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# Implementation of Dirlik's damage model for the vibration fatigue analysis

Gürzap İ. Demirel<sup>a,\*</sup>, Altan Kayran<sup>b</sup>

<sup>a</sup>*Defense Industries Research and Development Institute, Scientific and Technological Research Council of Turkey, Ankara 06261, Turkey*

<sup>b</sup>*Department of Aerospace Engineering, Middle East Technical University, Ankara 06800, Turkey*

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## Abstract

Mechanical fatigue is an important phenomenon when the structures are exposed to dynamic, fluctuating loadings. Especially aerospace structures are commonly exposed to random vibration loadings. In this paper, random vibration fatigue is studied both numerically and experimentally. For this purpose, a rectangular cross-section notched beam is designed in order to conduct the vibration fatigue analysis in the frequency domain by the Dirlik's damage model. Aluminum and steel notched beams are prepared in order to carry out the vibration fatigue tests to compare with the fatigue analysis results. The effect of modal damping on the vibration fatigue analysis results is particularly studied.

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*Keywords:* Random vibration fatigue; Dirlik's damage model, Vibration fatigue test, Damping ratio

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## 1. Introduction

The main focus of present paper is to implement Dirlik's damage model for the vibration fatigue analysis and to perform a comparative study with the experimental vibration fatigue tests. In general, mechanical fatigue analyses

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\* Corresponding author.

*E-mail address:* [gurzap.demirel@tubitak.gov.tr](mailto:gurzap.demirel@tubitak.gov.tr)

and experiments can be carried out in time domain or frequency domain. Today's delicate and advanced aerospace engineering structures are exposed to random vibration loads as well as other loads such as maneuvering or temperature etc. The random vibration loads can be examined and processed well in frequency domain rather than the time domain, [1, 2]. Moreover, if the loading frequency has a wide bandwidth as in random vibration loadings, the natural frequencies or resonance regions of the structures are disturbed with high probability, [3]. Hence, the present paper also examines the effect of modal damping ratio on the vibration fatigue analysis results because stress amplitudes in the resonance regions are affected strongly by the modal damping.

Random vibration fatigue analysis utilizes various damage counting methods such as Narrow Band, Steinberg, Tunna, Dirlik and Hancock etc., [4]. Among the methods, Dirlik's empirical formula for the cycle counting is the most superior in terms of accuracy and it is the most widely used frequency domain stress cycle counting method in vibration fatigue analysis [5, 6, 7]. Therefore, vibration fatigue analyses in the present paper are conducted with Dirlik' damage model and comparisons are made with the vibration fatigue test results.

For the purpose of this study, aluminum and steel rectangular cross-section beams are designed to perform the vibration fatigue tests. In order to obtain a more distinct fatigue life than other parts of the beams, notches added to the beam geometry. Firstly, in order to ensure the reliability of the finite element model, mesh refinement work is conducted. To decide on the frequency interval of the analyses, modal analyses are carried out and to obtain the appropriate damping ratios of the notched beams, which are input to the frequency response analysis, modal tests are carried out. Random vibration fatigue analysis results are compared with the vibration fatigue test results and possible causes of the discrepancies between the analysis and test results are discussed. Furthermore, the effect of different damping ratio on the vibration fatigue analysis results is particularly investigated.

### *1.1. Notched Beam Structure Studied*

The rectangular cross-section notched beam is designed in order to perform the vibration fatigue analyses and tests. For this specific study, the overall geometry and notched region are designed so that fatigue failure occurs in the notched region first so the control of the random vibration fatigue analysis and test is easy. Moreover, the design of the notched beam is made such that it can be connected to both the vibration table and the modal shaker; hence the geometry of the notched beam is decided as shown in Fig.1. However, as the test program progressed the modal shaker is not used at all.

