The Effects of Fourth Generation on the double Lepton Polarization in $B \to K\ell^+\ell^-$ decay

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Abstract

This study investigates the influence of the fourth generation quarks on the double lepton polarizations in the $B \to K\ell^+\ell^-$ decay. Taking $|V_{t's}V_{t'b}| \sim \{0.01 - 0.03\}$ with phase about $100°$, which is consistent with the $b \to s\ell^+\ell^-$ rate and the B_s mixing parameter Δm_{B_s}, we have found out that the double lepton(μ, τ) polarizations are quite sensitive to the existence of fourth generation. It can serve as an efficient tool to search for new physics effects, precisely, to indirect search for the fourth generation quarks(t', b').

PACS numbers: 12.60.–i, 13.30.–a, 14.20.Mr

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1 Introduction

Although the standard model (SM) of electroweak interaction has very successfully described all existing experimental data, it is believed that it is a low energy manifestation of a fundamental theory. Therefore, intensive search for physics beyond the SM is now being performed in various areas of particle physics. One possible extension is the SM with more than three generations.

Considering the recent experimental data, i.e., LEP II, two different interpretations already exist. The first one insists on the fact the fourth generation is ruled out by these experimental data. The second one claimes that the status of the fourth generation is subtle [1]. This approach illustrates that the experimental results make some constrains on the fourth generation parameters, i. e., masses(fourth neutrino mass has to be greater than the half of the Z boson mass) and mixing[2]. While an unstable neutrino with mass of 50 GeV is ruled out by LEP II bounds, the stable one may be ruled out by dark matter direct search experiments[2]. Many authors who support the existence of fourth generation studied those effects in various areas, for instance, Higgs and neutrino physics, cosmology and dark matter[3]–[7].

It is known that Democratic Mass Matrix approach [8], which is quite natural in the SM framework, favors the existence of the fourth SM family [9, 10]. The main restrictions on the new SM families come from the experimental data on the ρ and S parameters [10]. However, the common mass of the fourth quark ($m_{t'}$) lies between 320 GeV and 730 GeV with respect to the experimental value of $\rho = 1.0002^{+0.0007}_{-0.0004}$ [11]. The last value is close to the upper limit on heavy quark masses, $m_q \leq 700$ GeV $\approx 4m_t$, which follows from partial-wave unitarity at high energies [12]. Flavor–changing neutral current (FCNC) $b \rightarrow s(d)\ell^+\ell^-$ decays provide important tests for the gauge structure of the standard model (SM) at one–loop level. Moreover, $b \rightarrow s(d)\ell^+\ell^-$ decays are also very sensitive to the new physics beyond the SM. New physics effects manifest themselves in rare decays in two different ways, either through new combinations with the new Wilson coefficients or through the new operator structure in the effective Hamiltonian, which is absent in the SM. One of the efficient ways in establishing new physics beyond the SM is the measurement of the lepton polarization in the inclusive $b \rightarrow s(d)\ell^+\ell^-$ transition[13] and the exclusive $B \rightarrow K(K^*, \rho, \gamma)\ell^+\ell^-$ decays [14]–[22].

In this study, we investigate the possibility of searching for new physics in the double lepton polarization of the $B \rightarrow K\ell^+\ell^-$ using the SM with fourth generation of quarks(b', t'). The fourth quark (t'), like u, c, t quarks, contributes to the $b \rightarrow s(d)$ transition at the loop level. Note that the fourth generation effects have been widely studied in baryonic and semileptonic B decays [23]–[36].

The main problem in the description of exclusive decays is to evaluate the form factors, i.e., the matrix elements of the effective Hamiltonian between initial and final hadron states. It is obvious that in order to achieve the form factors the non pertubative QCD approach has to be used(see for example [15]).

The sensitivity of the branching ratio and various asymmetries to the existence of fourth generation quarks in the $B \rightarrow K^*\ell^+\ell^-$ decay[33] and $\Lambda_b \rightarrow \Lambda\ell^+\ell^-$ [35, 34]decay are investigated and found out that the branching ratio, lepton polarization and forward–backward asymmetry are all very sensitive to the fourth generation parameters ($m_{t'}, V_{t'b}V_{t's}^*$). Con-

sequently, it is natural to ask whether the double lepton polarizations are sensitive to the fourth generation parameters, in the $B \to K\ell^+\ell^-$ decays. In the present work, we try to answer this question.

The paper is organized as follows: In section 2, we try to include the fourth generation in the the effective Hamiltonian. In section 3, the general expressions for the longitudinal, transversal and normal polarizations of leptons are obtained. In section 4, we examine the sensitivity of these polarizations to the fourth generation parameters $(m_{t'},\ V_{t'b}V_{t's}^*\ )$.

