DETERMINATION OF ELASTIC CONSTANTS OF FRESH AND COMPACT ANIMAL BONE

A MASTER THESIS
FOR THE DEGREE OF
MASTER OF SCIENCE

BY
ERK INGER

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DETERMINATION OF ELASTIC CONSTANTS OF

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By

Erk Inger

January, 1976

I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and quality, as a thesis for the degree of Master of Science.

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ABSTRACT

"Determination of Elastic Constants of Fresh and Compact Animal Bone"

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Investigation of bone properties is important for the assesment of its biological and mechanical functions.

Elastic properties of bone is related to its constituents, mainly to its collagen constituent which is a network of protein fibrils. Arrangement of mineral crystals in callogen suggests that the fresh bone may approximately be described by an elastic material having a hexagonal symmetry, which contains five independent elastic coefficients.

In this study, the five elastic coefficients of the fresh compact bone are experimentally determined using strain gage technique by performing three independent tests, namely, pure tension, hydrostatic pressure and pure torsion tests. The technical constants, which are Young's Moduli, Poisson's ratios and shear moduli in different directions, are also determined by relating them to the elastic coefficients found already.

For obtaining the five elastic coefficients, a computer program, in which the experimental data is linearly approximated by using the method of least squares, is developed.

In the view of the discussions presented, it is concluded that the elastic constants obtained in this study can reliably be used in future studies in bioengineering.

Key words : bone, collagen, hexagonal, elastic, strain gage, Young's modulus, Poison's ratio, shear modulus.

ÖZET

"Taze ve Kompakt Hayvan Kemiğinin Elastik Sabitlerinin Bulunuşu"
İNCER, ERK
Yüksek Lisans Tezi; Mak. Müh. Bölümü
Tez Yönetici: Y. Prof.Dr. Yalçın MENGİ
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Kemik özelliklerinin araştırılması, kemiğin biyolojik ve mekanik fonksiyonlarının anlaşılması için önemlidir.

Kemiğin elastik özellikleri, başlıcası, organik protein lif şebekesinden meydana gelen kollojen elemanı olmak üzere, kemiği meydana getiren elemanlara bağlıdır. Kollojen içindeki mineral kristallerinin düzeni, taze kemiğin yapısının yaklaşık olarak hegzagonal simetriye sahip ve beş elastik sabiti içeren bir elastik malzeme ile temsil edileceğini gösterir.

Bu çalışmada, taze ve kompakt kemiğe ait beş elastik sabit, streyn geyç tekniğiyle, çekme, hidrostatik basınç ve burulma deneylerinden oluşan üç bağımsız test yapılarak, deneysel olarak bulundu. Aynı zamanda, çeşitli yönlerdeki, Young modülleri, Poisson oranları ve kayma modüllerinden oluşan tekniksel sabitler, zaten bulunmuş olan elastik sabitlere bağlı olarak elde edildi.

Elastik sabitlerin elde edilmesi için, deneysel verileri en küçük kareler metoduyla, yaklaşık olarak lineerleştiren bir bilgisayar programı geliştirilmiştir.

Yapılan tartışmaların ışığı altında, bu çalışmadaki bulunan elastik sabitlerin bio-mühendisliğin gelecekteki çalışmalarında güvenle kullanılabileceği sonucuna varıldı.

Anahtar Kelimeler: kemik, kollojen, hegzagonal, elastik, streyn geyç, Young modülü, Poisson oranı, kayma modülü.

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NOMENCLATURE

$\underline{A}$: Coefficient matrix

a_{ij} : Elements of the coefficient matrix

$\underline{A}^T$: Transpose of the coefficient matrix

b_1, b_2 : Constants defined in the text

C_{ij} : Elastic coefficients

$\underline{C}$: Load vector

$\bar{c}_i$: Elements of the load vector

c : Bar velocity

d_{ij} : Piezoelectric constants

d : Diameter of the bone specimen

E : Modulus of elasticity

F : Normal force due to gravity

f : Natural frequency

G : Gage, 10^9

G : Shear modulus

h : Height of rectangular cross section

I : Moment of inertia of cross section

k : Wave number

L : Length of the bone specimen

M_T : Applied torque

$\underline{m}$: Mass per unit length

m_1, m_2

m_3, m_4 : Constants defined in the text

(m) : Ordering number

n : Number of sets of experimental data

P : Hydrostatic pressure

P_i : Elements of polarization vector

p : Number of linear equations

q : Load per unit of length

r, θ, z : Coordinates of cylindrical system

S : Cross sectional area of the bone specimen

t : Time

u : Displacement function

w : Angular frequency

$\underline{X}$: Unknown vector

x_j : Elements of the unknown vector

x, y, z : Coordinates of Cartesian system

α : Phase angle

ε : Error function

ε_{ij} : Strain components

ε_{ij}^T : Strain components in pure tension test

ε_{ij}^H : Strain components in hydrostatic pressure test

$\varepsilon_{\theta z_i}$: Strain components in pure torsion test

λ : Wavelength

μ : Micro, 10^{-6}

$\nu_{z\theta}$, ν_{zr},

$\nu_{r\theta}$, ν_{rz} : Poison's ratios in several directions

ρ : Mass per unit volume

σ_j : Normal stress components

τ_{ij} : Stress components

τ_o : Applied shear stress in pure torsion test

Chapter I

INTRODUCTION

Bioengineering is a field of study in which engineers, doctors, physiologists and other related scientists cooperate in the development of instruments, procedures and techniques necessary to treate the problems of living systems. Mechanics, materials, physiology, medicine, surgery, pathology, prosthesis, dentistry, athletics, social and environmental fields are some of the common areas of study in bioengineering.

The human body consists of trillions of cells, organized into tissues, which are organized into organs and organs into systems. Each organ has a different structure adapted for definite functions in the body. In human beings, the organs are very complex and their functions highly varied. The quantitave analyses of the relationships between structure and its function, and application of the results to man in healt and disease show that these relationships are the function of the physicochemical properties of tissues and their constituents, changing in time and space. For this reason, the basic properties of living tissues are the major objectives of bioengineering. Bioengineering also involves surgical and medical researches which are arised from health and disease problems of human beings. For example, in the research related to the safety of highway drivers, one wishes to design systems that will lessen the terrible effects of impact by using energy absorbing devices.