## **2. Theory of random vibration fatigue**

The frequency domain fatigue analysis is the most suitable method for the random vibration fatigue analysis. The data which are impractical to handle in time domain can be handled easily in the frequency domain, [8]. Also, if the resonance region of the mechanical system is excited, the time domain analysis comes short in terms of dynamic behavior in the resonance region, [5]. In the frequency domain, the input is given in the form of a Power Spectral Density (PSD) of the loading and the structure is modelled by a linear frequency response function relating the input loading to the output stress at a particular location in the structure. The output from the model is expressed as a PSD; in this case it is the PSD of stress. The transition between the time and the frequency domain is done using the Fourier transformation. By using the Fourier transformation, a complex random signal in time domain can be converted into the frequency domain and back to the time domain easily. The Fourier transformation can be used with continuous time signals. However, in the digital world, the time histories are recorded in discrete forms. Therefore, discrete Fourier transformation is usually needed. Hence, discrete Fourier transformation was developed in 1965 by Cooley and Tukey and it is called as Fast Fourier Transformation (FFT), [9].

The frequency response function (FRF) is basically the mathematical relationship between the harmonic input and output of the dynamic structural systems. FRF gives the amplitude and the phase information of the output as a function of frequency and it is unique for the particular mechanical system. The linear structure responds to a sinusoidal force with a sinusoidal displacement at the same frequency. Therefore, one can predict the frequency response (FR) of the system by multiplying the load and the FRF in frequency domain calculations.

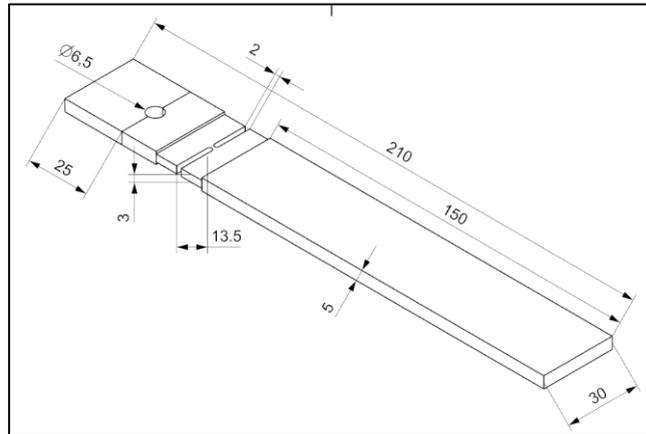


Fig. 1 Geometry of the notched beam, [mm]

PSD is used to characterize the strength of stationary random process in the frequency domain. In order to get the PSD profiles of the random vibration loading, Fast Fourier Transform (FFT) of the random load history is taken and then PSD profile is obtained by taking the modulus of the squared FFT and divided by the  $2T$ , where  $T$  stands for the sample period which can also be defined as  $1/f_s$  and  $f_s$  being the sampling frequency of the recorded signal.

In the vibration fatigue analysis, the moments of the PSD profiles are needed for the frequency domain cycle counting. The relevant spectral moments are computed from one sided PSDs. The  $n^{\text{th}}$  moment of area of the PSD is calculated by dividing the curve into small strips. The  $n^{\text{th}}$  moment of area of the strip is given by the area of the strip multiplied by the frequency raised to the power  $n$ . The  $n^{\text{th}}$  moment of area of the PSD is then found by summing the moments of all the strips. In theory, all possible moments are required to fully characterize the original process. However, in practice,  $m_0$ ,  $m_1$ ,  $m_2$  and  $m_4$  are sufficient to compute all of the information required for the subsequent fatigue analysis, [8].