2 The matrix element for the $B \to K\ell^+\ell^-$ decay in SM4

The matrix element of the $B \to K\ell^+\ell^-$ decay at the quark level is described by the $b \to s\ell^+\ell^-$ transition and the effective Hamiltonian at $O(\mu)$ scale can be written as

$$\mathcal{H}_{eff} = \frac{4G_F}{\sqrt{2}}V_{tb}V_{ts}^* \sum_{i=1}^{10} \mathcal{C}_i(\mu)\,\mathcal{O}_i(\mu)\ , \tag{1}$$

here the full set of the operators $\mathcal{O}_i(\mu)$ and the corresponding expressions for the Wilson coefficients $\mathcal{C}_i(\mu)$ in the SM are given in [37]. As has already been noted, the fourth generation is introduced in the same way as three generations in the SM, and so new operators do not appear clearly and the full operator set is exactly the same as in the SM. The fourth generation changes the values of the Wilson coefficients $C_7(\mu)$, $C_9(\mu)$ and $C_{10}(\mu)$, via virtual exchange of the fourth generation up type quark t'. The above mentioned Wilson coefficients are modified as:

$$C_7^{tot}(\mu) = C_7^{SM}(\mu) + \frac{\lambda_{t'}}{\lambda_t}C_7^{new}(\mu)\ ,$$

$$C_9^{tot}(\mu) = C_9^{SM}(\mu) + \frac{\lambda_{t'}}{\lambda_t}C_9^{new}(\mu)\ ,$$

$$C_{10}^{tot}(\mu) = C_{10}^{SM}(\mu) + \frac{\lambda_{t'}}{\lambda_t}C_{10}^{new}(\mu)\ , \tag{2}$$

where $\lambda_f = V_{fb}^*V_{fs}$ and the last terms in these expressions describe the contributions of the t' quark to the Wilson coefficients. $\lambda_{t'}$ can be parameterized as:

$$\lambda_{t'} = V_{t'b}^*V_{t's} = r_{sb}e^{i\phi_{sb}} \tag{3}$$

C_i's can also be re–written in the following form:

$$\lambda_t C_i \to \lambda_t C_i^{SM} + \lambda_{t'} C_i^{new}\ , \tag{4}$$

The unitarity of the 4×4 CKM matrix leads to

$$\lambda_u + \lambda_c + \lambda_t + \lambda_{t'} = 0. \tag{5}$$

One can $\lambda_u = V_{ub}^*V_{us}$ is very small compared with the others . Then, $\lambda_t \approx -\lambda_c - \lambda_{t'}$. And then

$$\lambda_t C_i^{SM} + \lambda_{t'} C_i^{new} = \lambda_c C_i^{SM} + \lambda_{t'}(C_i^{new} - C_i^{SM}) \tag{6}$$

It is clear that for the $m_{t'} \to m_t$ or $\lambda_{t'} \to 0$, $\lambda_{t'}(C_i^{new} - C_i^{SM})$ term vanishes, as required by the GIM mechanism.

In deriving Eq. (2), we factored out the term $V_{tb}^* V_{ts}$ in the effective Hamiltonian given in Eq. (1). The explicit forms of the C_i^{new} can be obtained from the corresponding expression of the Wilson coefficients in the SM by substituting $m_t \to m_{t'}$ (see [37, 38]). If the $\hat{s}$ quark mass is neglected, the above effective Hamiltonian leads to the following matrix element for the $b \to s \ell^+ \ell^-$ decay

$$
\mathcal{H}_{eff} = \frac{G_F \alpha_{em}}{2\sqrt{2}\pi} V_{tb} V_{ts}^* \left[C_9^{tot} \, \bar{s}\gamma_\mu (1 - \gamma_5) b \, \bar{\ell}\gamma_\mu \ell + C_{10}^{tot} \bar{s}\gamma_\mu (1 - \gamma_5) b \, \bar{\ell}\gamma_\mu \gamma_5 \ell \right.
$$
$$
\left. - 2 C_7^{tot} \frac{m_b}{q^2} \bar{s}\sigma_{\mu\nu} q^\nu (1 + \gamma_5) b \, \bar{\ell}\gamma_\mu \ell \right] , \tag{7}
$$

where $q^2 = (p_1 + p_2)^2$ and p_1 and p_2 are the final leptons four–momenta. The effective coefficient C_9^{tot} can be written in the following form

$$
C_9^{tot} = C_9 + Y(s) , \tag{8}
$$

where $s' = q^2/m_b^2$ and the function $Y(s')$ contains the contributions from the one loop matrix element of the four quark operators.

