THE EFFECT OF PROBLEM SOLVING METHOD WITH HANDOUT MATERIAL ON ACHIEVEMENT IN SOLVING CALCULUS WORD PROBLEMS

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ABSTRACT

THE EFFECT OF PROBLEM SOLVING METHOD WITH HANDOUT MATERIAL ON ACHIEVEMENT IN SOLVING CALCULUS WORD PROBLEMS

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This study was designed to investigate the effect of the Problem Solving Method with Handout Material (PSMHM) in comparison to the Traditional Method (TM) on achievement of the students in solving max-min word problems. Subjects for the study were 116 students enrolled in two classes of Math 153 Calculus I course at the Department of Mathematics and Mathematics Education, Middle East Technical University, Ankara in the academic year 1990-1991.

Three instruments:(i) General Achievement Test (GAT), includes precalculus knowledge, (ii) Pretest (PeT), and (iii) Posttest (PoT) include max-min word problems were developed and administered to the Experimental Group (EG) and Control Group (CG) students.

According to the GAT and PeT scores, no significant differences were found between the experimental and control groups at the beginning of the treatment. Two-way analysis of variance was used for the PoT scores in the present study. Results indicated that PSMHM increased the achievement of the EG students in solving max-min word problems. However, departments did not indicate any significant effect on the achievement of the students and no significant interaction was found between the treatment and the department.

Even though the sample size in this study is relatively small, the results show that a new instructional method can help students understand and learn the max-min word problems. Furthermore, the hypotheses and the questions of the research set up can lighten the other experimental investigations in the same area.

Key Words: Problem Solving, Maximum-Minimum Word Problems, Problem Solving Method with Handout Material, Traditional Method.

ÖZET

ÖĞRETİM MATERYALIN BİRLİKTE KULLANILDIĞI PROBLEM ÇÖZME YÖNTEMİNİN GENEL MATEMATİKTE SÖZEL PROBLEMLERİN ÇÖZÜMÜNDEKİ BAŞARIYA ETKİSİ

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Bu çalışma öğretim materyalin birlikte kullanıldığı "problem çözme yöntemini" geleneksel yöntemle karşılaştırarak öğrencilerin maximum-minimum sözel problemlerini çözmedeki başarıya etkisini araştırmak için yapılmıştır. Bu araştırma kapsamına 1990-1991 öğretim yılında Orta Doğu Teknik Üniversitesindeki Matematik Bölümü ve Matematik Öğretmenliği Programında Math 153 Genel Matematik (Calculus I) dersine kayıtlı iki karma sınıftan oluşan toplam 116 öğrenci alındı.

Geliştirilen üç ayrı ölçme aracı: (i) Genel matematik bilgisini içeren Genel Başarı Testi (GAT), Max-Min sözel problemlerini içeren (ii) Ön test (PeT), ve (iii) Son test (PoT), deney ve kontrol gruplarına uygulanmıştır.

Uygulamanın başında, GAT ve PeT puanlarına göre, deney ve kontrol grupları arasında anlamlı bir fark bulunmamıştır. Bu çalışmada PoT puanları için iki yönlü varyans analizi kullanılmıştır. Sonuçlar gösterdi ki PSMHM deney grubundaki öğrencilerin max-min sözel problemlerini çözmedeki başarısını arttırmıştır. Fakat bölümlerin başarıya önemli bir etkisi olmamıştır ve bölümlerle ve öğretim arasında da bir etkileşim bulunmamıştır.

Örneklemi nispeten küçük olan bu araştırmada elde edilen sonuçlar gösterdi ki yeni bir öğretim yöntemi Genel Matematikte sözel problemlerin daha iyi anlaşılmasında ve öğrenilmesinde etkili olabilmektedir. Ayrıca, bu çalışmada ortaya konulan hipotezler ve problemler ileride yapılacak bu konuyla ilgili bazı araştırmalara yol gösterecek ip uçları vermektedir.

Anahtar Sözcükler: Problem çözme, Maximum-Minimum sözel problemleri, Öğretim materyalin birlikte kullanıldığı problem çözme metodu, Geleneksel method.

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ABBREVIATIONS

CG	:	Control Group
EG	:	Experimental Group
HM	:	Handout Material
TM	:	Traditional Method
PSMHM	:	Problem Solving Method with Handout Material
GPWP	:	General Procedure for Solving Max-Min Word Problems
CoG	:	Comprehension Guide
Math	:	Mathematics
Math Ed	:	Mathematics Education
SPSS	:	Statistical Package for Social Sciences

CHAPTER I

INTRODUCTION

Mathematics education is one of the most important but the least understood subjects of an age. It is indeed far being merely a subset of mathematics, but a complex and dynamic endeavor because it must draw upon the knowledge and results of other disciplines continuously. Moreover, the role of mathematics in society is changing continuously.

The changing role of mathematics in society may require a different mathematics curriculum at all levels of schools, and universities. As the demands of society change, so do the essential competencies needed by individuals to live productively in that society. All students therefore will need competence in essential areas of mathematics, i.e problem solving, communicating mathematical ideas, mathematical reasoning, applying mathematics to everyday situations, etc e.g (NCSM,1989,P.388). An Agenda for Action stated that "problem solving must be the focus of school mathematics in the 1980's". It further suggested that " the mathematics curriculum should be organized around problem solving", that " mathematics teachers should create classroom environments in which problem solving can flourish", and that "appropriate curricular materials to teach problem solving should be developed for all grade levels" (NCTM, 1980) (cf, Krulik, & Rudnick, 1985). Keeping in view the ideas of Agenda for Action above, problem solving was taken into consideration and focused on while preparing instructional material in this study.

Mumme and Weissglass (1989, p.523) stated, on the other hand, that most of us have been successful in learning mathematics in traditional ways. We are also successful with many of our students. We feel good about being successful with the students who, like us, are able to perform using traditional approaches. The standards suggest, however, that success in the traditional approach does not assure understanding and that the traditional content does not adequately prepare all students for modern society. As a result, we are being asked to change both the content and the method of instruction. It is natural to have feelings about this change. Working through these feelings is essential to success.

The traditional approaches to mathematics education, and the techniques used to achieve the somewhat limited aims of the times when they are formulated, served their purpose very well indeed, and continued to do so for many years. However, in recent years, two factors have intervened to prove that the traditional approach, and the traditional mathematics course, cannot continue to satisfy the requirements of the present world or of the future. The two factors are as follows (Dienes ,& Golding, 1971, p.11):

* We must adapt ourselves toward the world of the tomorrows, i.e it is the responsibility of the teacher and of the educationalist to try to forecast the kind of world in which the child of today is to make his contribution as an adult, and to tailor his education accordingly.

* The world has been changing, modern educational psychologists and experimentalists have been discovering for more than they ever knew before about the way a child's thinking develops. These discoveries have made it obvious that there is an even greater need to change the teaching approaches than to change the course content. In this study, the content was not changed, but the approach and the method to teach max-min problems in calculus were varied.

Calculus has been a focus in our expanding knowledge of the universe, and a central part of college curriculum for more than a century. It is the key to understanding systems that change in all mathematical sciences, i.e natural and engineering sciences, economics and social sciences also. More specifically, students need calculus to understand the nature of the scientific laws and their applications in the modern world. In the last two decades, rapid large scale computing has increased, not lessened, the role of calculus in many of the outstanding problems of science and technology. It is worthwhile to emphasize that the issue should not be whether to replace calculus in the core curriculum, but rather how to transfuse and invigorate the pallid, passive calculus which is presently taught in so many colleges and universities (Ronald, 1986, p.250). Therefore we focus on the teaching methods or approaches to teach max-min problems in calculus.

Calculus is an excellent laboratory in which all students can begin to learn this important role of mathematics. The case for calculus remaining in the core of undergraduate mathematics is overwhelming for a wide variety of students. The structure of today's calculus course has changed little in the last several decades. Although various reforms have been proposed from time to time, a quick review of most popular calculus texts will show that little change has occurred. Many topics are included because an individual department wants it, and most attempts at changing the standard calculus courses have failed. Certainly, the time is overdue for a serious rethinking of the calculus we teach (Ronald, 1986, p.251). In this study calculus was chosen because of the followings:

* It is a base of all mathematics courses.
* It is taken by all science and engineering students.
* It is very important for teachers.

Orton (1986, p.661) stated that the details of methods of teaching which follow are based on the assumptions that we need to take a greater account of the needs of pupils, that this implies an intuitive approach and not a rigorous one.

Many instructors like Ramseyer feel that a change in teaching method together with a changed curriculum can help students to realize their potential more fully (cf, Cankoy, 1989, p.2).

The changes envisioned will require teachers both to develop a deeper understanding of mathematics on to construct new understanding of teaching and learning of mathematics. Working in a group and discussing the activity will then enrich everyone's understanding. Reflecting on a mathematical activity, teachers can therefore discuss such questions as the following (Mumme, & Weissglass, 1989, p.523):

* In what ways did the activity develop and extent students' mathematical understanding?
* How were students encouraged to think and reason?
* What view of mathematics is communicated by the activity?
* What are the implications for the students and the school of all teachers' using such activities ?

In today's mathematical education, there is a concern as to what techniques enables students to grasp concepts. This study was conducted to get the answers of the following questions:

* Does "new" mathematical instruction benefit the student in comparison to the traditional method?
* Does "new" mathematics impair the student's knowledge of the topic?

The purpose of the present study is to investigate the effect Problem Solving Method with Handout Material in terms of students' achievement in solving max-min word problems. The research was carried out at the Middle East Technical University (METU) in the academic year 1990-1991, on 116 freshmen students, at the Department of Mathematics and Mathematics Education enrolled in Math 153 Calculus I course. Mathematics Education is the option in Science Education department but Mathematics Education option is referred as Mathematics Education Department in the present study.

Word problems were chosen because of the followings:

* to assist students in seeing uses and applications of max-min principles in the real life.
* to improve reading comprehension.
* to make students enjoy solving word problems.

Taking into account the reasons above mentioned, i.e calculus and word problems, Handout Material (HM) was developed by using problem solving method in this study. More clearly, in preparing the HM, the general procedure for solving max-min word problems, the comprehension guide for some problems, and the problems with unfinished solution steps were developed and used during the treatment. The aim of developing the general procedure was to improve general understanding of word problem solving. The comprehension guide was utilized to build up reading comprehension of word problems. Stiff (1986, p.163) stated that comprehension guides help students understand written prose at three levels. The reader who correctly answer the questions what does the author say? what does the author mean by what is said? and which ideas expressed by the author? reads at the literal, interpretive, or applied levels of comprehension, respectively. Two of these levels of comprehension are important to an understanding of word problems: the literal and the applied, or operational levels. Comprehension guides give teachers the means to attack this reading problem as students improve their ability to solve word problems.

In this present study, the proficiency in solving word problems was tried to be gained to the students.

Conducting a research on the issue described above can enlighten mathematics educators to a certain degree on "what to do" and " how to do" in the teaching of word problems. To see the effect of Problem Solving Method with Handout Material and department on the achievement of Turkish freshmen students in solving max-min word problems was the main purpose of this study.

CHAPTER II

REVIEW OF LITERATURE

This chapter contains the review of literature relative to the mathematics and mathematics education, problem solving, and word problems.

2.1. Mathematics and Mathematics Education

2.1.1 General View on Mathematics

Mathematics education is far from being merely a subset of mathematics, it is a practical activity and not merely a theoretical study. Emenalo, and Okpora (1990, p.51) stated that mathematics and mathematics education are, however, indispensable in the effort of any nation toward achieving great heights in science and technology. Such fields as medicine, engineering, agriculture, business studies, computer science and ever the everyday life of layman, to mention but a few, require the use of mathematics for success. In fact, for each of these areas, the needed manpower resources can never be obtained in absence of mathematics. The increasing sophistication of society and associated demand for technological breakthrough make mathematics ever more indispensable.

The students we educate today can expect to change jobs many times during their life times. The jobs they hold will develop and change around them. Often, specific job skills will not transfer from one position to another. To prepare for mobility, students must develop a through understanding of mathematical concepts and principals; they must reason clearly and communicate effectively; they must recognize mathematical applications in the world around them; and they must approach mathematical problems with confidence. Individuals will need the fundamental skills that will enable them to apply their knowledge to new situations and to take control of their own lifelong learning (National Council of Supervisor of Mathematics (NCSM), 1989, p.389).

Good and Grouws (1987) report, such findings suggest that more classroom time is being spent on the routine computational skills than on understanding mathematical concepts (cf, Fiske, 1990, p.249). For this reason, new instructional strategies can be developed and applied to improve the understanding of max-min word problems.

NCSM (1989, p.391) stated that extensive professional development opportunities as well as new learning materials will be needed to implement the new instructional strategies. Similarly, Dienes, and Golding (1971, p.67) stated that the use of structured materials has been considered by a number of authorities, but perhaps Piaget, and then Dienes, have been most explicit in their conclusions.

The teaching of mathematics has three major areas of emphasis: (i) facts and skills, (ii) understanding, and (iii) problem solving. A further assumption of the model is that to do justice to the subject of mathematics, teaching approaches must be of a holistic nature, incorporating and integrating all three dimensions explained below:

Facts and Skills: It is on enabling students to complete standard textbook exercises and experience success on tests and examinations. A positive aspect of this emphasis is that students have clear objectives. They know what is expected of them, and teachers can easily evaluate their level of performance.

Understanding: It is on making mathematics meaningful. Learning is characterized by experiential approaches, real problem situations, and an emphasis on solution process.

Problem Solving: Whereas the first two dimensions deal with getting students to become familiar with and understand mathematical skills and concepts, problem solving gives them the opportunity to experience mathematical discovery and creativity. The emphasis is on making students better problem solvers by helping them realize the importance of (i) problem understanding and analysis, (ii) careful planning and organization, (iii) flexibility and open-mindedness, and (iv) continuous checking and evaluation of the solution process (Gadanidis, 1988, p.16).

2.1.2 Review of Literature on Calculus

Calculus has stood the test of time and has been taught to science and engineering students since formal study in these areas began. It is the foundation and wellspring for modern mathematics. There are just a few recent developments in calculus-based branches of mathematics. It is clear that mathematicians, both pure and applied, need to learn calculus early in their careers (Ronald, 1986, p.250).

One of the peculiar characteristics of a first year university calculus course is the extent of its reliance of specific mathematics skills that students are presumed to have mastered in high school. Calculus courses, and for that matter first-year courses in some of the physical sciences, are unforgiving: they depend heavily and directly upon the student's actual skills from high school mathematics courses. This circumference has naturally led to heightened concern about mathematics in the student transition from high school to college (Burton, 1989, p.350).

Prospective teachers, especially, should appreciate and master topics from high school mathematics that are used in calculus, such as arithmetic, fraction, algebra, geometry, analytic geometry, elementary functions theory, trigonometry, exponents and logarithms, curve sketching.

The report of the Committee on the Undergraduate Program in Mathematics (CUPM) (1987, p.782) stated that the majority of students who have taken calculus in high school and have not clearly earned advanced placement do not in either the standard Calculus I and Calculus II course. The students do not have the level of mastery of Calculus I topics to be successful if placed in Calculus II and are often doomed by attitude problems if placed in Calculus. An additional factor to consider is the negative effect that a group of students who are repeating most of the content of Calculus I has on the rest of the class as well as on the level of the instructor's presentations. What is needed are courses designed especially for students who have taken calculus in high school and have not earned advanced placement. These courses need to be designed so that they:

* acknowledge and build on the high school experiences of the students,
* provide necessary review opportunities to ensure an acceptable level of understanding of Calculus I topics,
* are clearly different from high school calculus courses, i.e students do not feel that they are essentially just repeating their high school course,
* result in an equivalent of a one semester advanced placement.

In addition to that a research into students' understanding of calculus suggest that there are many learning difficulties which current teaching methods are not helping students to overcome (Orton, 1986, p.659) .

Buchalter, and Stephens (1989, p.225) conducted a study to examine the effect of which various factor have upon students aptitude or readiness for calculus at the college level. One of the factors is the overall high school academic performance in mathematics. The result indicated the overall performance in high school mathematics significantly affected calculus aptitude.

2.2. Problem Solving

In all of the researche carried in the last few decades, it is seen that there is a general tendency to the problem solving in teaching mathematics, and both mathematics educators and cognitive psychologists have become increasingly concerned over the past decade with the problem solving processes used in attacking mathematical word problems. .

The authors of the standards, for example, have assumed that mathematics teachers are (a) concerned with problem solving, (b) willing to restructure their form of instruction, and (c) desire to use alternative methods of assesement (cf, Fiske, 1990, p.250). Mathematical knowledge or skills have little value if they cannot be applied to new or unfamiliar situations. For this reason, developing students' ability to apply mathematical knowledge and skills to problem-solving situations is widely recognized as one of the major goals of mathematics instruction (Carpenter, Corbitt, Kepner, Lindquist, & Reys, 1980, p.427) .

Owen and Sweller(1989) imply that attention allocated to general problem-solving strategies would be more appropriately diverted to instruction concerned with domain-specific knowledge and practice with worked examples and goal-modified problems. Because curricula in several countries are in the process of being modified to incorporate explicit consideration of the nature of general problem-solving strategies (cf,Lawson, 1990, p.403).

Good, Grouws, and Ebmeier (1983) define development in mathematics instruction as " the process whereby a teacher facilitates the meaningful acquisition of an idea by a learner". They further view development as consisting of five components: (a) attending to prerequisities, (b) attending to relationships, (c) attending to representation, (d) attending to perception, and (e) generality of concepts. As Grouws (1985) states, " I predict that research will show that the quality of development is highly correlated with students' acquisition of problem solving skills" (p.303) (cf, Kameenui, & Griffin, 1989, p.588).

An Agenda for Action (NCTM 1980) stated that the mathematics should be organized around problem solving (p.2) (cf,Fiske, 1990, p.249). On the other hand, Hubbard (1990, p.567) stated that there are many reasons why it is difficult for students to learn by attending mathematics lectures. These reasons stated by Hubbard as follows:

* The lecturer has to assume that the students posses a certain level of mathematical knowledge and experience. It is well documented that the knowledge and experience in most classes today is extremely variable.

* The lecturer has to set a pace for the development of

material since mathematics is highly sequential.

* Most lecturers insist that their students take notes (to keep them busy and hence quiet).

* One objective of a lecture should be to focus the students' attention on the main concepts.

* Lecturers work very hard at researching material to produce good lecture notes which they proceed to deliver to their students.

Krulik, and Rudnick (1981, p.37) presented some thoughts on problem solving and some specific suggestions for achieving the important goal of having all of our students become better problem solvers.

1. Problem solving should no be considered as a vehicle for practicing writing equations and /or other concepts , but should be presented as a unique concept, and then integrated throughout the entire mathematics program. Problem solving should be regarded as the basic skill of mathematics, and should be continualy emphasized.

2. The way to make students into better problem solvers is to expose them more and more problem solving. Teacher should build files of non-routine problems, and then spend time at all grade levels involving students in the problem solving process.

3. The problem solving process suggest that a set of heuristics be developed jointly by the teacher and students.

As a result of the review of certain literature on problem solving, it can be said that problem solving should not be limited to special lessons or units. Problem solving should be an integral part of all instructions (Carpenter, Corbitt, Kepner, Lindquist, and Reys, 1980, p.427).

2.3. Word Problems

The area of verbal problem solving has been a major concern of both researchers and educators for many years and has attracted more attention than any other topics in the mathematics curriculum (Suydam, & Weaver, 1975). This focus has been reinforced by the National Council of Teachers of Mathematics (1980). Although children are exposed to mathematical word problems as early as the first grade, verbal problem solving continues to be a trouble some aspect of mathematics instruction for many students and teachers (Ballew, & Cunningham, 1982; Rohrkemper, & Bershon, 1984). In fact, difficulty in mathematical word problem solving has been identified as a national phenomenon (Carpenter, Corbitt, Kepner, Lindquist, & Reys, 1981; Silver, 1985; Suydam, 1984) (cf, Kameenui, & Griffin, 1989, p.575).

Similarly, Sowder (1983, p.107) stated that a common area of both mathematics and science educators is students' ability to solve mathematical word problems.

Carpenter (1985) stated that word problems are difficult for children at all levels of mathematics.

But we should not abandon teaching word problems as teachers. We should try to find interesting and useful problems for students to solve and develop more effective strategies to help people learn how to solve verbal problems (Bell , 1980, p.652) since many students dislike word or story problems and they have not been very successful in solving them, teachers have had a difficult time going beyond teaching students specific approaches to certain types of problems. Students tend to memorize a certain way to work each type of problem and then are ill equipped to deal with new situations (Thaeler, 1986, p.682).

Bell (1980, p.652) stated that before we try to help students learn to solve verbal problems, we should accept a basic premise: problem solving cannot be taught in an algorithmic sense. We should not give students a list of steps (an algorithm) that will lead to a solution for each type of problem if we intent to teach problem solving rather than exercise solving. Since problem solving should not be taught algorithmically. A good way to help people learn how to solve verbal problems is to see that they get plenty of practice at trying to solve problems.

