STABILIZATION AND ORIENTATION CONTROL OF A TANK GUN

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ABSTRACT

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In this study, stabilization of a battle tank gun system in azimuth and elevation is considered. The equations of motion of the tank gun system has been formulated by accounting all of the motion freedoms of tank hull in addition to the azimuth and elevation motion freedoms. Then, mathematical model of the tank gun system is constructed by using these equations of motion. The motion of the tank hull is provided from a sample ground surface, which is provided mathematically. The mathematical models of sensor errors and hydraulic motors are also provided.

Two control methods are used for stabilization of the tank gun. First, the position control system by using the computed torque method is modeled. Then, a velocity feedback control system is also tried. Sample simulations for both of these systems are formed and the results are plotted, performance of these systems are compared and discussed.

Keywords : Gun Control Systems, Stabilization of a tank gun

ÖZ

TANK SİLAHININ DENGELENMESİ VE YÖN KONTROLÜ

Şahin, Süreyya
Yüksek Lisans, Makine Mühendisliği Bölümü,
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Bu çalışmada, bir tank silahının yanal ve yükseliş eksenlerinde dengede tutulması araştırılmıştır. Tank silah sisteminin hareket denklemleri tank gövdesinin mümkün olan hareketleri de düşünülerek çıkartılmıştır ve daha sonra matematik modeli oluşturulmuştur. Tank gövdesinin ilerleyebileceği örnek bir yüzey matematiksel olarak oluşturulmuş, ve bu yüzey tank gövdesinin hareketlerinin bulunmasında kullanılmıştır. Tank silah sisteminin kontrolü ele alınırken hız ve ivme ölçen cihazların hata modelleri ve bir hidrolik motor sisteminin modeli gözönüne alınmıştır..

Tank silah sisteminin dengede tutulmasına yönelik iki kontrol sistemi kullanılmıştır. Bu sistemlerden ilki tank silahının tork hesaplama yöntemi ile pozisyon kontrolünün sağlanmasıdır. Daha sonra, tank silahının hız geribesleme yöntemi ile kontrolü denenmiştir. Bu iki yöntem içinde örnek çozümler oluşturulmuş, ve sonuçlar yardımıyla bu iki sistemin performansı tartışılmıştır.

Anahtar Kelimeler : Tank silah Kontrol sistemleri, Tank silahının dengede tutulması

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TABLE OF CONTENTS

APPENDICES

YÖK

LIST OF FIGURES

LIST OF SYMBOLS

Symbols

$\vec{a}$	Acceleration Vector
$\vec{\alpha}$	Angular Acceleration Vector
$\hat{C}$	Transformation Matrix
D	Derivative Operator
$\vec{\nabla}$	Gradient
F	Coordinate Frame
$\vec{g}$	Gravitational Acceleration
$\hat{J}$	Inertia Matrix
$\vec{r}$	Position Vector
$\vec{u}$	Unit Vector
$\vec{\omega}$	Angular velocity vector

Superscripts

(.)	First time derivative
(..)	Second time derivative
(~)	Skew symmetric matrix
(-)	Column matrix
(^)	Matrix
(t)	Transpose
(→)	Vector

CHAPTER 1

INTRODUCTION

Tanks are mobile, protected weapon platforms. More specifically, they are automotive, tracked, armoured carriers of heavy direct-fire weapons. They stem from developments in two different fields. One is that of automotive vehicles and in particular of tracked tractors, which formed the basis of the construction of the first tanks as armoured vehicles capable of cross-country movement. The other is that of the development of heavy weapons, which had grown in importance in relation to individual, portable weapons but whose effectiveness was constrained by their limited mobility until it proved possible to overcome this by mounting them in tanks. Demands on increasing the battlefield mobility, that is, the ability of tanks to move when in actual or imminent contact with enemy forces, inevitably lead to the requirement of firing on the move, instead of having to stop every time they engage a target. This requirement call, in turn, for gun control systems which minimize the effects of vehicle motion on the main armament of tanks and in particular its ability to hit targets [1, 2].

In early tanks of the First World War, the guns were mounted so that they were free to pivot about their trunnions and gunners controlled them in elevation by pressing on a shoulder piece attached to the gun cradle. The free elevation type of mounting and shoulder control had continued to be used in light tanks up to the early stage of the Second World War. The heavier types of tanks were fitted with manually operated elevating gears from the late 1920s. Manually operated gears are a more precise if somewhat slower means of controlling the elevation of guns than shoulder pieces and they are relatively simple, light and inexpensive. Control of guns in azimuth was exercised in some of the early light tanks, by the gunner simply pulling or pushing against the sides of the turret, which was free to rotate when not locked in position. This primitive method was not suitable for heavier turrets than that used with light tanks and hence heavy tanks were fitted with traversing gears. The traversing gears consisted, in essence, of a hand crank driven pinion, which was attached to the turret and engaged with the internal teeth of a ring gear fixed to the hull. Manual operation of the traversing gears severely restricted the speed with which the turrets could be traversed, particularly as they grew in size and weight. Hence, the heavier types of tanks began to be fitted with powered traverse. Early types of powered traverse drives were of electrical type and although they were able to slew turrets more rapidly than hand traverse, they did not provide sufficiently precise control for accurately laying guns on target, which still had to be done manually. In 1940s, electrical motors have begun to be used not only for rapidly slewing the turret, but also for fine laying. During this period, hydraulic drive systems for the turret has also been fitted and widely used. The use of electrically or hydraulically powered systems for traversing turrets was followed by adoption of powered controls for the elevation of tank guns, which in turn lead to the stabilization of tank guns.

1.1 Gun Stabilization Systems

The effects of vehicle motion on the armament of the tank can be minimized by gun stabilization systems that are designed to maintain the spatial orientation of guns. Systems designed to do this are basically closed loop servo systems which control the orientation of the guns relative to the inertial space by employing gyroscopes to sense the motion of the guns relative to it and using position or velocity feedback signals provided by them.

The simplest type of stabilization systems have been confined to the elevation of guns and consist, in essence, of a single closed-loop servo system with a rate gyroscope mounted on the gun to sense its angular velocity. Any difference between this velocity and that commanded by the gunner causes the elevation servo-motor, or actuator, to rotate the gun so that the difference, or 'error', is minimized and if the gunner holds his controls steady the system maintains the gun at a fixed elevation relative to space.

Systems of this kind were used in most of the US tanks in which stabilization was originally introduced during the Second World War and also when it was first applied in Soviet T-54 tanks. The argument for stabilizing guns only about the elevation axis has been that disturbances about the other axes are much smaller and, therefore, less important. However, disturbances about the traverse axis of the turret are not insignificant and except for the original stabilization systems all the others have stabilized the turrets in azimuth in addition to stabilizing the gun in elevation.

A two axis stabilization system developed by Westinghouse was already fitted during the Second World War in some US M4 medium tanks but the effectiveness of this type of system was not established until the ones produced after the war for the British tanks. All the British produced systems have used electric drives and two rate gyroscopes mounted on the gun cradle to sense the angular motion of the gun in elevation and of the turret in azimuth. A block diagram of one of the two stabilized drives of these systems is shown in Fig 1-1, which illustrates their unique feature, namely the use of metadynes as power amplifiers. The diagram happens to be of the traverse drive but is equally illustrative, in principle, of the elevation drive. Stabilization systems used with electro-hydraulic power drives are basically similar but in place of the electric servo-motors they use hydraulic motors for traversing turrets and double acting hydraulic rams for elevating guns.[1]

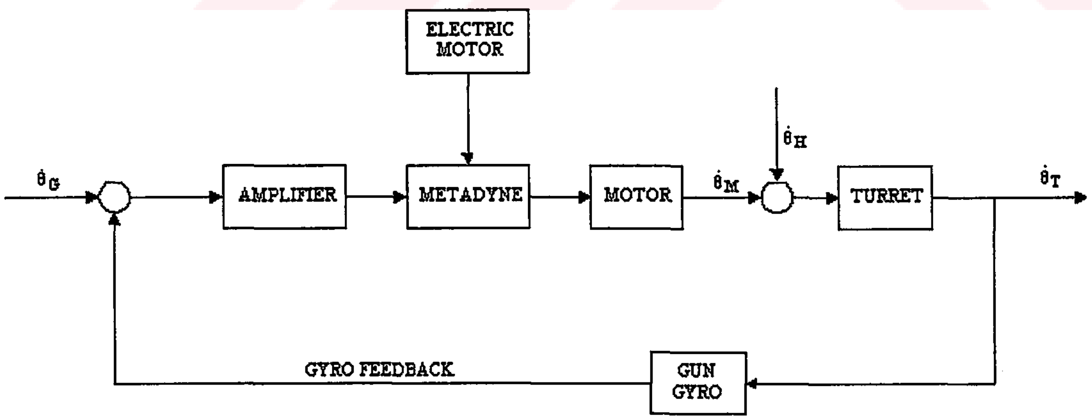

Figure 1-1 Simplified Block diagram showing the function of the gunner in closing the target to line of sight loop.

The basic two gyro control systems have proved reasonably effective and even if they do not always make it possible for gunners to aim accurately on the move, they can at least aim roughly, so that only relatively small adjustments have to be made when their tanks stop to fire. However, in the nature of things, the response of the basic systems is not sufficiently low level when tanks move at speed over rough ground. In consequence, more elaborate systems began to be developed in 1960s. These 'second generation' systems incorporate two additional gyros in feedforward open loops which respond to angular velocities of the vehicle and provide anticipatory commands to the azimuth and elevation drives, thereby approximately stabilizing the gun. Thus, one additional gyro is mounted in the hull to sense the angular rotation of the hull in the plane of the rotation of the turret, and generate feedforward commands to the traverse drive, as shown in Fig 1-2. The second of the additional gyros is mounted in the turret to sense the angular rotation of the turret in the elevation plane of the gun, and to generate feedforward commands to the elevation drive. As a result, the demand on the two gun mounted gyros is reduced to correcting the errors of the feedforward loops and the stabilization of the gun is considerably improved.

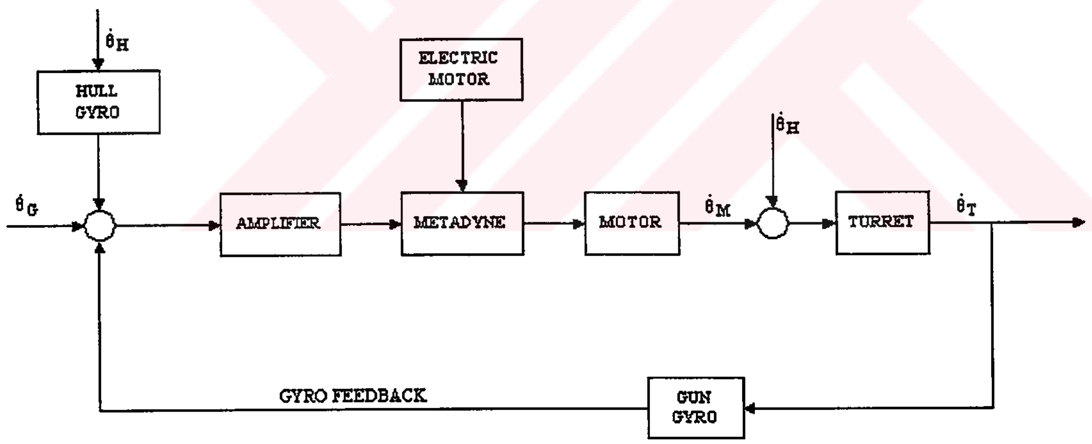

Figure 1-2 Simplified block diagram of an electro-hydraulic traverse drive with gyro feedback and an open-loop feedforward channel.

The use of feed-forward control was pioneered by the Cadillac Gage Company, following the revival of interest in stabilization in the United States in the late 1950s. In particular, it was first demonstrated in 1962 and which was adopted in 1969-1970 by the German and Belgian Armies for retrofitting their Leopard 1 tanks. Feed-forward loops have been incorporated since then in other tanks, including the Leopard 2 and the US M1, and even in the US M2 Infantry Fighting Vehicle.

Feed-forward loops have also been used in Soviet T-55 and T-62 tanks. But in their case the two primary closed loops incorporate rate integrating gyroscopes, instead of rate gyroscopes which are generally used elsewhere, and rate integrating gyroscopes are also used in their open loops. The advantage of using rate integrating gyroscopes is that they provide a memory of the gun position. Hence, the gun is driven back by the control system to its position before any disturbance no matter how severe the latter might be, which is not the case with control systems using rate gyroscopes.

A further refinement incorporated in some of the more recent stabilization systems is the addition of yet another rate gyroscope, to sense the roll of the turret. In other cases, including the Cadillac Gage 'Add-On' system, the gun azimuth gyroscope is mounted so that it senses the turret roll rate, as well as the turret azimuth rate, and together with the gun elevation gyroscope it can therefore provide signals for vehicle roll compensation as well as azimuth compensation.

Some of the more recent stabilization system also incorporate accelerometers, especially to measure the angular acceleration of the turret in azimuth, in order to produce signals which result in the artificial damping of mechanical resonances. The outcome of all refinements incorporated in the second-generation systems has been to reduce considerably gun-pointing errors and consequently to increase further the probability of hitting targets on the move. However, second-generation systems still only maintain the position of the tank guns in space and they do not provide gunners with all the aids which are possible. In particular, the gunners still have to track targets or, in other words, close the overall weapon-target loop by visual feedback, as indicated in Fig 1-3.

In fact, stabilization of the gun has helped by reducing motion of the image presented to him in the sight, which is in general mechanically linked to the gun. This has been of particular help to the gunners in observing targets on the move and has been one of the major practical advantages to result from the adoption of the basic stabilization systems. However, the image presented to the gunner can be made much more stable by decoupling his sight from the gun and stabilizing its head independently, since the inertia of the sight is much more smaller than that of a gun.

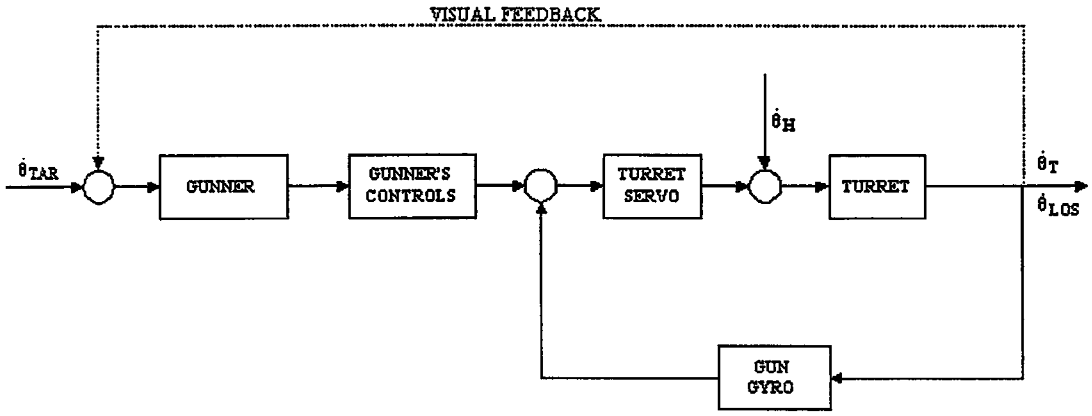

Figure 1-3 Simplified block diagram showing the function of the gunner in closing the target to line-of-sight loop

The high degree of line-of-sight stability achieved with independently stabilized sights raises the quality of the images which are provided by them and this, in turn, gives gunners a chance to detect targets more quickly and at longer ranges. Independently stabilized sights also make it possible to track targets more accurately, which increases hit probability. Moreover, the accuracy with which the line of sight is stabilized makes it possible to use it as an inertial reference for the gun and the turret, instead of the inertial sensors mounted on them. In fact, this is done whenever an independently stabilized sight is used and the gun and turret are then slaved to the sight, which results in a director-type fire control system. A simplified block diagram of the traverse part of such a system with an independently stabilized gunner's sight and with the turret slaved to it is shown in Fig. 1.4.

The slaving of the gun and the turret to an independently stabilized sight has not resulted in the elimination of the gun and turret feedback systems with their gun-mounted gyroscopes or, where feed-forward control is used, of the additional turret and hull gyroscopes. These have continued to be used to stabilize the guns but, since they can not do it to the high degree of accuracy to which the sight can be stabilized, there are errors between the position of the gun and the offsets from the line of sight required by the fire control system. However, the errors can be measured and the signals generated by them can be used to drive the gun into coincidence with the required offset position, increasing the accuracy with which the gun is pointed.

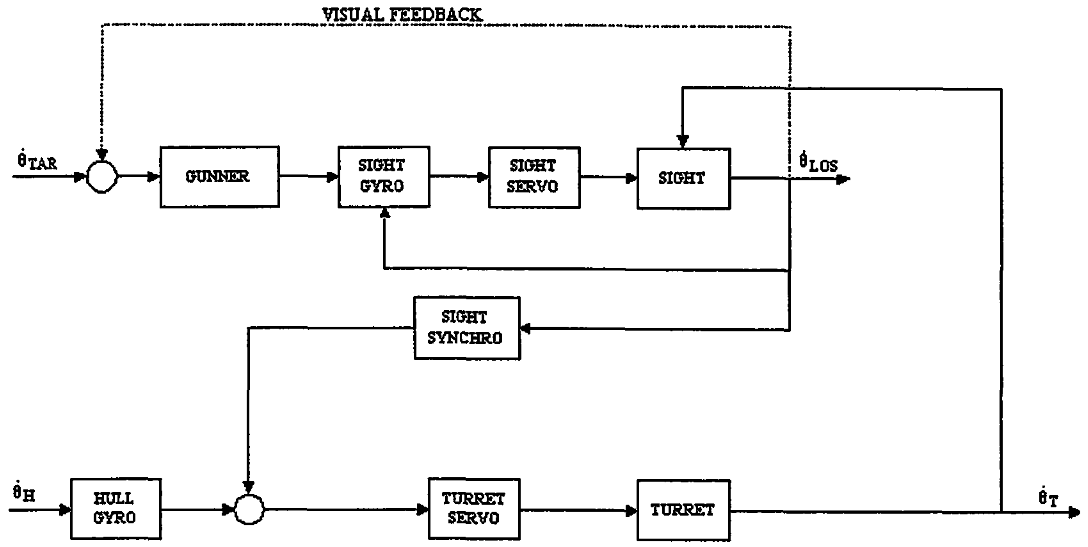

Figure 1-4 Simplified block diagram of a director type system with an independently stabilized gunner's sight and the turret/gun slaved to it. The gun offset command input from the fire control computer has been omitted, for the sake of clarity.

A director-type system based on the independently stabilized gunner's sight and slaved gun was first developed in mid 1960s by the Delco Electronics Division of General Motors. On the other hand, the Chrysler version of the same system has had a gunner's sight independently stabilized only in elevation. In azimuth the sight relies on the stabilization of the turret but the aiming mark, or graticule, is driven in azimuth by a separate servo system so that its motion as observed by the gunner is minimized. At the same time the turret is pseudo-directed in azimuth of its proper aiming angle while the gun is controlled in elevation as in a director-type system. In consequence, it was not considered cost effective to provide independent stabilization for the gunner's sight in azimuth as well as in elevation and it was argued that the stabilization of its head mirror only in elevation would make the sight independently stabilized about both axes and its performance characteristics are, inevitably, inferior to theirs in some respects.

A much simpler alternative to the director-type systems has been developed in Sweden by Bofors Aerotronics and in Britain by Marconi Command and Control Systems in the form of a pseudo-director system. With this system the sight remains slaved to the gun but the injected aiming mark, or graticule, is decoupled from it and stabilized. In consequence, when the gunner aims at a target he only drives the aiming mark. However, while he is laying the aiming mark on the target the fire control computer compares the position of the aiming mark with that of the gun and commands the appropriate gun offsets.

The advantage of the pseudo-director system is that it provides a highly stabilized aiming mark without the cost and complexity of an independently stabilized sight and can be applied readily to existing tanks with simple telescopic or periscopic sights mechanically linked to tank guns. In fact, Bofors Aerotronics produced their system, from 1983 onwards, for retrofitting British Centurion tanks. Unfortunately, the pseudo-director system does nothing to improve the stability of the image which the gunner sees. It does nothing, therefore, to improve his ability to detect and to identify targets on the move, which he needs to do in the first instance.

Because of the shortcomings of the alternatives, several more recently designed tanks have followed having director-type fire control systems with a gunner's sight independently stabilized in azimuth as well as elevation. They include the Leopard 2 and, more recently, the South Korean type 88, the Osorio developed by Engesa, the Italian C-1 Ariete and the Japanese TK-X.

Another refinement which has accompanied the development of director-type systems is the introduction of rate-aided tracking. This represents an advance on the compensation for the angular motion of tanks which, in principle, is all that the stabilization systems were intended to do until then. The introduction of rate-aided tracking means that it is no longer left entirely to the gunner to compensate for the relative translational motion between his tank and the target. In particular, rate-aided tracking compensates the direction of the gunner's line of sight for the translational motion of his own tank.

The addition of open-loop rate-aiding involves the provision of supplementary sensors in the form of accelerometers to measure the translational disturbances as well as the speed of the tank normal to the gunner's line of sight. It also involves processing of the signals, but this can be taken care of by the digital fire control computers. When rate-aided tracking is added the gunner still has to compensate, by himself, for any changes in the motion of the target. Nevertheless, it improves his performance considerably. The MBT-70 and XM-803 were actually the first tanks to incorporate rate aided tracking, which was developed for them by Delco Electronics and their example was followed by the Leopard 2, which became the first tank to go into service with it [1].

1.2 Aim and Scope of the Study

In this study, stabilization of a tank gun which has two degrees of freedom in azimuth and elevation is considered. Since the objective of the tank stabilization system is to provide the ability of hitting targets while the tank is on the move, disturbances during the motion of the vehicle should be taken into account.

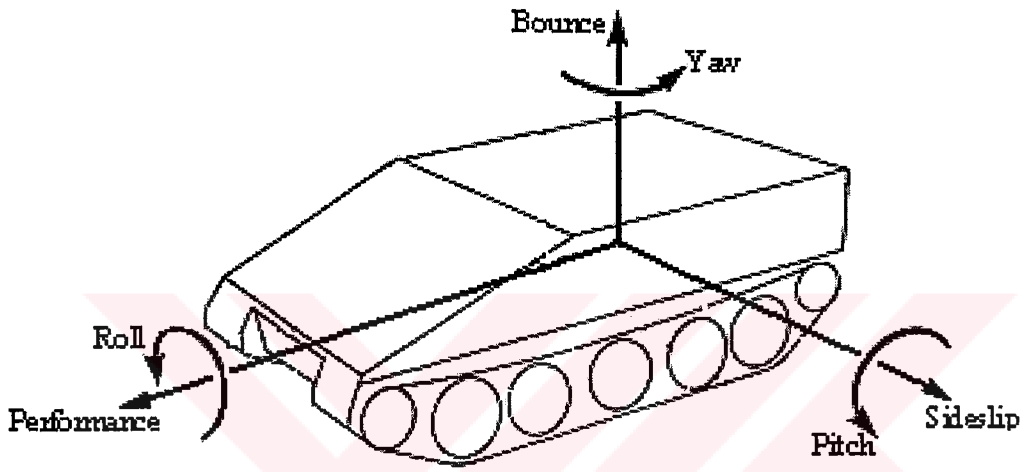

Figure 1-5 Motion freedoms of tank hull

Motion freedoms of a tank hull are shown in Fig 1-5 [3]. In Chapter 2, equations of motion of a tank gun system are derived including the possible motions of a tank illustrated in this figure. Newton-Euler equations are used for derivation and this system is arranged so that if the motion of the tank hull as well as the motor torques in azimuth and elevation are given, it is possible to find the tank gun angles.

In the tank gun stabilization system, tank hull-ground surface interactions has to be considered so that the gun is able to fire on the move. A surface model is generated mathematically in this study. Position, velocity and acceleration of tank hull is found by using this modeled surface.

The gun and hull motions are measured by some devices such as gyroscopes and accelerometers. Kinematic variables measured by these devices are contaminated with errors and noise associated with their electromechanical characteristics. Matlab/Simulink is used for obtaining the measured kinematic variables of the tank.

In a tank gun stabilization system, either hydraulic or electrical motors are used for traversing and elevating the gun. Sample hydraulic motor systems used in azimuth and elevation directions are modeled by using Matlab/Simulink and these models are used for obtaining the torques in the stabilization systems considered in Chapter 3.

After modeling the gun control equipments, the method that is to be used for stabilizing the tank gun is considered. Selection of the stabilization method is based on the requirement that the gun system should be able to hit or track its target as precisely as possible to increase its mobility. The method that is considered for this reason is the computed torque method. In this method, the disturbances from the tank hull which effect the tank gun stabilization system are measured, and the gunner's commands are known. Equations of motion of the tank gun system are rearranged to solve for torques which are necessary for driving the tank gun, and the computed torques are used in turn for finding the driving motor torques of the system by using the actuator models. Obtaining the motor torques in azimuth and elevation, they are used again to solve for the gun acceleration, velocity and position. The error between the desired position and the actual position is found and by using this comparison, the necessary torque required for obtaining the desired position of the tank gun is found. Since all of the possible disturbances which effect the gun system are taken into account in computing the driving torques, it is expected that the computed torque method will give good results for stabilizing the gun system. However, the application of this method may be difficult if the cost and the complexity of the system are taken into account because it requires a computer as well as plenty of sensors.

A second method with a PID controller using the velocity feedback is also considered in this study because this method is widely used in stabilization systems. In this method, tank hull motion is considered as disturbance and the motor torques are calculated by using the error between the desired velocity and actual velocity of the gun and processed through a PID block. Using the motor response and the tank hull motion, the gun acceleration, velocity and position are calculated. The main advantages of this system are its simplicity and its cost since less hardware is required in this system. It is expected that the results obtained by this method is not satisfactory compared with the computed torque method because the hull motion is not considered in the control method, but taken as a disturbance trying to deviate the gun system from its desired position.

Finally, regarding the above two methods, it is suggested that the computed torque method would be more applicable with the latest advances in quality of the sensors and the computer systems and the decrease in their costs, and their better results.

CHAPTER 2

DYNAMICS OF THE TANK GUN

Since tanks were originally built during the First World War, the great majority of them have had the same basic configuration. This has consisted of a hull with a driving compartment in the rear and in between a fighting compartment surmounted by a rotating turret mounting the tank's armament [1]. Although there are variations in tank components -such as a turretless tank-, the basic configuration is considered as a model in this study to keep the generality of the system. However, the basic configuration of the system is subjected to some simplifications and assumptions in deriving the equations of motion of the system:

- The basic configuration components, that is, tank hull, turret and barrel are assumed to be rigid bodies.
- Track effects, which give flexibility to the system are neglected. In turn, track is considered as rigid and is part of the tank hull.

Vehicle suspensions, which are used to isolate the hull from the accelerations due to ground irregularity and vehicle motion and in turn improve the ride comfort give additional degrees of freedom to the vehicle. The vehicle suspensions are neglected in this study to simplify the dynamics of the system. This simplification is reasonable since the disturbance from the tank hull, not directly from the ground, effects the gun system. And these disturbances can be included to the stabilization system since all degrees of freedom of the tank hull are taken into account. As a result of neglecting the vehicle suspension system, tank hull is assumed to be sliding on the ground. Thus, disturbances from the ground are modeled as if they directly effect the tank hull.

The resultant system is illustrated in Fig 2-1. In addition to performance (forward motion), roll, bounce (upward motion), yaw, sideslip and pitch motions of the tank hull, turret and barrel movements are also considered and the degrees of freedom of the system is 8. Coordinate frames are stated for this system and necessary frame transformations are also formulated in this chapter.

Newton-Euler formulation is used to obtain the equations of motion. In deriving the equations of motion, none of the possible offsets between the link rotation axes and center of gravities are neglected to keep the generality of the system. Kinematic equations for the tank turret and tank barrel are written in sections 2.3, and 2.4. All kinematic variables are expressed in their frames so that inertia matrices remain constant in deriving the equation of motion. Then, the Newton-Euler equations are written for the tank turret and barrel in sections 2.5 and 2.6, and rearranged in section 2.7. Thus, a mathematical model is constructed for the tank using results in section 2.7.

Finally, a sample surface, which will be treated as the source of disturbance from the tank hull to the gun system, is formulated in section 2.8.

2.1 Coordinate Frames

Coordinate frames are defined in order to explain the dynamics of the battle tank system properly. In this study, F_i represents the ith orthogonal, right handed, and equi-scaled coordinate frame and $\vec{u}_k^{(i)}$ represents the kth unit vector associated with this coordinate frame [4].

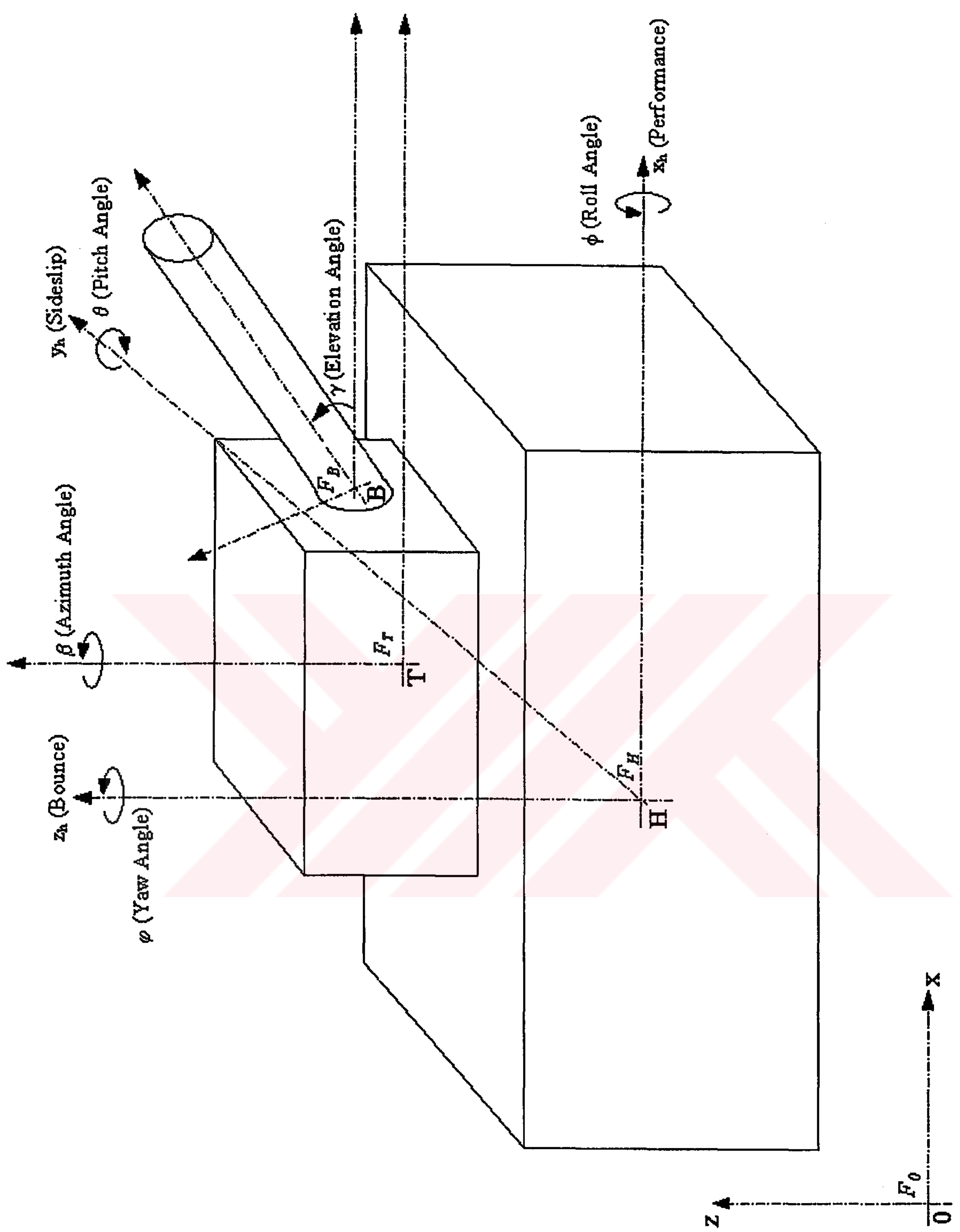

Figure 2-1 Motion freedoms of the tank system considered in this study.

The first coordinate frame that is used in formulating the dynamics of the tank gun system is the earth fixed frame, denoted by F_0, and the unit vectors associated with this coordinate frame are $\vec{u}_1^{(0)}$, $\vec{u}_2^{(0)}$, $\vec{u}_3^{(0)}$ respectively.

Motion of the tank hull is expressed by using 321 (yaw, pitch, roll) Euler angle sequence. As schematically illustrated by equation 2.1, two intermediate frames are defined for this purpose and they are denoted as F_m and F_n. F_h is defined as the tank hull frame. Last two frames are denoted as F_t and F_b. F_t is the tank turret frame and F_b is the tank barrel frame.

In this study the motion of the tank gun system, consisting of the tank turret and tank barrel, with respect to the earth fixed frame is also considered. Motion of the tank gun system is again expressed by using 321 Euler angle sequence. The frames which are required for this motion are denoted as F_a, F_c and F_b respectively as can be seen in equation 2.4. It is noted that these frames are used mainly for the conversion of the motion of the tank gun from earth fixed frame to the tank turret and barrel frames and vice versa. In the next section some frame transformation procedures used for this purpose are formulated [4].

2.2 Coordinate Frame Transformations

In this study, it is necessary to make frame transformations between coordinate frames for the purpose of formulating dynamics of the system properly. In modeling of stabilization systems, the command of gunner positioning the battle tank gun system is considered in earth fixed frame. However, the command of the gunner has to be expressed in tank gun frames so that it can be taken as reference in stabilization systems. In section 2.2.1, the transformation of the reference command from earth fixed frame to the tank gun frames is considered. In modeling of the stabilization systems, the output position and velocity of the tank gun is in the tank gun is directly obtained in the frame F_h, but it is desired that these variables are expressed in the earth fixed frame for comparing the results with the commanded desired values and also for simulating the outputs of the gyroscopes. Hence another procedure for transforming the position and velocity of the tank gun from tank gun frames to the earth fixed frame is also formulated in section 2.2.2.

2.2.1 Transformation from earth fixed frame to tank turret and tank barrel frames

The orientation of the tank barrel with respect to an earth fixed frame can be decomposed into sequential rotations by considering the motion of the tank hull as shown below:

$$F_0 \xrightarrow[\psi]{\vec{u}_3^{(0)}} F_m \xrightarrow[-\theta]{\vec{u}_2^{(m)}} F_n \xrightarrow[\phi]{\vec{u}_1^{(n)}} F_h \xrightarrow[\beta]{\vec{u}_3^{(h)}} F_t \xrightarrow[-\gamma]{\vec{u}_2^{(t)}} F_b \qquad (2.1)$$

where, ψ, θ, ϕ, are the yaw, pitch and roll angles of the tank hull, β is the azimuth angle of the tank turret, and γ is the elevation angle of the tank barrel as can be seen from Figure 2.1.

It is to be noted that the sign of the angles θ and γ are deliberately taken to be negative in order to give a sense of elevating the tank when positive pitch angle is specified for the tank hull and a positive elevation angle is specified to the tank barrel.

Using equation 2.1, transformation matrix from barrel frame to the earth fixed frame can be written as follows:

$$\hat{C}^{(0,b)} = \hat{C}^{(0,m)} \cdot \hat{C}^{(m,n)} \cdot \hat{C}^{(n,h)} \cdot \hat{C}^{(h,t)} \cdot \hat{C}^{(t,b)} \qquad (2.2)$$

Equation 2.2 can be rewritten by using the exponential rotation matrices:

$$\hat{C}^{(0,b)} = e^{\tilde{u}_3 \cdot \psi} \cdot e^{-\tilde{u}_2 \cdot \theta} \cdot e^{\tilde{u}_1 \cdot \phi} \cdot e^{\tilde{u}_3 \cdot \beta} \cdot e^{-\tilde{u}_2 \cdot \gamma} \qquad (2.3)$$

On the other hand, the orientation of the tank barrel can be rewritten by considering its motion in the earth fixed frame as follows:

$$F_0 \xrightarrow[\lambda]{\vec{u}_3^{(0)}} F_a \xrightarrow[-\mu]{\vec{u}_2^{(a)}} F_c \xrightarrow[\delta]{\vec{u}_1^{(c)}} F_b \qquad (2.4)$$

where λ, μ, δ are azimuth, elevation and roll angles of the tank gun, respectively. The transformation matrix from barrel frame to the earth fixed frame can be written as:

$$\hat{C}^{(0,b)} = e^{\tilde{u}_3 \cdot \lambda} \cdot e^{-\tilde{u}_2 \cdot \mu} \cdot e^{\tilde{u}_1 \cdot \delta} \tag{2.5}$$

It is desired to find the azimuth angle (β) of the turret frame and the elevation angle (γ) of the barrel frame for the given (ψ, θ, ϕ) angles of the tank hull and the azimuth and elevation angles (λ, μ) in earth fixed frame. It is noted that the roll angle (δ) in equation 2.4 is not found since the stabilization system is considered in azimuth and elevation directions only, but not in roll direction.

In order to find azimuth and elevation angles; equations 2.4 and 2.5 are equated to each other and rearranged. The following equation is obtained as a result:

$$e^{\tilde{u}_1 \cdot \delta} \cdot e^{\tilde{u}_2 \cdot \gamma} \cdot e^{-\tilde{u}_3 \cdot \beta} = e^{\tilde{u}_2 \cdot \mu} \cdot e^{\tilde{u}_3 \cdot (\psi - \lambda)} \cdot e^{-\tilde{u}_2 \cdot \theta} \cdot e^{\tilde{u}_1 \cdot \phi} \tag{2.6}$$

The right hand side of the equation 2.6 is a 3X3 matrix with known entries and is denoted by $\hat{D}$. Left hand side of the same equation is an unknown 3X3 matrix and its elements are formed in terms of azimuth and elevation angles and the roll angle. The desired angles can be found by equating the elements of the matrix equation 2.6.

Let the element in the first row and the third column of the matrix $\hat{D}$ be denoted by d_{13}. Then;

$$\bar{u}_1^t \cdot e^{\tilde{u}_1 \cdot \delta} \cdot e^{\tilde{u}_2 \cdot \gamma} \cdot e^{-\tilde{u}_3 \cdot \beta} \cdot \bar{u}_3 = d_{13} \tag{2.7}$$

By expanding the left side of the above equation, the element corresponding to d_{13} is obtained.

$$\sin(\gamma) = d_{13} \tag{2.8}$$

The value of the element d_{13} can be found by considering the right hand side of equation 2.6 as follows:

$$d_{13} = \bar{u}_1^t \cdot e^{\tilde{u}_2 \cdot \mu} \cdot e^{\tilde{u}_3 \cdot (\psi - \lambda)} \cdot e^{-\tilde{u}_2 \cdot \theta} \cdot e^{\tilde{u}_1 \cdot \phi} \cdot \bar{u}_3 \qquad (2.9)$$

Then, the following expression for d_{13} is obtained:

$$\begin{aligned} d_{13} = & [-\cos(\phi) \cdot \sin(\theta) \cdot \cos(\psi - \lambda) + \sin(\phi) \cdot \sin(\psi - \lambda)] \cdot \cos(\mu) \\ & + \cos(\phi) \cdot \cos(\theta) \cdot \sin(\mu) \end{aligned} \qquad (2.10)$$

The value of d_{13} can be found by using equation 2.10. Then using equation 2.8;

$$\cos(\gamma) = \sqrt{1 - d_{13}^2} \qquad (2.11)$$

Hence,

$$\gamma = a\tan 2\left(d_{13}; \sqrt{1 - d_{13}^2}\right) \qquad (2.12)$$

Elevation angle; γ, can be found by using the equations 2.10 and 2.11.

The azimuth angle in azimuth frame can be found in a similar manner. From equation 2.6;

$$\bar{u}_1^t \cdot e^{\tilde{u}_1 \cdot \delta} \cdot e^{\tilde{u}_2 \cdot \gamma} \cdot e^{-\tilde{u}_3 \cdot \beta} \cdot \bar{u}_2 = d_{12} \qquad (2.13)$$

where d_{12} is the element in the first row and the second column in the matrix $\hat{D}$. Expanding the left hand side of equation 2.13;

$$\sin(\beta) \cdot \cos(\gamma) = d_{12} \qquad (2.14)$$

where;

$$d_{12} = \sin(\phi)\cdot\cos(\theta)\cdot\sin(\mu) - \left[\cos(\phi)\cdot\sin(\psi-\lambda)+\sin(\phi)\cdot\sin(\theta)\cdot\cos(\psi-\lambda)\right]\cdot\cos(\mu) \tag{2.15}$$

and similarly,

$$\bar{u}_1^t \cdot e^{\tilde{u}_1\cdot\delta}\cdot e^{\tilde{u}_2\cdot\gamma}\cdot e^{-\tilde{u}_3\cdot\beta}\cdot\bar{u}_1 = d_{11} \tag{2.16}$$

Here, d_{11} is the element in the first row and the first column of the matrix $\hat{D}$. From equation 2.16;

$$\cos(\beta)\cdot\cos(\gamma) = d_{11} \tag{2.17}$$

and;

$$d_{11} = \cos(\theta)\cdot\cos(\psi-\lambda)\cdot\cos(\mu)+\sin(\theta)\cdot\sin(\mu) \tag{2.18}$$

It has to be considered that in general, the range of elevation angle remains limited between the fourth and the first quadrants in barrel coordinate frame. Hence the sign of $\cos(\gamma)$ term in equations 2.14 and 2.17 can be considered as positive. And the azimuth angle β can be found as:

$$\beta = a\tan 2(d_{12};d_{11}) \tag{2.19}$$

So, the azimuth angle β in the tank turret frame can be found by using the equations 2.15, 2.18 and 2.19.

