ENERGY ASSOCIATED WITH THE GIBBONS-MAEDA DILATON SPACE-TIME

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In order to obtain energy and momentum (due to matter and fields including gravitation) distributions of the Gibbons-Maeda dilaton spacetime, we use the Møller energy and/or momentum prescription both in Einstein's theory of general relativity and teleparallel gravity. We find the same energy distribution for a given metric in both of these different gravitation theories. Under two limits, we also calculate energy associated with two other models such as the Garfinkle-Horowitz-Strominger dilaton spacetime and the Reissner-Nordstrom spacetime. The energy obtained is also independent of the teleparallel dimensionless coupling constant, which means that it is valid in any teleparallel model. Our result also sustains (a) the importance of the energy-momentum definitions in the evaluation of the energy distribution for a given spacetime and (b) the viewpoint of Lessner that the Møller energy-momentum complex is a powerful concept of energy and momentum (c) the hypothesis of Vagenas that there is a connection between the coefficients of the energy-momentum expression of Einstein and those of Moller.

Keywords: Møller energy; black holes; general relativity; teleparallel gravity.

1. INTRODUCTION

Energy and/or momentum prescriptions introduced first by Einstein [1], were the foremost endeavor to solve the problem of energy localization. After that a large number of formulations of the gravitational energy, momentum and angular momentum have been proposed. Some of them are coordinate independent and others are coordinate-dependent. There lies a dispute on the importance of non-tensorial energy-momentum complexes whose physical interpretations have been questioned by a number of physicists, including Weyl, Pauli and Eddington. Also, there exists an opinion that the energy-momentum pseudo-tensors are not useful to find meaningful results in a given geometry. Chang, Nester, Chen [2] obtained that there exists a direct relationship between quasilocal and pseudotensor expressions; since every energy-momentum pseudotensor is associated with a legitimate Hamiltonian boundary term. Ever since the Einstein's energy-momentum complex was used for calculating energy and momentum in a general relativistic system, many attempts have been made to evaluate the energy distribution for a given space-time [3]. Except for the Møller definition these formulations only give meaningful results if the calculations are performed in *Cartesian* coordinates. Møller proposed a new expression for energy-momentum complex which could be utilized to any coordinate system. Next, Lessner [4] argued that the Møller prescription is a powerful concept for energy-momentum in general relativity.

Virbhadra [5], using the energy and momentum complexes of Einstein, Landau-Lifshitz, Papapetrou and Weinberg for a general non-static spherically symmetric metric of the Kerr-Schild class, showed that all of these energy-momentum formulations give the same energy distribution as in the Penrose energy-momentum formulation. In literature there are several papers on the calculation of the enegy-momentum distribution of the universe by using energy-momentum complexes [6].

Recently, the problem of energy-momentum localization has also been considered in teleparallel gravity [7]. Møller showed that a tetrad description of a gravitational field equation allows a more satisfactory treatment of the energy-momentum complex than does general relativity. Therefore, we have also applied the super-potential method by Mikhail *et. al.* [8] to calculate the energy of the central gravitating body. In Gen. Relat. Gravit. 36, 1255(2004); Vargas, using the definitions of Einstein and Landau-Lifshitz in teleparallel gravity, found that the total energy is zero in Friedmann-Robertson-Walker space-times. There are also new papers on the energy-momentum problem in teleparallel gravity. The authors obtained the same energy-momentum for different formulations in teleparallel gravity [9, 10, 11, 12].

In his new paper, Vagenas hypothesized [13] that there is a connection between the coefficients of the expression for the energy (when the energy and/or momentum complex of Einstein is employed) of the form

$$E(r) = \sum_{n=0}^{+\infty} \alpha_n^{(Einstein)} r^{-n} \tag{1}$$

and those of the expression for the energy (when the energy and/or momentum complex of Møller is employed) of the form

$$E(r) = \sum_{n=0}^{+\infty} \alpha_n^{(M\o ller)} r^{-n}. \tag{2}$$

The relation that materializes this connection between the aforementioned coefficients is given by

$$\alpha_n^{(Einstein)} = \frac{1}{n+1}\alpha_n^{(M\o ller)}. \tag{3}$$

Here, we will also check whether this hypothesis is true or not for our problem.