The other field of bioengineering is called prosthetics which replaces a missing body part structually, functionally and cosmetically. In designing prosthetic devices such as artificial legs and joint replacements, biochemical, physiological, histological and pharmacological properties of missing and replaced parts are to be taken into consideration. In all branches of bioengineering studies related to blood, tendon and skin properties, muscle mechanics, lung elasticity, arteries, heart, cartilage and bone characteristics are very important.

Among the several topics mentioned above, properties of bone is an important field of research, in particular, in examination of mechanical and biological functions of bone. Fractures, crippling injuries and malformations of the bone, in human beings, necessiate this work to free them from pain and suffering. Today, orthopedic surgeons can make bone grafts to replace damaged areas, transplant bony tissue and create new sockets for the end of the bones which have been injured or destroyed by disease. A number of bone banks have now been established, from which bone tissue can be grafted. The orthopedic specialists and bioengineers restored many crippled persons to useful activity who would otherwise have been permanently disabled.

The bone has four important biological functions. In addition to its obvious use of basic shape and framework of the body, muscles which are attached to bones permit them to function as levers for the body. Bones act also as protective device for bodily organs, as in the case of the skull protecting the brain. Further, bonny tissue is a storage depot for minerals which are not immediately needed by the body. Certain highly specialized bones of the middle ear aid in the maintenance of equilibrium.

Bone is covered by a thin, fibrous layer of cell called the periosteoum (see Fig.1). Osteoblasts or bone building cells are within this layer. The mineral component of bone , which contains a protein network called collagen, are deposited in this layers beneath the periosteoum. Bone tissue is porous due to a system of channels, called Haversian canals which provide pathways for blood vessels and nerves to travel to the interior of the bone. The interior of long bone is filled with marrow a soft, fatty substances.

45% of the total weight of bone consist of mineral substances, mainly calcium salts, 25% of it water constituents and the remainder is composed of organic material, chiefly a network of protein fibers, collagen. Because of its constituent materials, bone is extremely hard. Its elastic properties are due to the arrangement of mineral crystals within the collagen network.

FUKADA and YASUDA (1) describe collagen to be a kind of protein with a long sequence of polypeptide chains. Each chain makes a helical configuration and three strands of such helical chains are regularly combined in a long rodlike molecule. A number of hydrogen are formed between NH and CO groups of these chains. The direction of hydrogen bonds lies almost along the length of the molecule, which is the direction of the fiber axis in collagen fibrils. The crystallographic symmetry in crystals of collagen is supposed to be hexagonal.

Determination of strains in any skeletal bone under any induced stress is a different aspect of bioengineering. It has been suggested by several authors that piezoelectrical properties of bone may have important physiological functions. These piezoelectric

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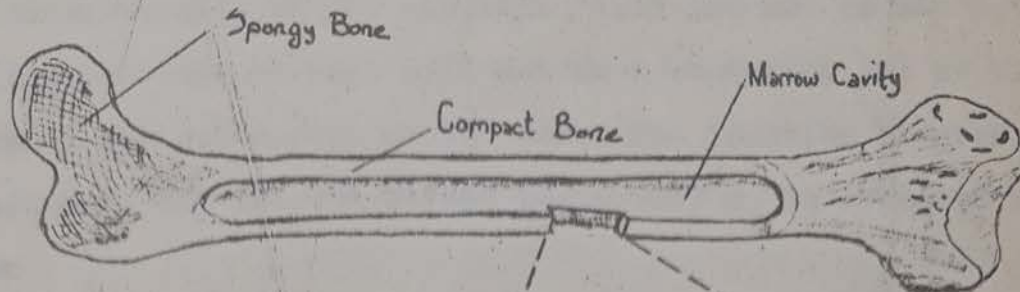
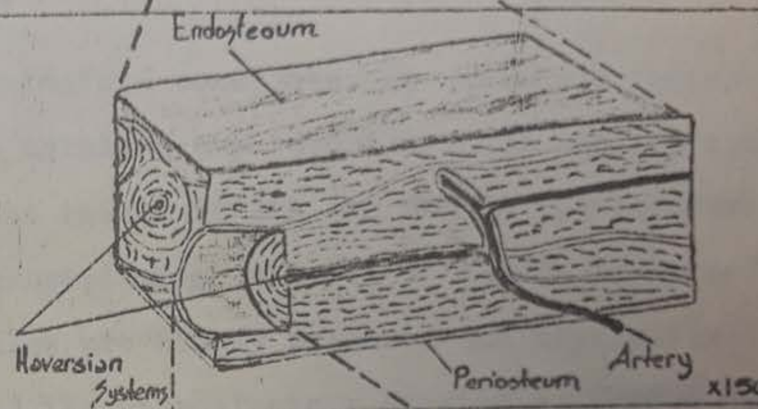
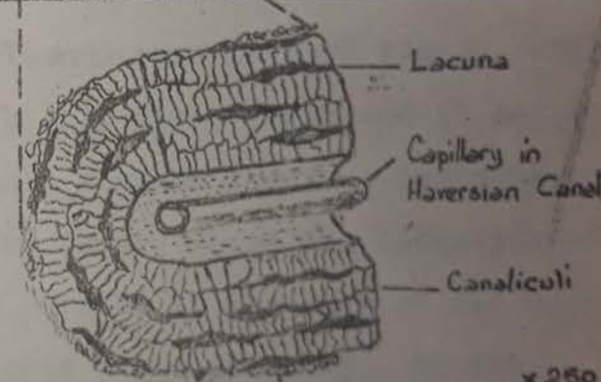


Enlarged section of compact bone showing the Haversian systems, blocks of concentric layers of bone deposited around a central canal.



Detail of Haversian system showing bone cells, osteocytes, in their chambers, lacunae. The lacunae communicate with the Haversian canal through the canaliculu.



Strip of one shell from Haversian system showing the two structural elements of bone, the collagen fibers and mineral crystal deposits.