It is noted that in the simulation of the stabilization systems, this formulation should be used for converting the reference position commands to the tank turret and tank barrel frames. In these simulations, reference velocity and acceleration commands are also given in the earth fixed frame. Hence, additional formulations are necessary for converting the velocity and acceleration commands from the earth fixed frame to the tank turret and tank barrel frames.

Let the angular velocity components of the tank hull $(\dot{\psi}, \dot{\theta}, \dot{\phi})$ be given. And also, let angular velocity component of the tank turret in the earth fixed frame $(\dot{\lambda})$ and the angular velocity of the tank barrel in earth fixed frame $(\dot{\mu})$ be known. Additionally, the angles defined for the tank hull (ψ, θ, ϕ), angular positions of tank turret (λ), and tank barrel (μ), are also known. It is desired to find the azimuth angular velocity component in turret frame $(\dot{\beta})$, and also the elevation angular velocity in barrel frame, $(\dot{\gamma})$.

Considering equation 2.1; the angular velocity of the tank barrel can be written as:

$$\vec{\omega} = \dot{\psi} \cdot \vec{u}_3^{(0)} - \dot{\theta} \cdot \vec{u}_2^{(m)} + \dot{\phi} \cdot \vec{u}_1^{(n)} + \dot{\beta} \cdot \vec{u}_3^{(h)} - \dot{\gamma} \cdot \vec{u}_2^{(t)} \tag{2.20}$$

This equation can be expressed in the earth fixed frame as:

$$\begin{aligned} \overline{\omega}^{(0)} = {} & \dot{\psi} \cdot \overline{u}_3 - \dot{\theta} \cdot \hat{C}^{(0,m)} \cdot \overline{u}_2 + \dot{\phi} \cdot \hat{C}^{(0,n)} \cdot \overline{u}_1 + \dot{\beta} \cdot \hat{C}^{(0,h)} \cdot \overline{u}_3 \\ & - \dot{\gamma} \cdot \hat{C}^{(0,t)} \cdot \overline{u}_2 \end{aligned} \tag{2.21}$$

On the other hand, the angular velocity of the tank barrel can also be written by using the equation 2.4 as:

$$\vec{\omega} = \dot{\lambda} \cdot \vec{u}_3^{(0)} - \dot{\mu} \cdot \vec{u}_2^{(a)} + \dot{\delta} \cdot \vec{u}_1^{(c)} \tag{2.22}$$

Equation 2.22 can be expressed in the earth fixed frame as:

$$\overline{\omega}^{(0)} = \dot{\lambda} \cdot \overline{u}_3 - \dot{\mu} \cdot \hat{C}^{(0,a)} \cdot \overline{u}_2 + \dot{\delta} \cdot \hat{C}^{(0,c)} \cdot \overline{u}_1 \tag{2.23}$$

The equations 2.21 and 2.23 are equal to each other. Hence, combining these equations and rearranging, it is obtained that

$$\dot{\gamma}\cdot\hat{C}^{(0,t)}\cdot\bar{u}_2-\dot{\beta}\cdot\hat{C}^{(0,h)}\cdot\bar{u}_3+\dot{\delta}\cdot\hat{C}^{(0,c)}\cdot\bar{u}_1=(\dot{\psi}-\dot{\lambda})\cdot\bar{u}_3+\dot{\mu}\cdot\hat{C}^{(0,a)}\cdot\bar{u}_2 -\dot{\theta}\cdot\hat{C}^{(0,m)}\cdot\bar{u}_2+\dot{\phi}\cdot\hat{C}^{(0,n)}\cdot\bar{u}_1 \quad (2.24)$$

where;

$$\hat{C}^{(0,m)}=e^{\tilde{u}_3\cdot\psi} \quad (2.25)$$

$$\hat{C}^{(0,n)}=e^{\tilde{u}_3\cdot\psi}\cdot e^{-\tilde{u}_2\cdot\theta} \quad (2.26)$$

$$\hat{C}^{(0,h)}=e^{\tilde{u}_3\cdot\psi}\cdot e^{-\tilde{u}_2\cdot\theta}\cdot e^{\tilde{u}_1\cdot\phi} \quad (2.27)$$

$$\hat{C}^{(0,t)}=e^{\tilde{u}_3\cdot\psi}\cdot e^{-\tilde{u}_2\cdot\theta}\cdot e^{\tilde{u}_1\cdot\phi}\cdot e^{\tilde{u}_3\cdot\beta} \quad (2.28)$$

$$\hat{C}^{(0,a)}=e^{\tilde{u}_3\cdot\lambda} \quad (2.29)$$

$$\hat{C}^{(0,c)}=e^{\tilde{u}_3\cdot\lambda}\cdot e^{-\tilde{u}_2\cdot\mu} \quad (2.30)$$

Hence, it is seen that only the angular velocity terms on the left hand side of equation 2.24 are unknowns. They can be found as follows:

Let

$$\hat{A}=[\bar{a}_1 \quad \bar{a}_2 \quad \bar{a}_3] \quad (2.31)$$

where

$$\bar{a}_1 = -\hat{C}^{(0,h)} \cdot \bar{u}_3$$

$$\bar{a}_2 = \hat{C}^{(0,t)} \cdot \bar{u}_2 \qquad (2.32)$$

$$\bar{a}_3 = \hat{C}^{(0,c)} \cdot \bar{u}_1$$

$$\bar{X} = \begin{bmatrix} \dot{\beta} \\ \dot{\gamma} \\ \dot{\delta} \end{bmatrix} \qquad (2.33)$$

and

$$\bar{P} = \left[(\dot{\psi} - \dot{\lambda}) \cdot \bar{u}_3 + \dot{\mu} \cdot \hat{C}^{(0,a)} \cdot \bar{u}_2 - \dot{\theta} \cdot \hat{C}^{(0,m)} \cdot \bar{u}_2 + \dot{\phi} \cdot \hat{C}^{(0,n)} \cdot \bar{u}_1\right] \qquad (2.34)$$

Then equation 2.24 can be written in the form

$$\hat{A} \cdot \bar{X} = \bar{P}$$

$\bar{X}$ can be found as;

$$\bar{X} = \hat{A}^{-1} \cdot \bar{P} \qquad (2.35)$$

$\dot{\beta}$ and $\dot{\gamma}$ can be found by extracting the first and the second elements of $\bar{X}$.

The angular accelerations ($\ddot{\beta}$ and $\ddot{\gamma}$) can be found by taking the derivative of the equation 2.24. It is considered that the angular components of the tank hull, $(\ddot{\psi}, \ddot{\theta}, \ddot{\phi})$ are known. The angular acceleration of the tank turret in the earth fixed frame $(\ddot{\lambda})$, and the angular acceleration of the tank barrel in the earth fixed frame $(\ddot{\mu})$ are also given. In addition to the angular acceleration terms stated above, the angular velocity components, $\dot{\psi}, \dot{\theta}, \dot{\phi}, \dot{\beta}, \dot{\gamma}, \dot{\lambda}, \dot{\mu}$, and the angles $\psi, \theta, \phi, \beta, \gamma, \lambda, \mu$, which are explained previously, are also known. Taking the derivative of the equation 2.24 and rearranging the terms, the following equation is found;

$$\begin{aligned}\ddot{\delta}\cdot\hat{C}^{(0,b)}\cdot\overline{u}_1-\ddot{\beta}\cdot\hat{C}^{(0,h)}\cdot\overline{u}_3+\ddot{\gamma}\cdot\hat{C}^{(0,t)}\cdot\overline{u}_2 &= (\ddot{\psi}-\ddot{\lambda})\cdot\overline{u}_3-\ddot{\mu}\cdot\hat{C}^{(0,a)}\cdot\overline{u}_2-\dot{\mu}\cdot\dot{\hat{C}}^{(0,a)}\cdot\overline{u}_2\\ &\quad -\dot{\delta}\cdot\dot{\hat{C}}^{(0,c)}\cdot\overline{u}_1-\ddot{\theta}\cdot\hat{C}^{(0,m)}\cdot\overline{u}_2-\dot{\theta}\cdot\dot{\hat{C}}^{(0,m)}\cdot\overline{u}_2\\ &\quad +\ddot{\phi}\cdot\hat{C}^{(0,n)}\cdot\overline{u}_1+\dot{\phi}\cdot\dot{\hat{C}}^{(0,n)}\cdot\overline{u}_1\dot{\beta}\cdot\dot{\hat{C}}^{(0,h)}\cdot\overline{u}_3\\ &\quad -\dot{\gamma}\cdot\dot{\hat{C}}^{(0,t)}\cdot\overline{u}_2\end{aligned} \tag{2.36}$$

where;

$$\dot{\hat{C}}^{(0,a)}=\dot{\lambda}\cdot\widetilde{u}_3\cdot e^{\widetilde{u}_3\cdot\lambda}=\dot{\lambda}\cdot\widetilde{u}_3\cdot\hat{C}^{(0,a)}$$

$$\dot{\hat{C}}^{(0,c)}=\dot{\lambda}\cdot\widetilde{u}_3\cdot e^{\widetilde{u}_3\cdot\lambda}\cdot e^{-\widetilde{u}_2\cdot\mu}-\dot{\mu}\cdot e^{\widetilde{u}_3\cdot\lambda}\cdot\widetilde{u}_2\cdot e^{-\widetilde{u}_2\cdot\mu}$$

$$\dot{\hat{C}}^{(0,m)}=\dot{\psi}\cdot\widetilde{u}_3\cdot e^{\widetilde{u}_3\cdot\varphi}=\dot{\psi}\cdot\widetilde{u}_3\cdot\hat{C}^{(0,m)}$$

$$\dot{\hat{C}}^{(0,n)}=\dot{\psi}\cdot\widetilde{u}_3\cdot e^{\widetilde{u}_3\cdot\psi}\cdot e^{-\widetilde{u}_2\cdot\theta}-\dot{\theta}\cdot e^{\widetilde{u}_3\cdot\psi}\cdot\widetilde{u}_2\cdot e^{-\widetilde{u}_2\cdot\theta}=\dot{\psi}\cdot\widetilde{u}_3\cdot\hat{C}^{(0,n)}-\dot{\theta}\cdot\hat{C}^{(0,n)}\cdot\widetilde{u}_2$$

$$\dot{\hat{C}}^{(0,h)}=\dot{\psi}\cdot\widetilde{u}_3\cdot\hat{C}^{(0,h)}-\dot{\theta}\cdot e^{\widetilde{u}_3\cdot\psi}\cdot\widetilde{u}_2\cdot e^{-\widetilde{u}_2\cdot\theta}\cdot e^{\widetilde{u}_1\cdot\phi}+\dot{\phi}\cdot\hat{C}^{(0,h)}\cdot\widetilde{u}_1$$

$$\begin{aligned}\dot{\hat{C}}^{(0,t)} &= \dot{\psi}\cdot\widetilde{u}_3\cdot\hat{C}^{(0,t)}-\dot{\theta}\cdot e^{\widetilde{u}_3\cdot\psi}\cdot\widetilde{u}_2\cdot e^{-\widetilde{u}_2\cdot\theta}\cdot e^{\widetilde{u}_1\cdot\phi}\cdot e^{\widetilde{u}_3\cdot\beta}\\ &\quad +\dot{\phi}\cdot e^{\widetilde{u}_3\cdot\psi}\cdot e^{-\widetilde{u}_2\cdot\theta}\cdot\widetilde{u}_1\cdot e^{\widetilde{u}_1\cdot\phi}\cdot e^{\widetilde{u}_3\cdot\beta}+\dot{\beta}\cdot\hat{C}^{(0,t)}\cdot\widetilde{u}_3\end{aligned}$$

The above equations can be written in matrix form:

$$\hat{B}\cdot\overline{Y}=\overline{R}$$

It is noted that from equation 2.35; the matrix $\hat{B}$ remains the same as given in equations 2.31 and 2.32. The contents of the other terms in equation 2.36 are given as follows:

$$\overline{Y} = \begin{bmatrix} \ddot{\beta} \\ \ddot{\gamma} \\ \ddot{\delta} \end{bmatrix} \quad (2.37)$$

$$\begin{aligned} \overline{R} = (\ddot{\psi} - \ddot{\lambda}) \cdot \overline{u}_3 - \ddot{\mu} \cdot \hat{C}^{(0,a)} \cdot \overline{u}_2 - \dot{\mu} \cdot \dot{\hat{C}}^{(0,a)} \cdot \overline{u}_2 - \dot{\delta} \cdot \dot{\hat{C}}^{(0,c)} \cdot \overline{u}_1 \\ - \ddot{\theta} \cdot \hat{C}^{(0,m)} \cdot \overline{u}_2 - \dot{\theta} \cdot \dot{\hat{C}}^{(0,m)} \cdot \overline{u}_2 + \ddot{\phi} \cdot \hat{C}^{(0,n)} \cdot \overline{u}_1 + \dot{\phi} \cdot \dot{\hat{C}}^{(0,n)} \cdot \overline{u}_1 \\ \dot{\beta} \cdot \dot{\hat{C}}^{(0,h)} \cdot \overline{u}_3 - \dot{\gamma} \cdot \dot{\hat{C}}^{(0,t)} \cdot \overline{u}_2 \end{aligned} \quad (2.38)$$

$$\overline{Y} = \hat{B}^{-1} \cdot \overline{R} \quad (2.39)$$

the angular acceleration terms in azimuth and elevation can be found by taking the first and the second elements of the column matrix $\overline{Y}$ [4, 5].

2.2.2 Transformation from tank turret and tank barrel frames to the earth fixed frame

This problem is formulated because of the necessity of obtaining the barrel position and velocity in the earth fixed frame and thus simulating the outputs of the measurement devices, i.e. gyroscopes. So, the angles λ, and μ have to be found for the given $\psi, \theta, \phi, \beta, \gamma$ angles defined in the previous sections.

Considering the equation 2.6 and rearranging; the following equation is obtained.

$$e^{\tilde{u}_3 \cdot \lambda} \cdot e^{-\tilde{u}_2 \cdot \mu} \cdot e^{\tilde{u}_1 \cdot \delta} = e^{\tilde{u}_3 \cdot \psi} \cdot e^{-\tilde{u}_2 \cdot \theta} \cdot e^{\tilde{u}_1 \cdot \phi} \cdot e^{\tilde{u}_3 \cdot \beta} \cdot e^{-\tilde{u}_2 \cdot \gamma} \quad (2.40)$$

The right hand side of the equation 2.40 is a 3X3 matrix with known entries and is denoted by $\widehat{D}$, Left hand side of the same equation is an unknown 3X3 matrix and its elements are formed in terms of λ, μ, δ. The desired angles can be found by following the same procedure as in section 2.2.1.

Let d_{31} be the element in the third row and the first column in the known matrix $\hat{D}$. Then,

$$\bar{u}_3^t \cdot e^{\tilde{u}_3 \cdot \lambda} \cdot e^{-\tilde{u}_2 \cdot \mu} \cdot e^{-\tilde{u}_1 \cdot \delta} \cdot \bar{u}_1 = d_{31} \qquad (2.41)$$

Expanding the left hand side of the above equation;

$$\sin(\mu) = d_{31} \qquad \text{and} \qquad \cos(\mu) = \sqrt{1 - d_{31}^2}$$

So,

$$\mu = a\tan 2(d_{31}; \sqrt{1 - d_{31}^2}) \qquad (2.42)$$

The value of the element c_{31} can be found by expanding the right hand side of the equation 2.40 as:

$$\begin{aligned} d_{31} &= \cos(\gamma) \cdot \cos(\beta) \cdot \sin(\theta) + (\cos(\gamma) \cdot \sin(\beta) \cdot \sin(\phi) \\ &\quad + \sin(\gamma) \cdot \cos(\phi)) \cdot \cos(\theta) \end{aligned} \qquad (2.43)$$

Hence, the elevation angle; μ can be found by using the equations 2.42 and 2.43. The azimuth angle λ can be found in a similar manner from equation 2.40:

$$\bar{u}_2^t \cdot e^{\tilde{u}_3 \cdot \lambda} \cdot e^{-\tilde{u}_2 \cdot \mu} \cdot e^{-\tilde{u}_1 \cdot \delta} \cdot \bar{u}_1 = d_{21} \qquad (2.44)$$

The term d_{21} in equation 2.44 is the element in the second row and the first column in the matrix $\hat{D}$. By using 2.44;

$$\cos(\mu) \cdot \sin(\lambda) = d_{21} \qquad (2.45)$$

Similarly;

$$\bar{u}_1^t \cdot e^{\tilde{u}_3 \cdot \lambda} \cdot e^{-\tilde{u}_2 \cdot \mu} \cdot e^{-\tilde{u}_1 \cdot \delta} \cdot \bar{u}_1 = d_{11}$$

and

$$\cos(\mu) \cdot \cos(\lambda) = d_{11} \tag{2.46}$$

Using equations 2.45 and 2.46, the following equation is obtained for the azimuth angle.

$$\lambda = a\tan 2(d_{21}; d_{11}) \tag{2.47}$$

where;

$$\begin{aligned} d_{21} = {} & (\cos(\gamma) \cdot \cos(\beta) \cdot \cos(\theta) - (\cos(\gamma) \cdot \sin(\beta) \cdot \sin(\phi) \\ & + \sin(\gamma) \cdot \cos(\phi)) \cdot \sin(\theta)) \cdot \sin(\psi) - (\cos(\gamma) \cdot \sin(\beta) \cdot \cos(\phi) \\ & - \sin(\gamma) \cdot \sin(\phi)) \cdot \cos(\psi) \end{aligned} \tag{2.48}$$

$$\begin{aligned} d_{11} = {} & (\cos(\gamma) \cdot \cos(\beta) \cdot \cos(\theta) - (\cos(\gamma) \cdot \sin(\beta) \cdot \sin(\phi) + \sin(\gamma) \\ & \cdot \cos(\phi)) \cdot \sin(\theta)) \cdot \cos(\psi) - (\cos(\gamma) \cdot \sin(\beta) \cdot \cos(\phi) \\ & - \sin(\gamma) \cdot \sin(\phi)) \cdot \sin(\psi) \end{aligned} \tag{2.49}$$

It is noted that the solution for the roll angle δ is not formulated since it will not be used in this work.

The angular velocity terms, $\dot{\lambda}, \dot{\mu}$ can also be found by using the same procedure in section 2.2.1. Rearranging equation 2.24;

$$\begin{aligned} \dot{\lambda} \cdot \bar{u}_3 - \dot{\mu} \cdot \hat{C}^{(0,a)} \cdot \bar{u}_2 + \dot{\delta} \cdot \hat{C}^{(0,b)} \cdot \bar{u}_1 = {} & \dot{\psi} \cdot \bar{u}_3 - \dot{\theta} \cdot \hat{C}^{(0,m)} \cdot \bar{u}_2 + \dot{\phi} \cdot \hat{C}^{(0,n)} \\ & \cdot \bar{u}_1 + \dot{\beta} \cdot \hat{C}^{(0,h)} \cdot \bar{u}_3 - \dot{\gamma} \cdot \hat{C}^{(0,t)} \cdot \bar{u}_2 \end{aligned} \tag{2.50}$$

is obtained. The unknown terms in the above equation are $\dot{\lambda}, \dot{\mu}, \dot{\delta}$ .By using this equation, the following matrix form can be obtained.

$$\hat{E} \cdot \overline{Z} = \overline{F} \tag{2.51}$$

where;

$$\hat{E} = \begin{bmatrix} e_1 & e_2 & e_3 \end{bmatrix} \tag{2.52}$$

$$\begin{aligned} e_1 &= \overline{u}_3 \\ e_2 &= -\hat{C}^{(0,a)} \cdot \overline{u}_2 \\ e_3 &= \hat{C}^{(0,c)} \cdot \overline{u}_1 \end{aligned} \tag{2.53}$$

$$\overline{Z} = \begin{bmatrix} \dot{\lambda} \\ \dot{\mu} \\ \dot{\delta} \end{bmatrix} \tag{2.54}$$

$$\overline{F} = \left[\dot{\psi} \cdot \overline{u}_3 - \dot{\theta} \cdot \hat{C}^{(0,m)} \cdot \overline{u}_2 + \dot{\phi} \cdot \hat{C}^{(0,n)} \cdot \overline{u}_1 + \dot{\beta} \cdot \hat{C}^{(0,h)} \cdot \overline{u}_3 - \dot{\gamma} \cdot \hat{C}^{(0,t)} \cdot \overline{u}_2\right] \tag{2.55}$$

Then, $\overline{Z}$ can be found from the equation 2.50. $\dot{\lambda}$ and $\dot{\mu}$ can be found by extracting the first and the second elements from $\overline{Z}$.

Completing the suitable procedures for coordinate frame transformations used in this study, dynamics of that tank model is discussed in the following sections [4, 5].

2.3 Kinematic Equations for the Tank Turret

The position of the tank turret with respect to the earth fixed frame is shown in figure 2.2.

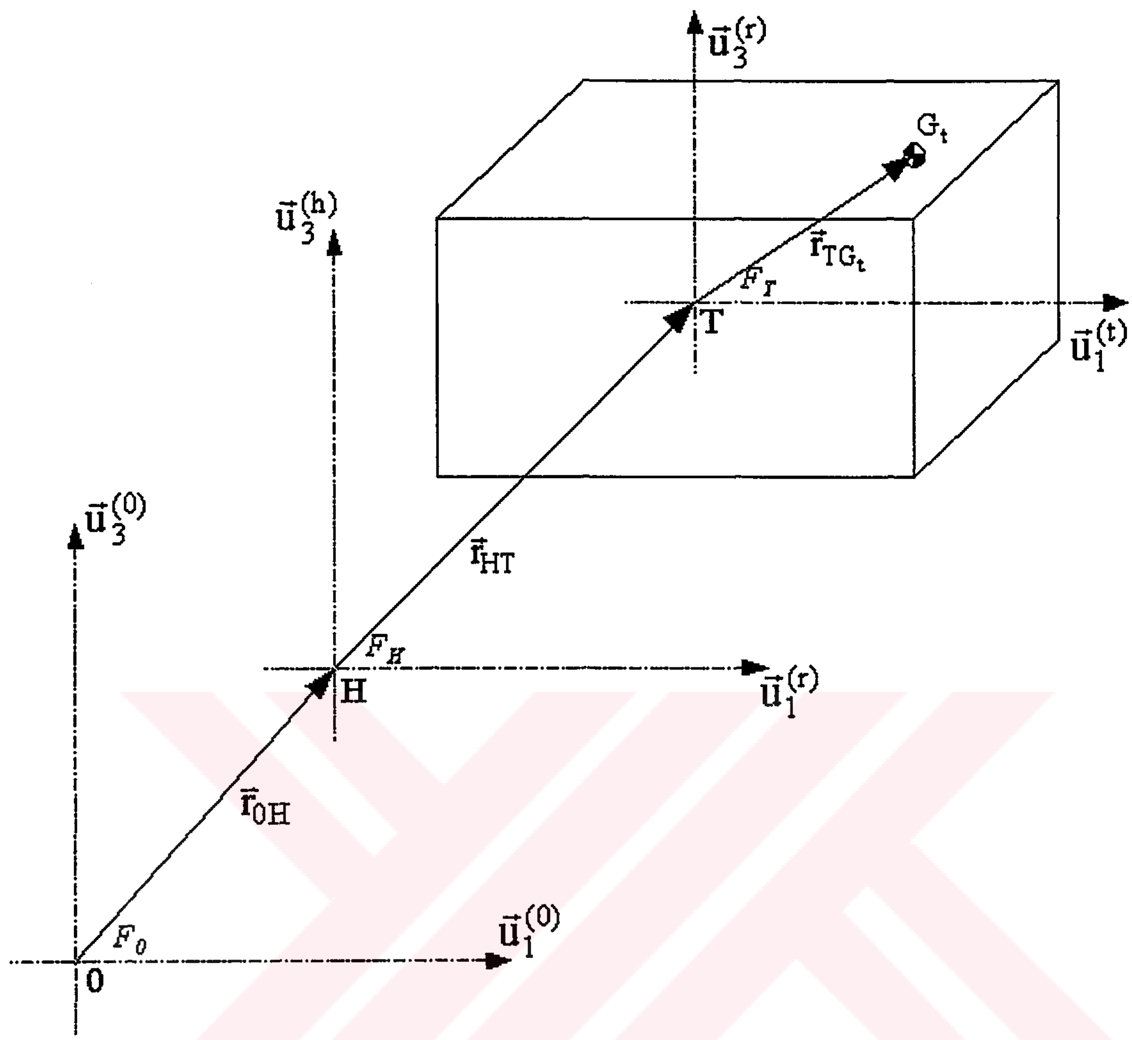

Figure 2-2 Position of the mass center of the tank turret with respect to inertial frame.

The position vector from the earth fixed frame to the to the turret frame can be written as:

$$\vec{r}_{0G_t} = \vec{r}_{0H} + \vec{r}_{HT} + \vec{r}_{TG_t} \tag{2.56}$$

Acceleration of the tank turret in the earth fixed frame can be found by taking the second derivative of 2.56.

$$D_0^2\vec{r}_{0G_t} = D_0^2\vec{r}_{0H} + D_0^2\vec{r}_{HT} + D_0^2\vec{r}_{TG_t} \tag{2.57}$$

Acceleration of the tank mass center of the tank turret can be rewritten in the general form:

$$\vec{a}_H = D_0^2 \vec{r}_{0H}$$

$$\vec{a}_{G_t} = D_0^2 \vec{r}_{0G_t}$$

where;

$$\vec{a}_H = \ddot{x} \cdot \vec{u}_1^{(0)} + \ddot{y} \cdot \vec{u}_2^{(0)} + \ddot{z} \cdot \vec{u}_3^{(0)} \tag{2.58}$$

$$\begin{aligned} \vec{a}_{G_t} = \vec{a}_H &+ \left[D_h^2 \vec{r}_{HT} + 2 \cdot \vec{\omega}_H \times (D_h \vec{r}_{HT}) + \vec{\alpha}_H \times \vec{r}_{HT} + \vec{\omega}_H \times (\vec{\omega}_H \times \vec{r}_{HT}) \right] \\ &+ \left[D_t^2 \vec{r}_{TG_t} + \vec{\omega}_t \times (D_t \vec{r}_{TG_t}) + \vec{\alpha}_t \times \vec{r}_{TG_t} + \vec{\omega}_t \times (\vec{\omega}_t \times \vec{r}_{TG_t}) \right] \end{aligned} \tag{2.59}$$

Since it is assumed that all the members of the tank are rigid;

$$D_h \vec{r}_{HT} = 0 \qquad \text{and} \qquad D_h^2 \vec{r}_{HT} = 0 \tag{2.60}$$

Also;

$$D_t \vec{r}_{TG_t} = 0, \qquad \text{and} \qquad D_t^2 \vec{r}_{TG_t} = 0 \tag{2.61}$$

Equation 2.59 is written in matrix form in the turret frame as follows:

$$\bar{a}_{G_t} = \bar{a}_H^{(t)} + \hat{C}^{(t,h)} \cdot \left[\tilde{\alpha}_h^{(h)} + \left(\tilde{\omega}_h^{(h)} \right)^2 \right] \cdot \bar{r}_{HT}^{(h)} + \left[\tilde{\alpha}_t^{(t)} + \left(\tilde{\omega}_t^{(t)} \right)^2 \right] \cdot \bar{r}_{TG_t}^{(t)} \tag{2.62}$$

where;

$$\bar{a}_H^{(t)} = \hat{C}^{(t,0)} \cdot \bar{a}_H^{(0)}$$

$$\hat{C}^{(t,0)} = e^{-\tilde{u}_3 \cdot \beta} \cdot e^{-\tilde{u}_1 \cdot \phi} \cdot e^{\tilde{u}_2 \cdot \theta} \cdot e^{-\tilde{u}_3 \cdot \psi}$$

$$\hat{C}^{(t,h)} = e^{-\tilde{u}_3 \cdot \beta}$$

It is noted that the transformation matrices illustrated above are found by employing the rotation sequences in equation 2.1.

Angular velocity and acceleration are necessary to find the linear acceleration of the mass center as can be seen from 2.62. First the angular velocity and acceleration of the tank hull are expressed as follows:

$$\vec{\omega}_h = \dot{\psi} \cdot \vec{u}_3^{(m)} - \dot{\theta} \cdot \vec{u}_2^{(n)} + \dot{\phi} \cdot \vec{u}_1^{(h)} \tag{2.63}$$

Equation 2.63 can be written in tank hull frame as:

$$\overline{\omega}_h^{(h)} = \overline{\omega}_h = \dot{\phi} \cdot \overline{u}_1 - \dot{\theta} \cdot \hat{C}^{(h,n)} \cdot \overline{u}_2 + \dot{\psi} \cdot \hat{C}^{(h,m)} \cdot \overline{u}_3 \tag{2.64}$$

where;

$$\hat{C}^{(h,n)} = e^{-\tilde{u}_1 \cdot \phi}$$

$$\hat{C}^{(h,m)} = e^{-\tilde{u}_1 \cdot \phi} \cdot e^{\tilde{u}_2 \cdot \theta}$$

Angular acceleration of the tank hull can be found as:

$$\vec{\alpha}_h = D_h \vec{\omega}_h = D_0 \vec{\omega}_h \tag{2.65}$$

From equation 2.65, it is seen that taking the derivative of the angular velocity of the tank hull is more suitable by using equation2.62. Hence;

$$\overline{\alpha}_h^{(h)} = \ddot{\phi} \cdot \overline{u}_1 - \ddot{\theta} \cdot \hat{C}^{(h,n)} \cdot \overline{u}_2 - \dot{\theta} \cdot \dot{\hat{C}}^{(h,n)} \cdot \overline{u}_2 + \ddot{\psi} \cdot \hat{C}^{(h,m)} \cdot \overline{u}_3 + \dot{\psi} \cdot \dot{\hat{C}}^{(h,m)} \cdot \overline{u}_3 \tag{2.66}$$

$$\dot{\hat{C}}^{(h,n)} = -\dot{\phi}\cdot\tilde{u}_1\cdot e^{-\tilde{u}_1\cdot\phi} = -\dot{\phi}\cdot\tilde{u}_1\cdot\hat{C}^{(h,n)}$$

$$\dot{\hat{C}}^{(h,m)} = -\dot{\phi}\cdot\tilde{u}_1\cdot\hat{C}^{(h,m)} + \dot{\theta}\cdot\hat{C}^{(h,m)}\cdot\tilde{u}_2$$

Rearranging 2.66 with derivative of rotation matrices;

$$\begin{aligned}\overline{\alpha}_h^{(h)} = \ddot{\phi}\cdot\overline{u}_1 - \left(\ddot{\theta}\cdot\hat{I} - \dot{\theta}\cdot\dot{\phi}\cdot\tilde{u}_1\right)\cdot\hat{C}^{(h,n)}\cdot\overline{u}_2 + \\ \left[\ddot{\psi}\cdot\hat{C}^{(h,m)} + \dot{\psi}\cdot\left(-\dot{\phi}\cdot\tilde{u}_1\cdot\hat{C}^{(h,m)} + \dot{\theta}\cdot\hat{C}^{(h,m)}\cdot\tilde{u}_2\right)\right]\cdot\overline{u}_3\end{aligned} \tag{2.67}$$

Then the angular velocity of the tank turret can be found as:

$$\vec{\omega}_t = \vec{\omega}_h + \vec{\omega}_{t/h}$$

$$\vec{\omega}_t = \vec{\omega}_h + \dot{\beta}\cdot\vec{u}_3^{(t)}$$

In column form, the previous equation is expressed as follows:

$$\overline{\omega}_t^{(t)} = \overline{\omega}_t = \hat{C}^{(t,h)}\cdot\overline{\omega}_h^{(h)} + \dot{\beta}\cdot\overline{u}_3 \tag{2.68}$$

where;

$$\hat{C}^{(t,h)} = e^{-\tilde{u}_3\cdot\beta}$$

Similarly;

$$\vec{\alpha}_t = D_t\vec{\omega}_t = D_0\vec{\omega}_t \tag{2.69}$$

$$\overline{\alpha}_t^{(t)} = \overline{\alpha}_t = \hat{C}^{(t,h)}\cdot\overline{\alpha}_h + \dot{\hat{C}}^{(t,h)}\cdot\overline{\omega}_h + \ddot{\beta}\cdot\overline{u}_3 \tag{2.70}$$

where;

$$\dot{\hat{C}}^{(t,h)} = -\dot{\beta} \cdot \tilde{u}_3 \cdot \hat{C}^{(t,h)}$$

Hence;

$$\overline{\alpha}_t^{(t)} = \hat{C}^{(t,h)} \cdot \left(\overline{\alpha}_h - \dot{\beta} \cdot \tilde{u}_3 \cdot \overline{\omega}_h\right) + \ddot{\beta} \cdot \overline{u}_3 \qquad (2.71)$$

The azimuth angular acceleration equation, should be arranged in such a way that it will be composed of an azimuth acceleration component and some other terms.Let

$$\overline{\alpha}_{t_1} = \overline{u}_3$$

$$\overline{\alpha}_{t_2} = \hat{C}^{(t,h)} \cdot \left(\overline{\alpha}_h - \dot{\beta} \cdot \tilde{u}_3 \cdot \overline{\omega}_h\right)$$

Then;

$$\overline{\alpha}_t = \ddot{\beta} \cdot \overline{\alpha}_{t_1} + \overline{\alpha}_{t_2} \qquad (2.72)$$

Acceleration of the turret mass center is also arranged in a similar manner to obtain a term with the azimuth angular component in addition to a component consisting of the other terms. So, by using the equation 2.62 with 2.72;

$$\bar{a}_{G_t} = \bar{a}_{G_{t1}} \cdot \ddot{\beta} + \bar{a}_{G_{t2}},$$

$$\bar{a}_{G_{t1}} = \overline{\alpha}_{t_1} \cdot \bar{r}_{TG_t}, \qquad (2.73)$$

$$\bar{a}_{G_{t2}} = \bar{a}_H^{(t)} + \hat{C}^{(t,h)} \cdot \left[\tilde{\alpha}_h^{(h)} + \left(\tilde{\omega}_h^{(h)}\right)^2\right] \cdot \bar{r}_{HT} + \left[\tilde{\alpha}_t^{(t)} + \left(\tilde{\omega}_t^{(t)}\right)^2\right] \cdot \bar{r}_{TG_t}$$

2.4 Kinematic Equations for the Tank Barrel

The position of the tank barrel with respect to the earth fixed frame is shown in figure 2.3. Hence, the position of the tank barrel can be written as:

$$\vec{r}_{0G_b} = \vec{r}_{0H} + \vec{r}_{HT} + \vec{r}_{TB} + \vec{r}_{BG_b} \tag{2.74}$$

Acceleration of the mass center of the tank barrel can be found by taking the second derivative of the position vector in 2.74.

$$D_0^2\vec{r}_{0G_b} = D_0^2\vec{r}_{0H} + D_0^2\vec{r}_{HT} + D_0^2\vec{r}_{TB} + D_0^2\vec{r}_{BG_b} \tag{2.75}$$

Equation 2.75 can be expanded as follows:

$$\begin{aligned} \vec{a}_{G_b} = \vec{a}_H &+ \left[D_H^2\vec{r}_{HT} + 2\cdot\vec{\omega}_H \times \left(D_h\vec{r}_{HT}\right) + \vec{\alpha}_H \times \vec{r}_{HT} + \vec{\omega}_h \times \left(\vec{\omega}_h \times \vec{r}_{HT}\right)\right] \\ &+ \left[D_t^2\vec{r}_{TB} + 2\cdot\vec{\omega}_t \times \left(D_t\vec{r}_{TB}\right) + \vec{\alpha}_t \times \vec{r}_{TB} + \vec{\omega}_t \times \left(\vec{\omega}_t \times \vec{r}_{TB}\right)\right] \\ &+ \left[D_b^2\vec{r}_{BG_b} + 2\cdot\vec{\omega}_b \times \left(D_b\vec{r}_{BG_b}\right) + \vec{\alpha}_b \times \vec{r}_{BG_b} + \vec{\omega}_b \times \left(\vec{\omega}_b \times \vec{r}_{BG_b}\right)\right] \end{aligned} \tag{2.76}$$

Above equation can be simplified due to the rigid body assumption. In addition to the zero terms in linear acceleration of the tank turret expressed in equations 2.60 and 2.61;

$$D_b\vec{r}_{BG_b} = 0 \qquad \text{and} \qquad D_b^2\vec{r}_{BG_b} = 0 \tag{2.77}$$

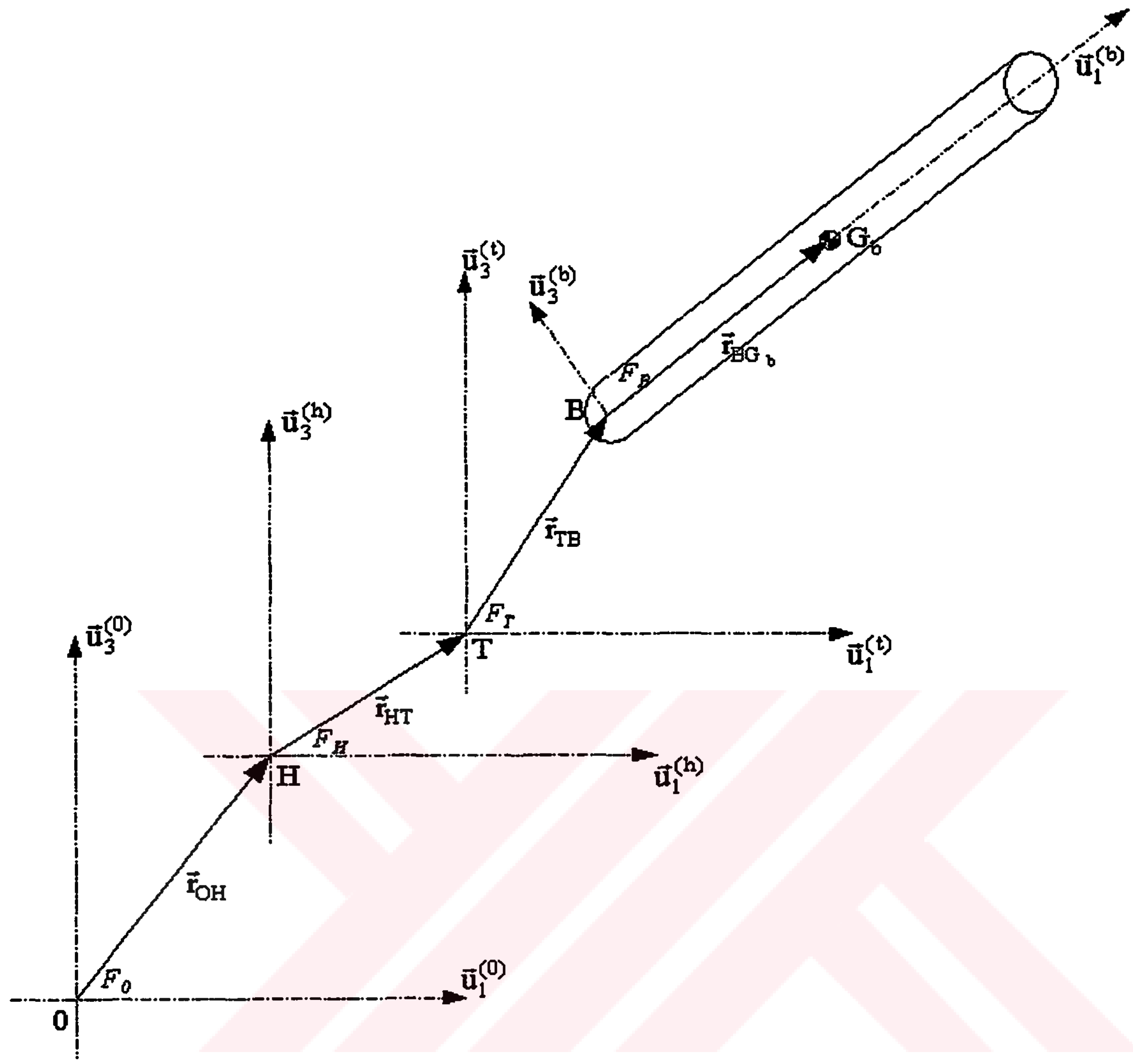

Figure 2-3 Position vector of tank barrel with respect to the earth fixed frame.