In addition to the short distance contributions, $Y_{per}(s')$ receives also long distance contributions, which have their origin in the real $c\bar{c}$ intermediate states, i.e., J/ψ, ψ', $\cdots$. In the present study we neglect the long distance contributions for the sake of simplicity.

Now, having the effective Hamiltonian, describing the $b \to s \ell^+ \ell^-$ decay at a scale $\mu \simeq m_B$, we can write down the matrix elements for the $B \to K \ell^+ \ell^-$ decay. The matrix element for this decay can be obtained by sandwiching the effective Hamiltonian between B and K meson states; which are parameterized in terms of form-factors which depend on the momentum transfer squared, $q^2 = (p_B - p_K)^2 = (p_+ - p_-)^2$. It follows from Eq.(7) that in order to calculate the amplitude of the $B \to K \ell^+ \ell^-$ decay the following matrix elements are required;

$$
\langle K \left| \bar{s}\gamma_\mu b \right| B \rangle, \langle K \left| \bar{s}i\sigma_{\mu\nu} q^\nu b \right| B \rangle, \langle K \left| \bar{s}b \right| B \rangle, \langle K \left| \bar{s}\sigma_{\mu\nu} b \right| B \rangle .
$$

These matrix elements are defined as follows [39, 40];

$$
\langle K(p_K) \left| \bar{s}\gamma_\mu b \right| B(p_B) \rangle = f_+ \left[(p_B + p_K)_\mu - \frac{m_B^2 - m_K^2}{q^2} q_\mu \right] + f_0 \frac{m_B^2 - m_K^2}{q^2} q_\mu, \tag{9}
$$

$$
\langle K(p_K) \left| \bar{s}\sigma_{\mu\nu} b \right| B(p_B) \rangle = -i \frac{f_T}{m_B + m_K} \left[(p_B + p_K)_\mu q_\nu - q_\mu (p_B + p_K)_\nu \right] . \tag{10}
$$

Note that the finiteness of Eq. (7) at $q^2 = 0$ is guaranteed by assuming that $f_+(0) = f_0(0)$.

The matrix elements $\langle K(p_K) \left| \bar{s}i\sigma_{\mu\nu} q^\nu b \right| B(p_B) \rangle$ and $\langle K \left| \bar{s}b \right| B \rangle$ can be derived from Eqs. (9) and (10) by multiplying both sides of these equations by q^μ and using the equations of motion, we get;

$$
\langle K(p_K) \left| \bar{s}b \right| B(p_B) \rangle = f_0 \frac{m_B^2 - m_K^2}{m_b - m_s}, \tag{11}
$$

$$
\langle K(p_K) \left| \bar{s}i\sigma_{\mu\nu} q^\nu b \right| B(p_B) \rangle = \frac{f_T}{m_B + m_K} \left[(p_B + p_K)_\mu q^2 - q_\mu (m_B^2 - m_K^2) \right]. \tag{12}
$$

As has already been mentioned, the form-factors entering Eqs.(9)-(12) represent the hadronization process. In order to calculate these form-factors information about the non-perturbative region of QCD is required. Therefore, for the estimation of the form-factors to be reliable, a nonperturbative approach is needed. Among the nonperturbative approaches, the QCD sum rule is more predictive in studying the properties of hadrons. The form-factors appearing in the $B \to K$ transition are computed in the framework of the light cone QCD sum rules[39, 40]. We will use the result of the work in [40] where radiative corrections to the leading twist wave functions and $SU(3)$ breaking effects are taken into account. As a result, the form-factors are parameterized in the following way [40];

$$f_i(q^2) = \frac{r_1}{1 - q^2/m_1^2} + \frac{r_2}{(1 - q^2/m_1^2)^2}, \tag{13}$$

where $1 = +$ or T, and

$$f_0(q^2) = \frac{r_2}{1 - q^2/m_{fit}^2}, \tag{14}$$

with $m_1 = 5.41 \text{GeV}$ and the other parameters as given in Table 1.

	r_1	r_2	m_{fit}^2
f_+	0.162	0.173	$--$
f_0	0.	0.33	37.46
f_T	0.161	0.198	$--$

Table 1: The parameters for the form-factors of the $B \to K$ transition are given in [40].