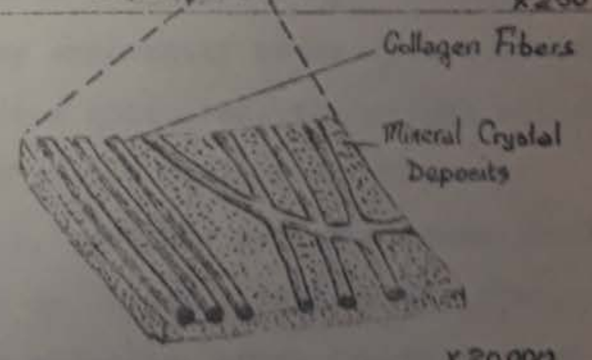


FIGURE 1. The structure of the bone.

properties of bone is explained by GJELSVIK (2). Bone is electrically polarized, when it is subjected to any load. The magnitude of the polarization depends on the electric field induced in the material. The loaded membrane of bone will act as a conductor and neutralize the induced surface charge on the bone. The relation between polarization vector P_i and induced stress components σ_j is epressed by the relation

$$P_i = d_{ij} \sigma_j , \quad (i=1-3) \; (j=1-6) \qquad (1.1)$$

where d_{ij}'s are the piezoelectric constants. In Eqn.(1.1) indical notation is used. In this notation any repeated index implies summation over the range of that index. FUKADA and YASUDA (1) observed the direct and converse piezoelectric effects of tendon of horse in which the collagen molecules are highly oriented and crystallized, and numerically they found its piezoelectric constants. SHAMOS and LAVINE (3) stated that the piezoelectric even in hard tissue, which is collagen in the case of bone, is highly directional and it has a maximum value for shear stress and a minimum for pure compressive and tensile stress. In long bones, like femur, where the direction of the collagen fibers may or may not be paralel to the axis of the bone, the maximum is found for the stresses directed at 45° to the collagen axis. The piezoelectricity apparently stems from a shearing stress on oriented long chains fibrous molecules, the actual effect being a displacement of charge due to the distortion of cross linkages in the molecular structure, probably hydrogen bonds. Thus a requirement for piezoelectricity in living tissue is the presence of well ordered asymmetric fibrous molecule, cross linked to form a uniaxial system which can be polarized by a shearing stress.

Elastic properties of bone are directly related with the collagen which is the major constituent of it. Hexagonal symmetry of crystallites of bone is attributed to the collagen whose crystals are supposed to be hexagonal. The hexagonal elastic material can be described by five independent coefficients. When it is referred to a cylinderical coordinate system (r, θ, z) in which the z-axis coincides with the symmetry axis of the material, the linear elastic stress-strain relations can be written as

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{13} & 0 & 0 & 0 \\ C_{13} & C_{13} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & G_{\perp} \end{bmatrix} \begin{bmatrix} \varepsilon_{rr} \\ \varepsilon_{\theta\theta} \\ \varepsilon_{zz} \\ 2\varepsilon_{\theta z} \\ 2\varepsilon_{zr} \\ 2\varepsilon_{r\theta} \end{bmatrix} = \begin{bmatrix} \tau_{rr} \\ \tau_{\theta\theta} \\ \tau_{zz} \\ \tau_{\theta z} \\ \tau_{zr} \\ \tau_{r\theta} \end{bmatrix} , \quad (1.2)$$

where $G_{\perp} = 1/2(C_{11}-C_{12})$ and C_{ij}'s are the elastic constants, ε_{ij}'s and τ_{ij}'s are the strain and stress components respectively. The literature contains extensive information for the static and dynamic mechanical properties of both human and animal bone by considering it, as an isotropic material, but there is not much information concerning these five elastic constants.

SIDNEY B. LANG (4) measured five elastic constants of dried bovine phalanx, dry femur and fresh bovine phalanx by an ultrasonic technique which is a dynamic test. He suggests that the crystallographic structure of the principal components of bone and its piezoelectric and pyroelectric behaviour shows that bone is a texture that has a hexagonal symmetry. In that study, samples are placed between two piezoelectric transducers. Transmitting transducer is exited by a pulse, transmitted signal is detected by the receiver transducer and it is amplified by a two stage wide band amplifier. The velocities were calculated from the transmission times and physical dimensions of the specimens. Time measurements of ultrasonic pulses through the material are obtained by means of a (0-100) μ sec delay potantiometer and an electronic time interval counter. This procedure is repeated in different directions of several specimens prepared from animal bones.

J. I. BRASE and J. SKORECKI (5) determined the modulus of elasticity of bone by a vibration method. The specimens are cut from tibiae of older beef cattle from the transverse and longitudinal vibrations of the specimen, the dynamic elastic moduli of bone has been obtained with the help of a small vibrator, oscillator and microscope.

In the transverse vibration experiment, the specimens (100 x 2.5 x 2.5) mm having uniform rectangular cross section are clamped at one end in a electromagnetic vibrator so that their long axis is at right angles to the vibration applied at the clamped end. A node will form near the clamped end and frequencies at which the resonance occurs will be given by the equation (see the Appendix A)

$$\operatorname{Cosh}(kL)\ \operatorname{Cos}(kL) = -1 \quad , \qquad (1.3)$$

where

$$k^4 = \frac{mw^2}{EI} \quad ,$$

and L is the length of the specimen, m is mass per unit length, w is angular frequency, E is Young's modulus and I is the moment of inertia of the cross section of the specimen. The frequency of induced vibration f is varied and noted at resonance. For each specimen as many modes as possible exited under magnification. The position of nodes found by plotting the whole mode i.e., the amplitude of vibration for various points along the beam. The expression which relates Young's modulus to the frequency is (see the Appendix A)

$$E = \frac{48\ \pi^2 \rho\ f^2 L^4}{(kL)^4\ h^2} , \tag{1.4}$$

where ρ is mass per unit volume and h is height of cross section. (kL) values are obtained by solving Eqn. (1.3) which is the frequency equation for the transverse motion of the beam. By inserting frequencies, which are obtained in the experiments, into Eqn. (1.4) Young's modulus of the specimen may be obtained.

For the longitudinal vibration experiment, specimens are clamped at one end in an electromagnetic vibrator so that, their long axis is parallel to the direction of vibration. The longitudinal vibration of a beam is similar to that of the column of air in an organ pipe. As it is explained in the Appendix B, the wave length corresponding to lowest mode is equal to (4L), where L is the length of the beam. For the lowest mode of longitudinal vibrations Young's modulus is related to frequency by (see the Appendix B)

$$E = 16\ \rho\ f^2\ L^2 , \tag{1.5}$$

which determines the value of Young's modulus when the frequency f is obtained in the experiment.

SWANSON, FREEMAN and DAY (6) examined the fatigue properties of bone specimens which are extracted cortices of human femura by performing rotating cantilever fatigue test. Having machined the specimen, a lathe is prepared and the specimen is replaced in the chuck. A weight is suspended by using a ball bearing which is attached to the free end of the test specimen so that the specimen is loaded as a cantilever. The specimen is then rotated by the lathe motor so that all the elements of bone are subjected to compressive and tensile stresses. The experiment continued until the specimen is fractured.