Then, equation 2.76 is expressed in matrix form in barrel frame as follows:

$$\bar{a}_{G_b} = \hat{C}^{(b,t)} \cdot \left[\bar{a}_H^{(t)} + \hat{C}^{(t,h)} \cdot \left[\tilde{\alpha}_h^{(h)} + \left(\tilde{\omega}_h^{(h)}\right)^2\right] \cdot \bar{r}_{HT}^{(h)} + \left[\tilde{\alpha}_t^{(t)} + \left(\tilde{\omega}_t^{(t)}\right)^2\right] \cdot \bar{r}_{TB}^{(t)}\right] \qquad (2.78)$$
$$+ \left[\tilde{\alpha}_b^{(b)} + \left(\tilde{\omega}_b^{(b)}\right)^2\right] \cdot \bar{r}_{BG_b}^{(b)}$$

where

$$\hat{C}^{(b,t)} = e^{\tilde{u}_2 \cdot \gamma}$$

Angular velocity and acceleration of the tank barrel can be found as follows:

$$\vec{\omega}_b = \dot{\psi}\cdot\vec{u}_3^{(m)} + \dot{\theta}\cdot\vec{u}_2^{(n)} + \dot{\phi}\cdot\vec{u}_1^{(h)} + \dot{\beta}\cdot\vec{u}_3^{(t)} - \dot{\gamma}\cdot\vec{u}_2^{(b)} \tag{2.79}$$

or in a more compact form:

$$\vec{\omega}_b = \vec{\omega}_h + \vec{\omega}_{t/h} + \vec{\omega}_{b/t} = \vec{\omega}_t - \dot{\gamma}\cdot\vec{u}_2^{(b)} \tag{2.80}$$

The angular velocity can be expressed in barrel frame;

$$\overline{\omega}_b^{(b)} = \hat{C}^{(b,t)}\cdot\overline{\omega}_t^{(t)} - \dot{\gamma}\cdot\overline{u}_2^{(b)} \tag{2.81}$$

Angular acceleration can be obtained by taking derivative of the velocity expression;

$$\vec{\alpha}_b = D_b\vec{\omega}_b = D_0\vec{\omega}_b \tag{2.82}$$

Equation 2.82 can be expressed in barrel frame as:

$$\overline{\alpha}_b^{(b)} = \hat{C}^{(b,t)}\cdot\left[\ddot{\beta}\cdot\overline{\alpha}_{t_1}^{(t)} + \overline{\alpha}_{t_2}^{(t)} + \dot{\gamma}\cdot\tilde{u}_2\cdot\overline{\omega}_t^{(t)}\right] - \ddot{\gamma}\cdot\overline{u}_2 \tag{2.83}$$

The angular acceleration term has to be decomposed such that it has some terms related with the azimuth and elevation acceleration components and some other terms. Let

$$\overline{\alpha}_{b_1} = \hat{C}^{(b,t)}\cdot\overline{\alpha}_{t_1}$$

$$\overline{\alpha}_{b_2} = -\overline{u}_2$$

$$\overline{\alpha}_{b_3} = \hat{C}^{(b,t)}\cdot\left[\overline{\alpha}_{t_2}^{(t)} + \dot{\gamma}\cdot\tilde{u}_2\cdot\overline{\omega}_t^{(t)}\right]$$

Then,

$$\overline{\alpha}_b = \overline{\alpha}_{b_1} \cdot \ddot{\beta} + \overline{\alpha}_{b_2} \cdot \ddot{\gamma} + \overline{\alpha}_{b_3} \tag{2.84}$$

The linear acceleration of the mass center of the tank barrel can be rearranged as follows:

$$\overline{a}_{G_{b1}} = \hat{C}^{(b,t)} \cdot \widetilde{\alpha}_{t_1}^{(t)} \cdot \overline{r}_{TB}^{(t)} + \widetilde{\alpha}_{b_1}^{(b)} \cdot \overline{r}_{BG_b}^{(b)}$$

$$\overline{a}_{G_{b2}} = \widetilde{\alpha}_{b_2}^{(b)} \cdot \overline{r}_{BG_b}^{(b)}$$

$$\overline{a}_{G_{b3}} = \hat{C}^{(b,t)} \cdot \left[\overline{a}_H^{(t)} + \hat{C}^{(t,h)} \cdot \left[\widetilde{\alpha}_h^{(h)} + \left(\widetilde{\omega}_h^{(h)} \right)^2 \right] \cdot \overline{r}_{HT}^{(h)} + \left[\widetilde{\alpha}_{t_2}^{(t)} + \left(\widetilde{\omega}_t^{(t)} \right)^2 \right] \cdot \overline{r}_{TB}^{(t)} \right] + \left[\widetilde{\alpha}_{b3}^{(b)} + \left(\widetilde{\omega}_b^{(b)} \right)^2 \right] \cdot \overline{r}_{BG_b}^{(b)}$$

Then the linear acceleration is written as:

$$\overline{a}_{G_b} = \overline{a}_{G_{b1}} \cdot \ddot{\beta} + \overline{a}_{G_{b2}} \cdot \ddot{\gamma} + \overline{a}_{G_{b3}} \tag{2.85}$$

2.5 Newton-Euler Equations for Tank Turret

Newton-Euler equations are used to solve the dynamics of the system. The forces and moments acting on the tank turret are defined as follows:

$$\overline{M}_{bt} = \overline{M}_{bt}^{(t)} = \begin{bmatrix} M_{bt1} \\ 0 \\ M_{bt3} \end{bmatrix} \quad , \quad \overline{M}_{ht} = \overline{M}_{ht}^{(t)} = \begin{bmatrix} M_{ht1} \\ M_{ht2} \\ 0 \end{bmatrix}$$

$$\overline{T}_\beta = \overline{T}_\beta^{(t)} = T_\beta \cdot \overline{u}_3 \quad , \quad \overline{T}_\gamma = \overline{T}_\gamma^{(t)} = T_\gamma \cdot \overline{u}_2$$

$$\overline{F}_{ht} = \overline{F}_{ht}^{(t)} = \begin{bmatrix} f_{ht1} \\ f_{ht2} \\ f_{ht3} \end{bmatrix} \quad , \quad \overline{F}_{bt} = \overline{F}_{bt}^{(t)} = \begin{bmatrix} f_{bt1} \\ f_{bt2} \\ f_{bt3} \end{bmatrix}$$

where $\overline{M}_{bt}$ is the moment acting from barrel to turret, $\overline{M}_{ht}$ is the moment acting from hull to turret, $\overline{T}_{\beta}$ and $\overline{T}_{\gamma}$ are the driving torques in azimuth and elevation respectively. $\overline{F}_{ht}$ is the force acting from hull to turret, and $\overline{F}_{bt}$ is the force acting from barrel to turret.

Gravitational acceleration is expressed in turret frame in 2.20.

$$\begin{aligned} \overline{g}^{(t)} &= \hat{C}^{(t,0)} \cdot (-g \cdot \overline{u}_3) \\ &= -g \cdot \hat{C}^{(t,0)} \cdot \overline{u}_3 \end{aligned} \tag{2.86}$$

Using the free body diagram in Fig 2-4, Newton-Euler equations are written around mass center of the tank turret:

$$m_t \cdot \overline{a}_{G_t} = \overline{F}_{ht} + \overline{F}_{bt} + m_t \cdot \overline{g}^{(t)}$$

$$\hat{J}_{Gt} \cdot \overline{\alpha}_t + \tilde{\omega}_t \cdot \hat{J}_{Gt} \cdot \overline{\omega}_t = \overline{M}_{bt} + \overline{M}_{ht} + \tilde{r}_{ht} \cdot \overline{F}_{ht} + \tilde{r}_{bt} \cdot \overline{F}_{bt} + \overline{T}_{\beta} + \overline{T}_{\gamma}$$

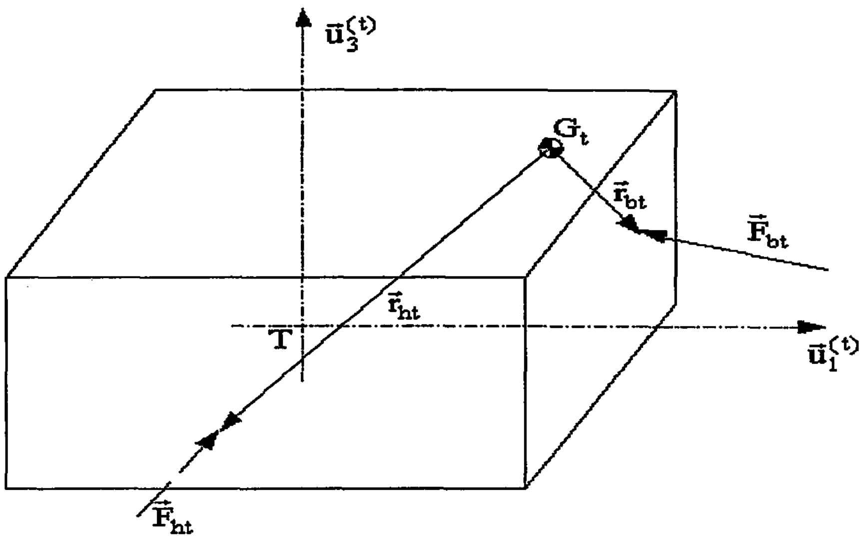

Figure 2-4 Free Body Diagram of the tank turret

The above equation set can be arranged so that it also includes azimuth and elevation angular acceleration terms. The resultant equations are:

$$\left(m_t \cdot \bar{a}_{G_{t1}}\right)\cdot\ddot{\beta} - \bar{F}_{ht}^{(t)} - \bar{F}_{bt}^{(t)} = m_t \cdot \left(\bar{g}^{(t)} - \bar{a}_{G_{t2}}\right) \quad (2.87)$$

$$\left(\hat{J}_{Gt} \cdot \bar{\alpha}_{t1}\right)\cdot\ddot{\beta} - \bar{M}_{bt}^{(t)} - \bar{M}_{ht}^{(t)} - \tilde{r}_{ht} \cdot \bar{F}_{ht}^{(t)} - \tilde{r}_{bt} \cdot \bar{F}_{bt}^{(t)} = (-\hat{J}_{Gt} \cdot \bar{\alpha}_{t2} - \tilde{\omega}_t \cdot \hat{J}_{Gt} \cdot \bar{\omega}_t) + \bar{T}_{\beta}^{(t)} + \bar{T}_{\gamma}^{(t)} \quad (2.88)$$

2.6 Newton-Euler Equations for the Tank Barrel

Forces acting on the barrel are shown in Fig 2-5[4, 5] and they are denoted as follows:

$\bar{F}_{tb}^{(b)} = -\hat{C}^{(b,t)} \cdot \bar{F}_{bt}^{(t)}$, $\bar{M}_{tb}' = -\hat{C}^{(b,t)} \cdot \bar{M}_{bt}^{(t)}$

$\bar{T}_{\gamma} = -T_{\gamma} \cdot \bar{u}_2$, $\vec{r}_{G_bB} = -\vec{r}_{BG_b}$

where $\overline{F}_{tb}$ represents the force acting from the turret to the barrel, $\overline{M}_{tb}$ is the moment acting from the tank turret to the tank barrel, and $\overline{T}_{\gamma}$ is the driving torque for elevating the tank barrel.

$$\overline{g}^{(b)} = \hat{C}^{(b,t)} \cdot \overline{g}^{(t)}$$

It is noted that the necessary terms tabulated above are expressed in barrel frame.

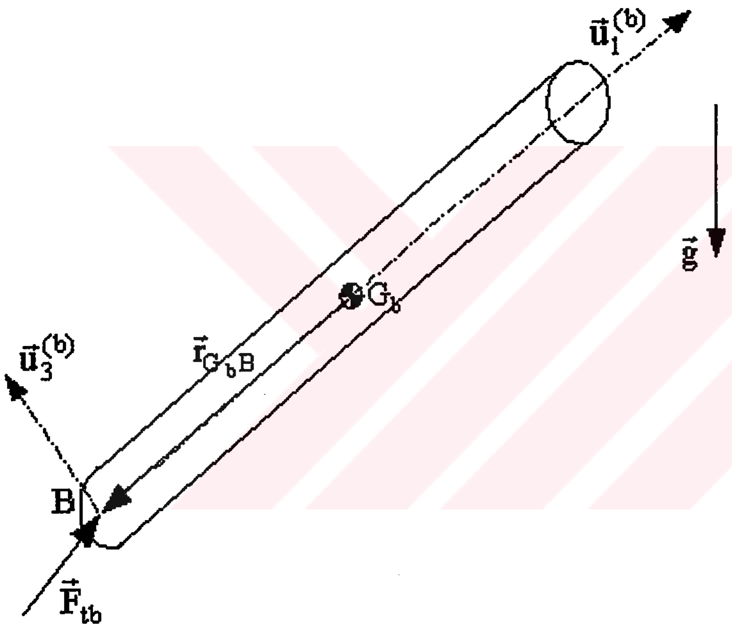

Figure 2-5 Free body diagram of the tank barrel

Newton-Euler equations are written about the mass center of the tank barrel as follows:

$$m_b \cdot \overline{a}_{G_b} = \overline{F}_{tb}^{(b)} + m_b \cdot \overline{g}^{(b)}$$

$$\hat{J}_{G_b} \cdot \overline{\alpha}_b + \widetilde{\omega}_b \cdot \hat{J}_{G_b} \cdot \overline{\omega}_b = \overline{M}_{tb} + \widetilde{r}_{G_bB} \cdot \overline{F}_{tb} + \overline{T}_{\gamma}$$

The above equation set can be rearranged so that it contains the azimuth and angular acceleration of the system.

$$\left(m_b \cdot \bar{a}_{G_{b1}}\right) \cdot \ddot{\beta} + \left(m_b \cdot \bar{a}_{G_{b2}}\right) \cdot \ddot{\gamma} - \bar{F}_{tb} = m_b \cdot \left(-\bar{a}_{G_{b3}} + \bar{g}^{(b)}\right) \tag{2.89}$$

$$\left(\hat{J}_{Gb} \cdot \bar{\alpha}_{b1}\right) \cdot \ddot{\beta} + \left(\hat{J}_{Gb} \cdot \bar{\alpha}_{b2}\right) \cdot \ddot{\gamma} - \bar{M}_{tb} - \tilde{r}_{GbB} \cdot \bar{F}_{tb} = \left(-\hat{J}_{Gb} \cdot \bar{\alpha}_{b3} - \tilde{\omega}_b \cdot \hat{J}_{Gb} \cdot \bar{\omega}_b\right) + \bar{T}_{\gamma} \tag{2.90}$$

2.7 Tank Gun Model

Newton-Euler equations for the tank turret and the barrel can be rearranged to solve for the acceleration, velocity and the position of the gun system. It has been seen that the equation sets are highly nonlinear and the analytical solution of the equation set is very difficult. So, numerical solution is considered in this study. First acceleration terms are treated as unknowns together with the force and moment terms. Velocity and position in azimuth and elevation directions are considered as known. Then the Newton Euler equations for the tank turret and barrel become:

$$\hat{H} \cdot \bar{U} = \bar{G} \tag{2.91}$$

where $\hat{H}$ is an 12X12 matrix with the elements given below:

$$\hat{H} = \left[\bar{h}_1 \quad \bar{h}_2 \quad \hat{h}_3 \quad \hat{h}_4 \quad \hat{h}_5 \quad \hat{h}_6\right]$$

$$\bar{h}_1^t = \left[(m_t \cdot \bar{a}_{G_t1})^t \quad (\hat{J}_{G_t} \cdot \bar{\alpha}_{t_1})^t \quad (m_b \cdot \bar{a}_{G_b1})^t \quad (\hat{J}_{G_b} \cdot \bar{\alpha}_{b1})^t\right]$$

$$\bar{h}_2^t = \left[\bar{0}^t \quad \bar{0}^t \quad (m_b \cdot \bar{a}_{G_b2})^t \quad (\hat{J}_{G_b} \cdot \bar{\alpha}_{b2})^t\right]$$

$$\hat{h}_3^t = \begin{bmatrix} -\hat{I}^t & -\tilde{r}_{ht}^{\ t} & \hat{0}^t & \hat{0}^t \end{bmatrix}$$

$$\hat{h}_4^t = \begin{bmatrix} -\hat{I}^t & -\tilde{r}_{bt}^{\ t} & (\hat{C}^{(b,t)})^t & (\tilde{r}_{G_bB} \cdot \hat{C}^{(b,t)})^t \end{bmatrix}$$

$$\hat{h}_5^t = \begin{bmatrix} 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\hat{h}_6^t = \begin{bmatrix} 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & \cos(\gamma) & 0 & -\sin(\gamma) \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & \sin(\gamma) & 0 & \cos(\gamma) \end{bmatrix}$$

The column with the unknowns are given as:

$$\overline{U}^t = \begin{bmatrix} \ddot{\beta} & \ddot{\gamma} & \overline{F}_{ht}^t & \overline{F}_{bt}^t & M_{ht1} & M_{ht2} & M_{bt1} & M_{bt3} \end{bmatrix}$$

and also;

$$\overline{G} = \begin{bmatrix} m_t \cdot (\overline{g}_t - \overline{a}_{G_t2}) \\ -(\hat{J}_{G_t} \cdot \overline{\alpha}_{t2} + \tilde{\omega}_t \cdot \hat{J}_{G_t} \cdot \overline{\omega}_t) + \overline{T}_\beta + \overline{T}_\gamma \\ m_b \cdot (\overline{g}_b - \overline{a}_{G_b3}) \\ -(\hat{J}_{G_b} \cdot \overline{\alpha}_{b3} + \tilde{\omega}_b \cdot \hat{J}_{G_b} \cdot \overline{\omega}_b) - \overline{T}_\gamma \end{bmatrix}$$

The acceleration in azimuth and elevation angular accelerations can be solved from 2.21

$$\overline{U} = \hat{H}^{-1} \cdot \overline{G} \tag{2.92}$$

$$\ddot{\beta} = \overline{U}(1) \quad \text{and} \quad \ddot{\gamma} = \overline{U}(2) \tag{2.93}$$

Obtaining the accelerations, azimuth and elevation position and velocities can be obtained by using the state space representation:

Let

$$
\begin{aligned}
s_1 &= \beta \\
s_2 &= \gamma \\
s_3 &= \dot{\beta} \\
s_4 &= \dot{\gamma}
\end{aligned}
$$

Since the acceleration terms are known the velocity terms can be easily found by integrating and the positions can be found by integrating the velocity terms at a given time step. The velocity and position terms are used in the next time step to find the acceleration terms. Hence the algorithm for finding the position, velocity and acceleration of the tank gun system for given torque values are constructed.

2.8 Surface Modeling for the Tank System

In this study, a mathematical model of a sample surface is used to account for the ground-tank hull interactions. A surface equation z=z(x,y) is obtained by using the following formula.

$$
\begin{aligned}
z(x,y) = 0.5 - 0.5\cdot\cos(0.05\cdot x) + 0.6\cdot\sin(0.012\cdot y) \\
+ 0.0005\cdot\sin(0.6\cdot x) + 0.001\cdot\cos(0.8\cdot y)
\end{aligned}
\tag{2.94}
$$

where x and y are functions of time. A surface plot of the above equation is illustrated in figure 2-1

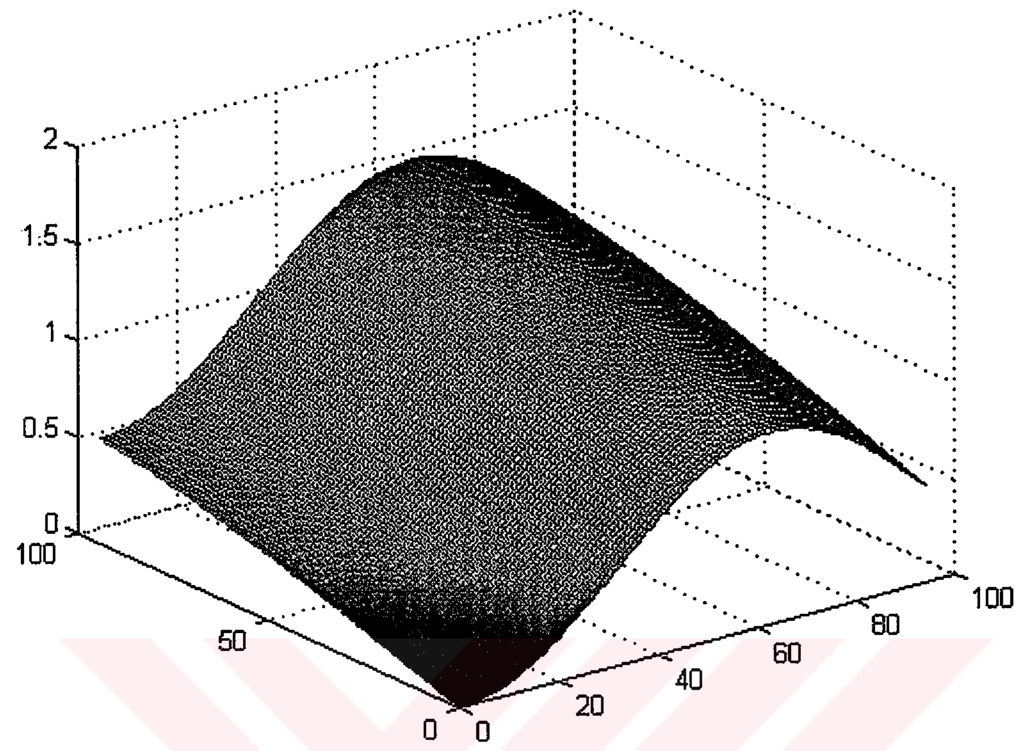

Figure 2-6 Mathematical model of a sample surface

The Linear acceleration terms can be calculated by taking the second derivatives of the position vector.

$$\begin{aligned}\ddot{z} = &\left[0.025\cdot\sin(0.05\cdot x)+0.0003\cdot\cos(0.6\cdot x)\right]\cdot\ddot{x}\\ &+\left[0.00125\cdot\cos(0.05\cdot x)-0.00018\cdot\sin(0.6\cdot x)\right]\cdot\dot{x}^2\\ &+\left[0.0072\cdot\cos(0.012\cdot y)-0.08\cdot\sin(0.8\cdot y)\right]\cdot\ddot{y}\\ &+\left[-0.0000864\cdot\sin(0.012\cdot y)-0.00064\cdot\cos(0.8\cdot y)\right]\cdot\dot{y}^2\end{aligned} \quad (2.95)$$

where; x, y, $\dot{x}$, $\dot{y}$, $\ddot{x}$, $\ddot{y}$ are functions of time.

Yaw, pitch and roll angles can be found. Using 2.23, a scalar field can be formed as:

$$\begin{aligned}\phi(x,y,z) = &z-0.5+0.5\cdot\cos(0.05\cdot x)-0.6\cdot\sin(0.012\cdot y)\\ &-0.0005\cdot\sin(0.6\cdot x)-0.001\cdot\cos(0.8\cdot y)\end{aligned} \quad (2.96)$$

Gradient of the above field is found by [6]:

$$\vec{\nabla}\phi = \frac{\partial\phi}{\partial x}\cdot\vec{i} + \frac{\partial\phi}{\partial y}\cdot\vec{j} + \frac{\partial\phi}{\partial z}\cdot\vec{k} \tag{2.97}$$

$$\frac{\partial\phi}{\partial x} = -0.5\cdot 0.05\cdot\sin(0.05\cdot x) - 0.005\cdot 0.6\cdot\cos(0.6\cdot x)$$

$$\frac{\partial\phi}{\partial y} = -0.6\cdot 0.012\cdot\cos(0.012\cdot y) + 0.001\cdot 0.8\cdot\sin(0.8\cdot y)$$

$$\frac{\partial\phi}{\partial z} = 1$$

Magnitude of the gradient is:

$$\left|\vec{\nabla}\phi\right| = \sqrt{\left(\frac{\partial\phi}{\partial x}\right)^2 + \left(\frac{\partial\phi}{\partial y}\right)^2 + \left(\frac{\partial\phi}{\partial z}\right)^2}$$

Then the unit normal of the surface, which is also considered as the third component of the tank hull can be found as follows:

$$\vec{u}_3^{(h)} = c_1\cdot\vec{i} + c_2\cdot\vec{j} + c_3\cdot\vec{k} \tag{2.98}$$

where;

$$c_1 = \frac{\frac{\partial\phi}{\partial x}}{\left|\vec{\nabla}\phi\right|} \quad , \quad c_2 = \frac{\frac{\partial\phi}{\partial y}}{\left|\vec{\nabla}\phi\right|} \quad , \quad c_3 = \frac{\frac{\partial\phi}{\partial z}}{\left|\vec{\nabla}\phi\right|}$$

Let

$$\vec{u}_1^{(h)} = a_1 \cdot \vec{i} + a_2 \cdot \vec{j} + a_3 \cdot \vec{k} \tag{2.99}$$

Then,

$$a_1^2 + a_2^2 + a_3^2 = 1 \tag{2.100}$$

$$\vec{u}_1^{(h)} \cdot \vec{u}_3^{(h)} = 0 \qquad \text{hence;} \qquad a_1 \cdot c_1 + a_2 \cdot c_2 + a_3 \cdot c_3 = 0 \tag{2.101}$$

Let η be the angle between the x-axis of the earth fixed frame and the performance direction of the tank hull;

$$a_2 = a_1 \cdot \tan(\eta) \tag{2.102}$$

Solving 2.100, 2.101, 2.102 for a_1, a_2, a_3 gives:

$$a_1 = \frac{1}{\sqrt{1 + \tan^2 \eta + \left(\frac{c_1 + c_2 \cdot \tan(\eta)}{c_3} \right)^2}}$$

$$a_2 = a_1 \cdot \tan(\eta)$$

$$a_3 = -\frac{a_1 \cdot (c_1 + c_2 \cdot \tan(\eta))}{c_3}$$

Similarly,

$$\vec{u}_2^{(h)} = \vec{u}_3^{(h)} \times \vec{u}_1^{(h)} \tag{2.103}$$

where,

$$\vec{u}_2^{(h)} = b_1 \cdot \vec{i} + b_2 \cdot \vec{j} + b_3 \cdot \vec{k}$$

Since all the unit vectors are known; transformation matrix from the hull frame to the earth fixed frame can be written as follows:

$$\hat{C}^{(0,h)} = \begin{bmatrix} \bar{u}_1^{(h/0)} & \bar{u}_2^{(h/0)} & \bar{u}_3^{(h/0)} \end{bmatrix} = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$$

On the other hand, the transformation matrix from tank hull frame to the earth fixed frame is already given in equation 2.27 as:

$$\hat{C}^{(0,h)} = e^{\tilde{u}_3 \cdot \psi} \cdot e^{-\tilde{u}_2 \cdot \theta} \cdot e^{\tilde{u}_1 \cdot \phi}$$

Hence,

$$a_3 = \bar{u}_3^t \cdot e^{\tilde{u}_3 \cdot \psi} \cdot e^{-\tilde{u}_2 \cdot \theta} \cdot e^{\tilde{u}_1 \cdot \phi} \cdot \bar{u}_1$$

$$\sin(\theta) = a_3$$

$$\cos(\theta) = \sqrt{1 - a_3^2}$$

$$\theta = a\tan 2(a_3; \sqrt{1 - a_3^2})$$

Similarly;

$$b_3 = \bar{u}_3^t \cdot e^{\tilde{u}_3 \cdot \psi} \cdot e^{-\tilde{u}_2 \cdot \theta} \cdot e^{\tilde{u}_1 \cdot \phi} \cdot \bar{u}_2$$

$$c_3 = \bar{u}_3^t \cdot e^{\tilde{u}_3 \cdot \psi} \cdot e^{-\tilde{u}_2 \cdot \theta} \cdot e^{\tilde{u}_1 \cdot \phi} \cdot \bar{u}_3$$

$$\cos(\theta)\cdot\sin(\phi)=b_3$$
$$\cos(\theta)\cdot\cos(\phi)=c_3$$

Hence,

$$\phi = a\tan 2(b_3;c_3)$$

And finally,

$$a_1 = \bar{u}_1^t \cdot e^{\tilde{u}_3\cdot\psi}\cdot e^{-\tilde{u}_2\cdot\theta}\cdot e^{\tilde{u}_1\cdot\phi}\cdot\bar{u}_1$$

$$a_2 = \bar{u}_2^t \cdot e^{\tilde{u}_3\cdot\psi}\cdot e^{-\tilde{u}_2\cdot\theta}\cdot e^{\tilde{u}_1\cdot\phi}\cdot\bar{u}_1$$

$$\cos(\theta)\cdot\cos(\psi)=a_1$$

$$\cos(\theta)\cdot\sin(\psi)=a_2$$

$$\psi = a\tan 2(a_2;a_1)$$

Hence, the above equations are used to find the yaw, pitch and roll angles of the tank hull. Obtaining the angular position components of the tank hull, angular velocity and acceleration components can be found by taking their time derivatives.

CHAPTER 3

GUN CONTROL SYSTEMS

In this chapter, the gun control equipment and gun control systems are discussed. In section 3.1, sensors used in tank gun control systems are considered. In section 3.2, hydraulic motors, which are used widely in tank gun stabilization systems, are discussed. Then, two methods, for gun control; mainly, computed torque method and velocity control by using PID controller are discussed.

3.1 Measurement devices

In tank gun stabilization systems, measurement devices are employed to measure the disturbances, which occur due to the ground-tank hull interaction. Also, these devices are used in measuring the tank turret and barrel orientation for the stabilization system. In this chapter some information about the measurement devices, which are widely used in tanks, is given. In section 3.1.2, error models of these devices and their implementation to the tank gun system are discussed.

3.1.1 Gyroscopes

Gyroscopes are the measurement devices, which are used for sensing the angular rate with respect to inertial frame. They are the devices exhibiting strong angular momentum. Angular momentum (or moment of momentum) is a vector property of any physical body that is spinning, with respect to inertial space, about an axis. In the absence of an applied torque- the rotational effect of a force about an axis- an angular momentum vector maintains a fixed orientation in inertial space, thereby providing a directional reference. By applying a calibrated torque to the spinning body, that is, to the gyro element, one may command the angular momentum to rotate relative to inertial space in a known manner. Measurement of the calibrated torque provides a measure of the rate of inertial rotation of the directional reference [7].

In this study, two types of gyroscopes distinguished by the nature of the reaction torque employed are explained briefly. The unit is called a rate gyro if the primary restraining torque is elastic. The term rate refers to the fact that a steady deflection angle of the gyro element relative to the case is a measure of a constant inertial angular velocity of the instrument about its sensitive or input axis. The unit is called an integrating gyro if the primary restraining torque is a damping reaction. The term integrating refers to the fact that, in this type, the deflection of the gyro element relative to the case is a measure of the integral of the inertial angular velocity of the instrument about its sensitive axis –that is, the change of angular attitude of the instrument [7].

Beginning from the first usage of the rate gyroscope at about 1920; it has been used to provide lead-angle data for antiaircraft fire-control sights; and later, used in the flight-control systems of aircraft and missiles, stabilization of a vehicle or some of its components. Early rate gyros were mechanically supported on ball bearings, used a linkage indicator, had an air-driven rotor, and depended upon a mechanical spring for elastic restraint. More accurate rate gyros have an electromechanical 'spring' formed by feeding the signal output of an integrating gyro back to the torque generator, which is the component by means of which a calibrated torque is applied to the gyro element. The gyro element is supported by flotation in the damping fluid, and the rotor is electrically driven [7].

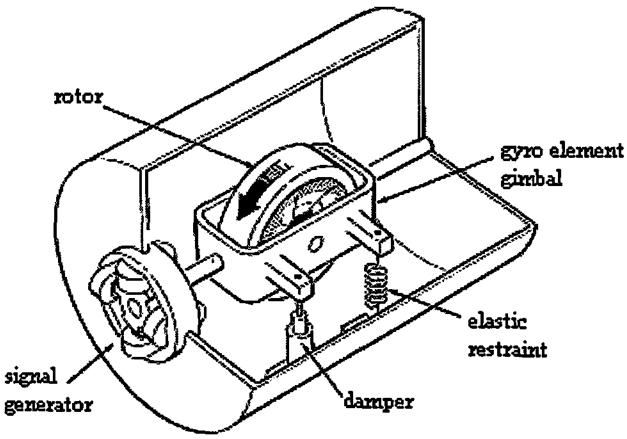

Figure 3-1 Basic elements of a rate gyro (Primarily elastic restraint).

It was realized that a good way of improving the performance of the rate gyro was to get rid of the spring. The resulting instrument, the integrating gyro, depends primarily upon the damping torque for restraint of the gyro element. In practice, the integrating gyro is always used with a powered follow-up, which in response to the signal generator output, rotates the case of the instrument about the input axis to keep the deflection angle between the gyro element and the case at null. In this manner, base motion effects produced by movement of the vehicle are eliminated, since the inertial rotation of the instrument about its input axis is maintained at zero unless the gyro is commanded to precess by means of its torque generator. Should the gyro be rotated, it will have a calibrated precession rate relative to inertial space about its input axis, which will be matched by rotation of the case of the instrument through the powered follow-up. In this manner the integrating gyro is used to give physical equipment very accurately calibrated inertial angular velocities [7].

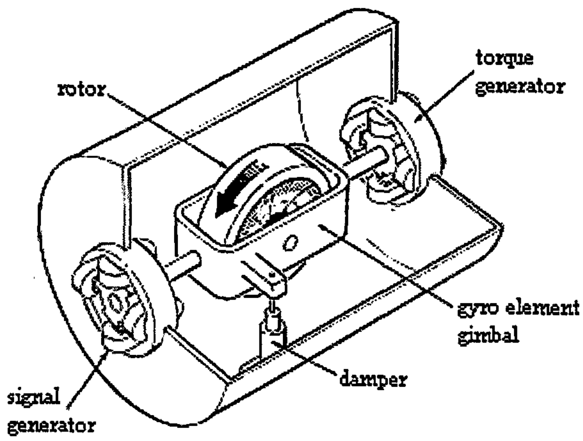

Figure 3-2 Basic elements of an integrating gyro (damping restraint).

3.1.2 Accelerometers and Potentiometers

An accelerometer contains a mass (called the 'proof mass') since the principle of acceleration measurement lies in measuring the force required to give the proof mass the acceleration being measured. In essence, Newton's second law of motion is used for measuring the acceleration of the proof mass.

In Figure 3.3 the spring element measures the force by deflecting proportionately. A displacement transducer converts the spring's deflection into a proportional voltage, since voltage indicating and recording devices are widely used in measurement systems [8].

A potentiometer is an electromechanical transducer that converts mechanical energy into electrical energy. The input to the device is in the form of a mechanical displacement, either linear or rotational. When a voltage is applied across the fixed terminals of the potentiometer, the output voltage, which is measured across the variable terminal and ground, is proportional to the input displacement, either linearly or according to some nonlinear relation [9].

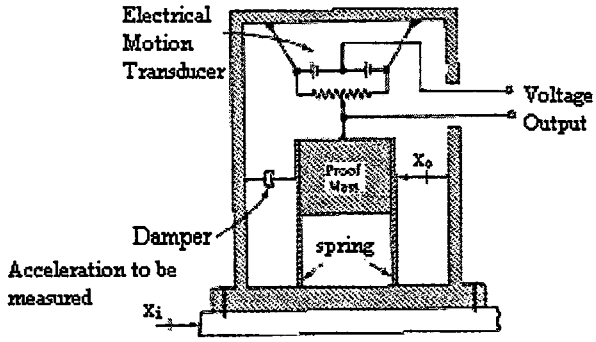

Figure 3-3 Components of an accelerometer

Rotary potentiometers are available commercially in single-revolution or multirevolution form, with limited or unlimited rotational motion. The potentiometers commonly are made with wirewound or conductive plastic resistance material. For precision control, the conductive plastic potentiometer is preferable, since it has infinite resolution, long rotational life, good output smoothness, and low static noise [9].

3.1.3 Error Modeling in measurement devices

In previous sections, brief information about gyroscopes and accelerometers were given. In this section, sources of errors which occur during making measurements with these devices due to their working conditions and the effects of these errors on the model to be formed for stabilizing to the gun system is considered.

There are many sources of errors in measurement devices. The gyro error that is taken into account in this study is the drift error, which is caused by acceleration, temperature, magnetic fields and noise. The gyro specific errors such as linearity, misalignment, scale factor etc. are not considered in modeling the gyro errors. Similarly, the accelerometer error that is modeled in this study is the accelerometer bias, which is caused by the operating conditions.

The accelerometer bias and gyro drift errors are modeled by means of adding Gaussian distributed random signals. The errors for accelerometers and gyros are considered as constant biases or drifts and constant white noise. Hence, the error equations for the sensor are [10]:

$$\varepsilon_a = \varepsilon_{a,bias} + w_a \tag{3.1}$$

$$\varepsilon_\omega = \varepsilon_{\omega,drift} + w_\omega \tag{3.2}$$

And the new values of the acceleration are:

$$a^* = a + \varepsilon_a \tag{3.3}$$

$$\omega^* = \omega + \varepsilon_\omega \tag{3.4}$$

3.2 Hydraulic Motors

In this section, power drives which are used to move the turret and gun are discussed. Specifically, electro-hydraulic drive systems are taken into account. Electro-hydraulic power drives have been used in modern battle tanks for many years and are still in widespread service. A pump produces high-pressure hydraulic fluid, which elevates and depresses the gun (by means of a hydraulic ram) and traverses the turret (via a hydraulic motor). The pump can be either driven directly by the vehicle main engine, or it may be electrically driven. With this latter arrangement, the pump can be located in the turret, which eliminates the need to pass high pressure hydraulic fluid through the hull to turret rotating interface, and power control can still be obtained using batteries, providing a useful silent watch capability. In the static firing mode, the hydraulic ram can be locked, to hold the gun firmly at the position[3].

Electro-hydraulic power drives are very powerful and yet capable of providing fine control for accurate, slow speed tracking. A clear advantage of using hydraulic systems is that they can hold large out-of-balance loads, which may exist about the traverse or elevation axes. In addition to their normal functions, hydraulic motors and actuators can also act as brakes. In the past, electro-hydraulic systems tended to be smaller than electrical systems, but this advantage is now being eroded [3]. The main drawbacks of the systems are:

- High pressure hydraulic poses a fire hazard in the turret. To protect the crew, halon-gas fire suppression systems are sometimes fitted and these are very effective, but obviously they occupy valuable space and increase the cost of the tank.
- The hydraulic fluid must be carefully filtered to ensure a smooth tracking performance.
- The pipework occupies room in the turret, tends to leak and is not simple to repair without introducing air and/or contaminants into the fluid
- The pump can be noisy and, like any mechanical device, prone to failure[3].

A spool valve for controlling the hydraulic system is considered in section 3.2.1. The model of this valve is used in driving a hydraulic ram in elevation and a positive displacement pump in traverse directions, and this is discussed in following sections.

3.2.1 Hydraulic Control valves

Hydraulic control valves are devices that use mechanical motion to control a source of fluid power. The most widely used valve is the sliding valve employing spool type construction. Spool valves are classified by the number of 'ways' flow can enter and leave the valve, the number of lands, and the type of center when the valve spool is in neutral position. Because all valves require a supply, a return, and at least one line to the load, valves are either three-way or four-way. A three-way valve requires a bias pressure acting on one side of an unequal area piston for direction reversal. Usually the head-side area is twice the rod-side area, and supply pressure acts on the smaller area to provide the bias force for reversal. A four-way valve would have two lines to the load. The number of lands on a spool vary from one in a primitive valve to the usual three or four, and special valves may have as many as six lands. If the width of the land is smaller than the port in the valve sleeve, the valve is said to have an open center or to be underlapped. A critical center or zero lapped valve has a land width identical to the port width and is a condition approached by practical machining. Closed center or overlapped valves have a land width greater than the port width when the spool is at neutral [11].

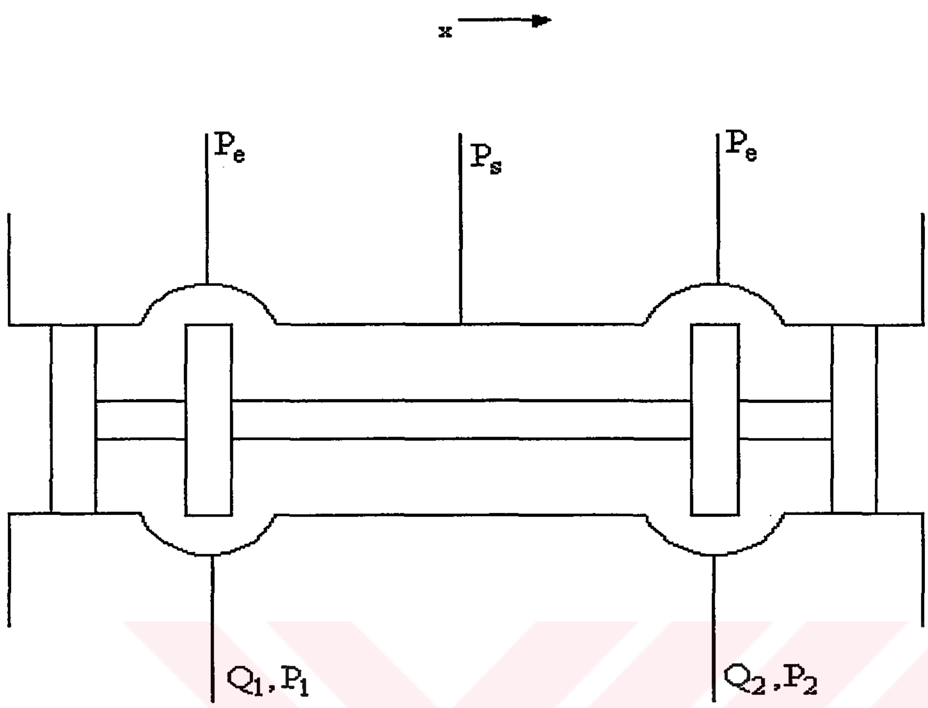

Figure 3-4 An open centered four way spool valve

A sample open centered four way spool valve is shown in Fig 3-4. In such a case, there is always leakage through the valves. Such leakage improves both the sensitivity and the linearity of the hydraulic motor to be controlled by the valve[12]. The flow equation for this kind of a spool valve can be written as:

$$Q = C_d \cdot w \cdot \sqrt{\frac{P_s - P_L}{\rho}} \cdot (\frac{x_0}{2} + x) - C_d \cdot w \cdot \sqrt{\frac{P_s + P_L}{\rho}} \cdot (\frac{x_0}{2} - x) \qquad (3.5)$$

and;

$$P_L = P_1 - P_2 \qquad (3.6)$$

where P_L is the load pressure, P_s is the supply pressure. The supply pressure is taken as constant in this study.

The following assumptions are made:

- The hydraulic fluid used in a system is incompressible,

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- The valve is symmetrical,
- The orifice area, (the width of the valve slot in the valve sleeve) at each port is proportional to the valve displacement x.

C_d is the discharge coefficient and it has a value range of 0.60-0.65 for sharp edged orifices. And it reaches to 1 as the edges of the orifice are rounded. w is the width of the orifice, x is the displacement of the spool ram and $x_0/2$ is the initial space in the orifice space.

A relation between the controller and an electrical current, which drives the control valve, can be given as[13]:

$$i = K_v \cdot E \tag{3.7}$$

where E is the output of the controller.

The control valve is driven by means of an electrical motor in an electrohydraulic system. The motor accepts a current and outputs a force in the following relationship[13]:

$$F = K_m \cdot i \tag{3.8}$$

Then a second relation between the valve opening x_v and the force equation is obtained by considering that the displacement is obtained by means of a spring[13].

$$x_v = \frac{F}{K_s} \tag{3.9}$$

3.2.2 Hydraulic Motors

In this section, positive displacement motors, which are widely used in control systems are considered. In these machines, hydraulic fluid passes through the inlet into a chamber which expands in volume and fills with fluid. The volume expansion causes shaft rotation in the motor. The volume of trapped fluid is then sealed from the inlet by some mechanical means and then transported to the outlet side where it is discharged [11].