The paper is organized as follows. In the next section, we introduce the Gibbons-Maeda dilaton spacetime. Next, in section 3, we give the energy-momentum definitions of Møller both in Einstein's theory of general relativity and the teleparallel gravity and Einstein complex in general relativity. Section 4 gives the calculations for the energy distribution associated with a given metric. Finally, section 5 is devoted to summarize and conclusions.

Notations and conventions: $c = h = 1$, metric signature $(+, -, -, -)$, Greek indices run from 0 to 3 and, Latin ones from 1 to 3. Throughout this paper, Latin indices (i, j, ...) number the vectors, and Greek indices (μ, ν,...) represent the vector components.

2. THE GIBBONS-MAEDA DILATON SPACETIME

The metric for the Gibbons-Maeda dilaton spacetime [14, 15] is

$$ds^2 = \frac{(r-r_+)(r-r_-)}{r^2-D^2}dt^2 - \frac{r^2-D^2}{(r-r_+)(r-r_-)}dr^2 - (r^2-D^2)(d\theta^2 + \sin^2\theta d\phi^2) \tag{4}$$

where $D = \frac{P^2-Q^2}{2M}$ and $r_\mp = M \mp \sqrt{M^2+D^2-P^2-Q^2}$. The parameters P and Q represent the black hole magnetic and electric charges, respectively. When $P = 0$, the metric returns the Garfinkle-Horowitz-Strominger dilaton spacetime which is given by the line-element

$$\begin{aligned} ds^2 = &\frac{(r-r_+)(r-r_-)}{r^2-(\frac{Q^2}{2M})^2}dt^2 - \frac{r^2-(\frac{Q^2}{2M})^2}{(r-r_+)(r-r_-)}dr^2 \\ &-(r^2-(\frac{Q^2}{2M})^2)(d\theta^2+\sin^2\theta d\phi^2) \end{aligned} \tag{5}$$

and, if $D = 0$, then the metric transforms into the Reissner-Nordstrom spacetime

$$ds^2 = \frac{(r-r_+)(r-r_-)}{r^2}dt^2 - \frac{r^2}{(r-r_+)(r-r_-)}dr^2 - r^2(d\theta^2+\sin^2\theta d\phi^2). \tag{6}$$

For the metric describing the Gibbons-Maeda dilaton spacetime, the non-vanishing components of the Einstein tensor $G_{\mu\nu}$ ($\equiv 8\pi T_{\mu\nu}$, where $T_{\mu\nu}$ is the energy-momentum tensor for the matter field described by a perfect fluid of density ρ and pressure p) are

$$G_{11} = \frac{(D^2-rr_+)(D^2-rr_-)}{(r_+-r)(r-r_-)(D^2-r^2)^2} \tag{7}$$

$$G_{22} = \frac{(D^2-rr_+)(D^2-rr_-)}{(D^2-r^2)^2} \tag{8}$$

$$G_{33} = \frac{(D^2-rr_+)(D^2-rr_-)\sin^2\theta}{(D^2-r^2)^2} \tag{9}$$

$$G_{00} = \frac{(r_+ - r)(r_- - r)}{(D^2 - r^2)^4}\left[D^2(D^2 + 2r^2 - 32r_-) + r_+(-3rD^2 + 2r_-D^2 + r^2 r_-)\right]. \quad (10)$$

The general form of the tetrad, h_i^μ, having spherical symmetry was given by Robertson [16]. In the Cartesian form it can be written as

$$h_0{}^0 = i\Xi, \qquad h_a{}^0 = \Sigma x^a, \qquad h_0{}^\alpha = i\Delta x^\alpha,$$
$$h_a{}^\alpha = \Upsilon \delta_a^\alpha + \Gamma x^a x^\alpha + \epsilon_{a\alpha\beta}\Re x^\beta \quad (11)$$

where $\Xi, \Upsilon, \Sigma, \Delta, \Gamma$, and $\Re$ are functions of t and $r = \sqrt{x^a x^a}$, and the zeroth vector h_0^μ has the factor $i^2 = -1$ to preserve Lorentz signature and the tetrad of Minkowski space-time is $h_a^\mu = \mathtt{diag}(i, \delta_a^\alpha)$ where (a=1,2,3).

