THE EFFECTS OF A FUNCTIONAL THINKING INTERVENTION ON FIFTH GRADE STUDENTS’ FUNCTIONAL THINKING SKILLS

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ABSTRACT

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The purpose of the study was to investigate the effects of a functional thinking intervention on 5^{th}-grade students' functional thinking skills. The sample of the current study was 43 fifth grade students in two public middle schools in Ankara, in which 20 of them constituted the experimental group in one school, and 23 of them constituted the control group in the other school. The sample was chosen by the convenience sampling method from public secondary schools in Ankara. While the control group students did not attend any intervention about functional thinking, the experimental group students participated in a functional thinking intervention lasting 12 hours (about 3 weeks). A Functional Thinking Test (FTT) was applied as a pre- and post-test to both groups. The quantitative data was supported by analyzing students' functional thinking strategies qualitatively. The statistical analyses were conducted to investigate whether there was an effect of the functional thinking intervention on students' functional thinking skills. The results of the study showed that there was not a significant mean difference between the experimental and control group at pre-test or post-test. However, the experimental group showed significant pre-to-post gains. Also, experimental group students were significantly better at being able to use variables in defining the function rule after the functional thinking intervention.

Keywords: Functional Thinking, 5th Grade Students, Early Algebra, Recursive Pattern, Covariational Thinking, Correspondence Thinking

ÖZ

FONKSİYONEL DÜŞÜNME UYGULAMASININ BEŞİNCİ SINIF ÖĞRENCİLERİNİN FONKSİYONEL DÜŞÜNME BECERİLERİNE ETKİLERİ

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Çalışmanın amacı, fonksiyonel düşünme uygulamasının 5. sınıf öğrencilerinin fonksiyonel düşünme becerilerine etkisini incelemektir. Araştırmanın örneklemini Ankara'da iki devlet ortaokulunda öğrenim gören 43 5. sınıf öğrencisi oluşturmuştur. Bunlardan bir okulda bulunan 20 öğrenci deney, diğer okulda bulunan 23 öğrenci kontrol grubunu oluşturmuştur. Katılımcılar, Ankara'daki devlet ortaokullarından uygun örnekleme kullanılarak seçilmiştir. Kontrol grubu öğrencileri fonksiyonel düşünme ile ilgili herhangi bir uygulamaya katılmazken, deney grubu öğrencileri 12 saat süren (yaklaşık 3 hafta) fonksiyonel düşünme uygulamasına katılmışlardır. Her iki gruba da ön ve son test olarak Fonksiyonel Düşünme Testi (FDT) uygulanmıştır. Nicel veriler, öğrencilerin fonksiyonel düşünme stratejilerinin nitel olarak analiz edilmesiyle desteklenmiştir. İstatistiksel analiz, fonksiyonel düşünme uygulamasının öğrencilerin fonksiyonel düşünme becerileri üzerinde bir etkisinin olup olmadığını araştırmak için yapılmıştır. Araştırmanın sonuçları deney ve kontrol grubu arasında son testte anlamlı bir fark olmadığını ortaya koymuştur. Bununla birlikte, deney grubu, ön test ve son test arasında anlamlı bir gelişme göstermiştir. Deney grubu öğrencileri, fonksiyonel düşünme uygulaması sonrasında fonksiyon kuralını

değişkenler kullanarak belirlemede anlamlı olarak daha iyi bir performans sergilemişlerdir.

Anahtar Kelimeler: Fonksiyonel Düşünme, 5. Sınıf Öğrencileri, Erken Cebir, Yinelemeli Örüntü, Kovaryans Düşünme, Birebir Eşleyerek Düşünme

To My Family

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ABBREVIATIONS

Control Group	CG
Experimental Group	EG
Functional Thinking Intervention	FTI
Functional Thinking Test	FTT
Ministry of National Education	MoNE

CHAPTER 1

INTRODUCTION

The National Council of Teachers of Mathematics (NCTM, 2000) remarks on the importance of algebra competence in daily life and for preparation for postsecondary education, and states that all students should learn algebra. Also, NCTM (2000) describes 'algebra' as a strand that includes major components of the curriculum. In the traditional curriculum, there is an "arithmetic-then-algebra" approach, so students learn numbers and operations in the elementary school. The algebra concepts are introduced in the middle and high school in an abstract way. However, some researchers defended the idea that arithmetic is not a required precondition for algebra (e.g., Carraher et al. 2006). Also, Cai and Knuth (2011) stated that approaching algebra as only symbolization and separating it from the arithmetic cause lack of comprehension for complex mathematical concepts. Blanton and Kaput (2011) argued that students need early experiences to deepen mathematical structures and relationships rather than isolated computation exercises so that early algebra combines computational, generality, and reasoning perspectives. Thus, students would become ready for later grades. NCTM (2000) supported the idea of blending algebra in the curriculum from pre-kindergarten to help students construct a strong basis for understanding more sophisticated works in algebra in the future. Also, Common Core State Standards for Mathematics (CCSSM, 2010) suggested that early algebra should be placed in mathematics education from kindergarten through K-12**.**

Blanton and Kaput (2005) expressed that the integration of algebraic reasoning into elementary grades provides an opportunity to build more complex and deeper mathematics into student's experiences from the beginning. There are five big ideas of algebra; equivalence and equations, generalized arithmetic, functional thinking, variable and quantitative reasoning (Blanton et al., 2011). Mainly, functional thinking “involves the generalization of relationships between covarying quantities, representing and justifying these relationships in multiple ways using natural language, variable notation, tables, and graphs” (Stephens et al., 2017, p.144). So, this important connection to early algebra and the scope of functional thinking make functions important in the early grades.

Blanton et al. (2011) defined functional thinking as a combination of generalizations, representations, justifications, and reasoning with relationships between quantities. Several researchers (e.g., Blanton, Brizuela et al., 2015; Blanton, Stephens et al., 2015; Stephens et al., 2015) defended that students can develop algebraic thinking, specifically functional thinking, in early grades if they are provided with appropriate instruction and environment.

The 2015 Grades 1-4 National Mathematics Curriculum (MoNE, 2015) included a sub-learning domain “transition to algebra” under the domain, number and operations. It contained objectives about variable, patterns, the meaning of equal sign, generalizations of operations, relationship between two quantities, and representation of these relationships. As the grade level increased, the number of objectives also increased (e.g., two objectives for 1st grade, three objectives for 2nd and 3rd grades, and four objectives for 4th grade). Turgut and Temur (2017) interpreted that the development of algebraic reasoning from first grade to fourth grade was aimed to be improved. In the Grades 1-8 National Mathematics Curriculum (MoNE, 2018), “transition to algebra” sub-learning domain was not mentioned. However, some of those objectives were covered under the number and operations and algebra learning domains.

Studies (Akkaya & Durmuş, 2006; Çelik & Güneş, 2013; Dede & Argun, 2003; Erbaş et al., 2009) mostly focused on middle and high school level and conducted in Turkey revealed students’ misconceptions and difficulties on algebra and algebraic thinking.. Students have been carrying their misconceptions about using variables, meaning of equal sign, solving equations from middle grades through high school.

Studies on functional thinking focused on generalizations of functional thinking (e.g., Türkmen & Tanışlı, 2019) and investigating functional thinking through linear function tables (e.g., Tanışlı, 2011).

Functional thinking intervention is accepted as an effective entry point to algebraic thinking in early grades (Carraher & Schliemann, 2007, as cited in Stephens et al., 2017). This study focuses on investigating fifth-grade students' functional thinking skills using a quasi-experimental design. The study, using contextual problems, provided students opportunities to discover relationships between two quantities, to define these relationships in different types of functional thinking approaches (covariational and correspondence relationships), and represent these relationships using different type of representations including verbal, symbolic, graphic forms.

1.1 Research Questions

This study was conducted with fifth-grade students who were enrolled in two public middle schools in Ankara, Turkey during the spring term of the 2018-2019 academic year. The study focused on answering the following research questions:

1. Is there a statistically significant mean difference between the functional thinking post-test scores of the 5th-grade students who attend the functional-thinking intervention and those who do not?
2. Is there a statistically significant mean difference between the functional thinking pre-test and post-test scores of the 5th-grade students who attend the functional-thinking intervention?
3. Is there a significant relationship between the two groups (5th-grade students who attend the functional-thinking intervention and those who do not) and the correctness in the functional thinking test items at pre-test and post-test?
4. How does 5th-grade students' functional thinking strategies differ in the functional-thinking test for those who attend the functional-thinking intervention, and who does not?

1.2 Significance of the Study

Kaput (2008) defined functional thinking as an essential part of algebraic thinking, and it has been accepted as one of the possible entries to algebra (Carraher & Schliemann, 2007). In Turkey, functional thinking starts with visual and number patterns in the elementary grades and continues as functions abstractly in high school (Kabael & Tanışlı, 2010). Functional thinking involves the generalization of relationships, multiple representations of these relationships, and reasoning and justification of generalizations (Blanton et al., 2011). Many studies show that students can define functional relationships (using covariational, correspondence thinking) and represent these relationships by using pictures, tables, graphs, words, and variables in early grades provided the appropriate environment and instruction (e.g., Blanton & Kaput, 2004; Blanton, Stephens et al., 2015, Isler et al., 2015; Strachota et al., 2016).

There is a lack of research in investigating the effects of a functional thinking intervention on students' functional thinking skills before formal algebra education in Turkey. This study aimed to focus on investigating fifth-grade students' functional thinking skills and the effects of a functional thinking intervention on students' functional thinking skills.

1.3 Definition of the Important Terms

Early Algebra: It is "an approach that elementary students are provided the time and space necessary to build their intuitive and informal ways of reasoning about the patterns and relationships they see in their everyday experiences as a basis for algebraic thinking" (Stephens et al., 2017, p. 143)

Algebraic Thinking: It is defined as "a habit of mind that permeates all of mathematics, and that involves student's capacity to build, justify and express conjectures about mathematical structure and relationships" (Blanton & Kaput, 2004, p. 142).

Functional Thinking: It is defined as “generalizing relationship between covarying quantities, expressing those relationships in words, symbols, tables, or graphs, and reasoning with these various representations to analyze function behavior” (Blanton et al., 2011, p. 47).

Covariational thinking: It involves analyzing how two quantities vary in relation to each other and keeping that variation explicit in the description of the function (Blanton et al., 2011, p. 52).

Correspondence thinking: It is “a correlation between two quantities expressed as a function rule.” (Blanton et al., 2011, p. 53)

1.4 Motivation for the Study

Algebra is perceived as an intimidating concept of mathematics. The general perception is that algebra is composed of symbols, formulas, and equations so it is hard, abstract, and apart from real life. Similarly, as part of algebra, functions can be a nightmare in the high school. During my own experience as a student and as a tutor, I observed students having difficulty in comprehending these concepts and they were not aware of the relationship between algebra and real life. The studies show that students can think algebraically before the formal operational stage. The functional thinking intervention has a potential to help students define and represent functional relationships in multiple ways in early grades. So, I decided to investigate fifth-grade students’ functional thinking ways, strategies, and representations they used to explain functional relationships before starting formal algebra education.

CHAPTER 2

LITERATURE REVIEW

The purpose of this study was to investigate the effects of functional thinking intervention on fifth-grade students' functional thinking skills. In the first section, the background of the study will be described. In the second part, functional thinking studies including studies conducted in Turkey will be presented.

2.1 Background of the Study

Functional thinking is an essential part of early algebra, so the present study was designed and conducted based on three related perspectives of early algebra. The first perspective was the core aspects of algebra defined by Kaput (2008). The other was five big ideas of algebraic thinking described by Blanton et al. (2011). Lastly, functional thinking levels will be presented based on Stephens et al. (2017).

2.1.1 Core Aspects and Strands of Algebra

Kaput (2008) explained algebra in light of two core aspects (Core Aspect A & Core Aspect B) and three strands (Strands 1, 2 & 3) that the core aspect is blended in (see Figure 2.1).

According to Kaput (2008), Core Aspects A refers to regularities and generalizations. Core Aspect B includes defining those generalizations "in conventional forms; algebraic notation, graphs and number lines, tables, and natural language forms" (p. 12). Kaput (2008) stated that Core Aspect B should be improved after Core Aspect A to strengthen algebraic reasoning.

Core Aspect A & B and Strands 1, 2 & 3 are strongly related with each other. Strand 1 involves generalized arithmetic, including numbers, operations, and their properties, particular number relationships, and quantitative reasoning. Also, it includes both conventional and invented computation strategies.

Figure 2. 1

Core Aspects and Strands in Kaput's Framework of Algebraic Reasoning

The Two Core Aspects
(A) Algebra as systematically symbolizing generalizations of regularities and constraints.
(B) Algebra as syntactically guided reasoning and actions on generalizations expressed in conventional symbol systems.
Core Aspects A & B Are Embodied in Three Strands
1. Algebra as the study of structures and systems astracted from computations and relations, including those arising in arithmetic (algebra as generalized arithmetic) and in quantitative reasoning.
2. Algebra as the study of functions, relations, and joint variation.
3. Algebra as the application of a cluster of modeling languages both inside and outside of mathematics.

Note. Reprinted from "What is Algebra? What is Algebraic Reasoning?" by J.J. Kaput, in J.J. Kaput, D. Carraher and M. Blanton (Eds), *Algebra in the Early Grades* (p. 11), 2008, Mahwah, NJ: Lawrence Erlbaum/Taylor & Francis Group.

Strand 2 is mostly related to functional thinking. Kaput (2008) defended that the scope of this strand is comprehensive. It includes a focus on change, linearity, the symbolization of functional relationships, and representations such as tables, graphs.

Kaput (2008) explained Strand 3 by three types of modeling. The first type of model is the number or quantity specific. In this model, the variable is accepted as unknown in the equation. The second type of modeling refers to Core Aspect A, which includes generalizations. This model uses one or more variables to define a function. In the third type of modeling, the variable is accepted as a parameter, and generalizations of relationships are compared with other situations.Kaput (2008) defended that generalizations and symbolization are significant parts of algebraic thinking.

2.1.2 Five Big Ideas of Algebraic Thinking

Blanton et al. (2011) stated algebraic thinking as an essential understanding for students in grades 3-5. Blanton et al. (2011) categorized an essential understanding of algebraic thinking as composed of five big ideas which are generalized arithmetic, equations, variables, quantitative reasoning, and functional thinking.

This study was designed based on the big idea of functional thinking. Functional thinking was defined as "generalizing relationships between covarying quantities, expressing those relationships in words, symbols, tables, or graphs, and reasoning with these various representations to analyze function behavior" (Blanton et al., 2011, p. 47). Also, the intervention of this study focused on the equations (big idea 2) and variables (big idea 3). Understanding the relational meaning of equal sign, "the same as" (Carpenter et al., 2003, as cited in Blanton et al., 2011; Stephens et al., 2013) is important in the early grades. The relational meaning of the equal sign requires students to decide "8 + 4 = 12 + 5 is false, because 8 + 4 and 12 + 5 are not equivalent" (p. 26). According to Blanton et al. (2011), the variable has five roles; representing a number in a generalization, a fixed unknown, varying quantity, a parameter, and arbitrary placeholder (pp. 32-34). In functional thinking, variables have the role of varying quantity to represent relationships between two quantities.

Functions are tools for expressing covariation between two quantities. There are different types of relationships defined in a function; *recursive patterns, covariational perspective,* and *correspondence rules* (Blanton et al., 2011). "Recursive patterns describe variation in a single sequence of values" (Blanton et al., 2011, p. 52). There are some limitations for defining functional relationships by recursive patterns. Firstly, a recursive pattern shows how to get a number in a sequence from the previous number. It does not include the independent variable, and explains the change in one quantity rather than covarying quantities. Thus, recursive patterns have limited applicability to explain functional relationships between two quantities. Covariational thinking includes "analyzing how two quantities vary in relation to each other and keeping that variation explicit in the description of the function" (Blanton et al., 2011, p. 52). Blanton et al. (2011, p. 53) defined a correspondence relationship as "a correlation between two quantities expressed as a function rule." They also stated that correspondence rules give information about specific function values not having a need to know other values (Blanton et al., 2011). Furthermore, Blanton et al. (2011) stated that natural language, algebraic notation, constructing tables and graphs are different representations for functional relationships that offer different perspectives. So, it is crucial to understand these representations and the connections between them. NCTM (2000) suggested that mathematics instruction should help all students to be able to "create and use representations to organize, record, and communicate mathematical ideas; select, apply, and translate among mathematical representations to solve problems; and use representations to model and interpret physical, social, and mathematical phenomena" (p. 67). Through representation standard, Blanton et al. (2011) claimed that a good understanding of functions requires representational fluency, in other words, a proficiency to represent in different ways and to traverse easily among these representations.

As a result, an essential part of algebraic thinking, functional thinking, includes generalizing relationships between covarying quantities and expressing these relationships in many ways of including as words, symbols, tables, and graphs.

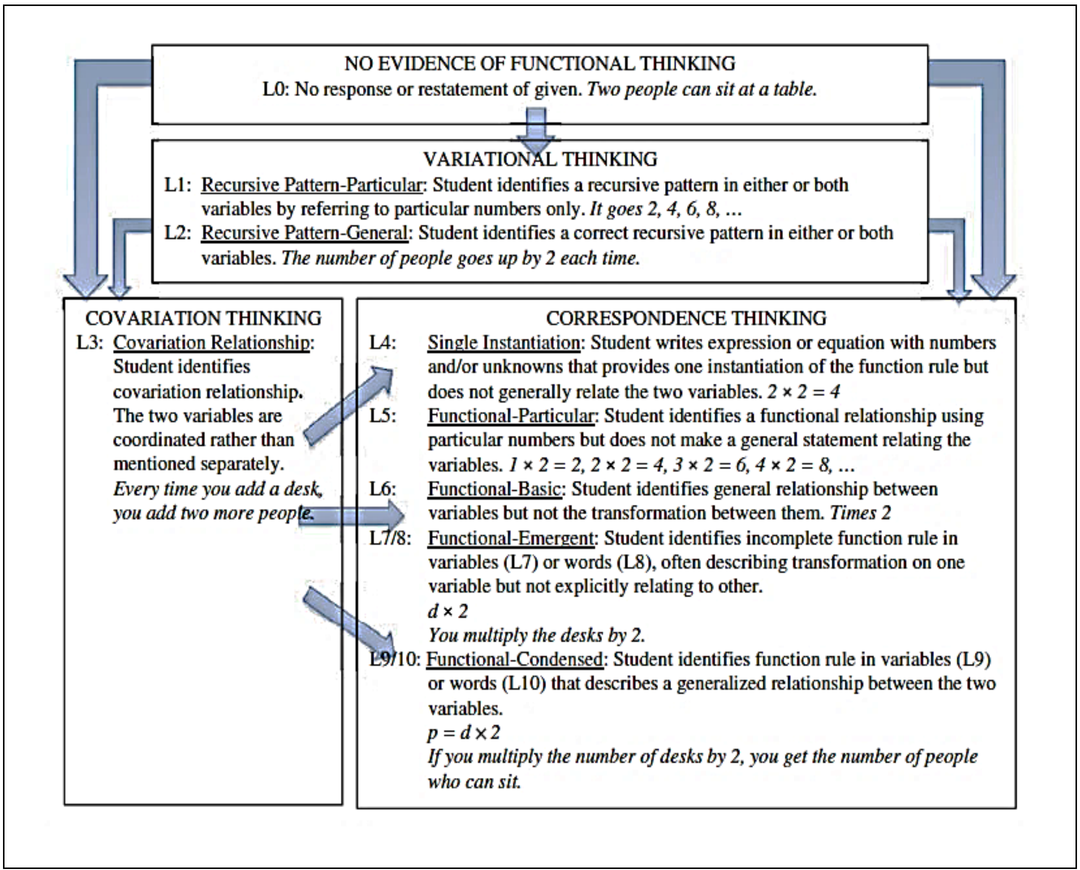
2.1.3 Levels of Sophistication of Functional Thinking

Stephens et al. (2017) focused on investigating students' improvement in generalizing and representing functional relationships as a part of a three-year longitudinal study on early algebra. They aimed to investigate the effects of an extensive early algebra intervention on students' algebraic thinking and readiness and focused on the context that revealed students' functional thinking and representations (tables, words, symbols, pictures, and graphs). The instructional sequence of the study was constructed in light of core aspects, big ideas, and practices of algebraic thinking. In grade 3, students worked on $y=mx$ and $y=x+b$

functions through recursive, covariational, and correspondence relationships using representations, including coordinate graphs. In grade 4, $y=x^2$ and $y=x^2 + b$ quadratic functional relationships were focused on. Students were introduced to exponential and piecewise functions in grade 5. Seven lessons of functional thinking intervention were performed in grades 3 and 4, and six lessons were for grade 5. All lessons were designed as small group works and whole-class discussions. Students' responses were analyzed through a coding schema that was based on the levels of sophistication describing students' generalization and representation of functional relationships (see Figure 2.2). The levels were categorized as three modes of functional thinking identified by Confrey and Smith (1991); recursive, covariational, and correspondence.

Figure 2. 2

Levels of sophistication describing grades 3-5 students' generalization and representation of functional relationships



Note. Reprinted from "A Learning Progression for Elementary Students' Functional Thinking" by A.C. Stephens, N. Fonger, S. Scrachota, I. Isler, M. Blanton, E. Knuth, and A.M. Gardiner, 2017, Mathematical Thinking and Learning, 19(3), p. 153.

Stephens et al. (2017) found that students succeeded in defining function rules in variables more than in words, so the categories of words are placed at a higher level (see Figure 2.2). The results were similar for the tasks on $y=mx$ and $y=mx+b$. There were four main results of the study; the first one is the "main path" that students progressed by skipping some levels. In the beginning, students' responses were at L2 (recursive pattern); those shifted towards L6 (Functional Basic), L9 (Functional Condensed in Variables, and L10 (Functional Condensed in Words) (see Figure 2.2). Secondly, students could define function rules in variables before they could define function rules in words. Thirdly, as the functions were getting complex, "symbolic advantage" became prominent (p. 159). Lastly, they concluded that the learning progression and levels were identified through the study could not involve all students' responses since there were many responses assessed in the "Other" category, especially for the task on $y=mx+b$.

In the present study, the aim was to investigate students' functional thinking skills before and after the intervention. Students were expected to define functional relationships and represent them in multiple ways. Students were expected to define recursive patterns and covariational relationships to some degree at the pre-test. So, after the functional thinking intervention, the aim was to help them define correspondence relationship and write function rules in words and variables (L9 and L10 in Figure 2.2). Therefore, students' written responses to both pre- and post-test were assessed by these levels.

2.2 Functional Thinking Studies

Functional thinking studies are based on defining functional relationships including recursive patterns, covariational, correspondence, relationships and using multiple representations such as pictures, tables, graphs, words, variables. Both intervention and non-intervention studies will be presented in this part.

Blanton, Stephens et al. (2015) designed a comprehensive intervention to develop students' algebraic thinking in elementary grades. There were 106 third grade students, 39 of them were experimental, and 67 of them were control students. There were 19 lessons taught once a week, throughout one year. Students' written responses were analyzed through correctness and strategies. The general result was that although there was no significant difference between the pre-test of two groups, the experimental group showed significant gain so that there was a significant mean difference at post-test between the groups. This result showed that comprehensive early algebra intervention was appropriate for third graders to engage in big ideas of algebra. Some related results about functional thinking of the study will be summarized in the following. Brady’s birthday task (see Figure 2.3) was asked for assessing functional thinking. There were recursive and covariational pattern, function rule in words and in variables strategies for a relationship, and drawing, recursive and functional rule for near value (p. 67). Except from item e, experimental group outperformed the control group at posttest. Item e could be solved in arithmetic way so control group students also showed gain. Although a few students (3% vs. 2% of the experimental and control group, respectively) defined the covariational relationship (e.g., “every time you add one more table, you add two more people”, p. 69) at the pre-test, 24% of the experimental group and 8% of the control group described a covariational relationship at post-test in the item c. More experimental group students defined functional rule in variables (e.g., A × 2 + 2 = B) than in words (e.g., Number of tables times two plus two equals number of people. (16% vs. 8%, respectively) at post-test in item d (p. 67). All in all, functional thinking intervention had positive effects on the development of students’ functional thinking skills even in third grade.

Figure 2. 3

Brady’s Birthday Task

Brady is having his friends over for a birthday party. He wants to make sure he has a seat for everyone. He has square tables.

He can seat 4 people at one square table in the following way:

If he joins another square table to the first one, he can seat 6 people.

Figure 2.3 (continued)

a) If Brady keeps joining square tables in this way, how many people can sit at 3 tables? 4 tables? 5 tables? Record your responses in the table below and fill in any missing information:

Number of tables	Number of people
1	
2	
3	
4	
5	

b) Do you see any patterns in the table? Describe them.
c) Find a rule that describes the relationship between the number of tables and the number of people who can sit at the tables. Describe your rule in words.
d) Describe your relationship using variables. What do your variables represent?
e) If Brady has 10 tables, how many people can he seat? Show how you got your answer.

Note. Reprinted from “The development of children's algebraic thinking: The impact of a comprehensive early algebra intervention in third grade” by Blanton, M., Stephens, A., Knuth, E., Gardiner, A. M., Isler, I., & Kim, J. S., 2015, *Journal for research in Mathematics Education*, *46*(1), 39-87.

Blanton and Kaput (2004) conducted a study to investigate elementary grade students’ expressions and representations from pre-kindergarten through fifth grade. “Eyes and Tails” task asked students, “If there was one dog, how many eyes would there be? Two dogs? Three dogs? 100 dogs? Do you see a relationship between the number of dogs and the number of eyes? How would you describe this relationship?” (p.136). Data were collected through students’ written works and teacher interviews. Pre-kindergarten students worked by counting visible objects (dog pictures, dots for eyes, and marks for tails). They found the number of eyes and tails for 3 and 4 dogs by counting instead of using pattern and predictions. Teachers constructed and recorded data on t-chart. Kindergarten students calculated the number of eyes and tails until ten dogs and recorded them on a t-chart by the teacher's help. Some students described the pattern as "counting by twos" and "every time we add one more dog, we get two eyes." (p.137). First-grade students defined multiplicative pattern as "double" (eyes) and "triple" (eyes and tails). Second graders recorded data up to 10 dogs on t-chart, defined the multiplicative relationship using natural language. Also, they could find the far value for the number of eyes and the number of tails for 100 dogs. Third graders defined the rule in words and variables and wrote "$n \times 2$" and "$2 \times n$". One of the 3rd-grade students drew a line graph to show the relationship. Fourth and fifth-grade students could define the functional relationship by using fewer data. Besides, a fourth-grade student wrote: "$\square \times 3 = n$" (p.140) to represent the relationship between the number of dogs and the total number of eyes and tails. This study showed that students could think covariationally, even in kindergarten. Also, even third-grade students could define the relationship by using symbols, variables and words. Blanton and Kaput argued that students have the potential for functional thinking in early grades.

Moreover, Blanton et al. (2017) performed a study to investigate first-grade students' development in variable notation using functional thinking contexts. Forty students participated in the study from two schools. Two cycles of the task-based instructional sequence were implemented, lasting 16 lessons in total. While the first cycle focused on $y=mx$, the second cycle focused on $y=x+b$. Data were collected by pre-, mid- and post-interviews with a similar task (p. 186). The results of the pre-interview that performed on dogs and noses ($y=x$) task were presented. There were six levels of sophistication in variable notation. The first two levels were defined as pre-variable. The first level was defined as "pre-symbolic" that students could not use any symbol or variable and conceptualize quantity "unknown." Hence, students have to "count and find a numerical value" (p. 189). At the second level, students used letters as a label to represent an object rather than a quantity. At the third level, students started to see symbols representing a variable. Students accepted the symbols as arbitrary. For instance, if a variable was represented by "c," children think that its only value should be “three” because of the order in the alphabet. At the fourth level, students accepted the variable as a fixed unknown like at level 3 but, here, the fixed value was determined randomly. At the fifth level, students comprehended “variables as varying unknowns” (p. 195). At the sixth level, students both realized “variables as varying unknowns” (p. 196) and used them in representing functional relationships. Level 6 was not observed in the pre-interview. This study suggested that if students were provided long-term experiences, they can develop symbol sense and overcome their misconceptions.

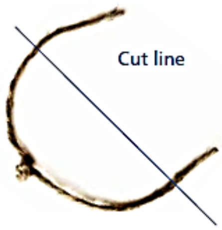
Similarly, Cañadas et al. (2016) worked with second-grade children on Brady’s birthday party problem (adapting from Blanton, Stephens et al., 2015) based on y=2x functional relationship in Spain. The lessons were based on whole group discussions, and small group works. Students were able to construct and organize data in a t-chart. Students defined both recursive (counting by twos) and functional relationships (doubling) for small numbers, but they shifted towards functional strategies for bigger numbers (e.g., 30; 40; 1,000,000). Also, through the intervention some students used variables to generalize the relationship. Cañadas et al. (2016) defended that classroom teaching activities based on functional thinking allow student to explore, define, represent and justify the relationship between variables.

Isler et al. (2015) presented a part of an early algebra teaching experiment in grades 3 through 5. There were six classes, two for each grade level, in the study. This part of the study was designed focusing on recursive, covariational, and correspondence thinking (Blanton et al., 2011) and multiple representations. Students' development was assessed by pre- and post-test. During the teaching experiment, as part of functional thinking, students worked on the string task (see Figure 2.4) in small groups. Five different colors of pieces of strings were given to students. The strings had a knot and folded over the knot. Students were expected to define the relationship between the number of cuts and the number of pieces. Students organized data on the table and initially engaged in recursive thinking. Through the questions asked, students realized the relationship by looking across the table. Students could define the correspondence rule as “L × 2+1=S” L represented the number of cuts, and S represented the number of string pieces.

Figure 2. 4

The String Task

Fold a piece of string at the knot. While it is folded, make 1 cut.



How many pieces of string do you have?

Fold a new piece of string at the knot. Make 2 cuts and find the number of pieces of string.

Repeat this for 3, 4, and 5 cuts, always remembering to use a new piece of string.

a. What can you say about the number of cuts? What can you say about the number of pieces of string?
b. Organze your information in a table. What do the variables represent?
c. What relationships do you see in the data? Use this information to predict the number of pieces of string you would have after 8 cuts.
d. Find a relationship between the number of cuts and the number of pieces of string.

   How would you describe your relationship in words?

   Describe your relationship using variables.

Note. Reprinted from “The string task: Not just for high school” by Isler, I., Marum, T., Stephens, A., Blanton, M., Knuth, E., & Gardiner, A. M., 2015, *Teaching Children Mathematics*, *21*(5), 282-292.

The Brady task (see Figure 2.3) was implemented in the written form at pre- and post-test. In the item b that asked to describing patterns in the table, while fourth and fifth-grade students showed an increase in the covariational relationship at post-test, third-grade students showed an increase in both covariational relationships and recursive patterns, but mostly in the recursive pattern. Moreover, in the item c (asking function rule in words) and in the item d (asking function rule in variables), there was no correspondence thinking (functional relationship in words and in variables) observed in the pre-test. Students defined function rule in words and variables at post-test. Fifth-grade students showed a higher increase compared to 3rd and 4th grade students in writing the function rule. The findings of this study supported the findings of other studies (e.g., Blanton & Kaput, 2004; Pinto & Canadas, 2018; Stephens et al., 2012) in that students can improve their functional thinking and use multiple representations.