An example of a positive displacement machine is a piston actuator and a schematic is shown in figure 3.5. Piston actuators are classified as limited travel actuators and are characterized by being simple in construction, available in a wide selection of rod configurations and types of mounting.

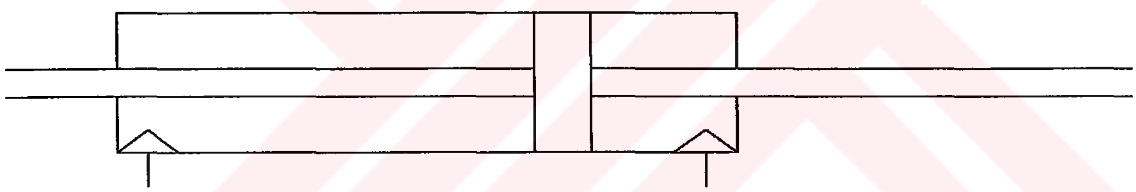

Figure 3-5 A double acting, double rod hydraulic piston actuator

The flow relation between the ram velocity and the flow rate is written as

$$Q_L = A \cdot \dot{y} \tag{3.10}$$

under the assumptions that;

- The piston is rigidly constructed,
- The hydraulic fluid used in a possible hydraulic motor system is incompressible,
- There is no leakage from the piston to the pot.

A scheme for the usage of hydraulic piston actuator for elevating and depressing the tank barrel is shown in figure 3.6. A relation between the elevation angle of the gun, and the hydraulic ram velocity can be found by using the loop closure equation[14].

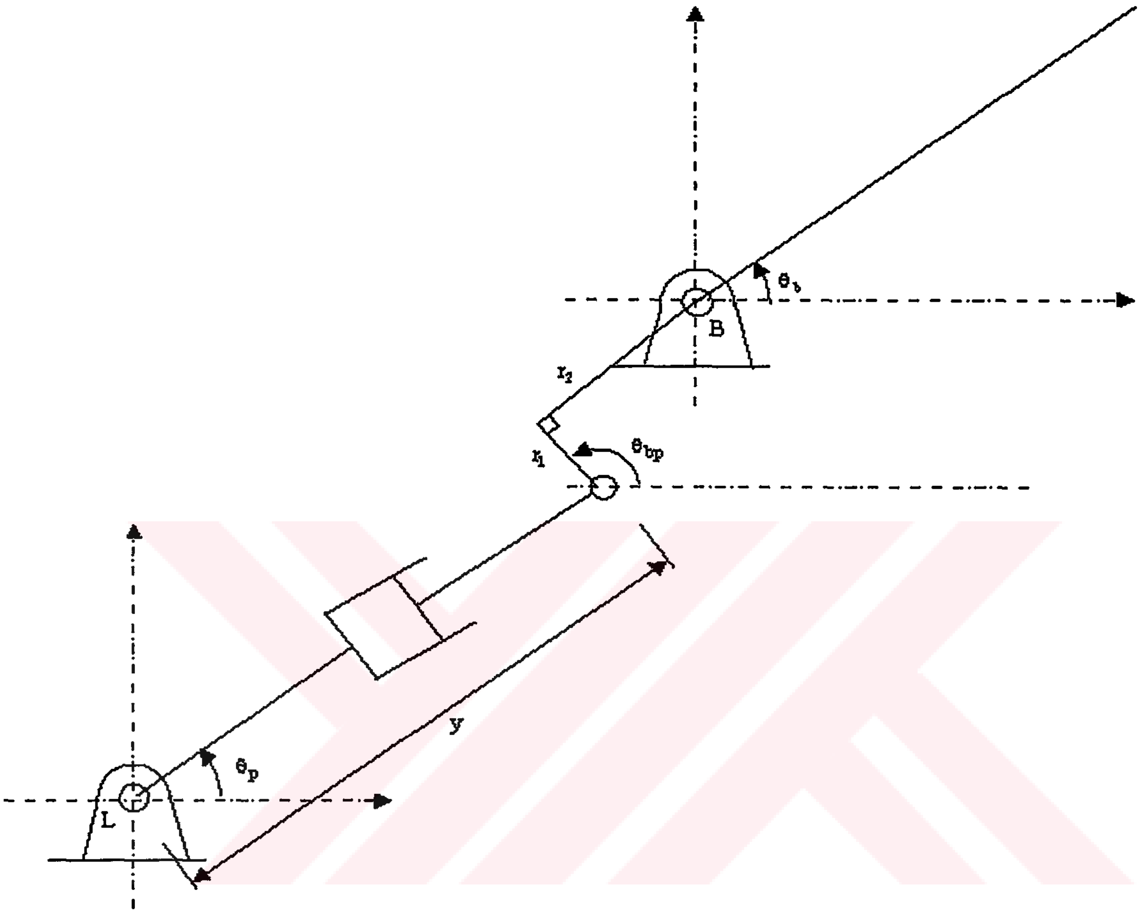

Figure 3-6 A hydraulic piston actuator system for elevating the tank gun

Using figure 3.6, loop closure equation can be written as follows:

$$y \cdot e^{j \cdot \theta_p} + r_1 \cdot e^{j \cdot \theta_{bp}} + r_2 \cdot e^{j \cdot \theta_b} = a_1 + j \cdot a_2 \tag{3.11}$$

where a_1 and a_2 are the horizontal and the vertical distances between the hydraulic ram frame origin (L) and the barrel frame origin (B);

$$\theta_b = \theta_{bp} - \frac{\pi}{2} \tag{3.12}$$

Taking the real and imaginary parts of eqn 3.11, and using eqn 3.12;

$$y \cdot \cos(\theta_p) - r_1 \cdot \sin(\theta_b) + r_2 \cdot \cos(\theta_b) = a_1 \tag{3.13}$$

$$y \cdot \sin(\theta_p) - r_1 \cdot \cos(\theta_b) + r_2 \cdot \sin(\theta_b) = a_2 \tag{3.14}$$

Since the displacement of the hydraulic ram y is to be found, eqns 3.13 and 3.14 can be rearranged as:

$$y \cdot \cos(\theta_p) = a_1 + r_1 \cdot \sin(\theta_b) - r_2 \cdot \cos(\theta_b) \tag{3.15}$$

$$y \cdot \sin(\theta_p) = a_2 + r_1 \cdot \cos(\theta_b) - r_2 \cdot \sin(\theta_b) \tag{3.16}$$

Then the ram displacement can be found by taking the square and then the addition of equation 3.15 and 3.16.

$$y = \sqrt{\left(a_1 + r_1 \cdot \sin(\theta_b) - r_2 \cdot \cos(\theta_b)\right)^2 + \left(a_2 + r_1 \cdot \cos(\theta_b) - r_2 \cdot \sin(\theta_b)\right)^2} \tag{3.17}$$

Ram velocity can be found by taking the derivative of equation 3.17;

$$\dot{y} = \frac{y_n}{y} \cdot \dot{\theta}_b \tag{3.18}$$

where

$$\begin{aligned} y_n = {} & a_1 \cdot \left(r_1 \cdot \cos(\theta_b) + r_2 \cdot \sin(\theta_b)\right) - a_2 \cdot \left(r_1 \cdot \sin(\theta_b) + r_2 \cdot \cos(\theta_b)\right) \\ & + 2 \cdot r_1 \cdot r_2 (1 - 2 \cdot (\cos(\theta_b))^2) \end{aligned}$$

So, given that the elevation angle and the angular velocity of the tank gun, it is possible to find the relevant hydraulic ram displacement. Another relation between the load pressure of the hydraulic system and the tank can be written as:

$$P_L = \frac{F}{A} \tag{3.19}$$

where F is the force acting on the hydraulic ram and can be found from

$$F = \frac{T}{r_1} \tag{3.20}$$

T is the torque required to rotate the tank gun.

Finishing the formulation for the hydraulic motor model necessary for depressing the tank barrel, the traverse drive used for rotating the tank turret in azimuth direction is considered. This drive consists of a positive displacement motor, which uses fluid under pressure and connected to the hydraulic traverse gears. Hence equations 3.10 through 3.20 are not valid for this drive. Instead the following formulas are used [11]:

$$T_g = D_m \cdot P_L \tag{3.21}$$

and

$$Q_L = D_m \cdot n_g \tag{3.22}$$

where D_m is an experimental constant and T_g is the output torque of the motor and n_g is the hydraulic motor shaft speed [11]. And for the traverse gear mesh;

$$T_L = \frac{n_g}{n_L} \cdot T_g \tag{3.23}$$

where, n_L is the velocity of the load and T_L is the load torque.

Completing the formulation for the hydraulic drive system of the tank turret; the modeling of these systems will be discussed in the next section.

3.3 Stabilization of the gun by using the computed torque method

In this section, the position control of the gun system by using computed torque method is considered for maintaining the orientation of the gun in spite of the yaw, pitch, roll, sideslip, bounce, performance motions of the vehicle. The tank gun model, which was derived in section 2.7 of chapter 2, is used as a plant to be controlled. Then the effects of the vehicle motion to the stabilization system as well as the error modeling of the measurement devices and the hydraulic motor model are included to the gun control system to see their effects on the gun control system.

Since it is desired that the tank is able to fire on the move, the effects of the motion of the tank should be taken into account in modeling a stabilization system. In this study, the motion of the tank due to tank hull-ground interaction is modeled by using the formulation in section 2.8 of chapter 2. The required variables for considering the tank hull motion in the control system are the accelerations in performance $(\ddot{x})$, sideslip $(\ddot{y})$, and bounce $(\ddot{z})$, directions; the yaw, pitch, roll angles (ψ, θ, ϕ); yaw, pitch and roll angular velocities $(\dot{\psi}, \dot{\theta}, \dot{\phi})$; and yaw, pitch, roll angular accelerations $(\ddot{\psi}, \ddot{\theta}, \ddot{\phi})$. In the stabilization model, the effects of the hull motion are used mainly in the tank gun model in which the azimuth and elevation angles of the tank gun are found and in finding the motion measured by the sensors, as shown in Figure 3-8.

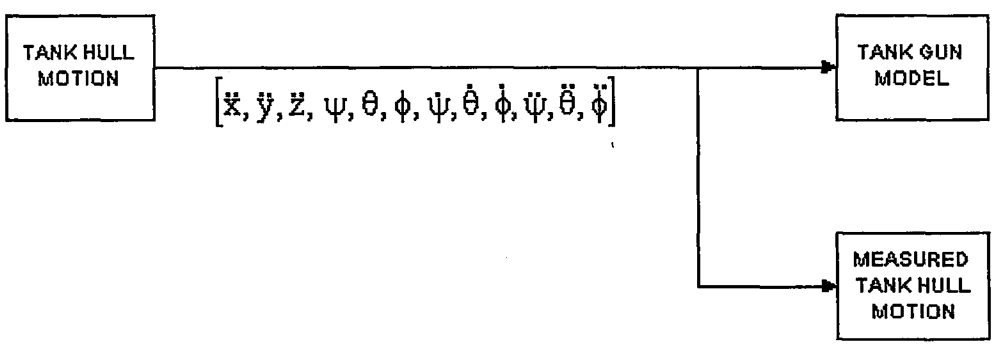

Figure 3-7 Effects of tank hull motion on the stabilization system.

In gun control by using the computed torque method, it is desired to measure the yaw, pitch, roll angles as well as their angular velocities and angular accelerations, and the accelerations in performance, sideslip and bounce directions so that the effect of these terms can be included in computing the torques necessary for driving the gun. Hence, the measured values of these terms have to be modeled in this method.

The measured performance, sideslip, bounce accelerations can be obtained by using the equations 3.1 and 3.3.

In simulation of the stabilization system, the measured yaw, pitch and roll angular velocities are obtained by using equations 3.2 and 3.4. Yaw, pitch and roll angles are obtained by integrating these angular velocities. The angular accelerations in these directions are obtained by taking the derivatives of yaw, pitch and roll angular velocities. However, as a result of taking the derivative of random generated signals, the amplitude of these signals become unrealistic. So, a first order low pass filter is used to obtain a more realistic signal. A first order transfer function in the following form is used:

$$G(s) = \frac{1}{\tau \cdot s + 1} \tag{3.24}$$

in order to obtain a smooth result for the angular acceleration terms.

In stabilizing the tank gun system by using the computed torque method, the azimuth and elevation angles have to be measured so that they could be compared with their reference commands. Potentiometers are employed for this purpose to measure the azimuth angle in tank turret frame and the elevation angle in tank barrel frame. The error characteristics of potentiometers are neglected in simulation of this stabilization system. The usage of the measured quantities of the tank hull motion is summarized in figure3-9.

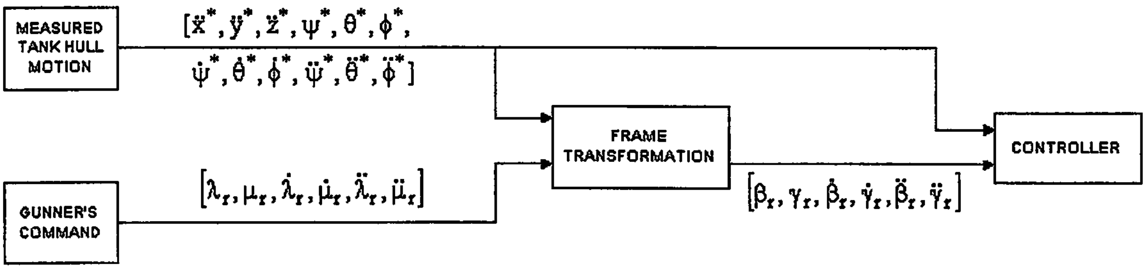

Figure 3-8 Usage of the measured variables of the tank hull in stabilization system.

The design of the controller in figure 3-9 is discussed in the following section.

In summary, the angular position, velocity and acceleration components of the tank hull $(\psi, \theta, \phi, \dot{\psi}, \dot{\theta}, \dot{\phi}, \ddot{\psi}, \ddot{\theta}, \ddot{\phi})$, linear acceleration terms of the tank hull $(\ddot{x}, \ddot{y}, \ddot{z})$, the azimuth angle in turret frame (β), and the elevation angle in the barrel frame (γ) are measured in the computed torque method.

3.3.1 Controller Design for the Computed Torque Method

Torques necessary for driving the gun system in azimuth and elevation can be found by considering the equations 2.87 through 2.90. These equations can be rearranged and be taken into the form

$$\hat{K} \cdot \overline{R} = \overline{N}$$

where;

$$\hat{K} = \begin{bmatrix} \overline{k}_1 & \overline{k}_2 & \hat{k}_3 & \hat{k}_4 & \hat{k}_5 & \hat{k}_6 \end{bmatrix}$$

the eelments of $\hat{K}$ are written as:

$$\overline{k}_1^t = \begin{bmatrix} \overline{0}^t & \overline{u}_3{}^t & \overline{0}^t & \overline{0}^t \end{bmatrix}$$

$$\overline{k}_2^t = \left[\overline{0}^t \quad \overline{u}_2{}^t \quad \overline{0}^t \quad -\overline{u}_2{}^t\right]$$

$$\hat{k}_3^t = \left[\hat{I}^t \quad \tilde{r}_{ht}{}^t \quad \hat{0}^t \quad \hat{0}^t\right]$$

$$\hat{k}_4^t = \left[\hat{I}^t \quad \tilde{r}_{bt}{}^t \quad \hat{0}^t \quad \hat{0}^t\right]$$

$$\hat{k}_5^t = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\hat{k}_6^t = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -\cos(\gamma) & 0 & \sin(\gamma) \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -\sin(\gamma) & 0 & \cos(\gamma) \end{bmatrix}$$

The unknowns are written as:

$$\overline{R}^t = \left[T_\beta \quad T_\gamma \quad \overline{F}_{ht}^t \quad \overline{F}_{bt}^t \quad M_{ht1} \quad M_{ht2} \quad M_{bt1} \quad M_{bt3}\right]$$

$\overline{N}$ can also be formed as:

$$\overline{N} = \begin{bmatrix} m_t\left(\overline{a}_{G_{t1}} \cdot \ddot{\beta} + \overline{a}_{G_{t2}} - \overline{g}^{(t)}\right) \\ \hat{J}_t \cdot \left(\overline{\alpha}_{t1} \cdot \ddot{\beta} + \overline{\alpha}_{t2}\right) + \tilde{\omega}_t \cdot \hat{J}_t \cdot \overline{\omega}_t \\ m_b\left(\overline{a}_{G_{b1}} \cdot \ddot{\beta} + \overline{a}_{G_{b2}} \cdot \ddot{\gamma} + \overline{a}_{G_{b3}} - \overline{g}^{(b)}\right) \\ \hat{J}_b \cdot \left(\overline{\alpha}_{b1} \cdot \ddot{\beta} + \overline{\alpha}_{b2} \cdot \ddot{\gamma} + \overline{\alpha}_{b3}\right) + \tilde{\omega}_b \cdot \hat{J}_b \cdot \overline{\omega}_b \end{bmatrix}$$

Then,

$$\overline{R} = \hat{K}^{-1} \cdot \overline{N}$$

and

$$T_\beta = \overline{R}(1),$$

$$T_\gamma = \overline{R}(2)$$

Hence, torques necessary for driving the tank gun system are found. Then the PID controller for the computed torque method can be formulated as follows:

Let the acceleration of the tank gun system be denoted by $\ddot{\tau}$, where τ is either the azimuth angle β, or the elevation angle γ. Then;

$$\ddot{\tau} = \ddot{\tau}_{des} + K_p \cdot (\tau_{des} - \tau) + K_d \cdot (\dot{\tau}_{des} - \dot{\tau}) + K_i \cdot \int_0^t (\tau_{des} - \tau) \cdot dt \qquad (3.25)$$

Let the error be defined as:

$$\varepsilon = (\tau_{des} - \tau)$$

Then, the laplace transform of equation 3.25 is;

$$(s^3 + K_d \cdot s^2 + K_p \cdot s + K_i) \cdot E(s) = 0 \qquad (3.26)$$

where the third order polynomial is the characteristic polynomial in equation 3.26. This polynomial can also be formed by the combination of a first and a second order polynomial;

$$D(s) = (s^2 + 2 \cdot \xi \cdot \omega_n \cdot s + \omega_n{}^2) \cdot (s + \eta \cdot \omega_n)$$

$$D(s) = s^3 + (2 \cdot \xi \cdot \omega_n + \eta) \cdot \omega_n \cdot s^2 + (1 + 2 \cdot \xi \cdot \eta) \cdot \omega_n{}^2 \cdot s + \eta \cdot \omega_n{}^3$$

The PID controller can be formed by the following relations;

$$K_d = (2 \cdot \xi \cdot \omega_n + \eta) \cdot \omega_n \quad (3.27)$$

$$K_p = (1 + 2 \cdot \xi \cdot \eta) \cdot \omega_n^2 \quad (3.28)$$

$$K_i = \eta \cdot \omega_n^3 \quad (3.29)$$

Hence the controller for the computed torque method can be found by using the procedure explained in this section.

3.3.2 Modeling of the hydraulic motors

In this section the modeling of the hydraulic motors when the computed torque method is used for gun stabilization is considered. In this method, since a torque is calculated for driving the tank gun system, the starting point will be the torque to be obtained from the hydraulic drive in both azimuth and elevation.

The linear actuator system in elevation is considered first. Since a torque is taken as the input in the computed torque system, this torque is converted to force by using equation 3.20 and then the load pressure is found by using 3.19. Also, by using 3.17 and 3.18; the velocity of the ram is found. Then the valve opening required for the computed torque is calculated by using the equations 3.5 and 3.10. Then this valve opening is converted into command signal required by the system by using equations 3.7, 3.8 and 3.9. Then this command is used to generate the actual current. The actual current is obtained by using a first order transfer function due to the electrical characteristics of the driving motor. The following equation is used for this purpose:

$$G(s) = \frac{1}{\tau \cdot s + 1} \quad (3.30)$$

Obtaining the current required for the system, the valve displacement can be found by using equation 3.8 and 3.9. An additional transfer function is introduced due to the mass of the spool. This is a second order transfer function illustrated below:

$$G(s) = \frac{\omega_n^{\ 2}}{s^2 + 2 \cdot \xi \cdot \omega_n + \omega_n^{\ 2}} \tag{3.31}$$

Hence, actual displacement of the spool valve is obtained by using equation 3.31. Then, equations 3.5 and 3.10 are used again to find the actual load pressure given by the actuator of the hydraulic system. Finally, equations 3.19 and 3.20 are used to find the corresponding motor torque of the system.

The modeling of the hydraulic motor system used for traversing the tank turret can be found in a similar procedure. The turret angular velocity and the computed torque are used for the starting point in this system. Then, the hydraulic flow rate and the load pressure are calculated by using equations 3.21, 3.22, and 3.23. Then the valve equation (3.5) is used to obtain the valve displacement corresponding to the computed torque. The command that is to be given to the system is found by using the equations 3.7, 3.8 and 3.9. Then, the actual current is obtained by using the equations 3.7 and 3.30 are used to find the actual current given to the system. Actual valve displacement is found by using the equations 3.9 and 3.31. Then, the actual load pressure is generated by using the equation 3.5. Finally, the actual torque output of the system is found by considering the equations 3.21, 3.22, and 3.23.

In summary, in the computed torque method, calculated torques are used to find a driving current, and this current is used again to generate the motor response of the system.

The block diagram of the stabilization system explained in this section is shown in figure 3-10. It is noted that the purpose of the last block which includes the transformation of the azimuth and elevation angles from gun frames to the earth fixed frame is to compare the results of the model with the reference angles. The details of the modeling of the blocks in figure 3-10 will be discussed in Chapter 4.

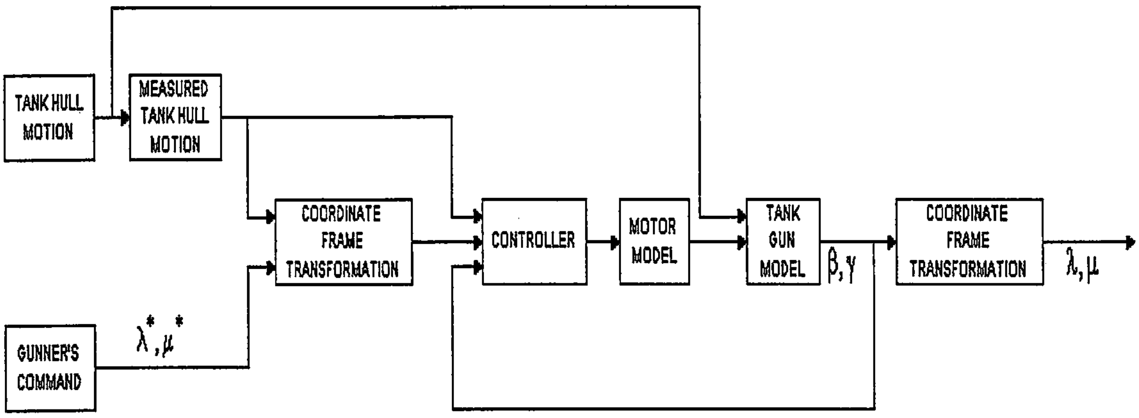

Figure 3-9 Simulation block Diagram of the gun control system by using the computed torque method.

3.4 Velocity Control of the Tank Gun by Using PID Controller

Another approach for stabilizing the tank gun is using PID Controller with velocity feedback by means of gyroscopes. This control method is chosen because of its simplicity and it is used widely in gun control systems as explained in Chapter 1.

The tank gun model is the same that was stated in the previous control method. Also, the same variables; accelerations in performance, sideslip and bounce directions as well as the yaw, pitch and roll angles, angular velocities and angular accelerations are used in order to consider the effects of the motion of the tank hull to the stabilization system. Hence, the same model considered in the previous control method for tank hull motion is used again. But this time, these variables are taken as disturbance to the tank gun model and they are not measured so that they could be used in the control system, which was the case in the previous control method.

The azimuth and elevation angular velocities with respect to the earth fixed frame $(\dot{\lambda}, \dot{\mu})$ are measured to provide the velocity feedback to the stabilization system. For this purpose, gyroscopes are used in tank turret and tank barrel. Error models of gyroscopes are taken into account by using the formulation in section 3.1.2.

The error between the reference angular velocity and the actual angular velocity is found in the earth fixed frame. Then, the error is fed to PID controller, and then the resultant signal is used in producing the motor torques required for elevating and traversing the tank gun. It is noted that; since the plant model that is considered in this study is highly nonlinear, it is not suitable to use the classical control material for finding the coefficients of the controller. Instead, a trial and error procedure will be used in determining the PID coefficients.

Since a current is generated from the result of the PID controller, the current is used to find the valve displacement, and then driving motor torques are generated. It is noted that the transfer functions, which are given in the previous section, are used again in the motor model.

The block diagram of the control system is shown in figure 3-11.

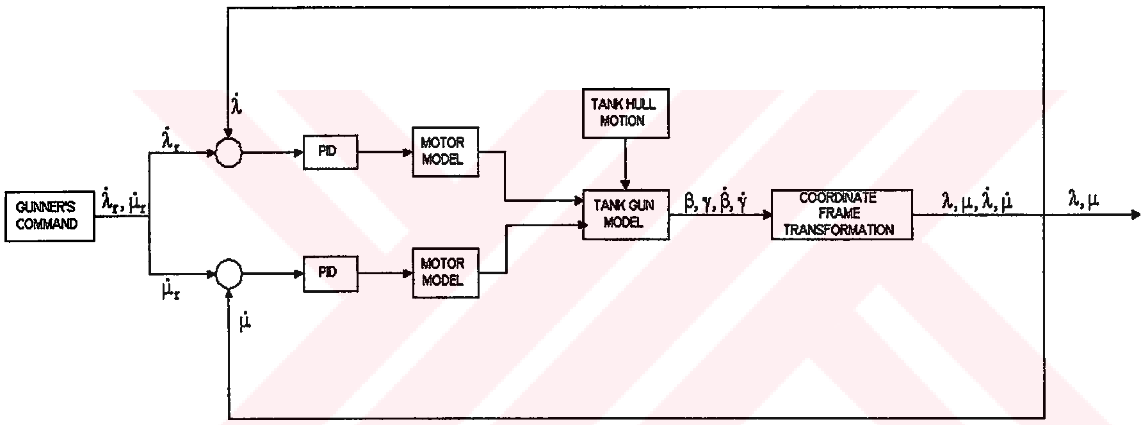

Figure 3-10 Simulation block diagram for the velocity control of the tank gun by using PID Controller.

The azimuth and elevation angles are transformed from the gun frames to the earth fixed frame for comparing with the reference angles. Also, azimuth and elevation angular velocities are transformed into the earth fixed frame since the error is calculated in this frame. This frame in turn, gives the effect of the gyroscopes because they measure angular rates with respect to the earth fixed frame. By using the transformation procedures in the control system, the stabilization system is completed.

CHAPTER 4

SIMULATION OF THE GUN CONTROL SYSTEMS

In this chapter, simulation programs of the gun control systems are given. Stabilization system by using the computed torque method is considered first and simulink version 2.2 model of this stabilization system is given in section 4.1. Then, simulink model of the velocity control of the tank gun by using the gyro feedback is given in section 4.2. Finally, results of these stabilization systems on a sample run will be shown in 4.3.

4.1 Model of the Gun Stabilization by Using the Computed Torque Method

The block diagram of the gun stabilization system by using the computed torque method was already given in Chapter 3. In this section the mathematical modeling of this control system will be discussed in detail. The blocks necessary for the simulation system will be given in detail first, and then the overall stabilization system will be discussed.

4.1.1 Modeling of the Reference Commands

The reference commands, which are given by the gunner in a tank gun stabilization system is considered in this section. In modeling, it is considered that the purpose of the gunner is to orient the tank gun from an initial position to a final position in a specified time interval and then holding the orientation of the gun at the final position. In moving the tank gun from the initial position to the final position, some mathematical relation will be given. This relation will be used for the gunner's command for moving the tank gun in azimuth and elevation directions. A half cycloidal relation is preferred for giving the reference motion of the tank This relation is advantageous in that it provides a smooth startup and end, which reflects the physical behavior of the tank gun. The half cycloidal motion is given as:

$$y(t) = L \cdot [\frac{t}{t_s} - \frac{1}{2 \cdot \pi} \cdot \sin(\frac{2 \cdot \pi \cdot t}{t_s})] \tag{4.1}$$

$$\dot{y}(t) = \frac{L}{t_s} \cdot [1 - \cos(\frac{2 \cdot \pi \cdot t}{t_s})] \tag{4.2}$$

$$\ddot{y}(t) = \frac{2 \cdot \pi \cdot L}{t_s^2} \cdot \sin(\frac{2 \cdot \pi \cdot t}{t_s}) \tag{4.3}$$

where t_s is the rise time and L is the displacement from initial position to the final position. The function y(t) and its derivatives are either the azimuth reference command for traversing the tank gun or the elevation reference for elevating the tank barrel. In simulink, the above equations are modeled in a subsystem, and then masked by using the mask property of the simulink. Thus, a dialog box is provided to the user in which it is possible to enter the desired orientation of the gun system. The dialog box and the block in which equations 4.1 through 4.3 are modeled are shown as follows:

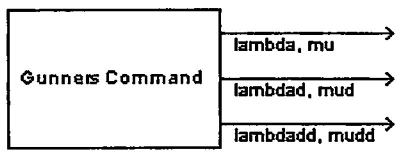

Figure 4-1 Block for generating the reference commands

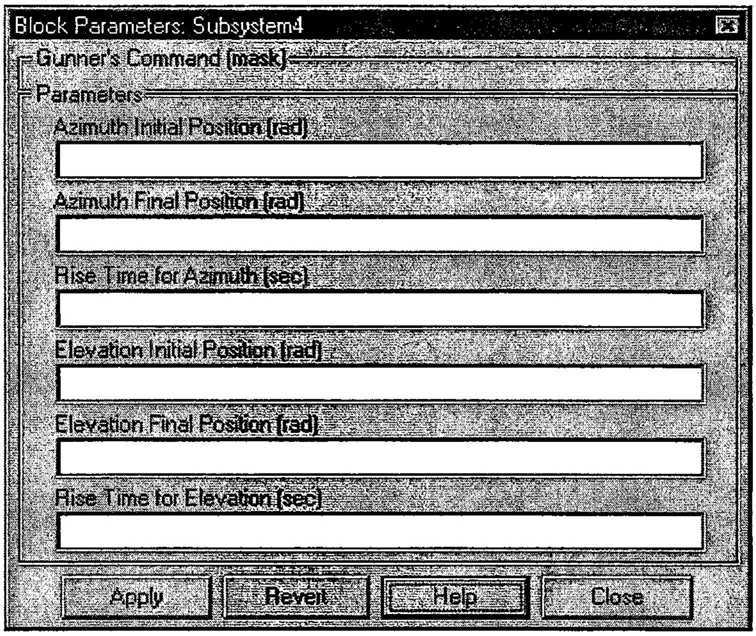

Figure 4-2 Dialog box for entering the reference commands.

The required parameters for the block are the initial and final values of the azimuth angle, rise time for traversing the tank turret from initial to final position, initial and final values of the elevation angle and the rise time for the elevation angle. Entering the required parameters, the block generates half cycloidal curves for moving the tank barrel from its initial position to final position, and then, remains at the final position for the rest of the time. The resultant block is shown in figure 4-2.

It is noted that the first output port of the block in figure 4-2 gives the azimuth and elevation angles in earth fixed frame (λ, μ), second output port gives the azimuth and elevation angular velocities $(\dot{\lambda}, \dot{\mu})$, and third output port gives the azimuth and elevation angular accelerations $(\ddot{\lambda}, \ddot{\mu})$, respectively.

4.1.2 Modeling of the Tank Hull Motion

The tank hull motion is modeled by using the formulation in section 2.8 of Chapter 2. For a given performance and sideslip motion $(x, y, \dot{x}, \dot{y}, \ddot{x}, \ddot{y})$; the acceleration in bounce direction of the tank hull are found by using the model in figure 4.3:

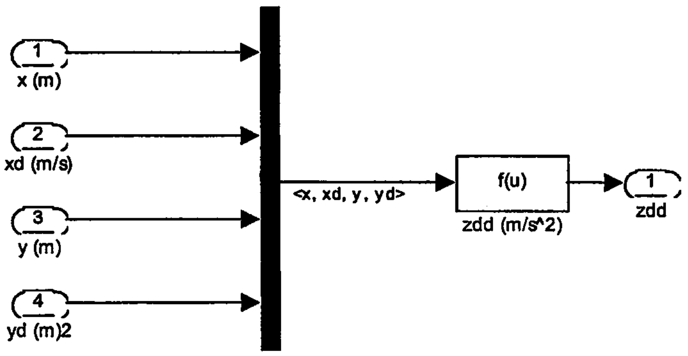

Figure 4-3 Acceleration in bounce direction

The roll, yaw and pitch angles of the tank hull can be found by using the transformation matrix from the tank hull frame to the earth fixed frame. Hence, the unit vectors associated with this matrix should be found. The third unit vector can be found by using the model in figure 4.4.

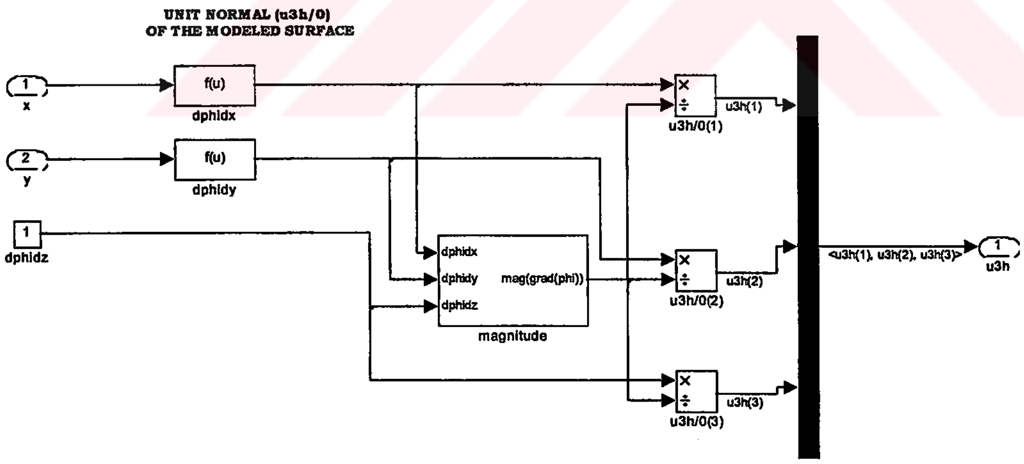

Figure 4-4 Third unit vector of the transformation matrix.

The subsystem denoted 'magnitude' includes the magnitude of the gradient function given by equation 2.103. This subsystem is illustrated in figure 4-5.

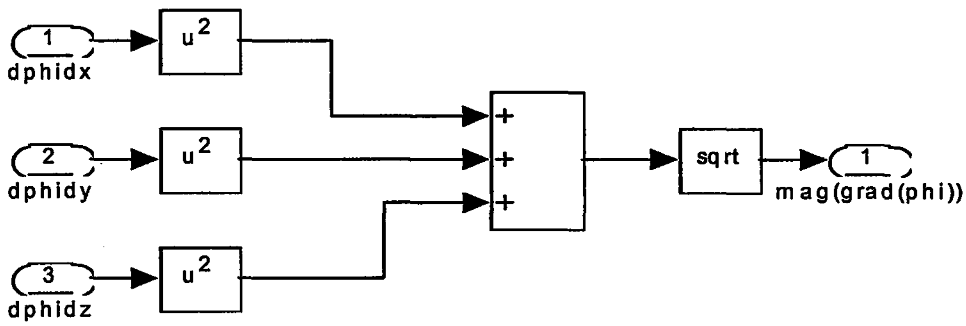

Figure 4-5 Subsystem used for finding the magnitude of the gradient

The first unit vector $(\vec{u}_1^{(h/0)})$ associated with the transformation matrix can be modeled as shown in figure 4-6:

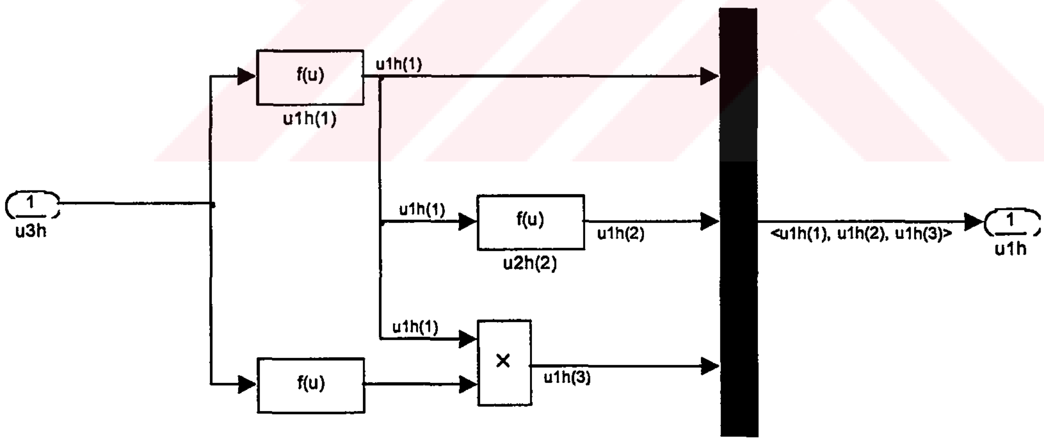

Figure 4-6 First unit vector of the transformation matrix.

Then, the second unit vector can be found by employing an S-Function block in Simulink. S-Functions use a template file which includes the necessary formulations. The equation (2.103) is used for finding $\vec{u}_2^{(h/0)}$. The final form for finding the transformation matrix from hull frame to the earth fixed frame is illustrated in figure 4.7.

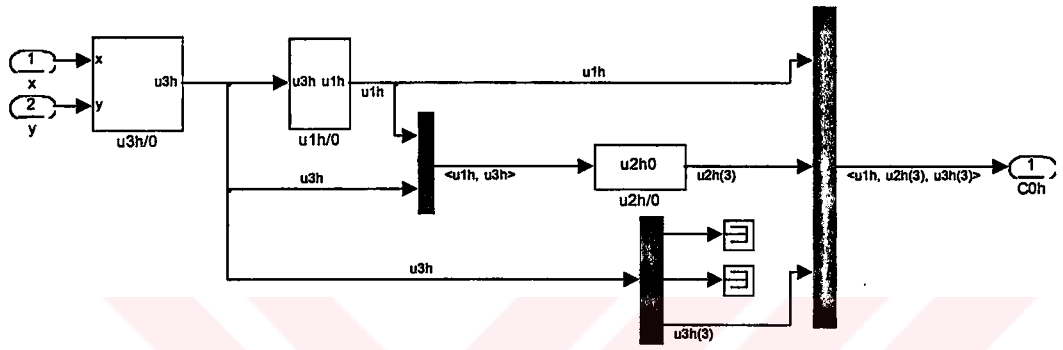

Figure 4-7 Transformation matrix from hull frame to the earth fixed frame.

It is noted that only the required components of the transformation matrix is taken out to find the yaw, pitch and roll angles of the tank hull. The yaw, pitch and roll angles can be found as shown in figure 4-8.

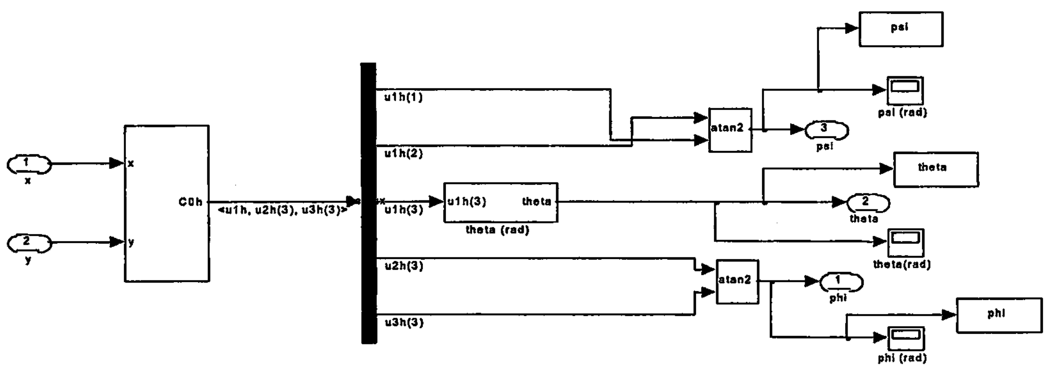

Figure 4-8 Rotation angles of the tank hull

The yaw, pitch and roll angular velocities, and angular accelerations are found by taking the time derivatives of the yaw, pitch and roll angles. In modeling by simulink, the derivative block of the simulink is used for finding the angular velocity and angular acceleration components of the tank hull. The simulink model is illustrated in figure 4-9.

It is noted that derivative blocks are employed in the subsystems named 'Angular Velocity' and 'Angular Acceleration'. The outputs of the model in figure 4.9 are the yaw, pitch, roll angles (ψ, θ, ϕ), their angular velocities $(\dot{\psi}, \dot{\theta}, \dot{\phi})$, angular accelerations $(\ddot{\psi}, \ddot{\theta}, \ddot{\phi})$, and the performance, sideslip, and bounce accelerations $(\ddot{x}, \ddot{y}, \ddot{z})$.