SIMKIN and ROBIN (7) described method of calculating the bending moment at failure and modulus of elasticity in bending of bone. Cortical bone specimens are tested in tension, compression and bending. The results were compared with the formulea developed for bending.

There are several types of experimental techniques available for the determination of static and dynamic properties of bone. Strain gage technique has the advantage of getting direct measurements from a living body, in which stimulating static or dynamic forces generated by the muscle action and the gravity.

EVANS (8) successfully bonded gages to an exposed area of tibia bone in living dogs. He recorded tibial strain during gait up to 36 hour. Bonding of the long term strain gages to living bone is accomplished by the use of methyl two cyano acrylate monomer adhesive.

LISSNER (9) applied the strain gages to bone cadavers to evaluate the effects of impact forces. ROBERTS (10) advised fabrication of prewired gage units which can function on bone, in cadavers for three months and living animals for three weeks. Then LANYON (11-14) analysed strains in sheep tibia and vertebrae for periods up to three weeks following gage installation. In 1973, Lanyon bonded rossette strain gages to living bone of sheep by using isobutyl two cyanoacrylate monomer as an adhesive. He examined the changing direction and magnitude of the maximum and minimum principal strains, the maximum shear strains and strain rate encountered during natural locomation of the sheep. BONFIELD and C.H.U. (15-16) used micro strain techniques in finding the Young's modulus of longitudinal bovine tibia compact bone specimen. The stress-strain relationships were determined either with a capaticance gage which allowed continuous strain measurements on the x-y recorder or with a Tuckerman optical gage using a loading-unloading technique. Advances in industrial strain gage technology now permit laboratory preparation of gage units which can be bonded to the bone of the living animals for minumum of three weeks.

MC LEISH and HABBOOBI (17) reported the experimental problems which arise when using electrical resistance strain gage to determine the stresses and loads in cadaveric bone. They suggest that in the living state bone is saturated with moisture but it is difficult to use strain gages in such conditions because the free moisture produces leakage currents which affect the results. Beside that, strain measurements from gages can not be interpretted strictly as data from living bone since osteocytes are killed during preparation of the surface of the bonding. However, if the bone is fully dried out, it shrinks becoming more brittle and showing a marked change in its tensile properties.

In the present study, wet bone is considered as an elastic material having a hexagonal symmetry. For this type of material elastic stifness matrix has five independent elastic coefficients. These coefficients for wet bone are found by using three independent tests, namely, pure tension, hydrostatic pressure and pure torsion tests. Deformation of wet bone specimens have been investigated by micro strain techniques. The cylindrical specimens of diameter 6mm and of length 50 mm are extracted from the femur of calf by hand sawing and machining it into cylindrical shape so that the axis of the specimen is paralel to longitudinal axis of femur. Water is used as the coolant during the machining process. The strain gages fabricated with terminals are bonded through the circumferance of the middle portion of the specimen. Three gages are bonded in this section in circumferential, axial directions and in the direction which makes an angle of 45° with its longitudinal axis. An adhesive called Ethicon Bucrylate, Isobutyle 2 Cyano Acrylate Monomer is used, in bonding process. Bonded gages and their terminals are coated by the coating materials M-Coat-C and M-Coat-B. Two arm bridge which is composed of a dummy gage and the gages attached to the specimen are used to measure the strains due to applied stresses. Pure tension, hydrostatic pressure and twisting moment are applied to the specimen and the strains in different directions are measured in each test. Then using the experimental data obtained in tension, hydrostatic pressure and torsion tests, the elastic coefficients of wet bone, which is determined.

In Chapter 2, the theory, on which the experimental procedure based, is discussed. It also includes derived relations and mathematical procedure used in computer programming and the method of least squares employed for numerical determination of elastic constants of bone.

Chapter 3 covers the instrumentation and experimental procedure for pure tension, hydrostatic pressure and torsion tests.

Experimental data and results are given in Chapter 4.

Chapter 5 is the final chapter which includes the discussion and conclusions reached in the view of the results and recommendations for the use of the elastic constants in the future studies.

Chapter 2

THEORETICAL ANALYSIS

2.1 Description of Specimen

The bone specimen is extracted from the compact portion of the calf femur by hand sawing so that the axis of the specimen is parallel to the longitudinal axis of femur (see Fig. 2, Chap. 3). The extracted portion of femur is machined into a compact circular cylindrical shape by lathe. The details concerning the preparation of the specimen are fully discussed in Chapter 3.

2.2 Theoretical Considerations

The crystal structure of the major components of bone suggests that it behaves as an elastic material having hexagonal symmetry. When the bone specimen is refered to a cylindrical coordinate system (r, θ, z) (see Fig. 2), in which the z-axis coincides with the symmetry axis of the specimen, the constitutive equations which relate stresses to strains, can be written as

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{13} & 0 & 0 & 0 \\ C_{13} & C_{13} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{\text{[illegible]}} \end{bmatrix} \begin{bmatrix} \epsilon_{rr} \\ \epsilon_{\theta\theta} \\ \epsilon_{zz} \\ 2\epsilon_{\theta z} \\ 2\epsilon_{zr} \\ 2\epsilon_{r\theta} \end{bmatrix} = \begin{bmatrix} \tau_{rr} \\ \tau_{\theta\theta} \\ \tau_{zz} \\ \tau_{\theta z} \\ \tau_{zr} \\ \tau_{r\theta} \end{bmatrix}, \quad (2.1)$$

where C_{ij}'s are the elastic coefficients, τ_{ij}'s and ϵ_{ij}'s are the the stress and strain components respectively, and $G_{\perp} = 1/2(C_{11}-C_{12})$.

Since there are five independent elastic coefficients, namely, C_{11}, C_{12}, C_{13}, C_{33}, C_{44}, in elastic stifness matrix, one needs five independent equations to determine them. These five independent equations can be obtained from three independent experiments, which are pure tension, hydrostatic pressure and pure torsion tests. In these experiments, stresses are computed from the applied forces and strains which are measured by means of resistance wire strain gages.

Details about bonding of strain gages, their positions on the specimen and the experimental techniques for pure tension, hydrostatic pressure and pure torsion tests are given in Chapter 3.

