20\,\hat{I}_2(2,1,2)m_c$$
$$+10\,\hat{I}_1^{[0,1]}(2,2,2)m_c+20\,\hat{I}_3^{[0,1]}(3,1,2)m_c-20\,\hat{I}_4(2,1,2)m_c+20\,\hat{I}_3(2,1,2)m_c$$
$$-40\,\hat{I}_4^{[0,1]}(3,2,1)m_c+40\,\hat{I}_3^{[0,1]}(3,2,1)m_c-5\,\hat{I}_1^{[0,2]}(3,2,2)m_c+40\,\hat{I}_3^{[0,1]}(2,2,2)m_c$$
$$+5\,\hat{I}_2(2,2,1)m_c+10\,\hat{I}_2^{[0,1]}(2,2,2)m_c-15\,\hat{I}_1(2,1,2)m_c-15\,\hat{I}_0^{[0,1]}(3,1,2)m_c$$
$$-10\,\hat{I}_0^{[0,1]}(2,2,2)m_c-20\,\hat{I}_3^{[0,2]}(3,2,2)m_c+10\,\hat{I}_1(1,3,1)m_b+20\,\hat{I}_4(1,2,2)m_b$$
$$-40\,\hat{I}_3^{[0,1]}(3,1,2)m_b+40\,\hat{I}_4^{[0,1]}(3,1,2)m_b-20\,\hat{I}_2(2,1,2)m_b+20\,\hat{I}_4^{[0,1]}(3,2,1)m_b$$
$$-10\,\hat{I}_2^{[0,1]}(3,2,1)m_b-20\,\hat{I}_3(1,2,2)m_b-10\,\hat{I}_2^{[0,1]}(2,3,1)m_b-40\,\hat{I}_3(1,3,1)m_b$$
$$+5\,\hat{I}_2^{[0,2]}(3,2,2)m_b+10\,\hat{I}_0(2,2,1)m_b-20\,\hat{I}_3(2,1,2)m_b-10\,\hat{I}_1(2,2,1)m_b$$
$$-10\,\hat{I}_2(2,2,1)m_b-10\,\hat{I}_0(1,3,1)m_b-20\,\hat{I}_4^{[0,2]}(3,2,2)m_b-40\,\hat{I}_3^{[0,1]}(2,3,1)m_b$$
$$-20\,\hat{I}_2(1,2,2)m_b+20\,\hat{I}_4(2,1,2)m_b-40\,\hat{I}_3(3,1,1)m_b-10\,\hat{I}_2(1,3,1)m_b$$
$$-10\,\hat{I}_2^{[0,1]}(3,1,2)m_b-20\,\hat{I}_3^{[0,1]}(2,2,2)m_b+40\,\hat{I}_4(1,3,1)m_b+40\,\hat{I}_4(3,1,1)m_b$$
$$+20\,\hat{I}_3^{[0,2]}(3,2,2)m_b-40\,\hat{I}_3(2,2,1)m_b+20\,\hat{I}_4^{[0,1]}(2,2,2)m_b+40\,\hat{I}_4^{[0,1]}(2,3,1)m_b$$
$$-20\,\hat{I}_3^{[0,1]}(3,2,1)m_b+40\,\hat{I}_4(2,2,1)m_b+25\,\hat{I}_0(3,2,1){m_c}^3-60\,\hat{I}_3(4,1,1){m_c}^3$$
$$+10\,\hat{I}_2(3,1,2){m_c}^2m_b+20\,\hat{I}_4(3,2,2){m_c}^5+5\,\hat{I}_0(3,2,2){m_c}^5-5\,\hat{I}_2(3,2,2){m_c}^5$$
$$-20\,\hat{I}_3(3,2,2){m_c}^5-5\,\hat{I}_1(3,2,2){m_c}^5-40\,\hat{I}_4^{[0,1]}(3,2,2){m_c}^3-10\,\hat{I}_2(2,2,2){m_c}^3$$
$$+40\,\hat{I}_4(3,2,1){m_c}^3+40\,\hat{I}_3^{[0,1]}(3,2,2){m_c}^3+20\,\hat{I}_4(3,1,2){m_c}^3+5\,\hat{I}_2(3,2,2){m_c}^4m_b$$
$$-20\,\hat{I}_4(3,2,2){m_c}^4m_b+20\,\hat{I}_3(3,2,2){m_c}^4m_b+5\,\hat{I}_1(3,2,2){m_c}^3{m_b}^2-5\,\hat{I}_0(3,2,2){m_c}^3{m_b}^2$$
$$+20\,\hat{I}_3(3,2,2){m_c}^3{m_b}^2-20\,\hat{I}_4(3,2,2){m_c}^3{m_b}^2+5\,\hat{I}_2(3,2,2){m_c}^3{m_b}^2-5\,\hat{I}_2(3,2,2){m_c}^2{m_b}^3$$
$$+20\,\hat{I}_4(3,2,2){m_c}^2{m_b}^3-20\,\hat{I}_3(3,2,2){m_c}^2{m_b}^3-10\,\hat{I}_2^{[0,1]}(3,2,2){m_c}^2m_b+20\,\hat{I}_0(2,3,1){m_c}^2m_b$$

$$C_V^{T-PT} = 10\,\hat{I}_1(3,2,2){m_b}^4{m_c}^2-20\,\hat{I}_1(1,1,2)+10\,\hat{I}_0^{[0,1]}(2,2,1)-10\,\hat{I}_2^{[0,2]}(3,2,1)$$
$$-10\,\hat{I}_0(3,2,2){m_b}^3{m_c}^3-10\,\hat{I}_1(3,2,2){m_b}^3{m_c}^3-10\,\hat{I}_1(1,2,1)-10\,\hat{I}_0(1,1,2)$$
$$-60\,\hat{I}_1(1,4,1){m_b}^3m_c-10\,\hat{I}_0^{[0,1]}(3,2,2){m_b}^3m_c-60\,\hat{I}_0(1,4,1){m_b}^3m_c+20\,\hat{I}_2(3,2,1){m_b}^3m_c$$
$$+20\,\hat{I}_1(3,2,1){m_b}^3m_c-10\,\hat{I}_2^{[0,1]}(3,2,2){m_b}^3m_c-10\,\hat{I}_1^{[0,1]}(3,2,2){m_b}^3m_c-60\,\hat{I}_2(1,4,1){m_b}^3m_c$$
$$+20\,\hat{I}_2(2,3,1){m_b}^3m_c-10\,\hat{I}_2(3,2,2){m_b}^3{m_c}^3+10\,\hat{I}_2(3,2,2)m_b{m_c}^5+10\,\hat{I}_1(3,2,2)m_b{m_c}^5$$