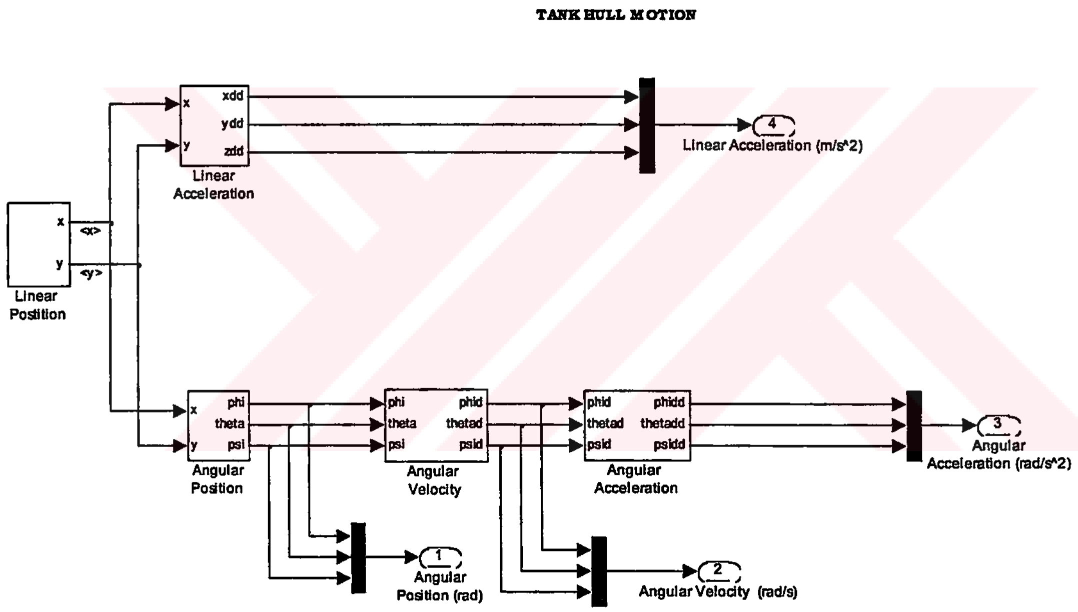

Figure 4-9 Simulink Model of Tank Hull Motion

4.1.3 Addition of the Sensor Error Models to The Tank Hull Motion

Random signal blocks of simulink are used to model the sensor errors. The sensor error outputs are simply added to the tank hull motion which were modeled in section 4.1.2. The resultant simulink model is shown in figure 4.11. The accelerometer errors are added to the performance, sideslip and bounce accelerations and the gyroscope errors are added to the angular velocity components of the tank hull. Then the integral of the outputs are taken to find the measured angles. Also, after taking the derivatives of the measured angular velocities and passing them through a filter, the measured angular accelerations are obtained. The model in figure 4-11 is taken in a subsystem in simulink shown in figure 4-10.

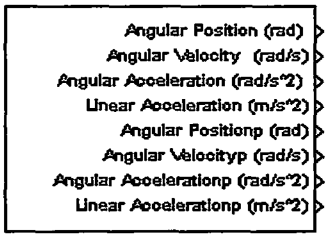

Figure 4-10 Simulink block of the tank hull motion

The first four output ports in figure 4-10 are for the tank hull motion. The remaining four output ports are for the measured tank hull quantities.

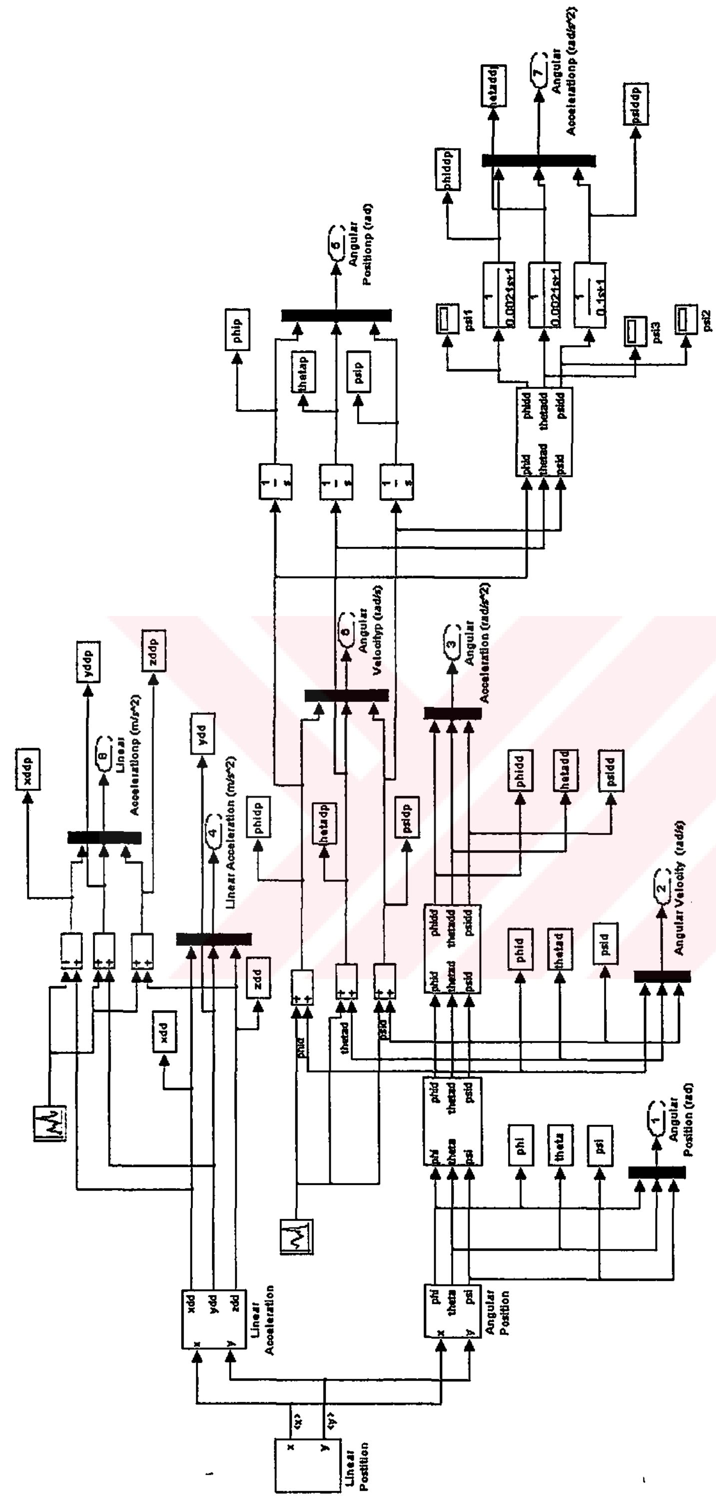

Figure 4-11 The modified simulink model of the tank hull motion. This model includes the hull motion as well as their measured quantities.

4.1.4 Modeling of Coordinate Frame Transformations

The formulation for the transformation of the azimuth and elevation angles, angular velocities and angular accelerations were given in section 2.1.1 of Chapter 2. Simulink model for the transformation of the azimuth and elevation angles from the earth fixed frame to the tank gun frames is given in figure 4.12.

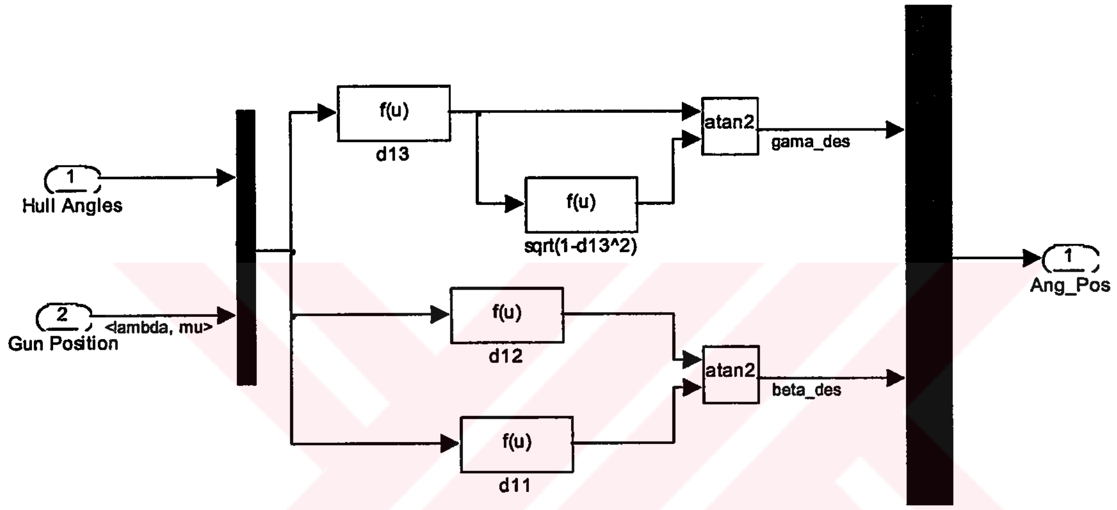

Figure 4-12 Simulink model for the transformation of the gun angles from the earth fixed frame to the gun frames

Also, the azimuth and elevation angular velocity components can be transformed from the earth fixed frame to the tank gun frames as shown in Figure 4-13.

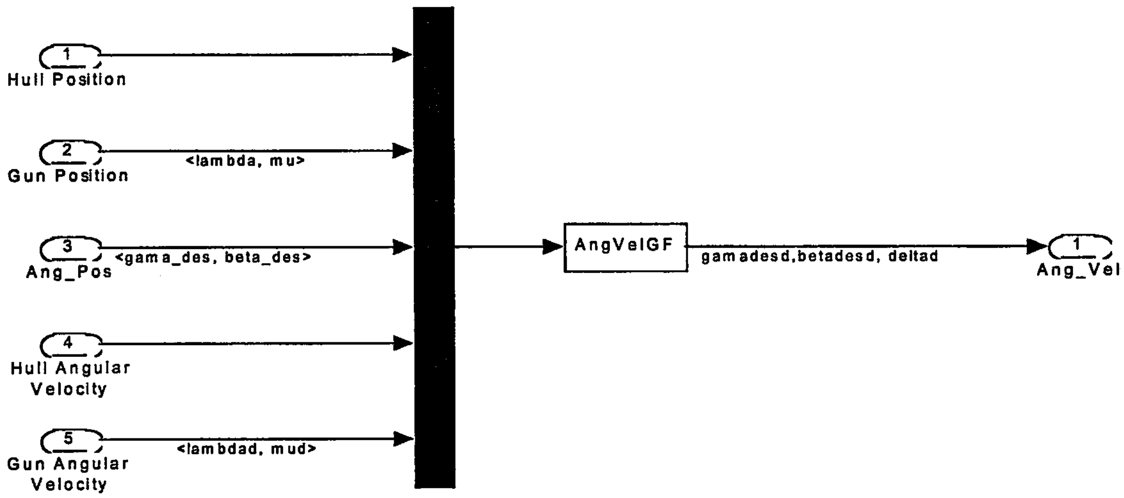

Figure 4-13 Transformation of angular velocities of the tank gun from earth fixed frame to the gun frames.

An S-Function block of Matlab is used again for finding the angular velocity of the tank gun. The code of this S-function is illustrated in appendix A. Obtaining the gun angles and angular velocities; the angular acceleration components of the tank gun can be transformed into the gun frames by using the simulink model shown in figure 4-14.

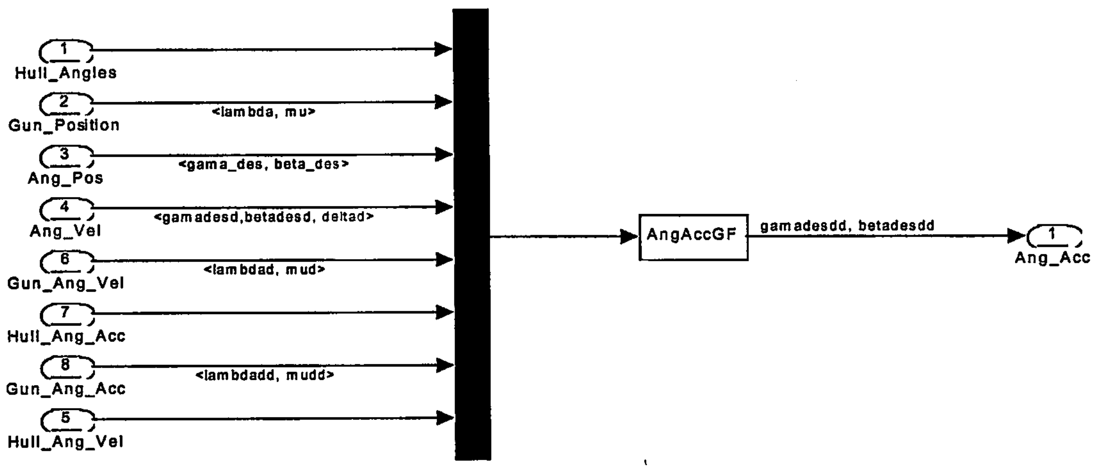

Figure 4-14 Transformation of azimuth and elevation angular accelerations from earth fixed frame to the gun frames

An S-Function is employed (AngAccGF) for finding the angular accelerations $(\ddot{\beta}, \ddot{\gamma})$. The code of this function is given in Appendix B. Completing the blocks necessary for the frame transformation, the overall subsystem to be used in the control system is illustrated in Figure 4-15.

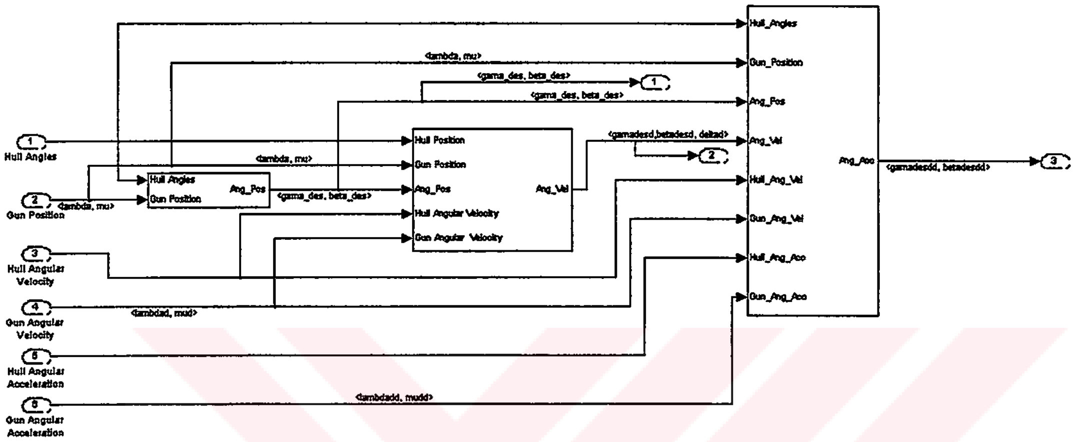

Figure 4-15 Transformation model from earth fixed frame to the gun frames

As can be seen from the figure 4-15; the models shown in figures 4-12, 4-13 and 4-14 are taken into subsystem blocks in simulink and then they are combined. Thus the given reference commands $(\lambda, \mu, \dot{\lambda}, \dot{\mu}, \ddot{\lambda}, \ddot{\mu})$ are transformed into the tank turret and barrel frames $(\beta, \gamma, \dot{\beta}, \dot{\gamma}, \ddot{\beta}, \ddot{\gamma})$ by using the measured tank hull motion $(\psi, \theta, \phi, \dot{\psi}, \dot{\theta}, \dot{\phi}, \ddot{\psi}, \ddot{\theta}, \ddot{\phi})$.

4.1.5 Controller Model

The simulink model of the controller is given in figure 4-16. It consists of two PID controllers for the azimuth and elevation positions of the tank gun, and an S-Function, which includes the code for the torque computation. The inputs are the azimuth and elevation angles, reference azimuth and elevation angles; and the measured tank hull motion. Error between the reference and real orientation of the tank hull is found and is sent to the PID controller so that the command acceleration is obtained. Then, this acceleration term is used together with the measured values of the tank hull motion for getting the torques. Thus, torques which is necessary by the system for making the error zero is obtained.

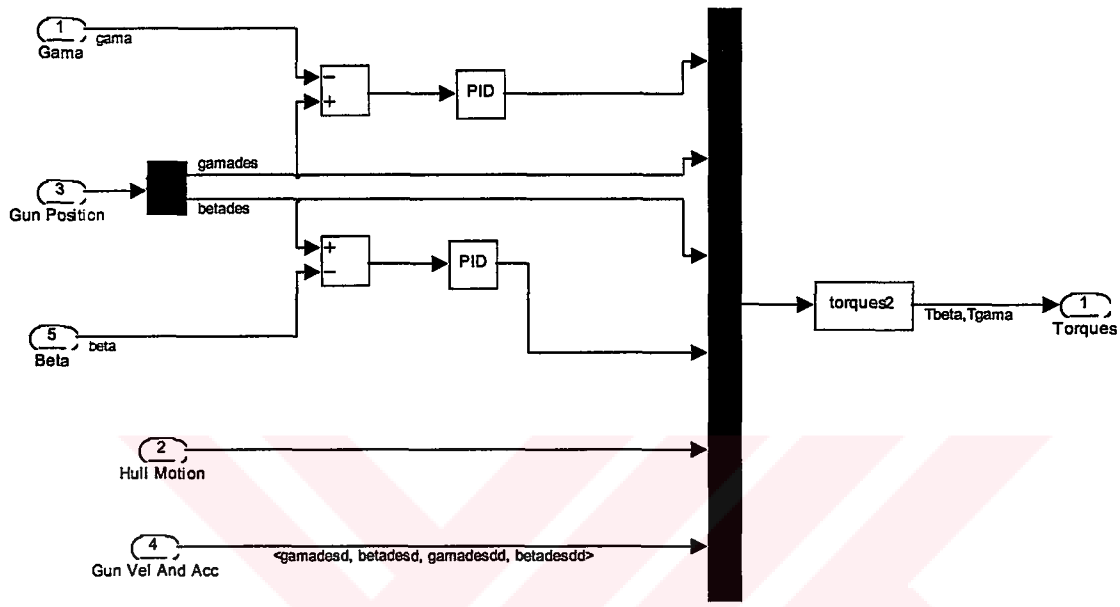

Figure 4-16 Simulink model of the controller for the computed torque method.

The code of the S-Function for computing the torques is illustrated in the appendix C.

4.1.6 Hydraulic Motor Model

Modeling of the hydraulic motors in this control method was already discussed in Chapter 3.

Simulink models of hydraulic motors for azimuth and elevation are shown in figure 4-17. For given computed torques, azimuth and elevation positions the motor torques can be found as shown in this figure.

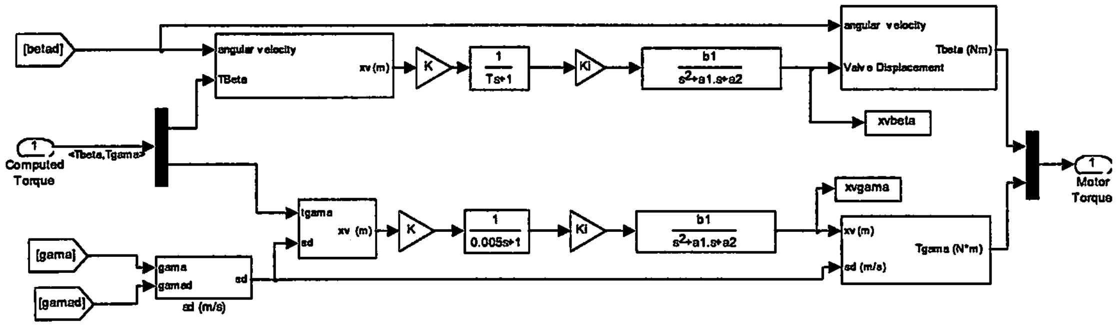

Figure 4-17 Motor models of the control system

The upper portion of this figure is relevant with the hydraulic motor used in azimuth control and the lower portion is for the motor used in elevation control. For given azimuth angular velocity and azimuth torque, the valve displacement is found by using the subsystem shown in figure 4-18. It is noted that a saturation block is used to limit the computed torque by the maximum torque that can be obtained by the motor. Also, the equations 3.21 and 3.22 are used in figure 4-18.

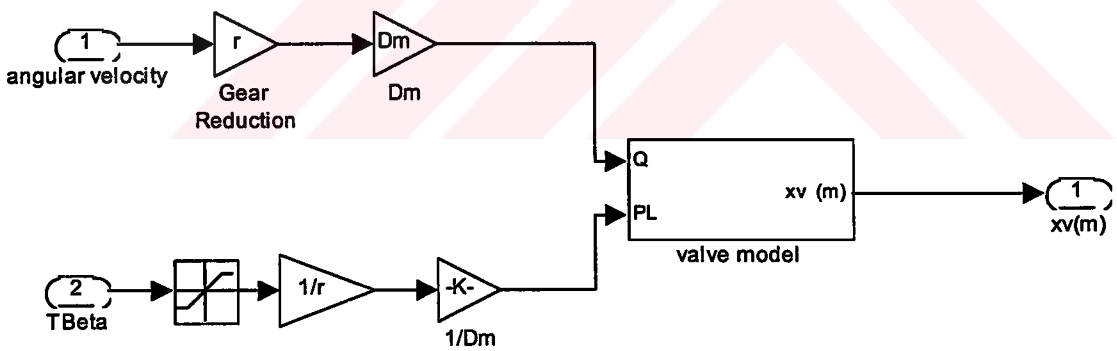

Figure 4-18 Valve displacement model in azimuth direction.

The subsystem named 'valve model' includes equation 3-5. This equation is used for calculating the valve displacement in azimuth direction for the given flow rate and the load pressure. The simulink model of equation 3-6 is shown in figure 4-19.

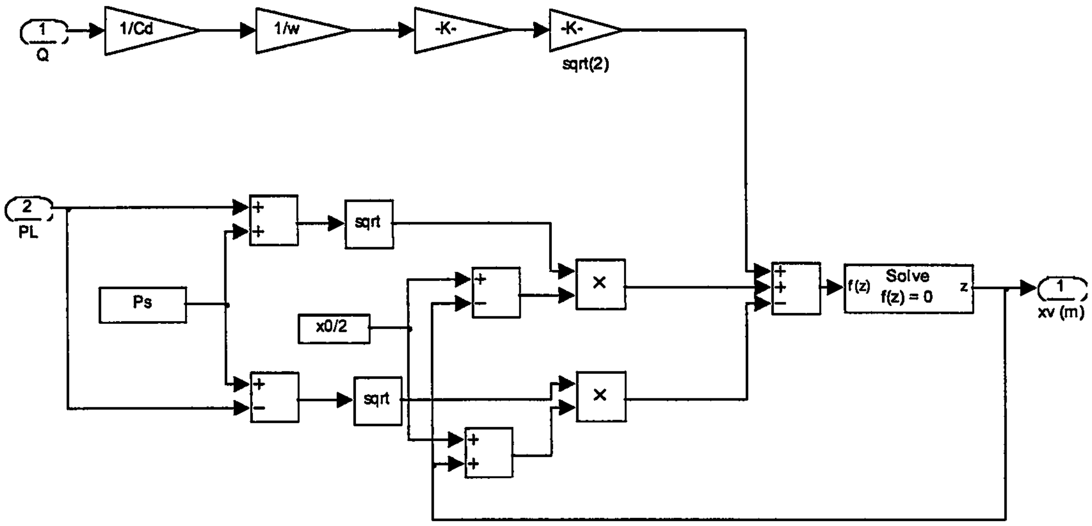

Figure 4-19 Simulink model for calculating the valve opening

After the valve opening is found, it is converted to the current by means of the equations 3-8 and 3-9. Then a transfer function is used for obtaining the real current as shown in figure 4-17. The current is converted to calculated valve displacement and the actual valve displacement is found by using a second transfer function. Then the azimuth torque is obtained by using a second subsystem. This subsystem is shown in figure 4-20.

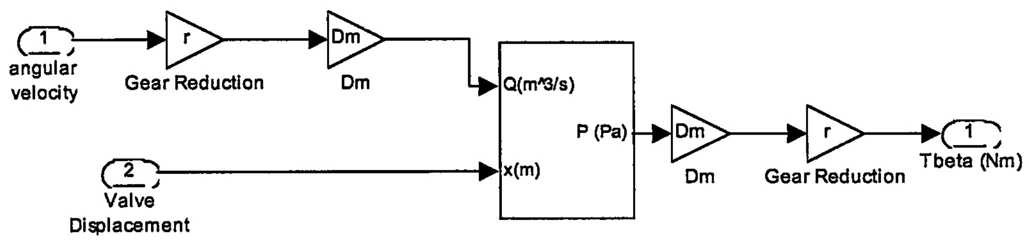

Figure 4-20 Computation of the azimuth torque for given valve displacement and angular velocity.

Since the valve displacement is given this time, equation 3-5 has to be resolved for the load pressure. The necessary simulink model for obtaining the load pressure is given in figure 4-21.

The load pressure is converted into azimuth torque required for the system as shown in figure4-20. Thus the modeling of the hydraulic motor in azimuth is completed.

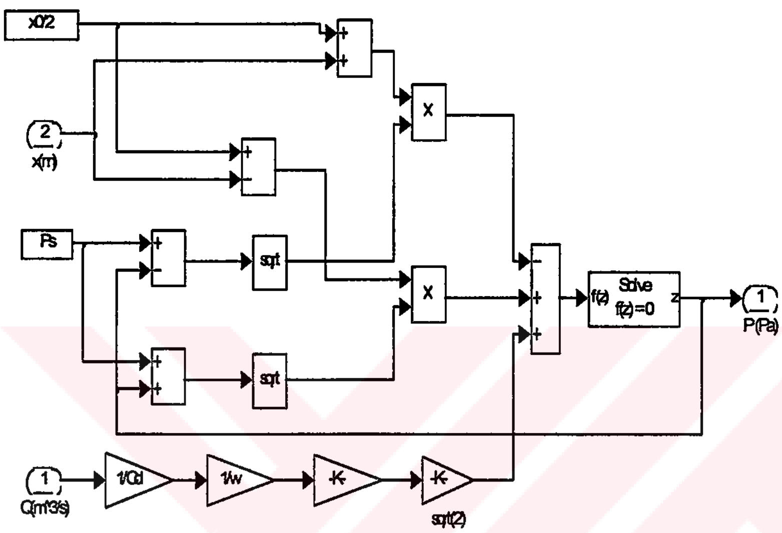

Figure 4-21 Simulink model for calculating the load pressure.

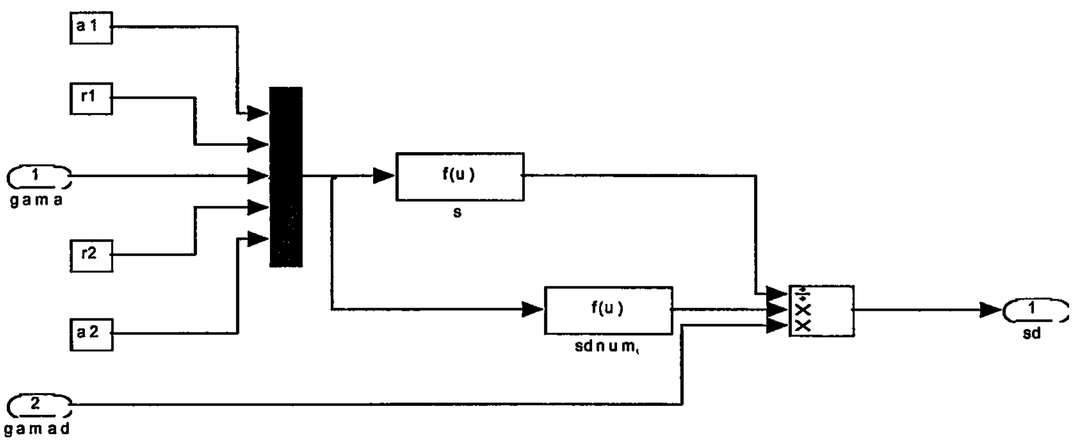

Figure 4-22 Simulink model of the linear actuator velocity.

The hydraulic motor model required in elevation direction is basically the same with that of modeled in azimuth direction. Instead of the angular velocity, the displacement velocity of the linear actuator is calculated as shown in figure 4-22. This calculation is actually the modeling of the equations 3-17 and 3-18. Another difference is that the equations 3-21, 3-22 and 3-23 are replace by 3-10, 3-19 and 3-20 in modeling of the elevation motor.

Thus both the azimuth and elevation motor torques can be found by using the model in figure 4-17.

4.1.7 Tank Gun Model

The procedure explained in section 2-7 of Chapter 2 is used for modeling the tank gun. The simulink model for the tank gun model is illustrated in figure 4-23. This model consists of an S-Function for calculating the azimuth acceleration and integral blocks for obtaining the azimuth and elevation angles.

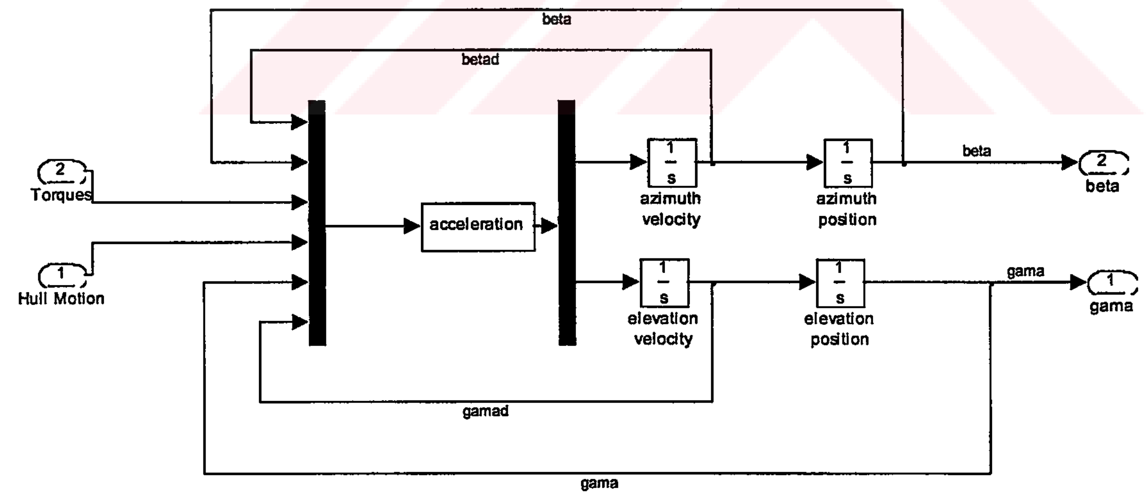

Figure 4-23 Tank gun model

The inputs to the tank gun model are the motion of the tank hull and the motor torques. These data, together with the azimuth and elevation angles and their angular velocities which were calculated at the previous time step are sent to the S-function block named 'acceleration'. The code of this block is illustrated in

Appendix D. Then the azimuth and elevation angular accelerations are calculated. The azimuth and elevation angles are found by double integration.

4.1.8 Coordinate Frame Transformation From Gun Frames to the Earth Fixed Frame

Additional frame transformation from the tank turret and barrel frames to the earth fixed frame for displaying and comparison purposes is done. Simulink model of this procedure is shown in figure 4-24.

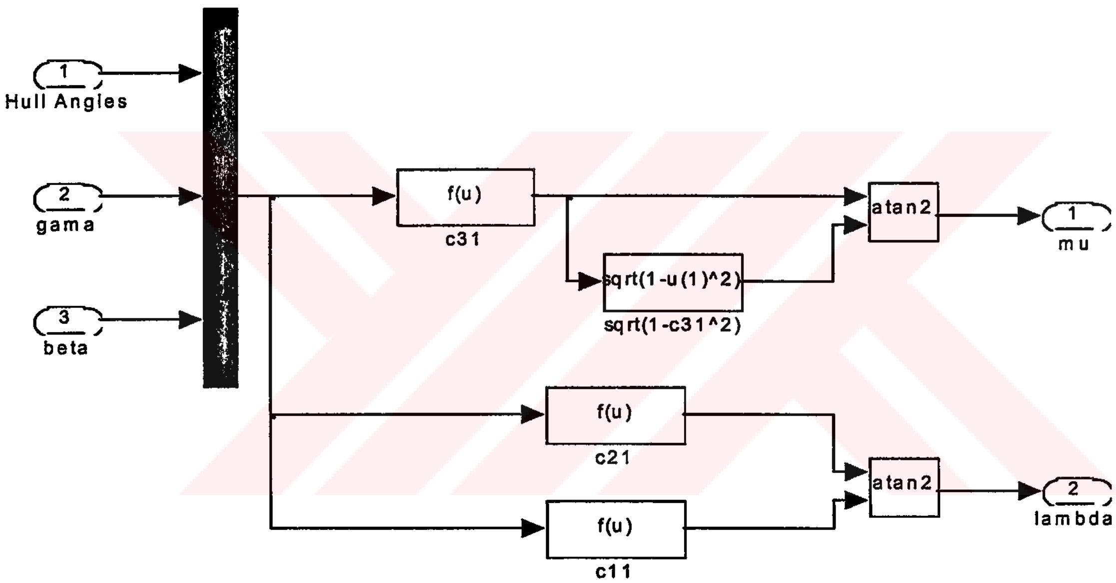

Figure 4-24 Transformation of azimuth and elevation angles from the gun frames to the earth fixed frame.

4.1.9 Simulink Model of the Gun Stabilization System

Completing the necessary subsystems required, the stabilization system by using the computed torque method is modeled in simulink as shown in Figure 4-25.

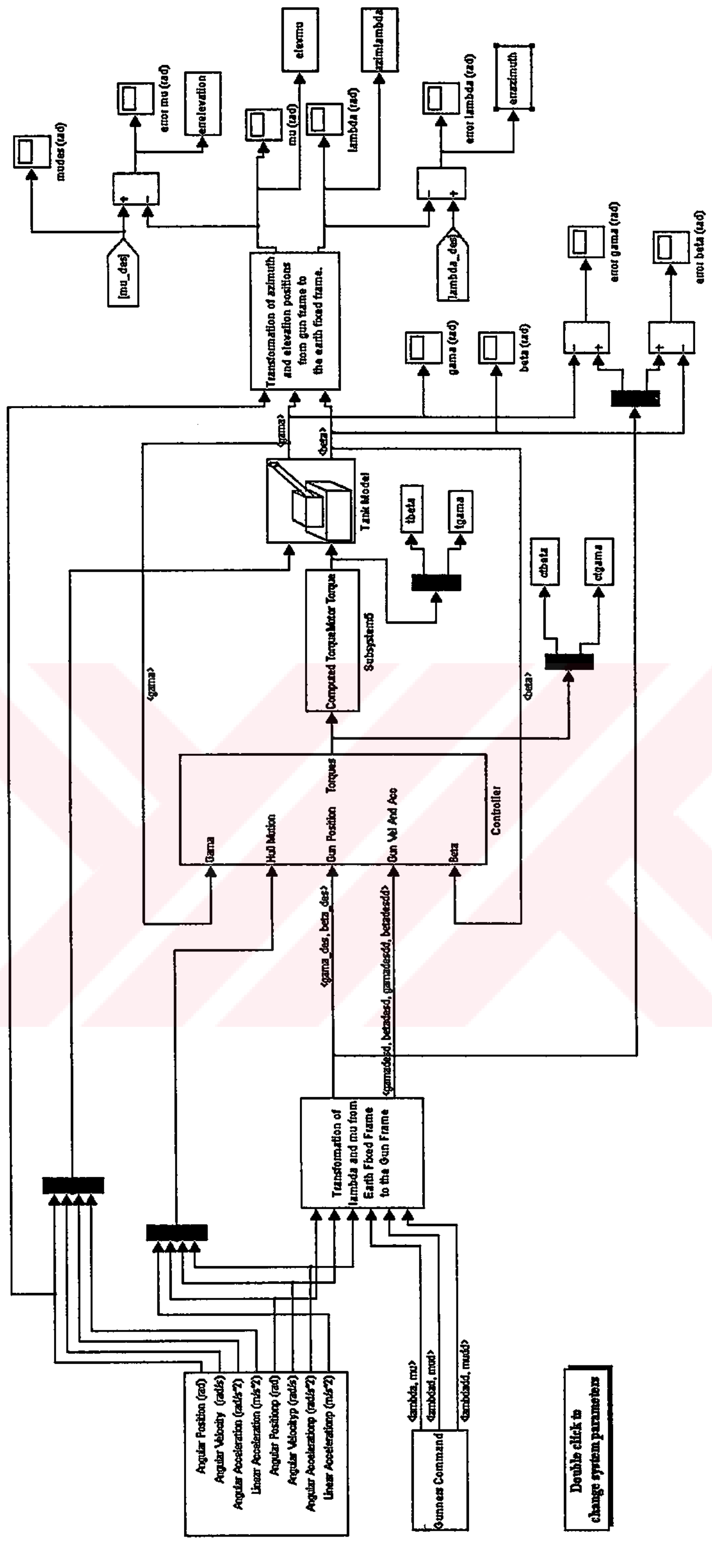

Figure 4-25 Simulink Model of the gun control system by using the computed torque method.

The mass parameters of the tank gun can be entered to the model by double clicking on the block labeled 'Double Click to Change System Parameters'. A graphical user interface shown in figure 4-26 is activated by this way.

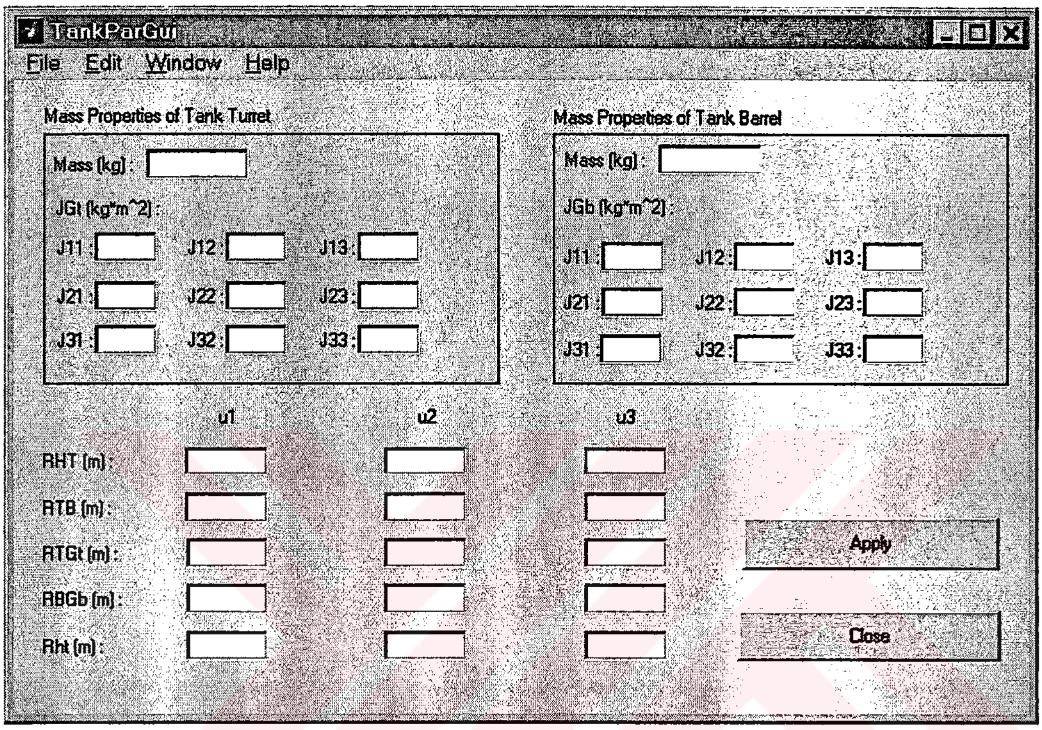

Figure 4-26 Graphical user interface for the tank gun system.

Thus inertia matrices as well as the geometric distances, which were specified in Chapter 2, can be entered to the program.

4.2 Velocity Control of the Gun by using PID Controller

The second control method that is considered in this study is the gun control system by using PID controller and a velocity feedback by means of a gyroscope. The simulink model of this control system is shown in Figure 4-27.

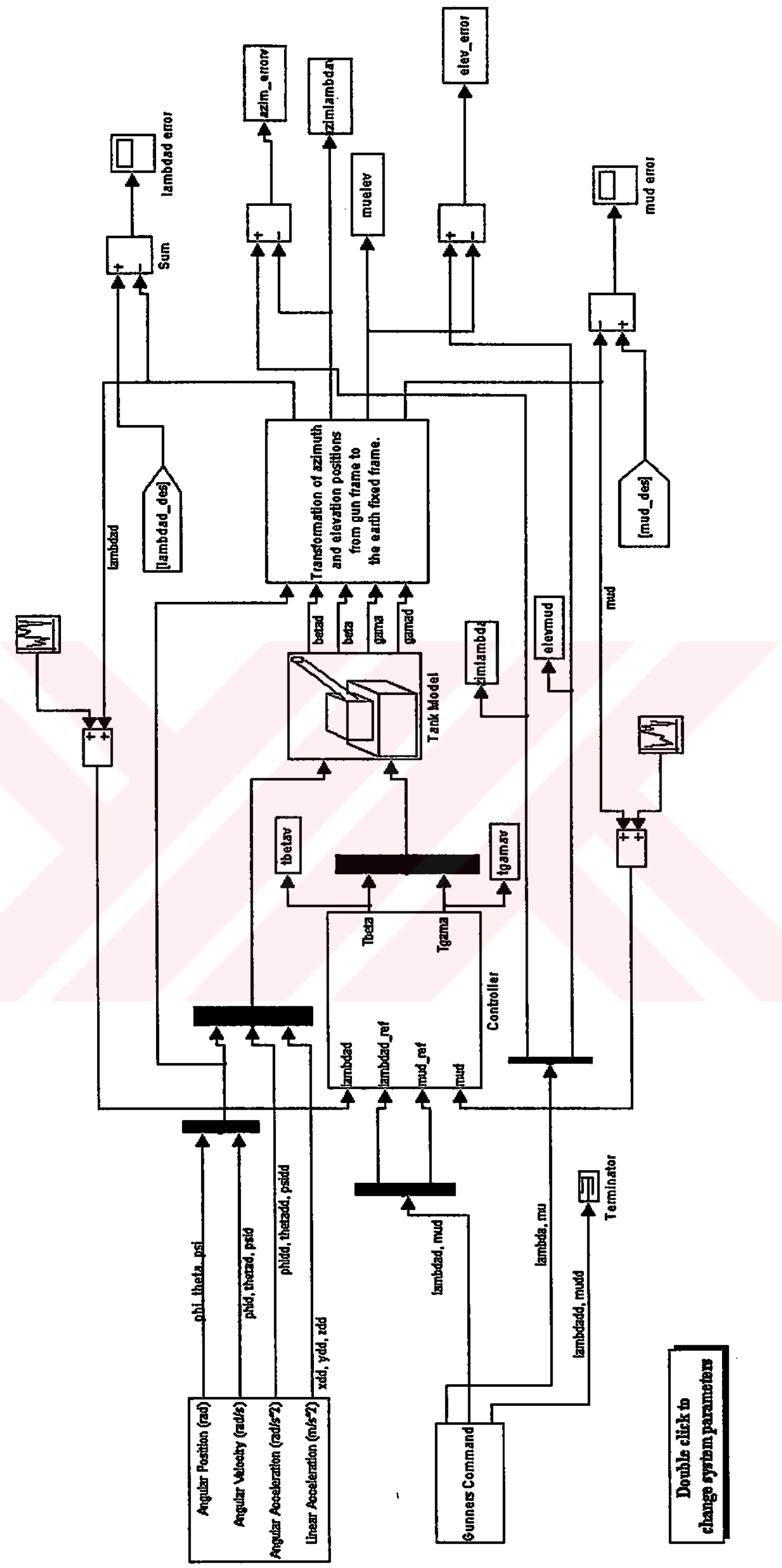

Figure 4-27 Simulink model for velocity control of the tank gun

The subsystems used in this control method are basically the same with the ones in the previous section. The Gunner's command and the Hull motion blocks are the same and are already explained in the previous section. However, the acceleration components of the gunner's command are not used this time and the position command of the gunner is used just for calculating the error term in position. Also, the tank hull motion is considered as a disturbance to the tank gun model. And it is not measured for using in the control system. This was the case in the previous section.

