A NOVEL FRACTIONAL ORDER VISCOELASTIC MATERIAL MODEL FOR SOFT BIOLOGICAL TISSUES AND ITS FINITE ELEMENT APPLICATION

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Background: Fractional order models have proven to be very useful for studying viscoelastic materials and material parameter identification. It has been shown that fractional order material models give better results compared to integer order models with fewer parameters in experimental data fit. Although fractional order linear viscoelastic material models are present in the literature, none of the investigators have implemented the developed material models into a finite element analysis package program due to cumbersome mathematics of fractional calculus. Linear fractional order viscoelastic models include a "*spring-pot*" element instead of a dashpot which is seen in integer order regular (Kelvin-Voigt, Maxwell) models. In addition, fractional order viscoelastic constitutive relations involve a generalized exponential function, called the Mittag-Leffler function. A fast computation scheme for the Mittag-Leffler function is offered specifically for finite element analysis applications [1]. The implementation of the fractional order viscoelastic material model into a finite element analysis package program yields future better success of analyzing soft tissues in interaction with their mechanical environments.

Aims: The aim of this study is to develop a viscoelastic material model for soft biological tissues involving fractional operators and to implement the developed material model into the finite element analysis package program Msc.Marc 2010®.

Methods: *In-vivo* experimental studies of bulk soft tissue indicate that the mechanical response of soft tissues are non-linear and viscoelastic. The time-dependent, viscoelastic, dissipation showing part of extended James-Green-Simpson material model including the two-term Prony series [2] is further modified to a less parameter fractional order nonlinear viscoelastic material model including the Mittag-Leffler function. The novel material model developed is implemented into the Msc.Marc 2010® software via the built-in user-subroutine "*uelastomer*". The verification of the user-subroutine developed is performed via Matlab R2009b® by employing the same parameters under the same loading condition. Two set of "normalized reaction force vs. time" data are obtained for verification under constant displacement (unit-step input, relaxation response), small strain loading condition, one from 3-D finite element analysis result on Msc.Marc 2010® and the other from 1-D analytical result on Matlab R2009b® (Figure 1). Further, the viscoelastic material mechanical responses which are creep, relaxation and hysteresis are simulated via finite element analysis (FEA) using the novel material model user-subroutine identified to the program.

Results: Two sets of relaxation response data obtained are shown to be perfectly fitted, therefore verifying the user-subroutine developed by us. Fractional order viscoelastic material models provide better fit capabilities to experimental creep, relaxation and hysteresis data compared to integer order one-term and two-term Prony series representation with less number of parameters (Figure 2).

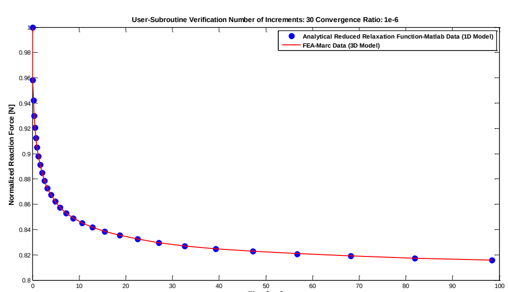

Figure 1: Relaxation response data obtained from (3-D) FEA and (1-D) analytical model.

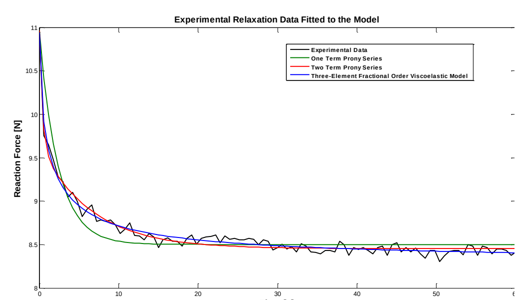

Figure 2: Comparison of three-element FOV model to integer order one term and two term Prony series representations.

Conclusions: Fractional order viscoelasticity (FOV) provides valuable understanding of tissue viscoelastic behavior. With the aid of implementing the fractional order viscoelastic material model into finite element analysis will lead to more realistic simulation and problem solving of soft tissue-mechanical environment interactions.

References:

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