Three critical dimensions were derived from the research on problem solving in mathematics that provide a framework for an analysis of word-problem solving instruction in basal mathematics curriculum programs: (i) the classification of word problems, (ii) the characteristics of word problems, and (ii) the feature of instructional strategies for solving word problems. However, Kilpatrick (1985) identifies five perspectives on teaching mathematical problem solving that have been advocated in recent years: (i) providing the learner with many apportunities to practice solving word problems; (ii) dividing word problems into component parts, then teach each part separately; (iii) requiring students to model the master problem solver; (iv) utilizing group problem solving sessions; and (v) facilitating students' metacognition abilities (cf, Kameenui, & Griffin, 1989, p.583).

Wheeler and McNutt (1983) note that word problems "require students to read several sentences, to decide how to organize the problem, and to solve or compute the problem they have created" (p.309). According to Ashlock, Johnson, Wilson, and Jones (1983), verbal problems " should involve a child in gathering, organizing, and interpreting information so that he can use mathematical symbols to describe real world relationship" (p.239) (cf, Kameenui, & Griffin, 1989, p.586). It is worthwhile to state that reading ability is clearly related to success in solving word problems presented in the written form. Some pupils who are recognized good readers experience difficulty with mathematical story problems. Reading story problems involving mathematics requires different skills and techniques than reading material in other curricular areas because problem solving requires careful, detailed, analytical reading (Collier, Calhoun, Lerch, & Harold, 1969, p327).

Comprehension guide is a key factor in improving reading in mathematics (Henrichs and Sisson 1980). A comprehension guide for a word problem consists of literal statements, the word problem, operational statements which are explained very briefly below:

Literal statements express factual information found in the world problem. Users of a guide should decide what the word problem actually says, then identify the important facts therein. Often, students read poorly at the literal level of comprehension because of a poor vocabulary. Sometimes students read too well, adding information that seems to follow but is not given. The use of comprehension guides gives students the opportunity to address both types of comprehension difficulties.

Operational statements express mathematical computations or procedures needed to solve the problem. Users of a guide should decide which mathematical concepts and operations are required to obtain solution. Word problems are often difficult because students do not try to combine known information found in the statement of the problem. Students must learn to construct solutions on the basis of all available information (cf, Stiff, 1986, p.163).

Stiff added that a teacher should follow three steps when preparing a comprehension guide for students' use.

1. The first step is to identify a word problem which is chosen from the mathematical text-books. They are the most

convenient and obvious source of problems.

2. Next, teachers should construct declarative statements to express the literal and operational content of the word problem. Literal and operational declarative statements are created by asking, respectively, each of two questions: What is the essential information given or asked for? and What computations or procedures must one use to solve this problem?

3. Not all declarative statements should express accurate information. An important step in creating a comprehension guide is to construct both true and false statements about the problem at the literal and operational levels.

Solving problems with inadequate and extraneous data tends to force pupils to examine the data critically before trying to solve the problem. Paying attention to details should be emphasized because solving everyday problems necessiates selecting and organizing data essential to the solution (Collier, & Lerch, 1969, p.338).

Ganadies (1988, p.16) stated that very often the most difficult aspect of solving *nonroutine problems* is getting started.

According to Polya (1962) teaching students to solve word problems is "the most important single task of mathematical instruction". For most students the part that is hardest to learn is the translation between natural language and standard mathematical notation. Polya (1957) provides the following description " In order to translate a sentences from English into French two things are necessary; First, we must understand thoroughly the English sentence. Second, we must be familiar with the form of expression peculiar to French language. The situation is very similar when we attempt to express in mathematical symbols a condition proposed in words. First, we must understand thoroughly the condition. Second, we must be familiar with the forms of mathematical expression. An English sentence is relatively easy to translate into French if it can be translated word by word. But there are English idioms which can not be translated into French word for word. If our sentence contain such idioms, the translation becomes difficult; we have to pay less attention to the separate words, and more attention to the whole meaning; before translating the sentence, we may have to rearrange it" (cf, Lochhead, & Others, 1985, p.5).

Central to most of the mathematics curriculum from algebra, to statistics, calculus and beyond is the concept of a variable. Most of the modern mathematics rests upon symbolization processes that allow letters to represent unknown, varying quantities. Those symbolization processes take on ever greater significance when viewed as a means for encoding word data into mathematical language. Letters in this context stand at the interface between meaning in language and meaning in mathematics so, for example, in algebra, x may be not only be thought of as a number, but also as a number or amount of some quantitative entity to which it refers such as number of books bought or number of dollars spent (Rosnick, 1982, p.3).

The studies by Clement, Lochhead, Monk (1981), Ehrlich, Soloway, Abbott (1983), and Kaput and Sims-Knight (1983) have shown that in the United States a very high percentage of mathematically trained college students lack basic fluency in the use of mathematics as a language for representing quantitative relationships. Most errors made by these students stem from an incorrect model for how mathematical expressions code information (Lochhead, & Others, 1985, p.3).

Muth (1984) studied to determine the relative importance of computational ability and reading ability to the solution of aritmetic word problems by taking 200 sixth graders (109 girls and 91 boys) from two middle schools located in a university community. The measures of task performances were the number of problems correctly answered, the number of problems correctly set up, and the amount of time spent taking the test. The experimental materials included the Comprehension Tests of Basic Skills (1976) and a 15 item arithmetic word problem test was constructed specifically for this study. The results correctly answered 58 % of the 15 problems. Results of the correlational analyses indicated that reading ability and computational ability were positively correlated ($p < 0.001$). In addition, reading ability was positively correlated with correct answers and setup ($p < 0.001$). Similarly, computational ability was positively correlated with correct answers and setups ($p < 0.001$).

Sherrill (1970) defined a printed mathematical word problems as follows: " Statements that have been presented in mathematics classrooms as being problems". An objective testing instrument of twenty such mathematical word problems was developed. Three groups of tenth grade mathematics subjects were identified: 1) Those subjects receiving one form of the word problems without any pictures (n=114); 2) Those subjects receiving the same form of the word problems with an inaccurately drawn picture (n=112); 3) Those subjects receiving the same form of the word problems with accurately drawn pictures (n=96). To test the hypothesis, the means of the groups were compared using single classification analysis of variance. The F ratio was significant at the 0.0001 level. This analysis indicated there was a significant difference between all the means. The order of the mean score, form highest to lowest, was groups 3, 1, and 2; that is accurate pictorial representation, followed by no pictures, followed by inaccurate pictures (cf, Sherrill, & Webb, 1974, p.560).

Similarly, Sherrill, and Webb (1974) selected ten problems from the Sherrill's (1970) study. The sample consisted of 80 preservice elementary school teachers who were enrolled in mathematics courses for prospective elementary school teachers. The data obtained from the study conducted by Sherrill (1970), the hypothesis tested was restated in the positive form. The comparison of the three means was made by single classification analysis of variance. The results of the test that the groups receiving the accurately drawn pictures performed significantly better than either of the other groups. This data seems to support the findings of Sherrill (1970).

Taking into account the observational facts given in the aforementioned literature, max-min word problems were selected in this study. Comprehension guide was prepared to remove reading and understanding difficulties with taking Stiff's (1986) comprehension guide model as a basis.

CHAPTER III

PRELIMINARY KNOWLEDGE: PROBLEM SOLVING AND ITS IMPORTANCE IN MATH EDUCATION

This chapter presents the problem solving method and its importance in math education including to the background information, components of essential mathematics, problem solving log, heuristics, and steps, specific abilities related to problem solving, and steps for solving word problems.

3.1. Background Information

To help today's students to get prepared for tomorrow's world, the goals of school mathematics must be appropriate for the demands of a global economy in an age of information. The National Council of Teachers of Mathematics' New Standards for School Mathematics (NCTM 1989) identifies five broad goals required to meet students' mathematical needs for the 21st century (cf, Steen, 1989) as follows:

*** To value mathematics.**

Students' experiences in school must bring them to believe that mathematics has value for them.

*** To reason mathematically.**

Students must learn to gather evidence, to make conjectures, to formulate models, to invent counter examples, and to build sound arguments.

*** To communicate mathematics**

Learning to read, to write, and to speak about mathematical topics is essential not only as an objective in itself-in order that knowledge learned can be effectively used - but also as a strategy for understanding. There is no better way to learn mathematics than by expressing arguments carefully in written form, by working in groups, and by arguing about strategies.

*** To solve problems**

Industry expects school graduates to be able to use a wide variety of mathematical methods to solve problems. Students must, therefore, experience a variety of problems- variety in context, in length, in difficulty ,and in methods.

*** To develop confidence**

The ability of individuals to cope with the mathematical demands of everyday life.

National Council Supervisor of Mathematics (NCSM) (1989, p.391) stated that students need modes of instruction that are suitable for increased emphasis on problem solving, applications, and the higher-order thinking skills.

Problem solving has been an important objective of mathematics instruction for many years, yet many people in general are not very good problem-solvers (Collier, & Lerch, 1969, p.328) .

Mathematical problem solving is important because a major goal of mathematics education is to prepare students for the future. Since the future is difficult to foresee, students must be taught to apply mathematical skills and concepts to novel situations. The National Council of Teachers of Mathematics (1980) has targeted problem solving as the focus of school mathematics of the 1980s. Similarly, the National Council of Supervisors of Mathematics (1977) stated that "learning to solve problems is the principal reason for studying mathematics "(p.2) (cf, McCoy, 1990, p.48).

Troutman and Lichtenberg(1974, p.590) stated that the development of the ability to solve problems is probably the most important aspect of one's education. Interpreted broadly, this ability can help people make choices that determine their complete life-style. Therefore, it is essential that students in our schools learn to sort out, assimilate, and select data that will enable them to make choices appropriate to them and will enhance their relationships to others.

Problem solving usually requires some creativity in analyzing, synthesizing, and evaluating situations, whereas exercise solving tends to require only routine application of previously learned facts and procedures (Polya 1973) (cf,Bell, 1980,p.652).

Troutman and Lichtenberg(1974, p.591) stated that several educators have attempted to analyze the act of problem solving and have generated models of these analyses. Five steps seem to be common to these models. The problem solver

- must be aware of the problem;
- must translate the problem into terms he can handle;
- must generate information and strategies necessary for solving the problem;
- must implement strategies necessary for solving the problem;
- must evaluate solutions for the problem.

Problem solving is appropriately considered a basic skill in the teaching of any mathematics course. Begle (1979) concurs with this philosophical position when he states that one of the primary justifications for teaching mathematics in the schools is its usefulness in solving many kinds of real world problems (cf, Grouws, & Thomas, 1981, p.307)

Mathematical problem solving can be analyzed into two major component processes: problem comprehension and problem solution (Mayer, 1985, 1986; Mayer, Larkin, & Kanade, 1984). Problem comprehension includes translation of each sentence of the problem into an internal representation and integration of the information to form a coherent structure, whereas problem solution involves the planning, monitoring, and the execution of the necessary computations. Although students experience many difficulties in the problem comprehension phase, most mathematics instruction focuses on problem solution, particularly execution (Carpenter, Corbitt, Kepner, Lindquist, & Reys, 1981; Mayer, 1985, 1986) (cf, Lewis, & Mayer, 1987, p.363).

3.2 Components of Essential Mathematics

The following twelve components of essential mathematics were stated (National Council Supervisor of Mathematics, 1989, p.389):

(a) Problem solving

Learning to solve problems is the principle reason for studying mathematics. Problem solving is the process of applying previously acquired knowledge to new and unfamiliar situations. Solving word problems in texts is one form of problem solving, but students also should be faced with nontext problems. Problem-solving strategies involve posing questions, analyzing situations, translating results, illustrating results, drawing diagrams, and using trial and error. Students should see alternate solutions to problems; they should experience problems with more than a single solutions.

(b) Communicating mathematical ideas

Students should learn the language and notions of mathematics. They should learn to receive mathematical ideas through listening, reading, and visualizing.

(c) Mathematical reasoning

Students should learn to make independent investigations of mathematical ideas.

(d) Applying mathematics to everyday situations

Students should be encouraged to take everyday situations, translate them into mathematical representations. They should observe how mathematics grow from the world around them.

(e) Alertness to the reasonableness of results

In solving problems, students should question the reasonableness of a solution or conjecture in relation to the original problem.

(f) Estimation

Students should acquire simple techniques for estimating such measurements as length, area, volume. They should be able to decide when a particular result is precise enough for the purpose at hand.

(g) Appropriate computational skills

Students should gain facility in using addition, subtraction, multiplication,and division with whole numbers and decimals.

(h) Algebraic thinking

Students should learn to use variables to represent mathematical quantities and expressions; they should be able to represent mathematical functions and relationships using tables, graphs, and equations.

(i) Measurement

They should be able to measure distance, mass(weight), time, capacity, temperature, and angles.

(j) Geometry

Geometric concepts should be explored in settings that involve problem solving and measurement

(k) Statistics

Students should plan and carry out the collection and organization of data to answer questions in their everyday lives.

(l) Probability

Taking into account the components of essential mathematics except probability, and statistics as fundamental while the Teaching\Learning material was established.

3.3. Problem Solving Log, Heuristics, and Steps

3.3.1. Problem Solving Log

The valuable tool that complements problem solving by example is the problem solving log (see figure 3.1) (Stiff, 1988, p.667) .

Problem Solving Log

Problem ID:---------------------------------

Record heuristics, anecdotes, and information needed to solve problem.

Understand the Problem	- clarify and identify information.
Devise a Plan	- identify operations and procedures.
Carry Out the Plan	- perform operations ; complete procedures.
Look back	- discuss, refine, modify.

Figure 3.1. A problem solving log.

3.3.2 Problem Solving Heuristics

Problem solving process should be completed using the heuristics, anecdotal comments, and distillations of group and class discussion for given problems. A list of heuristics (Polya, 1957,1962-65) must be available from which students may select (cf, Stiff, 1988, p.667) .

Problem Solving Heuristics

* Select appropriate notation
* Make a drawing, figure, or graph
* Identify wanted, given, and needed information
* Restate the problem
* Write a mathematical statement
* Draw from known information
* Construct a table
* Make a guess and check it
* Exhaust all possibilities
* Make a simpler problem
* Construct a physical model or experiment
* Work backwards
* Look for patterns
* Generalize
* Check the solution
* Find another way to solve it
* Find another result
* Study the solution process

Ungson (1985) conducted a study to develop a heuristic-oriented, self-instructional booklet for the teaching of problem solving strategies to adult college students. The approach used is the heuristic method of Polya's four phases of problem solving process. A two week exploratory use of the booklet was conducted with ten students. A pretest, posttest, questionnaire and informal interview were administered. The scoring of tests was based on criteria that reflected the quality of four phases of problem solving process. The results indicated that self-instructional booklets can be a useful adjunct to classroom instruction when combined with teacher reinforcement.

Truckson (1983) conducted a study to find out if college students' problem solving abilities increase when they are taught by heuristic and conventional methods. The secondary purpose of this study was to determine the effects of heuristic teaching and the instruction in the problem solving upon arithmetic achievement. The analysis from process measure assessments show that subjects taught by a heuristic method were more successful in obtaining the solution than the subjects taught by the conventional method.

3.3.3 Problem Solving Steps

Polya's (1957) four step outline of problem solving remains a useful way to categorize problem solving heuristics. Polya's four main steps are used repeatedly with sets of heuristics that teachers must identify for use with given problems.

<u>Polya's Problem Solving Steps:</u>

1.Understand the Problem

(a) restate the problem

(b) select appropriate notation

(c) make a sketch, a drawing, or table

2.Devise a Plan

(a) look for pattern

(b) make a simpler problem

(c) make a guess and check it

(d) use appropriate labels

3.Carry Out the Plan

(a) check special cases

(b) verify the details of the plan

4.Look Back

(a) generalize

(b) find another method of solution

(c) study the method of solution for future reference

Candless (1989) was investigated the effects of two forms of instruction specified on mathematical problem solving skills. One form of instruction specified explicitly three problem solving strategies. The other form provided a general recipe (Polya's four steps) to solving problems. Both forms of instruction were implemented through problem solving sessions. Changes in problem solving facility were measured by the difference between a problem solving pretest score and a problem solving posttest score for each subject. The dependent variables were Gain Score for Problems Solved Completely, Gain Score for Problems Solved Completely Correctly or Almost Correctly, and a Strategy Use Score, which assessed subject's use of designated strategies on the Posttest. The results indicated no statisticaly significant differences between the two forms of instruction on any of three measures.

In order to help students focus on other aspects of problem solving, it might help to put a diagram(see figure 3.2). The typical approach to problem solving is to translate the problem into mathematical terms and then attain a numerical solution. However, this is not enough. We must also involve students with such components as decision making, quality of the product, reasonableness of the results, implications for the use, practicality, and values (McGinty, and Mayerson, 1980, p.501).

Problem Solving

Decision making

|

Value judgements

|

Implications

Other possibilities

|

Satisfaction

Practicality

|

Reasonableness

Feasibility

Mathematical factors —— Method of solution —— Knowledge specific to the problem

Fig.3.2 Diagram for Problem Solving

The following Table 3.1 denotes the Polya's problem solving stages and their modified skills for the present study.

Table 3.1 : Stages and Problem Solving Skills.

Stages	Mathematics Problem Solving Skills
1. Understanding the Problem	a. Stating the unknown in the problem b. Stating the data in the problem c. Stating the condition in the problem
2. Devising a Plan	a. Deriving useful things from data b. Finding the connection between the data and the unknown
3. Carrying Out the Plan	a. Carrying out plan of the solution b. Solving the problem
4. Looking Back	a. Examining the solution

Moreover, Shoenfeld (1985) stated that a heuristic strategy is a technique or suggestion designed to help students better understand the problem. The systematic outline of the problem - solving strategy is given in Figure 3.3.

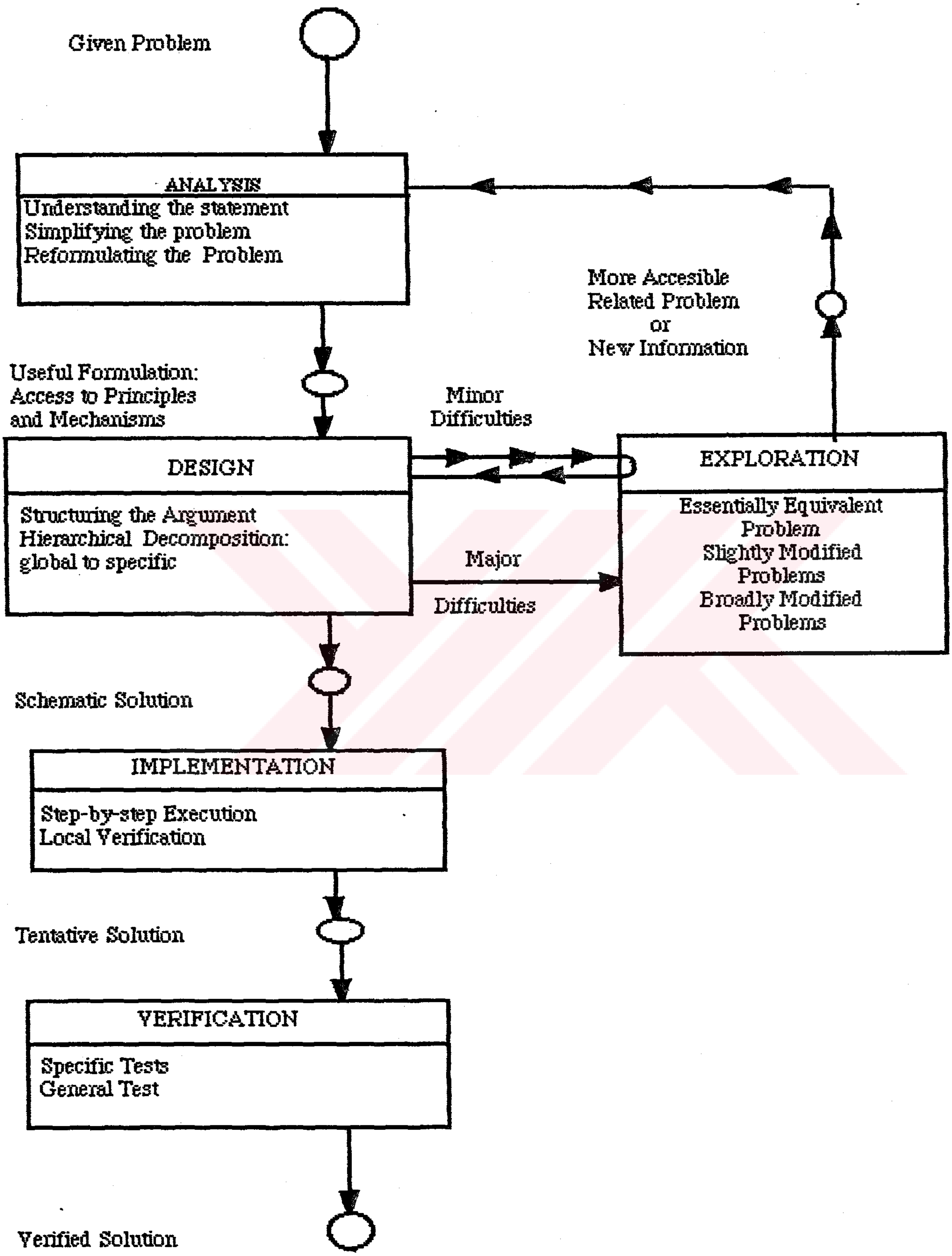


Figure 3.3: Schematic outline of the Shoenfeld's problem-solving strategy

Wooten (1990) conducted a study to investigate the teaching of mathematical problem solving strategies upon fifth grade students' abilities to solve routine and non-routine problems. The subjects included 50 fifth-grade classroom teachers and their students from 14 elementary schools. Teachers in the experimental group one received inservice on the teaching of problem solving strategies and materials to assist them. Teachers in the experimental group two and the comparison group received inservice on teaching routine word problems. Teachers in the experimental group two received problem solving strategy materials with no explanation and the comparison group did not receive any problem solving materials. Students were pretested using a routine problem solving test followed by a 13 week instructional segment in which the students in the experimental group one were taught specific problem solving strategies and the students in the experimental group two and the comparison group were taught specific routine objectives. Posttest included the routine problem solving test, the Romberg-Wearne Mathematical Problem Solving Test and the My Class Inventory Learning Environment Scale. The study employed an Analysis of Covariance for the routine test and Multivariate Analysis of Variance for the remaining tests. The results indicated significant differences between the two experimental groups and the comparison group on the routine test, but no significant difference between the two experimental groups.