The controller is changed as shown in figure 4-28. This time, the errors are found between the reference velocity and the actual velocity. Then a PID controller is used for controlling the hydraulic motors in azimuth and elevation directions. For a given PID command , the output torque is found. The models of the driving motors are the part of the model of the hydraulic motor used in the previous section. For a given current, control valve displacement is found and then, the resultant torque is calculated.

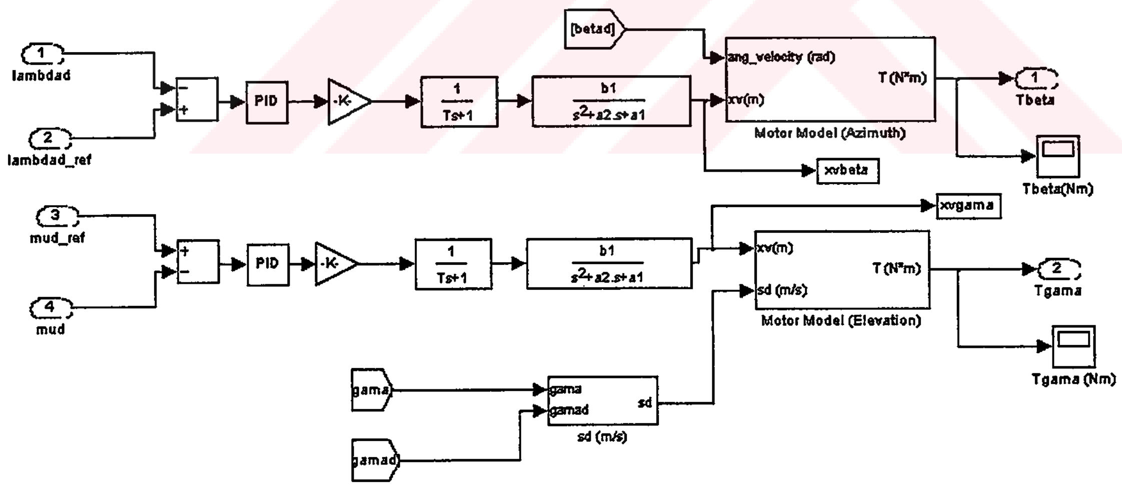

Figure 4-28 Controller and motor models for the stabilization system

The tank gun model is also the same with that of the previous section. However, two additional outputs are taken for azimuth and elevation angular velocities to be fed back to the controller. Also, since the outputs of the tank gun model are in gun frames, they have to be transformed into the earth fixed frame. The simulink model prepared for this purpose is illustrated in figure 4-29.

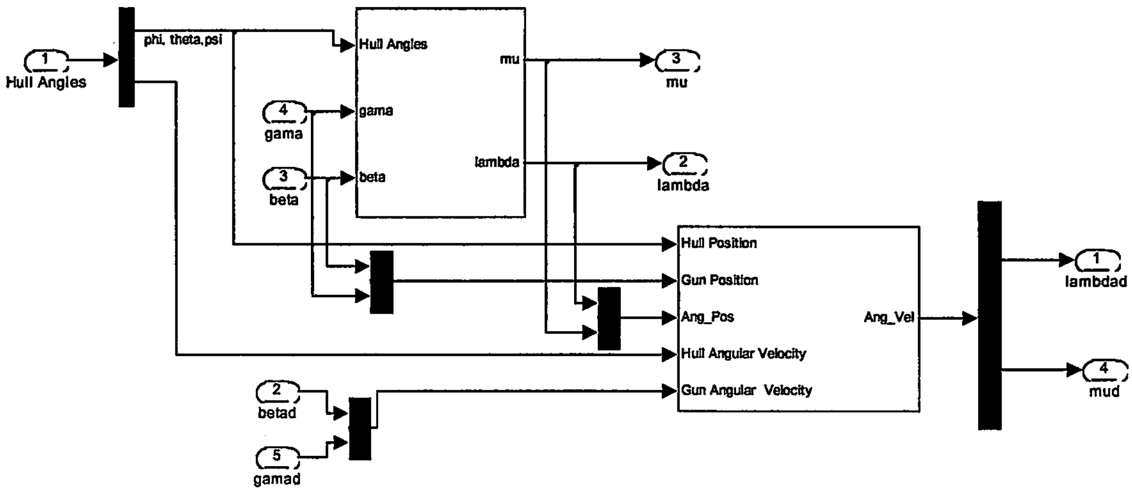

Figure 4-29 Coordinate frame from the gun frames to the earth fixed frame.

The first subsystem used in figure 4-29 is for transforming the azimuth and elevation angles to the earth fixed frame. The second subsystem used is for transforming the azimuth and elevation angles to the earth fixed frame. This subsystem is shown in figure 4-30. It basically consists of an S-Function Block named 'AngVel0F'. The code of this block is given in Appendix E.

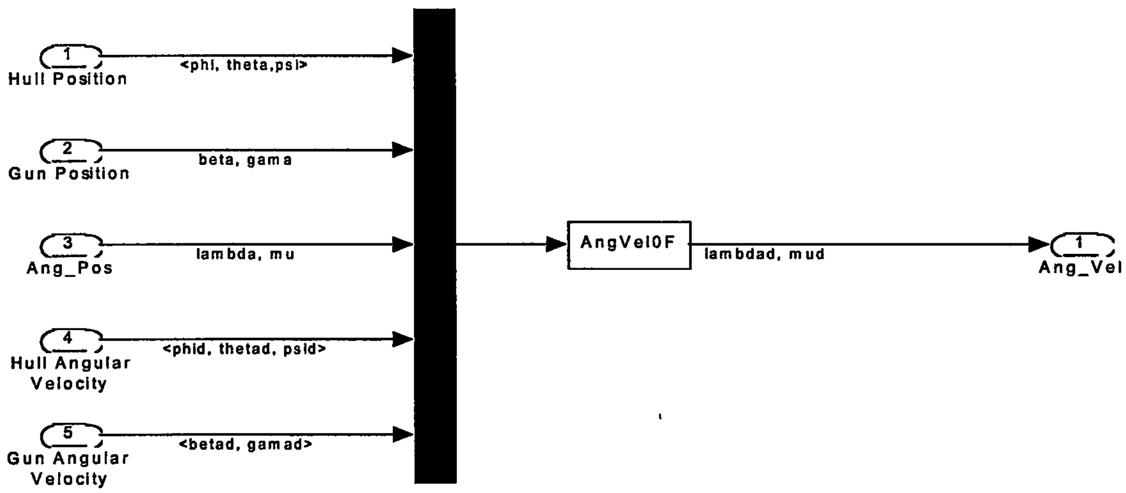

Figure 4-30 Transformation of the angular velocities to the earth fixed frame.

Finally, it is noted that the gyro drift errors are taken into account during the feedback of the gun angular velocities. Random signal blocks are used for this purpose. Thus the simulink model of the velocity control method by using a PID controller is completed.

4.3 Simulation Results

In this section, the simulation results of the tank gun control systems on a sample run will be given. At the beginning of this section, required parameters for the simulation of the tank gun control systems will be given. Then a sample run on both of the stabilization systems considered system will be discussed.

4.3.1 Tank Properties

Mass and geometric properties of a tank is needed for simulating the gun control systems. The tank, which is considered in this section, is Leopard 1. The mass and inertia properties of the tank turret and barrel are tabulated as:

Mass of the tank turret	:	7400kg
Mass of the tank barrel	:	2900kg

And

$$\hat{J}_{G_t} = \begin{bmatrix} 4222 & 145 & -381 \\ 145 & 7769 & 3.95 \\ -381 & 3.95 & 11195 \end{bmatrix} kg \cdot m^2$$

$$\hat{J}_{G_b} = \begin{bmatrix} 268 & -0.1 & 10.8 \\ -0.1 & 5317 & 0 \\ 10.8 & 0 & 5508 \end{bmatrix} kg \cdot m^2$$

where $\hat{J}_{G_t}$ is the inertia matrix of the tank turret with respect to its mass center and $\hat{J}_{G_b}$ is the inertia matrix of the tank barrel around its mass center. The geometric distances required by the simulink model are:

$$\overline{R}^t_{HT} = [1.24 \quad -0.27 \quad 0.83] m$$

$$\overline{R}^t_{TB} = [1.135 \quad 0 \quad 0] m$$

$$\overline{R}^t_{TG_t} = [-1.4 \quad -0.15 \quad 0.096] m$$

$$\overline{R}^t_{BG_b} = [0.935 \quad -0.788 \quad -0.591] m$$

It is noted that the definition of these geometric distances are already given in Chapter 2.The above parameters can be used in the graphical user interface explained in previous sections. Another important parameter that is to be used in a stabilization system is the range of the tank barrel in elevation. This range is -10°/23° for Leopard 1 tank. Hence, the elevation commands will be in this range during simulations.

4.3.2 Tank Hull Motion

The simulink model for finding the tank hull was explained in section 4.1. This model is used for calculating necessary parameters of the tank hull motion. It is required that the performance and sideslip positions are functions of time. Then, the tank hull parameters; namely, performance, sideslip and bounce accelerations $(\ddot{x}, \ddot{y}, \ddot{z})$, yaw, pitch and roll angles, angular velocities, and angular accelerations are found $(\psi, \theta, \phi, \dot{\psi}, \dot{\theta}, \dot{\phi}, \ddot{\psi}, \ddot{\theta}, \ddot{\phi})$. The velocity of the tank hull is given as:

$$V = 12 \text{ m/s} \tag{4.4}$$

this velocity is a typical velocity of the tank in an off road environment. The angle between the performance axis of the tank hull and the u_1 axis of the earth fixed frame is also required. This angle (η) is taken as constant and:

$$\eta = 45° \tag{4.5}$$

Then the velocity of the tank hull can be decomposed into the earth fixed frame as:

$$\vec{V} = (V \cdot \cos(\eta)) \cdot \vec{u}_1^{(0)} + (V \cdot \sin(\eta)) \cdot \vec{u}_2^{(0)} \tag{4.6}$$

Hence, the performance and sideslip components of the tank hull is found by integrating the components of the equation 4.6.

$$x(t) = V \cdot \cos(\eta) \cdot t \tag{4.7}$$

$$y(t) = V \cdot \sin(\eta) \cdot t \tag{4.8}$$

Then, using the simulink model for tank hull motion, the required parameters of the tank hull is obtained. They are plotted in the following graphs:

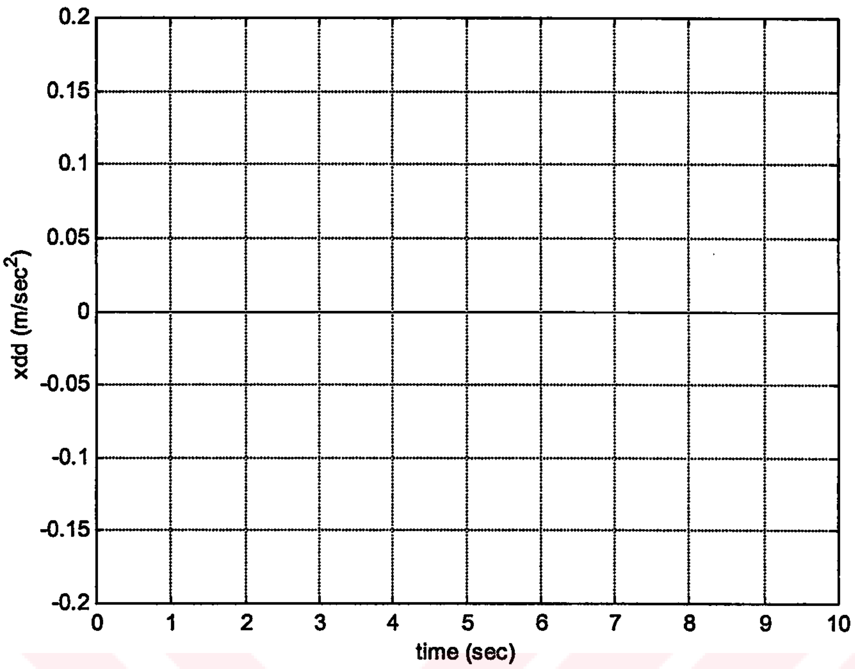

Figure 4-31 Acceleration of the tank hull in performance direction

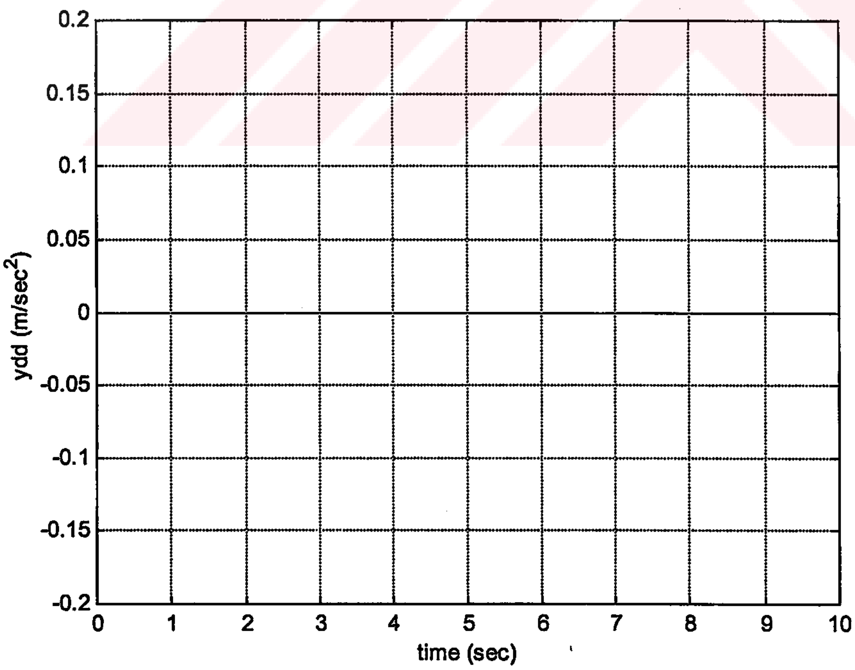

Figure 4-32 Acceleration of tank hull in sideslip direction

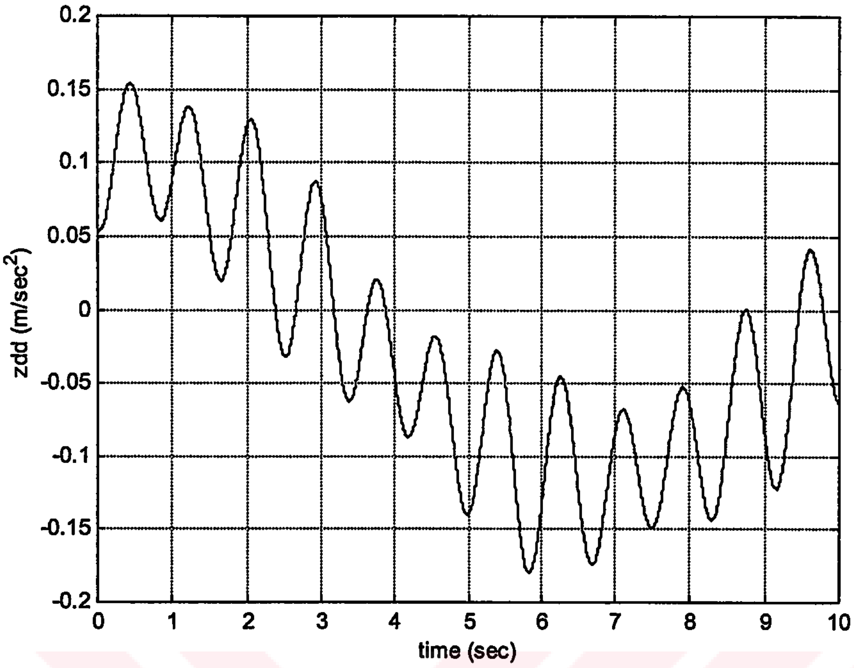

Figure 4-33 Acceleration of tank hull in bounce direction

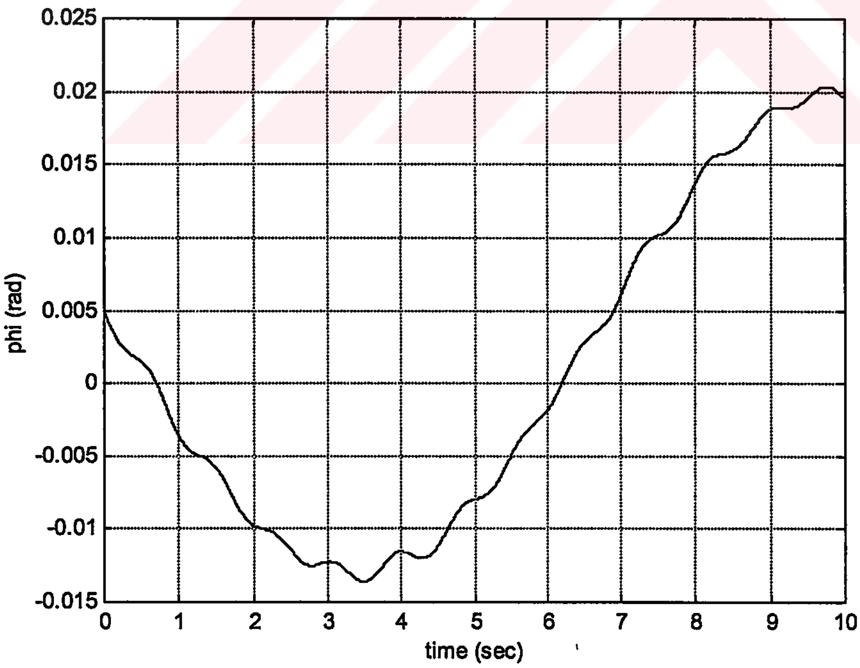

Figure 4-34 Roll angle

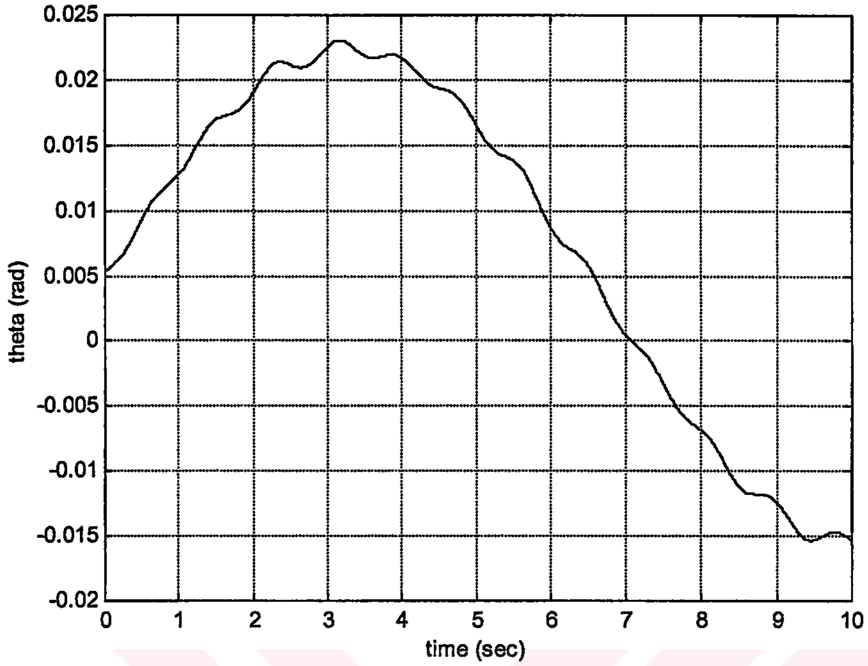

Figure 4-35 Pitch Angle

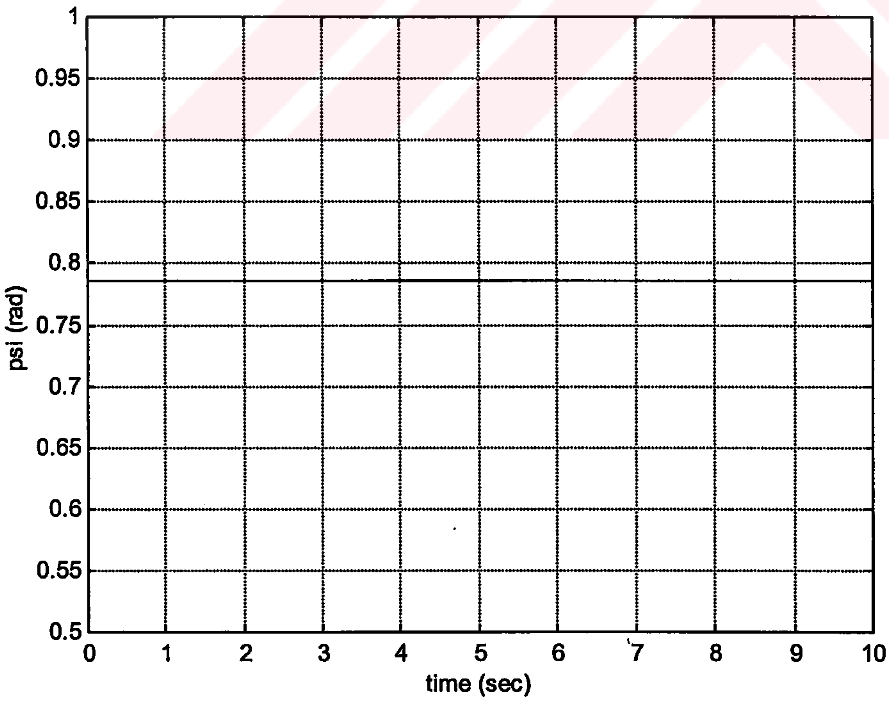

Figure 4-36 Yaw Angle

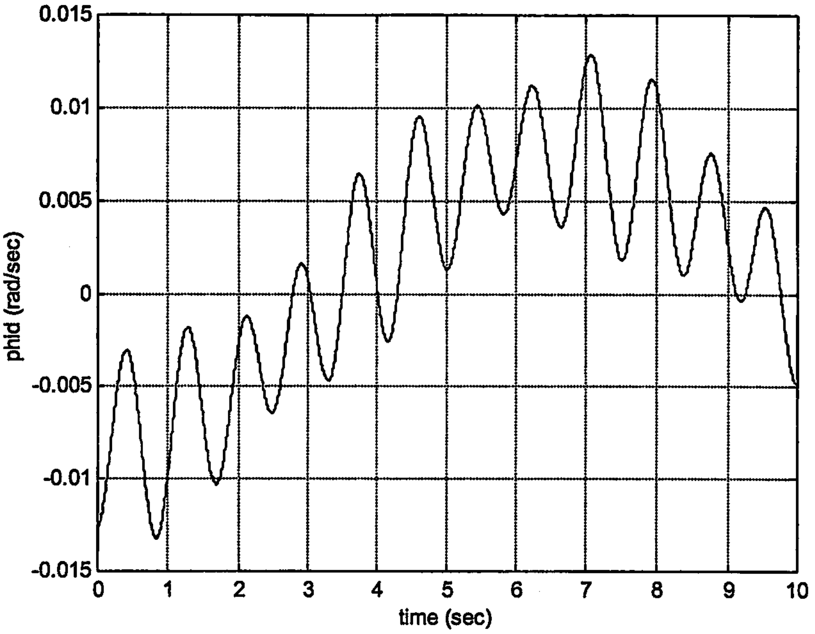

Figure 4-37 Roll angular velocity

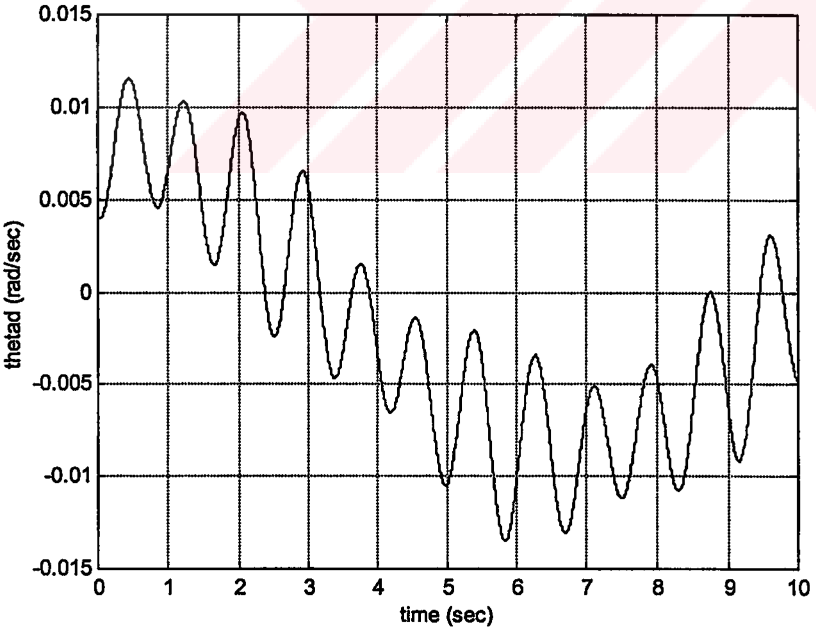

Figure 4-38 Pitch angular velocity

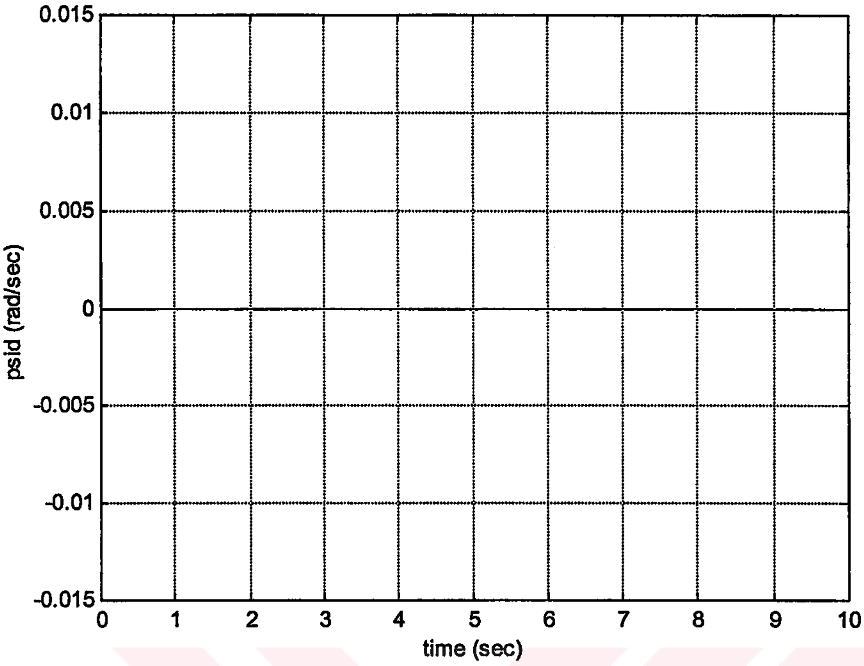

Figure 4-39 Yaw angular velocity

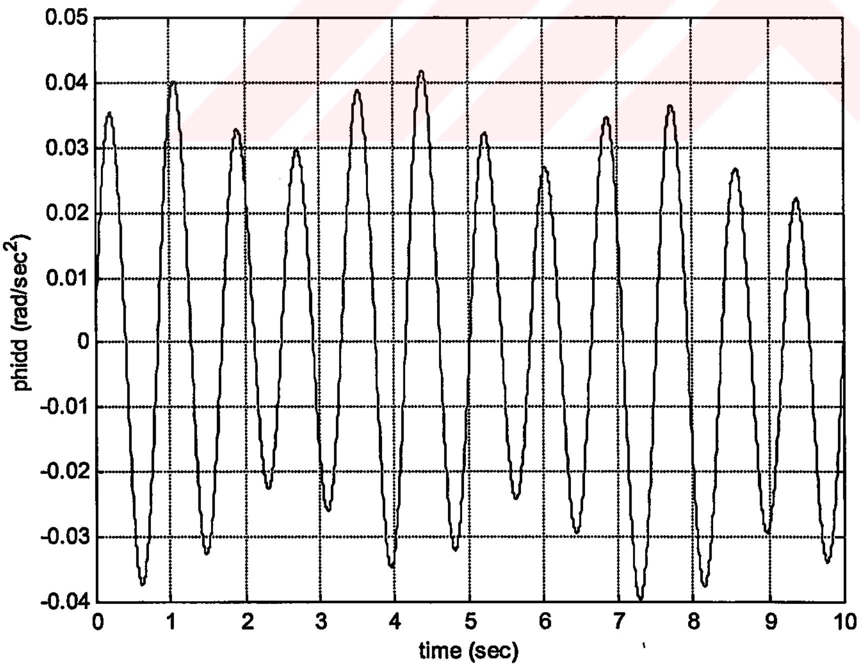

Figure 4-40 Roll angular acceleration

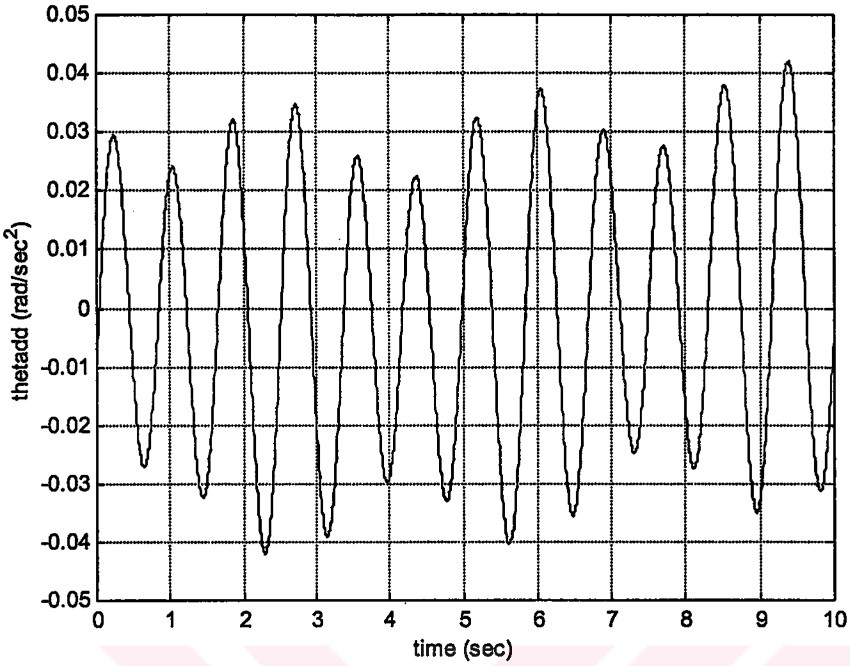

Figure 4-41 Pitch angular acceleration

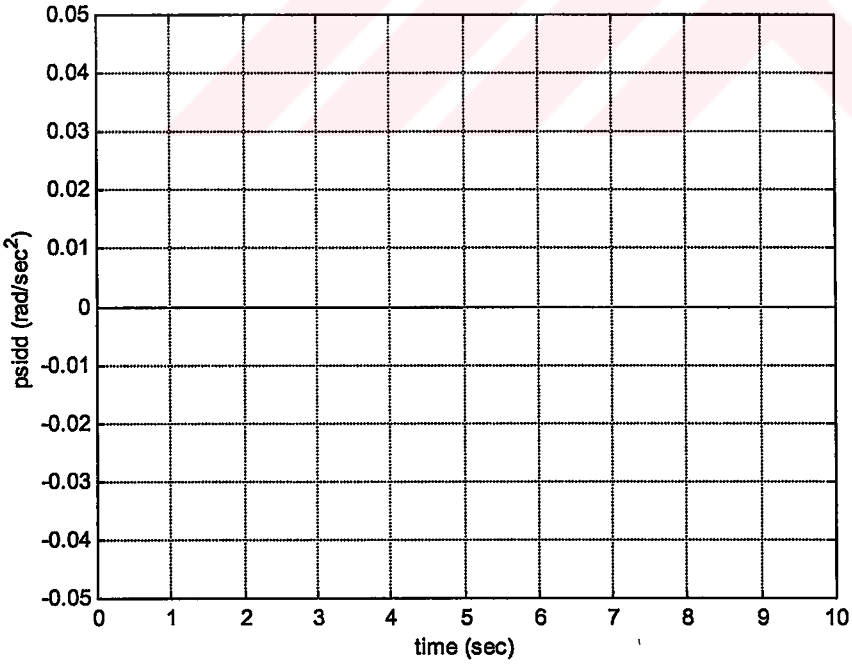

Figure 4-42 Yaw angular acceleration

T.C. YÜKSEKÖĞRETİM KURULU
DOKÜMANTASYON MERKEZİ

It is noted that the acceleration in performance and sideslip $(\ddot{x}, \ddot{y})$ are zero because the tank moves with constant velocity. Also, the yaw angle of the tank is 45° (0.785 rad) due to the fact that the angle between the $\vec{u}_1$ axis of the earth fixed frame and the hull performance axis (η) is taken as constant. So, the yaw angular velocity and angular acceleration terms are also zero.

4.3.3 Measured Values of the Tank Hull Motion

Measured values of the tank hull will be used in the computed torque method. Gyroscopes and accelerometers are used for measuring the tank hull motion given in the previous section. The sensor errors are already modeled by using simulink and explained in section 4.1. The sensor error data are given as follows:

Gyroscope drift error	:	0.01°/hr
Gyroscope white noise	:	$3 \cdot 10^{-5}$°$/\text{sec}/\sqrt{\text{Hz}}$
Accelerometer bias error	:	50μg
Accelerometer bias white noise	:	$10\mu g/\sqrt{\text{Hz}}$

The error data is used in the simulink model which was discussed in section 4.1. Then, the results are plotted as follows:

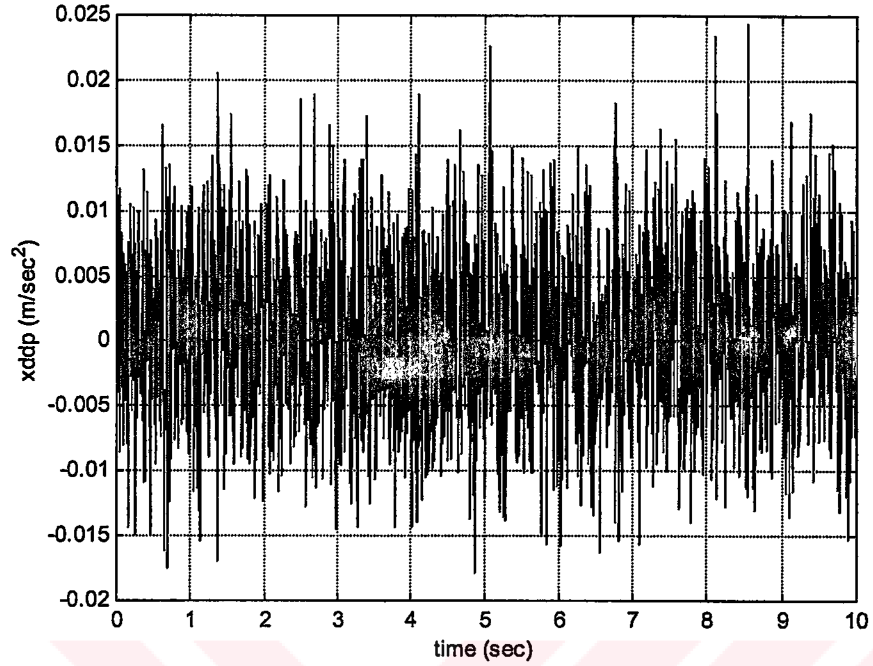

Figure 4-43 Measured performance acceleration of the tank

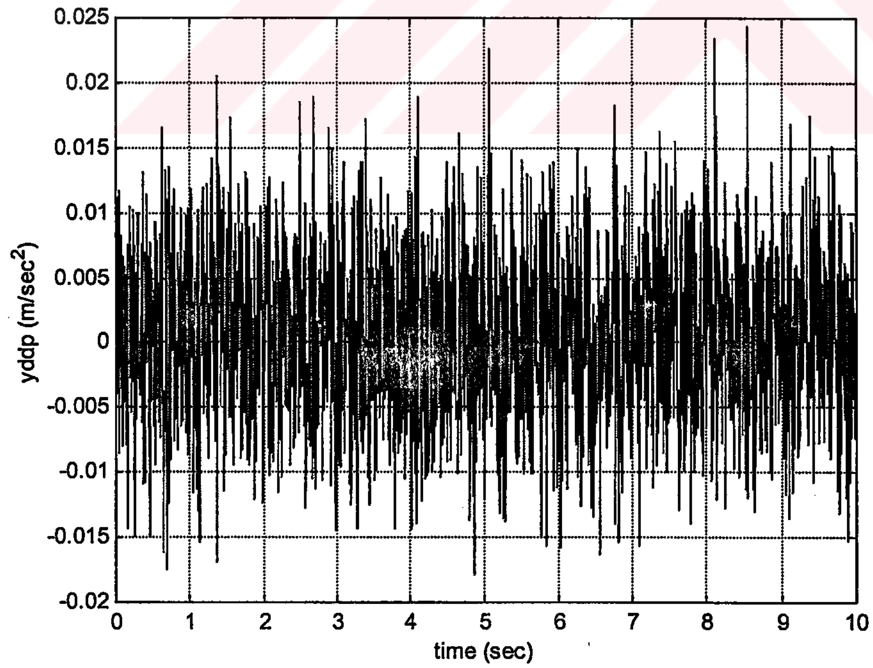

Figure 4-44 Measured sideslip acceleration

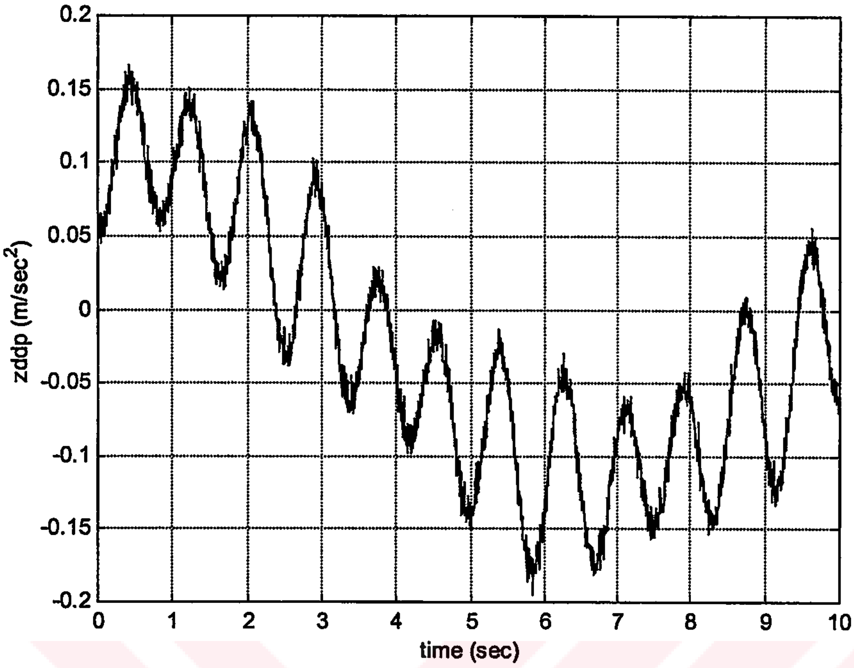

Figure 4-45 Measured bounce acceleration,

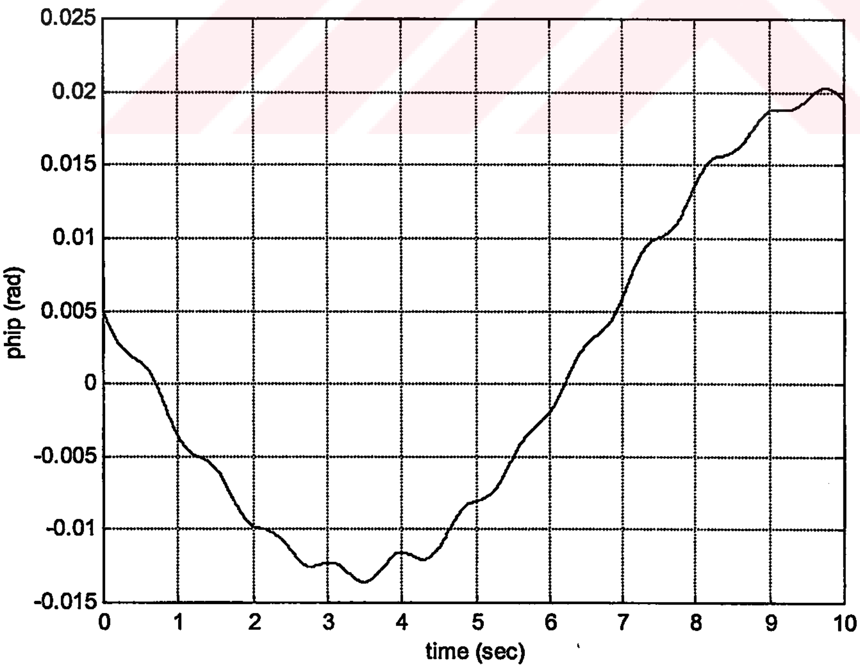

Figure 4-46 Measured roll angle.