$$
\begin{aligned}
&+10\,\hat{I}_0(3,2,2)m_b{m_c}^5+20\,\hat{I}_0(3,2,1){m_b}^3m_c-20\,\hat{I}_1(2,3,1){m_b}^3m_c+10\,\hat{I}_1(3,1,2){m_b}^3m_c\\
&+20\,\hat{I}_0(2,3,1){m_b}^3m_c-10\,\hat{I}_1(3,2,2){m_b}^2{m_c}^4+10\,\hat{I}_1^{[0,1]}(3,2,2){m_b}^4-20\,\hat{I}_1(3,2,1){m_b}^4\\
&+60\,\hat{I}_1(1,4,1){m_b}^4-10\,\hat{I}_0(3,1,2){m_c}^4-10\,\hat{I}_1(3,1,2){m_c}^4+10\,\hat{I}_1(3,2,1){m_c}^4\\
&+10\,\hat{I}_0(3,2,1){m_c}^4+10\,\hat{I}_2^{[0,1]}(3,2,1){m_b}^2+60\,\hat{I}_1(1,3,1){m_b}^2+20\,\hat{I}_1^{[0,1]}(3,2,1){m_b}^2\\
&-10\,\hat{I}_1(2,2,2){m_b}^4+20\,\hat{I}_0(2,3,1)m_b{m_c}^3-20\,\hat{I}_2^{[0,1]}(3,2,2)m_b{m_c}^3+20\,\hat{I}_1(2,2,2)m_b{m_c}^3\\
&+10\,\hat{I}_2(3,2,1)m_b{m_c}^3+20\,\hat{I}_2(2,3,1)m_b{m_c}^3+10\,\hat{I}_1(3,2,1)m_b{m_c}^3+20\,\hat{I}_2(2,2,2)m_b{m_c}^3\\
&+20\,\hat{I}_1(2,3,1)m_b{m_c}^3+30\,\hat{I}_0(4,1,1)m_b{m_c}^3+20\,\hat{I}_1^{[0,1]}(3,2,2){m_b}^2{m_c}^2+10\,\hat{I}_2(3,2,1){m_b}^2{m_c}^2\\
&-20\,\hat{I}_1(3,2,1){m_b}^2{m_c}^2-30\,\hat{I}_1(4,1,1){m_b}^2{m_c}^2+20\,\hat{I}_0(3,2,1)m_b{m_c}^3+30\,\hat{I}_1(4,1,1)m_b{m_c}^3\\
&+30\,\hat{I}_2(4,1,1)m_b{m_c}^3+20\,\hat{I}_0(2,2,2)m_b{m_c}^3-20\,\hat{I}_1(2,2,2){m_b}^2{m_c}^2-40\,\hat{I}_2(1,3,1)m_bm_c\\
&-30\,\hat{I}_2(2,2,1)m_bm_c-70\,\hat{I}_1(2,2,1)m_bm_c-10\,\hat{I}_2^{[0,1]}(3,2,1)m_bm_c-20\,\hat{I}_0^{[0,1]}(2,2,2)m_bm_c\\
&-20\,\hat{I}_2^{[0,1]}(2,2,2)m_bm_c+20\,\hat{I}_0(1,2,2)m_bm_c+10\,\hat{I}_0^{[0,2]}(3,2,2)m_bm_c+10\,\hat{I}_1^{[0,2]}(3,2,2)m_bm_c\\
&-20\,\hat{I}_0^{[0,1]}(3,2,1)m_bm_c+20\,\hat{I}_2(1,2,2)m_bm_c-20\,\hat{I}_1^{[0,1]}(2,3,1)m_bm_c-20\,\hat{I}_0(2,2,1)m_bm_c\\
&-20\,\hat{I}_2^{[0,1]}(3,1,2)m_bm_c+20\,\hat{I}_2(2,1,2)m_bm_c-40\,\hat{I}_0(1,3,1)m_bm_c-20\,\hat{I}_0^{[0,1]}(3,1,2)m_bm_c\\
&-20\,\hat{I}_2^{[0,1]}(2,3,1)m_bm_c-10\,\hat{I}_1^{[0,1]}(3,2,1)m_bm_c+20\,\hat{I}_0(2,1,2)m_bm_c+10\,\hat{I}_2^{[0,2]}(3,2,2)m_bm_c\\
&-10\,\hat{I}_0(1,2,1)-20\,\hat{I}_1(1,2,2){m_b}^2+10\,\hat{I}_0(2,2,1){m_b}^2+20\,\hat{I}_1^{[0,1]}(3,1,2){m_b}^2\\
&-30\,\hat{I}_1(2,1,2){m_b}^2+20\,\hat{I}_1^{[0,1]}(2,2,2){m_b}^2+30\,\hat{I}_2(2,2,1){m_b}^2-10\,\hat{I}_1^{[0,2]}(3,2,2){m_b}^2\\
&+10\,\hat{I}_0^{[0,1]}(3,1,2){m_c}^2-20\,\hat{I}_0(2,1,2){m_c}^2-10\,\hat{I}_0^{[0,1]}(3,2,1){m_c}^2-20\,\hat{I}_1(2,1,2){m_c}^2\\
&+60\,\hat{I}_1(2,2,1){m_b}^2-30\,\hat{I}_2(1,2,1)-20\,\hat{I}_0^{[0,1]}(3,2,2)m_b{m_c}^3+20\,\hat{I}_1(2,1,2)m_bm_c\\
&-20\,\hat{I}_0^{[0,1]}(2,3,1)m_bm_c+20\,\hat{I}_1(1,2,2)m_bm_c-10\,\hat{I}_1(3,1,1)m_bm_c-20\,\hat{I}_1^{[0,1]}(3,1,2)m_bm_c\\
&-20\,\hat{I}_1^{[0,1]}(2,2,2)m_bm_c-40\,\hat{I}_1(1,3,1)m_bm_c-20\,\hat{I}_1^{[0,1]}(3,2,2)m_b{m_c}^3-10\,\hat{I}_1^{[0,1]}(3,2,1){m_c}^2\\
&+10\,\hat{I}_1^{[0,1]}(3,1,2){m_c}^2-20\,\hat{I}_2(2,2,1){m_c}^2+10\,\hat{I}_0(3,1,1){m_c}^2+10\,\hat{I}_2^{[0,1]}(3,2,1){m_c}^2\\
&-10\,\hat{I}_2^{[0,1]}(3,1,2){m_c}^2+10\,\hat{I}_2(3,1,1){m_c}^2
\end{aligned}
$$

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\begin{aligned}
C_0^{T-PT} \;=\;&-15\,\hat{I}_0(2,2,2){m_c}^4{m_b}^2-15\,\hat{I}_0(4,1,1){m_c}^4{m_b}^2+5\,\hat{I}_0(3,1,2){m_c}^5m_b-5\,\hat{I}_0(3,2,1){m_c}^5m_b\\
&-5\,\hat{I}_0(3,2,2){m_c}^3{m_b}^5+5\,\hat{I}_0(3,2,2){m_c}^4{m_b}^4+5\,\hat{I}_0(3,2,2){m_c}^5{m_b}^3-5\,\hat{I}_0(3,2,2){m_c}^6{m_b}^2\\
&+30\,\hat{I}_0^{[0,1]}(2,2,2){m_c}^2{m_b}^2-15\,\hat{I}_0(2,1,2){m_c}^2{m_b}^2-15\,\hat{I}_0^{[0,2]}(3,2,2){m_c}^2{m_b}^2\\
&-20\,\hat{I}_0(2,2,1){m_c}^3m_b+10\,\hat{I}_0^{[0,1]}(3,2,1){m_c}^3m_b+10\,\hat{I}_0(2,1,2){m_c}^3m_b-30\,\hat{I}_0(1,4,1)m_c{m_b}^5\\
&-5\,\hat{I}_0^{[0,1]}(3,2,2)m_c{m_b}^5+10\,\hat{I}_0(3,2,1)m_c{m_b}^5+10\,\hat{I}_0(2,3,1)m_c{m_b}^5-10\,\hat{I}_0(3,2,1){m_c}^2{m_b}^4\\
&+30\,\hat{I}_0(1,4,1){m_c}^2{m_b}^4+15\,\hat{I}_0(4,1,1){m_c}^3{m_b}^3-10\,\hat{I}_0^{[0,1]}(3,2,2){m_c}^3{m_b}^3\\
&-10\,\hat{I}_0(2,3,1){m_c}^3{m_b}^3+10\,\hat{I}_0(2,2,2){m_c}^3{m_b}^3+15\,\hat{I}_0^{[0,1]}(3,2,2){m_c}^4{m_b}^2\\
&+10\,\hat{I}_0(1,2,2)m_c{m_b}^3+15\,\hat{I}_0(2,1,2)m_c{m_b}^3-20\,\hat{I}_0^{[0,1]}(3,1,2)m_c{m_b}^3+10\,\hat{I}_0^{[0,1]}(2,3,1)m_c{m_b}^3\\
&-5\,\hat{I}_0(2,2,1)m_c{m_b}^3-10\,\hat{I}_0^{[0,1]}(2,2,2)m_c{m_b}^3-20\,\hat{I}_0(1,2,2){m_c}^2{m_b}^2\\
&+15\,\hat{I}_0^{[0,1]}(3,2,1){m_c}^2{m_b}^2-5\,\hat{I}_0(3,1,1){m_c}^2{m_b}^2+30\,\hat{I}_0(1,3,1){m_c}^2{m_b}^2\\
&+15\,\hat{I}_0^{[0,1]}(3,1,2){m_c}^2{m_b}^2-30\,\hat{I}_0(2,1,1)m_cm_b+10\,\hat{I}_0^{[0,1]}(3,1,1)m_cm_b-5\,\hat{I}_0^{[0,2]}(3,2,1)m_cm_b\\
&-10\,\hat{I}_0^{[0,1]}(2,1,2)m_cm_b-50\,\hat{I}_0(1,2,1)m_cm_b+10\,\hat{I}_0(1,1,2)m_cm_b+5\,\hat{I}_0^{[0,2]}(3,1,2)m_cm_b
\end{aligned}
$$