In this section, it is assumed that the state of the deformation is in elastic range so that the state of applied stresses are below the yielding point.

2.3 Pure Tension Test

In pure tension test, the specimen is subjected to an axial force F , and axial and circumferential strains, ϵ^T_{zz} and $\epsilon^T_{\theta\theta}$ are measured by two strain gages which are bonded on the mid-portion of the specimen in axial and circumferential directions. For this type of loading, stress and strain components take the forms

$$\tau_{zz} = \frac{F}{S} = \frac{4F}{\pi d^2}\ , \quad \tau_{rr} = \tau_{\theta\theta} = \tau_{\theta z} = \tau_{rz} = \tau_{r\theta} = 0 \tag{2.2}$$

$$\epsilon^T_{rr} \neq 0\ , \quad \epsilon^T_{\theta\theta} \neq 0, \quad \epsilon^T_{zz} \neq 0, \quad \epsilon^T_{\theta z} = \epsilon^T_{rz} = \epsilon^T_{r\theta} = 0$$

where d and S are the diameter and cross sectional area of the specimen respectively, and supercript T designates the value of the quantitiy in pure tension test.

When the values of stresses and strains, given by Eqns. (2.2) are substituted in Eqn. (2.1), one obtains

$$
\begin{aligned}
C_{11}\,\varepsilon_{rr}^{T} + C_{12}\,\varepsilon_{\theta\theta}^{T} + C_{13}\,\varepsilon_{zz}^{T} &= 0 \\
C_{12}\,\varepsilon_{rr}^{T} + C_{11}\,\varepsilon_{\theta\theta}^{T} + C_{13}\,\varepsilon_{zz}^{T} &= 0 \\
C_{13}\,\varepsilon_{rr}^{T} + C_{13}\,\varepsilon_{\theta\theta}^{T} + C_{33}\,\varepsilon_{zz}^{T} &= \sigma \quad ,
\end{aligned}
\tag{2.3}
$$

where $\sigma = \frac{F}{S}$. It should be noted that in Eqns. (2.3) $\varepsilon_{\theta\theta}^{T}$, ε_{zz}^{T} and σ are measurable and known quantities, while ε_{rr}^{T} is the non-measurable radial strain which should be considered as an unknown variable.

2.4 Hydrostatic Pressure Test

In hydrostatic pressure test, the specimen is subjected to hydrostatic pressure P, and axial and circumferential strains, ε_{zz}^{H} and $\varepsilon_{\theta\theta}^{H}$ are measured by the two strain gages used in pure tension test. Stress and strain components under the applied hydrostatic pressure take the forms

$$
\begin{aligned}
&\tau_{rr} = \tau_{\theta\theta} = \tau_{zz} = -P \ , \quad \tau_{\theta z} = \tau_{rz} = \tau_{r\theta} = 0 \\
&\varepsilon_{rr}^{H} \neq 0 \ , \quad \varepsilon_{\theta\theta}^{H} \neq 0 \ , \quad \varepsilon_{zz}^{H} \neq 0 \ , \quad \varepsilon_{\theta z}^{H} = \varepsilon_{rz} = \varepsilon_{\theta r} = 0 \ ,
\end{aligned}
\tag{2.4}
$$

where supercript H, denotes the value of strain measurement in hydrostatic pressure test.

When the stress and strain quantities, indicated in Eqns. (2.4), are inserted in Eqn. (2.1), it yields

$$
\begin{aligned}
C_{11}\,\varepsilon^{H}_{rr} + C_{12}\,\varepsilon^{H}_{\theta\theta} + C_{13}\,\varepsilon^{H}_{zz} &= -P \\
C_{12}\,\varepsilon^{H}_{rr} + C_{11}\,\varepsilon^{H}_{\theta\theta} + C_{13}\,\varepsilon^{H}_{zz} &= -P \\
C_{13}\,\varepsilon^{H}_{rr} + C_{13}\,\varepsilon^{H}_{\theta\theta} + C_{33}\,\varepsilon^{H}_{zz} &= -P \ ,
\end{aligned}
\tag{2.5}
$$

In Eqns. (2.5), all the quantities, except the radial strain ε^{H}_{rr} are measureable.

2.5 Pure Torsion Test

In pure torsion test, the specimen is subjected to a twisting moment M_T and torsional strain, $\varepsilon_{\theta z}$, are measured directly by a strain gage which is bonded in a direction which makes an angle of 45^{o} with the longitudinal axis. In this test, stress and strain components take the forms

$$
\begin{aligned}
\tau_{rr} = \tau_{\theta\theta} = \tau_{zz} = \tau_{rz} = \tau_{r\theta} = 0 \ , \quad & \tau_{\theta z} = \tau_{o} \\
\varepsilon_{rr} = \varepsilon_{\theta\theta} = \varepsilon_{zz} = \varepsilon_{rz} = \varepsilon_{r\theta} = 0 \ , \quad & \varepsilon_{\theta z} \neq 0 \ ,
\end{aligned}
\tag{2.6}
$$

where $\tau_{o} = \dfrac{16 M_T}{\pi d^3}$ is applied torsional stress.

When stress and strain components in Eqns.(2.6) are substituted in Eqn. (2.1), a single equation

$$\tau_{o} = 2\, C_{44}\, \varepsilon_{\theta z} \ , \tag{2.7}$$

is obtained. Eqn. (2.7) determines the value of C_{44}, because the applied torsional stress τ_{o} and the corresponding torsional strain $\varepsilon_{\theta z}$ are measurable.