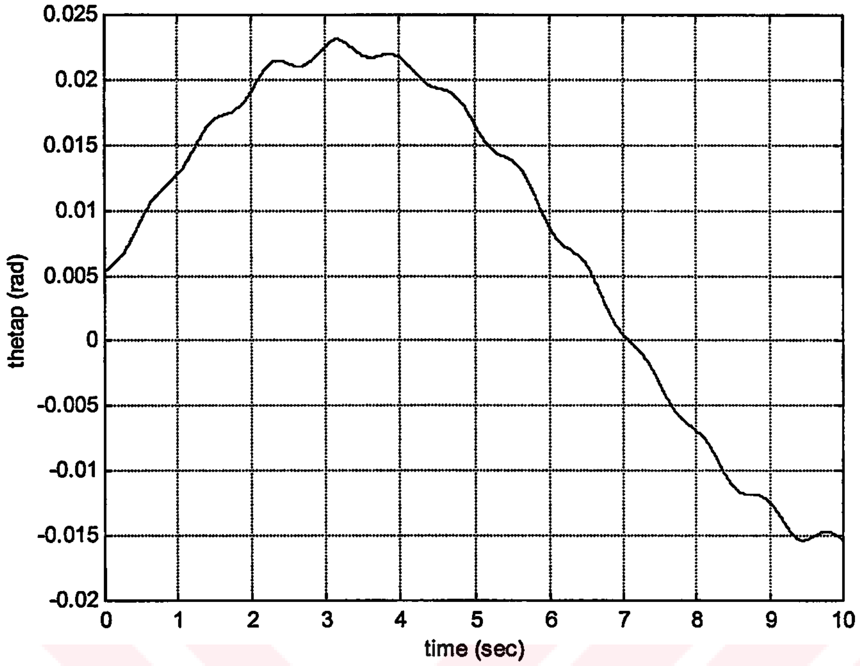

Figure 4-47 Measured pitch angle

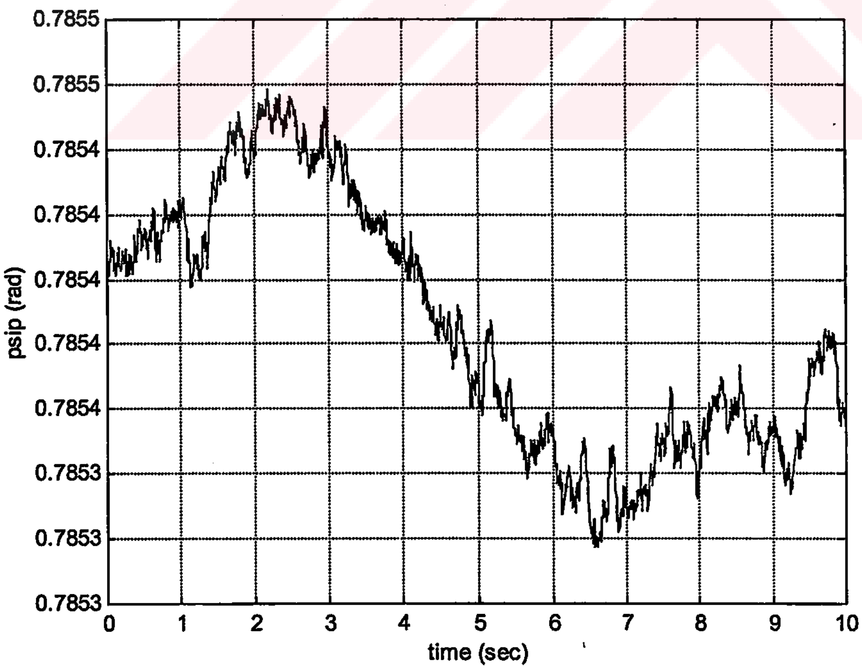

Figure 4-48 Measured yaw angle

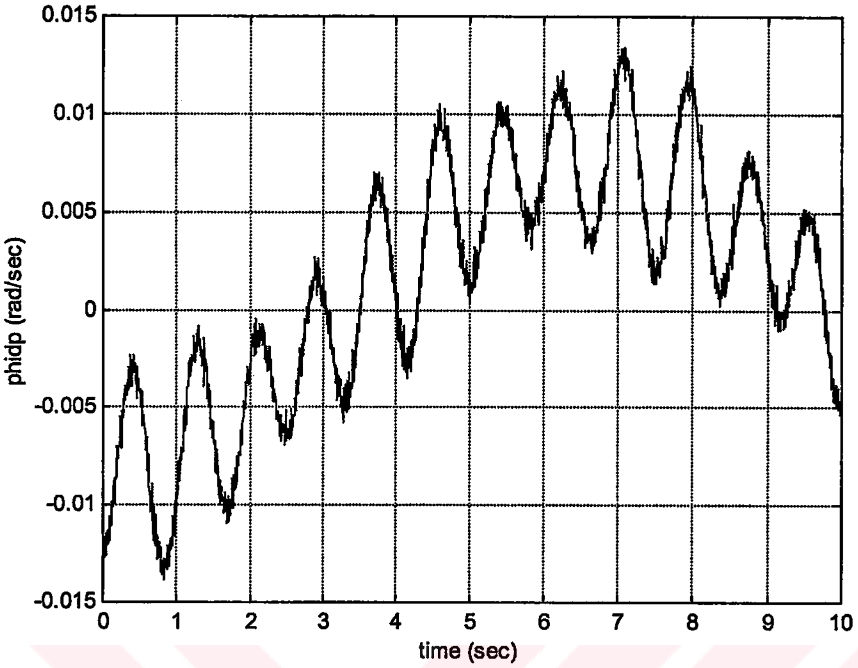

Figure 4-49 Measured roll angular velocity

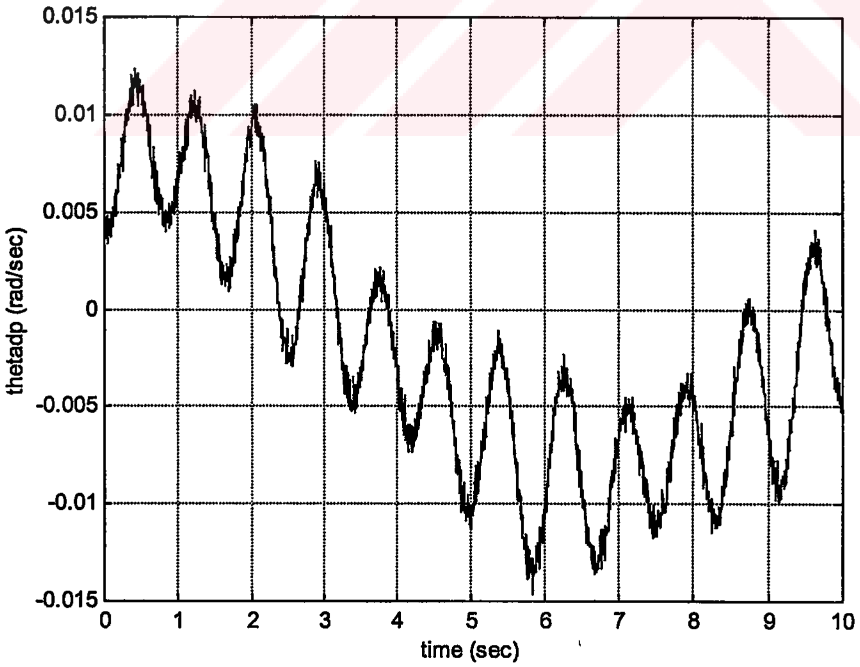

Figure 4-50 Measured pitch angular velocity

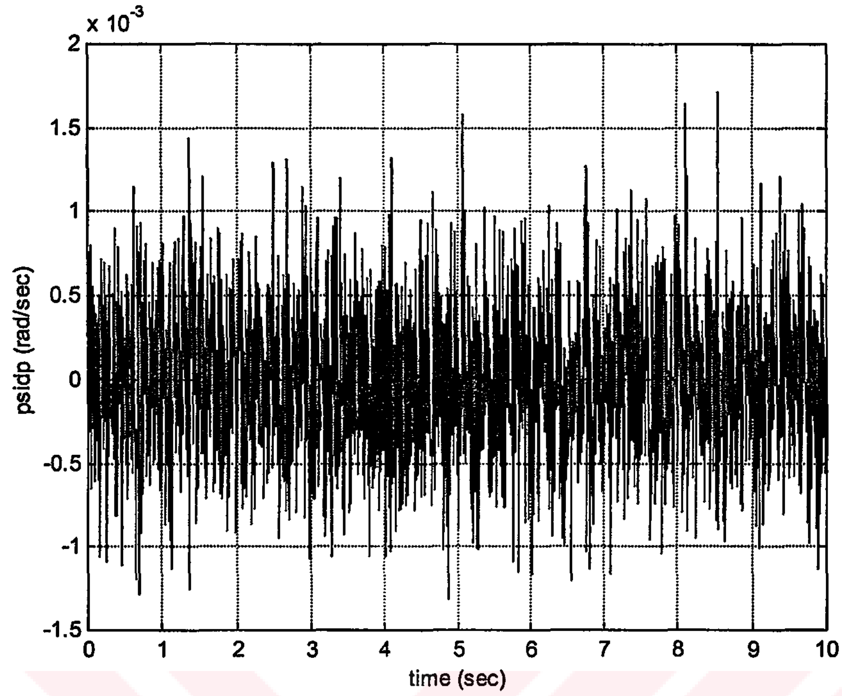

Figure 4-51 Measured yaw angular velocity

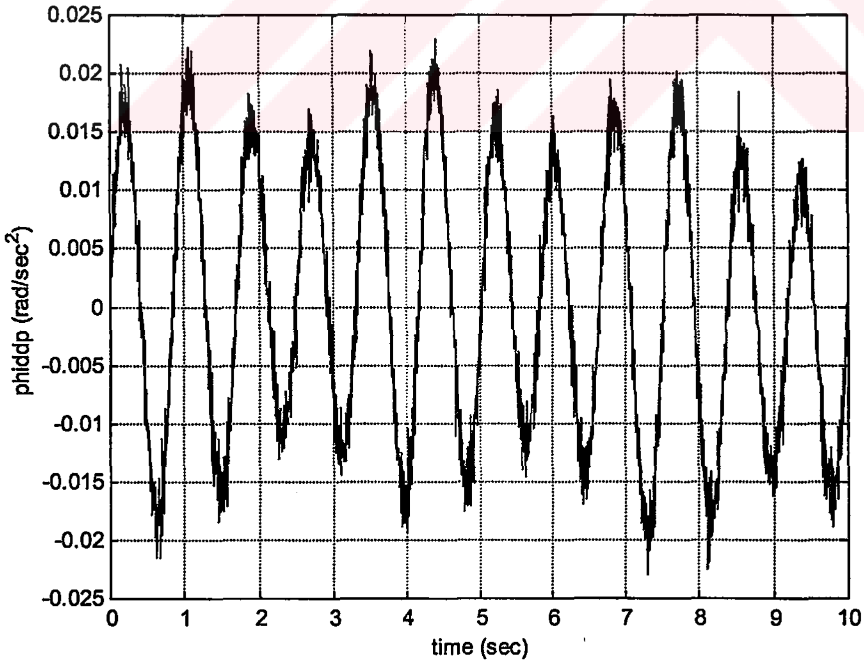

Figure 4-52 Measured roll angular acceleration

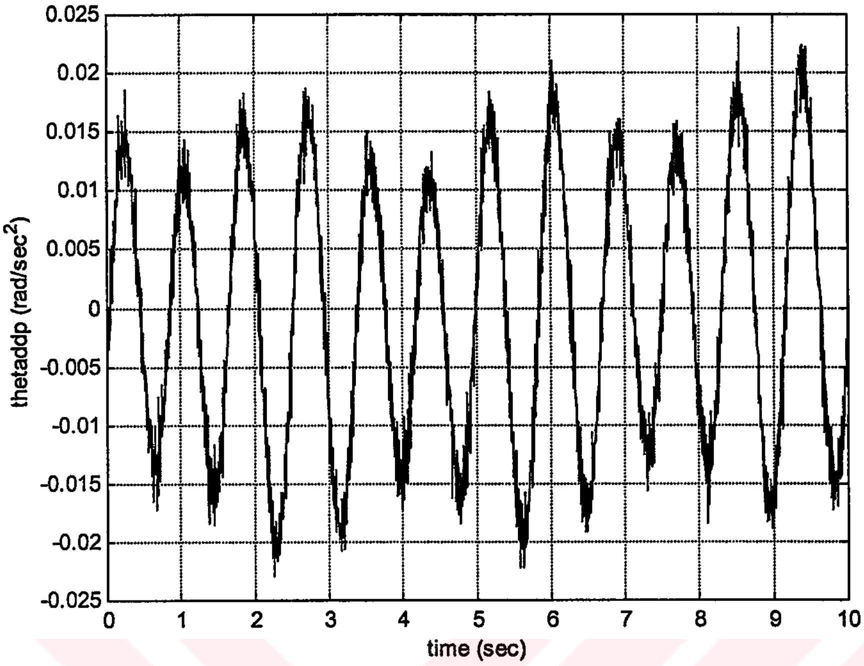

Figure 4-53 Measured pitch angular acceleration

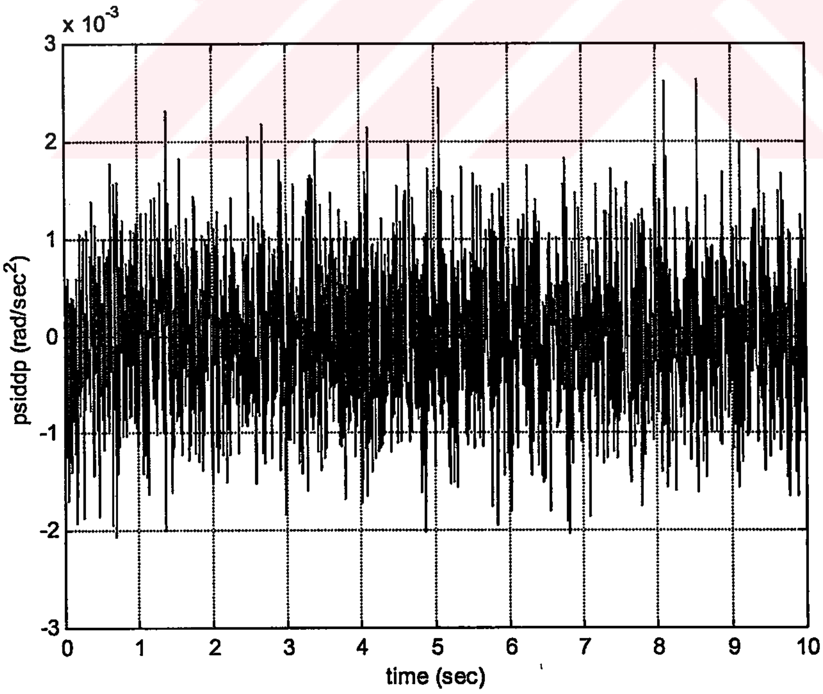

Figure 4-54 Measured yaw angular acceleration

It is noted that some low pass filters are used in obtaining the above results for the measured tank hull angular accelerations. The filters, which are used in this simulation, are:

$$G(s) = \frac{1}{0.0021 \cdot s + 1}$$

for obtaining the measured roll and pitch angular accelerations. The filter:

$$G(s) = \frac{1}{0.1 \cdot s + 1}$$

is used for obtaining the measured angular acceleration.

4.3.4 PID Parameters of the gun control systems

The selection of the PID parameters is based on the formulation given in chapter 3 for the computed torque method. The parameters are used in the PID controller block in the simulink model. The suitable values are found by trial and error in Simulink and they are tabulated as follows:

$$\omega_n = 8, \qquad \xi = 0.8, \qquad \eta = 3$$

or in terms of the PID parameters:

$$K_p = 371.2, \qquad K_i = 192, \qquad K_d = 126.4$$

When the second control method is considered, it is noted that there is no way to formulate the coefficients of the PID controller used in this method because the plant to be controlled is highly nonlinear. The only way to determine the suitable parameters is by using trial and error. The reference coefficients are chosen as that of the previous ones. Also, it is desired that the overshoot in the response of the system is to be minimized. The resultant parameters are tabulated as follows:

$K_p = 85,$ $K_i = 1400,$ $K_d = 8$

4.3.5 Hydraulic Motors

The parameters of the open centered spool valve used in simulink model of the hydraulic motors are written as:

$$C_d = 0.611$$

$$w=1.5 \text{ mm}$$

$$\rho = 890 \text{ kg/m}^3$$

$$x_0/2=6 \text{ mm}$$

$$P_s = 7 \text{ MPa}$$

The area of the actuator for elevating the tank barrel is taken as $A=0.003 \text{ m}^2$. Also, the parameters, which are required to find the velocity of the hydraulic ram, are given as follows:

$$a_1 = 1 \text{ m}$$

$$r_1 = 0.348 \text{ m}$$

$$a_2 = 0.3 \text{ m}$$

$$r_2 = 0.1 \text{ m}$$

Finally, the coefficient of the hydraulic motor used in azimuth and the gear reduction ratio used in driving the tank turret are:

$$D_m = 0.00012$$

$$r_g = 100$$

The constants given by the equations 3.8 and 3.9 are:

$$K_m = 0.3 \text{ N/mA}$$

$$K_s = 100000 \text{ N/m}$$

and finally, the additional relation between the controller command and the current has the gain:

$$K_v = 40 \text{ mA/V}$$

The transfer function, which were introduced in Chapter3 are given as follows:

$$G_i(s) = \frac{1}{0.005 \cdot s + 1}$$

and

$$G_v(s) = \frac{10000}{s^2 + 180 \cdot s + 10000}$$

4.3.6 Simulation Results of the Computed Torque Method:

The simulation results are based on input parameters defined previously. The sample run for the computed torque is given by the following reference commands:

Initial azimuth angle : 45°
Final azimuth angle : 95°
Rise time for the azimuth angle : 6 sec

And also,

Initial elevation angle : 0.3°

Final elevation angle : 20°

Rise time for the elevation angle : 8 sec

It is noted that the initial azimuth and elevation angles are taken as nonzero because although their initial angles are zero in their frames; in earth fixed frame, it is different due to the profile of the ground. The simulation results are shown as follows:

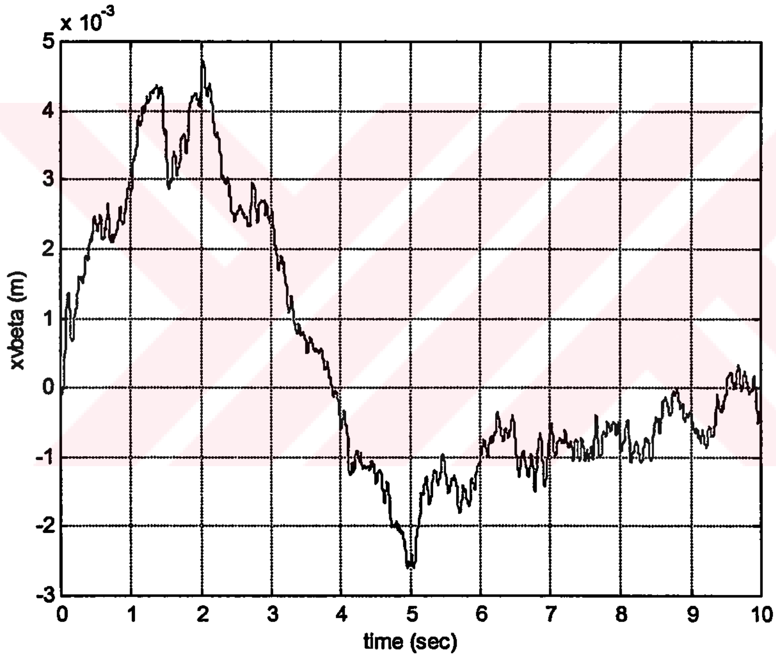

Figure 4-55 Valve displacement of the hydraulic motor for the azimuth direction in computed torque method

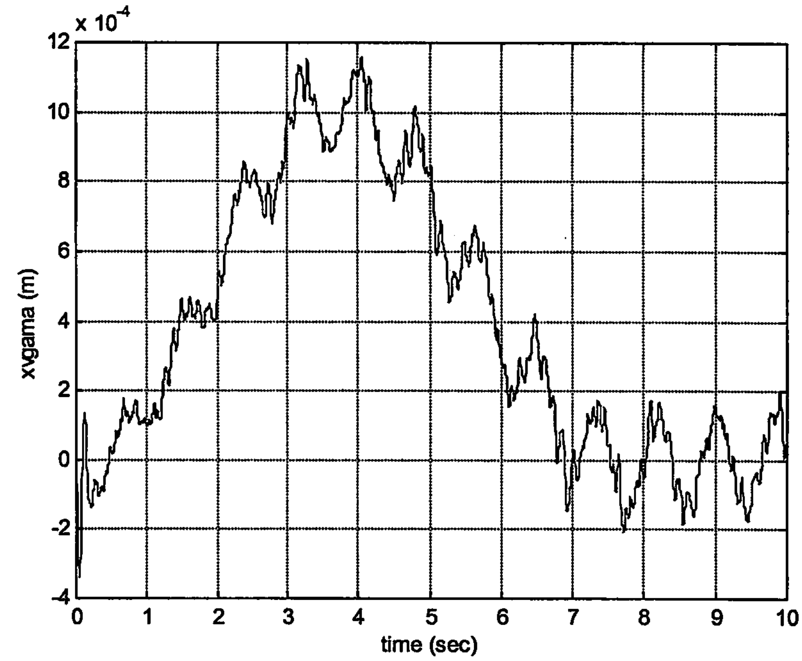

Figure 4-56 Valve displacement for the hydraulic motor in elevation direction in computed torque method.

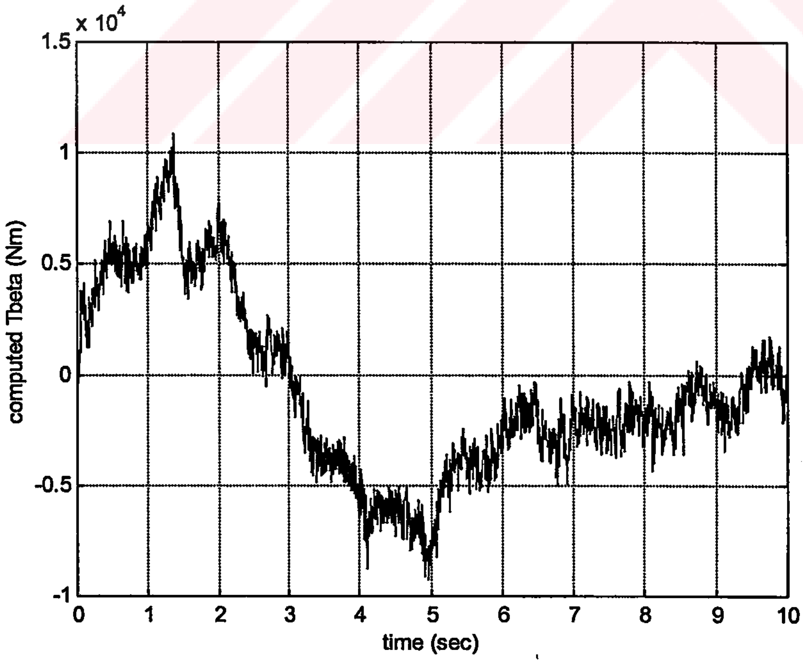

Figure 4-57 Computed torque for azimuth hydraulic motor

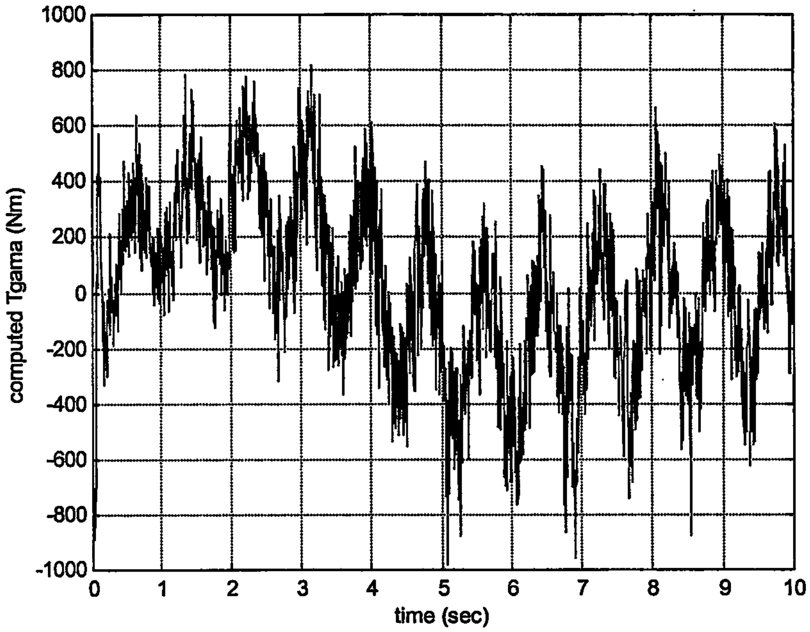

Figure 4-58 Computed Torque for the elevation

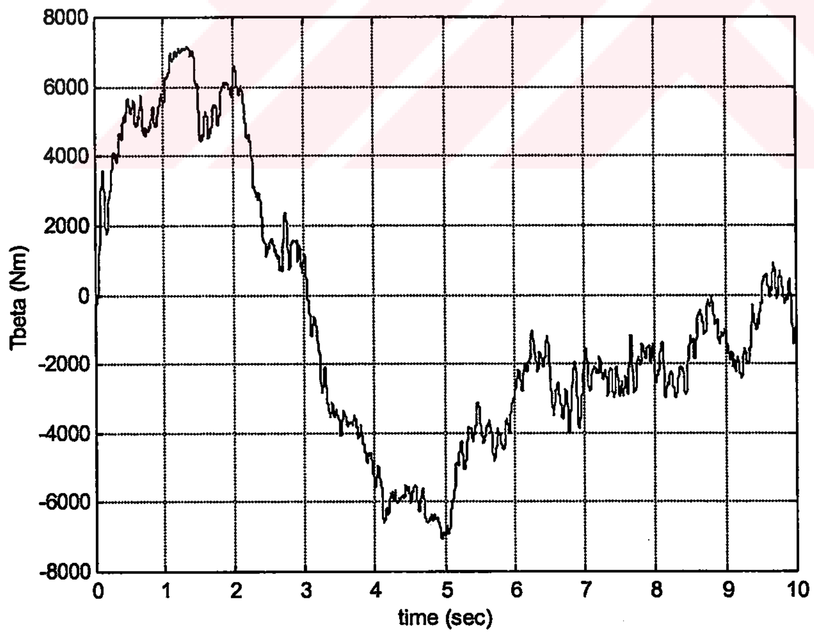

Figure 4-59 Motor Torque in azimuth direction in computed torque method

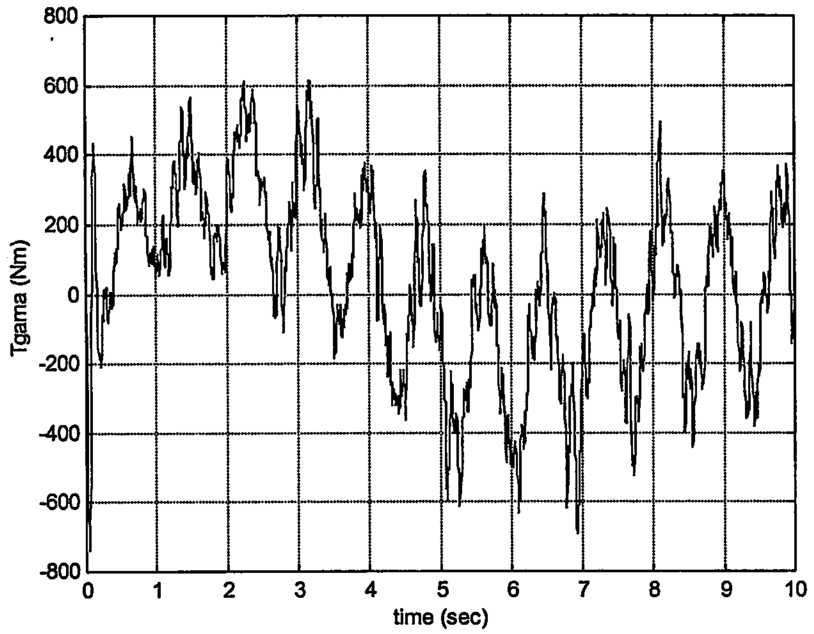

Figure 4-60 Motor Torque in elevation direction in computed torque method

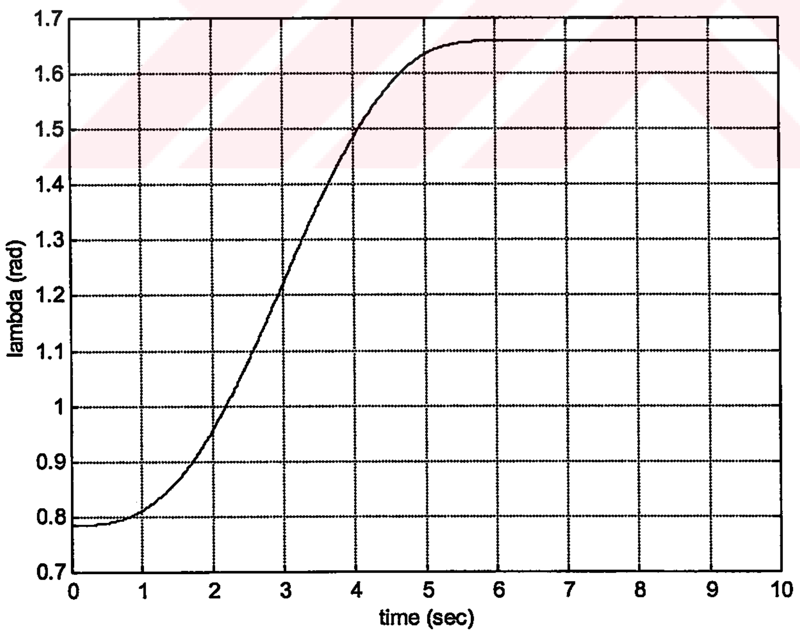

Figure 4-61 Azimuth angle in earth fixed frame in computed torque method

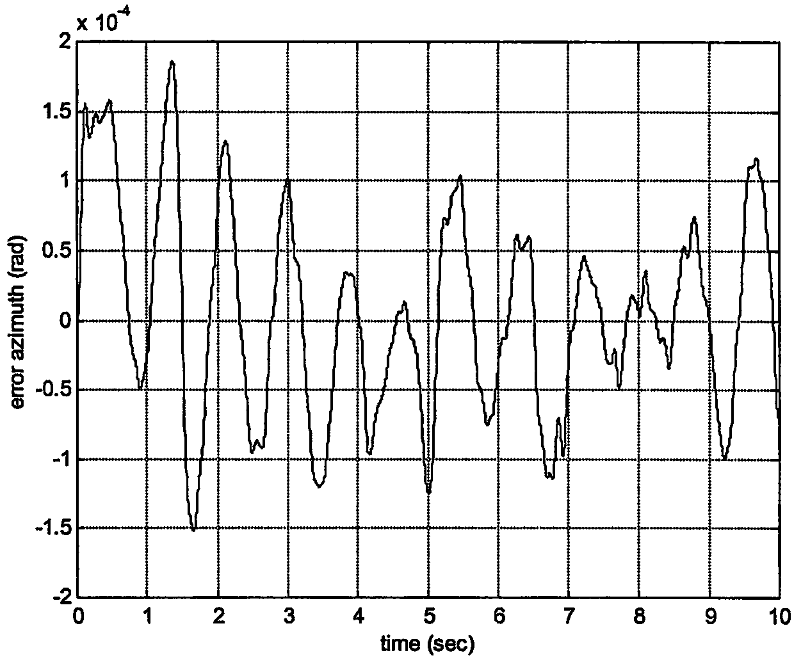

Figure 4-62 The error between the azimuth reference and the actual angles in computed torque method

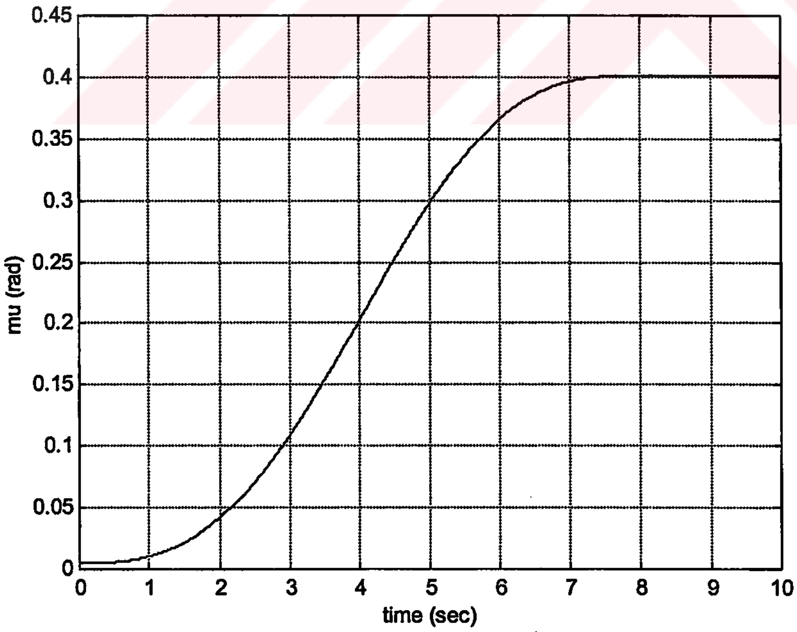

Figure 4-63 Elevation angle of the tank gun in computed torque method

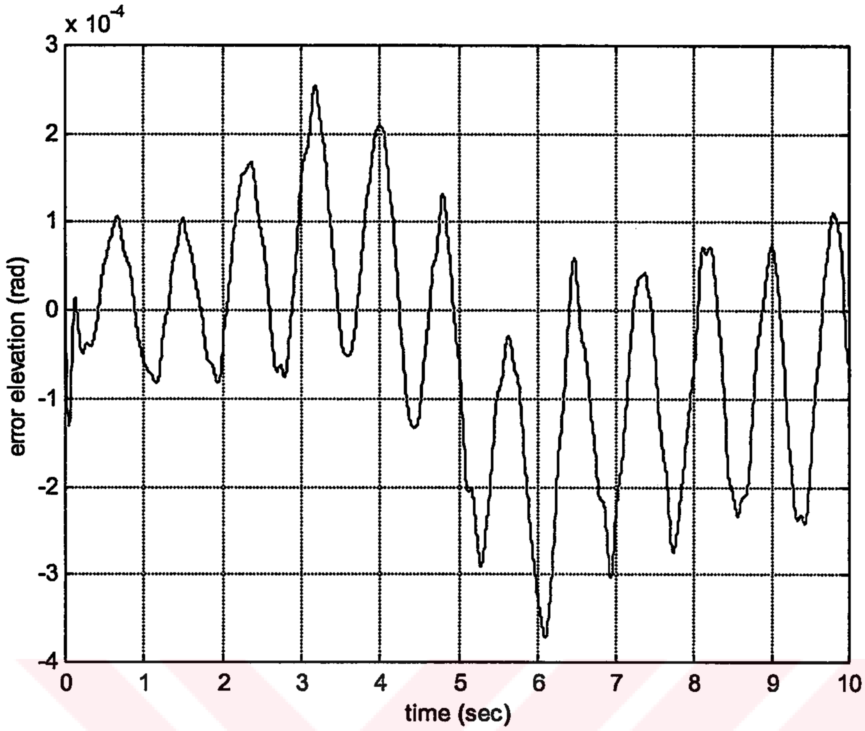

Figure 4-64 Error between the reference and the actual elevation angles in computed torque method

The valve displacement, and the torque graphs are also illustrated above to have a better idea of the stabilization system. The difference between the computed torque and the motor torque graphs are mainly due to the transfer functions used during the simulations.