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\begin{aligned}
&+20\,\hat{I}_0^{[0,1]}(2,2,1)m_cm_b - 70\,\hat{I}_0(1,3,1)m_cm_b{}^3 + 5\,\hat{I}_0^{[0,2]}(3,2,2)m_cm_b{}^3 + 5\,\hat{I}_0(3,1,1)m_cm_b{}^3 \\
&-20\,\hat{I}_0^{[0,1]}(1,2,1) - 5\,\hat{I}_0^{[0,2]}(3,2,1)m_b{}^2 - 15\,\hat{I}_0(1,1,2)m_b{}^2 + 30\,\hat{I}_0^{[0,1]}(2,1,2)m_b{}^2 \\
&-15\,\hat{I}_0^{[0,2]}(3,1,2)m_b{}^2 - 30\,\hat{I}_0^{[0,1]}(1,3,1)m_b{}^2 + 10\,\hat{I}_0(1,2,1)m_b{}^2 - 15\,\hat{I}_0^{[0,2]}(2,1,2) \\
&+20\,\hat{I}_0^{[0,1]}(1,2,2)m_b{}^2 - 15\,\hat{I}_0^{[0,2]}(2,2,2)m_b{}^2 + 20\,\hat{I}_0(2,1,1)m_b{}^2 + 30\,\hat{I}_0^{[0,1]}(2,1,2)m_c{}^2 \\
&-30\,\hat{I}_0^{[0,1]}(2,2,1)m_c{}^2 + 10\,\hat{I}_0^{[0,1]}(3,1,1)m_b{}^2 - 5\,\hat{I}_0^{[0,1]}(2,2,1)m_b{}^2 + 20\,\hat{I}_0(1,2,1)m_c{}^2 \\
&-15\,\hat{I}_0^{[0,2]}(3,1,2)m_c{}^2 + 15\,\hat{I}_0^{[0,2]}(3,2,1)m_c{}^2 - 20\,\hat{I}_0(1,1,2)m_c{}^2 + 15\,\hat{I}_0(2,1,1)m_c{}^2 \\
&-30\,\hat{I}_0^{[0,1]}(1,4,1)m_b{}^4 + 15\,\hat{I}_0(1,1,1) + 10\,\hat{I}_0(2,2,1)m_b{}^4 + 10\,\hat{I}_0^{[0,1]}(2,2,2)m_b{}^4 \\
&+10\,\hat{I}_0^{[0,1]}(3,2,1)m_b{}^4 - 5\,\hat{I}_0^{[0,2]}(3,2,2)m_b{}^4 + 15\,\hat{I}_0^{[0,1]}(3,1,2)m_c{}^4 - 5\,\hat{I}_0(1,2,2)m_b{}^4 \\
&-15\,\hat{I}_0(2,1,2)m_c{}^4 - 15\,\hat{I}_0^{[0,1]}(3,2,1)m_c{}^4 - 5\,\hat{I}_0(3,1,2)m_c{}^6 + 5\,\hat{I}_0(3,2,1)m_c{}^6 \\
&+15\,\hat{I}_0(2,2,1)m_c{}^4 + 20\,\hat{I}_0^{[0,1]}(1,1,2) + 15\,\hat{I}_0^{[0,2]}(2,2,1)
\end{aligned}
$$