Using the general coordinate transformation

$$h_{a\mu} = \frac{\partial \mathbf{X}^{\nu'}}{\partial \mathbf{X}^\mu} h_{a\nu} \quad (12)$$

where $\{\mathbf{X}^\mu\}$ and $\left\{\mathbf{X}^{\nu'}\right\}$ are, respectively, the isotropic and Schwarzschild coordinates (t, r, θ, ϕ). In the spherical, static and isotropic coordinate system $\mathbf{X}^1 = r\sin\theta\cos\phi$, $\mathbf{X}^2 = r\sin\theta\sin\phi$, $\mathbf{X}^3 = r\cos\theta$. We obtain the tetrad components of $h_a{}^\mu$ as

$$\begin{pmatrix} \frac{i\sqrt{r^2-D^2}}{\sqrt{(r-r_+)(r-r_-)}} & 0 & 0 & 0 \\ 0 & \sqrt{\frac{(r-r_+)(r-r_-)}{r^2-D^2}}s\theta c\phi & \frac{1}{\sqrt{r^2-D^2}}c\theta c\phi & -\frac{s\phi}{\sqrt{r^2-D^2}s\theta} \\ 0 & \sqrt{\frac{(r-r_+)(r-r_-)}{r^2-D^2}}s\theta s\phi & \frac{1}{\sqrt{r^2-D^2}}c\theta s\phi & \frac{c\phi}{\sqrt{r^2-D^2}s\theta} \\ 0 & \sqrt{\frac{(r-r_+)(r-r_-)}{r^2-D^2}}c\theta & -\frac{1}{\sqrt{r^2-D^2}}s\theta & 0 \end{pmatrix} \quad (13)$$

where $i^2 = -1$. Here, we have introduced the following notation: $s\theta = \sin\theta$, $c\theta = \cos\theta$, $s\phi = \sin\phi$ and $c\phi = \cos\phi$. For the Gibbons-Maeda dilaton spacetime, $g_{\mu\nu}$ is defined by

$$\begin{pmatrix} \frac{(r-r_+)(r-r_-)}{r^2-D^2} & 0 & 0 & 0 \\ 0 & \frac{D^2-r^2}{(r-r_+)(r-r_-)} & 0 & 0 \\ 0 & 0 & D^2-r^2 & 0 \\ 0 & 0 & 0 & (D^2-r^2)\sin^2\theta \end{pmatrix} \quad (14)$$

and its inverse $g^{\mu\nu}$

$$\begin{pmatrix} \frac{D^2-r^2}{(r-r_+)(r-r_-)} & 0 & 0 & 0 \\ 0 & \frac{(r-r_+)(r-r_-)}{r^2-D^2} & 0 & 0 \\ 0 & 0 & \frac{1}{D^2-r^2} & 0 \\ 0 & 0 & 0 & \frac{1}{(D^2-r^2)\sin^2\theta} \end{pmatrix}. \quad (15)$$

3. GRAVITATIONAL ENERGY AND/OR MOMENTUM DEFINITIONS TO BE USED

3.1. In general relativity

3.1.1. The energy and/or momentum complex of Møller In general relativity, it is given by [3]

$$M_\mu^\nu = \frac{1}{8\pi}\chi_{\mu,\alpha}^{\nu\alpha} \quad (16)$$

satisfying the local conservation laws:

$$\frac{\partial M^{\nu}_{\mu}}{\partial x^{\nu}} = 0 \tag{17}$$

where the antisymmetric super-potential $\chi^{\nu\alpha}_{\mu}$ is

$$\chi^{\nu\alpha}_{\mu} = \sqrt{-g}[g_{\mu\beta,\gamma} - g_{\mu\gamma,\beta}]g^{\nu\gamma}g^{\alpha\beta}. \tag{18}$$

The locally conserved energy-momentum complex M^{ν}_{μ} contains contributions from the matter, non-gravitational and gravitational fields. M^{0}_{0} is the energy density and M^{0}_{a} are the momentum density components. The momentum four-vector definition of Møller is given by

$$P_{\mu} = \int\int\int M^{0}_{\mu} dx dy dz. \tag{19}$$

Using Gauss's theorem, this definition transforms into

$$P_{\mu} = \frac{1}{8\pi}\int\int \chi^{0a}_{\mu}\mu_{\alpha} dS \tag{20}$$

where μ_a (where $a = 1, 2, 3$) is the outward unit normal vector over the infinitesimal surface element dS. P_i give momentum components P_1, P_2, P_3 and P_0 gives the energy.