4.3.7 Simulation results for the velocity control of the gun system

The same input parameters are used in this simulation with that of the computed torque method. The results are plotted as follows:

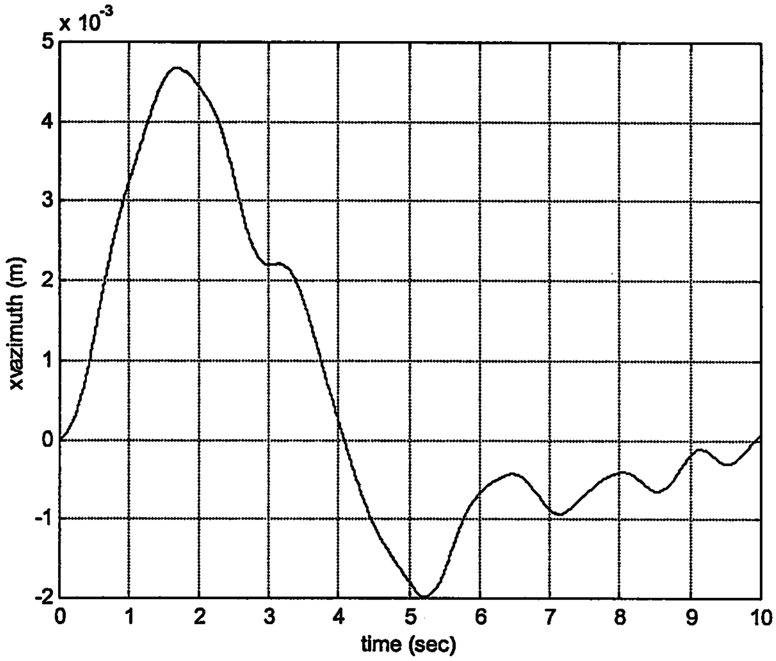

Figure 4-65 Valve displacement in azimuth motor

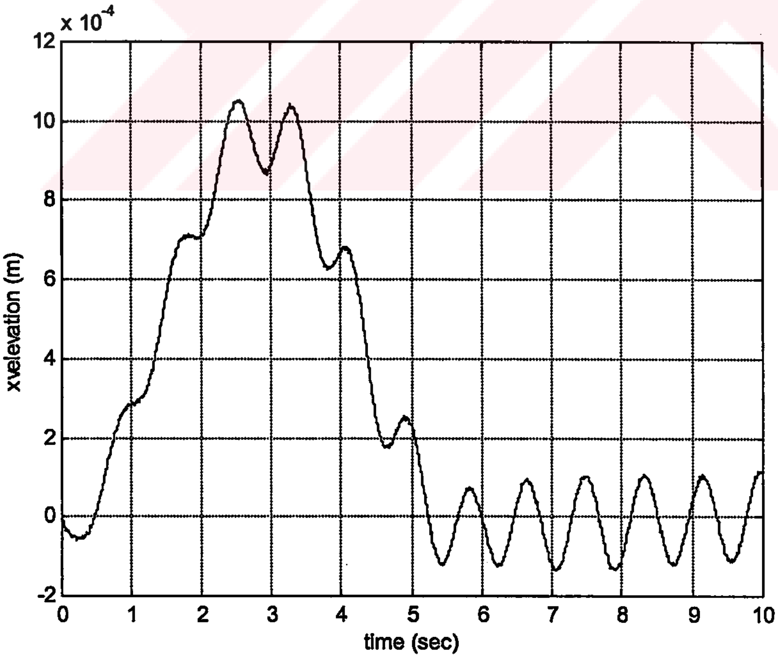

Figure 4-66 Valve displacement in elevation motor

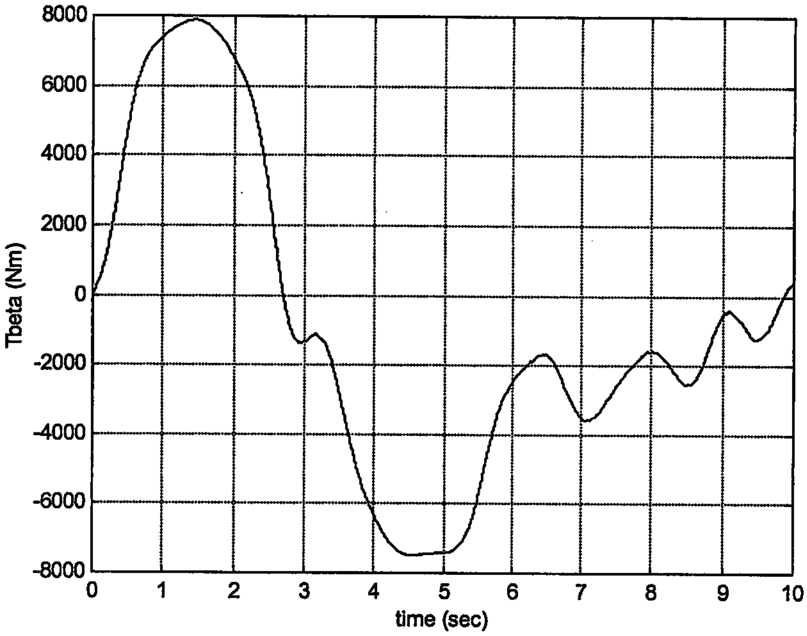

Figure 4-67 Azimuth motor torque

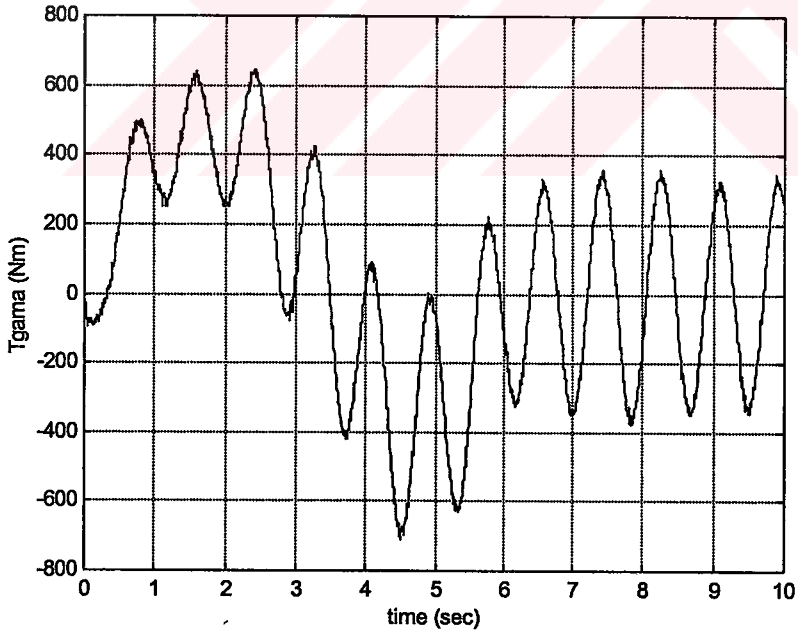

Figure 4-68 Elevation motor torque

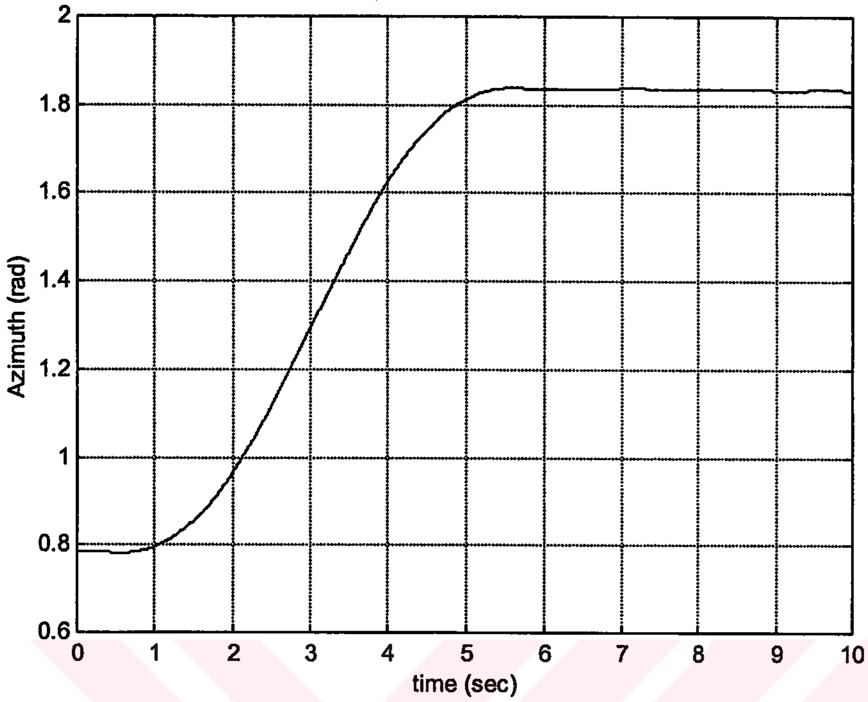

Figure 4-69 Azimuth angle in earth fixed frame

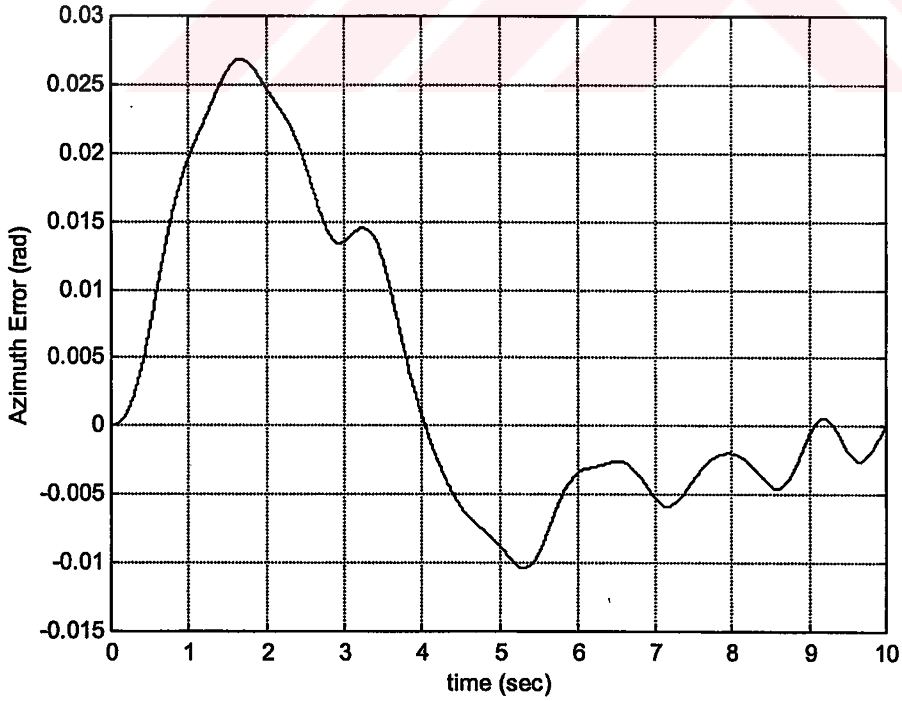

Figure 4-70 Error between the reference azimuth angle and the actual azimuth angle

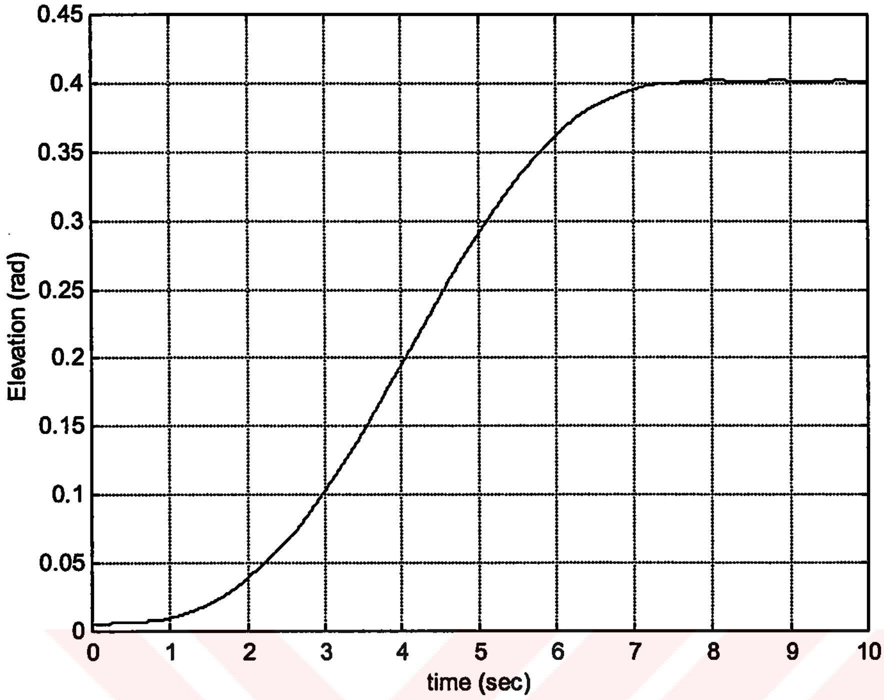

Figure 4-71 Elevation angle of the tank barrel

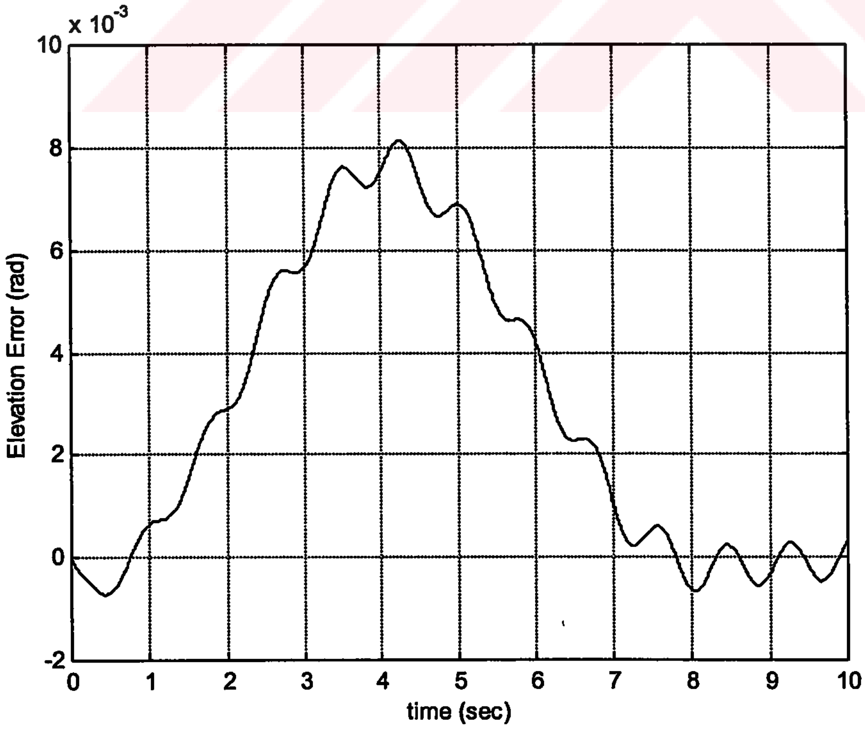

Figure 4-72 Error between the reference and the actual elevation angles

The results of the error graphs between the reference and the actual commands in figures 4-70 and 4-72 show that they are on the order of 10^{-2} radians, which is bigger compared to the error values of the control system with computed torque method.

Thus, in this chapter, simulink models of selected gun control methods; namely, computed torque method and a velocity control method by using a PID controller are done. And a sample simulation is made with both of the models and selected graphs are given to illustrate the performance of both of these systems.

CHAPTER 5

DISCUSSION AND CONCLUSION

In this thesis, the problem of controlling a tank gun while the tank is on the move is considered. The dynamics of the tank gun is solved for this purpose. The tank gun system is considered as having the ability of rotating in both the azimuth and elevation directions by means of its turret and barrel. Since the tank should be able to fire while it is on the move, the motion of the tank hull is not neglected. Hence eight degrees of freedom system is considered for the tank gun during the study. For the solution of this system, a formulation based on the Newton-Euler equations is used. Finishing the dynamics of the tank gun system, the gun control systems that are to be used in this study are considered. The control systems, which are used in this study, are the position control of the tank gun system by using the computed torque method, and the velocity control by using a PID controller. The error models of sensors, which are to be used in these systems, and the hydraulic drives for these systems, are formulated. Then the modeling methods are discussed for these methods. Finally, these control systems are simulated by using matlab simulink version 2.2.

The simulations, which are made for these control systems, are based on fixed time steps in Simulink. Runge Kutta 4th order solver of the simulink is used as the numerical integration method in simulations. It is concluded that there is a slight numerical error in both of these stabilization simulations due to the numerical solution of the tank model for the azimuth and elevation angles. Taking smaller time steps can decrease this source of error but this time the computer works slower. In this study, a fixed time interval of 0.005 seconds is taken and is considered as satisfactory for the simulation results.

The simulations in this study are based on the problem of orienting the tank gun from an initial position to a final position (this position may be the target of the tank gun) and then holding the gun at this position in the earth fixed frame.

In both of the systems, the same ground disturbance samples are used. However, in position control of the gun system by using the computed torque method, all of these disturbances are measured and are included in computing the torque necessary to drive the gun system. This torque value is sent to the motor model for calculating the control valve opening. This valve displacement is used for calculating the output torque of the system. Then the real hull motion is directed to the vehicle model together with the motor torque necessary to drive the gun system. In the velocity control of the gun control system by using gyro feedback, only the actual tank hull motion is used and it is treated as disturbance to the tank model. The motor torque is calculated by converting the output of the error term to the valve opening.

The control of the tank gun system by using the computed torque method is characterized by its complexity due to the frame transformations, torque calculations and motor model. However, the simulation results are rather satisfactory and the error terms are very small as can be seen from the simulation. This method never allows the error term to be increased as long as the necessary torque is provided. In the simulation results of this control system, the error terms are only related with the numerical solution and the bias and drift errors in the measurement devices, and the motor response delay due to the physical consideration of the control valve. If these error terms were coped with, then it would be possible to obtain zero error in the stabilization system. A disadvantage of this system arises from its complexity, it is not easy to implement this system to the tank gun system since it requires computer system in the battle tank. Another disadvantage may arise if an unexpected disturbance other than the present ones arises. This may be the improper working of the measurement devices, or addition of an unexpected damage in the components of the tank gun, for example; during the wartime. These unexpected errors may be coped by selecting good sensors, and by using powerful drives for the gun system and fast computer systems. These possibilities should be taken into account and when choosing the hardware:

- The sensors,
- The power drives,
- And the computer system should be selected carefully.

The clear advantage of the velocity control of the tank gun system by using gyro feedback is that its simplicity. However, the results of the tank gun system simulation show that the error terms are not as small as that of the computed torque method. This is unavoidable since the disturbances are fed directly to the tank gun system and are not treated as an input to the control system as in computed torque method. However, this method is simpler and requires less hardware compared with the computed torque method. For this reason it is cheaper – computer system, or plenty of sensors are not required.

Due to the simulation results plotted in Chapter 4, the computed torque method for stabilization of the tank gun is suggested. The following future works can be done with this control method to improve its performance and its cost.

Effects of different sensors and mass parameters can be observed on the model for the computed torque method to improve the system response.

In modern battle tanks, hydraulic as well as the electric power drives are used. Although hydraulic drives are used for both elevation and traverse directions, electric drives can also be used to see if better performance is obtained. In addition to the torque characteristics, electric drives are advantageous since it is safer than the hydraulic systems, have no viscosity problems. A combination of electric and hydraulic drives can also be tried if better results are expected.

Another future work can be combining the stabilization of the tank barrel with the line of sight stabilization. The main principles of this stabilization system are explained in chapter 1 and it is expected that good results can be obtained by slaving the tank barrel to the line of sight of the gunner. Hence a director type system can be formed as a result of considering the tank barrel and the line of sight of the gunner together.

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APPENDICES

A. Code For Obtaining Angular Velocity of the Tank Gun in Tank Gun Frames

```
function [sys,x0,str,ts] = AngVelGF(t,x,u,flag)

switch flag,

  %%%%%%%%%%%%%%%%%%
  % Initialization %
  %%%%%%%%%%%%%%%%%%
  case 0,
    [sys,x0,str,ts]=mdlInitializeSizes;

  %%%%%%%%%%%%%%%
  % Derivatives %
  %%%%%%%%%%%%%%%
  case 1,
    sys=[];

  %%%%%%%%%%
  % Update %
  %%%%%%%%%%
  case 2,
    sys=[];

  %%%%%%%%%%%
  % Outputs %
  %%%%%%%%%%%
  case 3,
    sys=mdlOutputs(t,x,u);

  %%%%%%%%%%%%%%%%%%%%%%%
  % GetTimeOfNextVarHit %
  %%%%%%%%%%%%%%%%%%%%%%%
  case 4,
    sys=[];

  %%%%%%%%%%%%%
  % Terminate %
  %%%%%%%%%%%%%
  case 9,
```

```
  sys=[];


  % Unexpected flags %
  %%%%%%%%%%%%%%%%%%%%
  otherwise
    error(['Unhandled flag = ',num2str(flag)]);

end

% end sfuntmpl

%
%=============================================================================
% mdlInitializeSizes
% Return the sizes, initial conditions, and sample times for the S-
function.
%=============================================================================
%
function [sys,x0,str,ts]=mdlInitializeSizes

sizes = simsizes;

sizes.NumContStates  = 0;
sizes.NumDiscStates  = 0;
sizes.NumOutputs     = 3;
sizes.NumInputs      = 12;
sizes.DirFeedthrough = 1;
sizes.NumSampleTimes = 1;   % at least one sample time is needed

sys = simsizes(sizes);

%
% initialize the initial conditions
%
x0  = [];

%
% str is always an empty matrix
%
str = [];

%
% initialize the array of sample times
%
ts  = [0 0];

% end mdlInitializeSizes

%=============================================================================
% mdlOutputs
% Return the block outputs.
%=============================================================================
%
function sys=mdlOutputs(t,x,u)
```

```
phi=u(1); theta=u(2); psi=u(3);
lambda=u(4); mu=u(5);
gamades=u(6); betades=u(7);
phid=u(8); thetad=u(9); psid=u(10);
lambdad=u(11); mud=u(12);
AngularVelocityInGunFrame;
sys =X;

% end mdlOutputs



%AngularVelocityInGunFrame

u1=[1; 0; 0]; u2=[0; 1; 0]; u3=[0; 0; 1];
u1tilda=tilda(u1); u2tilda=tilda(u2);
u3tilda=tilda(u3);
C0a=expm(u3tilda*lambda);
C0c=C0a*expm(-u2tilda*mu);
C0m=expm(u3tilda*psi);
C0n=C0m*expm(-u2tilda*theta);
C0h=C0n*expm(u1tilda*phi);
C0t=C0h*expm(u3tilda*betades);
a1=C0t*u2; a2=-C0h*u3; a3=C0c*u1;
A=[a1 a2 a3];
D=(psid-lambdad)*u3+(mud*C0a-thetad*C0m)*u2...
   +phid*C0n*u1;
X=inv(A)*D;
```

B. Code for obtaining the angular acceleration of the tank gun in tank gun frames

```
function [sys,x0,str,ts] =AngAccGF(t,x,u,flag)

switch flag,

  %%%%%%%%%%%%%%%%%%
  % Initialization %
  %%%%%%%%%%%%%%%%%%
  case 0,
    [sys,x0,str,ts]=mdlInitializeSizes;

  %%%%%%%%%%%%%%%
  % Derivatives %
  %%%%%%%%%%%%%%%
  case 1,
    sys=[];

  %%%%%%%%%%
  % Update %
  %%%%%%%%%%
  case 2,
    sys=[];

  %%%%%%%%%%%
  % Outputs %
  %%%%%%%%%%%
  case 3,
    sys=mdlOutputs(t,x,u);

  %%%%%%%%%%%%%%%%%%%%%%%
  % GetTimeOfNextVarHit %
  %%%%%%%%%%%%%%%%%%%%%%%
  case 4,
    sys=[];

  %%%%%%%%%%%%%
  % Terminate %
  %%%%%%%%%%%%%
  case 9,
    sys=[];

  %%%%%%%%%%%%%%%%%%%%
  % Unexpected flags %
  %%%%%%%%%%%%%%%%%%%%
  otherwise
    error(['Unhandled flag = ',num2str(flag)]);
```

```
end

% end sfuntmpl

%
%=============================================================
==========
% mdlInitializeSizes
% Return the sizes, initial conditions, and sample times for the S-
function.
%=============================================================
==========
%
function [sys,x0,str,ts]=mdlInitializeSizes

%
% call simsizes for a sizes structure, fill it in and convert it to
a
% sizes array.
%
% Note that in this example, the values are hard coded.  This is not
a
% recommended practice as the characteristics of the block are
typically
% defined by the S-function parameters.
%
sizes = simsizes;

sizes.NumContStates  = 0;
sizes.NumDiscStates  = 0;
sizes.NumOutputs     = 2;
sizes.NumInputs      = 20;
sizes.DirFeedthrough = 1;
sizes.NumSampleTimes = 1;   % at least one sample time is needed

sys = simsizes(sizes);

%
% initialize the initial conditions
%
x0  = [];

%
% str is always an empty matrix
%
str = [];

%
% initialize the array of sample times
%
ts  = [0 0];

% end mdlInitializeSizes

%
%=============================================================
==========
% mdlOutputs
% Return the block outputs.
```

```
%=====================================================================
==========
%
function sys=mdlOutputs(t,x,u)
phi=u(1); theta=u(2); psi=u(3);
lambda=u(4); mu=u(5);
gamades=u(6); betades=u(7);
gamadesd=u(8); betadesd=u(9);
lambdad=u(10); mud=u(11); deltad=u(12);
phidd=u(13); thetadd=u(14); psidd=u(15);
lambdadd=u(16); mudd=u(17);
phid=u(18); thetad=u(19); psid=u(20);
AngularAccelerationInGunFrame;
gamadesdd=Xa(1); betadesdd=Xa(2);
sys = [gamadesdd; betadesdd];

% end mdlOutputs



%AngularAccelerationInGunFrame

u1=[1; 0; 0]; u2=[0; 1; 0]; u3=[0; 0; 1];
u1tilda=tilda(u1); u2tilda=tilda(u2);
u3tilda=tilda(u3);

C0a=expm(u3tilda*lambda);
C0c=C0a*expm(-u2tilda*mu);
C0m=expm(u3tilda*psi);
C0n=C0m*expm(-u2tilda*theta);
C0h=C0n*expm(u1tilda*phi);
C0t=C0h*expm(u3tilda*betades);

C0ad=lambdad*u3tilda*C0a;
C0cd=C0ad*expm(-u2tilda*mu)-mud*C0a*u2tilda*expm(-u2tilda*mu);
C0md=psid*u3tilda*C0m;
C0nd=C0md*expm(-u2tilda*theta)-thetad*C0m*u2tilda*expm(-
u2tilda*theta);
C0hd=C0nd*expm(u1tilda*phi)+phid*C0n*u1tilda*expm(u1tilda*phi);
C0td=C0hd*expm(u3tilda*betades)+betadesd*C0h*u3tilda*expm(u3tilda*be
tades);
a1=C0t*u2; a2=-C0h*u3; a3=C0c*u1;
A=[a1 a2 a3];
D=(psidd-lambdadd+betadesd*C0hd)*u3-
((thetadd*C0m+thetad*C0md+gamadesd*C0td)...
   -(mudd*C0a+mud*C0ad))*u2+(phidd*C0n+phid*C0nd-deltad*C0cd)*u1;
Xa=inv(A)*D;
```

C. S-function for computing torques

```
function [sys,x0,str,ts] = Torques1(t,x,u,flag,mt,JGt11,JGt12,...

JGt13,JGt21,JGt22,JGt23,JGt31,JGt32,JGt33,mb,JGb11,JGb12,JGb13,...

JGb21,JGb22,JGb23,JGb31,JGb32,JGb33,RHR1,RHR2,RHR3,RRB1,RRB2,RRB3,..

RRGt1,RRGt2,RRGt3,RBGb1,RBGb2,RBGb3,Rht1,Rht2,Rht3)

%
% The following outlines the general structure of an S-function.
%
switch flag,

  %%%%%%%%%%%%%%%%%%
  % Initialization %
  %%%%%%%%%%%%%%%%%%
  case 0,
    [sys,x0,str,ts]=mdlInitializeSizes;

  %%%%%%%%%%%%%%%
  % Derivatives %
  %%%%%%%%%%%%%%%
  case 1,
    sys=[];

  %%%%%%%%%%
  % Update %
  %%%%%%%%%%
  case 2,
    sys=[];

  %%%%%%%%%%%
  % Outputs %
  %%%%%%%%%%%
  case 3,
    sys=mdlOutputs(t,x,u,mt,JGt11,JGt12,...

JGt13,JGt21,JGt22,JGt23,JGt31,JGt32,JGt33,mb,JGb11,JGb12,JGb13,...

JGb21,JGb22,JGb23,JGb31,JGb32,JGb33,RHR1,RHR2,RHR3,RRB1,RRB2,RRB3,..
.
   RRGt1,RRGt2,RRGt3,RBGb1,RBGb2,RBGb3,Rht1,Rht2,Rht3);

  %%%%%%%%%%%%%%%%%%%%%%%
  % GetTimeOfNextVarHit %
  %%%%%%%%%%%%%%%%%%%%%%%
  case 4,
    sys=[];
```

```
  %%%%%%%%%%%%%
  % Terminate %
  %%%%%%%%%%%%%
  case 9,
    sys=[];

  %%%%%%%%%%%%%%%%%%%%
  % Unexpected flags %
  %%%%%%%%%%%%%%%%%%%%
  otherwise
    error(['Unhandled flag = ',num2str(flag)]);

end

% end sfuntmpl

%
%=============================================================================
% mdlInitializeSizes
% Return the sizes, initial conditions, and sample times for the S-
function.
%=============================================================================
%
function [sys,x0,str,ts]=mdlInitializeSizes

%
% call simsizes for a sizes structure, fill it in and convert it to
a
% sizes array.
%
% Note that in this example, the values are hard coded.  This is not
a
% recommended practice as the characteristics of the block are
typically
% defined by the S-function parameters.
%
sizes = simsizes;

sizes.NumContStates  = 0;
sizes.NumDiscStates  = 0;
sizes.NumOutputs     = 2;
sizes.NumInputs      = 20;
sizes.DirFeedthrough = 1;
sizes.NumSampleTimes = 1;   % at least one sample time is needed

sys = simsizes(sizes);

%
% initialize the initial conditions
%
x0  = [];

%
% str is always an empty matrix
%
str = [];
```

```
%
% initialize the array of sample times
%
ts  = [0 0];

% end mdlInitializeSizes


%
%==============================================================================
==========
% mdlOutputs
% Return the block outputs.
%==============================================================================
==========
%
function sys=mdlOutputs(t,x,u,mt,JGt11,JGt12,...

JGt13,JGt21,JGt22,JGt23,JGt31,JGt32,JGt33,mb,JGb11,JGb12,JGb13,...

JGb21,JGb22,JGb23,JGb31,JGb32,JGb33,RHR1,RHR2,RHR3,RRB1,RRB2,RRB3,..
.
   RRGt1,RRGt2,RRGt3,RBGb1,RBGb2,RBGb3,Rht1,Rht2,Rht3)
input_data;
epgama=u(1);
gama=u(2);
beta=u(3);
epbeta=u(4);
xdd=u(5); ydd=u(6); zdd=u(7);
phi=u(8); theta=u(9); psi=u(10);
phid=u(11); thetad=u(12); psid=u(13);
phidd=u(14); thetadd=u(15); psidd=u(16);
gamad=u(17);
betad=u(18);
gamaddp=u(19);
betaddp=u(20);
betadd=betaddp+epbeta;
gamadd=gamaddp+epgama;
driving_torques1;
sys = y;

% end mdlOutputs




%driving_torques
%Driving_torques of a 2 DOF tank gun system
%This script calculates the driving torques of a 2 DOF tank gun
system
%whose kinematic variables are considered as input.


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% KINEMATIC EQUATIONS OF TANK HULL %%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%======= TRANSFORMATION MATRICES =======
Chn=expm(-u1tilda*phi);
```

```
Chm=Chn*expm(u2tilda*theta);
Ch0=Chm*expm(-u3tilda*psi);

%====== ANGULAR VELOCITY AND ACCELERATION ======
whh=phid*u1-thetad*Chn*u2+psid*Chm*u3; %Angular Velocity
%Angular acceleration:
alphahh=phidd*u1-(thetadd*I-thetad*phid*u1tilda)*Chn*u2...
   +(psidd*Chm+psid*(-1*phid*u1tilda*Chm+thetad*Chm*u2tilda))*u3;

%===== LINEAR ACCELERATION =====
ah0=xdd*u1+ydd*u2+zdd*u3;
aGhh=Ch0*ah0; %acceleration in hull frame

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% KINEMATIC EQUATIONS OF TANK TURRET %%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%====== TRANSFORMATION MATRICES =======
Cth=expm(-u3tilda*beta);
Ct0=Cth*Ch0;
%===== ANGULAR VELOCITY AND ACCELERATION ====
wtt=Cth*whh+betad*u3; %angular velocity
%angular acceleration components:
alphatt1=u3;
alphatt2=Cth*(alphahh-betad*u3tilda*whh);

%===== LINEAR ACCELERATION COMPONENTS ====
aGtt1=tilda(alphatt1)*RRGt;
aGtt2=Cth*(aGhh+(tilda(alphahh)+(tilda(whh))^2)*RHR)+...
   (tilda(alphatt2)+(tilda(wtt))^2)*RRGt;

%----- Gravitational acceleration -----
gt=Ct0*g;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% KINEMATIC EQUATIONS OF TANK BARREL %%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%===== TRANSFORMATION MATRICES =====
Cbt=expm(u2tilda*gama);

%====== ANGULAR VELOCITY AND ACCELERATION ======
wbb=Cbt*wtt-gamad*u2; %angular velocity
%angular acceleration components:
alphabb1=Cbt*alphatt1;
alphabb2=-u2;
alphabb3=Cbt*(alphatt2+gamad*u2tilda*wtt);

%====== LINEAR ACCELERATION COMPONENTS ======
aGbb1=Cbt*tilda(alphatt1)*RRB+tilda(alphabb1)*RBGb;
aGbb2=tilda(alphabb2)*RBGb;
aGbb3=Cbt*(Cth*(aGhh+((tilda(alphahh)+(tilda(whh))^2))*RHR)...
   +(tilda(alphatt2)+(tilda(wtt))^2)*RRB)+(tilda(alphabb3)+...
   (tilda(wbb))^2)*RBGb;

%------ Gravitational Acceleration ------
gb=Cbt*gt;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
%%%%%%%%%%%%%%% TORQUE COMPUTATION %%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
mht2=[1 0; 0 1; 0 0]; mbt2=[1 0; 0 0; 0 1];
mbt4=[-cos(gama) -sin(gama); 0 0; sin(gama) -cos(gama)];
A=[zeros(3,1) zeros(3,1) I I zeros(3,2) zeros(3,2)
   u3 u2 tilda(Rht) tilda(Rbt) mht2 mbt2
   zeros(3,1) zeros(3,1) zeros(3) -Cbt zeros(3,2) zeros(3,2)
   zeros(3,1) -u2 zeros(3) (-tilda(RGbB)*Cbt) zeros(3,2) mbt4];
B=[mt*(aGtt1*betadd+aGtt2-gt)
   JGt*(alphatt1*betadd+alphatt2)+tilda(wtt)*JGt*wtt
   mb*(aGbb1*betadd+aGbb2*gamadd+aGbb3-gb)

JGb*(alphabb1*betadd+alphabb2*gamadd+alphabb3)+tilda(wbb)*JGb*wbb];
U=inv(A)*B;
C=[1 0 0 0 0 0 0 0 0 0 0 0 0; 0 1 0 0 0 0 0 0 0 0 0 0 0];
y=C*U;
```

D. S-function for Finding Azimuth and Elevation Angles

```
function [sys,x0,str,ts] =
acceleration(t,x,u,flag,mt,JGt11,JGt12,...

JGt13,JGt21,JGt22,JGt23,JGt31,JGt32,JGt33,mb,JGb11,JGb12,JGb13,...

JGb21,JGb22,JGb23,JGb31,JGb32,JGb33,RHR1,RHR2,RHR3,RRB1,RRB2,RRB3,..
.
   RRGt1,RRGt2,RRGt3,RBGb1,RBGb2,RBGb3,Rht1,Rht2,Rht3)

switch flag,

  %%%%%%%%%%%%%%%%%%
  % Initialization %
  %%%%%%%%%%%%%%%%%%
  case 0,
    [sys,x0,str,ts]=mdlInitializeSizes;

  %%%%%%%%%%%%%%%
  % Derivatives %
  %%%%%%%%%%%%%%%
  case 1,
    sys=[];

  %%%%%%%%%%
  % Update %
  %%%%%%%%%%
  case 2,
    sys=[];

  %%%%%%%%%%%
  % Outputs %
  %%%%%%%%%%%
  case 3,
    sys=mdlOutputs(t,x,u,mt,JGt11,JGt12,...

JGt13,JGt21,JGt22,JGt23,JGt31,JGt32,JGt33,mb,JGb11,JGb12,JGb13,...

JGb21,JGb22,JGb23,JGb31,JGb32,JGb33,RHR1,RHR2,RHR3,RRB1,RRB2,RRB3,..
.
   RRGt1,RRGt2,RRGt3,RBGb1,RBGb2,RBGb3,Rht1,Rht2,Rht3);

  %%%%%%%%%%%%%%%%%%%%%%%
  % GetTimeOfNextVarHit %
  %%%%%%%%%%%%%%%%%%%%%%%
  case 4,
    sys=[];

  %%%%%%%%%%%%%
```

```
  % Terminate %
  %%%%%%%%%%%%%
  case 9,
    sys=[];

  %%%%%%%%%%%%%%%%%%%%
  % Unexpected flags %
  %%%%%%%%%%%%%%%%%%%%
  otherwise
    error(['Unhandled flag = ',num2str(flag)]);

end

% end sfuntmpl

%
%======================================================================
==========
% mdlInitializeSizes
% Return the sizes, initial conditions, and sample times for the S-
function.
%======================================================================
==========
%
function [sys,x0,str,ts]=mdlInitializeSizes

sizes = simsizes;

sizes.NumContStates  = 0;
sizes.NumDiscStates  = 0;
sizes.NumOutputs     = 2;
sizes.NumInputs      = 18;
sizes.DirFeedthrough = 1;
sizes.NumSampleTimes = 1;   % at least one sample time is needed

sys = simsizes(sizes);

%
% initialize the initial conditions
%
x0  = [];

%
% str is always an empty matrix
%
str = [];

%
% initialize the array of sample times
%
ts  = [0 0];

% end mdlInitializeSizes

%
%======================================================================
==========
% mdlOutputs
% Return the block outputs.
```

```
%=====================================================================
==========
%
function sys=mdlOutputs(t,x,u,mt,JGt11,JGt12,...

JGt13,JGt21,JGt22,JGt23,JGt31,JGt32,JGt33,mb,JGb11,JGb12,JGb13,...

JGb21,JGb22,JGb23,JGb31,JGb32,JGb33,RHR1,RHR2,RHR3,RRB1,RRB2,RRB3,..
.
   RRGt1,RRGt2,RRGt3,RBGb1,RBGb2,RBGb3,Rht1,Rht2,Rht3)
input_data;
betad=u(1); beta=u(2);
Tbeta=u(3); Tgama=u(4);
xdd=u(5); ydd=u(6); zdd=u(7);
phi=u(8); theta=u(9); psi=u(10);
phid=u(11); thetad=u(12); psid=u(13);
phidd=u(14); thetadd=u(15); psidd=u(16);
gama=u(17); gamad=u(18);
%ground_motion;
Gun_acceleration1;
sys =y1;

% end mdlOutputs



%Gun_acceleration
%This script gives the azimuth and elevation
%accelerations of a tank gun system

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% KINEMATIC EQUATIONS OF TANK HULL %%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%======= TRANSFORMATION MATRICES =======
Chn=expm(-u1tilda*phi);
Chm=Chn*expm(u2tilda*theta);
Ch0=Chm*expm(-u3tilda*psi);

%====== ANGULAR VELOCITY AND ACCELERATION ======
whh=phid*u1-thetad*Chn*u2+psid*Chm*u3; %Angular Velocity
%Angular acceleration:
alphahh=phidd*u1-(thetadd*I-thetad*phid*u1tilda)*Chn*u2...
   +(psidd*Chm+psid*(-1*phid*u1tilda*Chm+thetad*Chm*u2tilda))*u3;

%===== LINEAR ACCELERATION =====
ah0=xdd*u1+ydd*u2+zdd*u3;
aGhh=Ch0*ah0; %acceleration in hull frame

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% KINEMATIC EQUATIONS OF TANK TURRET %%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%====== TRANSFORMATION MATRICES =======
Cth=expm(-u3tilda*beta);
Ct0=Cth*Ch0;
%===== ANGULAR VELOCITY AND ACCELERATION ====
wtt=Cth*whh+betad*u3; %angular velocity
%angular acceleration components:
```

```
alphatt1=u3;
alphatt2=Cth*(alphahh-betad*u3tilda*whh);

%===== LINEAR ACCELERATION COMPONENTS ====
aGtt1=tilda(alphatt1)*RRGt;
aGtt2=Cth*(aGhh+(tilda(alphahh)+(tilda(whh))^2)*RHR)+...
   (tilda(alphatt2)+(tilda(wtt))^2)*RRGt;

%----- Gravitational acceleration -----
gt=Ct0*g;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% KINEMATIC EQUATIONS OF TANK BARREL %%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%===== TRANSFORMATION MATRICES =====
Cbt=expm(u2tilda*gama);

%====== ANGULAR VELOCITY AND ACCELERATION ======
wbb=Cbt*wtt-gamad*u2; %angular velocity
%angular acceleration components:
alphabb1=Cbt*alphatt1;
alphabb2=-u2;
alphabb3=Cbt*(alphatt2+gamad*u2tilda*wtt);

%====== LINEAR ACCELERATION COMPONENTS ======
aGbb1=Cbt*tilda(alphatt1)*RRB+tilda(alphabb1)*RBGb;
aGbb2=tilda(alphabb2)*RBGb;
aGbb3=Cbt*(Cth*(aGhh+((tilda(alphahh)+(tilda(whh))^2))*RHR)...
   +(tilda(alphatt2)+(tilda(wtt))^2)*RRB)+(tilda(alphabb3)+...
   (tilda(wbb))^2)*RBGb;

%------ Gravitational Acceleration ------
gb=Cbt*gt;

%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%% GUN ACCELERATION %%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%
Ctb=expm(-u2tilda*gama);
Tb=Tbeta*u3; Tg=Tgama*u2;
B=[JGt*alphatt1-mt*tilda(Rht)*aGtt1+mb*(tilda(Rbt)-
tilda(Rht))*Ctb*aGbb1
   JGb*alphabb1-mb*tilda(RGbB)*aGbb1];
G=[mb*(tilda(Rbt)-tilda(Rht))*Ctb*aGbb2
   JGb*alphabb2-mb*tilda(RGbB)*aGbb2];
Mb=[-1 0; 0 0; 0 -1; cos(gama) sin(gama); 0 0; -sin(gama)
cos(gama)];
Mh=[-1 0; 0 -1; 0 0; 0 0; 0 0; 0 0];
Ak=[B G Mb Mh];
Ck=[-(mb*(tilda(Rbt)-tilda(Rht))*Ctb*(aGbb3-gb)-
mt*tilda(Rht)*(aGtt2-gt)+...
      JGt*alphatt2+tilda(wtt)*JGt*wtt-Tb-Tg)
   -(-mb*tilda(RGbB)*(aGbb3-
gb)+JGb*alphabb3+tilda(wbb)*JGb*wbb+Tg)];
C=[1 0 0 0 0 0; 0 1 0 0 0 0];
U=inv(Ak)*Ck;
y1=C*U;
```

E. Code for Transforming the gun angular velocity to the earth fixed frame

```
function [sys,x0,str,ts] = AngVelGF(t,x,u,flag)

switch flag,

  %%%%%%%%%%%%%%%%%%
  % Initialization %
  %%%%%%%%%%%%%%%%%%
  case 0,
    [sys,x0,str,ts]=mdlInitializeSizes;

  %%%%%%%%%%%%%%%
  % Derivatives %
  %%%%%%%%%%%%%%%
  case 1,
    sys=[];

  %%%%%%%%%%
  % Update %
  %%%%%%%%%%
  case 2,
    sys=[];

  %%%%%%%%%%%
  % Outputs %
  %%%%%%%%%%%
  case 3,
    sys=mdlOutputs(t,x,u);

  %%%%%%%%%%%%%%%%%%%%%%%
  % GetTimeOfNextVarHit %
  %%%%%%%%%%%%%%%%%%%%%%%
  case 4,
    sys=[];

  %%%%%%%%%%%%%
  % Terminate %
  %%%%%%%%%%%%%
  case 9,
    sys=[];

  %%%%%%%%%%%%%%%%%%%%
  % Unexpected flags %
  %%%%%%%%%%%%%%%%%%%%
  otherwise
    error(['Unhandled flag = ',num2str(flag)]);
```

```
end

% end sfuntmpl

%
%=====================================================================
==========
% mdlInitializeSizes
% Return the sizes, initial conditions, and sample times for the S-
function.
%=====================================================================
==========
%
function [sys,x0,str,ts]=mdlInitializeSizes

sizes = simsizes;

sizes.NumContStates  = 0;
sizes.NumDiscStates  = 0;
sizes.NumOutputs     = 2;
sizes.NumInputs      = 12;
sizes.DirFeedthrough = 1;
sizes.NumSampleTimes = 1;   % at least one sample time is needed

sys = simsizes(sizes);

%
% initialize the initial conditions
%
x0  = [];

%
% str is always an empty matrix
%
str = [];

%
% initialize the array of sample times
%
ts  = [0 0];

% end mdlInitializeSizes

%=====================================================================
==========
% mdlOutputs
% Return the block outputs.
%=====================================================================
==========
%
function sys=mdlOutputs(t,x,u)
phi=u(1); theta=u(2); psi=u(3);
beta=u(4); gama=u(5);
lambda=u(6); mu=u(7);
phid=u(8); thetad=u(9); psid=u(10);
betad=u(11); gamad=u(12);
AngularVelocityInZerothFrame;
sys =result;
```

```
% end mdlOutputs



%AngularVelocityInZerothFrame
u1=[1; 0; 0]; u2=[0; 1; 0]; u3=[0; 0; 1];
u1tilda=tilda(u1); u2tilda=tilda(u2);
u3tilda=tilda(u3);
C0a=expm(u3tilda*lambda);
C0c=C0a*expm(-u2tilda*mu);
C0m=expm(u3tilda*psi);
C0n=C0m*expm(-u2tilda*theta);
C0h=C0n*expm(u1tilda*phi);
C0t=C0h*expm(u3tilda*beta);
a1=u3; a2=-C0a*u2; a3=C0c*u1;
A=[a1 a2 a3];
D=psid*u3-thetad*C0m*u2+phid*C0n*u1+betad*C0h*u3-gamad*C0t*u2;
Y=inv(A)*D;
lambdad=Y(1); mud=Y(2);
result=[lambdad; mud];
```


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