$$
\begin{aligned}
C_1^{T-PT} \;=\;& +10\,\hat{I}_0(3,1,1)m_cm_b + 10\,\hat{I}_1^{[0,1]}(2,2,2)m_cm_b - 10\,\hat{I}_2^{[0,1]}(2,2,2)m_cm_b \\
&-35\,\hat{I}_2(2,2,1)m_cm_b - 10\,\hat{I}_0^{[0,1]}(2,2,2)m_cm_b - 10\,\hat{I}_0(1,1,2) + 20\,\hat{I}_1(1,2,2)m_c{}^2 \\
&+15\,\hat{I}_0^{[0,1]}(4,1,1)m_c{}^2 + 25\,\hat{I}_2^{[0,1]}(3,1,2)m_c{}^2 - 15\,\hat{I}_0(2,1,2)m_c{}^2 - 5\,\hat{I}_1(3,1,1)m_c{}^2 \\
&-15\,\hat{I}_1^{[0,1]}(3,1,2)m_c{}^2 + 30\,\hat{I}_2^{[0,1]}(2,2,2)m_c{}^2 + 15\,\hat{I}_1(2,1,2)m_c{}^2 + 30\,\hat{I}_0^{[0,1]}(2,2,2)m_c{}^2 \\
&+5\,\hat{I}_2^{[0,1]}(3,2,1)m_c{}^2 - 15\,\hat{I}_1^{[0,1]}(3,2,1)m_c{}^2 + 15\,\hat{I}_1^{[0,2]}(3,2,2)m_c{}^2 - 15\,\hat{I}_0^{[0,2]}(3,2,2)m_c{}^2 \\
&+10\,\hat{I}_2(2,2,1)m_c{}^2 + 10\,\hat{I}_2(3,1,1)m_c{}^2 - 5\,\hat{I}_0(3,2,2)m_c{}^3m_b{}^3 - 5\,\hat{I}_2(3,2,2)m_c{}^3m_b{}^3 \\
&+5\,\hat{I}_2(3,2,2)m_c{}^2m_b{}^4 - 15\,\hat{I}_1(4,1,1)m_c{}^3m_b + 10\,\hat{I}_2(2,2,2)m_c{}^3m_b - 15\,\hat{I}_0^{[0,2]}(2,2,2) \\
&-15\,\hat{I}_1(2,1,2)m_cm_b - 5\,\hat{I}_2^{[0,1]}(3,2,1)m_cm_b + 5\,\hat{I}_0^{[0,2]}(3,2,2)m_cm_b - 10\,\hat{I}_0^{[0,1]}(3,2,1)m_cm_b \\
&-15\,\hat{I}_0^{[0,1]}(3,1,2)m_cm_b + 15\,\hat{I}_1^{[0,1]}(3,1,2)m_cm_b + 10\,\hat{I}_2(1,2,2)m_cm_b - 10\,\hat{I}_1^{[0,1]}(2,3,1)m_cm_b \\
&+5\,\hat{I}_1(3,1,1)m_cm_b + 30\,\hat{I}_2^{[0,1]}(2,1,2) - 20\,\hat{I}_1^{[0,1]}(2,1,2) + 10\,\hat{I}_1^{[0,2]}(3,1,2) \\
&-5\,\hat{I}_2^{[0,1]}(3,1,1) - 10\,\hat{I}_1(1,2,2)m_cm_b + 10\,\hat{I}_0(1,2,2)m_cm_b - 15\,\hat{I}_2^{[0,1]}(3,1,2)m_cm_b \\
&-15\,\hat{I}_2^{[0,2]}(3,1,2) + 30\,\hat{I}_2(1,3,1)m_b{}^2 + 20\,\hat{I}_2(2,2,1)m_b{}^2 + 5\,\hat{I}_2(3,1,1)m_b{}^2 \\
&-5\,\hat{I}_0^{[0,2]}(3,2,2)m_b{}^2 - 10\,\hat{I}_2^{[0,2]}(3,2,2)m_b{}^2 + 10\,\hat{I}_0^{[0,1]}(2,2,2)m_b{}^2 + 5\,\hat{I}_0(2,2,1)m_b{}^2 \\
&-30\,\hat{I}_0^{[0,1]}(1,4,1)m_b{}^2 - 10\,\hat{I}_1(2,2,1)m_b{}^2 - 5\,\hat{I}_0(1,2,2)m_b{}^2 + 5\,\hat{I}_1^{[0,2]}(3,2,2)m_b{}^2 \\
&+10\,\hat{I}_2^{[0,1]}(3,2,1)m_b{}^2 - 10\,\hat{I}_1^{[0,1]}(2,2,2)m_b{}^2 - 15\,\hat{I}_2(1,2,2)m_b{}^2 + 30\,\hat{I}_1^{[0,1]}(1,4,1)m_b{}^2 \\
&-5\,\hat{I}_1^{[0,1]}(2,2,1) - 5\,\hat{I}_2^{[0,1]}(2,2,1) + 20\,\hat{I}_0^{[0,1]}(1,2,2) - 15\,\hat{I}_2(2,1,2)m_b{}^2 - 30\,\hat{I}_2^{[0,1]}(1,4,1)m_b{}^2 \\
&+15\,\hat{I}_2^{[0,1]}(3,1,2)m_b{}^2 + 20\,\hat{I}_2^{[0,1]}(2,2,2)m_b{}^2 - 10\,\hat{I}_1^{[0,1]}(3,2,1)m_b{}^2 + 15\,\hat{I}_0^{[0,1]}(3,2,1)m_b{}^2 \\
&+5\,\hat{I}_1(1,2,2)m_b{}^2 + 15\,\hat{I}_1^{[0,2]}(2,2,2) - 15\,\hat{I}_2^{[0,2]}(2,2,2) + 10\,\hat{I}_0^{[0,1]}(3,1,1) \\
&+15\,\hat{I}_0^{[0,1]}(2,2,1) - 5\,\hat{I}_1^{[0,3]}(3,2,2) + 5\,\hat{I}_2^{[0,3]}(3,2,2) - 5\,\hat{I}_2(3,2,2)m_c{}^6 - 5\,\hat{I}_0(3,2,2)m_c{}^6 \\
&+5\,\hat{I}_1(3,2,1)m_c{}^4 - 10\,\hat{I}_0(3,2,1)m_c{}^4 - 15\,\hat{I}_2(4,1,1)m_c{}^4 - 15\,\hat{I}_2(2,2,2)m_c{}^4 \\
&+15\,\hat{I}_1(2,2,2)m_c{}^4 + 15\,\hat{I}_1(4,1,1)m_c{}^4 + 15\,\hat{I}_2^{[0,1]}(3,2,2)m_c{}^4 - 15\,\hat{I}_1^{[0,1]}(3,2,2)m_c{}^4 \\
&-15\,\hat{I}_0(2,2,2)m_c{}^4 + 5\,\hat{I}_2(3,2,2)m_c{}^5m_b + 5\,\hat{I}_0(3,2,2)m_c{}^5m_b - 5\,\hat{I}_1(3,2,2)m_c{}^5m_b \\
&+5\,\hat{I}_0(3,2,2)m_c{}^4m_b{}^2 - 5\,\hat{I}_1(3,2,2)m_c{}^4m_b{}^2 + 5\,\hat{I}_1(3,2,2)m_c{}^3m_b{}^3 + 5\,\hat{I}_1(3,2,2)m_c{}^6 \\
&-5\,\hat{I}_1(3,1,2)m_c{}^3m_b + 10\,\hat{I}_1^{[0,1]}(3,2,2)m_c{}^3m_b + 10\,\hat{I}_1(2,3,1)m_c{}^3m_b - 10\,\hat{I}_0^{[0,1]}(3,2,2)m_c{}^3m_b
\end{aligned}
$$

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\begin{aligned}
&+5\,\hat{I}_0(3,1,2){m_c}^3m_b - 5\,\hat{I}_1(3,2,1){m_c}^3m_b - 10\,\hat{I}_1(2,2,2){m_c}^3m_b + 5\,\hat{I}_2(3,1,2){m_c}^3m_b \\
&+15\,\hat{I}_2(4,1,1){m_c}^3m_b + 10\,\hat{I}_0(3,2,1){m_c}^3m_b - 10\,\hat{I}_2^{[0,1]}(3,2,2){m_c}^3m_b + 5\,\hat{I}_2^{[0,1]}(3,2,2){m_b}^4 \\
&-15\,\hat{I}_0(4,1,1){m_c}^4 + 5\,\hat{I}_1(3,1,2){m_c}^4 - 10\,\hat{I}_2(3,1,2){m_c}^4 - 5\,\hat{I}_0(3,1,2){m_c}^4 \\
&\hat{I}_0(2,2,2){m_c}^3m_b + 5\,\hat{I}_2(3,2,1){m_c}^3m_b - 10\,\hat{I}_2(2,3,1){m_c}^3m_b - 10\,\hat{I}_2(2,2,2){m_c}^2{m_b}^2 \\
&-20\,\hat{I}_2(3,2,1){m_c}^2{m_b}^2 + 30\,\hat{I}_0(1,4,1){m_c}^2{m_b}^2 + 10\,\hat{I}_1(3,2,1){m_c}^2{m_b}^2 - 5\,\hat{I}_0(3,2,1){m_c}^2{m_b}^2 \\
&+10\,\hat{I}_2^{[0,1]}(3,2,2){m_c}^2{m_b}^2 + 30\,\hat{I}_2(1,4,1){m_b}^4 - 15\,\hat{I}_2(4,1,1){m_c}^2{m_b}^2 + 30\,\hat{I}_2(1,4,1){m_c}^2{m_b}^2 \\
&+5\,\hat{I}_2(3,2,1)m_c{m_b}^3 + 5\,\hat{I}_1^{[0,1]}(3,2,2)m_c{m_b}^3 + 5\,\hat{I}_2(3,1,2)m_c{m_b}^3 - 30\,\hat{I}_2(1,4,1)m_c{m_b}^3 \\
&-10\,\hat{I}_2(2,3,1)m_c{m_b}^3 - 5\,\hat{I}_2(3,2,1){m_b}^4 - 30\,\hat{I}_0(1,4,1)m_c{m_b}^3 + 10\,\hat{I}_0(2,3,1)m_c{m_b}^3 \\
&+10\,\hat{I}_2^{[0,1]}(2,3,1)m_cm_b - 40\,\hat{I}_0(1,3,1)m_cm_b + 15\,\hat{I}_0(2,1,2)m_cm_b - 10\,\hat{I}_2(2,2,2){m_b}^4
\end{aligned}
$$

where

$$
\hat{I}_n^{[i,j]}(a,b,c) = \left(M_1^2\right)^i \left(M_2^2\right)^j \frac{d^i}{d\left(M_1^2\right)^i}\frac{d^j}{d\left(M_2^2\right)^j}\left[\left(M_1^2\right)^i \left(M_2^2\right)^j \hat{I}_n(a,b,c)\right] \;.
$$