3.4. Specific Abilities Related to Problem Solving

The following specific abilities related to solving problems were stated by Troutman, and Lichtenberg (1974, p.591):

* Finding characteristics of objects or mathematical ideas.

The more capable one becomes at identifying characteristics of an object or mathematical idea, the better his chances will be at determining how that object or idea relates to other objects or ideas. Activities requiring students to describe object, mathematical models, and ideas enable them to develop and reinforce this ability.

* Translating a mathematical communication into different forms.

One of the most effective ways to become familiar with subtleties of a mathematical communication is to translate it into as many forms as possible. Though there are many ways to accomplish a translation, the technique emphasized must in the general mathematics classroom in that of translating word problems into number sentences. This technique is essential to the problem solver.

* Finding similarities and differences.

Developing this ability requires students to compare mathematical ideas and find characteristics they do or do not share. Classroom experiences requiring students to use this ability can enable them to establish valuable insight related to classification systems, mathematical definitions and relationships.

* Determining sufficient, necessary, and equivalent conditions.

Whenever the student uses this ability, he is required to indicate criteria that constitute necessary, sufficient, or equivalent conditions for a mathematical idea.

* Making a generalization based on the observation of specific evidence.

An important ability that is typical of mathematical thinking is the ability to study a set of observations, to discern patterns existing among the observations, and to predict possible generalizations that lie the observation to classes of observations.

* Determining alternative strategies.

Problem situations traditionally offered in the general mathematics classroom encourage the notion that a problem can be solved by only one method and that the use of this method leads to a single solution. Unfortunately, reality is not so simple. Many problems that confront people can be studied from many points of view and can be solved using different strategies. As one develops the ability to design alternative strategies for solving a problem, he heightens his understanding of mathematical problems and increases his flexibility in using mathematical tools.

* Approximation

In fact, whether intentionally or not, the individual more often makes decisions based on approximations than on precise measures or calculations.

All these specific abilities related to the problem solving were tried to be earned to the students during the treatment.

3.5. Steps for Solving Word Problems

The practical use of mathematics should involve a continuous four-step process explained below (see figure 3.4) (Krulik, 1977, p.649):

1) Confronting the real-world problem.

2) Translating the problem into a suitable workable mathematical model.

3) Working out the solution.

4) Translating the solution to the mathematical model back into terms that reflect the original problem.

Krulik added, What do we do when we solve a problem?. Basically, we go through a series of four steps:

1 - Read the problem. Understand what is asked. What do we want to find? what data do we have? Draw a figure or a model. Introduce appropriate mathematical notation.

2 - Correlate the known data with the unknown. Is there a similar problem whose solution we know? Formulate several hypotheses. Develop a possible strategy to follow in an attempt to test each hypothesis.

3 - Carry out the strategy.

4 - Check the results.

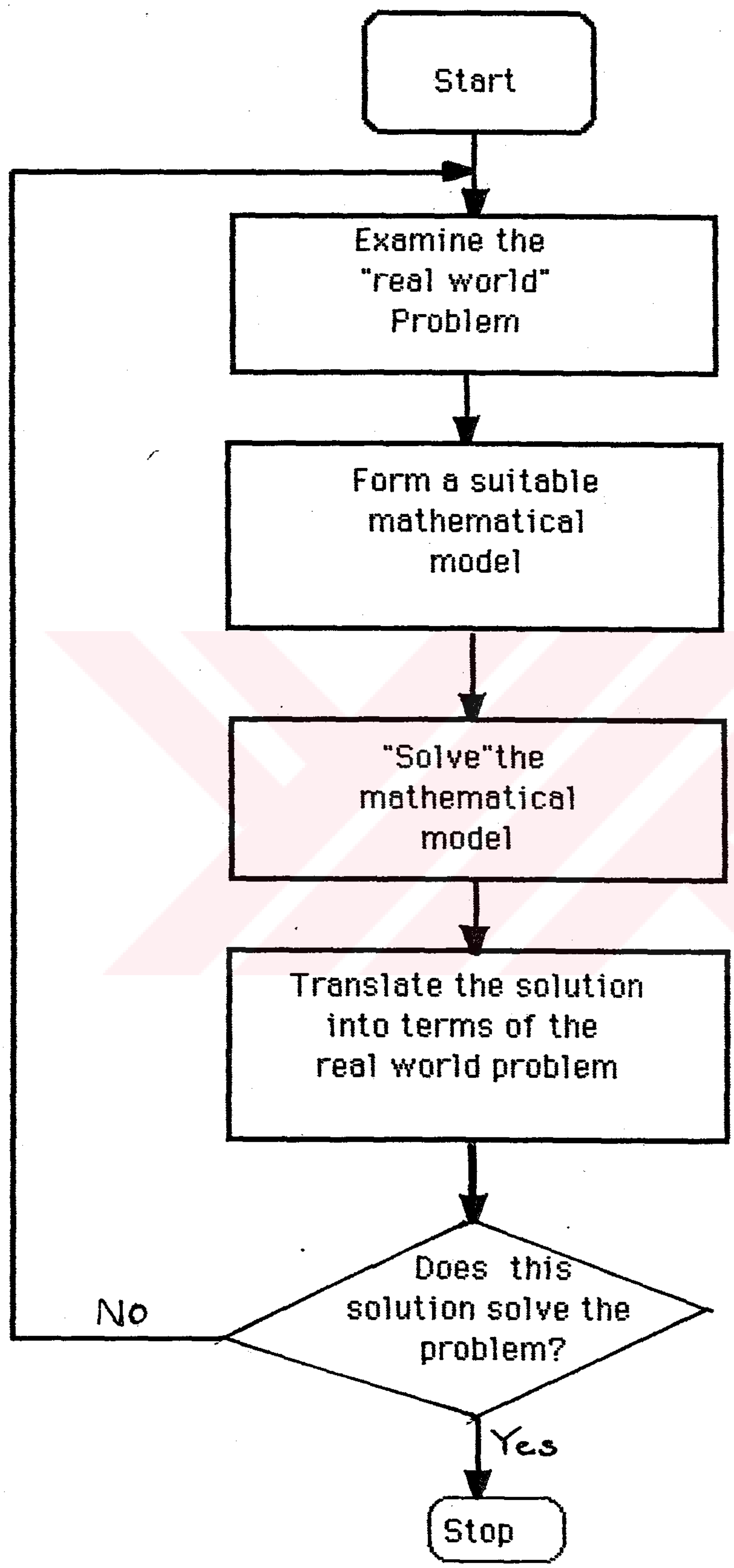


Figure 3.4: A four step problem solving straegy

We can better help our students learn to solve word problems. To help students in learning and teaching, the following steps are given below (Thaeler, 1986, p.698) :

Steps for Solving Word Problems

1) Read the problem until you have a clear mental picture of what is happening.

2) List all quantities that might be involved in the problem. Suggest a formula that might be helpful.

3) Write word equations, using the formula from the step 2 to help you.

4) Substitute the known quantities; create a variable when the need arises.

5) Solve the equation.

6) Do any additional calculations if the answer to the equation is not what was asked for in the problem.

7) Do the final check to see if the answer makes sense.

8) Try to solve the problem again by a different approach.

According to the steps of the word problem solving above, it can be concluded that mathematical word problems has three steps basically (e.g Chisko, 1985).

1) Decide what can you do or be done.

2) Do it.

3) Check the answer which is reasonable or not.

Accordingly, problem solving method was taken into consideration while preparing the handout material related to the max-min word problems in this present study.

CHAPTER IV

PROBLEMS, HYPOTHESES AND THE DESIGN OF THE STUDY

In this chapter, the definition of terms, the problems, the hypotheses, and the design of the study are presented.

4.1 Definition of Terms

Definitions of the terms as they are used in this study are as follows:

Traditional Method(TM)

In this present study, the Traditional Method refers to the method where students have been instructed only with lecture method.

Problem Solving Method with Handout Material(PSMHM)

Problem Solving Method with Handout Material is defined as an instructional method for increasing achievement in solving max-min word problems by giving the handout material which was prepared by taking Polya's problem solving stages.

Experimental Group(EG)

The experimental group refers to the group which was taught the max-min word problems by using a method which combines Problem Solving Method with Handout Material (PSMHM).

Control Group(CG)

The control group refers to the group which was taught the max-min word problems by using only the Traditional Method (TM).

Handout Material (HM)

The handout material was prepared by the researcher. This material presented max-min word problems on the basis of the problem solving stages of Polya which is given in the Appendix A.

4.2 The Main Problem and Subproblems

The purpose of the study is to provide answers to the following main problem and subproblems.

The Main Problem: The main problem of the study is stated as follows:

P: What is the effect of Problem Solving Method with Handout Material and department on the achievement of the students related to solving max-min word problems?

The Subproblems: The sub-problems of the study are stated as follows:

P1. What is the effect of PSMHM on the achievement of the students related to solving max-min word problems?

P2. What is the effect of department on the achievement of the students related to solving max-min word problems?

P3. Is there any interaction between treatment and department on the achievement of the students related to solving max-min word problems?

4.3 Hypotheses

The problems were tested by the following hypotheses:

Ho(1): There is no significant difference between the mean scores of the students taught by PSMHM, and those taught by TM with respect to the PoT scores.

Ho(2): There is no significant difference between the mean scores of the students at Mathematics, and the students at Mathematics Education with respect to the PoT scores.

Ho(3): There is no significant interaction between treatment (PSMHM or TM) and department (Math or Math Ed) with respect to the PoT scores.

4.4 Design of the Study

In this section, the subjects, the variables, the instruments, the procedures and activities, the assumptions and the limitations of the present study are discussed.

4.4.1 The Subjects of the Study

The study was conducted at the METU during the first semester of the 1990-1991 academic year in Math 153 Calculus I course. This course was taken by students from the Department of Mathematics and Mathematics education which was divided into two classes according to their alphabetic order of surname. In the present study, the classes were randomly assigned to experimental (EG) and control group (CG). The number of the subjects in both EG and CG and their distribution according to departments are shown in the Table 4.1.

Table 4.1: Distribution of subjects in the EG and CG according to departments

	Experimental Group	Control Group	Total
Mathematics Education	35	23	58
Mathematics	33	25	58
Total	68	48	116

4.4.2 Variables

The independent variables of this study are treatment (PSMHM or TM) and department (Math or Math Ed).

In the present study, department is treated as one of the independent variables since the students in Mathematics and Mathematics Education departments are different with respect to the high school branch, the high school mathematics performance, and university entrance examination scores.

when the branches of the high school are analyzed with respect to the department, the following differences are observed: 83.6 % of Mathematics students and 70.4 % of Mathematics Education students are coming from mathematics branch, 9.1 % of Mathematics students and 11.1 % of Mathematics Education students are coming from science branch, 13.0 % of Mathematics Education students are coming from literature branch but there are no Mathematics students coming from literature branch, 7.3 % of Mathematics and

5.6 % of Mathematics Education students are coming from other branches such as electronic (χ^2 = 7.99, p = 0.046). This indicates that students of Mathematics department are mainly mathematics oriented in the high school. However, students of Mathematics Education department are more heterogenous with respect to their high school branch.

When the high school mathematics performance of the Mathematics and Mathematics Education students are compared, it can be seen that Mathematics Education students are more successful than the Mathematics students in terms of high school mathematics performance since 62.3 % of Mathematics Education students and 32.7 % of Mathematics students had scores between 9 and 10; 32.1 % of Mathematics Education students and 50.9 % of Mathematics students had scores between 7 nd 8; 5.7 % of Mathematics Education students and 16.4 % of Mathematics students had scores between 5 and 6 (χ^2 = 10.07 , p = 0.0065)

These differences were also observed in the university entrance examination scores since the university entrance examination scores of Mathematics and Mathematics Education students were significantly different at the α = 0.05 level (t = 2.49, p = 0.014). The means of the university entrance examination scores of the Mathematics and Mathematics Education students were found as 489.98 and 504.78 and their standard deviations were found as 34.84 and 25.80 respectively. Larger standard deviation in Mathematics students implies that students of Mathematics are more heterogenous compared to Mathematics Education students in terms of university entrance examination scores ($F = 1.82$, $p = 0.033$).

Different characteristics observed in the students of two programs could create an interaction between treatment and department so that department is treated as an independent variable in this study.

The dependent variable is the students' achievement related to max-min word problems.

4.4.3 Instruments

In this study three measuring instruments were used.

1. General Achievement Test(GAT)
2. Pretest (PeT)
3. Posttest (PoT)

4.4.4 Procedure and Activities

A Pretest / Posttest and Experimental/ Control group design was used. The following procedures were followed in order to collect data.

1. GAT was administered to 136 subjects who were given 90 minutes to complete the test in the first semester of 1990-1991 academic year.
2. Two weeks later, PeT was administered to 116 subjects of 136 subjects who were allowed 30 minutes to complete.
3. One week later, 3 hours of treatment related to max-min word problems was done in the EG. In this period of time, EG

was treated with PSMHM by the researher but CG was treated with TM by the class teacher. Both groups received instruction on " Application of Derivative" before the treatment. The handout material was given to the EG students before the treatment (A description of the treatment with lesson plan is presented in Appendix A.1).

4. One day later, PoT was administered to 116 subjects who were allowed 30 minutes to complete.

In order to prevent students from remembering the test content, two strictly parallel instruments were used as Pretest (PeT) and Posttest (PoT) for measuring the achievement in solving max-min word problems in the present study.

4.4.5 Assumptions and Limitations

Following assumptions were made in regard to this study:

1. The administration of tests will be achieved under standard conditions.
2. All students in this study will answer the questions in accurately and sincerely.
3. The researcher and class teacher will be considered equal.
4. PeT and PoT are strictly parallel tests.
5. No outside event will occur during the treatment.

The following limitations were taken into consideration:

1. The scope of this study was limited to the students in the Math 153 Calculus I course at METU during the first semester of 1990-1991 academic year.

2. The study was also limited to one content area, i.e. max-min word problems .

3. This study was limited to 3 hours time duration except the time alloted for administering the measuring instruments.

4. Subjects were limited to 116 subjects.

CHAPTER V

DEVELOPMENT OF HANDOUT MATERIAL AND INSTRUMENTS FOR MEASUREMENT

In this chapter, the development of Handout material and instruments for measurement are presented.

5.1 Development of the Handout Material(HM)

Having come to the realization that the appropriate HM takes an important part in teaching word problems, HM used in this study was prepared to help students to develop the ability to solve the word problems by themselves.

Most students need guidance in learning to read a mathematical story problems meaningfully. After pupils have had time to read the problem situation and think about it, but before they compute the answer, teachers have found it good practice to ask such questions as:

- Does the problem involve combining groups? separating groups? comparing groups?
- What were you asked to find?
- What data were given that you can use?
- Will you need any data not given?
- Are there any data in the problem that you don't need?
- What will you do to find the answer?
- What do you already know that will help you?

- Can you draw a diagram or write a mathematical sentence to find out the relationship between what is given and what you are to find?

- About what do you think the answer will be? (Collier, & Lerch,1969,p.328)

5.1.1 The Parts of the Handout Material

In this study, the Handout material includes the followings:

1. A short review of the theorems related to max-min problems.
2. General procedure for solving max-min word problems.
3. Comprehension Guide for Problem 1 and stages for solution.
4. Comprehension Guide for Problem 2 and stages for solution.
5. Comprehension Guide for Problem 3 and stages for solution.
6. Problem 4, Problem 5, Problem 6, and Problem 7.

Each part of the HM was written on separate sheets and the copies were given to the EG students. The HM were prepared by taking the problem solving method into consideration. The solution stages in each problem were left incomplete and they were filled in through discussion with students.

5.1.2 General Procedure for Solving Max-Min Word Problems(GPWP)

General procedure for solving max-min word problems was used to teach procedures for attacking problems because the understanding of general procedure is important than getting the solution.

The purpose of GPWP is:

- to teach how we can start to think about max-min word problems.
- to help students develop a general plan for attacking a problem.
- to learn to solve problems in general. In solving problems, it is necessary to focus the attention on the process rather than on the answers of the problems.

5.1.3 Comprehension Guide (CoG)

Comprehension guide based on Polya's heuristics was used for clarifying problems and improving students' comprehension of word problems. Students were made familiar with the necessary stage which is the understanding the problem stage of Polya to solve problems.

In preparing the CoG we aimed:

- to help students read the the problems carefully.
- to help them translate the problem into familiar setting in which the relationships between known and unknown become clear and than explore the original setting.

- to check the student's understanding of what's to be found, and the meaning of data.
- to ask questions which help the students focus on the elements necessary for a solution.
- to help them solve the problem through discussion.
- to be useful to the problem solver if he works by himself.
- to develop a plan for assisting students to read a "word problem" meaningfully.
- to help students learn how to organize problems.

The CoG is used to understand the problem by identify the followings:

- the unknowns
- the data
- the conditions

After the verbal statement of the problem was understood, the students could find the unknown, the relevant and irrelevat data, and the conditions.

5.1.4 Characteristics of the Problems in the Handout Material

The Problems in the handout material were organized on the basis of such characteristics as context, common unit, and question form. These characteristics are identified in the Table 5.1

Table 5.1: The Characteristics of the Handout Material Problems

Problems	Characteristics of the Material Problems		
	Context	Question Form	Common Unit
1	Geometry (rectangle)	What	Lenght
2	Geometry (cube)	Find	Dimensions
3	Geometry (cylinder)	What	Dimensions
4	Geometry (rectangle+circle)	Find	Dimensions
5	Geometry (cone+sphere)	Find	Height
6	Motion	Could	Distance
7	Geometry (cylinder)	Find	Dimensions

5.2 Development of Measuring Instrument I: General Achievement Test

The General Achievement Test (GAT) was designed by the researcher to evaluate the prerequisite knowledge of the students about precalculus learning before starting the application of derivative content in Math 153 Calculus I. It was then used as a pretest in the present study (see Appendix B).

There were two purposes of the evelopment of GAT. First,it was used to see whether the students in both EG and CG were equivalent in terms of mthematics achievement.

Second, the test results were used to evaluate the students' mathematics background since they came from diverse high schools in different regions of Turkey.

The achievement of the students in GAT was refered as the knowledge of mathematics they have attained - the cognitive learning outcomes in mathematics.

The test "blueprint" was prepared. Ten major content areas and ten objective areas were identified in the Table 5.2. The test consisted of 50 multiple-choice objective items which were chosen from the University Entrance Examinations given to the students in the last ten years, i.e.1980 and 1990 in Turkey.

The analyses were carried out by using the Statistical Packages for Social Sciences (SPSS) (Nie,et.al., 1975). The descriptive statistics of GAT are shown in the Table 5.3.