Using the definition of the form factors given in Eqs.(9)-(12), we arrive at the following matrix element for the $B \to K \ell^+ \ell^-$ decay;

$$\mathcal{M}(B \to K\ell^+\ell^-) = \frac{G_F \alpha_{em}}{4\sqrt{2}\pi} V_{tb} V_{ts}^* \left\{ \bar{\ell}\gamma^\mu\ell \left[A(p_B + p_K)_\mu + Bq_\mu \right] \right. \tag{15}$$
$$\left. + \bar{\ell}\gamma^\mu\gamma_5\ell \left[C(p_B + p_K)_\mu + Dq_\mu \right].$$

The functions entering Eq.(15) are defined as;

$$A = (C_{LL}^{tot} + C_{LR}^{tot})f_+ + 2(C_{BR}^{tot} + C_{SL}^{tot})\frac{f_T}{m_B + m_K},$$
$$B = (C_{LL}^{tot} + C_{LR}^{tot})f_- - 2(C_{BR}^{tot} + C_{SL}^{tot})\frac{f_T}{(m_B + m_K)q^2}(m_B^2 - m_K^2),$$
$$C = (C_{LR}^{tot} - C_{LL}^{tot})f_+,$$
$$D = (C_{LR}^{tot} - C_{LL}^{tot})f_- .$$

where

$$C_{LL}^{tot} = C_9^{tot} - C_{10}^{tot}, \quad C_{LR}^{tot} = C_9^{tot} + C_{10}^{tot} .$$
$$C_{SL}^{tot} = -2m_s C_7^{tot}, \quad C_{BR}^{tot} = -2m_b C_7^{tot}. \tag{16}$$

Considering Eq. (15), we get the following result for the dilepton invariant mass spectrum of decay rate:

$$\frac{d\Gamma}{d\hat{s}}(B \to K\ell^+\ell^-) = \frac{G^2\alpha^2 m_B}{2^{14}\pi^5}|V_{tb}V_{ts}^*|^2 \lambda^{1/2}(1,\hat{r}_K,\hat{s})v\Delta(\hat{s}) , \tag{17}$$

where $\lambda(1,\hat{r}_K,\hat{s}) = 1 + \hat{r}_K^2 + \hat{s}^2 - 2\hat{r}_K - 2\hat{s} - 2\hat{r}_K\hat{s}$, $\hat{s} = q^2/m_B^2$, $\hat{r}_K = m_K^2/m_B^2$, $\hat{m}_\ell = m_\ell/m_B$, $v = \sqrt{1 - 4\hat{m}_\ell^2/\hat{s}}$ is the final lepton velocity, and $\Delta(\hat{s})$ is

$$\begin{aligned}
\Delta(\hat{s}) &= \frac{4m_B^2}{3}Re[24m_B^2\hat{m}_l^2(1-\hat{r}_K)D^\star C + \lambda m_B^2(3-v^2)|A|^2 + 12m_B^2\hat{m}_l^2\hat{s}|D|^2 \\
&+ m_B^2|C|^2\{2\lambda - (1-v^2)(2\lambda - 3(1-\hat{r}_K)^2)\}]
\end{aligned} \tag{18}$$

3 Double-Lepton Polarization

In this section, we will calculate the double–polarization asymmetries, i.e., when polarizations of both leptons have to be simultaneously measured. One can introduce a spin projection operator as follows:

$$\Lambda_1 = \frac{1}{2}(1 + \gamma_5 \not{s}_i^-) ,$$

$$\Lambda_2 = \frac{1}{2}(1 + \gamma_5 \not{s}_i^+) ,$$

for lepton ℓ^- and antilepton ℓ^+, where $i = L, N, T$ correspond to the longitudinal, normal and transversal polarizations, respectively. Firstly, we must define the orthogonal vectors s in the rest frame of leptons(where its vector is the polarization vector of the lepton):

$$s_L^{-\mu} = \left(0, \vec{e}_L^-\right) = \left(0, \frac{\vec{p}_-}{|\vec{p}_-|}\right) ,$$

$$s_N^{-\mu} = \left(0, \vec{e}_N^-\right) = \left(0, \frac{\vec{p}_K \times \vec{p}_-}{|\vec{p}_K \times \vec{p}_-|}\right) ,$$

$$s_T^{-\mu} = \left(0, \vec{e}_T^-\right) = \left(0, \vec{e}_N^- \times \vec{e}_L^-\right) ,$$

$$s_L^{+\mu} = \left(0, \vec{e}_L^+\right) = \left(0, \frac{\vec{p}_+}{|\vec{p}_+|}\right) ,$$

$$s_N^{+\mu} = \left(0, \vec{e}_N^+\right) = \left(0, \frac{\vec{p}_K \times \vec{p}_+}{|\vec{p}_K \times \vec{p}_+|}\right) ,$$

$$s_T^{+\mu} = \left(0, \vec{e}_T^+\right) = \left(0, \vec{e}_N^+ \times \vec{e}_L^+\right) , \tag{19}$$

here $\vec{p}_\mp$ and $\vec{p}_K$ are the three–momenta of the leptons $\ell^\mp$ and K meson in the center of mass frame (CM) of $\ell^-\ell^+$ system, respectively.