2.6 Reduced Form of Equations for Determination of Elastic Coefficients

Eqns. (2.3), (2.5) and (2.7) constitute seven equations for seven unknowns C_{11}, C_{12}, C_{13}, C_{33}, C_{44}, ε_{rr}^{T} and ε_{rr}^{H}. By eleminating ε_{rr}^{T} and ε_{rr}^{H}, number of equations can be reduced to five involving only five elastic coefficients. This elemination can be achieved first by solving the first of Eqns. (2.3) for ε_{rr}^{T} and first of Eqns. (2.5) for ε_{rr}^{H}, i.e,

$$\begin{aligned} \varepsilon_{rr}^{T} &= -\varepsilon_{\theta\theta}^{T}\frac{C_{12}}{C_{11}} - \varepsilon_{zz}^{T}\frac{C_{13}}{C_{11}} \\ \varepsilon_{rr}^{H} &= -\varepsilon_{\theta\theta}^{H}\frac{C_{12}}{C_{11}} - \varepsilon_{zz}^{H}\frac{C_{13}}{C_{11}} - \frac{P}{C_{11}} . \end{aligned} \tag{2.8}$$

When the first and second of Eqns. (2.6) are substituted into the remaining equations of Eqns. (2.3) and (2.5) respectively one gets

$$\begin{aligned} &[C_{11} - \frac{C_{12}^{2}}{C_{11}}]\ \varepsilon_{\theta\theta}^{T} + [C_{13} - \frac{C_{12}\,C_{13}}{C_{11}}]\ \varepsilon_{zz}^{T} = 0 \\ &[C_{13} - \frac{C_{12}\,C_{13}}{C_{11}}]\ \varepsilon_{\theta\theta}^{T} + [C_{33} - \frac{C_{13}^{2}}{C_{11}}]\ \varepsilon_{zz}^{T} = 0 \\ &[C_{11} - \frac{C_{12}^{2}}{C_{11}}]\ \varepsilon_{\theta\theta}^{H} + [C_{13} - \frac{C_{12}\,C_{13}}{C_{11}}]\ \varepsilon_{zz}^{H} = -P[1 - \frac{C_{12}}{C_{11}}] \\ &[C_{13} - \frac{C_{12}\,C_{13}}{C_{11}}]\ \varepsilon_{\theta\theta}^{H} + [C_{33} - \frac{C_{13}^{2}}{C_{11}}]\varepsilon_{zz}^{H} = -P[1 - \frac{C_{13}}{C_{11}}] . \end{aligned} \tag{2.9}$$

If the new unknown variables, x_1, x_2, x_3 and x_4 are defined by

$$x_1 = C_{11} - \frac{C_{12}^2}{C_{11}}$$

$$x_2 = C_{13} - \frac{C_{12}\,C_{13}}{C_{11}} \qquad (2.10)$$

$$x_3 = C_{33} - \frac{C_{13}^2}{C_{11}}$$

$$x_4 = \frac{C_{12}}{C_{11}} \quad ,$$

the system of four equations, Eqns. (2.9), takes the form

$$\underline{A}\ \underline{X} = \underline{C} \quad , \qquad (2.11)$$

where

$$\underline{A} = \begin{bmatrix} \varepsilon_{\theta\theta}^{T} & \varepsilon_{zz}^{T} & 0 & 0 \\ 0 & \varepsilon_{\theta\theta}^{T} & \varepsilon_{zz}^{T} & 0 \\ \varepsilon_{\theta\theta}^{H} & \varepsilon_{zz}^{H} & 0 & -P \\ 0 & \varepsilon_{\theta\theta}^{H} & \varepsilon_{zz}^{H} & P\,\dfrac{\varepsilon_{\theta\theta}^{T}}{\varepsilon_{zz}^{T}} \end{bmatrix} ,$$

$$\underline{X} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} , \qquad \underline{C} = \begin{bmatrix} 0 \\ \sigma \\ -P \\ -P\left(1+\dfrac{\varepsilon_{\theta\theta}^{T}}{\varepsilon_{zz}^{T}}\right) \end{bmatrix} .$$

Eqns. (2.10) imply that the elastic coefficients C_{11}, C_{12}, C_{13}, C_{33} can be determined by the relations

$$C_{11} = \frac{x_1}{1-x_4{}^2}$$

$$C_{12} = x_4 \cdot C_{11} \tag{2.12}$$

$$C_{13} = \frac{x_2}{1-x_4}$$

$$C_{33} = x_3 + \frac{C_{13}{}^2}{C_{11}} ,$$

when the unknowns x_i's $(i = 1\text{-}4)$ are obtained by solving the system Eqn. (2.11).

The fifth elastic coefficient C_{44} can independently be determined from Eqn. (2.7) which corresponds to pure torsion test.

2.7 Relations Between Elastic Coefficients and Technical Constants

The materials having hexagonal crystallographic structure can be also described by axial and radial Young's moduli E_{zz}, E_{rr}, three Poisson's ratios $\nu_{z\theta}$, ν_{rz}, $\nu_{r\theta}$ and shear modulus C_{44}, which are related to previously mentioned elastic coefficients C_{11}, C_{12}, C_{13}, C_{33} and C_{44}. In Poisson's ratio expressions the first and second indices indicate the directions of the applied tension and shortening respectively, e.g., $\nu_{z\theta}$ corresponds to shortening in θ-direction when the force is applied in z-direction. In what follows the expressions, which relate Young's moduli (E_{rr}, E_{zz}) and Poisson's ratios ($\nu_{z\theta}$, ν_{rz}, $\nu_{r\theta}$) to elastic coefficients (C_{11}, C_{12}, C_{13}, C_{33}), will be derived.

a. Determinations of $\nu_{z\theta}$ and E_{zz}

When a bone specimen is subjected to a axial stress ,
Eqn. (2.1) takes the form

$$
\begin{aligned}
C_{11}\ \varepsilon_{rr} + C_{12}\ \varepsilon_{\theta\theta} + C_{13}\ \varepsilon_{zz} &= 0 \\
C_{12}\ \varepsilon_{rr} + C_{11}\ \varepsilon_{\theta\theta} + C_{13}\ \varepsilon_{zz} &= 0 \\
C_{13}\ \varepsilon_{rr} + C_{13}\ \varepsilon_{\theta\theta} + C_{33}\ \varepsilon_{zz} &= \sigma\ .
\end{aligned}
\qquad (2.13)
$$

Elemination of ε_{rr} between the first and second of Eqns.(2.13) yields

$$\frac{\varepsilon_{\theta\theta}}{\varepsilon_{zz}} = - \frac{C_{13}}{C_{11} + C_{12}} ,$$

which implies that the expression for $\nu_{z\theta}$ should be

$$\nu_{z\theta} = \frac{C_{13}}{C_{11} + C_{12}} \ . \qquad (2.14)$$

It should be noted that $\nu_{z\theta}$ is equal to ν_{zr}, since on the plane of transverse cross section bone behavior is isotropic.