Table 5.3: The Results of Descriptive Statistics of the GAT

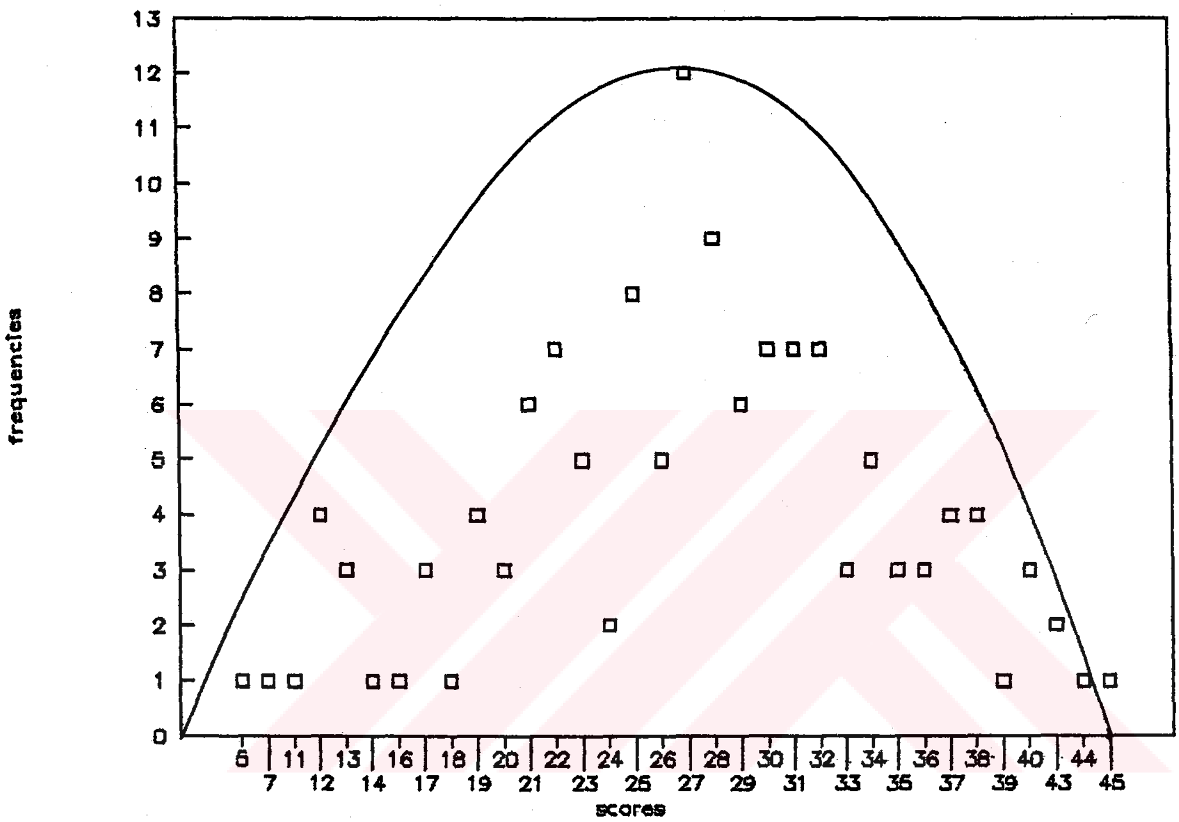
	DESCRIPTIVE STATISTICS
N of items	50
N of examines	136
Mean	27.19
Variance	58.75
Std-Dev	7.70
Skewness	-0.260
Kurtosis	0.058
Minimum	5
Maximum	45
Median	27
Mode	27
Alpha	0.857
SEM	2.884
Item Means	0.543
Mean Item-Tot	0.350
Mean Biserial	0.478

N: number of items SEM: standart error of measuremant
Alpha: estimates of reliability (KR20)

	1	2	3	4	5	6	7	8	9	10	
CONTENT / BEHAVIOUR	carry out algorithm	manipulate and compute rapidly and accurately	interpret symbolic data	put data into symbols	remember or recall definition, notations, operation, and concept	apply concepts to mathematical problems	analyse and determine the operations which may be applied	follow a line of reasoning	invent mathematical generalizations	discover relationships	Total
1. NUMBERS	3										3
2. POLYNOMIALS	2				1						3
3. ARITHMETIC OPERATION PROBLEMS				1			4				5
4. EQUATIONS		2			4						6
5. RELATIONS AND FUNCTIONS			2	5	2	1		2		1	13
6. TRIGONOMETRY	1	2	1		1						5
7. LOGARITHM									2		2
8. LINE EQUATIONS AND GRAPHS			2		1		1				4
9. LIMIT AND CONTINUITY			1			2					3
10. GEOMETRY					2	1			1	2	6
TOTAL	6	4	6	6	11	4	5	2	3	3	50

The distribution of GAT scores which was approximately normal but very slightly negatively skewed are given as seen in Figure 5.1.

Figure 5.1: Distribution of GAT scores



Using the Kuder-Richardson (KR20) and Split-Half Method, estimates of the reliability of the GAT were obtained. KR (20) and Split-Half reliabilities were found as 0.856 and 0.710 respectively.

Validity of the GAT was investigated by using content related evidence, criterian related evidence, and construct related evidence.

The content validity of the test has been checked by a calculus specialist in the department of mathematics at METU. The test blueprint was compared with the test content by the researcher also. The percentage of selecting the correct answer, the content area, and the objective of each item in GAT are indicated in the Table 5.4. According to the Table 5.4 , it can be concluded that the objective and the content number 1 were the most achievable one, but the objective number 10 or 5 and the content number 5 or 10 were the least achievable one.

The criterian related validity evidence was investigated by studying the correlations of the GAT scores with first midterm scores (Exam1) and second midterm scores (Exam2) in the Math 153 Calculus I course.

Means and standard deviations of the GAT, Exam1 and Exam2 were shown in the Table 5.5.

Table 5.5: Means and Standart Deviations of GAT, Exam1, and Exam2.

Variables	Mean	Standard-Deviation
GAT	27.19	7.66
Exam1	22.77	9.20
Exam2	25.51	10.23

In the table above the total scores of the exams were out of 40 and the score of GAT was out of 50.

The pearson product correlation coefficients of GAT score with Exam1 and Exam2 were given in the Table 5.6.

Table 5 .4: Percent Selecting the Correct Answer of GAT with Item Number/Correct Answer, Content Area, and Objective.

Percent Selecting the Correct Answer %	Item Number/ /Correct Answer	Content Area	Objective
98	1/A	1	1
97	4/A	1	1
93	3/D	2	1
91	5/A	1	1
89	11/C	4	2
86	10/C	3	6
84	14/B	5	7
82	16/D	2	5
79	27/A	7	9
78	2/E	2	1
72	17/C	4	5
71	26/D	7	9
70	9/E	3	6
70	34/C	5	4
68	18/E	4	5
68	21/C	6	5
68	29/A	8	5
65	7/C	3	6
65	15/D	5	8
62	6/B	3	6
61	20/A	4	5
61	32/B	5	5
59	12/E	4	2
59	41/B	9	7
57	42/E	5	3
56	22/A	6	2

Table : 5 .4: Percent Selecting The Correct Answer of GAT with Item Number/Correct Answer, Content Area, and Objective.

Percent Selecting the Correct Answer %	Item Number/ /Correct Answer	Content Area	Objective
55	39/E	9	7
53	36/A	8	6
52	13/A	5	8
49	44/E	5	3
49	46/C	10	7
47	8/B	3	4
45	37/C	5	4
43	33/C	5	5
43	40/A	9	3
42	25/A	6	1
42	28/D	8	3
41	35/B	5	4
32	24/E	6	3
31	43/E	5	4
27	30/A	8	3
24	45/E	10	9
20	50/B	10	5
18	23/E	6	2
18	49/B	10	5
16	48/B	10	10
14	47/D	10	10
10	19/S	4	5
10	38/E	5	4
9	31/E	5	10

Table 5.6: Correlations of the GAT scores with Exam 1 and Exam 2.

	Exam1	Exam2
Test	0.3704 p = 0.001	0.3105 p = 0.001

The correlation coefficients of GAT scores with Exam1 and Exam2 were significant at 0.05 level. In other words, there was a significant positive correlation.

Factor analysis was used in order to find a construct related evidence for GAT scores. Factor analysis was performed for the ten subscale scores of GAT. The means and standard deviations of ten subscales of GAT with respect to the sequences in the blueprint are shown in the Table 5.7.

Table 5.7. Means and Standard Deviations of the Subscales of GAT

Variable	Mean	Standard Deviation
T1	2.8676	0.3812
T2	2.5588	0.6751
T3	3.3603	1.2152
T4	3.6103	1.2363
T5	6.1985	2.7751
T6	2.2206	1.2334
T7	1.5221	0.7094
T8	1.8897	1.2029
T9	1.6029	1.1306
T10	1.3603	1.5476

T: Subscale of GAT

The first and the second eigenvalues were found as 3.57 and 1.25 respectively for the unrotated factor solution.

Results of factor analysis revealed that GAT was mainly two dimensional. The first dimension indicated the subscales (3,4,5,6,7,8,9,10) related to conceptual ability while the second dimension indicated the subscales (1,2) related to the computational ability. Varimax rotated factor matrix revealed almost the same structure for the GAT subscales except the subscale 7.

The initial and rotated factor loadings were reported in the Table 5.8 for the GAT.

The validity evidence obtained for GAT and the reliability estimates imply that scores obtained on this test are reliable and valid measures of the students general mathematics achievement.

Table 5.8: The Principle and Rotated Factor Structure of GAT

		Principal Factor Structure		
Eigen values	Dimensions	Factor1	Factor2	Communality
3.57459	T1	0.30068	0.70754	0.59103
1.25275	T2	0.47654	0.53410	0.512356
0.98828	T3	0.33003	0.01221	0.10907
0.87359	T4	0.63440	0.16075	0.42830
0.81684	T5	0.81988	-0.26528	0.74258
0.67628	T6	0.62942	-0.26563	0.46672
0.58664	T7	0.69594	0.24182	0.54282
0.45449	T8	0.69022	-0.00704	0.47645
0.44271	T9	0.69500	-0.19199	0.51988
0.33383	T10	0.48329	-0.45228	0.43813
		Varimax Rotated Factor Structure		
	Dimensions	Factor1	Factor2	
	T1	-0.08892	0.76362	
	T2	0.14972	0.69995	
	T3	0.28073	0.17395	
	T4	0.47168	0.45366	
	T5	0.84372	0.17529	
	T6	0.67839	0.08071	
	T7	0.48504	0.55458	
	T8	0.60323	0.33550	
	T9	0.69893	0.17716	
	T10	0.64380	-0.15380	

5.3 Development of Measuring Instrument II: Pretest (PeT) and Posttest (PoT)

5.3.1 Pretest

This test was prepared by selecting Max-Min problems from University Entrance Examinations conducted in Turkey between 1980 and 1990 years. PeT included 3 essay type items and the students were allowed 30 minutes to complete it. It was administered before the treatment (see Appendix C.1).

The PeT was prepared by the researcher. The purpose of this test was to assess the students prerequisite knowledge related to Max-Min principles and problem solving skills. Therefore, multiple choice type items were converted to essay type items. The score of the each question in PeT is out of 10 and the scoring key is presented in the Appendix D.1.

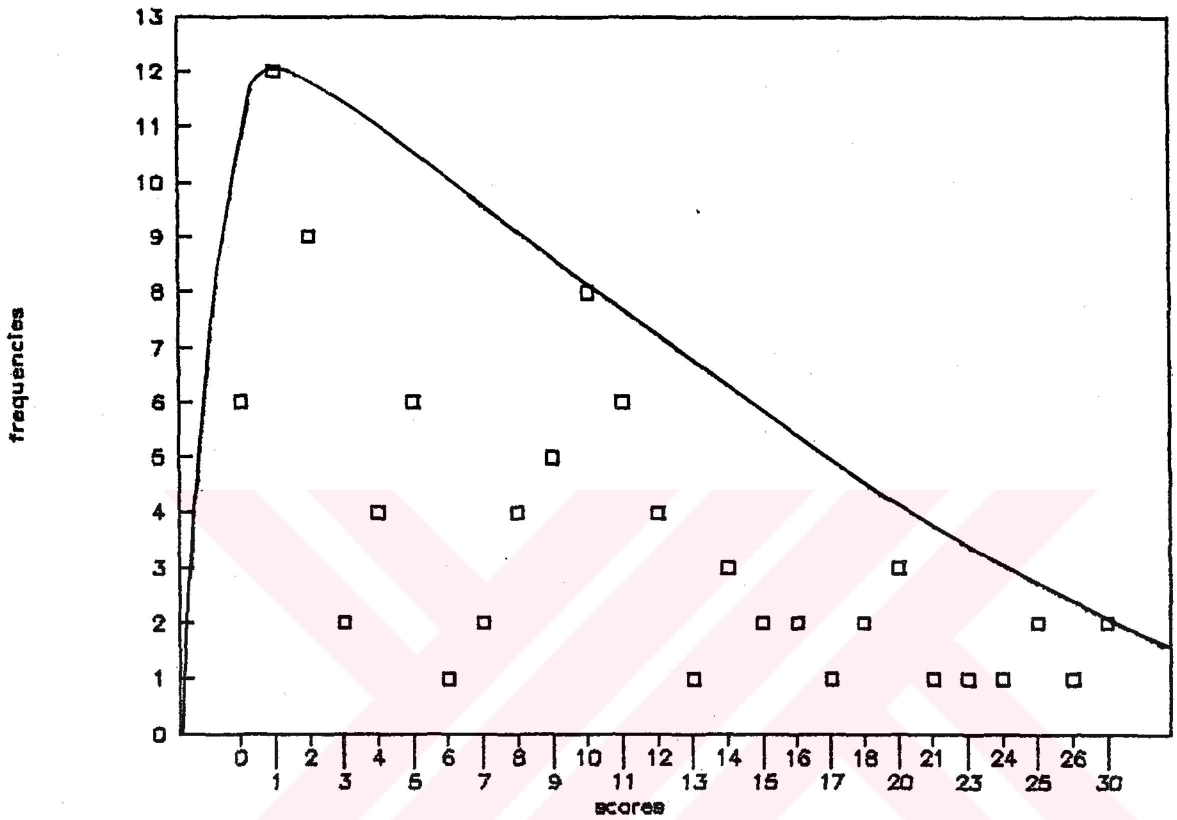
Context, common unit, and question form of the PeT questions are shown in the following Table 5.9.

Table 5.9: Characteristics of the PeT Questions.

Questions	Characteristics of the PeT Questions		
	Context	Question Form	Common Unit
1	Geometry (circle+trapezoid)	What	Height
2	Geometry (analytic geometry)	What	Distance
3	Geometry (rectangle)	What	Area

The distribution of PeT scores was positively skewed as seen in the Figure 5.2.

Figure 5.2: Distribution of PeT scores



Reliability estimate of the PeT was calculated by using analysis of variance (ANOVA) (Thorndike, 1982, p.156). The reliability estimate was found as 0.67 for this essay test.

A mathematician from the Department of Mathematics at the METU judged the content validity of PeT.

5.3.2 Posttest

The Posttest (PoT) was prepared by the researcher. This test which includes three essay type items and was administered to the EG and CG students after the treatment related to max-min word problems. 30 minutes was alloted to complete it (see Appendix C.2). The score of each question in PoT is out of 10 and the scoring key of PoT is given in the Appendix D.2.

The PoT was used:

- to see to what extent the problem solving behavioral criteria given are observable in student responses.
- to evaluate the effects of different treatments.

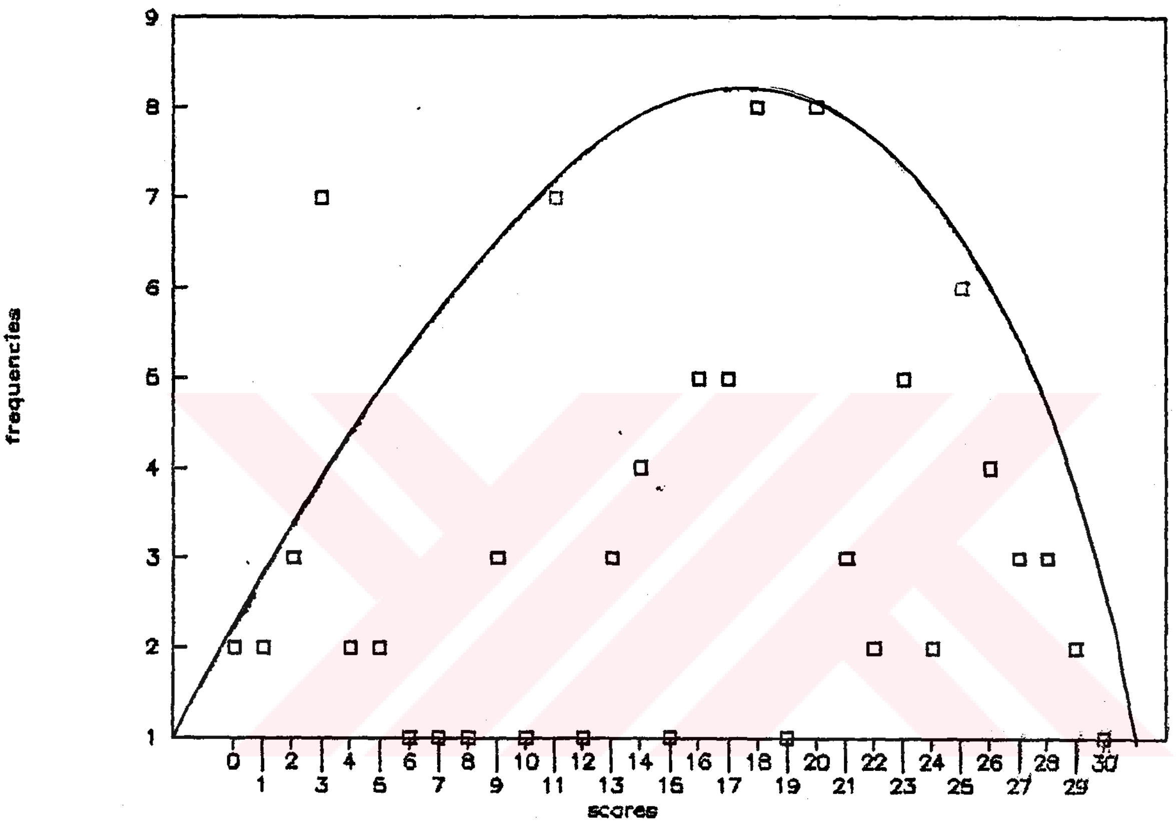
The Table 5.10 indicates the context, common unit, and question form of the PoT questions.

Table 5.10. The Characteristics of the PoT Questions.

Questions	Characteristics of the PoT Questions		
	Context	Question Form	Common Unit
1	Geometry (analytic geometry)	Find	Height
2	Geometry (rectangle+equilateral) triangle	Find	Dimension
3	Geometry (cube)	What	Dimension

The distribution of PoT scores was approximately normal but very slightly negatively skewed as seen in the Figure 5.3.

Figure 5.3: Distribution of PoT scores



Reliability estimate of the PoT was calculated by using ANOVA. The reliability estimate was found as 0.73.

The mathematician from the Department of Mathematics at the METU judged the content validity of PoT.

CHAPTER VI

ANALYSIS OF DATA AND RESULTS

In this chapter, analysis of data and the results of this analysis are indicated.

6.1 Analyses of Data

In the present study, the data collected from the measuring instruments were analyzed by the statistical technique called " two-way analysis of variance".

In order to see whether experimental and control groups are equivalent at the beginning, GAT and PeT scores of the experimental and control groups were compared with department by two-way analysis of variance.

The results of the two-way analysis of variance obtained from GAT are indicated in the Table 6.1.

Table 6.1: Analysis of Variance of the Data obtained from GAT

Source of variation	Sum of squares	Degrees of freedom	Mean square	F	p
Main Effects	151.471	2	75.736	1.288	0.280
Treatment(a)	35.609	1	35.609	0.605	0.438
Department(b)	109.811	1	109.811	1.867	0.175
Interaction	212.179	1	212.179	3.608	0.060
Explained	363.650	3	121.217	2.061	0.110
Residual	6528.472	111	58.815		

According to Table 6.1, it can be concluded that Factor A (treatment) and Factor B (department) have no significant effect on the achievement of students in GAT at $\alpha = 0.05$ which means that experimental and control groups are equivalent before the instruction. There is no interaction between treatment and department on the achievement of students in GAT also. The results of the two-way analysis of variance obtained from the PeT are indicated in the Table 6.2.

Table 6.2: Analysis of Variance of the Data obtained from PeT

Source of variation	Sum of squares	Degrees of freedom	Mean square	F	p
Main Effects	217.296	2	108.648	1.891	.156
Treatment(a)	140.540	1	140.540	2.446	.121
Department(b)	67.378	1	67.378	1.173	.281
Interaction	109.723	1	109.723	1.910	.170
Explained	327.019	3	109.006	1.897	.134
Residual	6376.668	111	57.447		

According to Table 6.2, it can be concluded that Factor A (treatment) and Factor B (department) have no significant effect on the achievement of students in PeT which means that experimental and control groups are equivalent before the instruction in terms of PeT scores at $\alpha = 0.05$. There is no interaction between treatment and department on the achievement of students in PeT also.

As a result of these two prior analyses, it was decided that only the PoT scores of the experimental and control group will be compared. Two-way analysis of variance was used to compare the effects of the two different instructional methods (PSMHM and TM) with two different departments (Math and Math Ed) on the achievement of students in solving max-min word problems.

In this study, statistical analyses were carried out by the use of Statistical Package for Social Sciences (SPSS) (Nie.et.al, 1975).

6.2 Results

In this section, the results of the study are presented. The hypotheses were tested by using two-way analysis of variance with achievement on PoT as the dependent variable, the treatment and the department as the independent variable.

<u>Hypothesis 1</u>: In order to test this hypothesis which has been stated that the differences between the mean scores of the students taught by PSMHM, and those taught by TM on PoT scores are not significant at $\alpha = 0.05$. F-test was used.

According to the Table 6.3, we conclude that Factor A (treatment) has a significant effect on the achievement of students solving max-min word problems in PoT($F = 5.093$).

The means of the experimental and the control groups are 15.03 and 10.96 respectively.