Besides the studies that had an intervention, Isler et al. (2017) conducted a study to investigate the effect of a grades 3-5 early algebra intervention on 6th grade students' success in functional thinking. There were 80 sixth grade students; 46 of them participated in the intervention across grades 3-5, and 34 of them were the control group. Brady's task (see Figure 2.3) was used as an assessment tool asked students the function rule "$2\times d = p$" (p. 435) in words and variables. Results showed that experimental group students used functional condensed in words (48% vs. 26% for the experimental and control groups, respectively) and functional condensed in variables (65% vs. 48% for the experimental and control groups, respectively) strategies more than the control group. Similarly, in the items that asked to define the function rule "$2\times d +2 = p$" (p. 435), experimental group students used functional condensed in words (24% vs. 12% for the experimental and control groups, respectively) and functional condensed in variables (43% vs. 26% for the experimental and control groups, respectively) strategies more than the control group. Also, this study found that students in both groups were more successful in defining function rule in variables than words. As a result, the experimental group outperformed the control group in all items. This study emphasized that students who were engaged with functional thinking activities in early grades were better in generalizing and representing functions at the middle school.

2.2.1 Functional Thinking Studies in Turkey

In this part, studies on functional thinking conducted in Turkey will be presented. There are not many studies that focused on functional thinking in Turkey.

Türkmen and Tanışlı (2019) conducted a study to investigate 3rd, 4th, and 5th-grade students' levels of generalizations of functional thinking in the early grades. There were 116 participants in total; 45 third grade students, 36 fourth grade students, and 35 fifth grade students. Data were collected by two open-ended tasks (Brady Task, see Figure 2.3) referred to $y=mx$ and $y=mx+n$ functional relationships. In Task 1, students were asked to define patterns and generalize the patterns whose rule was $y=2x$. In Task 2, students' functional thinking skills for the $y=2x+2$ functional relationship was examined. Students' functional thinking levels were determined by regarding levels of sophistication constructed by Blanton, Brizuela et al. (2015) and Stephens et al. (2017). Students' written responses were categorized based on seven levels and six sublevels. For the task $y=2x$, the majority of students (46.7 % of 3rd graders, 44.4 % of 4th graders, 34.2% of 5th graders) were found in the "Functional Particular-Multiplicative Relationship" level. At this level, students could define the multiplicative relationship between the number of tables and the number of people as *"1×2=2, 2×2=4, 3×2=6"*, but they could not generalize this functional relationship. Moreover, 31.4 % of fifth-grade students were at the "Emergent Functional–Variables" level (M ×2=K). Furthermore, 19.4% of fourth-grade students were in the "Primitive Functional-Words" level, and they could define the functional relationship on one of the variables as *"If we multiply the number of tables by 2, we get the result*" (p.354). Also, 2.8% of the fourth-grade students were at the highest level, 'Function as Object.' Those students could define the functional relationship by abstracting objects so they could describe the rule for other objects as *"How many socks are 50 pairs of socks?"* (p.362) In addition, 20% of third-grade students responded at the "Recursive Pattern-Particular" level. Those students just focused on the patterns limited to the numbers on the table and found the number of people as 2, 4, 6, 8, 10, 12, 14 for the number of tables. For the task $y=2x+2$, students were found to tend to ignore the constant term. The majority of the students (28.9% of 3rd graders, 33.3% of 4th graders, 22.9% of 5th graders) were in the "Functional Particular-Multiplicative Relationship" level. Those defined the function rule as "Four people sit on a table. If we multiply one by 4, we get 4; if we multiply four by 4, we get 16" (p.358). They defined a wrong multiplicative relationship. On the other hand, some students could define the $y=2x+2$ functional relationship. About 18% of 3rd grade, 25 % of 4th grade, and 14% of 5th grade students were defined the correct rule in the "Functional Particular-Multiplicative Relationship" level by regarding the constant term. For example, a fourth-grade student explained the rule by applying functional particular multiplicative relationship for 100 tables as "100× 2= 200 200+2=202". About 23% of 5th grade students responded at the "Emergent Functional-Variables." Lastly, about 3% of 4th grade students gave a response at the highest level, 'Function as Object.' To sum up, this study that was investigated in Turkey showed that students can think functionally, they can define function rules in variables in the early grades. Therefore, Türkmen and Tanışlı (2019) suggested that functional thinking and algebraic thinking should be placed in the curriculum in early grades.

Similarly, Tanışlı (2011) performed a study to investigate fifth-grade students' functional thinking ways by linear function tables. Task-based interviews were conducted using 16 tasks with four participants. Those tasks were presented in function tables in which square and triangles represented dependent and independent variables, respectively. The study focused on students' identifying patterns and their functional thinking ways. The researcher found out that students looked down columns on the table, so they focused on the constant difference between the dependent variable and independent variable recursively. Thus, they defined a recursive relationship. Moreover, it was examined that students looked at the table horizontally, and they focused on the difference between the dependent variable and the independent variable in each row. Therefore, in the first part, students could not define the functional relationship between variables. When students' functional thinking ways were examined, students were found to define correspondence relationship both additively and multiplicatively. For example, one of the students explained that “Whenever the number of triangles increases by 1, the number of square increases by 3” (p. 213), then he realized the multiplicative relationship and defined correspondence relationship between the number of triangles and the number of squares in natural language. He tested the function rule on the 60th step. In Turkey, fifth-grade students do not know to use variables as letters, so students defined the correspondence rules in natural language as “the number of triangles × 2 + 2= the number of squares” (p. 215). The study showed that fifth-grade students could think covariationally and define correspondence relationships in words and semi-symbolic forms that is using numbers and operations like ×,÷, +.Also, students used reasoning abilities in generalizing the functional relationships. So, this study suggested that teachers should support students to think functionally in multiple ways in order for them develop functional thinking in early grades.

Girit and Akyüz (2016) defined that generalization of patterns is important to develop algebraic thinking so they conducted a study about middle school (6th, 7th and 8th grades) students’ reasoning and strategies for generalizing patterns. 154 middle school students (48 6th graders, 59 7th graders, and 47 8th graders) participated in the study and answered 6 open-ended linear growth pattern problems including numeric, pictorial, and tabular representations. Students’ generalizations were assessed by two reasoning strategies; numerical reasoning and figural reasoning and two generalization strategies; using notations and using descriptive words. The results showed that 6th graders focused on numerical reasoning and descriptive words strategies by describing recursive pattern verbally (e.g., chairs increase by 3, p. 252). A few six graders (7%) used symbolic notation such as n·3, n·4+2, 4·n-1, n+2 (p.253). Seventh grade students tended to write algebraic expressions by trial and error. Unlike 6th graders, 7th grade students preferred to use formal notations and descriptive words strategies. Although the percentage of 8th grade students who wrote algebraic expression for generalizations was the same as 7th graders, 8th grade students wrote more correct expressions. In general, while the grade level increased, tendency to define algebraic notation also increased. However, most of the students who wrote algebraic expressions could not define correct generalizations. Lastly, students showed a tendency to give constant number for variables. Girit and Akyüz (2016) suggested that students should be provided opportunities to investigate relationship between patterns and variables by using different types of patterns. They also stated that the importance to ask questions to lead students define generalizations (e.g., asking far terms 50th, 100th).

2.3 Summary of the Literature Review

In the literature part, firstly, the theoretical background of the study, Kaput's (2008) algebraic thinking framework, and the place of functional thinking in this framework, were presented. Core Aspects A & B and Strands 1, 2 & 3 are strongly related to each other. Strand 2 refers to functional thinking and representations that were the basis of the present study. Then, the five big ideas of algebraic thinking that were described by Blanton et al. (2011) were presented. Big Idea 5 was defined as functional thinking, and it covered the scope of the study. Also, the relational meaning of the equal sign (Stephens et al., 2013) and the roles of variables that were covered in the big ideas defined by Blanton et al. (2011) were important for the design of this study. Next, the levels of sophistication describing generalizations and representations of functional relationships were presented based on Stephens et al. (2017). These levels were used to assess students' responses in addition to correctness in this study.

In the second part, intervention and non-intervention studies on functional thinking (e.g., Blanton et al., 2017; Blanton & Kaput, 2004; Cañadas et al. (2016); Isler et al., 2015; Isler et al., 2017) were reviewed. These studies revealed that students were able to describe functional relationships and represent them in multiple ways such as pictures, tables, graphs, words, and variables in early grades. Non-intervention studies revealed students' current understandings in functional thinking. They defined that students could define recursive and covariational relationships mostly; also, correspondence relationships. Generally, intervention studies were presented as a part of comprehensive early algebra intervention studies. They investigated students' level of functional thinking before the intervention, then they observed students' progression in defining and representing function rule after the intervention. These studies found that although students focused on recursive patterns at the beginning, they could describe function rule in words and in variables after the intervention.

In the last part, functional thinking studies in Turkey (e.g., Girit & Akyüz, 2016; Tanışlı, 2011; Türkmen & Tanışlı, 2019) were presented. Girit and Akyüz (2016) found that middle grade students tend to generalize patterns by algebraic expressions as the grades increase. Tanışlı (2011) investigated that fifth grade students could define covariational relationships and represent those relationships by words and semi-symbolic notations. Türkmen and Tanışlı (2019) found that fifth-grade students could define function rule in words and variables. Students were more successful in the $y=mx$ functional relationship than $y=mx+b$.

CHAPTER 3

METHODOLOGY

This chapter presents the design of the study, participants, context of the study, data collection methods, instrument, data analysis procedures, the validity and reliability of the study, assumptions of the study, limitations of the study, and ethics.

The present study aimed to investigate fifth-grade students' functional thinking skills. The following research questions were sought through this aim:

1. Is there a statistically significant mean difference between the functional thinking post-test scores of the 5th-grade students who attend the functional-thinking intervention and those who do not?
2. Is there a statistically significant mean difference between the functional thinking pre-test and post-test scores of the 5th-grade students who attend the functional-thinking intervention?
3. Is there a significant relationship between the two groups (5th-grade students who attend the functional-thinking intervention and those who do not) and the correctness in the functional thinking test items at pre-test and post-test?
4. How does 5th-grade students' functional thinking strategies differ in the functional-thinking test for those who attend the functional-thinking intervention, and who does not?

3.1 Research Design

The present study aimed to investigate the effect of functional-thinking instruction on 5th-grade students' functional thinking skills. To find answers to the research questions, both quantitative and qualitative methods were used. To Williams (2007), the quantitative method includes data collection that information can be measured by statistical methods to support or refute the hypothesis. Statistical tests were used to analyze the data and get quantitative results in the present study. Qualitative research aims to collect detailed information for the studied topic. Frankel et al. (2012) defined that “Qualitative data are collected in the form of words or pictures rather than numbers” (p. 427). In the present study, the Functional Thinking Test (FTT) was used as a pre- and post-test instrument. This test was comprised of open-ended questions. Students’ answers were analyzed by correctness and the strategies they used. The analysis of correctness was performed by using descriptive and inferential statistics. Therefore, the first, second, and third research questions were sought through quantitative research. Student strategies were coded according to a coding scheme prepared by the researcher. Thus, the fourth research question was sought by qualitative research.

The present study aimed to test hypotheses about the cause-and-effect relationship, so it was carried out with the control and experimental groups. These groups received a pre-test in the same week. Then, the experimental group attended a functional thinking intervention. At the end of the intervention, the groups received a post-test in the same week. The static group pretest-posttest design was used to assess the effect of these processes in the quantitative part of the study (Fraenkel et al., 2012). While the independent variable of the study is the functional thinking intervention, the dependent variable is students’ functional thinking skills. The design of the study was shown in Table 3.1.

Table 3. 1

The Symbolic Notation of The Static Group Pretest-Posttest Design

Groups	Pre-test	Intervention	Posttest
EG	O	FTI	O
CG	O	-	O

In Table 3.1, “EG” shows the experimental group, and “CG” shows the control group. In this study, the effect of functional thinking intervention on students’ functional thinking skills was investigated, so Functional Thinking Intervention

(FTI) was provided to the experimental group (EG). However, the control group (CG) did not attend any intervention process between the pre-test and post-test. To evaluate the effects of functional thinking intervention, an instrument, Functional Thinking Test (FTT), was used, which is shown as “O”. This instrument was used as both pre-test and post-test in both experimental and control groups.

3.2 Population and Sample

In this study, the target population was 5th-grade students in Ankara. By the convenience sampling method, two public schools were selected in Çankaya, Ankara. One of them was assigned as a control group, so these students participated in the pre-test and post-test in the same week with the experimental group. The school chosen as the control group applies foreign language intensive program to fifth grade students so students attend 13 lesson hours of English class each week. Also, students are enrolled to school according to their primary school GPAs, so students’ academic level might be higher than the average. The school principal assigned one of fifth-grade classes as the control group according to the schedule. The control school was a full-time school and had 520 students. The average class size was 26. All classes had two whiteboards and a smartboard. There were 42 teachers, and five of them were mathematics teachers. The school chosen as the experimental group was a full-time school as well and had 284 students. The average class size was 17. There were 26 teachers in this school, and 3 of them were mathematics teachers. There were 16 classes, and their physical conditions were similar. This school was similar to the control school about physical conditions. The principal was assigned one of fifth-grade classes as the experimental group. The experimental group students were administered the pre-test and post-test and the functional thinking intervention between the tests. Intervention was conducted by taking lessons from different teachers not to affect time schedule of teachers. This study was conducted during the Spring semester of the 2018-2019 academic year.

In the control group, there were 24 students; 12 girls and 12 boys. One of the students had a medical report during the pre-test, and another student did not participate in the post-test. Control group students interested in the study. Those asked questions to the researcher about context of the problems, variables, patterns and relationships between the variables at the end of the test. In the experimental group, there were 20 students; 7 girls and 13 boys. The experimental group was more diverse than the control group. There was a non-native student in the experimental group. The researcher provided materials by translating them into English and gave explanations in English throughout the study. Thus, she attended the intervention process actively. Also, there was an inclusive student. She did not participate in the intervention regularly because she participated in the *Individualized Education Program.* At the beginning of the intervention, the experimental group was not motivated sufficiently. However, students learned the flow of the lessons and became interested in the context of the activities in time. Some exit cards were applied as a competition to motivate the students.

The number of students who took the pre-test and the post-test in the experimental and control groups were presented in Table 3.2. The table shows that a total of 42 students composed the sampling of the study.

Table 3. 2

The Number of Students who took the Pre-test and the Posttest in the Experimental and Control Groups

Groups	Pre-test	Posttest	Pretest ∩ Posttest
Experimental	20	20	20
Control	23	23	22
Total	43	43	42

3.2.1 The Role of the Researcher

The researcher had a role as a designer and practitioner. The researcher designed the instruments and lesson plans. Also, she implemented the intervention and observed the classroom actively. She analyzed the data through quantitative and qualitative methods. The researcher did not have teaching experience at the time of the study.

3.3 Context of the Study

In the Grades 1-8 National Mathematics Curriculum, algebra is one of the learning domains, and objectives related to algebra starts at the 6th grade level for the first time and continues until the end of the middle grades (MoNE, 2018). Algebra is not explicitly mentioned in the curriculum before the 6th-grade level. However, there are some objectives related to the big five ideas of algebra: equivalence and equations, generalized arithmetic, functional thinking, variable, and quantitative reasoning (Blanton et al., 2011). Therefore, Table 3.3 presents the objectives related to functional thinking in the Grades 1-8 National Mathematics Curriculum provided by the Ministry of National Education (MoNE, 2018).

Table 3. 3

Objectives addressing functional thinking in Grades 1-8 (MoNE, 2018)

Grades	Numbering in the Curriculum	Objectives
1st Grade	M.1.2.3.1	Students find the rule of a pattern consisting of objects, a geometric object or figure, and completes the pattern by identifying the missing objects in the pattern.
2nd Grade	M.2.1.1.6	Students identify number patterns that have a constant difference, find the rule of the pattern, and complete the pattern by determining the missing item.
3rd Grade	M.3.1.1.7	Students expand and generate the number of patterns that have a constant difference.

Table 3.3 (continued)

5th Grade	M.5.1.1.3	Students find the required steps of the given number and figure patterns.
6th Grade	M.6.2.1.1	Students write an algebraic expression for the given verbal situation and write a verbal situation for the given algebraic expression.
	M.6.2.1.2	Students compute the value of the algebraic expression for different natural number values that the variable can take.
7th Grade	M.7.2.1.3	Students express the rule of the number patterns using letters and find the asked term of the pattern when the rule was expressed by letters
	M.7.2.2.2	Students identify linear equations with one unknown and construct a linear equation with one unknown corresponding to the given real-life situations.
	M.7.2.2.3	Students solve equations with unknown.
	M.7.2.2.4	Students solve the problems that require constructing linear equations with one unknown.
8th Grade	M.8.2.2.1	Students solve the problems that require constructing linear equations with one unknown.
	M.8.2.2.2	Students identify the coordinate system with its characteristics and show the coordinates.
	M.8.2.2.3	Students express how one of the variables changes in relation to the other using a table and an equation when there is a linear relationship between the variables.
	M.8.2.2.4	Students draw the graph of linear relationships.
	M.8.2.2.5	Students formulate equations, tables, and graphs for real-life situations involving linear relationships and interpret them.

Data collection tools and lesson plans that were used in this study were prepared according to the objectives defined by Blanton et al. (2018) that will be presented in Table 3.8.

3.4 Data Collection Method

The purpose of the study was to investigate 5^{th}-grade students' functional thinking skills and the effects of functional thinking intervention (FTI) on students' functional thinking through the Functional Thinking Test (FTT) that was constructed by the researcher. Through this purpose, students participated in a pre-test and a post-test. The pre-test and post-test were identical. Moreover, an intervention was applied to the experimental group students between the tests. The FTT and lesson plans used during the intervention were revised after the pilot study. The revisions will be mentioned in the pilot study section.

The pre-test and post-test aimed to investigate students' knowledge about functional thinking. After the pre-test, experimental group students attended a functional thinking intervention lasting two weeks. Then, both the experimental group and control group students participated in the post-test, which was identical to the pre-test, at about the same time. Students were allowed 40 minutes to take the pre-test and post-test.

Data were collected in the Spring semester of the 2018-2019 academic year (see Table 3.4). The data collection procedure started when the approvals were obtained from the University Human Subjects Ethics Committee (see Appendix C) and the Ministry of National Education (see Appendix D). After the written consent forms were collected from the parents, the data collection process started. The schedule of the data collection process was presented in Table 3.4 below.

Table 3. 4

The Schedule of The Data Collection Process

Time	Administration
04.02.2019 - 15.02.2019	Conducting the pilot study
18.03.2019 (1 hour for each group)	Applying pre-test to the control group Applying pre-test to the experimental group
19.03.2019 (2 Hours)	Implementing the 1st lesson plan of FTI
21.03.2019 (2 Hours)	Implementing the 2nd lesson plan of FTI
25.03.2019 (3 Hours)	Implementing the 3rd lesson plan of FTI
27.03.2019 (2 Hours)	Implementing the 4th lesson plan of FTI
29.03.2019 (2 Hours) 02.04.2019 (1 Hour)	Implementing the 5th lesson plan of FTI
05.04.2019 (1 hour for each group)	Applying post-test to the control group Applying post-test to the experimental group

3.5 Instrument

In the present study, to investigate students' knowledge of functional thinking, a pre-test and post-test were conducted. The instrument that was used in the present study is the Functional Thinking Test (FTT) (see Appendix A). This test was applied as both pre-test and post-test in the experimental group and control group. The instrument of the study will be presented in Section 3.5.1 in detail.

3.5.1 The Functional Thinking Test

Functional thinking includes generalization of relationships between co-varying quantities, representing these relationships in multiple ways, including tables, words, equations, and graphs. The present study aimed to investigate students' existing functional thinking and to investigate the effect of a functional thinking intervention on experimental students' functional thinking skills. Through this aim, the Functional Thinking Test (FTT) was designed. The FTT aimed to assess students' functional thinking skills so the main problems were related to $y=mx$ and $y=mx+b$ types of equations (see Appendix A). Through these questions, students were expected to identify data, organize the data in a table, define patterns in this table, define the rule of the relationship between two quantities in variables and words and draw the coordinate graph to show the relationship. The Functional Thinking Test and the intervention lessons were designed according to the instructional objectives in the framework that will be shared in Table 3.8. (Blanton et al. 2018) Also, the objectives in the Grades 1-8 National Mathematics Curriculum (MoNE, 2018), which were covered in the FTT, were given in Table 3.5.

Table 3. 5

Objectives in the 2018 Middle School Mathematics Curriculum Covered in The Functional Thinking Test

Objectives	Item in the FTT
M.3.1.1.7 Students expand and generate the number of patterns that have a constant difference. M.5.1.1.3 Students find the required steps of the given number and figure patterns.	1a, 2a,1b, 2b
M.7.2.2.2 Students identify linear equations with one unknown and construct a linear equation with one unknown corresponding to the given real-life situations. M.8.2.2.5 Students formulate equations, tables, and graphs for real-life situations involving linear relationships and interpret them.	1d, 2d 1e, 2e

Table 3.5 (continued)

M.7.2.2.3 Students solve equations with unknown. M.8.2.2.1 Students solve the problems that require constructing linear equations with one unknown.	1f, 2f
M.7.2.2.4 Students solve the problems that require constructing linear equations with one unknown. M.8.2.2.1 Students solve the problems that require constructing linear equations with one unknown.	2g
M.8.2.2.3 Students express how one of the variables changes in relation to the other using a table and an equation when there is a linear relationship between the variables.	1c, 2c
M.8.2.2.4 Students draw the graph of linear relationships.	1g

The test had two main problems, and each had seven sub-questions. The objectives of each question in the functional thinking test are given in Table 3.6 and Table 3.7.

For the content validity of the test, questions were reviewed by an expert studying in the early algebra field of mathematics education. Also, a pilot study was performed. According to feedbacks, the test was revised.

Item 1 asked students to define the *y=2x* functional relationship. This item was adapted from Stephens et al. (2017). Students were given a contextual problem about drawing circles and were asked questions to define, represent, and generalize the functional relationship using variables and words. Item 1, its sub-questions, and objectives addressed were given in Table 3.6.

Table 3. 6

Item 1 of The Functional Thinking Test and Objectives Addressed

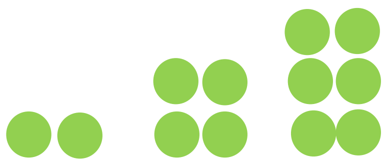
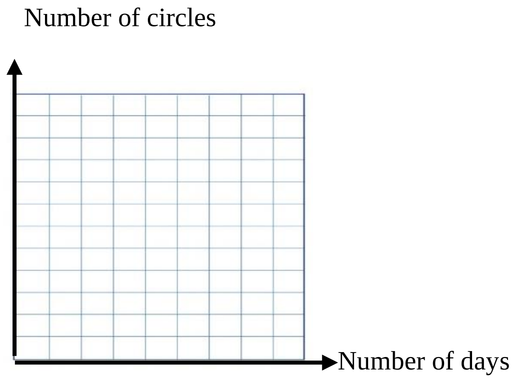
<table>
<tr><th>Item 1</th><th>Objectives</th></tr>
<tr><td>Each day in the class, Selin creates a picture by drawing circles joined together. Following are the pictures of circles that she drew on each day:

Day 1 Day 2 Day 3</td><td></td></tr>
<tr><td>a) How many circles are in her picture for Day 5?</td><td>Finding the value of unknown steps in a pattern</td></tr>
<tr><td>b) Organize your information in the given table.
<table><tr><th>Number of Days</th><th>Number of Circles</th></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr></table></td><td>Generating the data and organize in a function table</td></tr>
<tr><td>c) Which patterns do you see in the table? Describe.</td><td>Identifying patterns, define it in words, in variables</td></tr>
</table>

Table 3.6 (continued)

d) In your own words, describe the relationship between the number of days and the number of circles.	Identifying the function rule in words
e) Explain the relationship between the number of days and the number of circles by using variables (letters).	Identifying the function rule in variables
f) How many circles will be in the picture that Selin draws on the 100th day of the school? Show how you got your answer.	Using the function rule to predict far function values
g) Show the relationship between the number of days and the number of circles on the graph below. 	Constructing a coordinate graph to represent the relationship between two variables

Note. Adapted from “A Learning Progression for Elementary Students’ Functional Thinking” by A. C. Stephens, N. Fonger, S. Strachota, I. Isler, M. L. Blanton, E. Knuth, and A. M. Gardiner, 2017, Mathematical Thinking and Learning, 19(3), p.149.

Item 2 asked students to define the $y=3x+2$ functional relationship. This item was adapted from Blanton (2008). Some changes were made to the item after the pilot study. These changes were mentioned in detail in Section 3.5.2. Students were given a contextual problem about saving money and were asked to define, represent, and generalize the functional relationship by using variables and words. Item 2, its sub-questions, and objectives addressed were given in Table 3.7.

Table 3. 7

Item 2 of The Functional Thinking Test and Objectives Addressed

Item 2	Objectives
There are 2 TL in Mert's piggy bank at the beginning. Every week Mert's dad gives him 3 TL for helping with chores around the house. Mert is saving his money in his piggy bank to buy a bike.	
a) How much money is there in Mert's piggy bank in total at the end of Week 2? Week 3? Week 4?	Finding the value of unknown steps in a pattern
b) Organize your information in a table.	Generating the data and organize in a function table
c) Which patterns do you see in the table? Describe.	Identifying patterns, define it in words, in variables
d) In your own words, describe the relationship between the number of weeks and the total amount of the money in Mert's piggy bank.	Identifying the function rule in words
e) Explain the relationship between the number of weeks and the total amount of money in Mert's piggy bank by using variables (letters).	Identifying the function rule in variables
f) How much money will be in Mert's piggy bank in total at the end of the 30th week? Show how you got your answer.	Using the function rule to predict far function values
g) If a bike's cost is 95 TL, how many weeks will it take to have enough money for the bike?	Given the dependent variable, determining the independent variable

Note. Adapted from *Algebra and the elementary classroom: Transforming thinking, transforming practice* (p.179) by M. L. Blanton, 2008, Heinemann.

3.5.2 The Pilot Study of the Functional Thinking Test

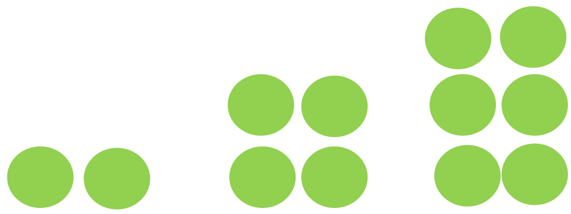
The study began with the pre-test that lasted a class hour. Thirty-six students attended to the pre-test. This test included open-ended questions; a total of 14 sub-questions under two main questions. The same test was applied as a post-test after FTI. At the end of the pilot study, some changes were required about the questions in the test. There were two main questions in the test; these questions were not changed in terms of their context. However, some wording of the questions changed to make them clearer. The changes made were detailed below.

The first main question was related to the $y=mx$ functional relationship and it had seven sub-questions. In the first question, item 1a was omitted because the shape included the answer. Item 1b was changed as “How many circles are in her picture for Day 5?” to observe students’ different strategies other than drawing to solve the question.

Figure 3. 1

Old Version of Item 1a and Item 1b in the FTT

1. Each day in the class, Selin creates a picture by drawing circles joined together. Following are the pictures of circles that she drew on each day:



Day 1 Day 2 Day 3

a) How many circles are in her Picture for Day 1? Day 2? Day 3?

b) Draw the Picture that she draws on Day 5.

Item 1d was not clear for some students so the expression was changed as “Which patterns do you see in the table? Please describe”. Also, students could define more than one strategy in this way.

Figure 3. 2

Old Version of Item 1d in the FTT

d) Describe the pattern in the table.

Item 1e and item 1f were questions asking students to write the rule that explains the relationship between the number of days and the number of circles. However, again, these were not clear to students, especially the old version of item 1f. Since, when students were asked to find the number of circles on any day of the school, students generally defined a constant number of the day and applied the function rule, so they gave a numerical result instead of writing the function rule in variables. Therefore, item 1e was changed as "Use words to describe the relationship between the number of the days and the number of circles." Item 1f was changed as "Use variables (letters) to describe the relationship between the number of days and the number of circles" correspondingly to make what students are expected to do clear.

Figure 3. 3

Old Version of Item 1e and Item 1f in the FTT

e)	Write the rule that explains the relationship between the number of days and the number of circles.
f)	How can you find the number of the circles that Selin draws on any day of the school?

The second main question was related to $y=mx+b$ functional relationship and it had seven sub-questions (see Appendix A). The changes in this question were similar to changes in the first main question.

Item 2d was changed as "Use words to describe the relationship between the number of the days and the number of circles," and item 2e was changed as "Use variables (letters) to describe the relationship between the number of days and the number of circles" parallel to the wording of item 1.

Figure 3. 4

Old Version of Item 2d and Item 2e in the FTT

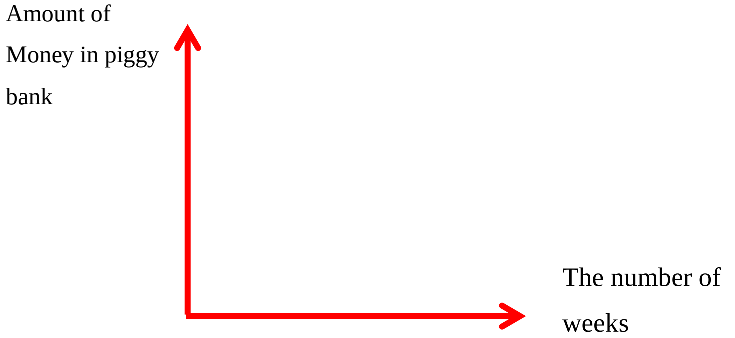
d) Write the rule that explains the relationship between the number of weeks and the amount of money.
e) How can you find the amount of money that Mert saves in any number of the week?

In the old version of item 2h, students were asked to draw a graph to show the relationship between the number of weeks and the amount of money in Mert's piggy bank. This question required the *y=3x+2* functional relationship and most of the students had difficulty in this type of function. Most of the students could not define the function rule. Many students ignored the amount of money in the piggy bank at the beginning so they defined the relationship as *y=3x* instead of the *y=3x+2*. In this case, students defined the points incorrectly. Therefore, this graph question was omitted from the second question and added to the first question which required the "*y=2x*" functional relationship.

Figure 3. 5

Old Version of Item 2h in the FTT

h) Show the relationship between the number of weeks and the amount of money in Mert's piggy bank.



The post-test was the same with the pre-test, so all changes detailed above were valid for the post-test.

3.6 Intervention

3.6.1 The Functional Thinking Intervention

The study aimed to investigate 5th grade students' functional thinking skills. Through this aim, a control group and an experimental group were chosen. Then, students were administered a pre-test to investigate their existing knowledge about functional thinking. After the pre-test, experimental group students attended the intervention lasting 12 lesson hours (about 3 weeks). There were five lesson plans to develop students' functional thinking skills (see Appendix B). Each activity was designed towards the objectives aimed at the types of functions; $y=x$, $y=2x$, $y=x+1$, and $y=2x+1$, *in* order. All lesson plans were developed by using three instructional methods: questioning, discussion, and cooperative learning. Students were asked to work in pairs. Each lesson had an activity sheet and an exit card. Some of these exit cards were solved by group work as a competition between groups and some of them were given as homework because of the time limitation. At the end of the intervention, the experimental and control group students attended the post-test at about the same time. The flow of the intervention was presented in Table 3.8. Details of the intervention will be presented in the following sections.