Random stress histories can be well manipulated and described by using the statistical parameters. This is due to the fact that any sample time history can only be regarded as one sample from an infinite number of possible samples that occur for the random processes. Figure 2 shows a one second piece taken from a typical wide band signal. Two of the most important statistical parameters are the number of zero crossings,  $E[0]$ , and the number of peaks,  $E[P]$ , in the signal. The irregularity factor ( $\gamma$ ) is defined as the number of upward zero crossings divided by the number of peaks, [8]. The first valid effort for estimating the fatigue damage from PSDs was studied by S.O. Rice in 1954. Rice discovered the important relationships for the number of upward mean crossings per second,  $E[0]$ , and the peaks per second,  $E[P]$ , in a random signal expressed only in terms of their spectral moments,  $m_n$ , [8]. In terms of spectral moments, upward mean or zero crossing is given by Equation (1), the number of peaks is given by Equation (2) and the irregularity factor is given by Equation (3).

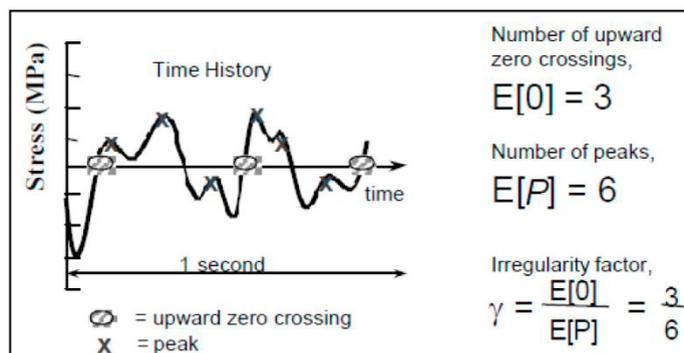


Figure 2 Expected zero, expected peak and the irregularity factor

$$E[0] = \sqrt{\frac{m_2}{m_0}} \tag{1}$$

$$E[P] = \sqrt{\frac{m_4}{m_2}} \tag{2}$$

$$\gamma = \frac{E[0]}{E[P]} = \sqrt{\frac{m_2^2}{m_0 * m_4}} \tag{3}$$

Mathematically, the most convenient way of storing the stress range histogram is in the form of PDF of the stress range, [10]. It is easy to transform from a stress range histogram to a PDF or backwards. In order to calculate the expected fatigue damage, E(D), firstly the PDF of the stress ranges, p(S), should be determined.

Multiplying the probability of the stress range p(S)\*dS with the total number of cycles (S<sub>t</sub>) in the histogram, total number of cycles N(S), for a given stress level S, can be obtained as shown by Equation (4).

$$N(S) = p(S) * dS * S_t \tag{4}$$

2.1. Stress cycle counting method from PSD in the frequency domain

Among all the stress cycle counting methods from PSD, Dirlik's empirical formula for the cycle counting is the most superior in terms of accuracy, [5]. Dirlik has derived an empirical closed form expression for the determination of the PDF of the cycle counting of stress ranges, which was obtained using extensive computer simulations to model the signals using the Monte Carlo technique, [11]. The fatigue damage (D) is calculated after the cycle counting by the Dirlik's method and given by Equation (5), where m and A are material properties which are the fatigue strength exponent obtained from the material S-N curve and the fatigue strength coefficient, respectively.

$$D = \left(\frac{1}{2^{m * A}}\right) * \int_0^\infty S^m * N(S) * dS \tag{5}$$

In Equation (5), histogram formula N(S) for stress cycles range is given by Equation (6), where v<sub>p</sub> and τ are the rate of peaks (number of expected peaks per unit time) and the exposure time, respectively.

$$N(S) = v_p * \tau * p(S) \tag{6}$$

The best correlation for p(S) proposed by Dirlik is given by Equation (7).

$$p(S) = \frac{\frac{D_1 * e^{-Z}}{Q} + \frac{D_2 * Z}{R^2} * e^{\frac{-Z^2}{2 * R^2}} + D_3 * Z * e^{-\frac{Z^2}{2}}}{2 * \sqrt{m_0}} \tag{7}$$

where, S is the stress range and other parameters in Equation (6) and (7) are given in Table 1.