Using the first and second of Eqns.(2.13), one can express $\varepsilon_{\theta\theta}$ and ε_{rr} in terms of ε_{zz}:

$$
\begin{aligned}
\varepsilon_{\theta\theta} &= - \frac{C_{13}}{C_{11} + C_{12}} \varepsilon_{zz} \\
\varepsilon_{rr} &= - \frac{C_{13}}{C_{11} + C_{12}} \varepsilon_{zz} \ .
\end{aligned}
\qquad (2.15)
$$

When Eqns.(2.15) are substituted into the third of Eqns. (2.13) one obtains

$$\frac{\sigma}{\varepsilon_{zz}} = C_{33} - \frac{2C_{13}^{\;2}}{C_{11} + C_{12}} \quad ,$$

which indicates that the expression for axial Young's modulus has the form

$$E_{zz} = C_{33} - \frac{2C_{13}^{\;2}}{C_{11} + C_{12}} \quad . \tag{2.16}$$

b. Determinations of $\nu_{r\theta}$, ν_{rz} and E_{rr}

When a bone specimen is subjected to a stress σ in radial direction, Eqn. (2.1) becomes

$$\begin{aligned} C_{11}\,\varepsilon_{rr} + C_{12}\,\varepsilon_{\theta\theta} + C_{13}\,\varepsilon_{zz} &= \sigma \\ C_{12}\,\varepsilon_{rr} + C_{11}\,\varepsilon_{\theta\theta} + C_{13}\,\varepsilon_{zz} &= 0 \\ C_{13}\,\varepsilon_{rr} + C_{13}\,\varepsilon_{\theta\theta} + C_{33}\,\varepsilon_{zz} &= 0 \;. \end{aligned} \tag{2.17}$$

Eleminating ε_{zz} and ε_{rr} between the second and third of Eqns. (2.17), one obtains

$$\frac{\varepsilon_{\theta\theta}}{\varepsilon_{rr}} = - \frac{(C_{33}C_{12} - C_{13}^{\;2})}{(C_{11}C_{33} - C_{13}^{\;2})}$$

and (2.18)

$$\frac{\varepsilon_{zz}}{\varepsilon_{rr}} = \frac{C_{13}(C_{12} - C_{11})}{(C_{13}^2 - C_{33}C_{11})} ,$$

which imply that

$$\nu_{r\theta} = \frac{C_{33}C_{12} - C_{13}^2}{C_{11}\,C_{33} - C_{13}^2}$$

(2.19)

$$\nu_{rz} = \frac{C_{13}(C_{12} - C_{11})}{C_{13}^2 - C_{33}\,C_{11}} .$$

If $\varepsilon_{\theta\theta}$ and ε_{zz} in Eqns. (2.18), which are expressed in terms of ε_{rr}, are substituted in the first equation of Eqns. (2.17), one obtains

$$\frac{\sigma}{\varepsilon_{rr}} = \frac{(C_{11}-C_{12})\ (C_{33}C_{11} + C_{33}C_{12} - 2C_{13}^2)}{(C_{11}C_{33} - C_{13}^2)} , \quad (2.20)$$

which may be considered as Young's modulus, E_{rr}, in radial direction.

The elastic coefficient C_{44}, which is obtained by pure torsion test, is equal to the shear modulus describing the shear deformation on the plane which is parallel to the longitudinal axis of the specimen. On the other hand, if the shear deformation occurs on the plane which is perpendicular to the longitudinal axis of the specimen, then Eqn. (2.1) implies that the shear modulus $G_{\perp}$ on the transverse plane is given by

$$G_{\perp} = \frac{1}{2} (C_{11} - C_{12}) \quad . \qquad (2.21)$$

2.8. Method of Least Squares Employed for Determination Elastic Coefficients

a. Method of Least Squares for Systems

The elastic coefficients C_{11}, C_{12}, C_{13} and C_{33} can be determined first by solving the system, Eqn. (2.11), for X_i (i 1-4), then substituting them into Eqns. (2.12). The coefficient matrix $\underline{A}$ and load vector $\underline{C}$ can be generated by using the applied stresses and strains measured for each direction in pure tension and hydrostatic pressure tests. Since in each test various strain values at various applied stresses and pressures are obtained, an overdetermined system of linear equations arise so that one should introduce method of least squares to obtain the optimum values of elastic coefficients.

In indicial notation, Eqn. (2.11) may be expressed as

$$\sum_{j=1}^{p} a_{ij} x_j = C_i \quad , \qquad (i = 1-p) \ , \qquad (2.22)$$

where a_{ij}'s are the elements of the coefficient matrix, C_i's are the components of the load vector, X_j's are the components of the unknown vector and p is the number of linear equations.

A norm for the total error for n sets of experimental data can be introduced in the form

$$\varepsilon = \sum_{m=1}^{n} \sum_{i=1}^{p} \left(C_i^{(m)} - \sum_{j=1}^{p} a_{ij}^{(m)} x_j\right)^2 , \qquad (2.23)$$

where $(\ )^m$ designates the value of $(\ )$ in m-th experiment.

For minimizing the value of the total error, first derivatives of the error expression with respect to x_i $(i = 1-p)$ should be set equal to zero; thus one obtains

$$\sum_{m=1}^{n} \sum_{i=1}^{p} C_i^{(m)} a_{is}^{(m)} = \sum_{m=1}^{n} \sum_{i=1}^{p} a_{is}^{(m)} \sum_{j=1}^{p} a_{ij}^{(m)} x_j , (s=1-p). \qquad (2.24)$$

In matrix form, this relation takes the form

$$\sum_{m=1}^{n} \underline{A}^{(m)^T} \underline{C}^{(m)} = \left[\sum_{m=1}^{n} \underline{A}^{(m)^T} \underline{A}^{(m)} \right] \underline{X} , \qquad (2.25)$$

where supercript T designates the transpose of coefficient matrix.

b. <u>Method of Least Squares for a Single Equation</u>

In the case of a single equation, the vectors $\underline{C}$, $\underline{X}$ and coefficient matrix $\underline{A}$ will be scalars. For this case, the general relation Eqn. (2.24), obtained for systems, reduces to

$$\sum_{m=1}^{n} \underline{A}^{(m)} \underline{C}^{(m)} = \left[\sum_{m=1}^{n} \underline{A}^{(m)^2} \right] \underline{X} , \qquad (2.26)$$

where A, C and X are scalars. When Eqn. (2.26) is applied to Eqn. (2.7), which corresponds to pure torsion test, the expression for determining the value of the constant C_{44} is found to be

$$C_{44} = \frac{\sum_{m=1}^{n} \tau_o^{(m)} \varepsilon_{\theta z}^{(m)}}{2 \sum_{m=1}^{n} (\varepsilon_{\theta z}^{(m)})^2} \tag{2.27}$$

Chapter 3

INSTRUMENTATION AND EXPERIMENTAL PROCEDURE

3.1 Preparation of Bone Specimen

Circular cylindrical specimen, 6 mm in diameter and 50 mm in length, are prepared for each experiment. The specimen, in required size, is extracted from the compact portion of calf by hand sawing. One end of bone sample is centered by a center drill, in drilling machine. The other end of specimen is clamped in the chuck of lathe and the centered end is supported by the tailstock centre of lathe. It is machined at very high speeds, 1500 r.p.m, and at very small feeds of lathe. Water is used as coolant during machining process. The machined specimens are kept in refrigerator.