Table 1.8

The Instructional Sequence of the Functional Thinking Intervention

Objectives Students should be able to	**Time**	**Materials**	**Instructional Methods**
1st Lesson Examine the role of the equal sign; the relational meaning of the equal sign "Identify a variable to represent an unknown quantity" "Examine the role of variable as a varying quantity" "Represent a quantity as an algebraic expression using variables" "Interpret an algebraic expression in a context"	2 class hours	Activity Sheet Boxes	Questioning Discussion Cooperative Learning Individual work

Table 3.8 (continued)

2nd Lesson			
"Generate data and organize in the function table" "Identify variables and their roles" "Identify a recursive pattern, describe in words" "Identify covariational relationship and describe in words" "Identify function rule and describe in words and variables" (The type of function: $y=x$) Use a function rule to predict near and far data		Activity Sheet Exit Card	Questioning Discussion Cooperative Learning Individual work
3rd Lesson			
"Generate data and organize in the function table" "Identify variables and their roles" "Identify recursive pattern, describe in words" "Identify covariational relationship and describe in words" "Identify function rule and describe in words and variables" (The type of function: $y=2x$ and $y=3x$) Use a function rule to predict near and far data "Construct a coordinate graph to represent problem data"	3 class hours	Activity Sheet Exit Card	Questioning Discussion Cooperative Learning Individual work
4th Lesson			
"Generate data and organize in the function table" "Identify variables and their roles" "Identify recursive pattern, describe in words" "Identify covariational relationship and describe in words" "Identify function rule and describe in words and variables" (The type of function: $y=x+1$) Use a function rule to predict near and far data	2 class hours	Activity Sheet Exit Card Ribbon Scissors	Questioning Discussion Cooperative Learning Individual work

Table 3.8 (continued)

5th Lesson			
"Generate data and organize in the function table" "Identify variables and their roles" "Identify recursive pattern, describe in words" "Identify covariational relationship and describe in words" "Identify function rule and describe in words and variables" (The type of function: $y=2x+1$ and $y=2x+2$) Use a function rule to predict near and far data	3 class hours	Activity Sheet Exit Card Ribbon Scissors	Questioning Discussion Cooperative Learning Individual work

Note. Adapted from Implementing a Framework for Early Algebra by M. Blanton et al., C. Kieran (ed.) *Teaching and Learning Algebraic Thinking with 5- to 12-Year-Olds,* ICME-13 Monographs, pp. 36-37, 2018, Springer Cham.

3.6.1.1 The First Lesson Plan

The objectives of the first lesson plan were presented in the Table 3.8. Two class hours were allocated for the first lesson.

The study aimed to define a functional relationship between two quantities in different ways: words, tables, variables, and graphs. Students were expected to define functional relationships in correspondence form that shows the relationship between two quantities in variables. Thus, this required students to understand the meaning of the equal sign, meaning of unknown and variable, and using them in an equation as prerequisite knowledge. Therefore, the intervention process was started by discussing the meaning of equal sign, unknown, and variable.

In the beginning, students were expected to find missing numbers in the given equalities. The researcher gave time to students to think. Then, the relational meaning of the equal sign was handled through the discussion of students' strategies. The researcher asked, such as "*What is the meaning of the symbols (dot, letter, boxes, line) here?*" and *"Why do we use them?"* (See Figure 3.6). Thus, students were aimed to comprehend the meaning of the unknown and the use of symbols to represent these unknowns and also the relational meaning of the equal sign, which means “the same as” (Stephens et al., 2013).

Figure 3. 6

The 1st Question in Task 1

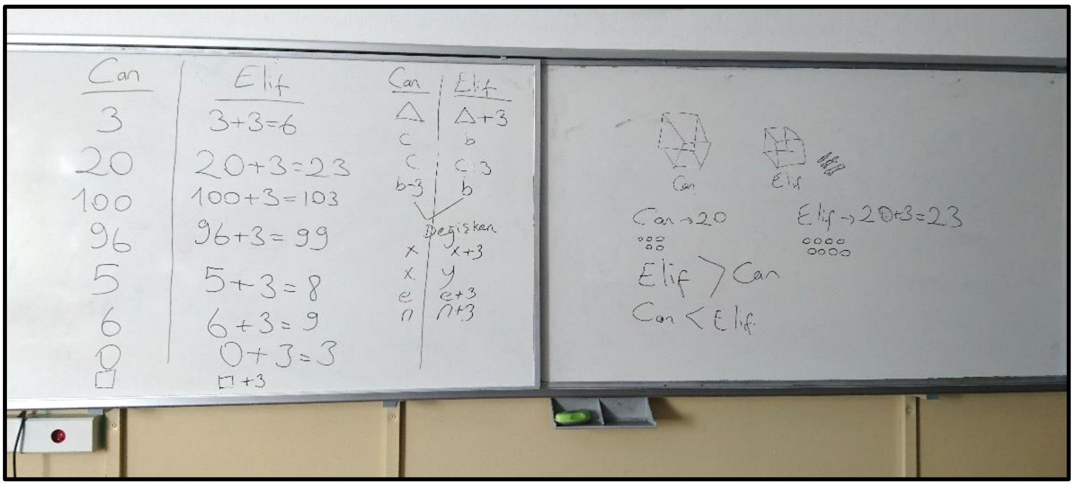
1) Find the unknown numbers in the given equations below.

$15+\square = 22$	$\_\_\_ - 37 = 48$
$8 + 42 = \ldots. + 8$	$9 \cdot 6 = 6 \cdot x$
$5+4= 3+\triangle$	$a + 11 = 13 + 7$
$53+ 27= n+ 23$	$118+ y = 62+ 119$

In the middle part of the lesson, the aim was to help students to have an initial sense of variables. The researcher brought two identical boxes into the class and asked *“Elif and Can have a box of candies. There is an equal number of candies in their boxes. Elif has three more candies in her hand. How can you define the number of candies they have?”* This task was originally used by Carraher et al. (2008) Students worked in pairs, and the researcher guided them to use different ways like drawing and table. Most of the students tended to define constant values for candies like if Can has ten candies, Elif has 13. Some pairs drew a certain number of candies for Can and three more for Elif. The researcher organized these numbers in a table (see Figure 3.7), and students realized that there was not a certain number of candies so that the unknown value can be shown by symbols like $\square$. In this case, If Can’s number of candies was defined by $\square$,Elif’s number of candies would be $\square$ +3. The researcher asked, *“Can you define the number of candies in another way?”*. Thus, students learned to use a letter to define the number of candies. One of the groups showed the number of candies Elif has by “*b*” and the number of Can’s candies by “*b-3*”.

Figure 3. 7

Students' answers for Task 2



The third question aimed students to define two variables representing two quantities in an equation. Students were expected to define the "*x + y=28*" equation in a word problem. Students found the different number of compositions that their sum equals 28, such as 14+14, 10+18. Then, the researcher wrote all the numbers on the board in a table. Students used symbols to represent the number of poetry books and the number of novels. The researcher asked students to write an equation by using variables. Students had difficulty in that part. So, the researcher guided students to write the equation by variables.

3.6.1.2 The Second Lesson Plan

By the second lesson plan, functional thinking-based activities were started. Students were expected to define the *y=x* functional relationship (see Table 3.8). Two class hours were allocated for this lesson plan.

The researcher reminded students of the meaning of the variable at the beginning of the lesson. Then, she asked students to work in pairs on the problem. Students were expected to provide the answer as 2 chickens give 2 eggs; 3 chickens give 3 eggs; 4 chickens give 4 eggs, and 5 chickens give 5 eggs (see Appendix B). The researcher asked students to write these findings in a given table. The researcher asked, *"What are the variables here?"* Students answered as "the number of chickens and the number of eggs." The researcher asked, *"Which patterns do you see in the table?"* and wrote all answers on the table. Most of the students defined recursive patterns as "+1". Then, the researcher asked, *"How many eggs can Ali get from 50 chickens or 100 chickens?"* to make students realize the relationship between the number of chickens and the number of eggs. Some of the students had difficulty in defining covariational and functional relationships, so the researcher guided students as *"As the number of chickens increases by…., the number of eggs increases by …"*. Then, students became aware of the covariational relationship between two variables. One of the students said, *"The number of chickens equals the number of eggs."* The researcher asked her to explain in detail and then to the class if they agree with her or not. Other students also agreed with this. The researcher aimed to have students define this functional relationship by using variables (letters). Then, the researcher asked, *"How can you define the number of eggs for any number of chickens?"*. Some of the students continued to assign a constant number like *"20 eggs for 20 chickens"*. Some of the students used symbols to define the functional relationship. However, most of the students answered as "$\square = \square$". The researcher asked students the meaning of this equality, and students explained that the number of chickens equals to the number of eggs. Therefore, these boxes represent the number of eggs for any number of chickens. One of the students defined this relationship as "$\square = \triangle$." The researcher dwelled on that the number of chickens and the number of eggs are different variables, so different symbols represented them. The researcher asked students, *"How can you define this relationship by using letters (variables)?"* Students wrote different equations like T=Y, a=b. The last question aimed to have students use the function rule for far values, so students find the answer as 50 eggs for 50 chickens. The researcher summarized the lesson, and the exit card was given as homework because of the time limitation.

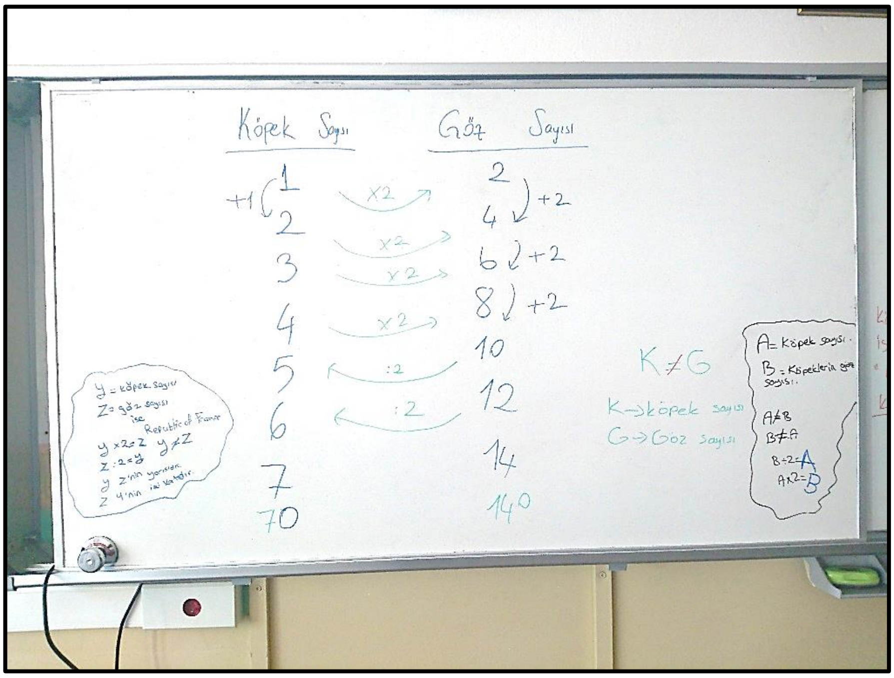
3.6.1.3 The Third Lesson Plan

Students were expected to define the $y=2x$ functional relationship (see Table 3.8). Three class hours were allocated for this lesson plan.

The researcher repeated the previous lesson by discussing the exit card given as homework. The flow of the lesson was similar to the second lesson. Students were familiar with the questions in the activity sheet and the researcher's expectations, so they concluded this activity quicker than the previous activity. Students worked in pairs. The researcher asked them to read the problem situation and discuss it with their pairs for the first item. The problem was, *"Imagine that you are an officer working in the dog shelter and want to find the number of eyes that dogs have. How many eyes does a dog have? How many eyes do two dogs have? How many eyes do three dogs have?"*(Blanton & Kaput, 2004, p.136). Most of the students gave correct answers. Then, the researcher asked students to organize their findings in a table. The researcher asked, *"What are the variables here?"* Students answered as *"the number of dogs and the number of eyes."* The researcher asked, *"Which patterns do you see in the table?"* and wrote all answers on the board. In the beginning, students defined recursive patterns as *"The number of dogs increases by one."*, *"The number of eyes increases by two."* The researcher asked, *"Is there any relationship between the number of dogs and the number of eyes?"* and gave time students to think. Some students defined that *"The number of the eyes is not equal to the number of eyes"* and *"The number of eyes is bigger than the number of dogs."* Then, they showed them these relationships by variables as $y \neq z$ (see Figure 3.8).

Figure 3. 8

Students' answers for Task 3



The researcher aimed students to define a covariational relationship and wrote, *"As the number of dogs increases by ...., the number of eyes....."* on the board. Then, students concluded the sentence correctly, "As the number of dogs increases by 1, the number of eyes increases by 2." The researcher asked students if there is any other relationship they realized. One of the students described that the number of eyes equals two times the number of dogs. Then, the researcher asked students to define this relationship by using variables (letters). One of the pairs used symbols as; " $\square \times 2 = \square$ ". Those explained that " $\square$ " represents the number of dogs, and $\square \times 2$ represents the number of eyes. Then, the researcher asked the students to discuss this. The researcher suggested that students put any constant number instead of boxes. Then students tried with different numbers, and they agreed that this equation is not correct for the functional relationship. Most of the students' answers were like *"K×2=G", "G=2×K". "G"* represents the number of eyes, and "*K"* represents the number of dogs. One of the students described the functional relationship differently; *"The number of dogs is half of the number of eyes."* Then, the researcher asked the students to write this relationship by using variables (letters). Students wrote "*G÷2=K", "K=G÷2"*. After this, students were expected to use the function rule by further values. The last question asked them to represent the relationship between the number of dogs and the number of eyes on a coordinate graph. Students were familiar with the picture graph and bar graph so they drew the bar graph at first. The coordinate graph was drawn on the activity sheet; the x-axis represented the number of dogs, and the y-axis was for the number of eyes. The researcher asked students to go back to the table, then, students and the researcher placed the values on the graph taken from the table. The researcher summarized the lesson and distributed exit cards. Students worked in groups of four. Each group decided on a group name. Then, they answered questions on the exit card in 20 minutes. Students were expected to define the *y=3x* functional relationship through the same steps in activity sheets. Questions were discussed with all groups, and the groups got a star for each correct answer. In the end, the group that received the maximum number of stars won the game.

3.6.1.4 The Fourth Lesson Plan

Students were expected to define the *y=x+1* functional relationship. The objectives of the fourth lesson plan were presented in Table 3.8. Two class hours were allocated for this lesson plan.

Students worked in pairs through the activity. The researcher distributed the activity sheet, a scissor, and a ribbon for each group. She explained that students were expected to cut the ribbon in the given number of cuts and record the number of pieces in the given table. The researcher observed students during the activity, and after being sure the cutting process was concluded, she asked them to define patterns in the table. A number of students described the recursive pattern as *"The number of cuts increases by one."* and *"The number of pieces increases by 1"*. The researcher asked, *"Is there any relationship between the number of cuts and the number of* *pieces?"* to have students realize the functional relationship. Then, students defined the functional relationship between the number of cuts and the number of pieces. They defined that the number of pieces is one more than the number of cuts. One of the groups described the functional relationship inversely as *"The number of cuts is one less than the number of pieces."*. The researcher asked students to write this relationship by using variables (letters). Students wrote *"P=K+1"* and *"K=P-1"*. The researcher asked, *"What do P and K represent in the equations?"*. Students explained that *"P"* showed the number of pieces, and *"K"* showed the number of cuts. After this, students were expected to use the function rule by further values. The last question was, *"At the end of a cutting process, 100 ribbon pieces were gotten. How many cuts were done?"*. Students used the function rule and got the answer as *"99 cuts"*. The researcher asked more questions like this. By the help of the questions, *"How many cuts were performed to get 201 pieces?"* and *"If we cut a rope in 48 times, how money pieces would be gotten?"* students used the function rule in different cases. The researcher distributed exit cards. This exit card asked three things they learned that day, two things they found interesting, and one thing they wanted to ask.

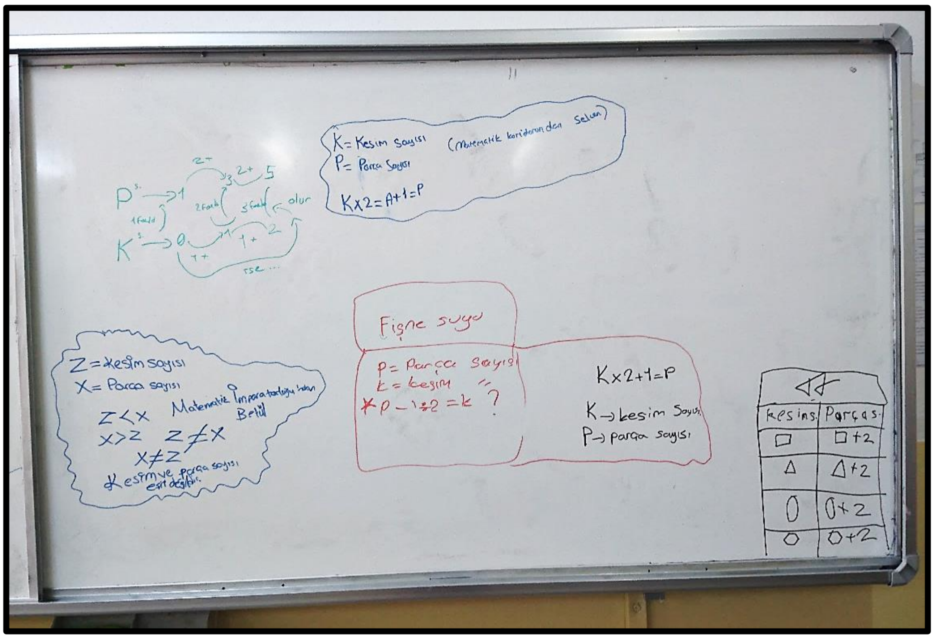
3.6.1.5 The Fifth Lesson Plan

Students were expected to define the *y=2x+1* functional relationship. The objectives of the fifth lesson plan were presented in Table 3.8.

Students worked in pairs through the activity. The researcher distributed the activity sheet, a scissor, and ribbons for each group. In this activity, the researcher distributed four different colors of ribbons; red, yellow, blue, pink to each group. There was a knot in the center of all ribbons. The researcher explained the cutting process. Then, all groups and the researcher cut the red ribbon together. The ribbon was folded from the middle then, the ribbon became double-deck. Then, the ribbon was cut once. The researcher asked students how many pieces there were. They got three pieces; one of them was a knotted piece. The researcher observed the groups and helped them during cutting and recording data in the table. Students cut blue ribbon two times, pink ribbon three times, and yellow ribbon four times in order in the same way. The researcher used different colors of ribbon to make it easier to decide the number of pieces. Students had difficulty in concluding the cutting process because of folding the ribbon. Students could define the recursive pattern in the table as *"The number of pieces increases by 2".* They could not find any covariational and functional relationship in the table. Therefore, the researcher drew a different table on the board. In the previous activities, tables were comprised of two columns for two variables. However, for this activity, one column for the number of cuts, one column for the number of pieces without a knot, one column for the number of knotted pieces, and one column for the total number of pieces were drawn as described in Isler et al., (2014). Firstly, students realized that the number of pieces increases by two (recursive pattern). The researcher asked students if there was a relationship between the number of cuts and the number of pieces. The researcher asked students if there was any other relationship they realized. One of the groups described that $z \neq x$ and $x > z$; "x" represents the number of pieces, and "z" represents the number of cuts (see Figure 3.9). Then, students defined a covariational relationship as *"As the number of cuts increases by one, the number of pieces increases by two."* One of the groups described that *"The number of the pieces equals two times the number of cuts"* (functional relationship). The researcher asked students to write this relationship by using variables. Students were familiar with this relationship from the previous activities so that they could write *"$K \times 2 = P$"* and *"$P = 2 \times K$"*. "K" showed the number of cuts, and "P" showed the number of pieces. After these, the researcher asked, "*What about the number of the knotted piece?*" Students realized that there is one knotted piece in all cases. Through discussions and the researcher's help, students could notice that one knotted piece were added in each case, the relationship between the number of cuts and the number of total pieces, P=2×K+1. They defined that the total number of pieces is equal to 1 more than two times of the number of cuts. As a result, they were able to write this relationship by using variables like "$K \times 2 + 1 = P$". However, some students wrote the equation as "$K \times 2 = A + 1 = P$" (see Figure 3.9). The researcher reminded students the meaning of the equal sign, and she suggested students put constant numbers instead of variables in the equation to see if that works. Then, students could understand the misconception in this equation regarding the equal sign and corrected it as “$K{\times}2{+}1{=}P$”. Moreover, one of the pairs defined the functional relationship inversely as “$P{-}1{\div}2{=}K$”. The researcher made students aware of the order of operations. Lastly, students were expected to use the function rule by further values. The last question was, *“How many pieces will we get at the end of the 20 cuts?”*. Thus, students used the function rule and found the answer as 41 pieces. This lesson plan activity was the hardest one for students. They were observed to have difficulty in using data in the table, noticing the functional relationship, and representing this relationship by using variables.

Figure 3. 9

Students’ answers for Task 5



The exit card was given as homework, but one class hour was allocated for this exit card. Defining the $y{=}2x{+}2$ functional relationship was expected from students in the exit card. It was comprised of two parts; in the first part, students were expected to find the number of people for the given number of tables. There was not any person sitting on the sides of the tables (see Figure 3.14). Students were able to define the functional relationship (*y=2x*) for this part. In the second part, it asked how the functional rule changes if people added on the sides of the tables. Most of the students had difficulty in defining the relationship between the number of tables and the number of people in this part. In the question, if there was one table, four people could sit. If there were two tables, six people could sit. However, students just focused on the first figure, and they wrote the rule as "$K=4 \times M$". "K" represented the number of people, "M" represented the number of tables. The researcher wrote this equation on the board and asked students to discuss it. The researcher asked students to organize data in the table. Some students drew further steps of the given figure of the pattern. Then, students realized that the number of people increased by two. They could define the recursive pattern in the table correctly, but they could not write the function rule. Then, the researcher asked students to ignore the people who sat at the beginning and at the end. So, they realized that the number of people equals two times the number of tables, then we add the two for the ignored people who were supposed to sit on the sides.

3.6.2 The Pilot Study of Functional Thinking Intervention

Before the main study, the tests and intervention were piloted with 37 5th grade students in one of the middle schools in Ceyhan, Adana at the beginning of the Spring semester of the 2018-2019 academic year. Five lesson plans were designed for the intervention. It was assumed that two class hours would be allocated for each lesson plan. To minimize the unexpected situations during the intervention of the main study, all lesson plans were implemented at the pilot study.

The study, which was composed of 5 lessons, took 12 class hours in total, including two class hours for the pre-test and post-test.

These lessons started with an activity about the meaning of the equal sign, unknowns, and variable concepts and continued with functional thinking activities. Each activity was designed towards the objectives aimed at the types of functions; $y=x$, $y=2x$, $y=x+1$, and $y=2x+1$ *in* order (See Table 3.8). The intervention process was planned as inquiry-based learning and group work. Activities aimed that students explore the functional relationships by the small group works and whole class discussions. Through these aims, the classroom was arranged for cooperative learning. Eight groups had four students, and one group was formed by five students. Each activity had an exit card that was related to a functional relationship studied during the lesson. However, because of time limitation, some of these exit cards were assigned as homework. These homework problems were discussed the next day at the beginning of the lesson. Finally, at the end of the 10 hours of the intervention process, students took a post-test. As described in Table 3.8, the instructional objectives were adopted from Blanton et al. (2018).

In the first lesson, students were given five equations, including missing numbers, and students were asked to find those numbers (see Figure 3.10). This question aimed to have students use the meaning of the equal sign. Also, the role of symbols representing an unknown number was discussed. One of the students found the missing numbers by using the structural meaning of the equal sign (e.g., numbers changed place so 8+42=42+8) and other students responded by using operational meaning of equal sign (e.g., 8+42=50+8). Therefore, three more items were added to the first question for the main study after the pilot study to help students understand the meaning of the unknown and the relational meaning of the equal sign (see Figure 3.10). Also, the balance concept was mentioned during the first activity to make students comprehend the meaning of the equal sign. In the pilot study, students had difficulty in understanding the relational meaning, the same as, structural meaning of the equal sign, and the difference between meaning of unknown and variable. Thus, some additional questions were added to the first activity in the main study. These additions were detailed below.

Figure 3. 10

Old Version of the 1st Question in Task 1

Find the unknown numbers in the given equations below.

$15+\square = 22$

$\_\_\_\_ - 37 = 48$

$8 + 42 = \ldots. + 8$

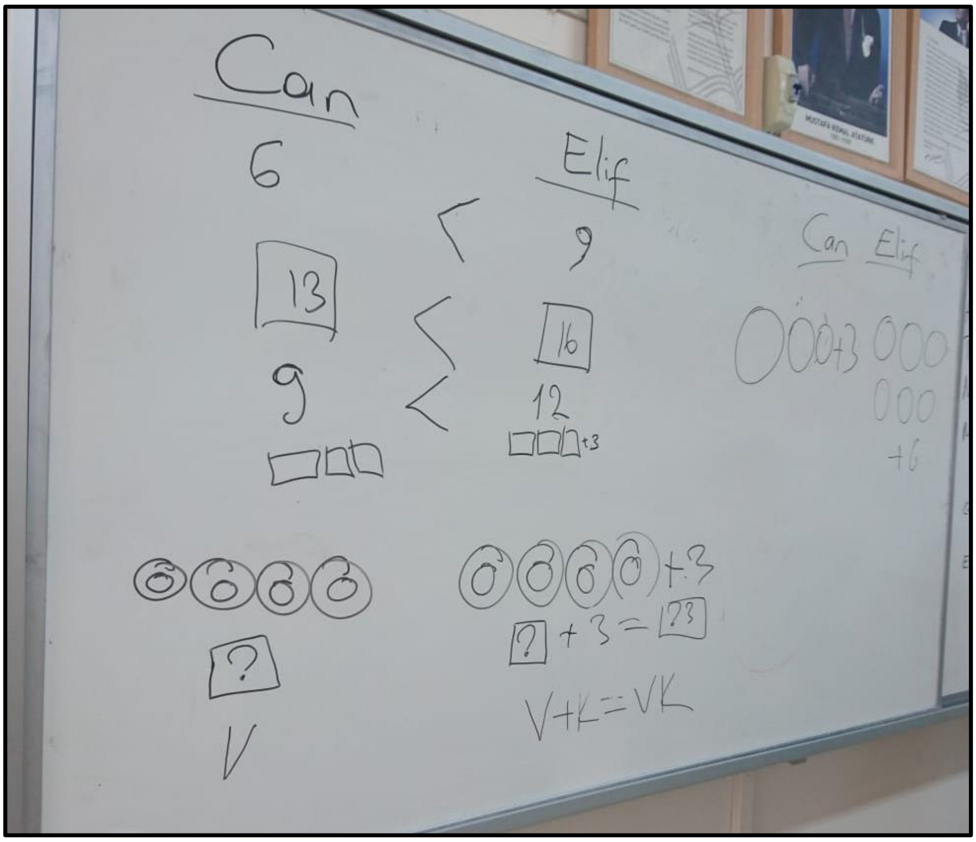
$a + 11 = 13 + 7$

$53+ 27= n+ 23$

In the second and third questions, students were expected to use variables to define varying quantities. Students were expected to progress from using symbols such as □, ○ to using letters e.g., c,x,y,n to define varying quantities. The context was the same for the two questions. Students worked on a word problem (see Appendix B). The researcher gave students time to think about the problem. In the beginning, students came up with values for the number of candies Can and Elif have. The researcher wrote all answers on the board. The majority of the groups described that "The number of Can's candies is less than the number of Elif's candies." The researcher made students realize multiple cases for the number of candies, and a class discussion was held to define symbols to represent the unknown number. Some groups drew boxes to use symbols, but they drew three boxes to represent the number of Can's candies and added as "+3" (see Figure 3.11).

Figure 3. 11

Students' answers for the 2nd question in Task 1



The researcher mentioned the unknown and variable concepts and explained the use of letters to represent the variable amounts. Then, students gave a response like "*Can: V and Elif: V+3, Can: BJK and Elif: BJK+3*".

In the third question, students were expected to use two different variables (see Figure 3.12). However, it was observed that the students continued to use the information in the second question. Since the same context was used, most of the students wrote only one case as "Can has six candies, and Elif has nine candies, and they have 15 candies in total." Therefore, that question was changed in terms of the context as "Tuna loves reading story and poetry books and he imagines setting up a library with books he read. He has 28 story and poetry books in total: a) Express the number of story and poetry books in Tuna's bookshelf in different ways and b) Write mathematical expression by using a variable that shows the number of story and poetry books in Tuna's bookshelf" was the new problem context used in the main study (see Appendix B).

Figure 3. 12

Old Version of the 3rd Question in Task 1

Elif and Can, each have a box of candies. There are 15 candies in total. According to this; a)Express the number of candies that Elif and Can has in different ways. (Picture, table…) b)Write the mathematical expression by using a variable (letter) that shows the number of candies Elif and Can each have. c)What do variables mean in the expression you wrote above?

The exit card problem was removed from the first activity (see Figure 3.13). The exit card was given as homework, but the majority of the students had difficulty in solving this problem. This problem required both multiplication and addition in an equation as "$2x+y=20$", so they had difficulty in defining the relationship between quantities and writing an algebraic expression. While most of the students could not solve the problem, others gave values to find the answer instead of using variables. Therefore, this problem was not used in the first activity of the main study.

Figure 3. 13

Exit Card of the First Lesson

If Alp has 20 TL to spend on 2 TL pencil and 1 TL eraser, how many ways can he spend all his money without receiving change? a)Write the mathematical expression by using variables (letter) that shows the relationship between the number of pencils, number of erasers and total money for Alp's shopping. b)What do variables mean in the mathematical expression you write above?

In general, students tended to define a single value for a varying quantity. Also, they had difficulty in symbolizing the varying quantity. The second, third, and fourth lessons were not changed in the main activity.

The fifth lesson, and the last lesson, was designed for $y=2x+1$ functional relationship. Students cut different colors of ribbons in different numbers of cuts and recorded the number of pieces they got in the given table. They had difficulty in completing the cutting process. Students were not able to define patterns in the table. There was an exit card for the function of $y=2x+2$ as homework. The questions in the exit card were in the same order with the tests and activities. Students were asked to define the relationships between the number of tables and the number of people who can sit at the tables at a birthday party (see Figure 3.14). Students generated the data, organized them in a table, and defined the functional relationship in words, and wrote an equation that is a rule for this relationship. Students had difficulty in defining the rule in words and in writing the function rule as $y=2x+2$. Therefore; in the main study, this problem was changed in the way that helps students to reach the function $y=2x+2$ through two parts; the first part problem required the functional relationship $y=2x$ (see Figure 3.14), in the second part, students were asked to define the change in the problem and to write the new rule as $y=2x+2$.

Figure 3. 14

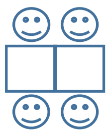
The Exit Card of The Fifth Lesson

PART A

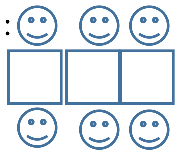
Nehir is planning for a birthday party. She wants to make sure she has a seat for everyone. She has square desks.

She can seat 2 people at one desk in the following way:

If she joins another desk to the first one , she can seat 4 people :



If she joins another desk to the second one , she can seat 6 people :

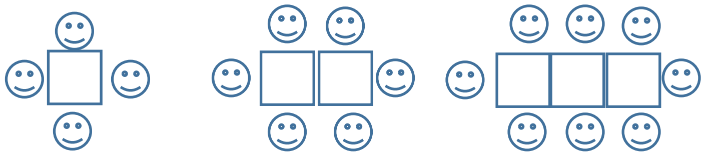


a) Is there a relationship between the number of desks and the number of people? If so, use words to write the rule that describes this relationship.

b) Use variables to write the rule that describes this relationship. What do these variables mean?

PART B

Nehir figured out she could seat more people if two people sat on the end of the rows of desks. For example, If Nehir had 2 desks, she can seat 6 people; if Nehir had 3 desks, she can seat 8 people.



f) How does new information affect the rule you wrote in part (c) and (d)?

g) Use words to write the new relationship between the number of desks and the number of people.