Table 1 Parameters in Equation (6) and (7)

$D_1 = \frac{2 * (x_m - \gamma^2)}{1 + \gamma^2}, \quad D_2 = \frac{1 - \gamma - D_1 + D_1^2}{1 - R}, \quad D_3 = 1 - D_1 - D_2, \quad Q = \frac{1,25 * (\gamma - D_3 - D_2 * R)}{D_1}$
$R = \frac{\gamma - x_m - D_1^2}{1 - \gamma - D_1 + D_1^2}, \quad Z = \frac{S}{2 * \sqrt{m_0}}, \quad x_m = \frac{m_1}{m_0} * \sqrt{\frac{m_2}{m_4}}, \quad v_p = \sqrt{\frac{m_4}{m_2}}, \quad \gamma = \frac{m_2}{\sqrt{m_0 * m_4}}$

### 3. Vibration fatigue analysis and test of notched beams

The material properties of the materials of the notched beams are given in Table 2.

Table 2 Mechanical material properties of aluminum and steel, [12]

	Aluminum	Steel
<b>Density</b>	2710 kg/m <sup>3</sup>	8000 kg/m <sup>3</sup>
<b>Elastic Modulus</b>	68.3 GPa	200 GPa
<b>Poisson's Ratio</b>	0.33	0.27

In order to decide on the frequency interval of the test and the analysis, modal analyses are carried out. Moreover, with the information received from the modal analysis test results, the misplacement of test devices (e.g. sensors) can be prevented. Before starting the vibration fatigue analysis, the mesh refinement work has been performed. Mesh refinement work assures the reliability of the finite element model. The result of the mesh refinement work is given in Table 3 which gives the maximum von Mises stress around the notch for different mesh sizes. In the rest of the present study, mesh number 7 is used in the finite element model of the notched beam.

Table 3 Result of the mesh refinement study

Mesh number	Mesh edge length around the notch (mm)	Maximum stress around notch (MPa)
1	8	34.2
2	4	30.5
3	2	31.5
4	1	32.8
5	0.5	38.3
6	0.25	40.6
7	0.125	40.9

The main aim of the preliminary modal analysis is to determine the frequency of interest. Hence, the modal effective mass fraction output is requested from the modal analysis. The modal effective mass fraction provides the information about the significance of the modes in the dynamic behavior of structures. For the specific beam geometry studied, the participation of 80-90% cumulative modal effective mass fraction is thought to be sufficient to represent the dynamic behavior of the notched beam. Eventually, the frequency interval is decided to be 1 Hz to 2048 Hz. However, since the vibration test table used in the present study allows tests from 4 Hz up to 2000 Hz, the frequency interval is decided to be 4 Hz-2000 Hz. Among the first 5 modes of the beam, only the 2<sup>nd</sup>, 4<sup>th</sup> and the 5<sup>th</sup> modes are out-of-plane displacement modes. Mode 1 is the in-plane displacement mode and mode 3 is the in-plane twisting mode and they are ineffective on the out-of-plane deformation of the beam according to the modal effective mass fraction results. Therefore, modes 1 and 3 are ignored in the rest of this study.

The modal damping ratios of the beam should be determined by carrying out with modal testing because the fatigue analysis results are very sensitive to the damping values, [10]. Without including the damping, in the resonance regions the notched beam unrealistically high stresses occur. However, in reality, there is the damping in dynamic systems. Therefore, in the present study modal testing is carried out with the impact hammer. Proper clamping is used to provide the necessary edge condition and with the use of accelerometers modal damping ratios of the beam are extracted. For this purpose, LMS Test.Lab software is used and the half-power bandwidth method is used while calculating the damping ratios, [13]. The modal damping ratio results of the modal tests are given in Table 4.