Hypothesis 2: In order to test this hypothesis which has been stated that the difference between the mean scores of the students in the Math Department, and those in the Math Ed Department on PoT is not significant at $\alpha = 0.05$. F-test was used, and the results are shown in the Table 6.3.

According to the results, we conclude that Factor B (department) has no significant effect on the achievement of students solving max-min word problems in PoT ($F = 1.996$).

The means of the mathematics and mathematics education students are 12.07 and 14.68.

Hypothesis 3: In order to test this hypothesis which has been stated that the interaction between treatment (PSMHM or TM) and department (Math or Math Ed) on the PoT scores is not significant at $\alpha = 0.05$, F-test was used ($F=0.097$), the results are presented in the Table 6.3.

This indicates that there is no interaction between treatment and department on the achievement of students solving max-min word problems.

Table 6.3: Analysis of Variance of the Data obtained from PoT

Source of variation	Sum of squares	Degrees of freedom	Mean square	F	p
Main Effects	631.157	2	315.578	3.698*	.028
Treatment(a)	434.536	1	434.536	5.093*	.026
Department(b)	170.352	1	170.352	1.998	.160
Interactions	8.293	1	9.293	0.097	.756
Explained	639.449	3	213.150	2.498	.063
Residual	9471.211	111	85.326		

* Significant at $\alpha = 0.05$

In the light of the findings obtained by statistical testing, the following conclusions were obtained.

According to the results of two-way analysis of variance from PoT, the followings were concluded:

* Factor A (treatment) has a significant effect on the achievement of students in solving max-min word problems.

* Factor B (department) has no significant effect on the achievement of students in solving max-min word problems.

* There is no interaction between treatment and department on the achievement of students in solving max-min word problems.

It can be concluded that PSMHM has a positive effect on the achievement of students in max-min word problems.

CHAPTER VII

DISCUSSIONS, IMPLICATIONS, RECOMMENDATIONS AND CONCLUSION

This chapter presents discussions of the results that were stated in the previous chapter. Implications, Recommendations and conclusions based on the discussion are stated.

7.1 Discussion

The purpose of the study was to assess the effect of the Problem Solving Method with Handout Material and the department on the achievement of students in solving max-min word problems.

Two-way analysis of variance was performed on the scores obtained from the PoT to evaluate the effect of treatment and department, and their combination on the achievement of the students in solving max-min word problems. The analysis indicated the followings:

- Factor A (Treatment) has a significant effect on the achievement of students solving max-min word problems at $\alpha = 0.05$
- Factor B (Department) has no significant effect on the achievement of students solving max-min word problems at $\alpha = 0.05$.
- There is no interaction between treatment and departmant on the achievement of students solving max-min word problems at $\alpha = 0.05$.

Results of this study indicated that the students who were treated with PSMHM were more successful than the ones who were treated with TM. Therefore, it can be said that the new teaching method had positive effect on the achievement of the students in solving word problems. 3 hours of treatment created 4.07 points mean difference between the experimental and the control groups. If more time had been dedicated to the treatment, it could be claimed that the subjects in the EG would have been more successful in the posttest.

As it was mentioned before, the PSMHM aims at developing three abilities of students: the ability to translate word problems into mathematical symbols, the ability to solve equations, and the ability to interprete the result. A word problem must be read carefully to translate word problem into mathematical symbols. The students' reading comprehension were improved by the use of comprehension guide. Otherwise, the students' attention would have focused only on working out the solution of the problem rather than on reading the problem since the lack of understanding a problem is a fundamental cause which creates great difficulty for the students in solving a word problem.

Ungson (1984) observed that instructional booklets can be a useful adjunct to classroom instruction when combined with teacher reinforcement. At the same time, her study verified the observation that students generally fail to look back on their solutions. For many students problem solving activity ends when an answer is obtained, showing that problem solving is viewed as a product and not a process. Findings of the present study support Ungson's research. The students in the EG were more successful than the students in the CG in solving the word problems correctly. This finding might indicate that EG students successfully looked back on their solution compared to CG students.

Hollander (1978, p.333) stated that successful problem solving strategies were attributed to (1) the comprehension of mathematical relationships as expressed through the words and symbols within the problem; (2) the strength of the students ability to employ abstract analytical reasoning; (3) the strength of the students ability to reason insightfully; (4) making neither exessive nor few references to the text relative to their peers; and (5) the minimum number of computational steps necessary for the solution.

In the present study, students were expected to be able to (1) organize the facts given; (2) decide what was to be found; (3) choose the appropriate arithmetical operations; (4) estimate a reasonable answer; and (5) execute the solution. Mean differences in favor of the EG students may imply that students treated with PSMHM had succesfully achieved the aforamentioned steps in the problem solving process.

When the response patterns of the experimental and the control groups were closely analyzed in the PoT, it was observed that the experimental group was more successful than the control group in indicating the complete solution of the word problems. The percentages of the complete solution of word problems in both groups are as follows; 27 % in EG and 21 % in CG for problem 1, 43 % in EG and 33 % in CG for problem 2, and 53 % in EG and 49 % in CG for problem 3 in which the figure is presented to the students. If a figure related to a problem is given to the students, the number of correct answers increases. The Polya's problem solving stages used in the handout material improved the students' problem solving skills. Giving the problem with its drawings helped students to solve the problem easily. This finding implies that the second stage of Polya's problem solving process is the most important stage. It is clearly observed that PSMHM developed the skills of the students to draw the figure related to the data given since EG was more successful than the CG while solving the "non-figure" word problems.

Webb, and Sherrill's (1974) study supports this finding also, that the presentation of an accurate pictures aids students achievement with mathematical word problems.

When the response patterns of the experimental and control group students related to the writing the correct equation of the given problem are analyzed closely, it was observed that 38 % in EG and 31 % in CG for problem 1, 23 %

in EG and 13 % in CG for problem 2, 18 % in EG and 13 % in CG students for problem 3 wrote the correct equation and got the partial grade. In this respect, EG was more successful than the CG while writing the correct equation of the given problem. It shows that reading comprehension plays an important role in writing the correct equation and in visualizing the problem situation since the percentage of correctly written equation implies that the problem situation is visualized or the fiqure of the given problem is drawn. This study is also consistent with Saygi (1990) study. She reported significant correlations among the Polya's problem solving skills on the prospective mathematics teachers. It was found that restating the problem, guessing the results, and solving the problems correlated with visualizing the problem situation. At the same time restating the problem correlated significantly with solving the problems respectively. Therefore, it can be concluded that subjects who understood the problem were able to devise a plan and carry out that plan. Moreover, Hart (1985) stated that lack of understanding of either mathematical or situational concepts is one of the factors which hinders the problem solving process related to the applied mathematics problems.

In the present study, comprehension guide was used to supply the understanding of the problems. It can be said that understanding of the problem is the most important step in solving word problems with respect to the above results related to the writing the correct equation of the given problem and solving the problem correctly. Muth (1984) observed that reading ability contributed to success in solving the arithmetic word problems. Reading ability accounted for about 14 % of the variance in correct answers. In short, this finding also support the hypothesis that reading ability plays a major role in the solution of arithmetic word problems.

Similar result was found by Saygi (1990). She found that reading comprehension contributes towards explaining the variance produced on mathematical problem solving skills. As a result, there is a need for the reorientation of the mathematics curriculum towards a goal of "understanding". Educators must emphasize general problem solving skills at all levels, especially understanding the problem that is required for the solution of word problems.

The results obtained from the mathematics and mathematics education students indicated no significant difference between the departments on the achievement of the students in solving max-min word problems. It can be concluded that PSMHM increased the achievement of both Mathematics and Mathematics Education students equally in solving max-min word problems even though these students have different educational background.

Briefly, it can be said that a subject's ability to solve mathematical word problems was appeared to be affected by the method of presentation rather than their educational background and departments. As a result, following Polya's problem solving stages during the treatment improves important role on the students' achievement in solving max-min word problems.

7.2 Implications

This study holds the following implications for educational practice:

1) The present study suggests that students' ability to transfer mathematical sentences to the mathematical symbol will be increased in the proportion to the degree to which problem solving method with handout material are used in the classroom.

2) Pupil ability to transfer heuristics of problem solving are important outcomes of instruction, problem solving method should have been integral part of the methodology used in the presenting word problems in the classroom.

3) Polya's problem solving stages and handout that best facilitate the acquisition of problem solving.

7.3 Recommendations and Conclusions

7.3.1 Conclusions

The aims of this study were given as follows:

- To encourage students to develop active problem solving starategies for which they can be find applicabilty beyond the specific problems or even class itself.

- To bring students one step closer to the goal of educating students to be literate people in a highly technological society.

- To develop a model that constructs representations of word problems solving with the information needed for successful solution. In this way, the students can be helped to transfer their problem solving skills to real life situations.

Students' perception of word problem solving as a tool can be strengthened to make intelligent decisions in their daily affairs with the new instructional methods.

There appeared to be achievement differences in the solution of max-min problems for EG and CG problem solvers after the treatment. As a result, this study conludes that the PSMHM has an effect on the achievement of the students to solve word problems.

Based on the findings of the study, the following conclusions were made within the context of this study:

- A subject's ability to solve the mathematical word problems is affected by the method of instruction.

- The representation of problems with figure aids student's achievement with word problems.

This study represents a beginning in this direction in Turkey, but there is still much to be done in developing problem solving in mathematics. As a result, further research is needed to verify the results and give futher evidence of generalizibility.

7.3.2 Recommendations

Results of this present study offer us the following recommendations:

1) A similar study should be conducted to test the effectiveness of PSMHM while increasing the duration of application of this method.

2) A similar study should be conducted with the teaching of other type of word problems (Age , interest,...).

3) A similar study should be conducted at other grade levels and other universities.

4) A similar study can be implemented using same instructors for each group (EG and CG)

5) A similar study should be conducted to test the effectiveness of PSMHM while decreasing the number of students in the class.

6) Intervening variables could be selected to determine if there are interactions between treatment and such factors as age ,sex, IQ, etc.

REFERENCES

<ANON>AM - MATH - MO, 1987. " A Report of the CUPM Panel on Calculus Articulation in the Transition from High School Calculus to College Calculus", Mathematical Monthly, Vol.94, No.8, pp.776-785.

Bell, F., H., 1980. " Posing and Solving Verbal Problems", Mathematics Teacher, pp.652-656

Buchalter, B., & Stephens, L., 1989. " Factors Influencing Calculus Aptititude", Int. J. Math. Educ. Sci. Technol, Vol.20, No.2, pp.225-227.

Burton, B.M., 1989. " The Effect of Prior Calculus Experience on Introductory College Calculus ", American Mathematical Montly, Vol.96, No.4, pp.350-354.

Candless, P.H., 1989. " Teaching Mathematical Problem Solving Strategies", Dissertation Abstracts Intenational, Vol.51, No.1, 1990, pp.101-A.

Cankoy, O., 1989. " Differences between Traditional Lecture Method and Mathematics Laboratory Instruction in terms of achievement related to a probability unit", Master Thesis (METU).

Carpenter, T.P., 1985. " How children solve simple word problems", Education and Urban Society, Vol.17, No.4, pp.417-425.

Carpenter, T.P., Corbitt, M.K., Kepner, H.S, Lindquist, M.M, and Reys, E.R., 1980. "Naep Note: Problem Solving", Mathematics Teacher, Vol.73, No.9, pp.427-433.

Chisko, M.A.,1985. " Developmental Math: Problem Solving and Survival", Mathematics Teacher, pp.592-596.

Collier, C.C., & Lerch, H.H., 1969. Teaching Mathematics in the Elementary School, Collier - Macmillan Canada.

Dienes, Z.P. and Golding, E.W.,1971. Approach to Modern Mathematics, Hender and Hender, New York.

Ellis, R., & Gulick, D., 1990. Calculus with Analytic Geometry, Orlonda, Florida.

Emenalo, S.I., and Okpara, E.N., 1990. "A Study Guide for Mathematics students", Int. J. Math. Educ. Sci. Technol., Vol.21, No.1, pp.51-59.

Fiske, B.M., 1990. " Are Teachers Prepared to Implement the Standards",Mathematics Teacher, Vol.83, No.4, pp.248-251.

Gadanidis, G., 1988. " Problem Solving: The Third Dimension in Mathemtics Teaching", Mathematics Teacher, Vol 81, pp.16-20.

Grouws, D.A., & Thomas, W.E., 1981. " Problem Solving: A Panoramic Approach", School Science and Mathematics, pp.307-315.

Hollander, S.K., 1978. " A Literature Review: Thought Processes Employed in the Solution of Verbal Arithmetic Problems", School Science and Mathematics, pp.327-334.

Hubbard, R., 1990. " Tertiary Mathematics without Lectures",Int. J. Math. Educ. Sci. Technol, Vol.21, No.4, pp.567-571.

Kameenui, J.E., Griffin, C.C., 1989. " The National Crisis in Verbal Problem Solving in Mathematics: A Proposal for Examinig the Role of Basal Mathematics", The Elementary School Journal, Vol.89, No.5, pp.575-592.

Krulik, S., 1977. "Problems, Problem Solving, and Strategy Games", Mathematics Teacher, Vol.70, pp.649-652.

Krulik, S. and Rudnick, J.A., 1985. " Developing Problem - Solving Skills", Mathematics Teacher.

Krulik, S., & Rudnick, J.A., 1981. " Suggestions for Teaching Problem Solving - A Baker's Dozen", School Science and Mathematics, pp.37-41.

Lawson, J.M., 1990. " The case for Instruction in the use of General Problem - Solving Strategies in Mathematics Teaching: A comment on Owen and Sweller", Journal for Research in Mathematics Education, Vol 21, No.5, pp.401-410.

Lewis, A.B., and Mayer, R.E., 1987. " Students' Miscomprehension of Relational Statements in Arithmetic Word Problems",Journal of Educational Psychology, Vol.79, No.4, pp.363-371.

Lochhead, J., & Others, 1985. " Representation of Mathematical Relationships in Four Countries: A Study of College Students' Mathematical Fluency", EXXON Education Foundation, New York, N.Y.

McCoy, P.L., 1990. " Literature Relating Critical Skills for Problem Solving in Mathematics and Computer Programming", School Science and Mathematics, Vol.90, No.1, pp.48-60.

McGinty, L.R. and Meyerson, N.L., 1980. " Problem Solving: Look Beyond the Right Answer", Mathematics Teacher, pp.501-503.

Mumme, J. and Weissglass, J., 1989. " The Role of Teacher in Implementing the Standards", Mathematics Teacher, Vol 82, No.7, pp.522-526.

Muth, D., K., 1984. " Solving Arithmetic Word Problems: Role of Reading and Computational Skills", Journal of educational Skills, Vol. 76, No.2,pp.205-210.

National Council of Supervisors of Mathematics, 1989. "Essential Mathematics for the Twenty - first Centruy:", Mathematics Teacher, pp.388-391.

Orton, T., 1986. "Introducing Calculus: an accumulation of teaching ideas", Int. J. Math. Educ. Sci. Technol., Vol.17. No.6.

Polya, G., 1957. How to Solve it, Princeton University Press

Ronald, G.D., 1986. "The Importance of Calculus in Core Mathematics", pp.250-251, 397.

Rosnick, P., 1982. " Students Conceptions of Semantically Laden Letters in Algebra. A Technical Report", Sponsoring Agency: National Science Foundation, Washington, D.C.

Saygi, M., 1990. " Assestment and analysis of prospective mathematics teachers problem solving skills for selected variables of math ability, reading comprehension and attitudes to ward mathematics", Doctor Thesis (METU).

Schoenfeld, H., A., 1985. Mathematical Problem Solving, Academic Press.

Silverman, A.R., 1985. Calculus with Analytic Geometry, Prentice - Hall, New Jersey.

Sowder, J.T., 1983. " Question Placement in Mathematical Word Problems", School Science and Mathematics, Vol.83, No.2, pp.107-111.

Steen, A.L., 1989. "Teaching Mathematics for Tomorrow's World", Educational Leadership.

Stiff, V.L., 1986. " Understanding Word Problems", Mathematics Teacher, Vol.79, No.3, pp.163-165, 215.

Stiff, V.L., 1988. "Problem Solving my Example", School Science and Mathematics, Vol.88, No.8, pp.666-675.

Thaeler, S.J., 1986. " A New Solution to an Old Problem - Solving Word Problems in Algebra", Mathematics Teacher, Vol.79, pp.682-689.

Thorndike, L.R., 1982. Applied Psychometrics, Houghton Mifflin Company Boston.

Troutman, P.A. and Lichtenberg, P.B., 1974. " Problem Solving in the General Mathematics Classrom", Mathematics Teacher, pp.590-597.

Truckson, E.B., 1983. " The Effects of Heuristic Teaching and Instruction on the Problem Solving Performance, Mathematics Achievement, and Attitudes of Junior College Arithmetic Students", Dissertation Abstracts International, Vol.43, No.11, pp.3532-A.

Ungson-Devito, M., 1984. " Development of a Self-Instructional Booklet for the Teaching of Problem Solving Strategies to Adult students", Dissertation Abstracts International, Vol.46, No.1, 1985, pp.95-A.

Webb, L.F., & Sherrill, J.M., 1974. " The effects of Differing Presentations of Mathematical Word Problems Upon the Achievement of Preservice Elementary Teachers" School Science and Mathematics, pp559-

Wooten, P.A., 1990. " The Effects of Problem Solving Strategy Instruction on fifth-graders' Problem Solving Achievement and Classroom Learning Environment", Dissertation Abstracts International", Vol.51, No.6, pp.1946-A.

APPENDICES

APPENDIX A

HANDOUT MATERIAL

MAXIMUM-MINIMUM THEOREM

Let f be continues on a closed, bounded interval [a,b]. Then f has a maximum and minimum value on [a,b].

Ex: The function $f(x) = x - x^3$ for $0 \leqslant x \leqslant 1$

1) $f(x) = x - x^3$ is continuous on the closed interval [0,1].

2) By the maximum-minimum thm, f has a maximum value on [0,1].

3) But the theorem does not tell us where in [0,1] the maximum value occurs, nor does it tell us how to find the maximum value.

THEOREM:

Let f be defined on [a,b]. If an extreme value of f on [a,b] occurs at a number c in (a,b) at which f has a derivative, then $f'(c) = 0$.

Not: Compute the values of f at all critical points in (a,b) and at the end points a and b. The largest of those values is the maximum value of f on [a,b];the smallest of those values is the minimum value of f on [a,b].

(Critical points of f if either $f'(c)=0$ or $f'(c)$ does not exist)

Ex: $f(x) = x - x^3$. Find the extreme values of f on [0,1], and determine at which numbers in [0,1] they occur.

1) It is continues on [0,1].

2) It is differentiable

3) Critical values of f are the values of x for which $f'(x) = 0$

4) $f'(x) = 1 - 3x^2$

5) $f'(x) = 0 \Longrightarrow x = (-1/3)\sqrt{3}$ or $x = (1/3)\sqrt{3}$

6) $x = (-1/3)\sqrt{3}$ is not in the interval [0,1].

7) $x = (1/3)\sqrt{3}$

GENERAL PROCEDURE FOR SOLVING MAX-MIN WORD PROBLEMS

Step 1: (a) read the problem carefully, (b) choose a letter for the quantity to be maximized or minimized, and (c) choose different or auxiliary variables for the other quantities appearing in the problem.

Step 2: express the quantity to be maximized or minimized in terms of the auxilary variables.

Step 3: choose one auxiliary variable, say x, to serve as master variable, and use the information given in the problem to express all the auxiliary variables in terms of x

Step 4: use the results of step (2) and (3) to express the given quantity to be maximized or minimized in terms of x alone.

Step 5: use the theories related to the max-min to find the desired max or min value.

Step 6: lable answers correctly, in terms of the units stated in the problem.

Step 7: reread the problem and see if your answer makes sense. This is called, "checking your answer".

Comprehension Guide for Question 1

Maximum Rectangular Region Area Problem

I. Check all items that correctly identify information contained in the problem and what is to be found.

—— 1. A landowner wants to use 2 miles of fencing.

—— 2. Fencing are used to enclose a rectangular region.

—— 3. Maximum area of the rectangular region is 2.

—— 4. the sum of side lengths of the rectangular region is 2.

What is to be found?

—— A. The perimeter of the rectangular region.

—— B. The area of the rectangular region.

—— C. The derivative of the area.

—— D. The maximum value of the area.

—— E. The side of the rectangular region.

A landowner wishes to use 2 miles of fencing to enclose a rectangular region of maximum area. What should the lengths of sides be?

II. Correctly identify operations or procedures needed to solve the problem.

—— 1. 2(width) + 2(length) = perimeter

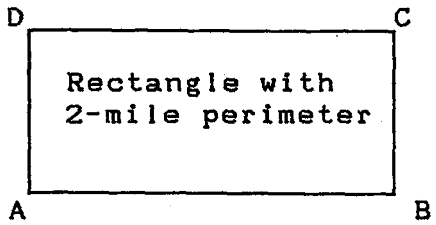
—— 2. Area = width * length

—— 3. width(1 - width) = area

—— 4. [width(1 - width)]′ = 0

Question 1: A landowner wishes to use 2 miles of fencing to enclose a rectangular region of maximum area. What should the lengths of sides be?

soln: Any rectangular region the landowner could enclose



1) x - length of the rectangle

2) y - width of the rectangle

3) The perimeter =

4) The boundaries of x and y

5) The area of the rectangle = A

6) The problem has now been reduced to find the maximum value of A on [0,1].