The longitudinal unit vectors are boosted to the CM frame of $\ell^-\ell^+$ by Lorenz transformation:

$$\left(s_L^{-\mu}\right)_{CM} = \left(\frac{|\vec{p}_-|}{m_\ell}, \frac{E\vec{p}_-}{m_\ell|\vec{p}_-|}\right) ,$$

$$\left(s_L^{+\mu}\right)_{CM} = \left(\frac{|\vec{p}_-|}{m_\ell}, -\frac{E\vec{p}_-}{m_\ell|\vec{p}_-|}\right) , \tag{20}$$

while the other two vectors remain unchanged.

We can now define the double–lepton polarization asymmetries as in [41]:

$$
P_{ij}(\hat{s}) = \frac{\left(\frac{d\Gamma}{d\hat{s}}(\vec{s_i}^{\,-}, \vec{s_j}^{\,+}) - \frac{d\Gamma}{d\hat{s}}(-\vec{s_i}^{\,-}, \vec{s_j}^{\,+})\right) - \left(\frac{d\Gamma}{d\hat{s}}(\vec{s_i}^{\,-}, -\vec{s_j}^{\,+}) - \frac{d\Gamma}{d\hat{s}}(-\vec{s_i}^{\,-}, -\vec{s_j}^{\,+})\right)}{\left(\frac{d\Gamma}{d\hat{s}}(\vec{s_i}^{\,-}, \vec{s_j}^{\,+}) + \frac{d\Gamma}{d\hat{s}}(-\vec{s_i}^{\,-}, \vec{s_j}^{\,+})\right) + \left(\frac{d\Gamma}{d\hat{s}}(\vec{s_i}^{\,-}, -\vec{s_j}^{\,+}) + \frac{d\Gamma}{d\hat{s}}(-\vec{s_i}^{\,-}, -\vec{s_j}^{\,+})\right)} \ , \tag{21}
$$

where $i, j = L, \ N, \ T$, and the first subindex i corresponds to lepton while the second subindex j corresponds to antilepton, respectively.

Now, regarding the aforementioned definitions, after doing the straight forward calculations we obtain the following results for $P_{ij}(\hat{s})$:

$$
\begin{align}
P_{LL} &= \frac{-4m_B^2}{3\Delta} Re[-24m_B^2\hat{m}_l^2(1-\hat{r}_K)C^\star D + \lambda m_B^2(1+v^2)|A|^2 \tag{22} \\
&\quad - 12m_B^2\hat{m}_l^2\hat{s}|D|^2 + m_B^2|C|^2(2\lambda - (1-v^2)(2\lambda + 3(1-\hat{r}_K)^2))], \tag{23} \\
P_{LN} &= \frac{-4\pi m_B^3\sqrt{\lambda\hat{s}}}{\hat{s}\Delta} Im[-m_B\hat{m}_l\hat{s}A^\star D - m_B\hat{m}_l(1-\hat{r}_K)A^\star C], \tag{24} \\
P_{NL} &= -P_{LN}, \tag{25} \\
P_{LT} &= \frac{4\pi m_B^3\sqrt{\lambda\hat{s}}}{\hat{s}\Delta} Re[m_B\hat{m}_lv(1-\hat{r}_K)|C|^2 + m_B\hat{m}_lv\hat{s}C^\star D], \tag{26} \\
P_{TL} &= P_{LT}, \tag{27} \\
P_{NT} &= -\frac{8m_B^2v}{3\Delta} Im[2\lambda m_B^2 A^\star C], \tag{28} \\
P_{TN} &= -P_{NT}, \tag{29} \\
P_{TT} &= \frac{4m_B^2}{3\Delta} Re[-24m_B^2\hat{m}_l^2(1-\hat{r}_K)C^\star D - \lambda m_B^2(1+v^2)|A|^2 - 12m_B^2\hat{m}_l^2\hat{s}|D|^2 \\
&\quad + m_B^2|C|^2\{2\lambda - (1-v^2)(2\lambda + 3(1-\hat{r}_K)^2)\}], \tag{30} \\
P_{NN} &= \frac{4m_B^2}{3\Delta} Re[24m_B^2\hat{m}_l^2(1-\hat{r}_K)C^\star D - \lambda m_B^2(3-v^2)|A|^2 + 12m_B^2\hat{m}_l^2\hat{s}|D|^2 \\
&\quad + m_B^2|C|^2\{2\lambda - (1-v^2)(2\lambda - 3(1-\hat{r}_K)^2)\}] \tag{31}
\end{align}
$$

4 Numerical analysis

In this section, we will analyze the dependence of the double–lepton polarizations on the fourth quark mass($m_{t'}$) and the product of quark mixing matrix elements ($V_{t'b}^* V_{t's} = r_{sb}e^{i\phi_{sb}}$). The challenging input parameters in the calculations are the form factors, which are related to the non–pertubative part of QCD. We will use the result of the study in [40] where radiative corrections to the leading twist wave functions and $SU(3)$ breaking effects are taken into account.