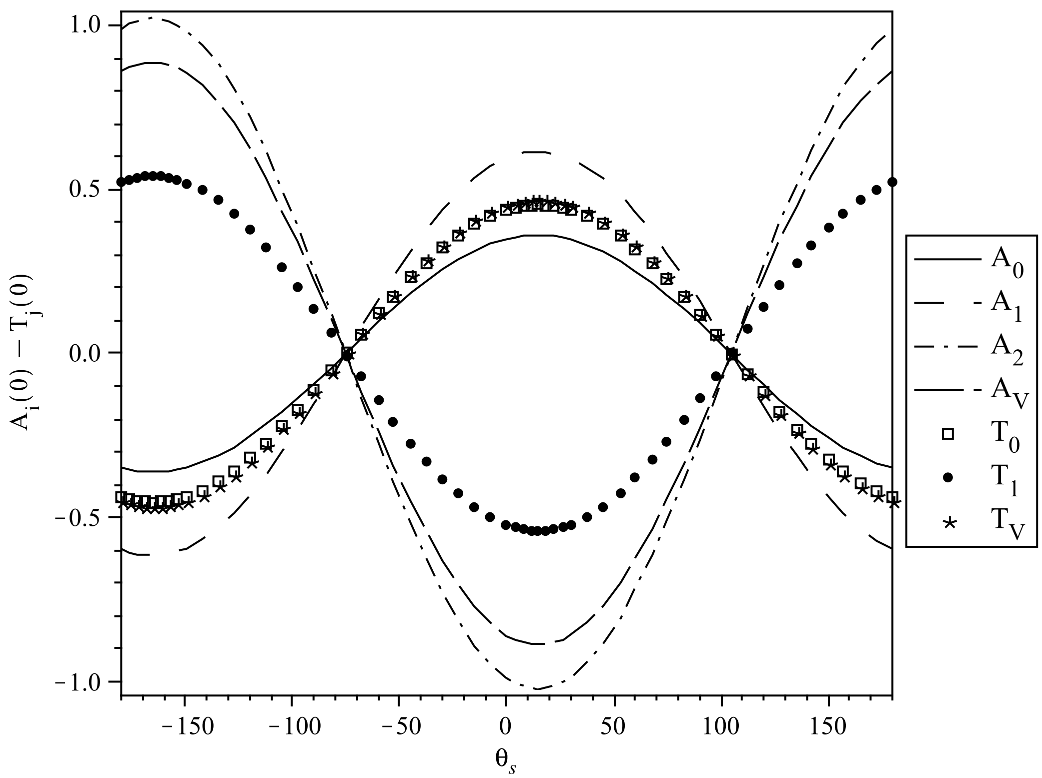

Figure 3: The dependence of the form factors on θ_s at $q^2 = 0$ for $B_c \to D_{s1}(2460)$ transition.

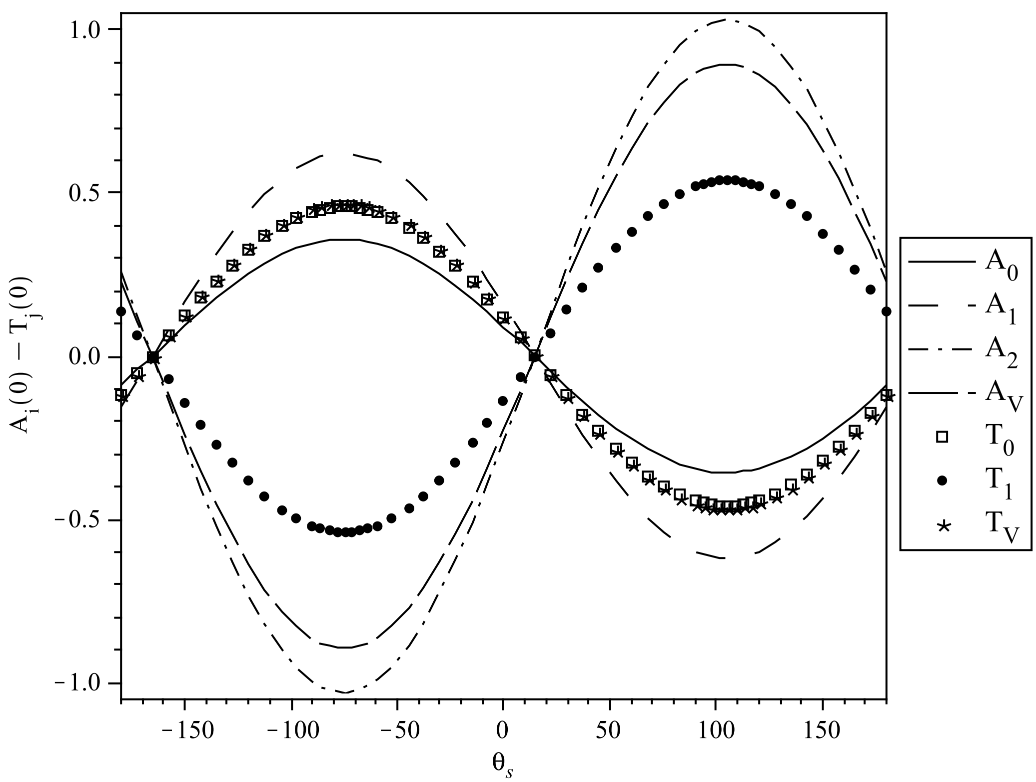

Figure 4: The dependence of the form factors on θ_s at $q^2 = 0$ for $B_c \to D_{s1}(2536)$ transition.