Comprehension Guide for Question 2

Metal Box Problem

I. Check all items that correctly identify information contained in the problem and what is to be found.

—— 1. A metal box(without top) is constructed from a square sheet.

—— 2. Sides of the cutting square are 10 inches.

—— 3. Sides of the square sheet of metal are 10 inches.

—— 4. Squares of the same size are cut from each corner of the sheet

What is to be found?

—— A. Side length of the square sheet of metal

—— B. Side length of the metal box

—— C. Volume of the metal box

—— D. Maximum value of the volume

A metal box(without top) is to be constructed from a square sheet of metal that is 10 inches on a side by first cutting square pieces of the same size from the corners of the sheet and then folding up the sides. Find the dimensions of the box with the largest volume that can be constructed.

II. Correctly identify the operations or procedures needed to solve the problem.

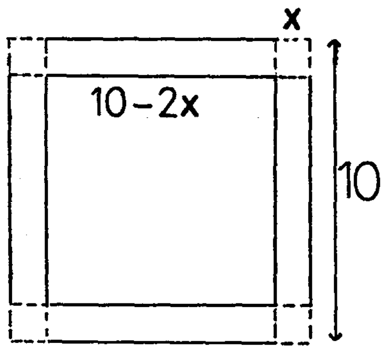
—— 1. 10 - 2(side length of the cutting square)

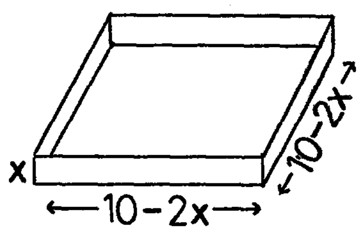
—— 2. Volume = high*(side length of the box)

—— 3. [high * (side length of the box)]' = 0

Question 2: A metal box(without top) is to be constructed from a square sheet of metal that is 10 inches on a side by first cutting square pieces of the same size from the corners of the sheet and then folding up the sides. Find the dimensions of the box with the largest volume that can be constructed.

Soln:





1) x - length (in inches) of each side of cutting square

2) Volume of box = V =

3) The boundaries of x

4) The problem reduces to finding the maximum value of V on [,]

To determine the critical points of V in (,) Comprehension Guide for Question 3

Manufacturing Tin Cans of Cylindirical Shape

I. Check all items that correctly identify information contained in the problem and what is to be found.

—— 1. A factory will produce tin cans

—— 2. The shape of the tin cans is cylindirical

—— 3. The amount of the material for each can is 150 π cm^2

What is to be found?

—— A. The volume of the cylindirical tin cans

—— B. The area of the cylindirical tin cans

—— C. The maximum value of the volume

—— D. The high of the cylindirical tin cans

—— E. The radius of the cylindirical tin cans

A factory will manufacture tin cans of cylindirical shape and it wants to use 150π cm^2 of material for each can. What would the dimensions of the can be so that its volume is largest?

II. Correctly identify the operations or procedures needed to solve the problem.

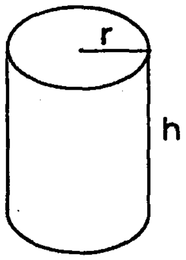
—— 1. Volume of tin cans = π $(radius)^2(high)$

—— 2. $150 = 2\pi(radius)(high)$

—— 3. Derivative of the volume

Question 3: A factory will manufacture tin cans of cylindirical shape and it wants to use 150π cm^2 of metal for each can. What would the dimensions of the can be so that its volume is largest?

soln:



1) r - radius of circle

2) h - high of the cylinder

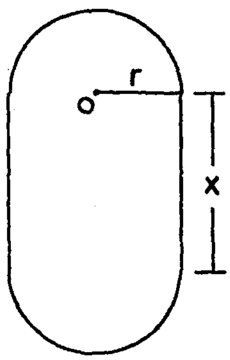
3) Volume of the cylinder = V =

4) Area of cylinder = A =

5) The problem reduces to finding the maximum value of V.

Question 4: An outdoor track is to be created in the shape shown in the figure and is to have a perimeter of 440 yards. Find the dimensions for the track that maximize the area of the rectangular portion of the field enclosed by the track.

Soln:



1) r - radius of two semicircles ($r>0$)

2) x - lenght of the rectangular portion

3) 2r - width of the rectangle

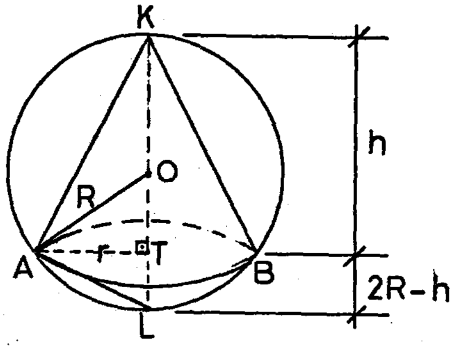
4) Area of the rectangular region = A =

5) Circumference of the field =

6) To find the maximum value of A, we first take the derivative of A.

Question 5: Find the high of the cone of maximum volume which can be inscribed in a sphere of radius R.

Soln:



1) R - radius of the sphere

h - high of the cone

2) Volume of the cone = V =

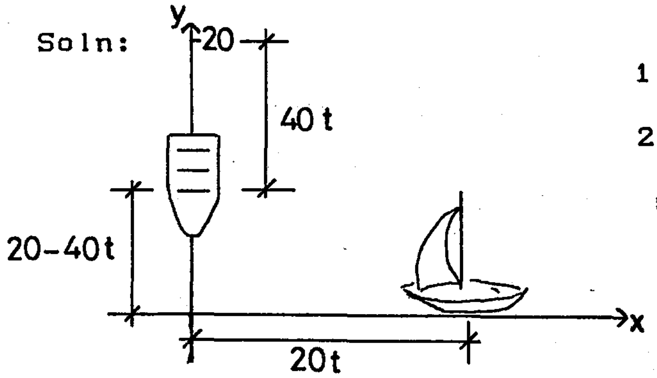
3) AKL perpendicular triangle

$|AT|^2 = |KT| * |LT|$

4) To find the maximum value of V

Question 6: At noon a sailboat is 20 kilometers south of a freighter. The sailboat is travelling east at 20 kilometers per hour, and the freighter is travelling south at 40 kilometers per hour. If visibility is 10 kilometers, could the people on the two ships ever see each other?

Soln:



1) time = t =0 at noon

2) any time t≥0

sailboat travelled 20t

freighter travelled 40t

3) At time t , the distance D (between the two ships)

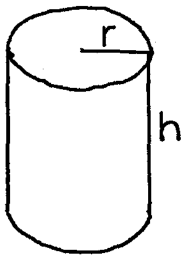
D =

4) To determine whether D ≤ 10 for any such value of t.

To find minimum value of D

Question 7: A manufacturer packages his product in 162 cm^3 cans that are right circular cylinders. If the material for the top costs five times as much per square cm^3 that used for the bottom and side, find the dimensions of the can if the cost of the material is to be minimum.

Soln:



1) r - radius of the can

h - height of the can

k - cost of the material for side and bottom

5k - cost of the top

C - total cost of material

2) C = total cost of material

C =

3) Volume of the can = V =

4) Find the minimum value of the cost

APPENDIX A.1

LESSON PLAN	COMMENTS
TOPIC: Maximum-Minimum Word Problems (Optimization unit).	For the university Math 153 Calculus I course.
APPERCEPTIVE BASIS (Previously learned knowledge) Review of precalculus contents, limit, continuity, and derivative.	These are the primary topics which the students need to be able to handle this topic
AIM: To review the maximum-minimum theorem and to apply the maximum-minimum theorem to solve some word problems.	
DO-NOW EXERCISES	
The function $f(x) = x - x^3$ for $0 \le x \le 1$ is given. Explain the properties of this function.	
1. $f(x) = x - x^3$ is continouos on the closed interval $[0,1]$.	
2. By the maximum - minimum theorem, f has a maximum value on $[0,1]$.	
3. But the theorem does not tell us where in $[0,1]$ the maximum value occurs, nor does it tell us how to find the maximum value.	This set of exercises will be distributed on spirit duplicator sheets

Ex: Let $f(x) = x - x^3$. Find the extreme values of f on [0,1], and determine at which numbers in [0,1] they occur.

1. It is continouos on [0,1]
2. It is differentiable
3. Critical values of f are the values of x for which $f'(x) = 0$
4. $f'(x) = 1 - 3x^2$
5. $f'(x) = 0 \Longrightarrow x = -\sqrt{3}/3$ or $x = \sqrt{3}/3$
6. $x = -\sqrt{3}/3$ is not in the interval [0,1]
7. An extreme value of f on [0,1] can occur only at one of the points 0 and 1 or at the critical point $\sqrt{3}/3$ in (0,1).
8. $f(x) = 0$, $f(1) = 0$, $f(\sqrt{3}/3) = 2\sqrt{3}/9$
9. Consequently the minimum value of f on [0,1] is 0, and it occurs at 0 and 1.
10. Maximum value of f on [0,1] is $2\sqrt{3}/9$, and it occurs at $\sqrt{3}/3$.

Notice also that these exercises review the theorems related to the max-min principles while at the same time permit the students to learn the properties of the theorems.

DEVELOPMENT AND METHODS

1. Give the pretest before the instruction

Pretest will elicit the prerequisite knowledge of the students.

2. Prepare the Teaching\learning material

This material will be prepared by the reseacher and given to the students before the instruction to increase the discussion time.

3. Prepare the comprehension guide for some problems.	This will improve the reading ability.
4. Indicate to the class the significance of the do-now exercises. That is, they have reinforced how can maximum- minimum values be found using theorems.	This must be carefully presented so as to reap the full impact intended from these exercises.
5. Discuss the general procedures for solving the word problem.	This is intended to teach the general procedures for solving the word problems.
6. Discuss the solutions of the given problems.	Discussion would be quite helpful here.
7. Give the posttest after the instruction.	Posttest is intended to see the problem solving skills of the students and the effect of instruction
DRILL: $f(x) = x^{2/3}$. Find the extreme values of f on [-1,1] and determine at which points they occur.	This is the simple exercises which only apply the theorems related to the max-min.
MEDIAL SUMMARY: 1. State the max-min theorem. 2. For what can we use the max-min theorem. 3. Can the max-min theorem be applied to any given function? 4. How can we solve the word or optimization problems?	These questions should elicit the key points of the previous part of lesson and thus serve as a summary to this point in the lesson.

APPLICATION AND DRILL

Students will be assigned to do their work by discussing.

1.A landowner wishes to use 2 miles of fencing to enclose a rectangular region of maximum area. What should be the lenghts of sides be?

2. A metal box (without top) is to be constructed from a square sheet of metal. Metal is 10 inches on a side of first cutting square pieces of the same size from the corners of the sheet and then folding up the sides. Find the dimensions of the box with the largest volume that can be constructed.

3. A factory will manufacture tin cans of cylindirical shape and it want to use 150π cm^2 of metal for each can. What would the dimensions of the can be so that its volume is largest?

4. An outdoor track is to be created in the shape shown in the figure and is to have a perimeter of 440 yards. Find the dimensions for the track that maximize that the area of the rectangular portion of the field enclosed by the track.

5. Find the high of the cone of maximum volume which can be inscribed in a sphere of radius R.

6. At noon a sailboat is kilometers south of a freighter. The sailboat is travelling east at 20 kilometers per hour, and the freighter is travelling

south at 40 kilometers per hour. If visibility is 10, could the people on the two ships ever see each other?

7. A manufacturer packages his product in 162 cm^3 cans that are right circular cylinders. If the material for the top costs five times as much per square cm^3 that used for the bottom and side, find the dimensions of the can if the cost of the material is to be minimum.

FINAL SUMMARY

This summary is intented to review the critical points of the lesson.

1. State in words the max-min theorems.

2. State in words the general procedure for solving the word problems.

APPENDIX B

KITAPÇIK NO:.........

GENEL BAŞARI TESTİ

GENEL AÇIKLAMA

Sevgili Öğrenciler,

Bu testin amacı, sizin Math 153 "Genel Matematik-Calculus- " dersini öğrenmeye başlamadan önce bu dersle ilgili ön bilgileriniz hakkında bilgi edinmektir. Vereceğiniz yanıtlar, Math 153 dersinin tasarlanmasına ışık tutacaktır. Bu nedenle yanıtlarınızı verirken dikkat ediniz.

Bu kitapçıkta çeşitli matematik konularını içeren çoktan seçmeli 50 soru vardır. Tüm soruları yanıtlamaya çalışırken aşağıdaki notları göz önünde bulundurunuz:

* Soruları dikkatlice okuduktan sonra 5 yanıttan size en doğru görüneni **cevap kağıdının** üzerine çarpı işareti koyarak gösteriniz.

* Soru kitapçığı üzerine herhangi birşey yazmayınız.

* Soru kitapçığının üzerinde bulunan kod numarasını cevap kağıdına yazmayı unutmayınız.

* Sınav süresi yaklaşık 90 dakikadır.

İlgileriniz için teşekkür ederiz.

BAŞARILAR

1) $1 + \cfrac{1 + \cfrac{1 + \cfrac{1}{2}}{\cfrac{1}{2}}}{\cfrac{1}{2}} =$ işleminin sonucu nedir?

A) 9 B) 3 C) $\frac{1}{2}$ D) $\frac{1}{4}$ E) $\frac{1}{8}$

2) $\frac{1}{x} + \frac{2}{x-1} - \frac{3x+1}{x^2-1}$ işleminin kısaltılmış biçimi aşağıdakilerden hangisidir ?

A) $-\frac{2}{x^2-1}$ B) $\frac{1}{x-1}$ C) $\frac{2}{x+1}$ D) $\frac{1}{x^2-1}$ E) $\frac{1}{x(x+1)}$

3) $\frac{3x^3y - 18x^2y + 27xy}{6x^2y^2 - 54y^2}$ ifadesinin kısaltılmış biçimi aşağıdakilerden hangisidir ?

A) $\frac{2y(x+3)}{x-3}$ B) $\frac{x(x+3)}{y(x-3)}$ C) $\frac{y(x+3)}{2x(x-3)}$

D) $\frac{x(x-3)}{2y(x+3)}$ E) $\frac{x(x-3)}{2y}$

4) $\frac{3^4a^{5-x}}{3^2a^{1-2x}}$ ifadesinin kısaltılmış biçimi aşağıdakilerden hangisidir?

A) $9a^{x+4}$ B) $6a^{x+4}$ C) $6a^{6-3x}$ D) $9a^{6-x}$ E) $2a^{6-3x}$

5) $\left(\frac{a^x}{a^y}\right)^{x-y} \left(\frac{a^y}{a^x}\right)^{x-y}$ işleminin sonucu nedir?

A) 1 B) a C) a^x D) a^y E) a^{x-y}

6) Bir manavda iki boy elma vardır. Küçük boy elmaların tanesi 120 gr, büyük boy elmaların tanesi 200 gr dır. Bu manavdan tam bir kilo elma alan bir kişi en çok kaç tane elma almış olabilir ?

A) 8 B) 7 C) 6 D) 5 E) 4

7) Kenar uzunlukları 2 nin katı olan, eşkenar üçgen biçimindeki bir bahçenin çevresine, bir köşegenden başlayarak 2 m ara ile ağaç dikiliyor. Dikilen toplam ağaç sayısı 21 olduğuna göre, bahçenin bir kenarı kaç m dir ?

A) 18 B) 16 C) 14 D) 12 E) 10

8) 

Hızı saatte a km olan bir hareketli A kentinden, hızı saatte b km olan diğer bir hareketli B kentinden aynı anda birbirlerine doğru hareket ederlerse 2 saat sonra karşılaşıyorlar. İki hareketli aynı koşullarla aynı anda, aynı yönde hareket etselerdi kaç saat sonra A kentinden hareket eden diğerine yetişecekti ? (a>b).

A) $\frac{2(a-b)}{a+b}$ B) $\frac{2(a+b)}{a-b}$ C) $\frac{a+b}{2(a-b)}$ D) $\frac{a-b}{a+b}$ E) $\frac{a+b}{a-b}$

9) Bir babanın yaşı, iki çocuğun yaşları toplamından 33 büyüktür. 3 yıl sonra babanın yaşı, çocukların yaşları toplamının 2 katı olacağına göre baba bugün kaç yaşındadır ?

A) 52 B) 54 C) 55 D) 56 E) 57

10) Bir paranın önce $\frac{1}{4}$ ünü, sonra kalanın $\frac{1}{3}$ ünü harcayınca geriye 8100 TL kaldığına göre, bu paranın tümü kaç liradır ?

A) 12150 B) 14600 C) 16200 D)18300 E)20550

11) $\dfrac{1}{x} + \dfrac{x}{x+1} + \dfrac{x-1}{x} = \dfrac{4}{3}$ eşitliğini sağlayan x değeri aşağıdakilerden hangisidir ?

A) $\dfrac{1}{4}$ B) $\dfrac{1}{3}$ C) $\dfrac{1}{2}$ D) $\dfrac{3}{4}$ E) $\dfrac{2}{3}$

12) $9 < | 2x - 7 | < 13$ eşitsizliğinin çözüm kümesindeki tamsayıların toplamı kaçtır ?

A) 14 B) 13 C) 12 D) 10 E) 7

13) $f(ab) = f(a) + f(b)$ olduğuna göre $f(1)$ in değeri hangisidir ?

A) 0 B) 1 C) a D) b E) ab

14) $(f \circ g)(x) = \dfrac{x}{x^2+1}$ ve $g(x) = x + 1$ olduğuna göre $f(x)$ fonksiyonu aşağıdakilerden hangisidir ?

A) $\dfrac{x+1}{x^2+2x+2}$ B) $\dfrac{x-1}{x^2-2x+2}$ C) $\dfrac{x^2+1}{x+1}$ D) $\dfrac{x^2+1}{x}$ E) $\dfrac{x}{x+1}$

15) $f(x) = 2^{3x-1}$ olduğuna göre $f(2x)$ in $f(x)$ cinsinden ifadesi aşağıdakilerden hangisidir ?

A) $3f(x)$ B) $3[f(x)]^2$ C) $2f(x)$ D) $2[f(x)]^2$ E) $2[f(x)]^3$

16) $(a + b - c)^2 - (a - b + c)^2$ ifadesinin en sade şekli aşağıdakilerden hangisidir ?

A) $2a(c - a)$ B) $4b(c - a)$ C) $4c(a - b)$

D) $4a(b - c)$ E) $2c(a - b)$

17) $ax^2 - 6x - 9 = 0$ denkleminin kökleri $x_1 = x_2$ ise, a nın değeri nedir ?

A) 1 B) 2 C) -1 D) -2 E) -3

18) $|x^2 + 1| \leq 3$ ün çözüm kümesi aşağıdakilerden hangisidir?

A) R B) $R-[-2,2]$ C) $[-2,2]$ D) $R-[-\sqrt{2},\sqrt{2}]$ E) $[-\sqrt{2},\sqrt{2}]$

19) Denklemi $y = x^2 - ax + 1$ olan parabol veriliyor. a nın hangi pozitif değeri için, başlangıç noktasından parabole çizilen teğetler birbirine dik olur?

A) 4 B) $\sqrt{3}$ C) 3 D) $\sqrt{2}$ E) 2

20) $\dfrac{(2 - x)(x + 3)}{x} > 0$ eşitsizliği aşağıdaki aralıkların hangisinde sağlanır ?

A) $-\infty < x < -3$ B) $2 < x < 3$ C) $-3 < x < 0$

D) $-3 < x < -2$ E) $3 < x < \infty$

21) Aşağıdakilerden hangisi sin 40^oye eşittir ?

A) $\sin 220^o$ B) $\cos 130^o$ C) $\cos(-50^o)$ D) $\sin(-40^o)$ E) $\sin 50^o$

22) $0 < x < \dfrac{\pi}{2}$, $\tan x = \dfrac{4}{3}$ olduğuna göre,

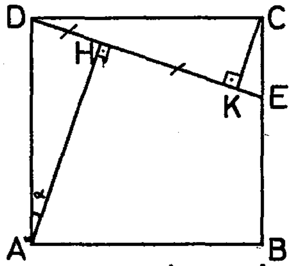
$\dfrac{\sin^3 x - \cos^3 x}{1 + \dfrac{1}{2}\sin 2x}$ ifadesinin değeri kaçtır?

A) $\dfrac{1}{5}$ B) $\dfrac{2}{5}$ C) $\dfrac{3}{5}$ D) $\dfrac{4}{5}$ E) 1

23) $\cos^2 \dfrac{\pi}{8} + \sin^2 \dfrac{3\pi}{8} = \dfrac{a}{2}$ olduğuna göre, a nın değeri aşağıdakilerden hangisidir ?