We use the next–to–leading order logarithmic approximation for the resulting values of the Wilson coefficients C_9^{eff}, C_7 and C_{10} in the SM [42, 43] at the re–normalization point $\mu = m_b$. It should be noted that in addition to the short distance contribution, C_9^{eff}

r_{sb}	0.005	0.01	0.02	0.03
$m_{t'}(GeV)$	739	529	385	331

Table 2: The experimental limit of $m_{t'}$ for $\phi_{sb} = \pi/3$[34]

r_{sb}	0.005	0.01	0.02	0.03
$m_{t'}(GeV)$	511	373	289	253

Table 3: The experimental limit of $m_{t'}$ for $\phi_{sb} = \pi/2$[34]

receives long distance contributions from the real $\bar{c}c$ resonant states of the J/ψ family. In the present research, we do not take the long distance effects into account.

The input parameters used in this analysis are as follows:
$|V_{tb}V_{ts}^*| = 0.0385$, $(C_9^{eff})^{sh} = 4.344$, $C_{10} = -4.669$, $\Gamma_B = 4.22 \times 10^{-13}$ GeV. In order to perform quantitative analysis of the double–lepton polarizations, the values of the new parameters($m_{t'}$, r_{sb}, ϕ_{sb}) are needed. Using the experimental values of $B \to X_s\gamma$ and $B \to X_s\ell^+\ell^-$, the restriction on $r_{sb} \sim \{0.01 - 0.03\}$ has been obtained [27, 34] for $\phi_{sb} \sim \{0 - 2\pi\}$ and $m_{t'} \sim \{200, 600\}$ (GeV)(see table 2). Considering the B_s mixing, which is in terms of the Δm_{B_s}, ϕ_{sb} is sharply restricted ($\phi_{sb} \sim \pi/2$) [23].

Before performing numerical analyses, we would like to add a few words about lepton polarizations. From explicit expressions of the lepton polarizations one can easily see that they depend on both $\hat{s}$ and the new parameters($m_{t'}$, r_{sb}). Therefore, it may experimentally be difficult to study these dependencies at the same time. For this reason, we eliminate the q^2 dependence by performing integration over $\hat{s}$ in the allowed region, i.e., we consider the averaged double–lepton polarization asymmetries. The average gained, here, over $\hat{s}$ is defined as:

$$\langle P_{ij}\rangle = \frac{\int_{4\hat{m}_\ell^2}^{(1-\sqrt{\hat{r}_K})^2} P_{ij}\frac{d\mathcal{B}}{d\hat{s}}d\hat{s}}{\int_{4\hat{m}_\ell^2}^{(1-\sqrt{\hat{r}_K})^2} \frac{d\mathcal{B}}{d\hat{s}}d\hat{s}} .$$

Our quantitative analyses indicate that some of the $\langle P_{ij}\rangle$ are less sensitive to the fourth generation parameters; i.e, the maximum deviation from the SM3 are $\sim 5\%$. We do not present those dependencies on fourth generation parameters with relevant figures. We present our analysis for strongly dependent functions in a series of figures where the black

r_{sb}	0.005	0.01	0.02	0.03
$m_{t'}(GeV)$	361	283	235	217

Table 4: The experimental limit of $m_{t'}$ for $\phi_{sb} = 2\pi/3$[34]

"DOTS" in figures show the experimental limit on $m_{t'}$, considering the 1σ level deviation from the measured branching ratio of $B \to X_s \ell^- \ell^+$(see Table 2,3,4). From these figures, we deduce the following results:

- Taking the fourth generation into account , the value of $\langle P_{LL} \rangle$ shows weak dependency for μ channel and alters τ channel at most about 20% compared to the SM3 the prediction, while it increases for , $\phi_{sb} = 90°$, 120°. However, it both increases and decreases for the $\phi_{sb} = 60°$ for both channels.

- In the SM3 the non–zero values of $\langle P_{LN} \rangle$ and $\langle P_{NT} \rangle$, as well as $\langle P_{NL} \rangle$ and $\langle P_{TN} \rangle$, have their origin in the higher order QCD corrections to the C_9^{eff}. Since these functions are proportional to the lepton mass and imaginary part of the C_9^{eff}, results of both are negligible. However, they seem to exceed the SM value sizeably. This is because of the new weak phase and new contribution to the Wilson coefficients coming out of the fourth generation. Furthermore, for fixed values of r_{sb}, their magnitude decreases by increasing the ϕ_{sb} in the the experimentally allowed region.