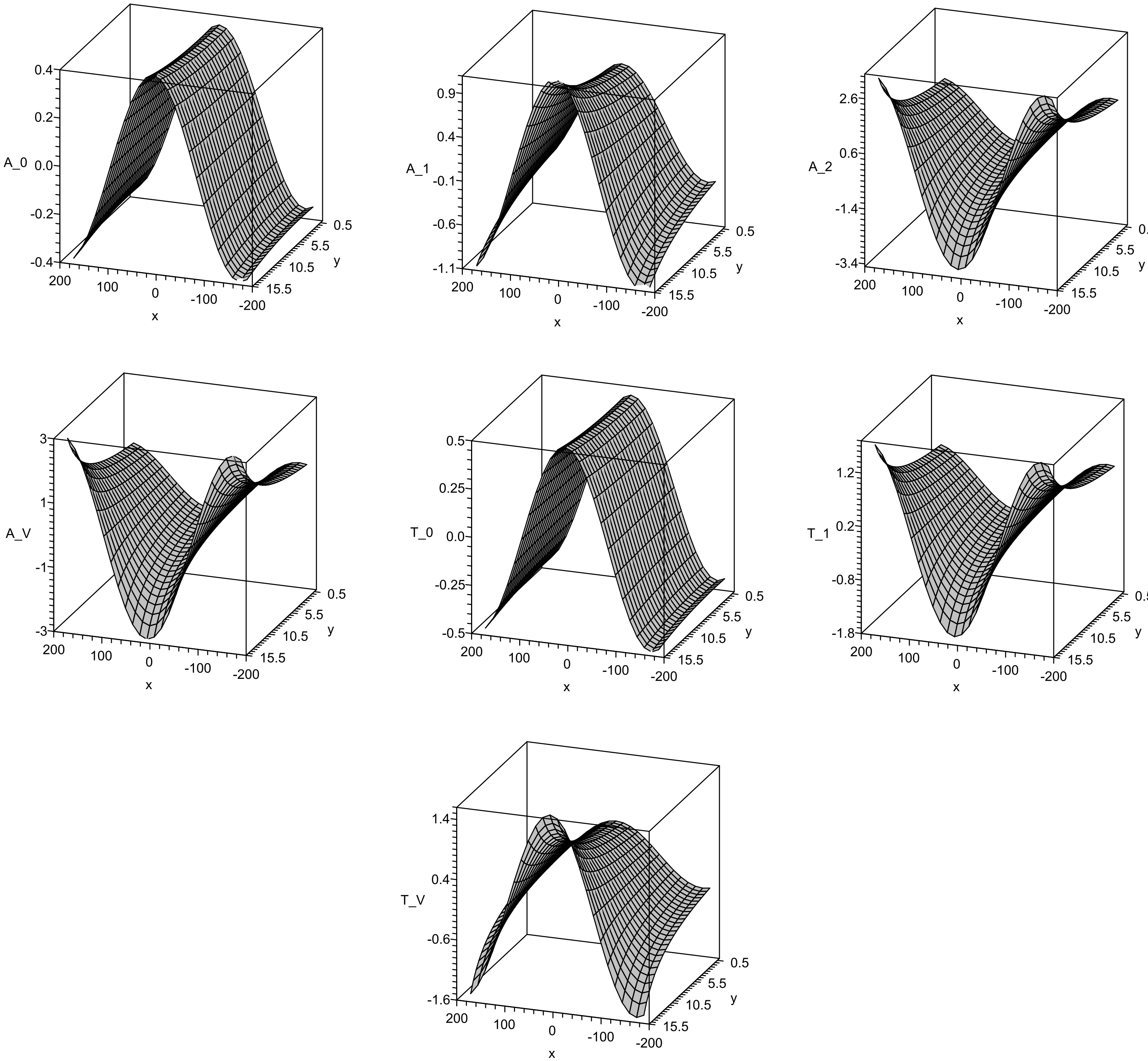

Figure 5: The dependence of the transition form factors on q^2 and θ_s for $B_c \to D_{s1}(2460)$ transition. In these figures, $x = \theta_s$ and $y = q^2$.

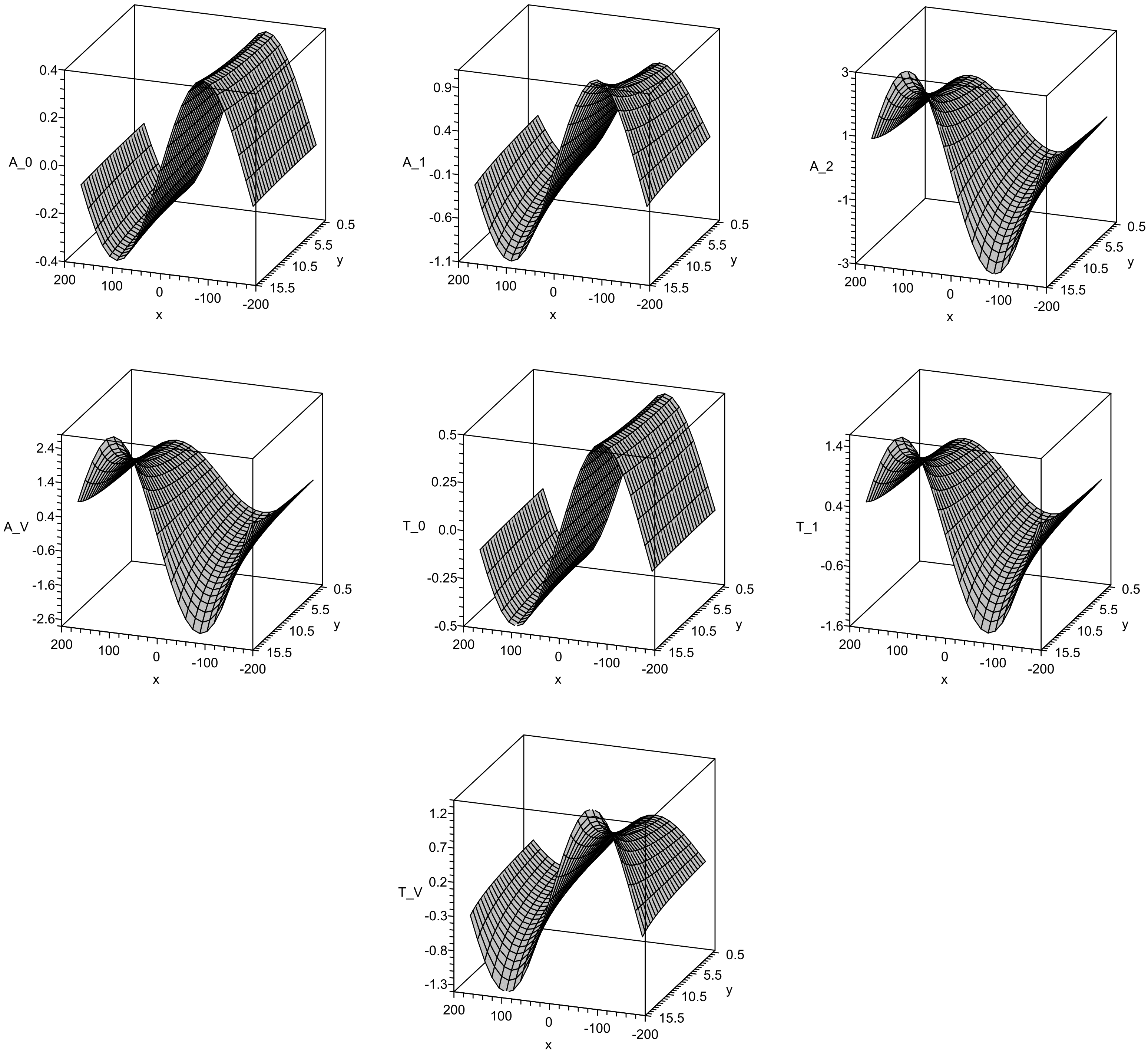

Figure 6: The dependence of the transition form factors on q^2 and θ_s for $B_c \to D_{s1}(2536)$ transition. In these figures, $x = \theta_s$ and $y = q^2$.