3.1.2. Einstein's energy and/or momentum prescription The formulation [1, 3] is defined as

$$\Theta^{\nu}_{\mu} = \frac{1}{16\pi} H^{\nu\alpha}_{\mu,\alpha} \tag{21}$$

where

$$H^{\nu\alpha}_{\mu} = \frac{g_{\mu\beta}}{\sqrt{-g}}\left[-g(g^{\nu\beta}g^{\alpha\xi} - g^{\alpha\beta}g^{\nu\xi})\right]_{,\xi} \tag{22}$$

Θ^{0}_{0} is the energy density, Θ^{0}_{a} are the momentum density components, and Θ^{a}_{0} are the components of energy-current density. The Einstein energy and momentum density satisfies the local conservation laws

$$\frac{\partial \Theta^{\nu}_{\mu}}{\partial x^{\nu}} = 0 \tag{23}$$

and the energy-momentum components are given by

$$P_{\mu} = \int\int\int \Theta^{0}_{\mu} dx dy dz. \tag{24}$$

P_{μ} is called the momentum four-vector, P_a give momentum components P_1, P_2, P_3 and P_0 gives the energy.

3.2. In teleparallel Gravity

The teleparallel theory of gravity (the tetrad theory of gravitation) is an alternative approach to gravitation and corresponds to a gauge theory for the translation group based on Weitzenböck geometry [17]. In the theory of teleparallel gravity, gravitation is attributed to torsion [18], which plays the role of a force [19], and the curvature tensor vanishes identically. The essential field is acted by a nontrivial tetrad field, which gives rise to the metric as a by-product. The translational gauge potentials appear as the nontrivial item of the tetrad field, so induces on space-time a teleparallel structure which is directly related to the presence of the gravitational field. The interesting place of teleparallel theory is that, due to its gauge structure, it can reveal a more appropriate approach to consider some specific problems. This is the case, for example, in the energy and momentum problem, which becomes more transparent.

Møller modified general relativity by constructing a new field theory in teleparallel space. The aim of this theory was to overcome the problem of the energy-momentum complex that appears in Riemannian space [20]. The field equations in this new theory were derived from a Lagrangian which is not invariant under local tetrad rotation. Saez [21] generalized Møller theory into a scalar tetrad theory of gravitation. Meyer [22] showed that Møller theory is a special case of Poincare gauge theory [23, 24].

In teleparallel gravity, the super-potential of Møller is given by Mikhail *et al.* [8] as

$$U_{\mu}^{\nu\beta} = \frac{(-g)^{1/2}}{2\kappa} P_{\chi\rho\sigma}^{\tau\nu\beta}[\Phi^{\rho} g^{\sigma\chi} g_{\mu\tau} - \lambda g_{\tau\mu}\xi^{\chi\rho\sigma} - (1-2\lambda) g_{\tau\mu}\xi^{\sigma\rho\chi}] \tag{25}$$

where $\xi_{\alpha\beta\mu} = h_{i\alpha} h^{i}{}_{\beta;\mu}$ is the con-torsion tensor and $h_{i}{}^{\mu}$ is the tetrad field and defined uniquely by $g^{\alpha\beta} = h_{i}^{\alpha} h_{j}^{\beta} \eta^{ij}$ (here η^{ij} is the Minkowski space-time). κ is the Einstein constant and λ is free-dimensionless coupling parameter of teleparallel gravity. For the teleparallel equivalent of general relativity, there is a specific choice of this constant.

Φ_{μ} is the basic vector field given by

$$\Phi_{\mu} = \xi^{\rho}{}_{\mu\rho} \tag{26}$$

and $P_{\chi\rho\sigma}^{\tau\nu\beta}$ can be found by

$$P_{\chi\rho\sigma}^{\tau\nu\beta} = \delta_{\chi}^{\tau} g_{\rho\sigma}^{\nu\beta} + \delta_{\rho}^{\tau} g_{\sigma\chi}^{\nu\beta} - \delta_{\sigma}^{\tau} g_{\chi\rho}^{\nu\beta} \tag{27}$$

with $g_{\rho\sigma}^{\nu\beta}$ being a tensor defined by

$$g_{\rho\sigma}^{\nu\beta} = \delta_{\rho}^{\nu}\delta_{\sigma}^{\beta} - \delta_{\sigma}^{\nu}\delta_{\rho}^{\beta}. \tag{28}$$

The energy-momentum density is defined by

$$\Xi_{\alpha}^{\beta} = U_{\alpha,\lambda}^{\beta\lambda} \tag{29}$$

where comma denotes ordinary differentiation. The energy is expressed by the surface integral;

$$E = \lim_{r\to\infty} \int_{r=constant} U_{0}^{0\zeta} \eta_{\zeta} dS \tag{30}$$

where η_{ζ} (with $\zeta = 1, 2, 3$) is the unit three-vector normal to surface element dS.