Note. Adapted from “The development of children's algebraic thinking: The impact of a comprehensive early algebra intervention in third grade” by M. Blanton, A. Stephens, E. Knuth, A. M. Gardiner, I. Isler, and J. S. Kim, 2015, *Journal for research in Mathematics Education*, *46*(1), pp.85-86.

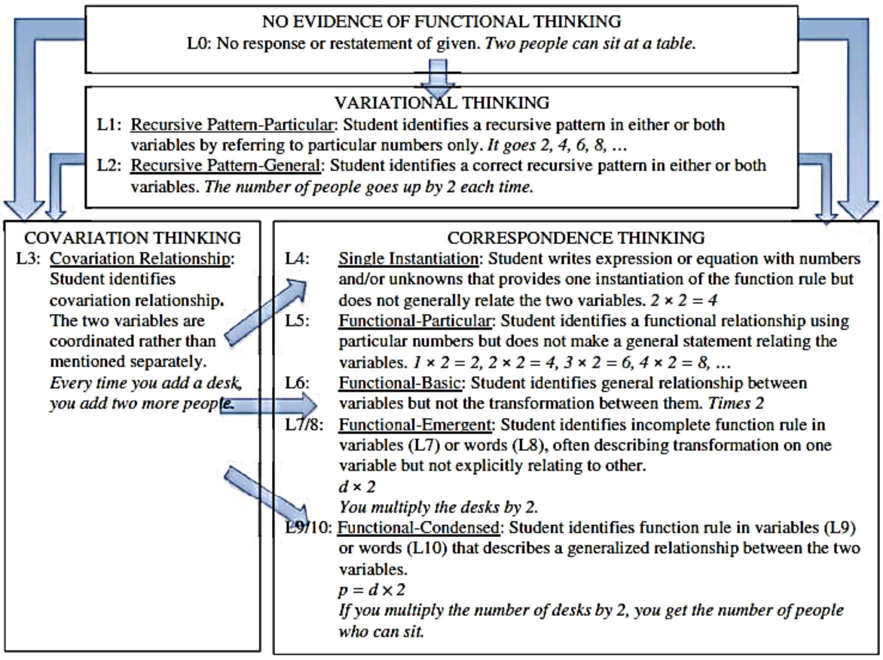
To sum up, it was observed that students had difficulty in comprehending the meaning of variables and the meaning of the equal sign in the pilot study. The flow of the lesson plans for functional thinking was the same across the lessons, so students were familiar with the expectations of activities. The last activity required more time for the cutting process and describing functional relationships.

3.7 Data Analysis

Data were analyzed by both the qualitative and quantitative methods. By the qualitative part of the data analysis, students' answers were assessed through correctness and strategies. Therefore, a coding scheme document was created based on Stephens et al. (2017) (see Figure 3.15).

Figure 3. 15

Levels of Sophistication Describing Grades 3-5 Students' Generalization and Representation of Functional Relationships



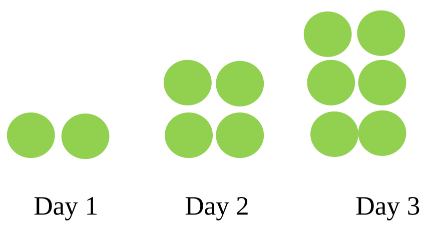
Note. Reprinted from "A Learning Progression for Elementary Students' Functional Thinking" by A.C. Stephens, N. Fonger, S. Strachota, I. Isler, M. Blanton, E. Knuth, and A.M. Gardiner, 2017, *Mathematical Thinking and Learning, 19*(3), p. 153. Copyright 2018 by Taylor & Francis.

Items that asked to define patterns, describe function rule in words and variables were coded according to these levels (see Figure 3.15). For some items, codes varied according to the structure of the item. Items that asked to find near value, far value and represent the relationship on the graph were coded by emerging codes and also using codes from the related literature. However, some students answered questions in the way that did not correspond to any level and was not of interest to the study or the response was not discernible. Those answers were coded as “Other (O)” (see an example in Table 3.8). Moreover, “Answer Only (AO)” code was used in the case that students gave only answer without showing their work. “No Response (NR)” code was used when students did not respond to the item. For some items, in the case of getting similar answers, new codes emerged. These codes were discussed and negotiated among coders. As an example, in Item 1c, students were asked to define patterns they saw in the table. Table 3.8 presents an example from the coding scheme.

Table 3. 8

Coding Scheme for Item 1c

Each day in the class, Selin creates a picture by drawing circles joined together. Following are the pictures of circles that she drew on each day:



Which patterns do you see in the table? Describe.

Strategy Code	Response
Functional Condensed-Words (FC-W)	Gün sayısı ile ikiyi çarpar isek Daire sayısını buluruz. Daire sayısından iki bölersek Gün sayısını buluruz.

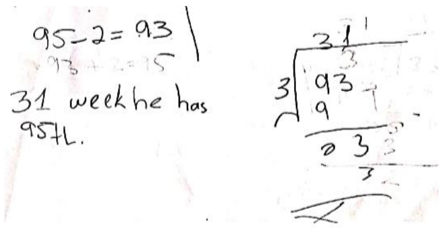
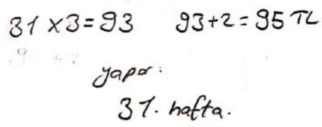
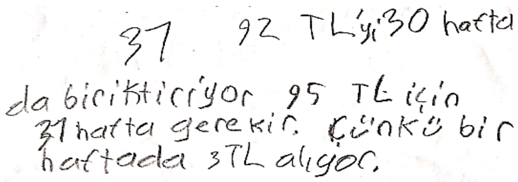
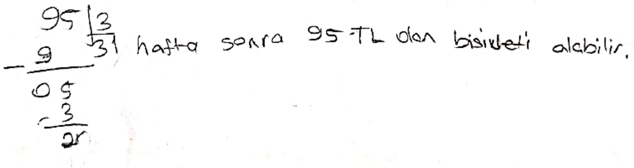
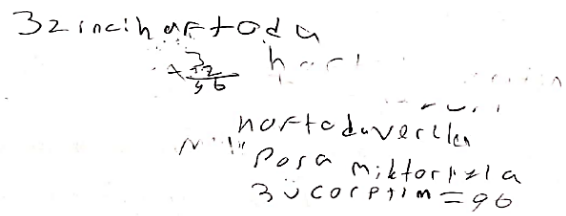
Table 3.8 (continued)

Functional Emergent-Words (FE-W)	Gün sayısı Daire sayısının ÷2 Daire sayısı gün sayısının x2
Functional Basic (FB)	x2 ve ÷2 örüntüleri vardır
Covariational Relationship (CR)	gün sayısı 1er 1er artarken daire sayısı 2şer 2şer artar
Recursive Pattern-General (RP-G)	Daire sayısı 2 artıyor.
Recursive Pattern-Particular (RP-P)	+2 +2 +2 +2 +2 2 4 6 8 10 12
Other (O)	Daire örüntüsü vardır. ikiden yukarıya giden örüntüler vardır

For item 1c, there was no new emerging code. However, in the item 2g, "If a bike's cost is 95 TL, how many weeks will it take to have enough money for the bike?", students were expected to find the value for the dependent variable given the value for the independent variable using the function rule. There were no strategy codes for this item in Stephens et al. (2017) levels of sophistication. Therefore, students' answers were examined and some strategies were utilized from the literature. "Unwinding (U)" and "Guess and Test" strategies were adopted from Blanton, Stephens et al. (2015). Also, new strategies were defined by examining the common responses used by students, (e.g., PI and D3 codes) (see Table 3.9).

Table 3. 9

Coding Scheme for Item 2g

Strategy Code	Response
Unwinding (U)	
Guess and Test	
From the Previous item-2f (PI)	
Dividing by 3 (D3)	
Other (O)	

In the quantitative part of the analysis was performed by statistical tests at IBM SPSS Statistics 24. To investigate whether there was a significant difference between experimental group students' test scores and control group students' test scores independent samples T-test was performed. To investigate whether there was a significant difference between experimental group students' pre-test and post-test scores, paired samples T- test were conducted. Lastly, Chi-Square test of Independence was conducted for analysis at the item level.

3.8 Validity and Reliability

Validity and reliability are essential terms in the selection and design of the instruments for all studies in the literature. The validity of the study is appropriateness, meaningfulness, correctness, and usefulness of the researcher's inferences on the data. Reliability refers to the consistency of the scores and answers across time, locations, and researchers (Frankel et al., 2012). The instrument FTT's validity and reliability were conducted by the researcher. These will be explained in detail next. The items in the instrument and tasks in the lesson plans were adapted from the literature and they were reviewed by a mathematics education researcher to increase the validity of the instrument

3.8.1 Internal Validity

According to Fraenkel et al. (2012), "any relationship observed between two or more variables should be unambiguous as to what it means rather than being due to something else" (p. 166). There were some threats to internal validity in this study, such as subject characteristics, mortality, location, instrumentation, testing, and implementation (Fraenkel et al., 2012).

The subject characteristic could be a threat to internal validity, there were two different classes in which one is control group, and the other one is the experimental group in the study. The participant schools were selected by the convenience sampling method. The control and experimental group assignments were done based on talking with the principals of the schools and their schedules. The schools were located in the same district, and their physical characteristics were similar. On the other hand, one of the schools, the control school, accepted students according to their grade-point average. The other one has not that criteria, so their classrooms had students at varying levels in terms of academic achievement.

The location threat was controlled. Both classes had approximately the same environment during the study.

The mortality threat is the loss of participants. To prevent this, the researcher requested the students to participate in the pre-test and post-test. All students in the experimental group attended both pre-test and post-test. However, two students in the control group had medical reports during the test implementation. So, one of the students could not participate pre-test, and the other could not participate post-test.

Another threat to internal validity is testing. To minimize this threat, there were three weeks between pre-test and post-test, which were identical in both experimental and control groups.

The instrumentation threat includes data collector characteristics, data collector bias, and instrument decay. In the study, the same researcher collected pre-test and post-test data, implemented the intervention, and analyzed the pilot study and the main study data. Therefore, data collector characteristics and data collector bias threats were eliminated. Fraenkel et al. (2012) defined that "Instrument decay threat occurs if the nature of the instrument (including the scoring procedure) is changed in some way" (p. 170). Essay type questions were used in the instrument. A coding scheme was prepared, and all students' answers were scored according to it by the researcher. Students' answers were evaluated by one by for each item.

The implementation is a threat to internal validity. Fraenkel et al. (2012) defined that instructor characteristics, and different instructors may affect post-test scores. To prevent this threat, the intervention and data collection processes were applied by the same researcher at the pilot study and the main study.

3.8.2 External Validity

According to Creswell (2012), external validity is “the validity of the cause-and-effect relationship being generalizable to other persons, settings, treatment variables, and measures” (p. 303). The present study was carried out by convenience sampling method. Because a nonrandom sampling method was used, the generalization of results to population might be limited.

“Ecological generalizability refers to the degree to which the results of a study can be extended to other settings or conditions” (Fraenkel et al., 2012, p. 105). The findings can be generalized to other groups having similar characteristics.

3.8.3 Reliability

Reliability is the consistency of data gotten with instruments over time, circumstances, and location (Fraenkel et al., 2012). The reliability of the qualitative part of the data, coding strategies, was checked by the inter-rater agreement method. To supply the reliability of FTT, a pilot study was performed. To assess the reliability of the coding, the interrater agreement was sought randomly selecting 20% of the data and coding it independently by a second coder who was an academician working on early algebra for the pilot study. The same process was repeated for the main study. The second recorder was a master student who studied early algebra in her research. In the cases where the agreement between two coders was lower than 80%, the codes were discussed and revisions were reflected to the analysis until 80% agreement between the two coders was reached. The instrument was considered as appropriate and reliable to measure proposed variables.

Moreover, Cronbach Alpha values were calculated for the quantitative part of the instrument. There are 14 items in the instrument that were assessed by correctness. Cronbach Alpha value for pre-test was 0.657, and this showed that the instrument was adequately reliable (Taber, 2018). Cronbach Alpha value for post-test was 0.813, so the instrument had high reliability (Taber, 2018) (see Table 3.10).

Table 3. 10

Cronbach's Alpha Value for Functional Thinking Test for Pre- and Posttest

Test Type	Cronbach's Alpha Value
Pretest	.657
Posttest	.813

3.9 Assumptions and Limitations

3.9.1 Assumptions

In the present study, it was assumed that standard conditions were supplied for the implementation of instruments. Moreover, it was assumed that all participants reflected their own opinions, and they did not affect each other.

3.9.2 Limitations

Creswell (2012) defines that "in the convenience sampling, the researcher selects available participants for the study. Therefore, convenience sampling makes it difficult to defend the population representativeness" (p. 145). The schools were selected according to the researcher's convenience, so this could limit the generalizability of the results. The results can be generalized for just other groups having the same characteristics. Moreover, the generalization of the results might be limited because of the limited sample size of the study.

In the present study, there were two groups; experimental and control groups. These groups were selected from different schools in the same district in Ankara. While the physical conditions of the schools were similar, the characteristics of students, due to control school's enrolling students based on previous GPAs, might have been different.

In the present study, the intervention lasted 12 hours, but this duration was used mainly in functional thinking activities. It was observed that the first lesson plan could be extended to have students comprehend the meaning of variables and the equal sign. In future studies, this could be extended if needed.

3.10 Ethics

The collected data, participants' names and personal information were kept confidential. Participants were coded by assigning numbers, so the interrater coder did not have any personal information about the participants in the analysis.

Before the data collection process, official permissions were received from the Human Research Ethics Committee and MoNE (see Appendices C and D). Moreover, a parental approval form was prepared that included the information about the process and permissions from both the control group and the experimental group parents were obtained. Students were asked to participate in the study, and their permissions were obtained orally.

CHAPTER 4

RESULTS

This chapter presents the results of the descriptive and inferential statistics analysis and findings in detail to respond to the research questions below.

1. Is there a statistically significant mean difference between the functional thinking posttest scores of the 5^{th}-grade students who attend the functional-thinking intervention and those who do not?
2. Is there a statistically significant mean difference between the functional thinking pretest and posttest scores of the 5^{th}-grade students who attend the functional-thinking intervention?
3. Is there a significant relationship between the two groups (5^{th}-grade students who attend the functional-thinking intervention and those who do not) and the correctness in the functional thinking test items at pretest and posttest?
4. How does 5^{th}-grade students' functional thinking strategies differ in the functional-thinking test for those who attend the functional-thinking intervention, and who does not?

4.1 Inferential Statistics Results of Functional Thinking Test

Students' answers were analyzed in terms of both correctness and strategy. Answers were assessed as "0" in the case of incorrect answers and no response, and "1" in the case of correct answers.

4.1.1 The Results of Pretest

To investigate whether there was a significant mean difference between the experimental group and control group's pretest scores, an independent samples T-test was conducted. Before the T-test, assumptions were checked and presented in the following sections.

4.1.1.1 Assumptions of the Independent Samples T-test for the Pretest

There are five assumptions of the independent sample T-test; the level of measurement, random sampling, independence of observations, normal distribution, homogeneity of variance (Pallant, 2011).

4.1.1.1.1 **Level of Measurement**

According to Pallant T-test requires continuous scale as dependent variable instead of discrete ones. In the current study, the dependent variable was the students' pretest scores, so this assumption was satisfied.

4.1.1.1.2 **Random Sampling**

According to Pallant (2011), "this is often not the case in real-life research" (p. 205). Therefore, this assumption was not sought in this study.

4.1.1.1.3 **Independence of Observation**

Pallant (2011) explained that measurements and observations should not be affected by each other and other measurements. (p. 205). In the current study, it was assumed that the measurements were not affected by each other.

4.1.1.1.4 **Normal Distribution**

In this study, the sample size of both groups was smaller than 50 so Shapiro-Wilk Test was conducted to check the normality assumption.

Table 4. 1

The result of the Shapiro-Wilk Test for the Pretest

	Statistics	df	Sig.
Experimental Group	.943	20	.274
Control Group	.944	23	.215

p >.05

According to the test of normality, for both groups, experimental and control group p > α. (α = .05). So, both groups had normal distribution.

4.1.1.1.5 **Homogeneity of Variance**

Pallant (2011) explained that “the variability of scores for each of the groups is similar” (p. 206). To test this, Levene’s test for equality of variance was performed.

Table 4. 2

The result of the Levene’s Test for the Pretest

	F	Sig.	t	df
Equal variances assumed	.000	.986	-1.518	41
Equal variances not assumed			-1.512	39.514

p >.05

According to Levene’s test p> α (.986 > .05). So, the variances of the groups are equal.

The five assumptions of the independent sample T-test were satisfied, so the data was found appropriate for the test.

4.1.1.2 The Results of Independent Samples T-test for the Pretest

The Functional Thinking Test included 14 items in total. Thus, the maximum point was 14 in the pretest that students could get. Twenty students in the experimental group and 23 students in the control group were administered the pretest. Table 4.3 presents the descriptive statistics of the pretest for both groups.

Table 4. 3

Descriptive Statistics of Scores in the Pretest for Both Groups

	Experimental Group	Control Group
N	20	23
Minimum	0	3
Maximum	8	10
Mean	4.75	5.65
Standard Deviation	1.99	1.89

As seen from the Table 4.3, the experimental group students' mean score in pretest (M= 4.8, SD = 2.0) was lower than the control group students' mean score in pretest (M= 5.7, SD = 1.9).

To investigate whether there was a significant mean difference between experimental group and control group students' pretest scores, Independent samples T-test was conducted. The result of the T-test was presented in Table 4.4 below.

Table 4. 4

The results of the Independent Samples T-test for the Pretest

		t-test for Equality of Means						
							95% Confidence Interval of the Difference	
		t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Pretest	Equal variances assumed	-1.518	41	.137	-.90217	.59441	-2.10260	.29825
	Equal variances not assumed	-1.512	39.514	.138	-.90217	.59657	-2.10835	.30401

An independent samples T-test was conducted to compare the pretest scores for experimental and control groups. There was no significant difference in scores for the experimental group (*M*= 4.75, *SD* = 1.99) and the control group (*M*= 5.65, *SD* = 1.89; t (41) = −1.52, *p* = .14, two tailed). The magnitude of the differences in the means (mean difference = −.90, 95% *CI*: -2.10 to .30) was small (eta squared = .05).

4.1.2 The Results of Posttest

To investigate whether there was a significant mean difference between the experimental group and control group posttest scores, independent samples T-test was conducted. Before the T-test, assumptions were checked and presented in the following sections.

4.1.2.1 Assumptions of the Independent Samples T-test for the Posttest

There are five assumptions of the independent sample T-test; the level of measurement, random sampling, independence of observations, normal distribution, homogeneity of variance (Pallant, 2011).

4.1.2.1.1 **Level of Measurement**

Pallant (2011) defined that dependent variable should be continuous. In the current study, the dependent variable was the students' posttest scores, so this assumption was satisfied.

4.1.2.1.2 **Random Sampling**

According to Pallant (2011), "this is often not the case in real-life research" (p. 205). Therefore, this assumption was not sought in the study.

4.1.2.1.3 **Independence of Observation**

In the current study, it was assumed that the measurements were not affected by each other.

4.1.2.1.4 **Normal Distribution**

In this study, the sample size of both groups was smaller than 50, so the Shapiro-Wilk Test was conducted to check the normality assumption.

Table 4. 5

The Result of the Shapiro-Wilk Test for the Posttest

	Statistics	df	Sig.
Experimental Group	.951	20	.376
Control Group	.949	23	.282

$p > .05$

According to the test of normality, each group's p values (.376 for the experimental group and .282 for the control group) are bigger than the alpha value (.05). So, both groups had a normal distribution for posttest scores.

4.1.2.1.5 **Homogeneity of Variance**

To test this assumption, Levene's test for equality of variance was performed. Table 4.6 presents the results of the Levene's test.

Table 4. 6

The Result of the Levene's Test for the Posttest

	F	Sign.	t	df
Equal variances assumed	5.775	.021	1.001	41
Equal variances not assumed			.968	30.254

$p < .05$

According to Levene's test $p < \alpha$ (.021 < .05). The significance value violated the homogeneity of variance assumption. Therefore, a non-parametric alternative of the t-test for independent samples, Mann-Whitney U test was conducted.

4.1.2.2 Mann Whitney U Test for the Posttest

The assumptions of the Mann-Whitney U test are independence of observation and random sampling (Pallant, 2011). These assumptions were supplied by the posttest scores for both groups and detailed in section 4.2.1.

Table 4. 7

The Results of the Mann-Whitney U Test for the Posttest

	Mann-Whitney U	Z	Asymp.Sig. (2-tailed)
Posttest	199.500	-.750	.453

The Mann-Whitney U Test revealed no significant mean difference in the posttest scores of the experimental group ($M = 23.5$, $n = 20$) and the control group ($M = 20.7$, $n = 23$), $U = 199.5$, $z = -.60$, $p = .45$, $r = .11$.

4.1.3 The Result of the Paired Samples T-test

To investigate whether there is a significant difference between experimental students' pretest and posttest scores, paired samples t-test was planned. Before the T-test, the assumptions were checked.

4.1.3.1 Assumptions of the Paired Samples T-test

There are three assumptions for paired-samples t-test; the level of measurement, independence of observation, normality.

4.1.3.1.1 **Level of Measurement**

Paired-samples t-test requires one categorical independent variable and one continuous dependent variable (Pallant, 2011). In the study, the dependent variable was test scores measured at different times (pretest and posttest scores). So, this assumption was satisfied.

4.1.3.1.2 **Independence of Observation**

Pallant (2011) explained that "data must be independent of one another; that is, each observation or measurement must not be influenced by any other observation or measurement" (p.205). In the current study, it was assumed that the measurements were not affected by each other.

4.1.3.1.3 **Normal Distribution**

The sample should have a normal distribution. In the study, the sample size was less than 50, so the Shapiro-Wilk Test was conducted to check the normality assumption.

Table 4. 8

The Results of the *Shapiro-Wilk Test for the Tests*

	Statistic	df	Sig.
Pretest Scores	.943	20	.274
Posttest Scores	.951	20	.376

$p > .05$

The Shapiro-Wilk test indicated that both tests' p values (.274 for pretest and .376 for posttest) are bigger than the alpha value (.05). Therefore, the data had a normal distribution.

All assumptions were satisfied, so data was appropriate for the paired-samples t-test.

4.1.3.2 The Results of the Paired Samples T-test

Students participated in a pretest, an intervention process, and a posttest throughout the study. To investigate whether there was a significant mean difference between students' pretest and posttest scores paired samples t-test was conducted. Table 4.12 presents the descriptive statistics for the tests.

Table 4. 9

Descriptive Statistics of Test Scores of Experimental Group

	Pretest	Posttest
N	20	20
Mean	4.75	7.05
Standard Deviation	2.00	3.53

As seen in Table 4.9, the mean of experimental group increased from 4.75 to 7.05 after the intervention.

Table 4. 10

The Results of Paired-Samples T-Test for the Experimental Group

		Paired Differences							
					95% Confidence Interval of the Difference				
		Mean	Std. Deviation	Std. Error Mean	Lower	Upper	t	df	Sig. (2-tailed)
Pair 1	Posttest - Pretest	2.30000	3.65772	.81789	.58813	4.01187	2.812	19	.011

A paired-samples t-test was conducted to evaluate the impact of the intervention on students' scores on the Functional Thinking Test (FTT). There was a statistically significant increase in test scores from pretest (M = 4.75, SD = 2.00) to posttest (M = 7.05, SD = 3.53), t (19) = 2.81, $p < .05$. The mean difference in test scores was 1.16, with a 95% confidence interval ranging from .59 to 4.01. The eta squared statistic (.29) indicated a large effect size.

The result shows that there is a difference between pre-test and post-test mean scores of experimental group. The functional thinking intervention help experimental group students to develop functional thinking.

4.1.4 The Results of Chi-Square Test for Independence

The Functional Thinking Test (FTT) had 14 items. A Mann-Whitney U test confirmed that there was no significant mean difference in the posttest scores of the experimental and control groups. To investigate whether there was a significant difference between the groups at the item level, Chi-Square tests for independence were conducted.

4.1.4.1 The Results of Chi-Square Test for Independence for Pretest

In the pretest 14 items were analyzed by Chi-Square test and results were presented in detail below. In the case of violating 'minimum expected cell frequency' assumption, Pallant (2011) suggested using Fisher's Exact Probability Test values.

Table 4. 11

The Results of the Chi-Square Test for Independence for Item 1a at Pretest

	Value	df	Asymptotic Significance (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	3.709	1	.054		
Continuity Correction	1.758	1	.185		
Likelihood Ratio	4.853	1	.028		
Fisher's Exact Test				.092	.092
Linear-by-Linear Association	3.623	1	.057		
N of Valid Cases	43				

$p > .05$

A Chi-square test for independence (with Fisher's Exact Test) indicated that there is no significant difference between the experimental and control groups for Item 1a in the pretest. p-value is bigger than the alpha value (.05).

Table 4. 12

The Results of the Chi-Square Test for Independence for Item 1b at Pretest

	Value	df	Asymptotic Significance (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	2.412	1	.120		
Continuity Correction	.684	1	.408		
Likelihood Ratio	3.174	1	.075		
Fisher's Exact Test				.210	.210
Linear-by-Linear Association	2.356	1	.125		
N of Valid Cases	43				

$p > .05$

A Chi-square test for independence (with Fisher's Exact Test) indicated that there is no significant difference between the experimental and control groups for Item 1b in the pretest. p-value is bigger than the alpha value (.05).

Table 4. 13

The Results of the Chi-Square Test for Independence for Item 1c at Pretest

	Value	df	Asymptotic Significance (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	.020	1	.889		
Continuity Correction	.000	1	1.000		
Likelihood Ratio	.020	1	.889		
Fisher's Exact Test				1.000	.595
Linear-by-Linear Association	.019	1	.890		
N of Valid Cases	43				

$p > .05$

A Chi-square test for independence (with Fisher's Exact Test) indicated that there is no significant difference between the experimental and control groups for Item 1c in the pretest, $p > .05$.

Table 4. 14

The Results of the Chi-Square Test for Independence for Item 1d at Pretest

	Value	df	Asymptotic Significance (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	6.982	1	.008		
Continuity Correction	5.201	1	.023		
Likelihood Ratio	7.912	1	.005		
Fisher's Exact Test				.011	.009
Linear-by-Linear Association	6.820	1	.009		
N of Valid Cases	43				

$p < .05$

A Chi-square test for independence (with Fisher's Exact Test) indicated that control group students significantly outperformed experimental group students for item 1d at pretest $p < .05$.

For the item 1e, there is no statistics constructed because both experimental group and control group students' answers were incorrect. Therefore, the item 1e is constant.

Table 4. 15

The Results of the Chi-Square Test for Independence for Item 1f at Pretest

	Value	df	Asymptotic Significance (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	.487	1	.485		
Continuity Correction	.066	1	.798		
Likelihood Ratio	.497	1	.481		
Fisher's Exact Test				.669	.403
Linear-by-Linear Association	.475	1	.490		
N of Valid Cases	43				

$p > .05$

A Chi-square test for independence (with Fisher's Exact Test) indicated that there is no significant difference between the experimental and control groups for Item 1f in the pretest, $p > .05$.

For the item 1g, there is no statistics constructed because both experimental group and control group students' answers were incorrect. Therefore, the item 1g is constant.

Table 4. 16

The Results of the Chi-Square Test for Independence for Item 2a at Pretest

	Value	df	Asymptotic Significance (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	.064	1	.801		
Continuity Correction	.000	1	1.000		
Likelihood Ratio	.064	1	.801		
Fisher's Exact Test				1.000	.541
Linear-by-Linear Association	.062	1	.803		
N of Valid Cases	43				

$p > .05$

A Chi-square test for independence (with Fisher's Exact Test) indicated that there is no significant difference between the experimental and control groups for Item 2a in the pretest, $p > .05$.

Table 4. 17

The Results of the Chi-Square Test for Independence for Item 2b at Pretest

	Value	df	Asymptotic Significance (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	.096	1	.756		
Continuity Correction	.000	1	1.000		
Likelihood Ratio	.097	1	.755		
Fisher's Exact Test				1.000	.569
Linear-by-Linear Association	.094	1	.759		
N of Valid Cases	43				

$p > .05$

A Chi-square test for independence (with Fisher's Exact Test) indicated that there is no significant difference between the experimental and control groups for Item 2b in the pretest, $p > .05$.

Table 4. 18

The Results of the Chi-Square Test for Independence for Item 2c at Pretest

	Value	df	Asymptotic Significance (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	1.422	1	.233		
Continuity Correction	.774	1	.379		
Likelihood Ratio	1.437	1	.231		
Fisher's Exact Test				.349	.190
Linear-by-Linear Association	1.389	1	.239		
N of Valid Cases	43				

$p > .05$

A Chi-square test for independence (with Yates Continuity Correction) indicated that there was no significant difference between the experimental and control groups for Item 2c, $\chi^2(1, n = 43) = .77$, $p = .379$, $phi = -.40$.

For the item 2d, there was no statistics were computed because both experimental group and control group students' answers were incorrect. Therefore, the item 2d is constant.

For the item 2e, there was no statistics were computed because both experimental group and control group students' answers were incorrect. Therefore, the item 2e is constant.

Table 4. 19

The Results of the Chi-Square Test for Independence for Item 2f at Pretest

	Value	df	Asymptotic Significance (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	.078	1	.780		
Continuity Correction	.000	1	1.000		
Likelihood Ratio	.078	1	.780		
Fisher's Exact Test				1.000	.515
Linear-by-Linear Association	.076	1	.782		
N of Valid Cases	43				

$p > .05$

A Chi-square test for independence (with Yates Continuity Correction) indicated that there was no significant difference between the experimental and control groups for Item 2f, $\chi^2(1, n = 43) = .00$, $p = 1.0$, *phi* =.043.

Table 4. 20

The Results of the Chi-Square Test for Independence for Item 2g at Pretest

	Value	df	Asymptotic Significance (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	1.608	1	.205		
Continuity Correction	.897	1	.343		
Likelihood Ratio	1.632	1	.201		
Fisher's Exact Test				.336	.172
Linear-by-Linear Association	1.571	1	.210		
N of Valid Cases	43				

$p >.05$

A Chi-square test for independence (with Yates Continuity Correction) indicated that there was no significant difference between the experimental and control groups for Item 2g, $\chi^2(1, n = 43) = .90$, $p = .34$, *phi* =.193.

Consequently, the results show that there was a significant difference between the experimental and control groups in only item 1d at pretest so that control group students significantly outperformed experimental group students for item 1d at pretest

4.1.4.2 The Results of Chi-Square Test for Independence for Posttest

In the post-test 14 items were analyzed by Chi-Square test and results were presented in detail below. In the case of violating 'minimum expected cell frequency' assumption, Pallant (2011) suggested using Fisher's Exact Probability Test values.

Table 4. 21

The Results of the Chi-Square Test for Independence for Item 1a at Post-test

	Value	df	Asymptotic Significance (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	.527	1	.468		
Continuity Correction	.016	1	.900		
Likelihood Ratio	.531	1	.466		
Fisher's Exact Test				.590	.446
Linear-by-Linear Association	.514	1	.473		
N of Valid Cases	43				

$p > .05$

A Chi-square test for independence (with Fisher's Exact Test) indicated that there was no significant difference between the experimental and control groups for Item 1a in the pretest, (.59 > .05).

Table 4. 22

The Results of the Chi-Square Test for Independence for Item 1b at Post-test

	Value	df	Asymptotic Significance (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	1177	1	.278		
Continuity Correction	.005	1	.944		
Likelihood Ratio	1.558	1	.212		
Fisher's Exact Test				.465	.465
Linear-by-Linear Association	1.150	1	.284		
N of Valid Cases	43				

$p > .05$

A Chi-square test for independence (with Fisher's Exact Test) indicated that there was no significant difference between the experimental and control groups for Item 1b in the pretest, (.47 > .05).