A) $\dfrac{1}{2}$ B) $\dfrac{\sqrt{2}}{2}$ C) $2 + \sqrt{2}$ D) 1 E) $\sqrt{2} - 1$

24) Aşağıdaki şekilde ABCD bir kare olduğuna göre



$m(\widehat{CKE}) = 90^o$

$m(\widehat{DHA}) = 90^o$

$m(\widehat{DAH}) = \alpha^o$

$|DH| = |HK|$

tanα nın değeri kaçtır ?

A) $\dfrac{\sqrt{2}}{2}$ B) $\dfrac{\sqrt{3}}{2}$ C) $\dfrac{3}{4}$ D) $\dfrac{2}{3}$ E) $\dfrac{1}{2}$

25) $\dfrac{1}{\cos^2 x} + \dfrac{1}{\sin^2 x} = 8$ denkleminin dar açı olan çözümü nedir ?

A) $\dfrac{\pi}{8}$ B) $\dfrac{\pi}{6}$ C) $\dfrac{\pi}{5}$ D) $\dfrac{\pi}{4}$ E) $\dfrac{\pi}{3}$

26) $\log_3 5 = a$ olduğuna göre

$\log_5 15$ in değeri nedir ?

A) $\dfrac{1}{a-1}$ B) $\dfrac{a}{a-1}$ C) $\dfrac{a-1}{a}$ D) $\dfrac{a+1}{a}$ E) $\dfrac{a}{a+1}$

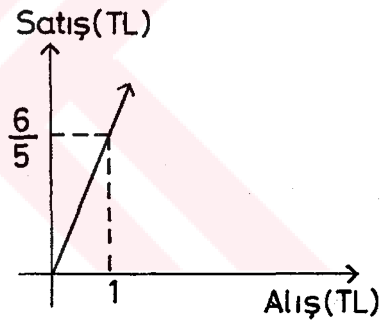
27) $\ln(xy) = 2a$

$\ln\left(\dfrac{x}{y}\right) = 2b$

olduğuna göre x in pozitif değeri nedir ?

A) e^{a+b} B) e^{b-a} C) e^{a-b} D) $e^{-(a+b)}$ E) e^{ab}

28) Yandaki doğrusal grafik bir malın maliyeti ile satış fiyatı arasındaki bağıntıyı göstermektedir. A(1 , 6/5) noktası bu doğru üzerinde olduğuna göre, 18000 TL ye satılan bir maldan kaç TL kar edilir ?

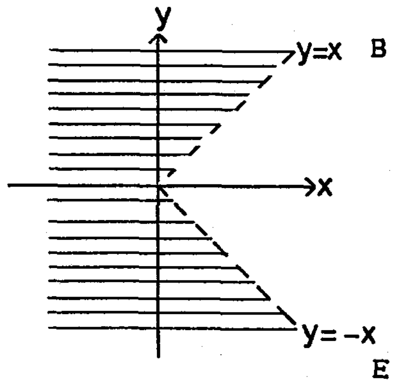


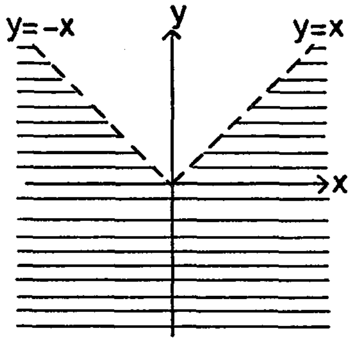
A) 1000 B) 1500 C) 2000 D) 3000 E) 3600

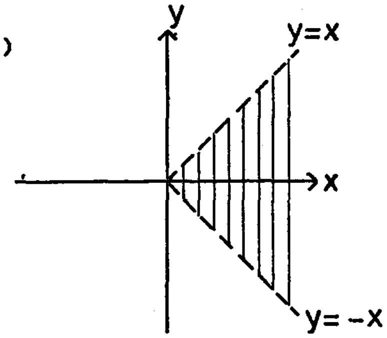
29) $2x + 3y - 4 = 0$ ve $x - 2y + 6 = 0$ doğrularının kesim noktasından geçen ve x- eksenine paralel olan doğrunun denklemi hangisidir ?

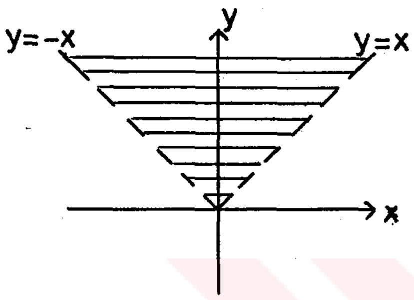
A) $y = \dfrac{16}{7}$ B) $y = \dfrac{8}{7}$ C) $y = -2$ D) $y = -1$ E) $y = 0$

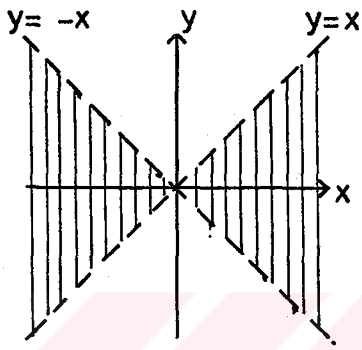
30) $x - |y| < 0$ bağıntısını sağlayan düzlemsel taralı bölge aşağıdakilerden hangisidir ?

A)


B)


C)


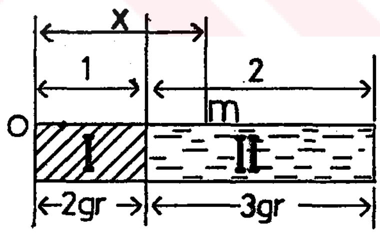
D)


E)


31) Şekildeki çubuk, aynı kalınlıkta ve homojen yapıda I,II parçalarından oluşmaktadır. Bu parçaların uzunlukları sırayla 1 ve 2 birim, ağırlıkları ise 2 ve 3 gr dır. Bu çubukla ilgili olarak,
f: x—— "x uzunluğunda OM parçasının ağırlığı" biçiminde bir fonksiyon tanımlanıyor. Buna göre, f(x) in [2,3] aralığındaki ifadesi aşağıdakilerden hangisidir ?

A) $\dfrac{3x + 2}{2}$ B) $\dfrac{3x - 1}{2}$ C) $\dfrac{3x - 4}{3}$

D) $\dfrac{3x - 2}{3}$ E) $\dfrac{3x + 1}{2}$



32) $y = \sqrt{3 - |x + 4|}$ fonksiyonunun tanım aralığı aşağıdakilerden hangisidir ?

A) $-3 \le x \le 4$ B) $-7 \le x \le -1$ C) $3 \le x \le 4$

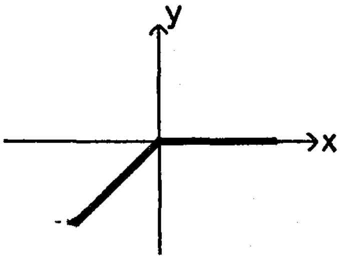
D) $-4 \le x \le -3$ E) $1 \le x \le 7$

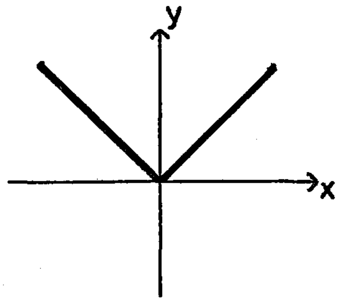
33) $f: R \longrightarrow R$, $f(x) = |x| - [\![x]\!]$ fonksiyonu veriliyor. $[-3,-2)$ aralığında fonksiyon aşağıdakilerden hangisi ile ifade edilebilir ?

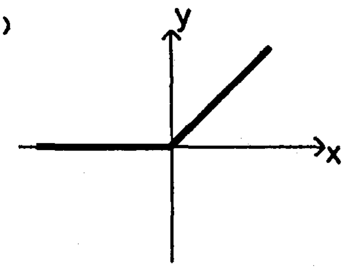
A) $f(x) = -x-2$ B) $f(x) = x+2$ C) $f(x) = -x+3$

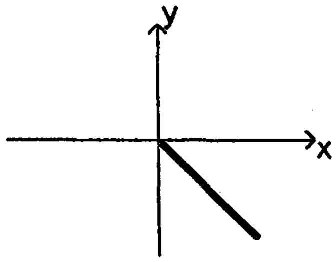
D) $f(x) = x+3$ E) $f(x) = x-2$

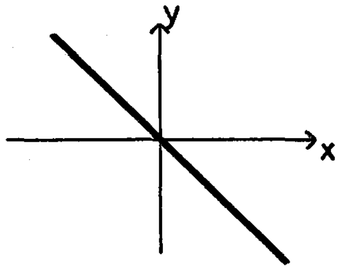
34) $2y = x + |x|$ fonksiyonunun grafigi aşağıdakilerden hangisidir ?

A)


B)


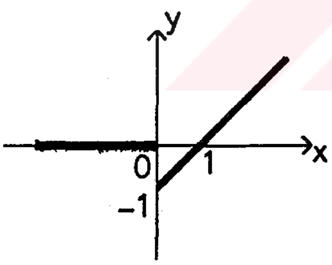
C)


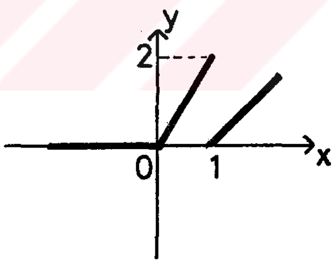
D)


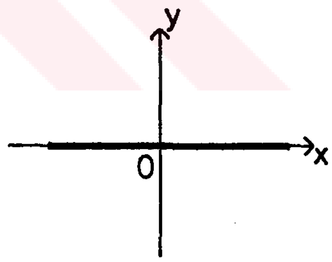
E)


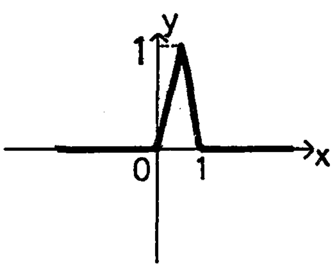
35) $f(x) = \begin{cases} -1 & x < 0 \\ x - 1 & x \geq 0 \end{cases}$ $\quad g(x) = \begin{cases} 1 & x < 0 \\ x + 1 & 0 \leq x \leq 1 \\ 0 & 1 \leq x \end{cases}$

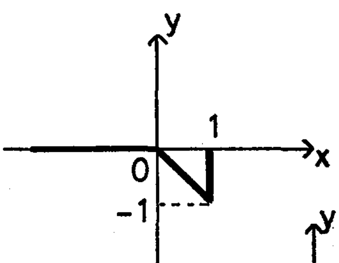
olduğuna göre, $(f + g)(x)$ in grafiği aşağıdakilerden hangisidir ?

A)


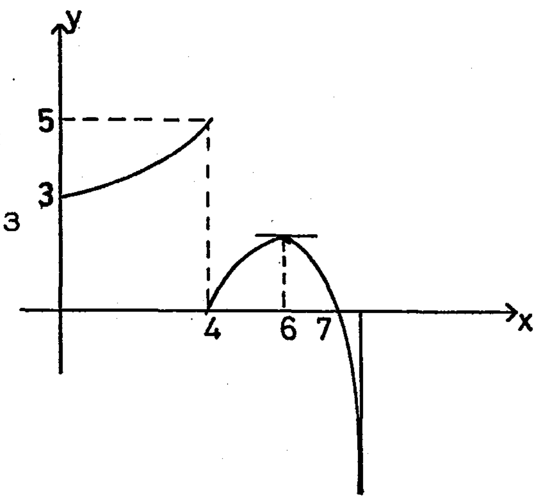
B)


C)


D)


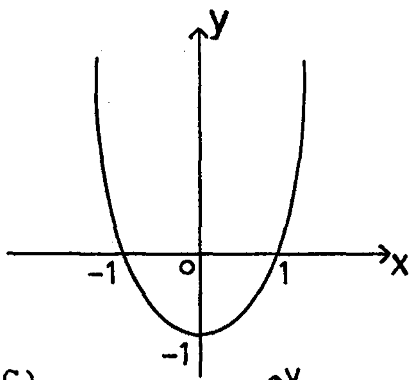
E)


36) Bir $y = f(x)$ fonksiyonunun grafigi yanda verilmiştir. $f[f(x)]=3$ olduğuna göre x in degeri nedir ?



A) 7 B) 6 C) 5 D) 4 E) 3

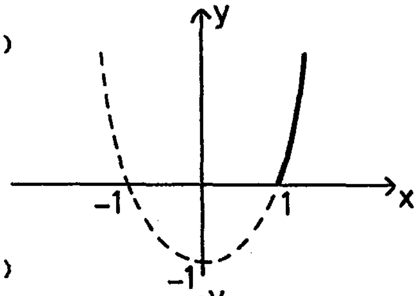
37) Şekildeki eğri f(x) fonksiyonunun grafiği olduğuna göre,



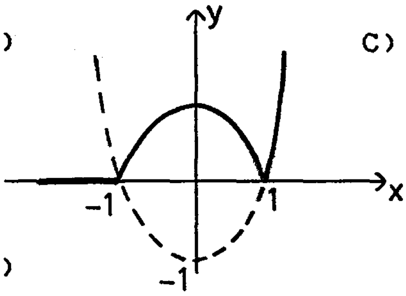
$$y = \frac{1}{2}\left(|f(x)| + f(x)\right) \text{ in}$$

grafiği aşağıdakilerden hangisidir?

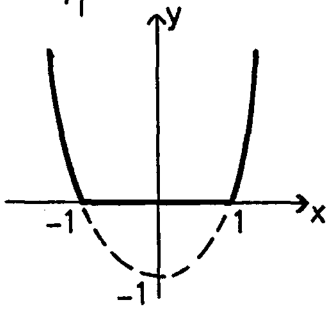
A)



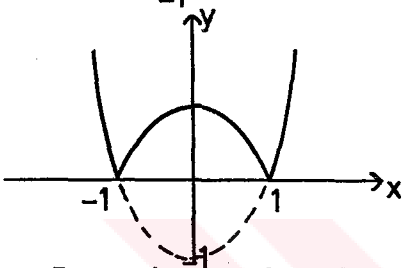
B)



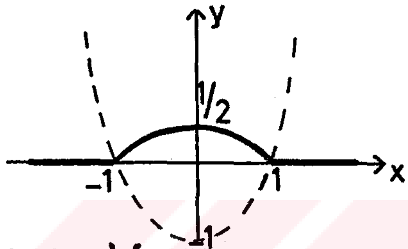
C)



D)



E)

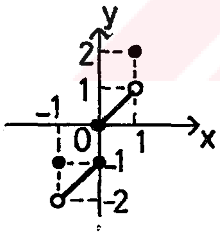


38) R reel sayılar kümesinde $\forall a \in Z$ için aşağıdaki biçimde bir fonksiyon tanımlanıyor.

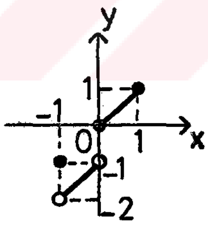
$M: x \longrightarrow M(x) = x - a \quad (a \leq x < a+1)$

$f(x) = 2x - M(x)$ in $[-1,1]$ kapalı aralığındaki grafiği aşağıdakilerden hangisidir ?

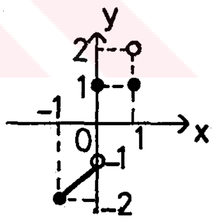
A)



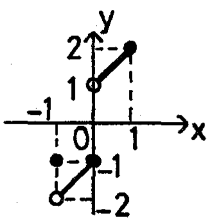
B)



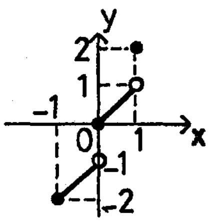
C)



D)



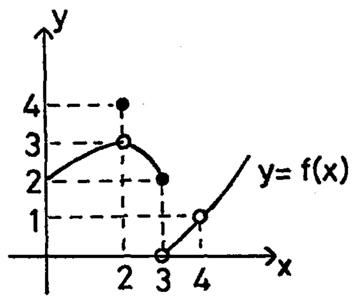
E)



39) $\lim\limits_{x \to 1} \dfrac{x\cos(\pi x) + 1}{x-1}$ değeri nedir ?

A) 1 B) $\frac{1}{2}$ C) 0 D) $-\frac{1}{2}$ E) -1

40) f, grafiği yanda verilen bir fonksiyondur. Bu fonksiyonunun x in 2, 3, 4 değerlerinden bazıları için var olan limitleri toplamı kaçtır ?



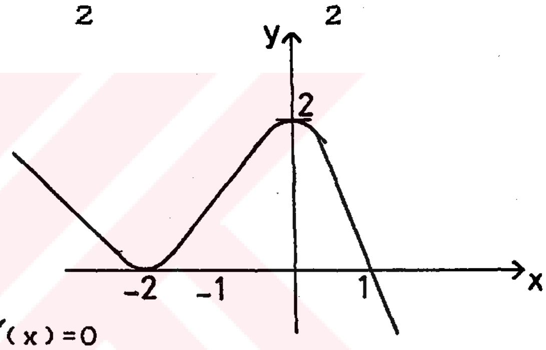
A) 4 B) 5 C) 6 D) 7 E) 8

41)
$$f(x) = \begin{cases} x + 1 & x \le 1 \\ 3 - ax^2 & 1 < x \end{cases}$$

biçiminde tanımlanan fonksiyonunun sürekli olması için a nın değeri ne olmalıdır ?

A) 2 B) 1 C) 0 D) $\frac{1}{2}$ E) $\frac{3}{2}$

42) Yandaki şekil 3. dereceden bir f(x) polinomunun grafiği olduğuna göre aşağıdakilerden hangisi yanlıştır ?

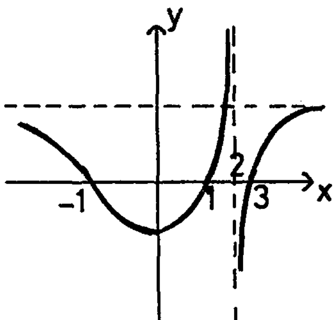


A) x=-2 için f(x)=0 B) x=-2 için f′(x)=0

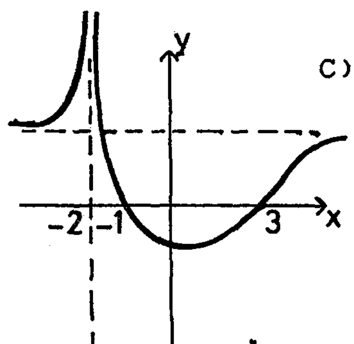
C) x=0 için f(x)=2 D) x=1 için f(x)=0 E) x=-1 için f′(x)<0

43) $y = \dfrac{(x+3)(x-1)}{(x-2)^2}$ fonksiyonunun grafiği aşağıdakilerden hangisi olabilir ?

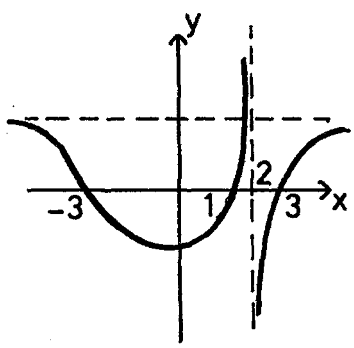
A)



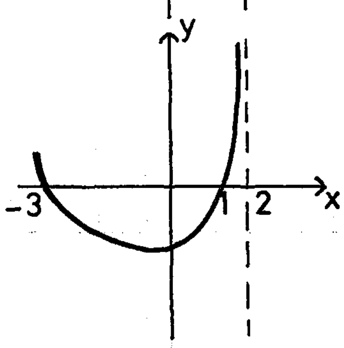
B)



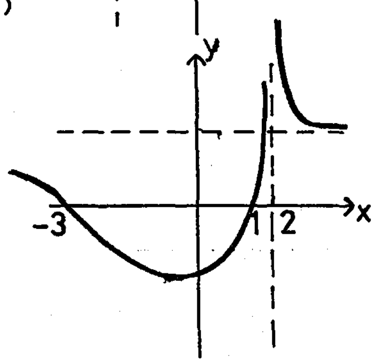
C)



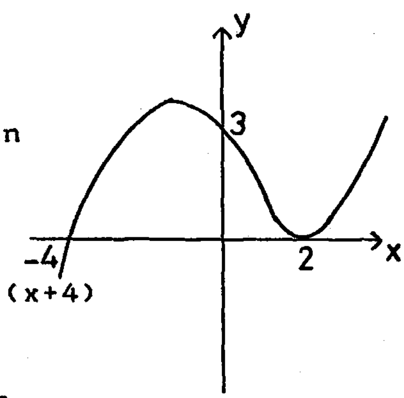
D)



E)



44) Yandaki eğri aşağıdaki fonksiyonlardan hangisinin grafiği olabilir ?



A) $y = 3\,(x-2)^2\,(x+4)$ B) $y = \dfrac{1}{16}\,(x-2)^2\,(x+4)$

C) $y = \dfrac{4}{3}\,(x+2)^2\,(x-4)$ D) $y = \dfrac{3}{4}\,(x+2)^2\,(x-4)$

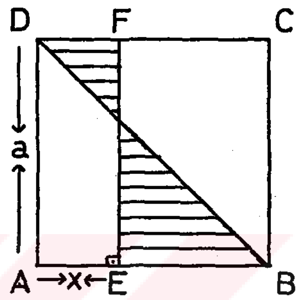
E) $y = \dfrac{3}{16}\,(x-2)^2\,(x+4)$

45) Yandaki şekilde ABCD bir karedir.