4. CALCULATIONS

4.1. In general relativity

The aim of this section is to evaluate the energy and momentum distributions associated with the Gibbons-Maeda dilaton metric, using the Møller and Einstein energy and/or momentum prescriptions. First, we have to evaluate the super-potentials in the contexts of the Møller and Einstein's complexes. There is the required non-zero super-potential of Møller

$$\chi_0^{01} = \frac{(Mr - P^2 - Q^2)\sin\theta}{r}. \tag{31}$$

By substituting this super-potential, as given above, into equation (16), one gets the following energy distribution

$$M_0^0 = \frac{M^2 \sin\theta[(P^2-Q^2)^2(P^2+Q^2) - 4rM(P^2-Q^2)^2 + 4M^2(P^2+Q^2)r^2]}{\pi[(P^2-Q^2)^2 - M^2r^2]^2} \tag{32}$$

while the momentum density distributions take the form

$$M_1^0 = 0 \tag{33}$$

$$M_2^0 = 0 \tag{34}$$

$$M_3^0 = 0. \tag{35}$$

Therefore, if we substitute the result (31) into equation (20), we get the total energy of the Gibbons-Maeda dilaton spacetime that is contained in a *sphere* of radius r

$$E_{(Moller)}(r) = M - \frac{P^2}{r} - \frac{Q^2}{r} \tag{36}$$

which is also the energy (mass) of the gravitational field that a neutral particle experiences at a finite distance r. Additionally, if we use equations (33-34) in (19), we can find the momentum components which are given by

$$P_1^{(Moller)} = P_2^{(Moller)} = P_3^{(Moller)} = 0. \tag{37}$$

In order to use the Einstein energy-momentum complex, we have to transform the line element (4) in quasi-Cartesian coordinates. According to

$$x = r\sin\theta\cos\phi, \tag{38}$$

$$y = r\sin\theta\sin\phi, \tag{39}$$

$$z = r\cos\theta, \tag{40}$$

one gets

$$\begin{aligned} ds^2 = {} & \frac{(r-r_+)(r-r_-)}{r^2-D^2}dt^2 - \frac{r^2-D^2}{r^2}(dx^2+dy^2+dz^2) \\ & -\frac{r^2-D^2}{r^4}\left(\frac{r^2-(r-r_+)(r-r_-)}{(r-r_+)(r-r_-)}\right)(xdx+ydy+zdz)^2. \end{aligned} \tag{41}$$

Using above metric transformed into quasi-Cartesian coordinates in equations (21), (22) and (24), one gets the expressions for the energy

$$E_{(Einstein)}(r) = M - \frac{P^2}{2r} - \frac{Q^2}{2r} \tag{42}$$

and for the momentum components

$$P_1^{(Einstein)} = P_2^{(Einstein)} = P_3^{(Einstein)} = 0. \tag{43}$$

By comparing the results presented in this section, it is easy to see that the hypothesis of Vagenas is true for the expression associated with the Gibbons-Maeda dilaton spacetime. The corresponding relation is obtained as

$$\alpha_1^{(Einstein)} = \frac{1}{2}\alpha_1^{(M\o ller)}. \tag{44}$$

which is the case of $n = 1$ for equation (3).

4.2. In teleparallel Gravity

In this part of the paper, we calculate the Møller energy associated with the Gibbons-Maeda dilaton spacetime in teleparallel gravity. Since the intermediary mathematical exposition are lengthly, we give only the final result. After making the required calculations [25, 26], the required non-vanishing component of $U_\mu^{\nu\beta}$ is

$$U_0^{01} = \frac{(Mr - P^2 - Q^2)\sin\theta}{\kappa r}. \tag{45}$$

Substituting this result in the energy integral (30), we have the following energy distribution

$$E(r) = M - \frac{P^2}{r} - \frac{Q^2}{r}. \tag{46}$$

This is the same as obtained in general relativity by using the Møller energy and/or momentum complex. It is also independent of the teleparallel dimensionless coupling constant, which means that it is valid not only in teleparallel equivalent of general relativity but also in any teleparallel model. This result also sustains that the hypothesis of Vagenas is true.