- Regarding the fourth generation, the value of $\langle P_{NN} \rangle$ changes 3–4 times for μ channel and at most about 25% for τ channel compared with the SM3 prediction, while it increases for , $\phi_{sb} = 90°$, 120°. But, it is both increases and decreases for the $\phi_{sb} = 60°$ in both channels.

- The situation for $\langle P_{TT} \rangle$ is similar to the $\langle P_{NN} \rangle$, if the μ channel is considered . But τ channel depicts weak dependence on the fourth generation parameters(at most $\sim 5\%$ deviation from the SM3 predictions).

- The value of $\langle P_{LT} \rangle$ changes about two times for μ channel and at most about 5% for τ channel compared with the SM3 prediction, while it increases for , $\phi_{sb} = 90°$, 120°. But, it both increases and decreases for the $\phi_{sb} = 60°$ in both channels.

Finally, let us briefly discuss whether it is possible to measure the lepton polarization asymmetries in experiments or not. A required number of the events (i.e., the number of $B\bar{B}$ pair) in terms of the branching ratio $\mathcal{B}$ at $n\sigma$ level, $\langle P_{ij} \rangle$ and the efficiencies of the leptons s_1 and s_2 are given by the expression

$$N = \frac{n^2}{\mathcal{B} s_1 s_2 \langle P_{ij} \rangle^2} ,$$

Typical values of the efficiencies of the τ–leptons range from 50% to 90% for their various decay modes[44]. It should be noted, here, that the error in τ–lepton polarization is estimated to be about $(10 \div 15)\%$ [45]. So, the error in measurement of the τ–lepton asymmetries is approximately $(20 \div 30)\%$, and the error in obtaining the number of events is about 50%.

Looking at the expression of N, it can be understood that in order to detect the lepton polarization asymmetries in the μ and τ channels at 3σ level, the minimum number of required events are (for the efficiency of τ–lepton we take 0.5):

- for $B \to K\mu^+\mu^-$ decay

$$N = \begin{cases} 3.5 \times 10^7 & \text{(for } \langle P_{LL} \rangle, \langle P_{LT} \rangle) \text{ ,} \\ 5.0 \times 10^8 & \text{(for } \langle P_{TL} \rangle) \text{ ,} \\ 2.0 \times 10^{11} & \text{(for } \langle P_{LN} \rangle) \text{ ,} \end{cases}$$

- for $B \to K\tau^+\tau^-$ decay

$$N = \begin{cases} (1.0 \pm 0.5) \times 10^9 & \text{(for } \langle P_{LL} \rangle, \langle P_{LT} \rangle, \langle P_{TL} \rangle, \langle P_{NN} \rangle) \text{ ,} \\ (5.0 \pm 2.5) \times 10^8 & \text{(for } \langle P_{TT} \rangle) \text{ ,} \\ (4.0 \pm 2.0) \times 10^{10} & \text{(for } \langle P_{LN} \rangle, \langle P_{NL} \rangle) \text{ ,} \\ (3.0 \pm 1.5) \times 10^{11} & \text{(for } \langle P_{NT} \rangle, \langle P_{TN} \rangle) \text{ .} \end{cases}$$

On the other hand, the number of $B\bar{B}$ pairs, that are produced at LHC are about $\sim 10^{12}$. As a result of the comparison of these numbers and N, we conclude that except $\langle P_{LN} \rangle$ in the $B \to K\mu^+\mu^-$ decay and $\langle P_{NT} \rangle$, $\langle P_{TN} \rangle$ in the $B \to K\tau^+\tau^-$ decay, all double lepton polarizations can be detectable at LHC. The numbers for the $B \to K\mu^+\mu^-$ decay presented above demonstrate that $\langle P_{LL} \rangle$ and $\langle P_{LT} \rangle$ for the $B \to K\mu^+\mu^-$ decay might be accessible to B factories after several years of running.

To sum up, in this study we present the most general analyses of the double–lepton polarization asymmetries in the $B \to K\ell^+\ell^-$ decay using the SM with the fourth generation of quarks. In our analyses, we have used the experimental results of the branching ratio for the $B \to X_s\mu^+\mu^-$ decay and B_s mixing to control the fourth generation parameters. We have found out that some of the double–lepton polarization functions which are already accessible to LHC depict the strong dependency on the fourth generation quark mass and product of quark mixing. The study of such strong dependent double–lepton polarization asymmetries can serve as a good test for the predictions of the SM and for the indirect search for the fourth generation up type quarks t'.