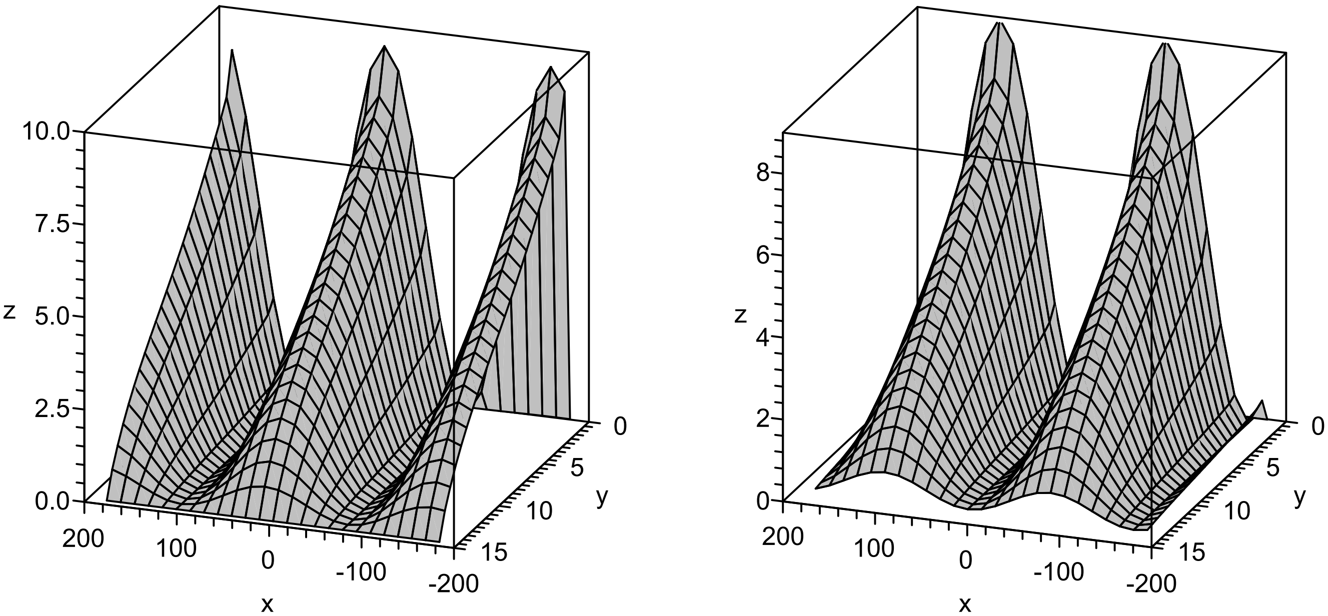

Figure 7: The decay width for $B_c \to D_{s1}\mu^+\mu^-$ with respect to θ_s and q^2. The left figure shows decay width of $B_c \to D_{s1}(2460)\mu^+\mu^-$ and right figure belongs to $B_c \to D_{s1}(2536)\mu^+\mu^-$. In these figures, $x = \theta_s$, $y = q^2$ and $z = \Gamma(B_c \to D_{s1}\mu^+\mu^-) \times 10^{-20}$.

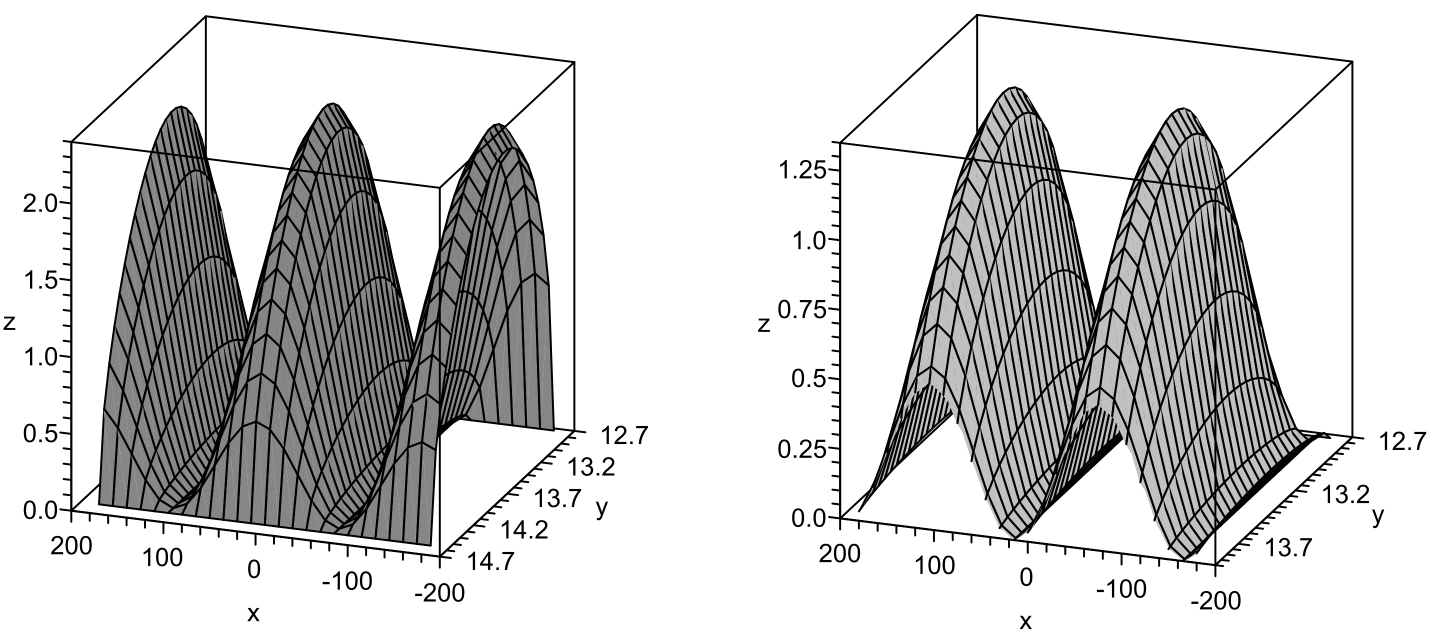

Figure 8: The decay width for $B_c \to D_{s1}\tau^+\tau^-$ with respect to θ_s and q^2. The left figure shows decay width of $B_c \to D_{s1}(2460)\tau^+\tau^-$ and right figure belongs to $B_c \to D_{s1}(2536)\tau^+\tau^-$. In these figures, $x = \theta_s$, $y = q^2$ and $z = \Gamma(B_c \to D_{s1}\tau^+\tau^-) \times 10^{-21}$.