$EF \perp AB$

$|AE| = x$

$|AD| = a$



olduğuna göre, taralı alanların toplamının ifadesi aşağıdakilerden hangisidir?

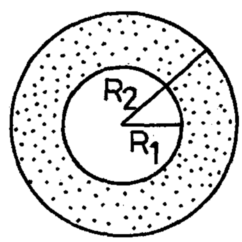
A) $x^2 + ax + a^2$ B) $2x^2 - ax + \dfrac{a^2}{2}$ C) $x^2 + 2ax + \dfrac{a^2}{4}$

D) $2x^2 + 2ax + a^2$ E) $x^2 - ax + \dfrac{a^2}{2}$

46) Yandaki şekilde $R_1 + R_2 = 6$ cm

ve $R_2 - R_1 = k$ cm olduğuna göre,

iki çember arasında kalan halkanın alanı kaç cm² dir ?

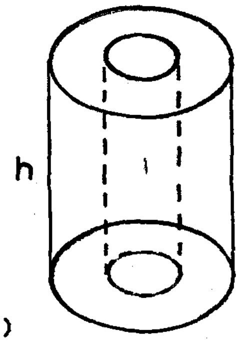


A) $3\pi k$ B) $4\pi k$ C) $6\pi k$ D) $8\pi k$ E) $9\pi k$

47) Boyu eninin iki katı uzunluğunda olan dikdörtgen şeklindeki bir kartonun tümü kullanılarak 16 cm^3 hacminde, kare prizma şeklinde kapaksız bir kutu yapılıyor. Kare prizmanın taban kenarı, verilen kartonun enine eşit olduğuna göre kullanılan kartonun alanı kaç cm² dir ?

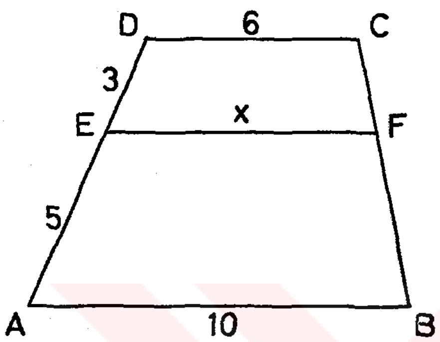
A) 128 B) 96 C) 64 D) 32 E) 16

48) İç içe geçirilmiş ve yükseklikleri eşit, dik silindir biçimindeki iki kaptan dıştakinin çapı içtekinin çapının iki katıdır. İçteki kap ağzına kadar su ile dolu iken tabanına çok yakın bir delik açılırsa, ikisi arasındaki boşlukta su hangi yüksekliğe çıkar?
(içteki kabın kalınlığı önemsenmiyecek)



A) $\frac{h}{2}$ B) $\frac{h}{4}$ C) $\frac{h}{3}$ D) $\frac{2h}{3}$ E) $\frac{3h}{4}$

49)



ABCD bir yamuk EF//AB

$|AB| = 10$ cm

$|AE| = 5$ cm

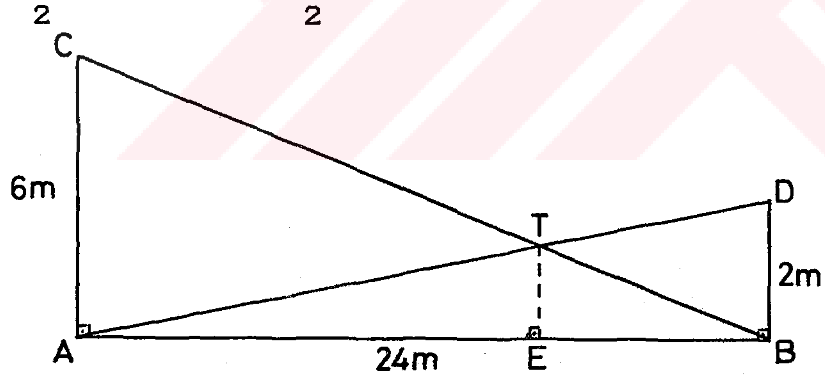
$|DC| = 6$ cm

$|ED| = 3$ cm

Yukarıda verilenlere göre $x = |EF|$ kaç cm dir?

A) $\frac{17}{2}$ B) $\frac{15}{2}$ C) 7 D) 8 E) 9

50)



Yukarıdaki şekilde

AC//TE//DB $\perp$ AB

$|AC| = 6$ m

$|DB| = 2$ m

$|AB| = 24$ m olduğuna göre, $|EB|$ kaç m dir ?

A) 4 B) 6 C) 8 D) 9 E) 12

SURNAME :
NAME :

KITAPÇIK NO:.........

CEVAPLAR

	A	B	C	D	E		A	B	C	D	E
(1)	( )	( )	( )	( )	( )	(26)	( )	( )	( )	( )	( )
(2)	( )	( )	( )	( )	( )	(27)	( )	( )	( )	( )	( )
(3)	( )	( )	( )	( )	( )	(28)	( )	( )	( )	( )	( )
(4)	( )	( )	( )	( )	( )	(29)	( )	( )	( )	( )	( )
(5)	( )	( )	( )	( )	( )	(30)	( )	( )	( )	( )	( )
(6)	( )	( )	( )	( )	( )	(31)	( )	( )	( )	( )	( )
(7)	( )	( )	( )	( )	( )	(32)	( )	( )	( )	( )	( )
(8)	( )	( )	( )	( )	( )	(33)	( )	( )	( )	( )	( )
(9)	( )	( )	( )	( )	( )	(34)	( )	( )	( )	( )	( )
(10)	( )	( )	( )	( )	( )	(35)	( )	( )	( )	( )	( )
(11)	( )	( )	( )	( )	( )	(36)	( )	( )	( )	( )	( )
(12)	( )	( )	( )	( )	( )	(37)	( )	( )	( )	( )	( )
(13)	( )	( )	( )	( )	( )	(38)	( )	( )	( )	( )	( )
(14)	( )	( )	( )	( )	( )	(39)	( )	( )	( )	( )	( )
(15)	( )	( )	( )	( )	( )	(40)	( )	( )	( )	( )	( )
(16)	( )	( )	( )	( )	( )	(41)	( )	( )	( )	( )	( )
(17)	( )	( )	( )	( )	( )	(42)	( )	( )	( )	( )	( )
(18)	( )	( )	( )	( )	( )	(43)	( )	( )	( )	( )	( )
(19)	( )	( )	( )	( )	( )	(44)	( )	( )	( )	( )	( )
(20)	( )	( )	( )	( )	( )	(45)	( )	( )	( )	( )	( )
(21)	( )	( )	( )	( )	( )	(46)	( )	( )	( )	( )	( )
(22)	( )	( )	( )	( )	( )	(47)	( )	( )	( )	( )	( )
(23)	( )	( )	( )	( )	( )	(48)	( )	( )	( )	( )	( )
(24)	( )	( )	( )	( )	( )	(49)	( )	( )	( )	( )	( )
(25)	( )	( )	( )	( )	( )	(50)	( )	( )	( )	( )	( )

APPENDIX C

APPENDIX C.1

ÖN TEST

GENEL AÇIKLAMA

Sevgili öğrenciler,

Bu testin amacı, sizin Math 153 " Genel Matematik - Calculus- " dersinde türevin uygulaması konusundaki ön bilgilerinizi öğrenmektir. Vereceğiniz yanıtlar, konunun kapsam ve öğretim yönteminin tasarlanmasında bize ışık tutacaktır. Bu nedenle, yanıtlarınıza dikkat ediniz.

Bu soru kağıdında türevin uygulaması konusuyla ilgili 3 soru vardır. Bu soruları yanıtlamaya çalışırken aşağıdakileri notları göz önünde bulundurunuz:

* Her soruyu dikkatlice okuyunuz.

* Açıklamalarınızı ve cevaplarınızı ekdeki boş kağıtlara ayrıntılı bir şekilde yazınız.

* Soruda istenenleri ve değişkenleri kendi cümlelerinizle yazınız.

* Çözümlerinizde izlediğiniz yolu, ara adımları ile gösteriniz.

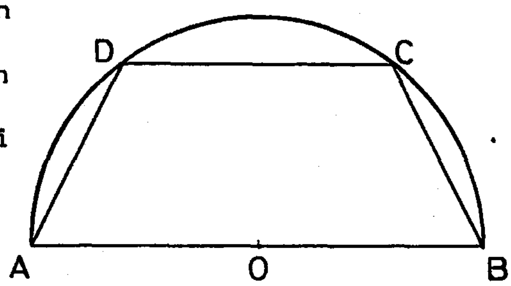
* Yazdıklarınızı silmeyiniz. Yanlış yaptığınızı düşünürseniz üzerini çiziniz.

* Soru kağıdının üzerine herhangi birşey yazmayınız.

İlgileriniz için teşekkür ederiz.

BAŞARILAR

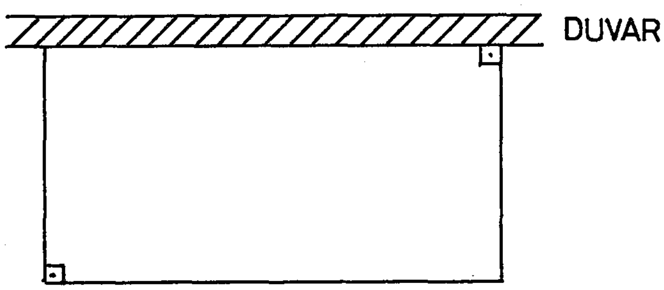
1) $|AB| = 2$ birim olan bir yarı çemberin içine çizili ABCD yamuğunun alanının en büyük degerini aldığında yüksekligi ne olur ?



2) $y = \dfrac{4}{x}$ fonksiyonunun başlangıç noktasına <u>en yakın</u> olan noktasının, başlangıç noktasına uzaklığı ne kadardır?

3) Şekildeki gibi dikdörtgen biçiminde ve bir kenarında duvar bulunan bir bahçenin üç kenarına bir sıra tel çekilmiştir.

Kullanılan telin uzunluğu 80 m olduğuna göre, bahçenin alanı <u>en fazla</u> ne kadar olabilir ?



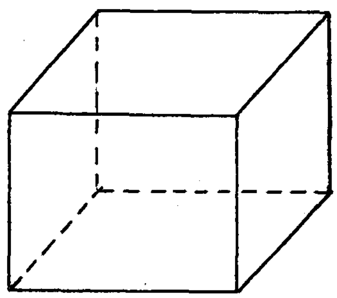
APPENDIX C.2

POSTTEST

1) Find the points on the graph $y = 4 - x^2$ that are closest to the point (0,2).

2) Suppose a window has the shape of a rectangle with an equilateral triangle attacked at the top. Assuming that the perimeter of the window is 12 feet, find the dimensions that allow the maximum amount of light to enter.

3) An open box having a square base is to be constructed from 108 m^2 of material. What should be the dimensions of the box to obtain a maximum volume?



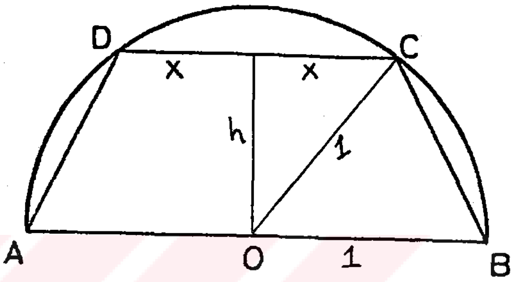
APPENDIX D

APPENDIX D.1

SCORING KEY OF PRETEST QUESTIONS

1. |AB| = 2 birim olan yarı çemberin içine çizili ABCD yamuğunun alanının en büyük degerini aldıgında yüksekligi ne olur?

Çözüm:



1) $|DC| = 2x$ olsun

2) OHC dik üçgeninde

$$|OH| = \sqrt{1 - x^2}$$

2

3) Yamuğun alanı

$$S = \frac{|AB| + |CD|}{2} * h$$

2

4) $$S = \frac{2 + 2x}{2} \sqrt{1 - x^2}$$

1

5) S'nin x'e göre türevi,

$$S' = \sqrt{1 - x^2} + (1 + x)\frac{-2x}{2\sqrt{1 - x^2}}$$

2

6) S alanı en büyük degerini aldıgında

$$S' = 0 \Longrightarrow -2x^2 - x + 1 = 0$$

$$x_{1,2} = \frac{1 \pm 3}{-4}$$

$x > 0$ oldugundan $x = 1\backslash 2$ dir.

$S'' = -4x - 1 < 0$ $\quad x = 1\backslash 2$ de

$x = 1\backslash 2$ max noktasıdır.

2

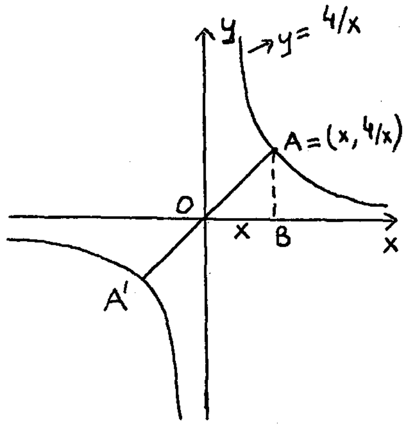
7) Buna göre yamuğun yüksekligi

$$H = \sqrt{1 - x^2} = \sqrt{1 - (1\backslash 2)^2} = \sqrt{3}\backslash 2$$

1

2. $y = 4\backslash x$ fonksiyonunun başlangıç noktasına en yakın olan noktasının, başlangıç noktasına uzaklığı ne kadardır.

Çözüm:



2

1) $y = 4\backslash x$ denkleminin grafiğin başlangıç noktasına en yakın noktası A (ya da A′) olsun.

2) $|OA|$ en küçük $\Longleftrightarrow$ $|OA|^2$ en küçük olur.

1

3) $|OA|^2 = x^2 + y^2 = x^2 + 16\backslash x^2$

4) $|OA|^2$ en küçük $\Longleftrightarrow$ $x^2 + 16\backslash x^2$ en küçük olmalıdır.

5) $f(x) = x^2 + 16\backslash x^2$

1

6) $f'(x) = 2x - 2*16\backslash x^3$

2

7) $f'(x) = 0 \Longrightarrow x - 16\backslash x^3 = 0$

$x = 16\backslash x^3 \Longrightarrow x^4 = 16$

$x = \pm 2$

2

8) $f''(x) = 2 + \dfrac{32*3x^2}{x} = 2 + \dfrac{96}{x^2} > 0$ min

1

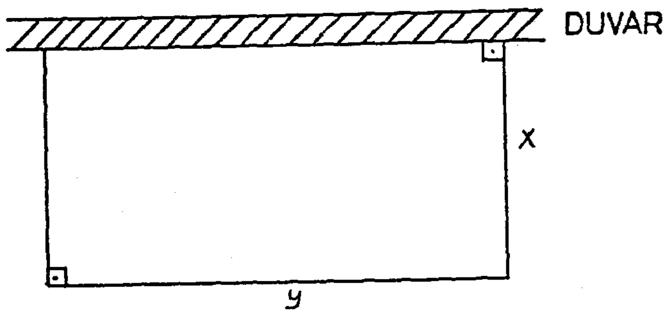
9) A (x,y) noktasında $x > 0$ olduğundan $x = 2$ ve $y = 2$.

10) $|OA| = \sqrt{2^2 + 2^2} = \sqrt{8} = 2\sqrt{2}$

1

3. Şekildeki gibi dikdörtgen biçiminde ve bir kenarında duvar bulunan bir bahçenin üç kenarına bir sıra tel çekilmiştir. Kullanılan telin uzunluğu 80 m olduğuna göre, bahçenin alanı en fazla ne kadar olabilir?

Çözüm:



1) x = dikdörtgenin genişliği
   y = dikdörtgenin uzunluğu   1

2) Verilen bahçenin çevresi $Ç = 2x + y = 80$ 1

3) Bahçenin alanı $S = x*y$ 1

4) $2x + y = 80 \Longrightarrow y = 80 - 2x$ 1

5) $S(x) = x * (80 - 2x)$ 1

6) $S'(x) = 80 - 2x + x * (-2x) = 80 - 4x$ 1

7) $S'(x) = 0 \Longrightarrow 80 - 4x = 0 \Longrightarrow x = 20$ 1

8) $S''(x) = -4 < 0 \Longrightarrow x = 20$ max   1

9) $y = 80 - 2x = 80 - 2 * 20 = 40$ 1

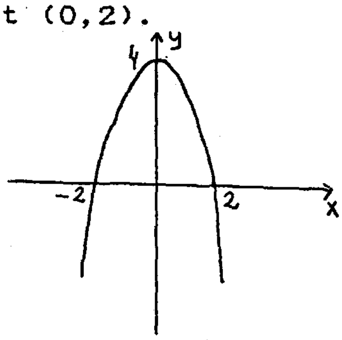
10) $S = x * y = 800\ m^2$ 1

APPENDIX D.2

SCORING KEY OF POSTTEST QUESTIONS

1. Find the points on the graph $y = 4 - x^2$ that are closest to the point (0,2).

Soln:



1

1) Take a point on the curve $y = 4 - x^2$ call that point (x,y)

Distance between (x,y) and $(0,2)$

$D = \sqrt{(x - 0)^2 + (y - 2)^2}$ minimize the distance

$D^2 = x^2 + (y - 2)^2$

$D^2(x) = x^2 + (4 - x^2 - 2)^2$

$D^2(x) = f(x)$ if $D^2(x)$ is min then $f(x)$ is min

2

2) $f(x) = x^4 - 3x^2 + 4$ 1

3) $f'(x) = 4x^3 - 6x = 2x(2x^2 - 3)$ 2

4) $f'(x) = 2x(2x^2 - 3) = 0 \Longrightarrow x = 0$ and $x_{1,2} = \mp\sqrt{3}\backslash\sqrt{2}$ 2

5) $f''(x) = 12x^2 - 6 = 6(2x^2 - 1)$

$f''(0) = -6 < 0$ local max

$f''(\mp\sqrt{3}\backslash\sqrt{2}) = 12 > 0$ local min

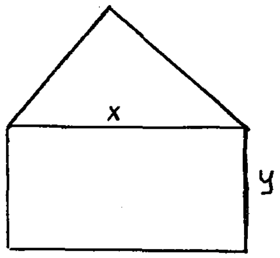
The closest point to (0,2)

$x_{1,2} = \mp\sqrt{3}\backslash\sqrt{2}$, $y_{1,2} = 5\backslash2$

2

2. Suppose a window has the shape of a rectangle an equilateral triangle attacked at the top. Assuming that the perimeter of the window is 12 feet, find the dimensions that allow the maximum amount of light to enter.

Soln:



1

1) x = side length of equilateral triangle

y = a side lenght of rectangle

Perimeter of the window $P = 2y + 3x = 12$

$y = (12 - 3x)\backslash 2 \quad 0 < x < 4$ 2

2) Area of the window $A = xy + x^2\sqrt{3}\backslash 4$

$A(x) = x\,(12 - 3x)\backslash 2 + x^2\sqrt{3}\backslash 4$ 2

3) $A'(x) = (12 - 6x)\backslash 2 + 2x\sqrt{3}\backslash 4$ 2

4) $A'(x) = 0 \Longrightarrow 12 - 6x + \sqrt{3}x = 0$ 1

$x = 12\backslash(6 - \sqrt{3})$

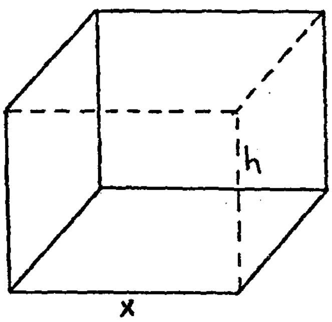
5) $A''(x) = -3 + \sqrt{3}\backslash 2 < 0 \quad x = 12\backslash(6 - \sqrt{3})$ local max 1

6) Triangle side length $x = 12\backslash(6 - \sqrt{3})$ 1

Height of the rectangle $y = (18 - 6\sqrt{3})\backslash(6 - \sqrt{3})$

3. An open box having a square base is to be constructed from $108\ m^2$ of material. What should be the dimensions of the box to obtain a maximum volume?

Soln:



1) x = side length of the cube

y = height of the cube

2) Area of the base = x^2

Volume of the box = $x^2 h = V$ 1

3) Surface area (except the top) = $x^2 + 4xh = 108$ 2

4) $h = (108 - x^2)\backslash 4x$

$V(x) = 27x - x^3\backslash 4$ 2

5) $V'(x) = 27 - 3x^2\backslash 4$

6) $V'(x) = 0 \Longrightarrow 27 - 3x^2\backslash 4 = 0 \Longrightarrow x = \pm 6$ 2

7) $V''(x) = -6x\backslash 4 < 0$ at $x = 6$

$x = 6$ is a local max 2

8) $h = 72\backslash 24 = 3$ m , $x = 6$ m 1