Table 4. 23

The Results of the Chi-Square Test for Independence for Item 1c at Post-test

	Value	df	Asymptotic Significance (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	.034	1	.853		
Continuity Correction	.000	1	1.000		
Likelihood Ratio	.034	1	.854		
Fisher's Exact Test				1.000	.597
Linear-by-Linear Association	.033	1	.855		
N of Valid Cases	43				

$p > .05$

A Chi-square test for independence (with Fisher's Exact Test) indicated that there was no significant difference between the experimental and control groups for Item 1c in the pretest, (1 > .05).

Table 4. 24

The Results of the Chi-Square Test for Independence for Item 1d at Post-test

	Value	df	Asymptotic Significance (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	.111	1	.739		
Continuity Correction	.000	1	.994		
Likelihood Ratio	.112	1	.738		
Fisher's Exact Test				1.000	.498
Linear-by-Linear Association	.109	1	.741		
N of Valid Cases	43				

$p > .05$

A Chi-square test for independence (with Yates Continuity Correction) indicated that there was no significant difference between the experimental and control groups for Item 1d, $\chi^2(1, n = 43) = .0$, $p = .99$, *phi* =.51.

Table 4. 25

The Results of the Chi-Square Test for Independence for Item 1e at Post-test

	Value	df	Asymptotic Significance (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	7.406	1	.006		
Continuity Correction	5.622	1	.018		
Likelihood Ratio	7.787	1	.005		
Fisher's Exact Test				.012	.008
Linear-by-Linear Association	7.234	1	.007		
N of Valid Cases	43				

$p < .05$

A Chi-square test for independence (with Yates Continuity Correction) indicated that there was no significant difference between the experimental and control groups for Item 1e, $\chi^2(1, n = 43) = 5.6$, $p = .018$, $phi = - .415$.

Table 4. 26

The Results of the Chi-Square Test for Independence for Item 1f at Post-test

	Value	df	Asymptotic Significance (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	.010	1	.919		
Continuity Correction	.000	1	1.000		
Likelihood Ratio	.010	1	.919		
Fisher's Exact Test				1.000	.720
Linear-by-Linear Association	.010	1	.920		
N of Valid Cases	43				

p > .05

A Chi-square test for independence (with Fisher's Exact Test) indicated that there was no significant difference between the experimental and control groups for Item 1f in the pretest, (1 > .05).

There was no statistic computed for item 1g constructed because both experimental group and control group students' all answers were incorrect. Therefore, the item 1g was constant.

Table 4. 27

The Results of the Chi-Square Test for Independence for Item 2a at Post-test

	Value	df	Asymptotic Significance (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	2.618	1	.106		
Continuity Correction	1.695	1	.193		
Likelihood Ratio	2.637	1	.104		
Fisher's Exact Test				.127	.096
Linear-by-Linear Association	2.557	1	.110		
N of Valid Cases	43				

$p > .05$

A Chi-square test for independence (with Yates Continuity Correction) indicated that there was no significant difference between the experimental and control groups for Item 2a $\chi^2(1, n = 43) = 1.70$, $p = .193$, $phi = -.247$.

Table 4. 28

The Results of the Chi-Square Test for Independence for Item 2b at Post-test

	Value	df	Asymptotic Significance (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	3.866	1	.049		
Continuity Correction	2.668	1	.102		
Likelihood Ratio	3.923	1	.048		
Fisher's Exact Test				.094	.051
Linear-by-Linear Association	3.776	1	.052		
N of Valid Cases	43				

$p > .05$

A Chi-square test for independence (with Pearson Chi-Square) indicated that there was no significant difference between the experimental and control groups for Item 2b $\chi^2(1, n = 43) = 2.67$, $p = .102$, $phi = -.300$.

Table 4. 29

The Results of the Chi-Square Test for Independence for Item 2c at Post-test

	Value	df	Asymptotic Significance (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	3.866	1	.049		
Continuity Correction	2.668	1	.102		
Likelihood Ratio	3.923	1	.048		
Fisher's Exact Test				.094	.051
Linear-by-Linear Association	3.776	1	.052		
N of Valid Cases	43				

$p > .05$

A Chi-square test for independence (with Yates Continuity Correction) indicated that there was no significant difference between the experimental and control groups for Item 2c $\chi^2(1, n = 43) = 2.67$, $p = .102$, *phi* = - .300.

Table 4. 30

The Results of the Chi-Square Test for Independence for Item 2d at Post-test

	Value	df	Asymptotic Significance (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	3,800[a]	1	,051		
Continuity Correction	2,275	1	,131		
Likelihood Ratio	4,034	1	,045		
Fisher's Exact Test				,081	,065
Linear-by-Linear Association	3,712	1	,054		
N of Valid Cases	43				

$p > .05$

A Chi-square test for independence (with Fisher's Exact Test) indicated that there was no significant difference between the experimental and control groups for Item 2d in the post-test, (.81> .05).

Table 4. 31

The Results of the Chi-Square Test for Independence for Item 2e at Post-test

	Value	df	Asymptotic Significance (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	6.507	1	.011		
Continuity Correction	4.301	1	.038		
Likelihood Ratio	8.419	1	.004		
Fisher's Exact Test				.016	.016
Linear-by-Linear Association	6.355	1	.012		
N of Valid Cases	43				

p < .05

A Chi-square test for independence (with Fisher’s Exact Test) indicated that there was a significant difference between the experimental and control groups for Item 2e in the post-test, (.016 > .05). The experimental group outperformed the control group by using variables in function rule.

Table 4. 32

The Results of the Chi-Square Test for Independence for Item 2f at Post-test

	Value	df	Asymptotic Significance (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	.102	1	.750		
Continuity Correction	.000	1	1.000		
Likelihood Ratio	.101	1	.750		
Fisher's Exact Test				1.000	.502
Linear-by-Linear Association	.099	1	.753		
N of Valid Cases	43				

p > .05

A Chi-square test for independence (with Yates Continuity Correction) indicated that there was no significant difference between the experimental and control groups for Item 2f $\chi^2(1, n = 43) = .00$, $p = 1.00$, $phi = -.49$.

Table 4. 33

The Results of the Chi-Square Test for Independence for Item 2g at Post-test

	Value	df	Asymptotic Significance (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	.111	1	.739		
Continuity Correction	.000	1	.994		
Likelihood Ratio	.112	1	.738		
Fisher's Exact Test				1.000	.498
Linear-by-Linear Association	.109	1	.741		
N of Valid Cases	43				

$p > .05$

A Chi-square test for independence (with Pearson Chi-Square) indicated that there was no significant difference between the experimental and control groups for Item 2g χ^2 $(1, n = 43) = .111$, $p = .74$, $phi = .51$.

Consequently, Chi-square test for independence test results showed that there was a significant difference between the experimental group and the control group in item 1e and item 2e at the post-test. In both items, experimental group students significantly outperformed control group students in defining function rule by variables.

4.2 Descriptive Results of the Functional Thinking Test

In order to investigate the fourth research question "How does 5th grade students' functional thinking differ in the functional thinking test for those who attend the functional thinking intervention and who does not?", both experimental group and control group students' responses in the Functional Thinking Test (FTT) were examined in detail. There were two main items in FTT. Both Item 1 and Item 2 had 7 sub-questions. For many items, a correctness (correct (1)/incorrect (0)) code and a strategy code were assigned. Coding schemes varied according to the structure of the items. For items that asked to define patterns in the table, function rule in words and in variables, levels of sophistication for generalizing functional relationships (Stephens el al., 2017) were used. For items asking to find near and far value by using function rule and other strategies, emerging codes and strategies from literature such as Blanton, Stephens et al. (2015) were used. Apart from these, "Answer Only (AO)", "No Response (NR)" and "Other (O)" codes were utilized. If a student did not give an answer, it was coded as "No Response (NR)" for both correctness and strategy. If a student wrote only the result without showing their work, it was coded as "Answer Only (AO)". Lastly, if a student provided a response that did not correspond to any level, was not of interest to the study or the response was not discernible, then "Other" code was assigned. Also, if a student used more than one strategy in the response, the most sophisticated strategy code was assigned as the strategy code.

Item 1

Item 1 was about $y = 2x$ functional relationship. Students were supposed to define functional relationship between two variables and represent this relationship by table, words, algebraic expressions and graph. Item 1a asked students to determine the unknown step of the pattern. Item 1b asked students to organize a table to record data. Item 1c asked students to define patterns they see in the table. Students were expected to explain relationship between two variables by words in Item 1d. Students were supposed to define this relationship by using variables in Item 1e. Item f asked students to find the value for further step. Students were expected to show the relationship between two variables on the coordinate graph in item 1g.

Item 1a

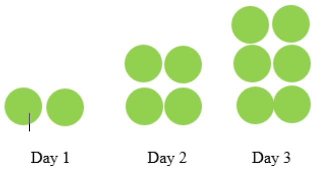
Item 1a (Figure 4.1) asked finding the 5^{th} step of the given pattern.

Figure 4. 1

Item 1a in FTT

Item 1

Each day in the class, Selin creates a picture by drawing circles joined together. Following are the pictures of circles that she drew on each day:



a) How many circles are in her picture for Day 5?

The performance of the students is given in Table 4.34.

Table 4. 34

The Percentage of Correctness of EG and CG in Item 1a in FTT

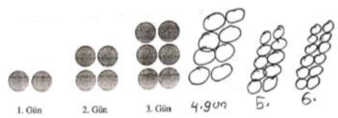
Item 1a Correctness	Experimental Group N=20		Control Group N=23	
	Pre	**Post**	**Pre**	**Post**
Correct (1)	85,00 %	90,00%	100,00%	95,65%
Incorrect (0)	10,00 %	5,00%	0,00%	4,35%
No Response (NR)	5,00 %	5,00%	0,00%	0,00%

In the Item 1a, the experimental group and control group students had similar performances in both pretest and posttest. Item 1a was answered predominantly correct in both groups. In the experimental group, there was an increase (5%) and 90% of students gave correct response at posttest. In the control group, while all of students gave correct answer at pretest, approximately 96% of those gave correct answer at posttest. Consequently, experimental and control students could successfully determine the 5th step of the given pattern.

Also, students were assigned strategy codes for Item 1a. Figure 4.2 provides the coding scheme for Item 1a that was used to categorize student strategies, the description of the codes, and an example of students' written work. In each of these coding schemes, strategies were listed from the most sophisticated to the least sophisticated.

Figure 4. 2

Coding scheme for Item 1a in FTT

Strategy Code	Description	Example
Function Rule (FR)	Student finds the result by applying the function rule.	5×2=10
Recursive Pattern General (RP-G)	"Student identifies a correct recursive pattern in either or both variables."	Student defines and shows the pattern as "+2" or "increasing by twos".
Recursive Pattern Particular (RP-P)	"Student identifies a recursive pattern by referring to particular numbers only."	It goes 2, 4, 6, 8,10
Drawing (D)	Student draws the other circles to reach the number of circles drawn in the fifth day.	
Other (O)	Student produces a strategy that differs from above or the strategy is not discernible.	6+4=10
Answer Only (AO)	Student writes just the answer without showing their work.	Day 5: 10 circles
No Response (NR)	Student does not give an answer.	

Note. Adapted from "A Learning Progression for Elementary Students' Functional Thinking" by A. C. Stephens, N. Fonger, S. Strachota, I Isler, M. Blanton, E. Knuth, A. M. Gardiner, 2017, *Mathematical Thinking and Learning*, 19(3), p. 153.

The percentages of each strategy used by both experimental and control groups are in Table 4.35. All strategies were accepted as correct, in the case of giving answer as 10.

Table 4. 35

The Percentage of Strategies used by EG and CG in Item 1a in FTT

Item 1a Strategy	Experimental Group N=20		Control Group N=23	
	Pre	Post	Pre	Post
Function Rule	10.00%	20.00%	13.04%	26.09%
Recursive Pattern General	.00%	.00%	13.04%	4.35%
Recursive Pattern Particular	5.00%	15.00%	17.39%	17.39%
Drawing	40.00%	15.00%	26.09%	21.74%
Other	10.00%	.00%	.00%	.00%
Answer Only	30.00%	45.00%	30.43%	30.43%
No Response	5,.0%	5.00%	.00%	.00%

For Item 1a, Function Rule (FR) was the most sophisticated strategy. The percentage of FR strategy increased at posttest in both experimental and control groups. While Recursive Pattern General (RP-G) strategy was not observed in the experimental group in both pre-test and post-test, the percentage of RP-G decreased from approximately 13% at pre-test to approximately 4% at post-test in the control group. Five percent of the experimental group used Recursive Pattern Particular (RP-P) strategy at pre-test; its percentage increased to 15% at post-test. The percentage of students who used RP-P in the control group remained the same (about 17%) at pre-test and posttest. Drawing(D) was the most used strategy in both groups at pre-test. 40% of the experimental students used a drawing to find the 5th step of the pattern at pre-test, this decreased to 15% at post-test. In the control group, about 26% of the students used a drawing while this decreased to about 22% at the post-test. While Answer Only (AO) increased from 30% to 45% in the experimental group at post-test, the control group remained the same about 30% at pre-test and post-test. All students answered the question in the control group at both pre-test and post-test. Five percent of experimental group did not respond the question in both tests.

Item 1b

Item 1b (Figure 4.3) asked students to organize the information in the given table.

Figure 4. 3

Item 1b in FTT

b) Organize your information in the given table

Number of Day	Number of Circle

Students were assigned only correctness code for this item. Item 1a was answered predominantly correct in both the experimental and control groups. The performance of students in percentage is presented in Table 4.36.

Table 4. 36

The Percentage of Correctness of EG and CG in Item 1b in FTT

Item 1b Correctness	Experimental Group N=20		Control Group N=23	
	Pre	Post	Pre	Post
Correct (1)	90.00 %	95.00%	100.00%	100.00%
Incorrect (0)	.00 %	.00%	.00%	.00%
No Response (NR)	10.00 %	5.00%	.00%	.00%

As seen in the table, none of students gave incorrect answer among students who answered the question. All students could organize the table correctly in the control group at both pretest and posttest. The percentage of students who gave correct answer in the experimental group increased from 90% at pretest to 95% at posttest. The percentage of students who did not give an answer in the experimental group decreased by 5% at posttest. Consequently, both experimental and control group students could successfully organize information about variables in the given table.

Item 1c

Item 1c (Figure 4.4) asked students to define the patterns they see in the table.

Figure 4. 4

Item 1c in FTT

c) Which patterns do you see in the table? Describe.

Students were assigned a correctness code and a strategy code for this item. The performance of students in percentage for Item 1c is given in Table 4.37.

Table 4. 37

The Percentage of Correctness of EG and CG in Item 1c in FTT

Item 1c Correctness	Experimental Group N=20		Control Group N=23	
	Pre	Post	Pre	Post
Correct (1)	80.00 %	85.00%	78.26%	86.96%
Incorrect (0)	5.00 %	5.00%	21.74%	13.04%
No Response (NR)	15.00 %	10.00%	.00%	.00%

The majority of the experimental and control group students described the patterns correctly. The percentage of correct answer increased at posttest in both groups. The percentage of incorrect answers of the control group was more than the experimental group at both pretest and posttest. In general, students could define the patterns that they saw in the table.

Figure 4.5 provides the coding scheme for Item 1c that was used to categorize student strategies, the description of the codes, and an example of students' written work. Functional Condensed-Words (FC-W), Functional Emergent-Words (FE-W), Functional Basic (FB), Functional Particular (FP), Covariational Relationship (CR), Recursive Pattern General (RP-G) and Recursive Pattern Particular (RP-P) strategies were coded as correct for this item.

Figure 4. 5

Coding scheme for Item 1c in FTT

Strategy Code	Description	Example
Functional Condensed-Words (FC-W)	"Student identifies function rule in words that describes a generalized relationship between two variables."	The number of circles is two times the number of days
Functional Emergent-Words (FE-W)	"Student identifies incomplete function rule in words, often describing transformation on one variable but not explicitly relating other."	We multiply the number of days by two
Functional Basic (FB)	"Student defines general relationship between variables but not the transformation between them."	Two times, half
Functional Particular (FP)	"Student defines a functional relationship using particular numbers but does not make a general statement relating the variables."	2×2=4, 3×2=6,…
Covariational Relationship (CR)	"Student identifies covariation relationship. The two variables are coordinated rather than mentioned separately."	Each day number of circles increases by 2. As the number of the day goes up by 1, the number of the circles goes up by 2.
Recursive Pattern General (RP-G)	"Student identifies a correct recursive pattern in either or both variables."	Increasing by twos The number of circles goes up by 2
Recursive Pattern Particular (RP-P)	"Student identifies a correct recursive pattern in either or both variables by referring to particular number only."	It goes 2, 4, 6, 8,10

Figure 4.5 (continued)

Other (O)	Student produces a strategy that differs from above or the strategy is not discernible.	1=2,2=4,3=6,4=8
No Response (NR)	Student does not give an answer.	

Note. Adapted from "A Learning Progression for Elementary Students' Functional Thinking" by A. C. Stephens, N. Fonger, S. Strachota, I Isler, M. Blanton, E. Knuth, A. M. Gardiner, 2017, *Mathematical Thinking and Learning*, 19(3), p. 153.

The percentages of each strategy used in item 1c by both experimental and control groups are presented in Table 4.38.

Table 4. 38

The Percentage of Strategies used by EG and CG in Item 1c in FTT

Item 1c Strategy	**Experimental Group N=20**		**Control Group N=23**	
	Pre	**Post**	**Pre**	**Post**
Functional Condensed-Words	.00%	15.00%	.00%	13.04%
Functional Emergent-Words	5.00%	5.00%	8.70%	4.35%
Functional Basic	.00%	15.00%	.00%	.00%
Functional Particular	.00%	.00%	.00%	4.35%
Covariation Relationship	25.00%	20.00%	17.39%	8.70%
Recursive Pattern General	45.00%	25.00%	52.17%	56.52%
Recursive Pattern Particular	5.00%	5.00%	.00%	.00%
Other	5.00%	5.00%	21.74%	13.04%
No Response	15.00%	10.00%	.00%	.00%

In the experimental group, frequency of no response for Item 1c decreased from 15 % at pretest to 10 % at posttest. All students gave a response to Item 1c in the control group. By posttest, students in the experimental group used more sophisticated strategies (FC-W, FE-W, FB, and FP) in response to these items than they had at pretest (30% vs. 10% at post-test and pre-test, respectively). Twenty-five percent of the experimental group continued to use Recursive Pattern -General (RP-G) strategy at posttest while this was 45% at pretest. However, approximately 57% of the control group continued to use RP-G strategy at posttest while this was about 52% at pretest. Moreover, 20% of the experimental group continued to use Covariation Relationship strategy at posttest; while it was 25% at pretest. In contrast, approximately 9% of the control group continued to use CR strategy at posttest while this was about 17% at pretest. In addition, no student used Functional Basic (FB) strategy at pretest or posttest in the control group. In contrast, 15% of the experimental group used FB strategy at posttest while none used at pretest. While neither experimental group nor control group used Functional Condensed-Words (FC-W) strategy at pretest, frequency of FC-W strategy was similar in the experimental group and control group at posttest, 15% and about 13%, respectively. The percentage of other strategy remained same in the experimental group from pre-test to post-test. In the control group, the percentage of other strategy decreased from approximately 21% to approximately 13% at posttest. Responses to item 1c such as, “visual pattern”, “circle pattern” “There are patterns going up from 2” were assessed in the other category.

Item 1d

Item 1d (Figure 4.6) asked students to describe the relationship between the variables in their own words.

Figure 4. 6

Item 1d in FTT

d) In your own words, describe the relationship between the number of days and the number of the circles.

Students were assigned two codes for this item; a correctness code and a strategy code. The performance of students in percentage for Item 1d is given in Table 4.39.

Table 4. 39

The Percentage of Correctness of EG and CG in Item 1d in FTT

Item 1d Correctness	Experimental Group N=20		Control Group N=23	
	Pre	Post	Pre	Post
Correct (1)	5.00%	30.00%	39.13%	34.78%
Incorrect (0)	75.00%	65.00%	60.87%	65.22%
No Response (NR)	20.00%	5.00%	.00%	.00%

Students in both groups gave incorrect answer predominantly for Item 1d at both pretest and posttest. The percentage of correct answer of the control group remained the same at pretest and posttest as approximately 39%. In contrast, the percentage of correct answer of the experimental group increased from 5% at pretest to 30% at posttest. While all control group students respond to the item 1d, the percentage of no response decreased from 20% at pre-test to 5% at posttest in experimental group.

Figure 4.7 provides the coding scheme for Item 1d that was used to categorize student strategies, the description of the codes, and an example of students' written work.

Figure 4. 7

Coding scheme for Item 1d in FTT

Strategy Code	Description	Example
Functional Condensed-Words (FC-W)	"Student identifies function rule in words that describes a generalized relationship between two variables."	The number of circles is two times the number of days. The number of the days is half of the number of the circles
Functional Emergent-Words (FE-W)	"Student identifies incomplete function rule in words, often describing transformation on one variable but not explicitly relating other."	We multiply the number of days by two.

Figure 4.7 (continued)

Functional Basic (FB)	"Student defines general relationship between variables but not the transformation between them."	Two times Half Double
Functional Particular (FP)	"Student defines a functional relationship using particular numbers but does not make a general statement relating the variables."	2×2=4, 3×2=6,4×2=8…
Single Instantiation (SI)	"Student writes expressions with number or unknowns that provides one instantiation of the function rule but does not generally relate the two variables."	2×2=4 3×2=6
Covariational Relationship (CR)	"Student identifies covariation relationship. The two variables are coordinated rather than mentioned separately."	As the number of the day goes up by 1, the number of the circles goes up by 2. Each day number of circles increases by 2.
Recursive Pattern General (RP-G)	"Student identifies a correct recursive pattern in either or both variables."	"increasing by twos"
Recursive Pattern Particular (RP-P)	"Student identifies a correct recursive pattern in either or both variables by referring to particular number only."	It goes 2, 4, 6, 8,10 increasing by twos
Restatement of given (RS)	"Student rewrites the given numbers in the question."	2,4,6
Other (O)	Student produces a strategy that differs from above or the strategy is not discernible.	We add each day by two.
No Response (NR)	Student does not give an answer.	

Note. Adapted from "A Learning Progression for Elementary Students' Functional Thinking" by A. C. Stephens, N. Fonger, S. Strachota, I Isler, M. Blanton, E. Knuth, A. M. Gardiner, 2017, *Mathematical Thinking and Learning*, 19(3), p. 153.

Functional Condensed-Words (FC-W) strategy was accepted as the correct strategy for this item. The percentages of each strategy used by the experimental and control groups are in Table 4.40.

Table 4. 40

The Percentage of Strategies used by EG and CG in Item 1d in FTT

Item 1d Strategy	Experimental Group N=20		Control Group N=23	
	Pre	Post	Pre	Post
Functional Condensed-Words	5.00%	30.00%	39.13%	34,78%
Functional Emergent-Words	.00%	5.00%	4.35%	4,35%
Functional Basic	10.00%	.00%	.00%	.00%
Functional Particular	5.00%	5.00%	.00%	.00%
Single Instantiation	5.00%	.00%	4.35%	.00%
Covariation Relationship	20.00%	15.00%	13.04%	26,09%
Recursive Pattern General	15.00%	20.00%	21.74%	17,39%
Recursive Pattern Particular	5.00%	.00%	4.35%	4,35%
Restatement of Given	.00%	5.00%	.00%	.00%
Other	15.00%	15,00%	13.04%	13.04%
No Response	20.00%	5,00%	.00%	.00%

The percentage of no response decreased from 20% at pretest to 5% at posttest in the experimental group. All students in the control group gave an answer both at pretest and posttest. While the percentage of CR strategy increased from about 13% at pretest to about 26% at posttest in the control group, it decreased from 20% at pretest to 15% at posttest in the experimental group. None of the control students used FP and FB strategies at pretest or posttest. The same frequency, 5% of the experimental students used FP at pretest and posttest. The use of FB strategy by the experimental group was 10% at the pretest none used at posttest. In the control group, the percentage of FC-W decreased from about 39% at pretest to about 35% at posttest. However, in the experimental group, the percentage of FC-W increased from 5% at pretest to 30% at posttest. In contrast to control group, the percentage of functional relationship strategies (FC-W, FE-W, FB, FP) increased from 20% at pre-test to 40% at post-test. Consequently, experimental group students were found to use more sophisticated strategies than the control group at post-test. Moreover, the percentage of other strategies remained the same for both groups from pre-test to post-test. Responses to item 1d such as, "each day two circles", "Circles are multiplying by 2" were assessed in the other category.

Item 1e

Item 1e (Figure 4.8) asked students to describe the relationship by using letters as variables.

Figure 4. 8

Item 1e in FTT

e) Explain the relationship between the number of the days and number of the circles by using variables (letters).

Students were assigned two codes for this item; a correctness code and a strategy code. The performance of students in the percentage for Item 1e is given in Table 4.41.

Table 4. 41

The Percentage of Correctness of EG and CG in Item 1e in FTT

Item 1e Correctness	**Experimental Group N=20**		**Control Group N=23**	
	PRE	**POST**	**PRE**	**POST**
Correct (1)	.00%	45.00%	.00%	8.70%
Incorrect (0)	25.00%	35.00%	52.17%	65.22%
No Response (NR)	75.00%	20.00%	47.83%	26.09%

The percentage of NR decreased highly from 75% at pre-test to 20% at post-test in the experimental group and from about 48% at pre-test to about 26% at post-test in the control group. The majority of the students gave a response at post-test. The percentage of correctness increased at post-test in both groups at post-test. While none of the experimental students could give a correct response at pre-test, half of the students gave a correct answer at post-test. In the control group, similarly none of the students were able to give a correct answer at pretest, while about 9% of the students did correctly at post-test.

In addition, students were assigned a strategy code for their answers. Figure 4.9 provides the coding scheme for Item 1e that was used to categorize student strategies, the description of the codes, and an example of students' written work.

Figure 4. 9

Coding scheme for Item 1e in FTT

Strategy Code	Description	Example
Functional Condensed-Words (FC-W)	"Student identifies function rule in words that describes a generalized relationship between two variables."	The number of circles is multiple of number of days
Functional Condensed – Variable (FC-V)	"Students write a complete rule in variables. Student uses at least one variable."	G×2=D, D÷2=G, G×2= Number of circles, D÷2= Number of days
Functional Basic (FB)	"Student defines general relationship between variables but not the transformation between them."	Two times Half
Functional Particular (FP)	"Student defines a functional relationship using particular numbers but does not make a general statement relating the variables."	2×2=4, 3×2=6,…
Covariational Relationship (CR)	"Student identifies covariation relationship. The two variables are coordinated rather than mentioned separately."	Each day number of circles increases by 2. As the number of the day goes up by 1, the number of the circles goes up by 2.

Figure 4.9 (continued)

Recursive Pattern General (RP-G)	"Student identifies a correct recursive the pattern in either or both variables."	The number of circles increases by two. The number of days increases by one.
Recursive Pattern Particular (RP-P)	"Student identifies a recursive pattern by referring to particular numbers only."	It goes 2, 4, 6, 8,10
Other (O)	Student produces a strategy that differs from above or the strategy is not discernible.	(Day) W x 2 Q(Circle)
No Response (NR)	Student does not give an answer.	

Note. Adapted from "A Learning Progression for Elementary Students' Functional Thinking" by A. C. Stephens, N. Fonger, S. Strachota, I. Isler, M. Blanton, E. Knuth, A. M. Gardiner, 2017, *Mathematical Thinking and Learning*, 19(3), p. 153.

Functional Condensed-Variables (FC-V) strategy was accepted as the correct strategy for this item. The percentages of each strategy used by the experimental and control groups in Item 1e are in Table 4.42.

Table 4. 42

The Percentage of Strategies used by EG and CG in Item 1e in FTT

Item 1e Strategy	**Experimental Group N=20**		**Control Group N=23**	
	Pre	**Post**	**Pre**	**Post**
Functional Condensed-Words	.00%	10.00%	.00%	4.35%
Functional Condensed Variables	.00%	45.00%	.00%	8.70%
Functional Basic	5.00%	.00%	.00%	.00%
Functional Particular	.00%	.00%	13.04%	13.04%
Covariation Relationship	.00%	.00%	.00%	4.35%
Recursive Pattern General	5.00%	.00%	4.35%	.00%
Recursive Pattern Particular	5.00%	.00%	13.04%	4.35%
Other	10.00%	25.00%	21.74%	39.13%
No Response	75.00%	20.00%	47.83%	26.09%

In both groups, the percentage of no response decreased at posttest. In the experimental group, 75% of the students could not give an answer at pretest while this was 20% at posttest. In the control group, the NR frequency was about 48% and 26% respectively at pretest and posttest. None of the experimental students used FP or FB strategies at posttest while 5% used FB at pretest. In the control group, approximately 13% used FP strategy at both pretest and posttest, and none of the control students used FB strategy at pretest or posttest. Regarding FC-V strategy, none of the experimental or control students used it at pretest. Approximately 9% of the control group used FC-V strategy at posttest. In contrast, %45 of the experimental group used FC-V strategy at posttest. Consequently, experimental students were more successful than control students in defining the functional relationship by using variables.

Item 1f

Item 1f (Figure 4.10) asked students to find the value for the 100th step.

Figure 4. 10

Item 1f in FTT

f) How many circles will be in the picture that Selin draws on the 100th day of the school? Show how you got your answer.

Students were assigned a correctness code and a strategy code for Item 1f. The performance of students in percentage for Item 1f is given in Table 4.43.

Table 4. 43

The Percentage of Correctness of EG and CG in Item 1f in FT

Item 1f Correctness	Experimental Group N=20		Control Group N=23	
	Pre	Post	Pre	Post
Correct (1)	90.00%	95.00%	82.61%	95.65%
Incorrect (0)	5.00%	.00%	17.39%	4.35%
No Response (NR)	15.00%	10.00%	.00%	.00%

Students could give correct answer predominantly. In the experimental group NR decreased from 15% at pretest to 10% at posttest gave NR, while all control group students gave a response at both pretest and posttest. Eighty five percent of the experimental group and approximately 87% of the control group could give a correct response at posttest.

In addition, students were assigned a strategy code for their answers. Figure 4.11 provides the coding scheme for Item 1f that was used to categorize student strategies, the description of the codes, and an example of students' written work.

Figure 4. 11

Coding scheme for Item 1f in FTT

Strategy Code	Description	Example
Function Rule (FR)	Student finds the result by using the function rule.	$100 \times 2 = 200$
Other (O)	Student produces a strategy that differs from above or the strategy is not discernible.	$100 \div 5 = 20$ $20 \times 10 = 200$
Answer Only (AO)	Student writes only answer without showing her/his work	200
No Response (NR)	Student does not give an answer.	

The percentages of each strategy used by the experimental and control groups in Item 1f are in Table 4.44.

Table 4. 44

The Percentage of Strategies used by EG and CG in Item 1f in FTT

Item 1f Strategy	Experimental Group N=20		Control Group N=23	
	Pre	Post	Pre	Post
Function Rule	80.00%	85.00%	73.91%	95.65%
Other	10.00%	.00%	17.39%	4.35%
Answer Only	5.00%	10.00%	8.70%	.00%
No Response	5.00%	5.00%	.00%	.00%

While all students gave an answer to Item 1f in the control group, 5% of the experimental group did not give an answer at pre-test and post-test. The majority of the students in both the experimental and control groups used function rule strategy, that means, they used the function rule to find the result for 100th day at pretest and at posttest. Both groups showed increase in using FR strategy (5% vs. approximately 22% of experimental and control group, respectively) at post-test.