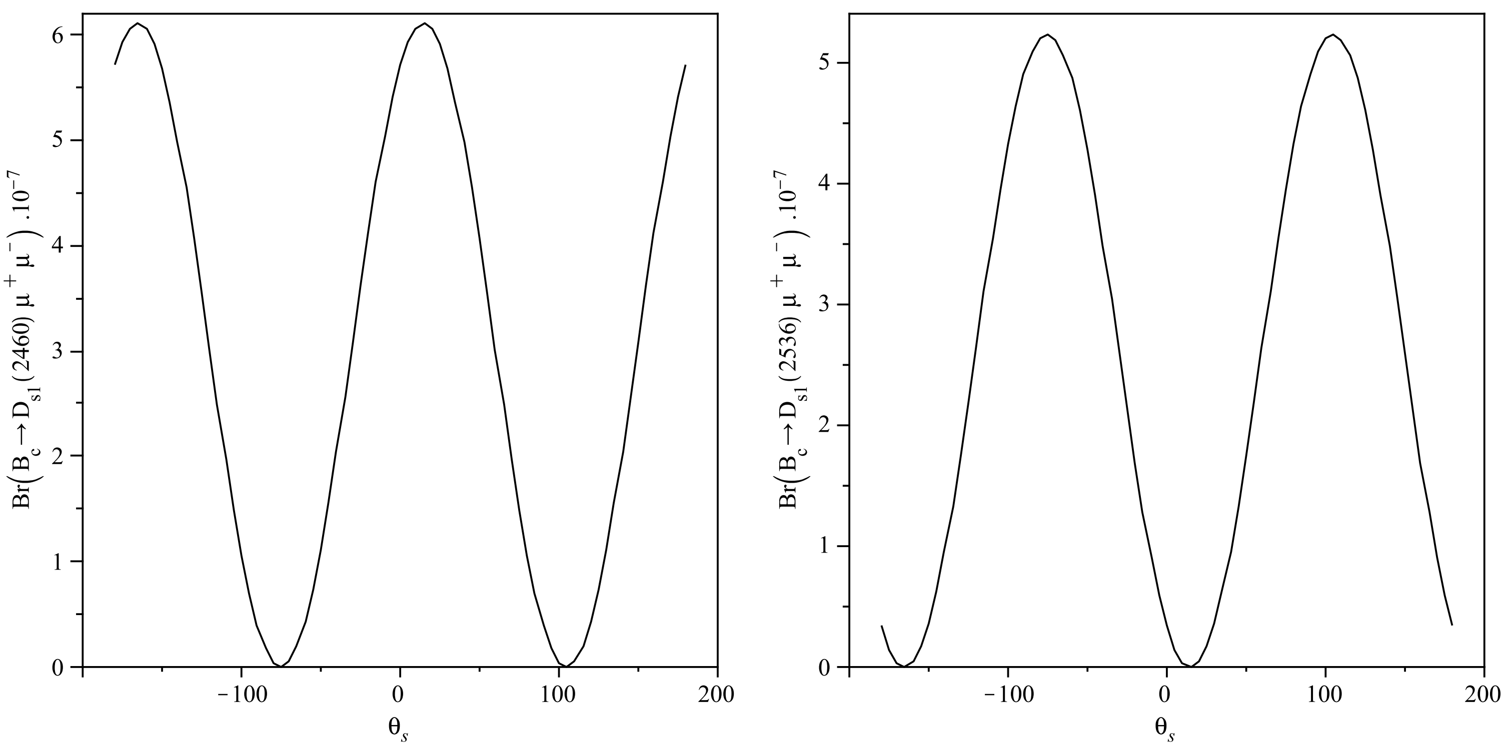

Figure 9: The branching ratios of $B_c \to D_{s1}\mu^+\mu^-$ with respect to θ_s.

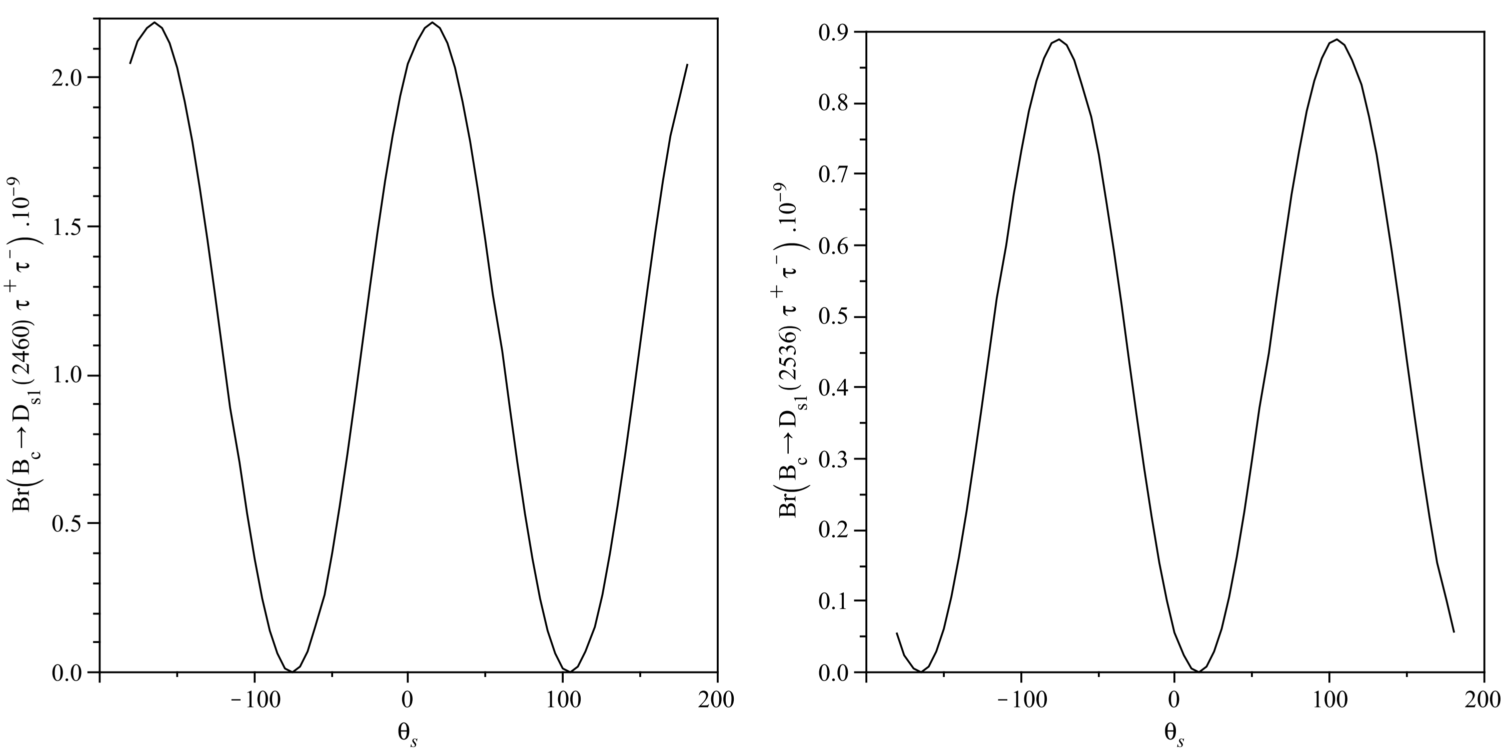

Figure 10: The branching ratios of $B_c \to D_{s1}\tau^+\tau^-$ with respect to θ_s.

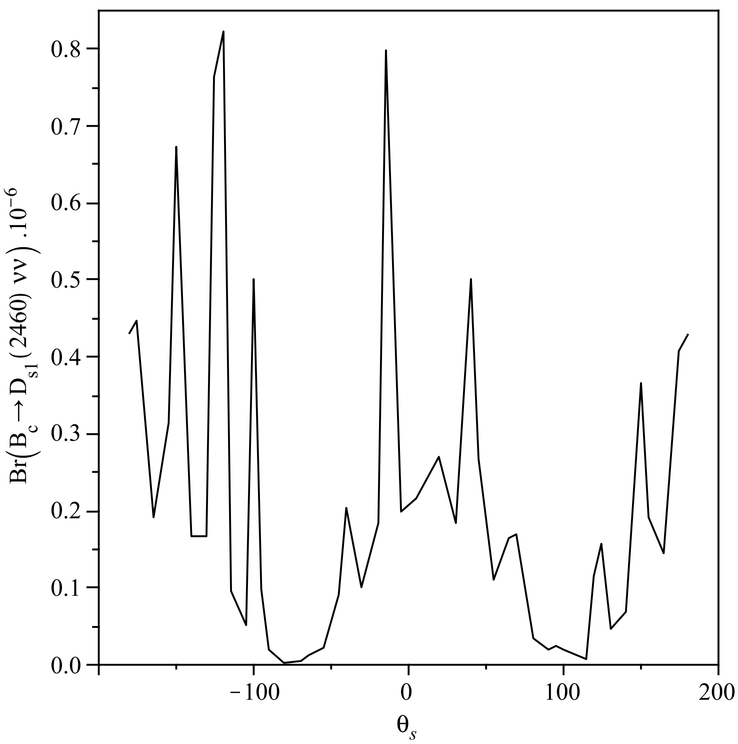

Figure 11: The same as Figs. 9, but for $B_c \to D_{s1}\nu\bar{\nu}$.