Table 4 Modal damping ratio results

	Aluminum Notched Beam			Steel Notched Beam		
Mode no.	Analysis frequency, [Hz]	Test frequency, [Hz]	Damping ratio, ( $\zeta$ ), [%]	Analysis frequency, [Hz]	Test frequency, [Hz]	Damping ratio, ( $\zeta$ ), [%]
2	39.64	38.49	1.14	39.50	39.79	2.16
4	664.10	635.05	0.48	661.40	641.04	1.80
5	1916.79	1762.75	0.46	1908.91	1746.37	1.12

In order to perform the vibration fatigue analysis of the notched beam, the magnitude of the stress PSD profile to unit load input should be determined. For this purpose, finite element model of the notched beam is prepared based on the outcome of the mesh refinement work. In order to extract the frequency response corresponding to the unit acceleration loading, the base acceleration load ( $1 \text{ mm/s}^2$ ) in the out-of-plane direction is applied to the region 25 mm from the left side of the beam as shown Figure 3.

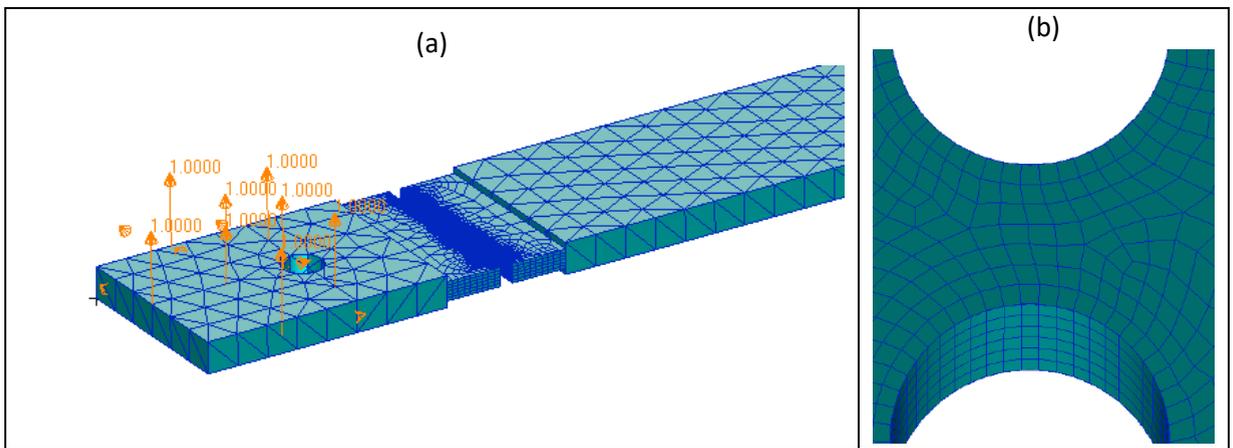


Fig. 3 a) Finite element model of the notched beam and the base acceleration applied [ $\text{mm/s}^2$ ], b) Finite element mesh around the notched region

The modal damping ratios of the notched beam, given in Table 4, are introduced into the finite element analysis performed by MSC Nastran utilizing the “Modal Damping” table through which one can give the damping ratios of the modes which have higher modal effective mass fractions. The frequency response analysis to unit acceleration load input is then performed by MSC Nastran. The stress frequency response of the notched beam to the unit load is used in the vibration fatigue analysis for all nodes in the finite element model. In order to show an example, the stress frequency response of the notched beam to unit load is extracted from the node which is the middle node at the top element at the notched region, as shown in Fig. 4. The von Mises stress frequency response of the example node in the aluminum notched beam analysis model is given in Fig. 5. As expected, the resonance regions of the stress frequency response and the natural frequencies are consistent.