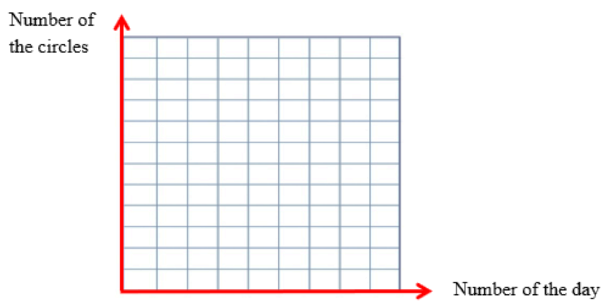
Item 1g

In item 1g, students were expected to represent the relationship between two variables on a coordinate graph (see Figure 4.12).

Figure 4. 12

Item 1g in FTT

g) Show the relationship between the number of the days and number of the circles on the graph below.



The percentage of correctness of the students was presented in Table 4.45

Table 4. 45

The Percentage of Correctness of EG and CG in Item 1g in FTT

Item 1g Correctness	Experimental Group N=20		Control Group N=23	
	Pre	**Post**	**Pre**	**Post**
Correct (1)	.00%	.00%	.00%	.00%
Incorrect (0)	85.00%	90,00%	100,00%	95.65%
No Response (NR)	15.00%	10.00%	.00%	4.35%

Students gave incorrect response in both at pre-test and post-test. While no response decreased (from 15% to 10%) in the experimental group, it increased in the control group from 0% to approximately 4%. Students were assigned strategy codes for Item 1g. Figure 4.13 provides the coding scheme for Item 1g that was used to categorize student strategies, the description of the codes, and an example of students' written work.

Figure 4. 13

Coding scheme for Item 1g in FTT

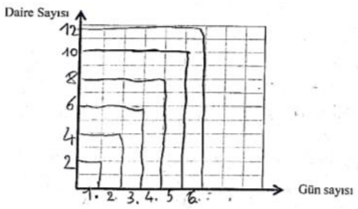
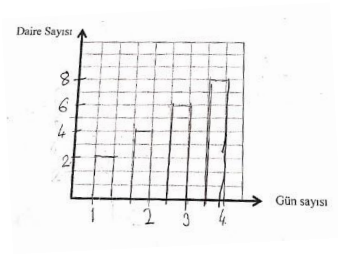
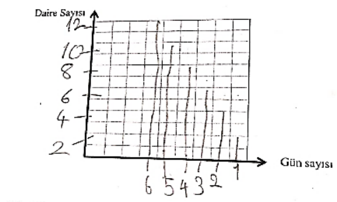
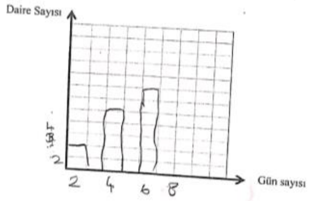
Strategy Code	**Description**	**Example**
Points Matched Correctly (PM)	Values are placed on the axes and matched correctly by representing points.	Points (1,2), (2,4), (3,6)…
Axes Matched Correctly (AxM)	Values are placed on the axes but matched the values without representing points.	

Figure 4.13 (continued)

Bar Graph (BG)	Student defines correct values for axes, places them on the axes and draws a bar graph.	
Other (O)	Student produces a strategy that differs from above or the strategy is not discernible.	 
No Response (NR)	Student does not give an answer.	

In Item 1g, students were expected to represent the functional relationship between number of days and number of circles on a coordinate graph.PM strategy was accepted as correct strategy. Percentages of each strategy used by the experimental and control groups in Item 1g are in Table 4.46.

Table 4. 46

The Percentage of Strategies used by EG and CG in Item 1g in FTT

Item 1g Strategy	Experimental Group N=20		Control Group N=23	
	Pre	Post	Pre	Post
Axes Matched	10.00%	55.00%	8.70%	17.39%
Bar Graph	30.00%	5.00%	39.13%	30.43%
Other	45.00%	30.00%	52.17%	47.38%
No Response	15.00%	10.00%	.00%	4.35%

There was no student used Points Matched strategy at pretest and posttest. Although approximately 17% of control group used Axes Matched Correctly (AxM) strategy, more than half of the experimental group students used AxM at post-test. They could not represent the points on the graph. Students could define the relationship and they placed values on the axes correctly. Furthermore, the percentage of Bar Graph (BG) strategy decreased in the experimental group from 30% at pretest to 10% at posttest while in the control group, it was about 39% at pretest and about 30% at posttest. However, the majority of the students in the experimental and control groups used Other (O) strategy at pretest (see Figure 4.13 for examples). The percentage of O strategy remained the same (about 47%) in the control group at pretest and posttest. In contrast, in the experimental group, the percentage of O decreased from 45% at pretest to 30 % at posttest.

Consequently, experimental group students were more successful in representing the functional relationship on a coordinate graph at posttest.

Item 2

Item 2 was about $y=3x+2$ functional relationship. Students were supposed to define the functional relationship between two variables and represent this relationship using a table, words, variables and graph. Item 2a asked students to determine the unknown steps of the pattern. Item 2b asked students to organize a table to record data. Item 2c asked students to define the patterns they see in the table. Students were expected to explain the relationship in words in Item 2d. Students were supposed to define this relationship by using variables in Item 2e. Item 2f asked students to use the function rule to predict a far function value. The most sophisticated strategy was using inverse function rule to find further function values in Item 2g.

Item 2a

Item 2a (Figure 4.14) was about finding the unknown steps of the given pattern.

Figure 4. 14

Item 2a in FTT

> **Item 2**
> There are 2 TL in Mert's piggy bank in the beginning. Every week Mert's dad gives him 3 TL for helping with chores around the house. Mert is saving his money in his piggy bank to buy a bike.
>
> **a)** How much money is there in Mert's piggy bank in total at the end of the Week 2? Week 3? Week 4?

Students' answers were analyzed by correctness and strategy. The performance of students in percentage is given in Table 4.47.

Table 4. 47

The Percentage of Correctness of EG and CG in Item 2a in FTT

Item 2a Correctness	**Experimental Group N=20**		**Control Group N=23**	
	PRE	**POST**	**PRE**	**POST**
Correct (1)	25.00%	50.00%	21.74%	26.09%
Incorrect (0)	60.00%	40.00%	69.57%	73.91%
No Response (NR)	15.00%	10.00%	8.70%	.00%

The percentage of correct answer increased at posttest in both groups; however, the experimental group showed greater performance. In the experimental group, the percentage of correct answer increased from 25% at pretest to 50% at posttest. In the control group, this was about 22% to 26% from pretest to posttest.

In addition, students were assigned a strategy code for Item 2a. Figure 4.15 provides the coding scheme for Item 2a that was used to categorize student strategies, the description of the codes, and an example of students' written work.

Figure 4. 15

Coding scheme for Item 2a in FTT

Strategy Code	Description	Example
Function Rule (FR)	Student finds the result by using the function rule.	2×3=6 6+2=8, 3×3=9 9+2=11, 4×3=12 12+2=14
Single Instantiation (SI)	Student finds the result by using function rule as multiplying by 3 then adding two for just one of the 2nd, 3rd and 4th week.	2×3=6 6+2=8 3×3=9 9+2=11 4×3=12 12+2=14
Incorrect Function Rule (I-FR)	Student uses incorrect function rule which is multiplying number of the week by 3 to find the amount of money in the piggy bank.	2×3=6,3×3=9,4×3=12
Recursive Pattern (RP)	Student finds the result by adding 3 recursively.	2+3=5 TL, 2+3+3=8 TL ,2+3+3+3= 11 TL ,2+3+3+3+3=14 TL
Other (O)	Student produces a strategy that differs from above or the strategy is not discernible.	2×7=14 14×3=42, 3×7=21 21×3=63, 4×7=28 28×3=84
Answer Only (AO)	Student writes only answer without showing her/his work	2nd week: 8, 3rd week: 11, 4th week: 14
No Response (NR)	Student does not give an answer.	

All strategies were accepted as correct in the case of giving the answer "8 TL for the second week; 11 TL for the third week and 14 TL for the fourth week". The percentages of each strategy used by both groups in Item 2a are in Table 4.48.

Table 4. 48

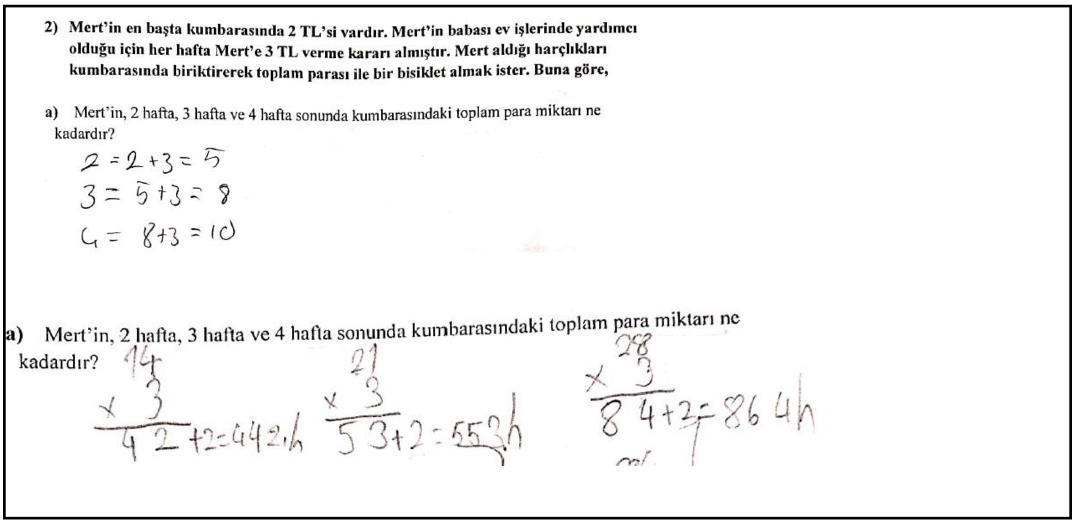
The Percentage of Strategies used by EG and CG in Item 2a in FTT

Item 2a Strategy	Experimental Group N=20		Control Group N=23	
	Pre	Post	Pre	Post
Function Rule	.00%	15.00%	.00%	4.35%
Single Instantiation	.00%	.00%	13.04%	.00%
Incorrect F. Rule	10.00%	.00%	.00%	.00%
Recursive Pattern	5.00%	.00%	17.39%	26.09%
Other	30.00%	15.00%	34.78%	47.83%
Answer Only	40.00%	60.00%	21.74%	21.74%
No Response	15.00%	10.00%	8.70%	.00%

FR was the most sophisticated strategy for this item. The percentage of FR strategy increased in both groups at posttest. In the experimental group, no student used FR strategy at pretest but 15% of the students used it at posttest. SI strategy was not seen in the experimental group at pretest or posttest but approximately 13% of the control group students used it at pretest, no control student used it at posttest. I-FR strategy, which the students used an incorrect function rule, was used just at pretest in both groups (10% of the experimental students and about 4% of the control students). The percentage of RP strategy was higher in the control group than the experimental group (about 17% vs. 5% at pretest and 0% vs. about 26% for control and experimental group students., respectively). Most of the control and experimental group students' strategies were coded as Other (see Figure 4.16). Also, most students in the experimental group gave an answer with no work shown (AO strategy) at pretest (40%) and posttest (60%) while this about 22% for control group student both at pretest and posttest.

Figure 4. 16

Students' Answers from Other Category for Item 2a


2) Mert'in en başta kumbarasında 2 TL'si vardır. Mert'in babası ev işlerinde yardımcı olduğu için her hafta Mert'e 3 TL verme kararı almıştır. Mert aldığı harçlıkları kumbarasında biriktirerek toplam parası ile bir bisiklet almak ister. Buna göre,

a) Mert'in, 2 hafta, 3 hafta ve 4 hafta sonunda kumbarasındaki toplam para miktarı ne kadardır?

2 = 2+3 = 5
3 = 5+3 = 8
4 = 8+3 = 10

a) Mert'in, 2 hafta, 3 hafta ve 4 hafta sonunda kumbarasındaki toplam para miktarı ne kadardır?

14 × 3 = 42 +2 = 44 2.h
21 × 3 = 53 +2 = 55 3h
28 × 3 = 84 +2 = 86 4h

To sum up, students could realize that the amount of money increased for each week but most of them could not realize the relationship or function rule between the number of weeks and the amount of money. So, for item 2a Answer Only and Other strategies were used predominantly.

Item 2b

Item 2b (Figure 4.17) asked students to draw a table and organize information in the table. Different than item 1, the table template was not provided to students in item2.

Figure 4. 17

Item 2b in FTT

b) Organize your information in a table

Students were only assigned correctness code for this item. The percentage of students' performance is presented in Table 4.49.

Table 4. 49

The Percentage of Correctness of EG and CG in Item 2b in FTT

Item 2b Correctness	Experimental Group N=20		Control Group N=23	
	Pre	Post	Pre	Post
Correct (1)	10.00%	45.00%	13.04%	17.39%
Incorrect (0)	65.00%	50.00%	82.61%	82.61%
No Response (NR)	25.00%	5.00%	4.35%	.00%

Students had difficulty in giving correct answer for Item 2a, in which they were asked the total amount of money in the second week and in the third week , so they also had difficulty in constructing a table and organizing information in that table. Approximately 17% of the control group students gave a correct answer at posttest, which was about 13% at pretest. In the experimental group, the percentage of correct answer increased from 10% at pretest to 45% at posttest. In conclusion, the experimental group students showed a better performance in constructing a table and organizing information in that table at posttest. But, still, half of the experimental group students (and about 83% of the control group students) were not able to construct a correct table at posttest.

Item 2c

Item 2c (Figure 4.18) asked students to describe patterns that they see in the table.

Figure 4. 18

Item 2c in FTT

c) Which patterns do you see in the table? Describe.

Students were assigned two codes for this item; a correctness and a strategy code. The percentage of students' performance is presented in Table 4.50.

Table 4. 50

The Percentage of Correctness of EG and CG in Item 2c in FTT

Item 2c Correctness	Experimental Group N=20		Control Group N=23	
	Pre	**Post**	**Pre**	**Post**
Correct (1)	30.00%	55.00%	47.83%	82.61%
Incorrect (0)	35.00%	40.00%	47.83%	17.39%
No Response (NR)	35.00%	5.00%	4.35%	.00%

For Item 2c, the percentage of No Response code decreased from pretest to posttest in both groups (the experimental group from 35% to 5%; the control group from about 4% to 0%). The percentage of correctness increased in both groups (the experimental group from 30% to 55%; the control group from about 48% to 83%). The majority of the students realize the pattern in the table at posttest.

Also, students were assigned a strategy code for their answer. Figure 4.19 provides the coding scheme for Item 2c that was used to categorize student strategies, the description of the codes, and an example of students' written work.

Figure 4. 19

Coding scheme for Item 2c in FTT

Strategy Code	Description	Example
Functional Emergent-Words (FE-W)	"Students identifies incomplete rule in words, often describing transformation on one variable but not explicitly relating to other."	We multiply the number of weeks by 3 then add 2.
Functional Basic (FB)	"Student defines general relationship between variables but not the transformation between them."	2 more than 3 times

Figure 4.19 (continued)

Covariational Relationship (CR)	"Student identifies a covariational relationship. The two variables are coordinated rather than mentioned separately."	As the number of the week goes up by 1, the amount of the money goes up by 3. Each week the amount of money increases by 3.
Recursive Pattern General (RP-G)	"Student identifies a recursive pattern in either or both variables."	The amount of money goes up by 3 each time.
Recursive Pattern Particular (RP-P)	"Student identifies a recursive pattern in either or both variables by referring to particular numbers only."	5,8,11,14
Other (O)	Student produces a strategy that differs from above or the strategy is not discernible.	2×7=14 days 14×3=42 TL
No Response (NR)	Student does not give an answer.	

Note. Adapted from "A Learning Progression for Elementary Students' Functional Thinking" by A. C. Stephens, N. Fonger, S. Strachota, I Isler, M. Blanton, E. Knuth, A. M. Gardiner, 2017, *Mathematical Thinking and Learning*, 19(3), p. 153.

The percentages of each strategy used by the experimental and control groups in Item 2c are in Table 4.51. For this item Functional Condensed-Words (FC-W), Functional Basic (FB), Covariational Relationship (CR), Recursive Pattern-General (RP-G) and Recursive Pattern-Particular (RP-P) strategies were accepted as correct.

Table 4. 51

The Percentage of Strategies used by EG and CG in Item 2c in FTT

Item 2c Strategy	Experimental Group N=20		Control Group N=23	
	Pre	Post	Pre	Post
Functional Emergent Words	.00%	.00%	4.35%	4.35%
Functional Basic	.00%	5.00%	.00%	4.35%
Covariation Relationship	25.00%	10.00%	13.04%	17.39%
Recursive Pattern-General	5.00%	40.00%	26.09%	52.17%
Recursive Pattern-Particular	.00%	.00%	4.35%	4.35%
Other	35.00%	40.00%	47.83%	17.39%
No Response	35.00%	5.00%	4.35%	.00%

In general, students could not define the functional relationship between the number of weeks and amount of saved money. The majority of both experimental and control group students explained the pattern by using RP-G; this increased from pretest to posttest (in the experimental group 5% to 40% and in the control group about 26% to about 52%). About same ratio of students in both groups used FB strategy at posttest while none of the student used it at pretest. At posttest, the percentage of Covariational Relationship (CR) strategy was higher for the control group than experimental group (approximately 17% vs. 10%, respectively). Lastly, most of the control and experimental students' strategies for this item were coded as Other (35% at pretest and 40% at posttest in the experimental group; about 48% at pretest and 17% at posttest in the control group). The O strategy included "There is addition and multiplication" and "7× the number of days +2".

To sum up, control group students used sophisticated strategies more frequently in item 2c (e.g., CR, FC-W). Many students in both groups could realize that the amount of total money in the piggy bank increases by 3 (RP-G strategy).

Item 2d

Item 2d (Figure 4.20) asked students to describe the relationship between the number of the weeks and total amount of money in words.

Figure 4. 20

Item 2d in FTT

d) In your own words, describe the relationship between number of the weeks and total amount of the money in Mert's piggy bank.

Students were assigned two codes for this item; a correctness and a strategy code. The percentage of students' performance is presented in Table 4.52.

Table 4. 52

The Percentage of Correctness of EG and CG in Item 2d in FTT

Item 2d Correctness	Experimental Group N=20		Control Group N=23	
	Pre	Post	Pre	Post
Correct (1)	.00%	25.00%	.00%	4.35%
Incorrect (0)	55.00%	55.00%	73.91%	82.61%
No Response (NR)	45.00%	20.00%	26.09%	26.09%

Students gave incorrect answer predominantly at pretest and posttest. While the percentage of incorrect answer remained the same in both groups, the percentage of NR decreased from pretest to posttest. Therefore, the percentage of correct answer increased from pretest to posttest. Students had difficulty in defining the functional relationship by words. Experimental group showed better performance at posttest (25%) while none of the students could describe it at pretest. In the control group, also none of the students were able to describe the rule in their own words, about 13% did at posttest. In addition to correctness, students' answers were assessed by strategy codes. Figure 4.21 provides the coding scheme for Item 2d that was used to categorize student strategies, the description of the codes, and an example of students' written work.

Figure 4. 21

Coding scheme for Item 2d in FTT

Strategy Code	Description	Example
Functional Condensed-Words (FC-W)	"Students identifies function rule in words that describes a generalized relationship between the two variables."	If we multiply the number of the weeks by 3 we find the earned money. Then, we add 2 TL that is in the beginning.
Functional Emergent-Words (FE-W)	"Students identifies incomplete rule in words, often describing transformation on one variable but not explicitly relating to other."	We multiply the number of weeks by 3 then add 2.
Functional Basic (FB)	"Student identifies general relationship between variables but not the transformation between them."	2 more than 3 times
Single Instantiation (SI)	"Student writes expressions with number or unknowns to define the rule but does not generally relate the two variables."	$4 \times 3=12$ $12+2=14$
Covariational Relationship (CR)	"Student identifies a covariational relationship. The two variables are coordinated rather than mentioned separately."	As the number of the week goes up by 1, the amount of the money goes up by 3. Each week the amount of money increases by 3.
Recursive Pattern General (RP-G)	"Student identifies a recursive pattern in either or both variables."	The amount of money goes up by 3 each time.
Recursive Pattern Particular (RP-P)	"Student identifies a recursive pattern in either or both variables by referring to particular numbers only."	5, 8, 11, 14
Other (O)	Student produces a strategy that differs from above or the strategy is not discernible.	It starts by taking more than four times the number of days and decreases by four times one by one.

Figure 4.21 (continued)

No Response (NR)	Student does not give an answer.

Note. Adapted from "A Learning Progression for Elementary Students' Functional Thinking" by A. C. Stephens, N. Fonger, S. Strachota, I Isler, M. Blanton, E. Knuth, A. M. Gardiner, 2017, *Mathematical Thinking and Learning*, 19(3), p. 153.

The percentages of each strategy used by EG and CG in Item 2d in FTT are in Table 4.53.

Table 4. 53

The Percentage of Strategies used by EG and CG in Item 2d in FTT

Item 2d Strategy	Experimental Group N=20		Control Group N=23	
	Pre	Post	Pre	Post
Functional Condensed-Words	.00%	25.00%	.00%	4.35%
Functional Emergent-Words	.00%	.00%	4.35%	.00%
Functional Basic	.00%	5.00%	.00%	.00%
Single Instantiation	.00%	0.00%	4.35%	.00%
Covariation Relationship	20.00%	15.00%	13.04%	26.09%
Recursive Pattern General	5.00%	0.00%	8.70%	13.04%
Recursive Pattern Particular	.00%	0.00%	4.35%	0.00%
Other	30.00%	35.00%	39.13%	43.48%
No Response	45.00%	20.00%	.00%	.00%

For Item 2d, FC-W strategy was accepted as correct. While 25% of the experimental group used Functional Condensed-Words (FC-W) at post-test , approximately 4% of the control group used. All students in the control group gave a response item 2d. In the experimental group, the percentage of no response decreased from 45% at pretest to 20% at posttest. Some students used RP strategies. (5% at pretest only in the experimental group, about 26% at pretest and 22% at posttest in the control group) Covariational strategy decreased from 20% at pretest to 15% at posttest in the experimental group. However, it doubled in the control group from about 13% at pretest to 26% at posttest. None of the students used SI strategy at posttest while one of the control students did at pretest. FB strategy was not used at posttest neither in the experimental nor in the control group but 10% of the experimental group students used at pretest. Other category was used predominantly in both groups at posttest (35% vs. approximately 43% of experimental and control groups, respectively). The O strategy included “The amount of money equals one more than two times number of weeks” and it start as one more two times and that number increase continually”. Consequently, students had difficulty in using functional thinking strategies.

Item 2e

Item 2e (Figure 4.22) asked students to define functional relationship using letters as variables.

Figure 4. 22

Item 2e in FTT

e) Explain the relationship between number of the weeks and total amount of the money in Mert’s piggy bank by using variables (letters).

Students were assigned a correctness and a strategy code for Item 2e. The percentage of students’ performance is presented in Table 4.54.

Table 4. 54

The Percentage of Correctness of EG and CG in Item 2e in FTT

Item 2e Correctness	Experimental Group N=20		Control Group N=23	
	Pre	Post	Pre	Post
Correct (1)	.00%	25.00%	.00%	.00%
Incorrect (0)	15.00%	50.00%	60.87%	56.52%
No Response (NR)	85.00%	25.00%	39.13%	43.48%

FC-V strategy was accepted as correct for item 2e. There was no correct answer in the control group at either pretest or posttest. In contrast, the percentage of correct answer increased from 0% at pretest to 25% at posttest in the experimental group. While the percentage of no response increased in the control group (from about 39% at pretest to about 43% at posttest), it decreased in the experimental group (from 85% at pretest to 25% at posttest). Students had difficulty in representing the functional relationship by variables. In addition to correctness, students' answers were assessed by strategy codes. Figure 4.23 provides the coding scheme for Item 2e that was used to categorize student strategies, the description of the codes, and an example of students' written work.

Figure 4. 23

Coding scheme for Item 2e in FTT

Strategy Code	Description	Example
Functional Condensed – Variable (FC-V)	"Students write complete rule in variables that describes a generalized relationship between the two variables."	(H×3)+2=P, H×3+2=P, H×3+2=amount of money, number of week×3+2=P
Functional Emergent-Words (FE-W)	"Students identifies incomplete rule in words, often describing transformation on one variable but not explicitly relating to other."	We multiply the number of weeks by 3 then add 2.
Functional Particular (FP)	"Student defines a functional relationship using particular numbers but there is no general explanation or rules by variables."	1×3+2=5, 2×3+2=8, 3×3+2=11….
Single Instantiation (SI)	"Student writes expressions with number or unknowns to define the rule but does not generally relate the two variables."	4×3=12 12+2=14
Covariational Relationship (CR)	"Student identifies a covariational relationship. The two variable are coordinated rather than mentioned separately."	As the number of the week goes up by 1, the amount of the money goes up by 3. Each week the amount of money increases by 3.

Figure 4.23 (continued)

Recursive Pattern General (RP-G)	"Student identifies a correct recursive pattern in either or both variables."	increasing by threes The amount of money goes up by 3
Recursive Pattern Particular (RP-P)	"Student identifies a recursive pattern in either or both variables by referring to particular numbers only."	It goes *5,8,11,14*
Other (O)	Student produces a strategy that differs from above or the strategy is not discernible	Bx2+1, $\square$ x2+1
No Response (NR)	Student does not give an answer.	

Note. Adapted from "A Learning Progression for Elementary Students' Functional Thinking" by A. C. Stephens, N. Fonger, S. Strachota, I Isler, M. Blanton, E. Knuth, A. M. Gardiner, 2017, *Mathematical Thinking and Learning*, 19(3), p. 153.

The percentages of each strategy used by the experimental and control groups in Item 2e are in Table 4.55.

Table 4. 55

The Percentage of Strategies used by EG and CG in Item 2e in FTT

Item 2e Strategy	**Experimental Group N=20**		**Control Group N=23**	
	Pre	**Post**	**Pre**	**Post**
Functional Condensed-Variable	.00%	25.00%	.00%	.00%
Functional Particular	.00%	.00%	.00%	4.35%
Single Instantiation	.00%	.00%	.00%	4.35%
Covariation Relationship	.00%	.00%	8.70%	.00%
Recursive Pattern General	.00%	.00%	4.35%	.00%
Recursive Pattern Particular	.00%	.00%	13.04%	4.35%
Other	15.00%	50.00%	21.74%	39.13%
No Response	85.00%	25.00%	39.13%	43.48%

For Item 2e, Functional Condensed-Variable (FC-V) strategy was coded as correct. In contrast to the control group, the percentage of NR decreased from 85% at pretest to 25% at posttest in the experimental group. Most of the answers were coded in the "Other" category. Control group students used a greater number of strategies than experimental group students did. No control students used FC-V strategy in pretest or posttest. On the contrary, a quarter of the experimental group students used this strategy at posttest while none used at pretest.

Item 2f

Item 2f (Figure 4.24) asked students to use the function rule to predict far function values.

Figure 4. 24

Item 2f in FTT

f) How much money will be in Mert's piggy bank in total at the end of the 30 weeks? Show how you got your answer.

Students were assigned a correctness and a strategy code for Item 2f. The percentage of students' performance is presented in Table 4.56.

Table 4. 56

The Percentage of Correctness of EG and CG in Item 2f in FTT

Item 2f Correctness	Experimental Group N=20		Control Group N=23	
	Pre	Post	Pre	Post
Correct (1)	35.00%	35.00%	34.78%	30.43%
Incorrect (0)	50.00%	55.00%	65.22%	69.57%
No Response (NR)	15.00%	10.00%	.00%	.00%

In this item, students were expected to find far function value as 92 for their response to be coded as correct. Students had difficulty in defining the function rule so most of the students could not give a response correctly. The percentage of correct answer remained the same in the experimental group (35%). There was a slight decrease in the correct answer in the control group from pretest (about 35%) to posttest (about 30%). Students' performance was similar at pretest and at posttest in both groups

In addition to correctness, students' answers were assigned strategy codes. Figure 4.25 provides the coding scheme for Item 2f that was used to categorize student strategies, the description of the codes, and an example of students' written work.

Figure 4. 25

Coding scheme for Item 2f in FTT

Strategy Code	Description	Example
Function Rule (FR)	Student finds the correct response by using the function rule.	30×3=90 90+2=92
Incorrect Function Rule (I-FR)	Student gives response by using an incorrect function rule, multiplying by 3	30×3=90
Other (O)	Student produces a strategy that differs from above or the strategy is not discernible	30×7=350 350×3=1050 TL
Answer Only (AO)	Student writes only answer without showing her/his work	92 TL 90 TL
No Response (NR)	Student does not give an answer.	

FR strategy was the expected strategy for Item 2f. Students were expected to find the amount of money at the end of 30 weeks by using the function rule. But in the case of giving 92 TL by using AO or O strategy, the answer was also accepted as correct. The percentages of each strategy used by both groups in Item 2f are in Table 4.57.

Table 4. 57

The Percentage of Strategies used by EG and CG in Item 2f in FTT

Item 2f Strategy	Experimental Group N=20		Control Group N=23	
	Pre	Post	Pre	Post
Function Rule	25.00%	25.00%	34.78%	30.43%
Incorrect Function Rule	25.00%	15.00%	13.04%	30.43%
Other	30.00%	35.00%	47.83%	30.43%
Answer Only	5.00%	15.00%	4.35%	.00%
No Response	15.00%	10.00%	.00%	4.35%

I-FR was higher in the control group than the experimental group at posttest (15% vs. about 30% while it was 25% vs. about 13% at pretest in the control and experimental groups, respectively). This strategy was used by students who ignored the 2 TL in the piggy bank at the beginning so they defined the rule as “the amount of money equals to 3 times the number of weeks”. Students’ answers were coded as “O” predominantly. The O category included responses such as writing recursive pattern as increasing by 3 until 30th week. The percentage of FR strategy remained the same as 25% in the experimental group. However, it decreased from approximately 35% to about 30% in the control group from pretest to posttest.

Item 2g

Students were expected to work backward in the equation in Item 2g (Figure 4.26).

Figure 4. 26

Item 2f in FTT

g) If a bike cost is 95 TL, how many weeks will it take to have enough money for the bike?

Students were assigned a correctness and a strategy code for this item. Students’ performance is presented in Table 4.58.

Table 4. 58

The Percentage of Correctness of EG and CG in Item 2g in FTT

Item 2g Correctness	Experimental Group N=20		Control Group N=23	
	Pre	Post	Pre	Post
Correct (1)	25.00%	30.00%	43.48%	34.78%
Incorrect (0)	35.00%	55.00%	47.83%	60.87%
No Response (NR)	40.00%	15.00%	8.70%	4.35%

If students gave "31" as a response, it was accepted as a correct answer. Almost all control group students gave a response at posttest. Although 40% of the experimental group students did not give any response at pretest, this decreased to 15% at posttest. While 30% of the experimental students gave a correct answer, approximately 35% of the control group students answered correctly.In addition to correctness, students' answers were coded by strategy codes. Figure 4.27 provides the coding scheme for Item 2g that was used to categorize student strategies, the description of the codes, and an example of students' written work.

Figure 4. 27

Coding scheme for Item 2g in FTT

Strategy Code	Description	Example
Unwinding (U)	Student finds the correct result by working backwards in the equation.	95-2=93 93÷3=31
Guess and Test (GT)	Student works forward in the equation by substituting.	31×3=93 93+2=95
From the Previous item (2f) (PI)	Student finds the amount of money at the end of 30 weeks as 92 TL in the item 2f.	Because the amount of money increases by 3 each week, 95 TL is gotten at the end of the 31 weeks.