5   Acknowledgment

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Figure captions

Fig. (1) The dependence of the $\langle P_{LL} \rangle$ for the $B \to K\tau^+\tau^-$ decay on the fourth generation quark mass $m_{t'}$ for three different values of $\phi_{sb} = \{60°,\ 90°,\ 120°\}$ and $r_{sb} = \{0.01,\ 0.02,\ 0.03\}$.

Fig. (2) The dependence of the $\langle P_{LN} \rangle$ on the fourth generation quark mass $m_{t'}$ for three different values of $\phi_{sb} \sim \{60°,\ 90°,\ 120°\}$ and $r_{sb} = \{0.01,\ 0.02,\ 0.03\}$ for μ lepton.

Fig. (3) The same as in Fig. (2), but for the τ lepton.

Fig. (4) The dependence of the $\langle P_{NT} \rangle$ on the fourth generation quark mass $m_{t'}$ for three different values of $\phi_{sb} \sim \{60°,\ 90°,\ 120°\}$ and $r_{sb} = \{0.01,\ 0.02,\ 0.03\}$ for μ lepton.

Fig. (5) The same as in Fig. (4), but for the τ lepton.

Fig. (6) The dependence of the $\langle P_{LT} \rangle$ on the fourth generation quark mass $m_{t'}$ for three different values of $\phi_{sb} \sim \{60°,\ 90°,\ 120°\}$ and $r_{sb} = \{0.01,\ 0.02,\ 0.03\}$ for μ lepton.

Fig. (7) The dependence of the $\langle P_{NN} \rangle$ on the fourth generation quark mass $m_{t'}$ for three different values of $\phi_{sb} \sim \{60°,\ 90°,\ 120°\}$ and $r_{sb} = \{0.01,\ 0.02,\ 0.03\}$ for μ lepton.

Fig. (8) The same as in Fig. (7), but for the τ lepton.

Fig. (9) The dependence of the $\langle P_{TT} \rangle$ on the fourth generation quark mass $m_{t'}$ for three different values of $\phi_{sb} \sim \{60°,\ 90°,\ 120°\}$ and $r_{sb} = \{0.01,\ 0.02,\ 0.03\}$ for μ lepton.

Figure 1:

Figure 2:

Figure 3:

Figure 4:

Figure 5:

Figure 6:

Figure 7:

Figure 8:

Figure 9:

Legend:
- SM
- $r_{sb}=0.01, \phi_{sb}=60$
- $r_{sb}=0.01, \phi_{sb}=90$
- $r_{sb}=0.01, \phi_{sb}=120$
- $r_{sb}=0.02, \phi_{sb}=60$
- $r_{sb}=0.02, \phi_{sb}=90$
- $r_{sb}=0.02, \phi_{sb}=120$
- $r_{sb}=0.03, \phi_{sb}=60$
- $r_{sb}=0.03, \phi_{sb}=90$
- $r_{sb}=0.03, \phi_{sb}=120$

Y-axis: $\langle P_{NT} \rangle (B \rightarrow K\tau^+\tau^-) \times 10^3$

X-axis: $m_{t'}$ (Gev)

Figure axes:

Y-axis: $\langle P_{LT} \rangle (B \to K \mu^+ \mu^+) \times 10^3$

X-axis: $m_{t'}$ (Gev)

Legend:
- SM
- $r_{sb}=0.01, \phi_{sb}=60$
- $r_{sb}=0.01, \phi_{sb}=90$
- $r_{sb}=0.01, \phi_{sb}=120$
- $r_{sb}=0.02, \phi_{sb}=60$
- $r_{sb}=0.02, \phi_{sb}=90$
- $r_{sb}=0.02, \phi_{sb}=120$
- $r_{sb}=0.03, \phi_{sb}=60$
- $r_{sb}=0.03, \phi_{sb}=90$
- $r_{sb}=0.03, \phi_{sb}=120$

Legend:
SM
$r_{sb}=0.01,\phi_{sb}=60$
$r_{sb}=0.01,\phi_{sb}=90$
$r_{sb}=0.01,\phi_{sb}=120$
$r_{sb}=0.02,\phi_{sb}=60$
$r_{sb}=0.02,\phi_{sb}=90$
$r_{sb}=0.02,\phi_{sb}=120$
$r_{sb}=0.03,\phi_{sb}=60$
$r_{sb}=0.03,\phi_{sb}=90$
$r_{sb}=0.03,\phi_{sb}=120$

Y-axis: $\langle P_{TT}\rangle(B\rightarrow K\mu^{+}\mu^{-})\times10^{3}$

X-axis: $m_{t'}$ (Gev)