5. SUMMARY AND DISCUSSIONS

We evaluated the energy distribution associated with the Gibbons-Maeda dilaton spacetime using Møller and Einstein's energy and/or momentum complexes and the teleparallel gravity analog of the Møller energy-momentum formulation.

The localization of energy-momentum in general relativity has been debated since the beginning of relativity. The energy-momentum pseudotensors are not tensorial object and one is forced to use *Cartesian* coordinates. Because of these reasons, this topic was not considered exactly for a long time. However, after Virbhadra, Rosen, Chamorro and Aguirregabiria's works [6], this subject was re-opened. In addition to this, Virbhadra underlined that although the energy-momentum complexes are not tensorial objects, they do not disturb the principle of general covariance as the equations defining the local conservation laws with these objects are covariant. In another study, Chang, Nester and Chen obtained that there exists a direct relationship between quasilocal and pseudotensor expressions; ever since energy-momentum pseudotensor is associated with a legitimate Hamiltonian boundary term.

In general relativity, several studies have been devoted to calculate the energy (due to matter plus fields including gravitation) distribution for a given space-time. For example; Chamorro-Virbhadra and Xulu showed, considering the general relativity analogs of Einstein and Møller's definitions, that the energy of a charged dilation black hole depends on the value h which controls the coupling between the dilation and the Maxwell fields

$$E_{(Einstein)} = M - \frac{Q^2}{2r}(1-h^2), \tag{47}$$

$$E_{(Moller)} = M - \frac{Q^2}{r}(1-h^2). \tag{48}$$

In addition, Virbhadra and Xulu obtained that the energy distribution in the sense of Einstein and Møller disagree in general. Next, Lessner showed that the Møller energy-momentum complex is a powerful concept of energy and momentum and Vagenas hypothesized that there is a connection between the coefficients of the expression for the energy (when the energy and/or momentum complex of Einstein is employed) of the form (1) and those of the expression for the energy (when the energy and/or momentum complex of Møller is employed) of the form (2). The relation that materializes this connection between the aforementioned coefficients is given by (3).

Using Møller complex, we found the same energy associated with the Gibbons-Maeda dilaton spacetime both in general relativity (GR) and teleparallel gravity (TG) is given by

$$E_{GR}^{(Moller)}(r) = E_{TG}^{(Moller)}(r) = M - \frac{P^2}{r} - \frac{Q^2}{r} \tag{49}$$

and using Einstein's energy-momentum definition in general relativity, we obtained the energy of the Gibbons-Maeda dilaton spacetime as

$$E_{GR}^{(Einstein)}(r) = M - \frac{P^2}{2r} - \frac{Q^2}{2r}. \tag{50}$$

The result supports that the energy distribution in the sense of Einstein and Møller disagree in general. Under two limits of the metric (4), the Gibbons-Maeda dilaton (GM) spacetime can be reduced to the Garfinkle-Horowitz-Strominger (GHS) dilaton spacetime and the Reissner-Nordstrom spacetime (RN), respectively. Therefore, using our results, one can easily find that

$$E_{GHS} = \lim_{P\to 0} E_{(Einstein)}(r) = M - \frac{Q^2}{2r} \tag{51}$$

$$E_{GHS} = \lim_{P\to 0} E_{(Moller)}(r) = M - \frac{Q^2}{r} \tag{52}$$

$$E_{RN} = \lim_{D\to 0} E_{(Einstein)}(r) = M - \frac{Q^2}{r} \tag{53}$$

$$E_{RN} = \lim_{D \to 0} E_{(Moller)}(r) = M - \frac{2Q^2}{r} \tag{54}$$

and at large distances ($r \to \infty$), one gets

$$E_{GM} = E_{RN} = E_{GHS} = M \tag{55}$$

by using Einstein and Møller's complexes.

Furthermore, this paper sustains (a) the results by Virbhadra and Xulu, (b) the viewpoint of Lessner, (c) the hypothesis of Vagenas, (d) the importance of the energy-momentum definitions in the evaluation of the energy distribution of a given space-time, and (e) the Møller energy-momentum definitions which allows to make calculations in any coordinate system. Finally, in teleparallel gravity the energy obtained is independent of the teleparallel dimensionless coupling constant, which means that it is valid not only in the teleparallel equivalent of general relativity, but also in any teleparallel model.

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