Figure 4.27 (continued)

Dividing by 3 (D3)	In the beginning, student divides 95 by 3 and s/he does not interpret the remainder.	95 | 3 93 | 31 − 02
Other (O)	Student produces a strategy that differs from above or the strategy is not discernible	95-1=94÷2=47
Answer Only (AO)	Student writes only answer without showing her/his work	31 30
No Response (NR)	Student does not give an answer.	

Unwinding (U) strategy was the expected strategy for Item 2g. This strategy requires to work backwards in the equation as "95-2=93 TL 93÷3=31 weeks". The percentages of each strategy used by the experimental and control groups in Item 2g are in Table 4.59.

Table 4. 59

The Percentage of Strategies used by EG and CG in Item 2g in FTT

Item 2g Strategy	**Experimental Group N=20**		**Control Group N=23**	
	Pre	**Post**	**Pre**	**Post**
Unwinding	.00%	5.00%	13.04%	8.70%
Guess and Test	.00%	5.00%	4.35%	17.39%
Dividing by 3	5.00%	.00%	4.35%	.00%
Previous Item	10.00%	5.00%	13.04%	.00%
Other	10.00%	35.00%	39.13%	60.87%
Answer Only	30.00%	35.00%	17.39%	8.70%
No Response	40.00%	15.00%	8.70%	4.35%

The Unwinding (U) strategy was the most sophisticated strategy. In the control group, the percentage of using "U" strategy was higher than the experimental group.

"Guess and Test" strategy was the application of the function rule by substituting one value "31". In the experimental group, AO and O strategies were used predominantly (about 30% at pretest and 35% at posttest for AO and 10% at pretest and 35% at posttest for O). Most of the students' answers (about 61%) in the control group were coded as "O" at posttest. Other stagey included responses such as "95÷ 6=16" and "32th week since 30th week he gets 90 TL, at 31st 93 TL, and at 32nd week 96 TL and 1 TL of him will remain in her pocket".

As a result, students were more successful in Item1 than Item 2. Item 1 asked students to define $y=2x$ functional relationship. Students could organize table, realize the patterns on the table and define functional relationship between the number of days and the number of circles. Both experimental and control group students showed similar performances in defining the functional relationship by their words (30% and 34% in experimental and control group, respectively). However, experimental group students had better performance in defining the functional relationship by variables. In addition, students were asked to draw a coordinate graph to represent functional relationship in item 1. Students tended to draw bar graph at pretest. In contrast to control group, more than half of the experimental group students were able to draw a coordinate graph at posttest item 1. Item 2 asked students to define $y=3x+2$ functional relationship. Students had difficulty in organizing the table, realizing patterns in the table and defining functional relationship between the number of the weeks and the amount of money in the piggy bank. Most of the students ignored "2 TL in the piggy bank at the beginning" so they defined the functional relationship as $y=3x$. In item 1d, control group students (34%) were better than the experimental group (30%) in defining function rule in words. In contrast to Item 1, experimental group students (25%) had better performance than control group in defining the functional relationship by words item 2 at posttest. Moreover, experimental group students were more successful in defining the functional relationship in variables for both items.

CHAPTER 5

DISCUSSION AND IMPLICATIONS

The purpose of the present study was to investigate the effects of functional thinking intervention on students' functional thinking skills. In this chapter, the findings will be summarized and discussed. Also, recommendations and implications will be presented.

12.1 The Effects of Intervention on Student's Functional Thinking

The findings showed that there was no significant difference between the pre-test or post-test scores of the experimental and control group students. Although the control group's mean (M=5.65) was higher than the experimental group's mean (M=4.75) at the pre-test, the experimental group's mean (M = 7.05) was higher at the post-test than the control group (M = 6.39). In contrast to the control group, the experimental group showed a statistically significant gain between the tests. These findings support the other studies (e.g., Blanton, Isler et al. 2019; Blanton, Stephens, et al., 2015) in that the experimental group showed significant development in defining functional relationships after the intervention.

Control group students did not receive any intervention about functional thinking. According to Grades 1-8 National Mathematics Curriculum, (MoNE, 2018), students worked on the geometric concepts (constructing basic geometric constructions; line, line segment, ray, types of angles and defining place of the points by unit and direction. Therefore, control group students' development could not be explained by the curriculum. However, control group students were interested in the content of the study during the pre-test; they asked the researcher questions so they might have searched for variables and students might have become more familiar with the items in the post-test, which could have affected the results of the study.

Blanton, Stephens et al. (2015); Blanton et al. (2018); Pinto and Canadas (2018) performed intervention studies and found that extensive early algebra instructions developed students’ algebraic thinking skills, including functional thinking. In addition, non-intervention studies (e.g., Blanton & Kaput, 2004; Tanışlı, 2011; Türkmen & Tanışlı, 2019) revealed that students were able to think functionally in even early grades and they could engage in functional thinking activities.

Moreover, the descriptive results of the present study showed that the experimental group students used more sophisticated strategies in defining functional relationships than the control group students did at post-test. At the item level, in item 1e and 2e, experimental group students significantly outperformed the control group in writing the function rule in variables. Similarly, Blanton, Stephens et al. (2015) mentioned that experimental students used more algebraic strategies than the control group did at post-test.

12.2 Generalizations and Representations of Functional Relationships

Functional thinking is one of the three strands defined by Kaput (2008) and one of the five big ideas of algebraic thinking (Blanton et al., 2011). Blanton et al. (2018) defined functional thinking “to include generalizing relationships between co-varying quantities and representing, justifying, and reasoning with these generalizations through natural language, variable notation, drawings, tables, and graphs” (p. 33). The present study aimed to investigate effects of functional thinking intervention on students’ functional thinking skills. Students were asked to define $y=2x$ and $y=3x+2$ functional relationships. In general, students were more successful in defining the $y=2x$ functional relationship than $y=3x+2$. Similarly, Türkmen and Tanışlı (2019) and Blanton, Brizuela et al. (2015) reported that students could define $y=mx$ functional relationship easier than $y=mx+b$. On the other hand, some items could be responded using arithmetic strategies instead of the function rule. So, control group students also were successful on those items.

In item 1a and item 2a, students were asked to find near value for the given patterns. Item 1a included a visual pattern increasing by two circles for each day. Item 2a asked to define the number pattern increasing by 3 TL for each week by starting 2. Almost all students found near values correctly for item 1a by using drawing (15% of the experimental group, approximately 22 % of the control group) and recursive patterning (15% of the experimental group, approximately 23 % of the control group) strategies, also by function rule (20% of the experimental group, approximately 26% of the control group) at post-test. Most of the students wrote only answer as 10 circles at the post-test (45% vs. about 30% for experimental and control groups, respectively). However, item 2a was harder to find near values for students. Most of the students could not find the near values correctly. Some students considered that the amount of money (2 TL) in the piggy bank that was saved in the first week (instead of 5 TL). Therefore, they answered for the second week as 5 TL (instead of 8), the third week as 8 TL (instead of 11) and so on. Some of the students ignored the 2 TL in the piggy bank at the beginning; therefore, they found as 6 TL for the second week, 9 TL for the third week, and 12 TL for the fourth week by using the rule $y = 3x$ instead of $y = 3x + 2$.

In item 1b and item 2b, students were expected to complete or construct the function table. Almost all of the experimental and control group students organized the table correctly in item 1b that required $y=2x$ functional relationship at posttest. Item 2b was harder for students to construct and organize the table. Although a few experimental students completed this item correctly at the pre-test, nearly half of the experimental students constructed and organized the table at post-test. This result was consistent with the study conducted by Isler et al. (2014/2015) and Stephens et al. (2012) which stated third, fourth, and fifth-grade students could construct tables representing functional relationships if appropriate experiences are provided. In addition, Yeşildere-İmre et al. (2017) found that while middle grade students were focusing on arithmetic generalizations for figural patterns, they tended to define algebraic generalizations for patterns presenting in the table. Therefore, tables helped students to realize patterns and relationship between variables and to generalize those relationships algebraically.

In item 1c and item 2c, students were asked to describe patterns they saw in the table. Almost half of the experimental group students and more than half of the control group students defined the recursive pattern-general (L2) as "The number of circles increases by 2" for item 1c at the pre-test. This was not surprising because recursive patterns are focused on in the Grades 1-8 National Mathematics Curriculum (MoNE, 2018) through 5th grade. Although in the control group, the percentage of using recursive pattern increased at post-test, it decreased in the experimental group. More than half of the experimental group students defined covariational and functional relationships at post-test. These findings were consistent with the study conducted by Stephens et al. (2012), which asserted that a classroom teaching experiment based on early algebra helped students regard the covariational and functional relationships between two co-varying variables. On the other hand, for item 2c, the percentage of the recursive pattern (the amount of money increasing by 3) increased at the post-test in both the experimental and control groups. Students had difficulty in describing the covariational and functional relationship between co-varying variables (the number of weeks and the amount of money in the piggy bank). The control group students were more successful than the experimental group in defining patterns.

In item 1d and item 2d, students were asked to define the function rule in words. The control group was more successful in defining the function rule in words (39% vs. 5% for control and experimental group, respectively) at the pre-test in item 1d. In the control group, the percentage of using covariational relationship (L3) increased at post-test. However, in the experimental group, the percentage of writing the function rule (functional condensed in words (L10)) increased (from 5% to 30%) at the posttest. This result was similar to the “main path”, that is, students tended to define the recursive relationship at the beginning. Then, they shifted towards correspondence thinking (Stephens et al., 2017). Similarly, in item 2d, while the control group showed an increase (13% vs. 26% for pre-test and post-test, respectively) in the covariational relationship (L3), the experimental group showed an increase (0% vs. 25% for pre-test and post-test, respectively) in writing the function rule (functional condensed in words (L10)).

Item 1e and item 2e asked students to write the function rule in variables. As expected, there were no students who defined the function rule in variables at the pre-test. In item 1e, the experimental group showed development between the pre- and post-test (from 0% to 45%) in defining the function rule by variables. On the other hand, it was surprising that two students from the control group could write a correct equation for item 1e at post-test. In item 2e, all students struggled with the function rule. Stephens et al. (2017) found that students were more successful in writing the function rule in variables than words. On the other hand, in the current study, for the experimental group, the percentage of defining the function rule in variables (25%) was equal to defining function rule in words (25%) for item 2e at post-test. That is, students who defined the function rule in words could write an equation. It was observed that one of the experimental students used variables to represent quantities, but he could not write an equation for the relationship between variables, and defined a recursive pattern using variables such as “$Q \xrightarrow{+3} X$”. This showed that students needed more practice with variables to define the relationship between two quantities in the equation form. In addition, for item 2e, some experimental students wrote the function rule in a different way as “P-2÷3=H” instead of “(P-2)÷3=H”. Those students were able to define the functional relationship between the amount of money and the number of weeks but they ignored the order of operations. In addition, one of the experimental students could find near values and construct the table correctly in item 2. However, he could not write function rule and use the equal sign correctly, such as “H × 2 = T + 2 = P” for the item 2e at post-test. Similarly, Strachota et al. (2016) investigated that while a fourth-grade student could define function rule in the level of functional condensed in variables (L9) for *y=5x* functional relationship, for the more challenging task, he responded in the lower level as "x · 5 = y + 2" instead of "x · 5 +2= y". They reported that his operational view of the equal sign (Stephens et al., 2013) affected how he wrote the function rule.

In item 1f and item 2f, students were asked to find far values. In item 1f, students could define *y=2x* functional relationship in words or variables; therefore, students were more successful in finding the far value. Experimental and control group students showed similar performance at the pre-test, and also both groups showed improvement at post-test. However, the control group used the function rule more than the experimental group did (96% vs. 85% for the control and experimental group, respectively) at post-test. Similarly, Blanton, Stephens et al. (2015) did not find significant difference between the non-intervention and intervention students in finding the far function value. They explained that items asking near and far values could be solved by arithmetic ways rather than algebraic ways so non-intervention students could find correct results. In contrast to item 1f, students struggled with item 2f. So, the percentage of using the function rule was lower than 1f (25% vs. 30% for the experimental and control group, respectively) at post-test. Also, there was not a significant difference between the groups. Similarly, Stephens et al. (2012) working with third through fifth-grade students on the Brady task problem that required students to define *y=2x+2* functional relationship. Stephens et al. (2012) reported that both control and experimental groups showed development at post-test and there was no significant difference between the control and experimental groups in predicting far values at post-test. In the present study, there was no change in the experimental group in terms of using the function rule to find the far value, and there was a little decrease (approximately 4%) in the control group at post-test. While "Answer Only (AO)" increased (from 5% to 15%) in the experimental group at posttest, “Incorrect Function Rule”, that is using $y=3x$ instead of $y=3x+2$, increased (from 13% to 30%) in the control group at post-test.

In item 1g, students were asked to construct a coordinate graph to show the relationship between the number of days and the number of circles. In the pre- and post-test, there were no students who constructed this graph correctly. In the pre-test, the bar graph was a strategy used by both groups. In the Grades 1-8 National Mathematics Curriculum (MoNE, 2018), until the 7th-grade level, students are familiar with the bar graph. So, this result was anticipated. Moreover, a few students in both groups placed points on the x and y axes correctly, but they matched the axes without indicating points (coded as Axes Matched Correctly, AxM) in the pre-test. In the post-test, more than half of the experimental group students placed the points correctly on the axes but matched the axes without indicating points. Although those students realized the relationship between the number of days and the number of circles, they could not represent it in the coordinate graph. During the intervention, graph representation was handled only in the third lesson plan, so this may not have been sufficient for students to interiorize graphs for representing functional relationships.

In item 2g, the value of the dependent variable was given, and the value of the independent variable was asked. In contrast to Blanton, Stephens et al. (2015), the percentage of the correct answer in the control group was higher than the experimental group. In this item, it was aimed that students could use reversibility and use the “unwinding” strategy that was accepted as a more algebraic strategy for solving equations before students were taught equation solving at the middle school (Blanton, Stephens et al., 2015, p. 57). Blanton, Stephens et al. (2015) found that while 11% of the intervention group used the “Unwinding” strategy correctly, no control group students used this strategy at post-test. In the present study, it was surprising that both in the pre-test (0% vs. 13% for the experimental and control group, respectively) and post-test (5% vs. 9% for the experimental and control group, respectively), more students in the control group used this strategy. The reason that the experimental group might not have used this strategy was 35% of the experimental students were found to give the correct answer without showing their work (“Answer Only” strategy) at post-test.

As a result, there was no significant mean difference between the experimental and control group at pre-test, bu, the control group’s mean was higher than the experimental group at the pre-test. Although there was no significant mean difference between the experimental and control groups at the post-test, the experimental group showed a higher performance at post-test and significant pre-to-post gains.

12.3 Implications

In this part, implications and recommendations for future studies will be presented. As mentioned above, many studies (e.g., Blanton, Brizuela et al., 2015; Cañadas et al., 2016; Stephens et al., 2017) reported that students could define, represent, and generalize function rules in words and variables in the case of providing appropriate environment. The present study likewise found that functional thinking intervention helped fifth-grade students to gain an algebraic approach to functional relationships. In the present study, although students could define the $y=2x$ functional relationship by using multiple representations, students had difficulty in defining the function rule in both words and variables for $y=3x+2$. However, it was observed that students in this study needed more practice to comprehend the use of variables and the relational meaning of the equal sign.

Also, in the Grades 1-8 National Mathematics Curriculum (MoNE, 2018), the meaning of the equal sign is involved starting from first grade and students are expected to work with visual and number patterns in different grade levels. Then, they meet formal algebra , specifically variables, in 6th grade; equations in 7th grade and functional thinking take part in 8th grade and beyond. Çelik and Güneş (2013) found that 7th and 8th grade students had difficulty in comprehending literal symbols as unknown, variable and generalized number. Although 9th grade students were expected to use diverse roles of litteral symbols in the mathematics curriculum, most 9th grade students could not understand the variable role of literal symbols. Dede and Argün (2003) assessed the reasons of students' difficulties in terms of the structure of algebra, students' readiness level and the missing in teaching of algebra. However, the present study showed that students could use variables and equations to generalize functional relationships after the intervention, so students were capable of thinking algebraically in early grades. In conclusion, functional thinking could help students develop algebraic thinking if it was introduced early on in the curriculum especially through contextual problems like that were used in this study; therefore, curriculum developers could consider the results of this study in that regard. Kaya and Keşan (2014) argued that algebraic thinking and reasoning starts from elementary grades and continues with algebra instruction so providing learning settings that develop students' algebraic thinking is important. Functional thinking activities based on contextual problems and multiple representations would be an effective way to improve students' algebraic thinking and reasoning in early grades. Also, Carraher et al. (2006) defended that functions is a comprehensive topic that unite the other subtopics of algebra, and it should be included in the curriculum in early grades. Moreover, Carraher and Schliemann (2007, as cited in Stephens et al., 2017) explained functional thinking as an essential way to algebra since it includes generalizations of relations between variables, representing relationships by tables, graphs, words, and algebraic notation and reasoning.

All in all, implementation is one of the key points to develop students' algebraic thinking, also functional thinking, so teachers' approach and knowledge play an essential role. Blanton (2008) suggested four instructional goals for teachers, which are "representing, questioning, listening and generalizing" (p. 94). Teachers can foster the classroom environment (using group works, contextual problems, whole-class discussions) so that students are able to represent algebraic situations and also ask questions to help students justify their generalizations. Teachers' role is essential in planning algebraic thinking, specifically functional thinking activities and implementing them effectively. Therefore, pre-service mathematics teachers and elementary teachers should be asked to prepare lesson plans and activities based on algebraic thinking including functional thinking for early grades in the methods courses. Both preservice and inservice teachers should be made aware regarding the role of early algebra including the focus on the meaning of the equal sign, different roles of variables, and equations, and functional thinking.

There are some recommendations for future studies in light of the findings of the present study. As mentioned before, the background characteristics of the students should be regarded as important in experimental studies. Moreover, the scope of functional thinking intervention should include algebraic concepts like the relational meaning of the equal sign, using variables as letters to help students represent function rules correctly and meaningfully. In addition to written responses of students, pre-, mid- and post- interviews might help understand students' thinking and progress deeper at the beginning and end, and during the course of the intervention. Future studies can also include the teacher perspective, teachers can receive training to provide the functional thinking intervention themselves. Also, the national and international studies on early algebra and functional thinking did not focus on integrating technology at interventions. Technological tools would help teachers and students in teaching and learning algebraic thinking in early grades. So, it is suggested that future studies can focus on integrating technology perspective.

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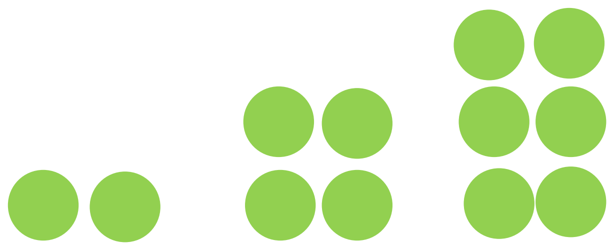
APPENDICES

A. FUNCTIONAL THINKING TEST

AD-SOYAD:

1. Selin, her gün okulda dairelerden oluşan bir resim çiziyor. Selin'in ilk üç günde çizdiği resimler aşağıdaki gibidir:



1.Gün 2. Gün 3. Gün

a) Selin'in, 5. günde çizeceği resimdeki daire sayısını bulunuz.

b) Elde ettiğiniz verileri yandaki tabloya yazınız.

Gün sayısı	Daire sayısı

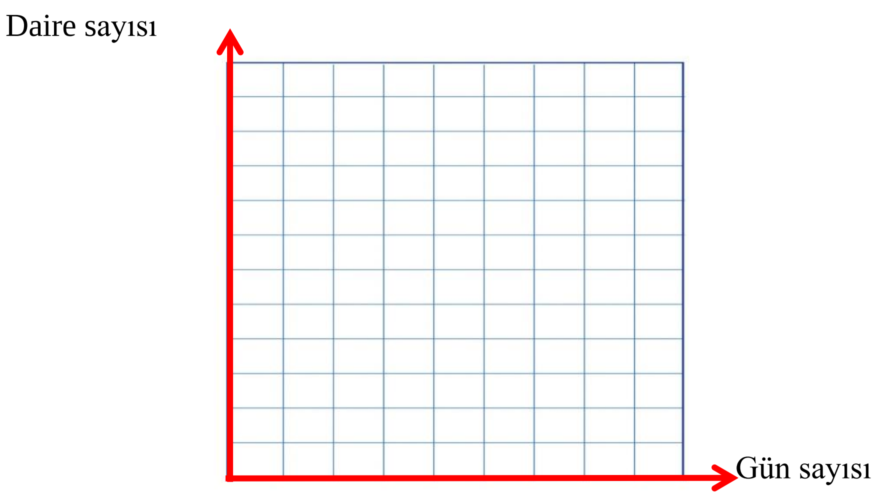
c) Tabloda hangi örüntüler vardır? Açıklayınız.

d) Gün sayısı ile daire sayısı arasındaki ilişkiyi açıklayan kuralı sözcüklerle açıklayınız.

e) Gün sayısı ile daire sayısı arasındaki ilişkiyi açıklayan kuralı değişken kullanarak yazınız.

f) Selin'in, okulun 100. gününde çizeceği resimde kaç tane daire olmalıdır?

g) Gün sayısı ile daire sayısı arasındaki ilişkiyi grafikle gösteriniz.



2. **Mert'in en başta kumbarasında 2 TL'si vardır. Mert'in babası ev işlerinde yardımcı olduğu için her hafta Mert'e 3 TL verme kararı almıştır ve Mert aldığı harçlıkları kumbarasında biriktirerek toplam parası ile bir bisiklet almaya karar veriyor. Buna göre;**

a) Mert'in; 2 hafta, 3 hafta ve 4 hafta sonunda kumbarasındaki toplam para miktarı ne kadardır?

b) Elde ettiğiniz bilgileri tablo oluşturarak düzenleyiniz.

c) Tabloda hangi örüntüler vardır? Açıklayınız.

d) Hafta sayısı ile Mert'in kumbarasındaki toplam para miktarı arasındaki ilişkiyi sözcüklerle açıklayınız.

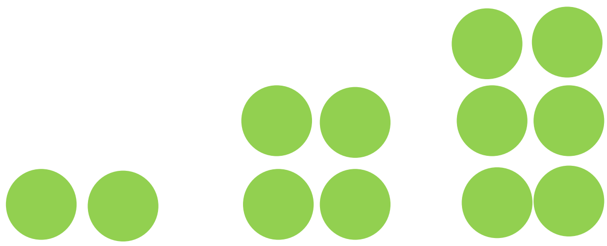
e) Hafta sayısı ile Mert'in kumbarasındaki toplam para miktarı arasındaki ilişkiyi değişken kullanarak yazınız.

f) Mert, 30 hafta sonunda kumbarasındaki toplam para miktarı ne kadar olur?

g) Mert'in almak istediği bisikletin fiyatı 95 TL ise Mert kaç haftanın sonunda bu bisikleti satın alabilir?

Name - Surname:

1) Each day in the class, Selin creates a picture by drawing circles joined together. Following are the pictures of circles that she drew on each day:



Day 1 Day 2 Day 3

a) How many circles are in her picture for Day 5?

b) Organize your information in the given table.

Number of Days	**Number of Circles**

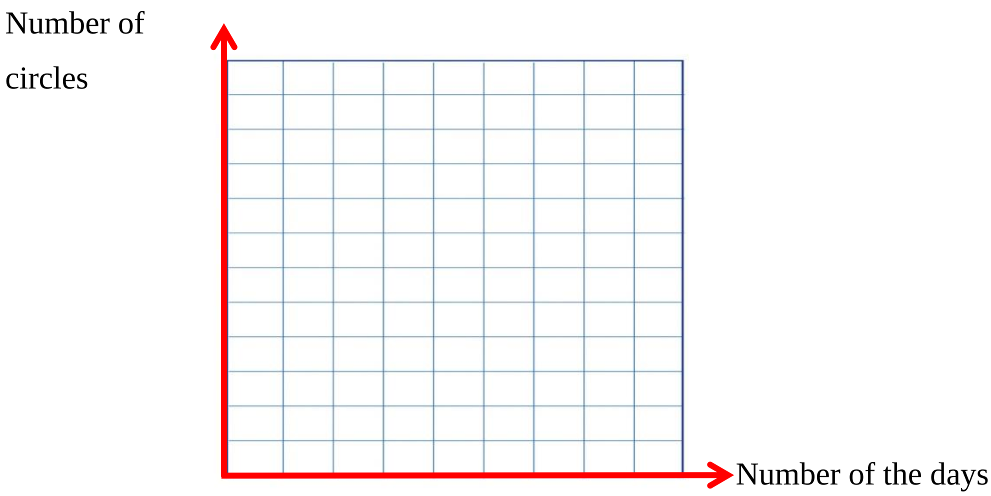
c) Which patterns do you see in the table? Describe.

d) In your own words, describe the relationship between the number of days and the number of circles.

e) Explain the relationship between the number of days and the number of circles by using variables (letters).

f) How many circles will be in the picture that Selin draws on the 100th day of the school? Show how you got your answer.

h) Show the relationship between the number of days and the number of circles on the graph below.



2) There are 2 TL in Mert's piggy bank at the beginning. Every week Mert's dad gives him 3 TL for helping with chores around the house. Mert is saving his money in his piggy bank to buy a bike.

a) How much money is there in Mert's piggy bank in total at the end of the Week 2? Week 3? Week 4?

b) Organize your information in a table.

c) Which patterns do you see in the table? Describe.

d) In your own words, describe the relationship between the number of weeks and the total amount of the money in Mert's piggy bank.

e) Explain the relationship between the number of weeks and the total amount of money in Mert's piggy bank by using variables (letters).

f) How much money will be in Mert's piggy bank in total at the end of the 30th week? Show how you got your answer.

g) If a bike's cost is 95 TL, how many weeks will it take to have enough money for the bike?

B. LESSON PLANS FOR FUNCTIONAL THINKING INTERVENTION

DERS PLANI 1:

Bu derste hedeflenen kazanımlar,

* Bilinmeyen bir niceliği göstermek için bir değişken tanımlama
* Değişkenin rolünü değişen bir nicelik olarak inceleme
* Bir niceliği değişken kullanarak cebirsel ifade ile temsil etme
* Cebirsel bir ifadeyi bağlam içinde yorumlama

Öğrencilerden beklenen ön bilgiler:

* Bir eşitlikte bilinmeyen sayıyı hesaplayabilir.
* Bilinmeyeni □, △ gibi sembollerle ifade edebilir.

Öğretim Tekniği: Grup çalışması, Tartışma, Keşfetme

Materyal: Etkinlik kağıtları, çıkış kartları

Süre: 80 dk.

GİRİŞ (5-10 dk.):

- Derse etkinlik kağıdındaki ilk soru ile başlanır.
- İlk soruda öğrencilerden eşitliklerin doğru olması için gerekli sayıları bulmaları istenir.
- Eşitliğin anlamı üzerinde durulmalıdır.
- Öğrenciler eşitlik sembolünün 'sonuç' anlamına odaklanarak yanlış cevaplar verebilirler. (8+42=50 ya da 53+27= n+23 n=80, n=103 olarak cevaplayabilirler. 'a+11=13+7' ifadesinin yanlış olduğunu, eşitlikten sonra toplama işlemi olmaz gibi cevaplar verilebilir.)
- Bu cevapların gelmesi durumunda eşitlik sembolünün anlamı denge ve eşit kollu terazi kavramları üzerinden anlatılabilir.
- Öğrenciler eşitliği "işlem yapmak" (53+27= n+23 n=80) ya da eşitliğin iki tarafının eşitliğini kontrol etmek için iki taraftaki işlemlerin sonuçlarını karşılaştırır (53+27= n+23 eşitliğinde 53+27=80 bu nedenle eşitliğin diğer tarafının da 80 olması gerektiği için 80-23= 57 n=57). Öğrencilerden eşitlik sembolünün ilişkisel anlamını kavramaları beklenir (53+27= n+23 eşitliğinde eşitlik sembolü terazi gibi düşünülerek sol taraftaki 27 sayısı 4 azalarak 23 olmuş bu durumda dengenin bozulmaması için sol taraftaki 53 değeri 4 artmalıdır yani n= 57).

- Öğrencilere bu sorudaki kutu, çizgi, nokta ve harflerin neyi ifade ettiği sorulmalıdır. Bu sembollerin görevi nedir sorusu sorularak öğrencilerin bilinmeyeni ifade etmek için çeşitli semboller kullanıldığını ve ‘*bilinmeyen*(unknown)’ kavramını kavraması amaçlanmaktadır.

GELİŞME (25-30 dk):

- Etkinliğin ana kısmı 2. Ve 3. Sorudan oluşmaktadır.
- Öğrencilerin 3-4 kişilik gruplar halinde çalışmaları istenir.
- Öğrencilere 2. Soru için düşünme süresi verilir bu sırada gruplar gözlemlenir.
- 2. Sorunun ilk kısmında (a) öğrencilerin problemdeki bilinmeyen şeker sayılarını resim kullanarak (kutu, nokta , daire…vb.) çizmeleri ya da tahmini değerleri tablo halinde yazmaları ve buradan bilinmeyen değeri ifade etmek için sembol kullanmaya geçilebilir.
- Tablo oluşturmada öğrencilere yardımcı olmak için önce öğrencilerden şeker sayıları için tahminler alınır ve tabloya yazılır.
- Farklı öğrencilerden farklı sayı tahminleri gelecektir. Bu durumda şeker sayısın bilinmediğini ve herkesin farklı tahminini genellemek için başka bir şekilde ifade edilmesi gerektiği vurgulanır.
- Öğrencilerden gelen farklı sayısal cevaplar üzerinde durularak “ Herkesin farklı tahminleri var ve hepsi farklı ve biz hem Elif’in hem de Can’ın şeker sayısını bilmiyoruz. Bilinmeyen bir niceliği ifade etmek için ne yaparız?” şeklinde öğrenciler sembol kullanmaya teşvik edilebilir.
- Öğrencilerin Can’ın sahip olduğu şeker sayısını $\square$, Elif’in sahip olduğu şeker sayısını $\square+3$ olarak ifade etmesi beklenir
- Burada $\square$ ifadesinin yerine *x,y,z,n,a,b,c* gibi değişkenler kullanmaya geçiş yapmak için ‘Şeker sayısını daha farklı şekilde nasıl ifade edebiliriz?’sorusu yöneltilebilir.
- Son durumda Öğrencilerin Can’ın sahip olduğu şeker sayısını n , Elif’in sahip olduğu şeker sayısını $n+3$ olarak ifade etmesi beklenir.
- İlk sorudaki sembollerin kullanımına ve anlamına yönelik ‘ *Kullandığımız kutu ve harfler neyi ifade etmektedir?*’ sorusu yöneltilmelidir.
- 1. Soruda bilinmeyeni ifade ederken kutu ve harflerin kullanıldığına vurgu yapılarak bu soruda da bilinmeyen şeker sayısını ifade etmek için farklı semboller kullanılabileceği üzerinde durulur.
- Öğrencilerden Elif’in şeker sayısını a,b,c,n Can’ın şeker sayısını a+3,b+3, c+3, n+3 şeklinde bilinmeyen kullanarak ifade etmeleri beklenmektedir.
- Aşağıdaki tablo sayılardan sembol ve harf (bilinmeyen) kullanımına geçerken kullanılabilir.

Elif'in şeker sayısı	Can'ın şeker sayısı
1	1+3=**4**
2	2+3=**5**
5	5+3=**8**
10	10+3=**13**
$\square$	$\square$ **+3**
N	**N+3**
a	**a+3**

- 3. Soru için öğrencilere düşünme süresi verilir. Grup içinde tartışmaları istenir.
- Tablo ve resim kullanmaları için yönlendirme yapılabilir.
- Öğrencilere şeker sayıları içi tahmin yapmaları istenir. Toplamı 28 olan farklı durumları söylemeleri beklenir (14 ve 14, 18 ve 10 gibi).