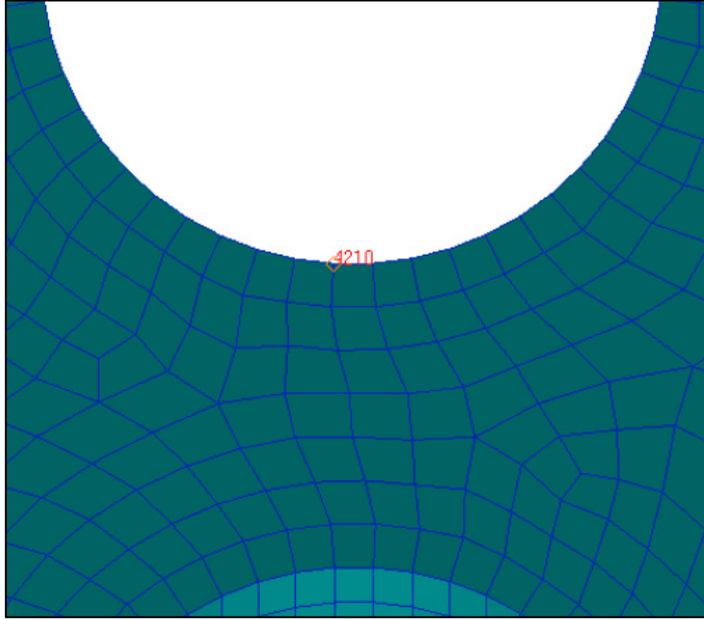


Fig. 4 The finite element node (4210) where the von Mises stress frequency response is calculated

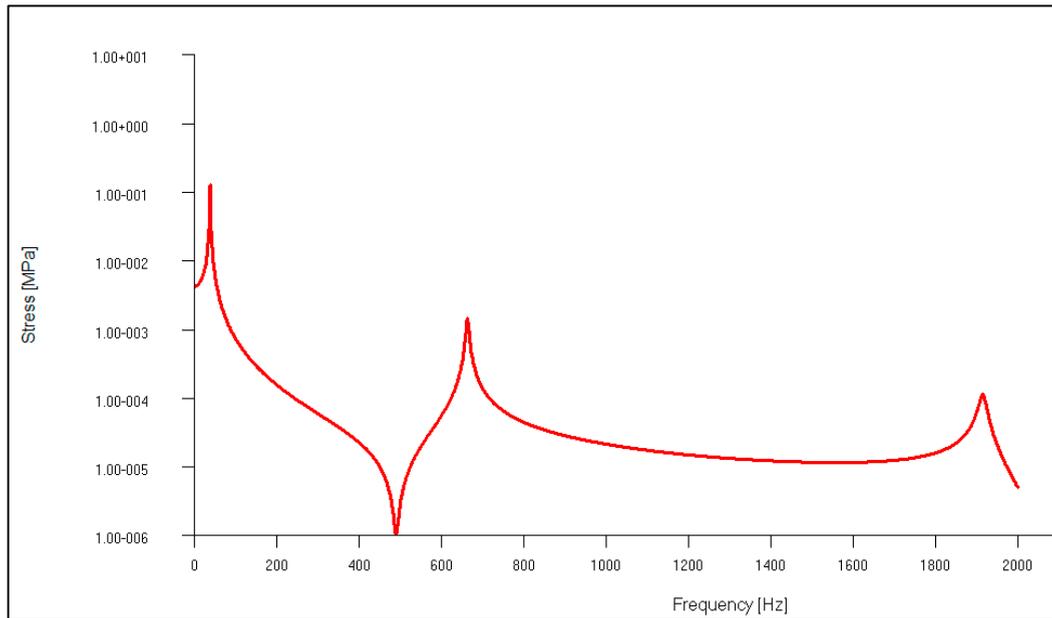


Fig. 5 Von Mises stress response of the example node corresponding to the unit load, aluminum notched beam

Following the completion of the frequency response analysis corresponding to unit base vibration input for all the nodes in the finite element model, the vibration fatigue analyses are conducted for both the aluminum and steel notched beams. For the vibration fatigue analysis, nCode DesignLife program is used as the fatigue analysis solver, [14]. In the vibration fatigue analysis, the beam is held horizontally; however, the mean stress effect of the gravity is neglected in this study. As mentioned before, the frequency interval of the PSD profile is taken as 4-2000 Hz.