Hikaye Kitabı Sayısı	Şiir Kitabı Sayısı	Toplam Kitap sayısı
5	**23**	**5+23=28**
7	21	7+21=28
10	18	10+18=28
14	14	14+14=28
$\square$	$\triangle$	$\square$+ $\triangle$=28
a	**b**	**a+b=28**

- Burada hikaye ve şiir kitapları için için farklı değişkenler kullanmaları gerekmektedir.
- Kullanılan sembollerin görevi ve anlamına yönelik '*Kullandığımız sembol ve harfler neyi ifade etmektedir?*' sorusu sorulmalıdır.
- Önceki soruda kullanılan sembollerin ve harflerin bilinmeyeni, bu soruda kullanılan sembol ve harflerin değişkeni temsil ettiği üzerinde tartışılmalıdır.
- Kullanılan değişkenlerin ifade ettiği niceliği tanımlamaları önemlidir.

BİTİŞ (5 dk.)

- Derste yapılan etkinliklerin amaçları ve sonuçları özetlenir.
- Bilinmeyen(unknown) ve değişken(variable) kavramları tekrar edilir.
  - * *Bir eşitlikte sabit bir sayı değerinin yerini tutan sembol ve harflere bilinmeyen (n,n+3) gibi),*
  - * *Birden fazla bilinmeyen değeri ifade eden sembol ve harfler değişken (x+y=28) olarak tanımlanmaktadır.*

ETKİNLİK KAĞIDI 1

1) Aşağıdaki eşitliklerde bilinmeyen sayıları bulunuz.

$15+\square = 22$	$\_\_\_ - 37 = 48$
$8 + 42 = ... + 8$	$9 . 6=6. x$
$5+4= 3+\triangle$	$a + 11 = 13 + 7$
$53+ 27= n+ 23$	$118+ y = 62+ 119$

2) Elif ve Can'ın birer kutu şekeri vardır. İkisinin kutusunda da eşit sayıda şeker vardır. Elif'in elinde 3 tane daha şeker olduğuna göre;

a) Elif ve Can'ın şeker sayılarını farklı şekillerde ifade edebilir misiniz?(Resim, tablo,..)

b) Elif ve Can'ın sahip olduğu şeker sayılarını sembol (şekil, harf) kullanarak matematiksel olarak ifade ediniz.

3) Tuna öykü ve şiir kitaplarını okumayı çok seven bir çocuktur. Tuna, okuduğu öykü ve şiir kitaplarından oluşan bir kütüphane kurmayı hayal etmektedir. Şu anda Tuna'nın toplam 28 tane öykü ve şiir kitabı olduğuna göre;

a) Tuna'nın kitaplığındaki öykü ve şiir kitabı sayılarını farklı şekillerde ifade edebilir misiniz? (Resim, tablo,..)

b) Tuna'nın kitaplığındaki öykü ve şiir kitabı sayılarını <u>değişken kullanarak</u> matematiksel olarak nasıl ifade edebilirsiniz?

DERS PLANI 2

Bu derste hedeflenen kazanımlar;

* Elde ettiği verileri tablo kullanarak düzenleyebilme
* Değişkenleri ve değişken rollerini sözel olarak tanımlayabilme
* Özyinelemeli örüntü (recursive pattern) sözel olarak tanımlayabilme
* Birlikte değişimsel ilişki (covariational relationship) sözel olarak tanımlayabilme
* Fonksiyon kuralını sözel ve sembolik olarak tanımlayabilme($y=x$ ilişkisini kurabilme ,sözel ve sembolik olarak tanımlayabilme)

Öğrencilerden beklenen ön bilgiler:

* Bilinmeyen bir niceliği göstermek için bir değişken tanımlama
* Değişkenin rolünü değişen bir nicelik olarak inceleme
* Bir niceliği değişken kullanarak cebirsel ifade ile temsil etme
* Cebirsel bir ifadeyi bağlam içinde yorumlama

Öğretim Tekniği: Keşfetme, Tartışma, Grup Çalışması

Materyal: Etkinlik kağıtları, Çıkış kartları

Süre: 80 dk

Başlangıç (10 dk.):

* Önceki derste öğrenilen bilinmeyen ve değişken kavramlarının anlamına ve kullanımına yönelik tekrarlar yapılır.
* Eşittir işaretinin anlamı üzerine tekrar yapılır.

Gelişme (60 dk.):

- Çalışma kağıtları dağıtılır.
- Öğrenciler 3-4 kişilik gruplara ayrılır.
- Öğrencilerin problem üzerinde konuşması, anlaması ve problemi tartışmaları için zaman verilir.
- Öğrencilerin 2 tavuğun 2 yumurta, 3 tavuğun 3 yumurta, 4 tavuk için 4 yumurta ve 5 tavuk için 5 yumurta örüntüsünü belirlemesi gerekmektedir.
- Tablo ile çalışma konusunda karışıklık yaşanması durumunda tablo düzenleme kısmında yardımcı olunabilir.

- Tablodan da yararlanarak örüntünün nasıl ilerlediği sorulabilir. Öğrenciler sadece yumurta sayısının ardışık olarak arttığını ve kuralı +1 olarak tanımlayabilirler bu durumda ileri adımlar sorulabilir. 50 tavuk, 100 tavuk gibi.
- Tavuk sayısı ve yumurta sayısı arasındaki ilişkiyi tanımlamakta zorlanmaları durumunda *"Tavuk sayısı .... artarken, yumurta sayısı .... artar."* İfadesi ile birlikte değişimsel ilişki (covariational relationship) tanımlamaları konusunda yönlendirme yapılabilir.
- Öğrenciler tabloyu dikey olarak yorumlayarak özyinelemeli(recursive pattern) tanımlayabilirler. Bu durumda tabloya yatay olarak bakmaları konusunda yönlendirme yapılabilir.
- Bu aşamada öğrencilerden "Tavuk sayısı ve yumurta sayısı eşittir" ya da "Yumurta sayısı tavuk sayısının 1 katıdır." ilişkisini bulmaları beklenir.
- Sözel olarak ifade edilen ilişkiyi değişken olarak ifade etmeye geçerken öncelikle semboller kullanılabilir. △, □
- Tavuk sayısı T ile yumurta sayısı Y ile gösterilebilir. Öğrencilerin T=Y ya da t=y fonksiyonel gösterime ulaşmaları önemlidir.
- Yazılan eşitliğin genel bir fonksiyon kuralı olduğu üzerinde durulmalıdır.

Bitiş (10dk.):

- Ders içindeki etkinlikler, etkinliğin amacı ve ulaşılan sonuçlar özetlenir;
  - Birlikte artış gösteren, iki değer için de artış miktarı aynı olan değişkenler arasındaki ilişkiyi gösteren ifade "*y=x*"dir.
- Çıkış kartı dağıtılır ve çözüm için süre verilir.

ETKİNLİK KAĞIDI 2

1) Ali amca organik yumurta satmak için bir tavuk çiftliği kurmak istemektedir. Ali amca yaptığı araştırma sonucunda bir tavuğun günde 1 tane yumurta verdiğini öğrenir. Buna göre;

a) Ali amca çiftliğe 2 tavuk alırsa günde kaç tane yumurta elde eder? 3 tavuk ? 4 tavuk? 5 tavuk?

b) Elde ettiğiniz verileri tabloya yerleştiriniz.

Tavuk Sayısı	***Yumurta Sayısı***

c) Tabloda hangi örüntüler vardır? Açıklayınız.

d) Tavuk sayısı ile yumurta sayısı arasındaki ilişkiyi nasıl ifade edebilirsiniz?

e) 50 tavuk alırsa bir günde kaç tane yumurta elde eder?

DERS PLANI 3:

Bu derste hedeflenen kazanımlar;

∗ Elde ettiği verileri tablo kullanarak düzenleyebilme
∗ Değişkenleri ve değişken rollerini sözel olarak tanımlayabilme
∗ Özyinelemeli örüntü (recursive pattern) sözel olarak tanımlayabilme
∗ Birlikte değişimsel ilişki (covariational relationship) sözel olarak tanımlayabilme
∗ Fonksiyon kuralını sözel ve sembolik olarak tanımlayabilme($y=2x$ *ve* $y=3x$ ilişkisini kurabilme, sözel ve sembolik olarak tanımlayabilme)
∗ Değişkenler arasındaki fonksiyonel ilişkiyi grafik üzerinde gösterebilme

Öğrencilerden beklenen ön bilgiler:

∗ Öğrenciler elde ettiği verileri tabloya yerleştirebilir
∗ Tablodaki örüntüyü ifade edebilir.
∗ Fonksiyon kuralını sözel ve sembolik olarak tanımlayabilme($y=x$ ilişkisini kurabilme, sözel ve sembolik olarak tanımlayabilme)

Öğretim Tekniği: Keşfetme, Tartışma

Süre: 80 dk.

Başlangıç (5 dk.):

- Önceki dersle ilgili tekrar yapılmalıdır.
- Önceki derse ait değerlendirme sorusu ile ilgili kavram yanılgılarına yönelik geri dönüşler yapılabilir**.** $y=x$ fonksiyonuna yönelik grafik ya da tablo verilip hangi fonksiyonel ilişkiyi gösterdiği sorulabilir. Öğrencilerin $y=x$ fonksiyonunu anlamlandırması ve kavraması bu ders için ön koşul becerisi olması açısından önemlidir.

Gelişme (25 dk):

- Etkinlik kağıtları dağıtılır ve öğrencilerin problemi anlamaları için süre verilir
- Problemi açıklamaları istenir. Verilen, istenen, bilinmeyen … vb.
- Öğrencilerin problemi nasıl yorumladıklarını ve izledikleri yollar gözlemlenir.

* *Bu aşamada 1 köpek kaç göze sahiptir? 2 köpeğin kaç tane gözü vardır? 3 köpeğin kaç tane gözü vardır şeklindeki sorular öğrencilerin sayısal olarak birden çok değerin olduğunu ve bu sayıların nasıl bir örüntü şeklinde ilerlediği konusunda yardımcı olur.*

- Öğrencilerden gelen tüm sayı ve olasılıklar tahtaya yazılır.
- Öğrencilerin bu değerleri tabloya yazmaları istenir.
- Tablodaki değerler arasındaki örüntü üzerinde konuşulmalıdır.

  * Bu kısımda öğrencilerin göz sayısını gösteren sütundaki sayılar arasında +2 (özyinelemeli örüntü) şeklinde ilerleyen bir örüntü tanımlaması beklenmektedir.
  * Köpek sayısı ile göz sayısı arasında ilişki olup olmadığı sorusu yöneltilebilir. Yukarıdan aşağıya olan bir ilişki yerine sağdan sola olan (köpek sayısı ile göz sayısı) ilişkiye odaklanmaları sağlanabilir.

• Öğrencilerin "göz sayısı köpek sayısının 2 katıdır" ilişkisine ulaşması amaçlanır. Bu ilişkiye yönlendirmek amacıyla "Köpek sayısı 1 artarken, göz sayısı …. Artar" ifadesi yardımcı olabilir.

• Sembol kullanarak ifade etmeleri için n tane köpeğin göz sayısını ya da herhangi bir sayıdaki köpeğin sahip olduğu göz sayısını nasıl ifade edebiliriz sorusu yöneltilebilir.

• Öğrenciler *2xn* ya da *2xK* ifadelerini kullanabilirler.

• Bu durumda 100 tane köpeğin kaç tane göze sahip olduğu sorusu yöneltilerek kuralı uygulaması beklenir.

• Fonksiyonel ilişkiyi göstermek için önceki derste öğrenilen y=x fonksiyonundan yola çıkılarak y=2x sonucuna ulaşmaları gereklidir. Bunu sağlamak için bir önceki adımda yazdıkları *2xn* ya da *2xK* ifadelerinin neye eşit olduğu sorulabilir.

• Değişken kavramına geçiş kısmında önce ■ ve ▲ sembolleri kullanılıp daha sonra harf kullanımına gidilebilir.

• Son olarak öğrencilerin *y=2x* ilişkisini koordinat grafik üzerinde göstermeleri istenir. Bu kısımda tabloya dönülerek köpek sayısına karşılık gelen göz sayısı incelenerek tabloya yerleştirilebilir.

* Öğrencilerden beklenen *y=2x* ilişkisini kullanarak değerleri belirlemeleridir.

Bitiş (10 dk.):

- Ders içindeki etkinlik ve ulaşılan sonuçlar tekrar edilir.
- *y=2x* fonksiyonunun anlamı ve gösterimi üzerinde tekrar durulur.
- Çıkış kartları dağıtılır ve çözmeleri için süre verilir.

ETKİNLİK KAĞIDI 3

1) Köpek barınağında çalışan bir görevli olduğunuzu düşünün ve köpeklerin sahip olduğu göz sayısını bulmak istiyorsunuz.

a) Bir köpek kaç tane göze sahiptir? 2 Köpeğin kaç gözü vardır? 3 köpeğin kaç gözü vardır?

b) Elde ettiğiniz verileri tabloya yerleştiriniz.

Köpek sayısı	Göz sayısı

c) Tablodaki veriler arasında bir örüntü var mıdır? Var ise bu örüntüyü tanımlayınız.

d) Köpek sayısı ile göz sayısı arasında bir ilişki var mıdır? Var ise bu ilişkiyi açıklayınız.

e) Bu barınakta 100 köpek varsa, göz sayısı kaçtır?

f) Barınakta bulunan herhangi bir sayıdaki köpeğin sahip olduğu göz sayısını nasıl ifade edersiniz? (Bu barınakta *n* tane köpek varsa, göz sayısını nasıl ifade edersiniz.?)

g) Köpek sayısı ve toplam göz sayısı arasındaki ilişkiyi grafik üzerinde gösteriniz.

ÇIKIŞ KARTI:

Barınakta çalışan bir görevli olduğunuzu düşünün. Barınaktaki köpeklerin sahip olduğu toplam göz ve kuyruk sayısını bulmak istiyorsunuz. Buna göre barınakta bir köpek varsa toplam göz ve kuyruk sayısı kaçtır? 2 köpeğin sahip olduğu toplam göz ve kuyruk sayısı? 3 köpek?

a) ***Elde ettiğiniz verileri tablo kullanarak düzenleyiniz. Tablodaki örüntüyü nasıl ifade edebilirsiniz?***

b) ***Köpek sayısı ile toplam göz ve kuyruk sayısı arasındaki ilişkiyi nasıl ifade edersiniz?***

c) ***n tane köpeğin sahip olduğu toplam göz ve kuyruk sayısını nasıl ifade edersiniz?***

d) 100 köpeğin sahip olduğu kuyruk ve göz sayısı kaçtır?

DERS PLANI 4:

Bu derste hedeflenen kazanımlar;

* Elde ettiği verileri tablo kullanarak düzenleyebilme
* Değişkenleri ve değişken rollerini sözel olarak tanımlayabilme
* Özyinelemeli örüntü (recursive pattern) sözel olarak tanımlayabilme
* Birlikte değişimsel ilişki (covariational relationship) sözel olarak tanımlayabilme
* Fonksiyon kuralını sözel ve sembolik olarak tanımlayabilme($y=x+1$) ilişkisini kurabilme, sözel ve sembolik olarak tanımlayabilme)
* Bağımlı değişkene ait bir değer verildiğinde bağımsız değişkene ait değeri hesaplayabilir.

Öğrencilerden beklenen ön bilgiler:

* Öğrenciler elde ettiği verileri tabloya yerleştirebilir
* Tablodaki örüntüyü ifade edebilir.
* Değişken içeren ifadeler yazabilir.
* Fonksiyon kuralını sözel ve sembolik olarak tanımlayabilme($y=x$, $y=2x$ ilişkisini kurabilme, sözel ve sembolik olarak tanımlayabilme)

Öğretim Tekniği: Grup çalışması, Keşfetme, Tartışma

Materyal: Etkinlik kağıtları, değerlendirme kartları, kurdele, makas

Süre: 80 dk.

Başlangıç(5 dk.):

- Önceki ders ile ilgili tekrar yapılır. $y=ax$ fonksiyonunun anlamı, değişkenler arasındaki ilişki tekrar edilebilir.
- Çıkış kartındaki soru ile ilgili farklı çözüm ya da kavram yanılgıları var ise bunlar tekrar edilir.

Gelişme (25-30 dk):

- Öğrencilerden 4 kişilik gruplar oluşturmaları istenir.
- Her gruba kurdele ve 1'er tane de makas dağıtılır.
- Etkinlik kağıtları dağıtılır.
- Öğrencilerin ellerindeki kurdeleyi kesmeden önce ellerinde kaç parça kurdele olduğu sorulur. Daha sonra kurdeleyi 1 kez kestiklerinde elde ettikleri parça sayısı sorulur. Bulguları verilen tabloya kaydetmeleri istenir.
- Bu işlemler öğrencilerin 5 kesim yapmasına kadar devam eder.
- Oluşturulan tablodaki örüntüyü açıklamaları beklenir. Bu kısımda öğrenciler "*Parça sayısı +1 olarak ilerliyor*." şeklinde bir örüntü tanımlayabilirler.
- Değişkenlerin neler olduğuna dikkat çekilir. Değişkenler (kesim sayısı ve parça sayısı) arasında ilişki olup olmadığı sorularak tablo üzerindeki yatay ilişkiye odaklanmaları sağlanmalıdır.
- Öğrencilerden beklenen ilişki tanımı "*Parça sayısı kesim sayısının bir fazlasıdır*." şeklindedir.
- Öğrencilerin sembolik ifadeye geçmeleri için "*Herhangi bir sayıda yapılan kesim sonucunda elde edilen parça sayısını nasıl ifade edebilirsiniz?*" ya da "*n tane kesim sonucunda elde edilen parça sayısını nasıl ifade edebilirsiniz?*" şeklindeki sorular yöneltilebilir.
- Parça sayısı (P), Kesim sayısı (K) olarak tanımlanabilir. Ulaşılacak sonuç "*P=K+1*" ya da "*y=x+1*"

Bitiş(5 dk.) :

- Ders içindeki etkinlik, ders içinde öğrenilmesi amaçlanan kazanımlar tekrar edilir.
- Öğrencilerin ders ile ilgili geri bildirimlerini almak için 3-2-1 değerlendirme kartları dağıtılır.

Not: Bu dersin değerlendirme kısmı bir sonraki dersin sonunda iki fonksiyonu çeşidini içeren bir çıkış kartı ile birlikte yapılacaktır.

ETKİNLİK KAĞIDI 4

1) Elinizdeki kurdeleleri istenilen sayıda kesiniz ve elde ettiğiniz parça sayısını tabloya yazınız.

Kesim sayısı	**Parça sayısı**
0	
1	
2	
3	
4	
5	

2) Tablodaki örüntüyü tanımlayınız.

3) Kesim sayısı ile parça sayısı arasındaki ilişkiyi açıklayınız?

4) Kesim sayısı ile parça sayısı arasındaki ilişkiyi değişken kullanarak matematiksel olarak nasıl ifade edebilirsiniz?

5) Bir kurdeleyi "n" defa kestiğimizde elde ettiğimiz parça sayısını nasıl ifade edebilirsiniz?

6) Bir kurdele belirli bir sayıda kesiliyor ve 100 tane parça elde ediliyor ise bu durumda kaç kesim yapılmıştır?

DERS PLANI 5:

Bu dersin kazanımları;

* Elde ettiği verileri tablo kullanarak düzenleyebilme
* Değişkenleri ve değişken rollerini sözel olarak tanımlayabilme
* Özyinelemeli örüntü (recursive pattern) sözel olarak tanımlayabilme
* Birlikte değişimsel ilişki (covariational relationship) sözel olarak tanımlayabilme
* Fonksiyon kuralını sözel ve sembolik olarak tanımlayabilme($y=2x+1$) ilişkisini kurabilme, sözel ve sembolik olarak tanımlayabilme)
* Bağımlı değişkene ait bir değer verildiğinde bağımsız değişkene ait değeri hesaplayabilir.

Öğrencilerden beklenen ön bilgiler:

* Öğrenciler elde ettiği verileri tabloya yerleştirebilir
* Tablodaki örüntüyü ifade edebilir.
* Değişken içeren ifadeler yazabilir.
* Fonksiyon kuralını sözel ve sembolik olarak tanımlayabilme($y=x$, $y=2x$, $y=x+1$ ilişkisini kurabilme, sözel ve sembolik olarak tanımlayabilme)

Öğretim Tekniği: Grup çalışması, Keşfetme, Tartışma

Materyal: Etkinlik kağıtları, değerlendirme kartları, kurdele, makas

Süre: 80 dk.

Başlangıç (5dk.):

- Önceki derste kurdele kesimi süreci, bu sürecin sonunda elde edilen sonuçlar ve ders sonundaki değerlendirme kartlarından yola çıkılarak tekrar yapılır.
- Kavram yanılgısı olan noktalar varsa kısa sürede geri dönüşler yapılarak yeni ders başlangıcı yapılır.

Gelişme (65 dk.):

- Önceki derste yapılan kurdele kesme etkinliği hatırlatılarak bu sefer düğümlü kurdeleler kullanarak kesme işlemi yaparsak nasıl bir ilişki ortaya çıkar şeklinde bir giriş yapılabilir.
- Öğrenciler grup çalışması yapmak için 4 kişilik gruplara ayrılır.
- Her gruba 4 farklı renkte (mavi, kırmızı, pembe, sarı) kurdeleler ve birer makas verilir.
- Çalışmaları sırasında kullanacakları etkinlik kağıtları dağıtılır.
- Her kurdelede 1 düğümün olmasına dikkat çekilir.
- Hiç kesim yapmadan kaç parça olduğu sorulur.
- İlk kesim tüm sınıfla birlikte yapılır.
- Her gruptan kırmızı kurdeleyi almaları ve düğüm yerinden ikiye katlamaları istenir. Daha sonra düğüm dışındaki bir noktadan 1 kere kesim yapılması istenir ve kaç parça kurdele elde edildiği üzerine konuşulur. Her grubun 3 parça elde ettiğinden emin olunmalıdır.
- Bulunan değerler tabloya yazılır.
- Mavi kurdele için aynı şekilde katlanarak 2 kesim, pembe kurdele için 3 kesim ve sarı kurdele için 4 kesim yapmaları gerektiği anlatılır.
- Öğrencilerin buldukları sonuçları tabloya yazmaları beklenmektedir.
- Grup çalışması sırasında gruplar gözlemlenir.
- Daha sonra tablodaki veriler arasında bir örüntü olup olmadığı var ise tanımlamaları istenir.
- Örüntüyü sadece parça sayısına odaklanarak yani tabloda yukarıdan aşağıya ilerleyen yinelemeli bir örüntü olarak tanımlayabilirler.
- Kesim sayısı ve parça sayısı arasında ilişki olup olmadığı sorulur.
- Kesim sayısı ve parça sayısı arasındaki ilişkiye dikkat çekmek için *"Kesim sayısı .... artarken elde edilen parça sayısı .... şeklinde artar."* İfadesi kullanılabilir.
- Herhangi sayıda yapılan kesimden elde dilen parça sayısını nasıl ifade edebilecekleri ya da n tane kesim sonucunda kaç parça elde edilir şeklinde sorular

yöneltilerek aradaki ilişkiyi sembolik olarak ifade etmeleri konusunda yardımcı olunabilir.

- Parça sayısı (P), Kesim sayısı (K) ile gösterilebilir. İlişkiyi açıklayan sembolik gösterim "*P=2xK+1*" ya da "*y=2x+1*" olarak tanımlanabilir.
- Kuralı bulduktan sonra 20 kesim sonucunda kaç tane parça elde edileceği sorusunun cevabını kuralı kullanarak vermeleri beklenir.

Bitiş (10 dk.):

- Ders içindeki etkinlik süreci ve ulaşılan fonksiyonda "*y=2x+1*" değişkenlerin anlamı, değişkenler arasındaki ilişki tekrar edilerek çıkış kartları dağıtılır.
- Çıkış kartların çözümü için zaman verilir.

ETKİNLİK KAĞIDI 5

a) 1 Kesim yapıldığında kaç parça elde edilir?
- 2 Kesim yapıldığında kaç parça elde edilir?
- 3 Kesim yapıldığında kaç parça elde edilir?
- 4 Kesim yapıldığında kaç parça elde edilir?

b) Elde ettiğiniz verileri tablo oluşturarak düzenleyiniz.

c) Tablodaki veriler arasında bir örüntü var mı? Var ise bu örüntüyü tanımlayınız.

d) Kesim sayısı ve parça sayısı arasında ilişki var mıdır? Var ise bu ilişkiyi nasıl tanımlarsınız?

e) Bu ilişkiyi değişken kullanarak nasıl ifade edebilirsiniz?

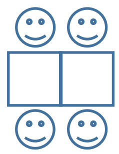
f) 20 defa kesim yapıldığında kaç parça elde edilir?

ÇIKIŞ KARTI

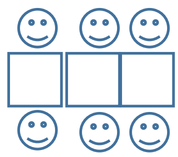
Nehir arkadaşlarını davet ettiği bir doğum günü partisi planlamaktadır. Partiden önce herkes için yeterli sayıda oturma yeri olup olmadığından emin olmak istiyor. Nehir kare şeklindeki masalara sahiptir.

Bir masada şekildeki gibi 2 kişi oturmaktadır.

Nehir bir masa daha eklediğinde; 2 masada şekildeki gibi 4 kişi oturmaktadır.



Nehir ikinci masaya bir masa daha eklerse 3 masada şekildeki gibi 6 kişi oturmaktadır.



a) Aşağıdaki tabloyu doldurarak farklı sayıdaki masalara oturabilecek kişi sayısını gösteriniz.

Masa Sayısı	Kişi sayısı
1	2
2	4
3	
4	
5	
6	
7	

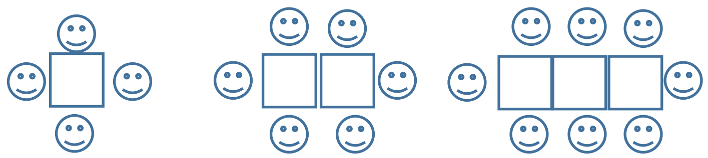
b) Oluşturduğunuz tabloda bir örüntü var mı? Var ise bu örüntüyü tanımlayınız.

c) Masa sayısı ile kişi sayısı arasında bir ilişki var mıdır? Var ise bu ilişkiyi sözcüklerle nasıl tanımlarsınız?

d) Bu ilişkiyi <u>değişken kullanarak</u> nasıl ifade edebilirsiniz? Bu değişkenler neyi ifade ediyor?

e) Bu parti için 100 masa birleşik olarak (yukarıdaki gibi) dizilirse kaç kişi partiye katılabilir?

Nehir masanın uçlarına iki kişinin daha oturması durumunda daha fazla kişiyi davet edebileceğini fark etmiştir. Örneğin, eğer Nehir şekildeki gibi 2 masayı birleştirirse 6 kişi oturabiliyor; 3 masayı birleştirirse 8 kişi oturabiliyor.



f) Yeni durum c ve d şıklarında yazdığın kuralı nasıl etkiler?

g) Yeni durumda masa sayısı ve kişi sayısı arasındaki ilişkiyi sözcüklerle nasıl tanımlarsınız?

h) Yeni durumda masa sayısı ve kişi sayısı arasındaki ilişkiyi <u>değişken kullanarak</u> nasıl ifade edebilirsiniz? Bu değişkenler neyi ifade ediyor?

i) Yeni durumda bu parti için 100 masa birleşik olarak (yukarıdaki gibi) dizilirse kaç kişi partiye katılabilir?

j) Yeni durumda bu partiye 100 kişinin katılabilmesi için kaç tane masa gereklidir?

C. APPROVAL OF THE UNIVERSITY HUMAN SUBJECTS ETHICS COMMITTEE

UYGULAMALI ETİK ARAŞTIRMA MERKEZİ
APPLIED ETHICS RESEARCH CENTER

ORTA DOĞU TEKNİK ÜNİVERSİTESİ
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19 ARALIK 2018

Konu: Değerlendirme Sonucu

Gönderen: ODTÜ İnsan Araştırmaları Etik Kurulu (İAEK)

İlgi: İnsan Araştırmaları Etik Kurulu Başvurusu

Sayın Dr.Öğretim Üyesi Işıl İŞLER BAYKAL

Danışmanlığını yaptığınız Gülnur AKIN'ın **"5. Sınıf Öğrencilerinin Fonksiyonel Düşünme Becerilerinin İncelenmesi"** başlıklı araştırması İnsan Araştırmaları Etik Kurulu tarafından uygun görülerek gerekli onay **2018-EGT-200** protokol numarası ile araştırma yapması onaylanmıştır.

Saygılarımla bilgilerinize sunarım.

Prof. Dr. Tülin GENÇÖZ
Başkan

Prof. Dr. Ayhan SOL
Üye

Prof. Dr. Ayhan Gürbüz DEMİR (4.)
Üye

Prof.Dr. Yaşar KONDAKÇI İZİNLİ
Üye

Doç. Dr. Emre SELÇUK
Üye

Doç. Dr. Pınar KAYGAN
Üye

Dr. Öğr. Üyesi Ali Emre TURGUT
Üye

D. APPROVAL OF THE MINISTRY OF NATIONAL EDUCATION

T.C.
MİLLÎ EĞİTİM BAKANLIĞI
Temel Eğitim Genel Müdürlüğü

Sayı : 70297673-605.01-E.668444 10.01.2019
Konu : Tez Önerisi

ORTA DOĞU TEKNİK ÜNİVERSİTESİ
(Öğrenci İşleri Daire Başkanlığına)

İlgi: a) Orta Doğu Teknik Üniversitesinin, Genel Müdürlüğümüzde 02/01/2019 tarihinde ve 88486 sayıda işlem gören yazısı.
b) Millî Eğitim Bakanlığının 22/08/2017 tarihli ve 2017/25 sayılı Genelgesi

Üniversiteniz Matematik ve Fen Bilimleri Anabilim Dalı yüksek lisans programı öğrencisi Gülnur AKIN'ın Dr. Öğretim Üyesi Işık İşler BAYKAL'ın danışmanlığında yürütmekte olduğu "5. Sınıf Öğrencilerinin Fonksiyonel Düşünme Becerilerinin İncelenmesi" konulu proje başvurusu hakkındaki ilgi (a) yazı ve ekleri Genel Müdürlüğümüzde oluşturulan komisyon tarafından incelenmiştir.

Söz konusu araştırmanın eğitim ve öğretimi aksatmayacak şekilde **gönüllülük esasına** dayalı olarak uygulanması, uygulamalarda sadece yazımız ekinde gönderilen **mühürlü anketin** kullanılması ve elde edilen kişisel verilerin **(doğum yeri vb)** gizliliği hususuna dikkat edilmesi, araştırma sonucunda elde edilen raporun basılı ve dijital ortamda Genel Müdürlüğümüze teslim edilmesi gerekmektedir.

Bu çerçevede araştırmanın **video kaydı yapılmaması** şartıyla Genel Müdürlüğümüze bağlı ortaokullarda yürütülmesi uygun bulunmuştur.

Bilgilerinizi ve gereğini rica ederim.

Dr. Cem GENÇOĞLU
Bakan a.
Genel Müdür

Ekler:
1-İlgi (a) yazı ve ekleri
2-Mühürlü Anket (4 sayfa)

DAĞITIM:
Gereği:
Orta Doğu Teknik Üniversitesi
(Mühürlü anket 4 sayfa)

Bilgi:
Ankara, Adana
Valiliklerine (İl Millî Eğitim Müdürlükleri)
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