Moreover, after few preliminary fatigue analysis iterations, which are done in order to verify that the test time is reasonable, the amplitude of the white noise is taken as  $100000 \text{ (mm/s}^2\text{)}^2/\text{Hz}$ . After supplying the FR of the unit base vibration load and the PSD profile, the time setting of the analysis is set to 1 second in nCode DesignLife and the Dirlik's method is used for the PSD cycle count method in the vibration fatigue analysis. The material selection is also done from the material library of the nCode DesignLife program. The nCode DesignLife has a material library from which one can select the ultimate tensile strength (UTS) for different alloys of aluminum. In this study, in order to select the appropriate aluminum and steel UTS, the tensile strength tests are conducted and the results of the tensile tests are given in Table 5.

Table 5 UTS values of the specimens

	Test #	1	2	3	4	5	6	7	Mean	Standard Deviation
Aluminum	UTS (MPa)	322.9	320.3	316.6	319.4	320.5	321.3	321.3	320.3	1.97
Steel	UTS (MPa)	648.8	644.8	640.7	648.1	638.0	685.6	669.6	653.7	17.4

According to Table 5, the mean UTS value of the aluminum specimens is 320 MPa and the mean UTS value of the steel specimens is 654 MPa. However, the nCode DesignLife material library has the option specifying the UTS in multiples of 100 MPa. Corresponding to the material selection done, the S-N curves are also available for the materials with different UTS values to be used in the vibration fatigue analysis. Hence, two vibration fatigue analyses are conducted for each material. For the aluminum material, UTS values of 300 MPa and 400 MPa are selected. For the steel, UTS values of 600 MPa and 700 MPa are selected. The roughness option is selected as polished; because in order to observe the crack initiation properly, the notched region of the beam is polished in the present experimental vibration fatigue test study. The results of the vibration fatigue analyses of the aluminum and steel notched beam are given in Table 6.

Table 6 Vibration fatigue analyses results of aluminum and steel notched beams

	Aluminum		Steel	
	300 MPa UTS	400 MPa UTS	600 MPa	700 MPa
Fatigue Life Results	207.7 s	23260 s	94.71 s	1388 s

In order to verify the fatigue analysis results of the aluminum and steel notched beams, vibration fatigue tests are conducted, as shown in Fig. 6. Vibration fatigue tests are continued until the occurrence of crack initiations which are monitored with the human eye in the present study. When the crack initiation occurs, the test is terminated and the time of the test is recorded. Total of 10 aluminum and 13 steel notched beams are tested. The times recorded from the beginning of the test to the observed crack initiations are given in Table 7.

Table 7 Crack initiation times of the aluminum and steel notched beams

	<b>Aluminum Notched Beam</b>	<b>Steel Notched Beam</b>
<b>Specimen #</b>	<b>Life (s)</b>	<b>Life (s)</b>
<b>1</b>	825	480
<b>2</b>	840	398
<b>3</b>	895	445
<b>4</b>	435	673
<b>5</b>	730	476
<b>6</b>	869	500
<b>7</b>	876	597
<b>8</b>	855	521
<b>9</b>	1120	387
<b>10</b>	880	475
<b>11</b>	-	341
<b>12</b>	-	408
<b>13</b>	-	413
<b>Mean</b>	832.5	470.3
<b>Standard Deviation</b>	170.4	90.1

Table 8 compares the results of experimental vibration fatigue analysis results and vibration fatigue analysis results performed for two different UTS values which are integer multiples of 100 MPa. It is seen that the selected UTS value is highly effective on the vibration fatigue analysis results. Experimentally determined fatigue life are encompassed within the fatigue lives obtained by the vibration fatigue analyses corresponding to the UTS values which are integer multiples of 100 MPa. In this regard, it can be inferred that vibration fatigue test and analysis results agree considerably well.



Fig. 6 The vibration fatigue test setup of the notched beams

Table 8 Comparison of fatigue life obtained by the vibration fatigue test and analysis

Fatigue Life Results	Aluminum Notched Beam			Steel Notched Beam		
	Analyses		Test	Analyses		Test
	300 MPa UTS	400 MPa UTS	320.3 MPa UTS	600 MPa UTS	700 MPa UTS	653.7 MPa UTS
	207.7 s	23260 s	832.5 s	94.71 s	1388 s	470.3 s

Moreover, in order to examine the effect of changing the damping ratio values, damping values of 0.015, 0.02 and 0.025 values are selected arbitrarily. With these damping ratios, same vibration fatigue analysis procedure is followed for the steel notched beams with 600 MPa UTS. It should be noted that for the high participating modes 2, 4 and 5, the same damping ratio is given in the finite element based frequency response analysis to study the effect of damping ratio on the fatigue life. The results of vibration fatigue analyses for different damping ratios are given in Table 9 which shows that the damping ratio significantly affects the vibration fatigue analysis results. Hence, determination of accurate damping ratio is very crucial in vibration fatigue analysis in order to get close fatigue life results to the experimentally determined fatigue life results.

Table 9 Fatigue life analysis results corresponding to different damping ratios

Damping Ratio (%)	Life (s)
1.5	9.812
2	57.1
2.5	278.7

#### 4. Conclusion

In this study, the vibration induced mechanical fatigue analyses and tests of aluminum and steel notched beams are conducted. It is noted that although there is some discrepancy between the fatigue lives obtained by the vibration fatigue analysis and tests, Dirlik's damage model estimates the fatigue life reasonably well when compared to the experimentally determined fatigue life. The possible reasons for the difference between the vibration fatigue test and analysis results are mainly attributed to the following:

- The selected UTS value is highly effective on the vibration fatigue analysis results. During the vibration fatigue analyses, the nCode program allows one to select the UTS values as multiples of 100 MPa with the corresponding S-N data. However, the real UTS values are 320.3 MPa for aluminum and 653.7 MPa for steel. As Table 7 shows, the selected UTS value is highly effective on the vibration fatigue analysis results.
- During the vibration fatigue tests, crack initiation is spotted by the eye inspection. Since the formation of micro cracks cannot be detected accurately with eye inspection, the actual crack initiation time and the observed crack initiation time may differ. It is highly probable that the real crack initiation times in the vibration fatigue tests are much earlier. Hence, the fatigue life determined in the vibration fatigue tests is longer than the real fatigue life corresponding to the actual crack initiation times.
- The damping ratio is highly effective on the vibration fatigue analysis results, as particularly studied in this paper. It is seen that the vibration fatigue analysis is very sensitive to the stress frequency response profile of the mechanical systems because the damping ratio affects the stress amplitudes in the resonance regions drastically. It is known that the damping ratio estimation in modal testing always has large error bands. Hence, one should determine the damping ratios accurately for each natural frequency for more accurate estimation of the fatigue life numerically.

It is noticed that the eye inspection method of crack initiation falls short in vibration fatigue testing. Since, the real time eye monitoring is insufficient to detect crack initiation in vibrating parts, another method should be implemented in order to determine the accurate crack initiation time. One option is to use high speed camera to catch the start of the crack initiation while the beam is still vibrating. Or, the test could be stopped at frequent intervals and the notched region could be checked for the onset of crack initiation. Vibration based crack initiation detection can be also a reasonable option. By means of a highly sensitive accelerometer mounted on the notched beam, vibration amplitudes can be monitored to detect the crack initiation. However, this study requires many tests and cross check for reliability. Other options could be to use non-destructive inspection (NDI) methods. For instance, periodic penetrant testing, acoustic emission testing, electromagnetic testing, infrared and thermal imaging testing, optical microscopy testing with stereoscopic microscope etc. can be implemented. However, the use of these NDI methods make the tests longer because to perform the NDI tests, vibration tests have to be paused frequently.

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