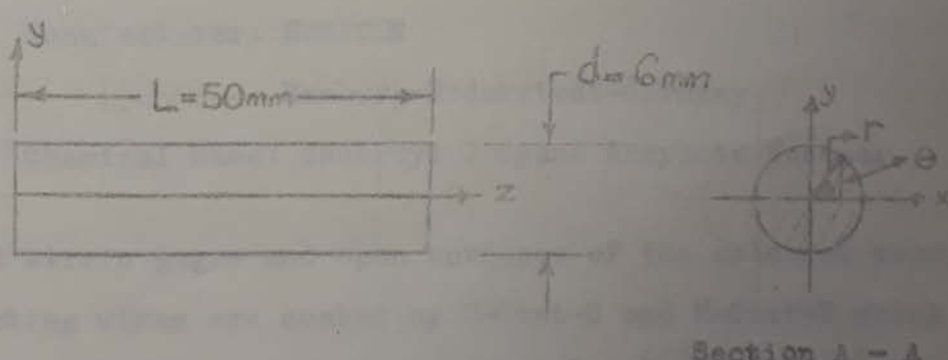


FIGURE 2 The bone specimen referred to (r, θ, z) coordinate system.

3.2 Gage, Adhesive and Coating Material

The following available strain gages with fabricated terminals are used in assembling two-arm bridge circuits;

Manufacturer: MICRO MEASUREMENTS

Romulus, Michigan U.S.A.

Gage Type: EA-41-031 DE - 120

Resistance in Ohms: 120 ± 0.4 %

Gage Factor at 75°F: 1.99 ± 1.0 %

Temperature Range: Up to 400°F for static measurements

500°F for dynamic strain.

Strain Limits: 30 000-50 000 microstrain, tension or compression

Fatigue Life: Over 10^7 cycles at 1400 microstrain

The adhesive ETHICON BUCRYLATE, used in bonding process, has the specifications

Manufacturer: ETHICON

Hamburg-Noderstedt-Germany

Chemical Name: Isobutyl 2 Cyano Acrylate Monomer

Bonded strain gages and open surfaces of the soldered terminals and connecting wires are coated by M-Coat-G and M-Coat-B which are manufactured by Micro Measurements, Romulus-Michigan, U.S.A.

3.3 Gage Bonding and Gage Coating Processes

Positions, for three strain gages, in axial, longitudinal directions and in the direction making an angle of 45° with longitudinal axis are marked through the circumference of the middle portion of bone specimen. Marked surface is scraped clean, swabbed dry and degreased as much as possible by using ether. After complete evaporation of ether, the surface of the bone specimen is flooded with adhessive Ethicon and the gage is replaced to its required position by the help of a scotch tape which is attached to the upper surface of gages. Bonding area is covered by a piece of gelatine and it is pressed by finger almost two minutes. The scotch tape and gelatine are relased, after the bonding process is completed (See Fig. 3 and 4).

In hydrostatic pressure test, the gages have to be in contact with oil which transmits pressure. Since the coating material M-coat G has an excellent chemical resistance to oil, the bonded gages are coated by this coating material. M coat G is a two part 100 % solid compound, packaged in collapsible metal tubes. The resin component is white and the curing agent is green. One part of resin is mixed by two parts of curing agent either in weight or in volume and the gages are coated by this mixture. Gage terminals and connecting wires are coated by M-Coat B.

3.4 Two Arm Bridge Assembly

The two terminals of the each of three gages are connected to the black and white terminals of three channels of the switching and balancing unit. A compansating gage is bonded on a dummy bone specimen and the terminals of this gage is connected to the white and green terminals of the switching and balancing unit as it is shown in the Fig. 5.

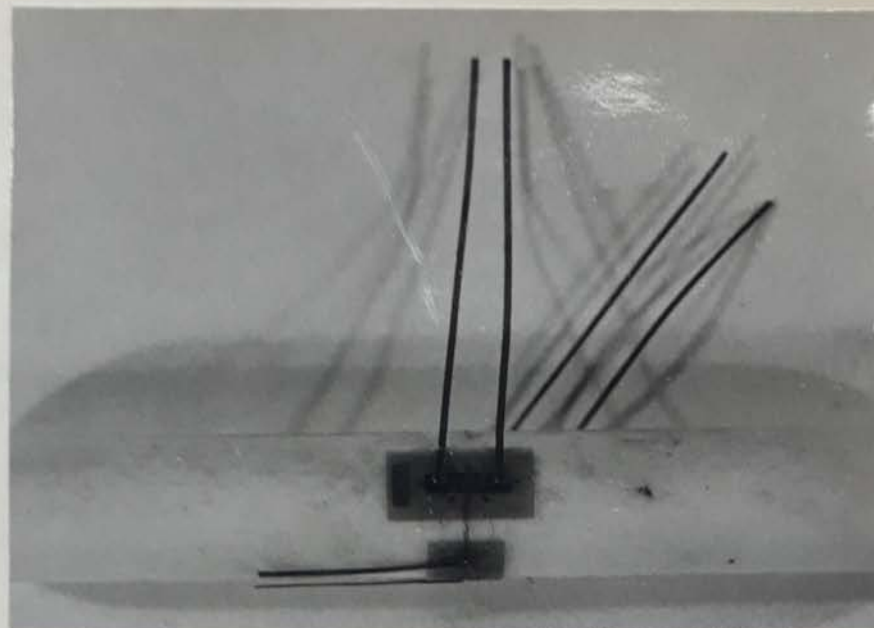


FIGURE 3 Strain gages bonded on the specimen in the axial and circumferential directions.

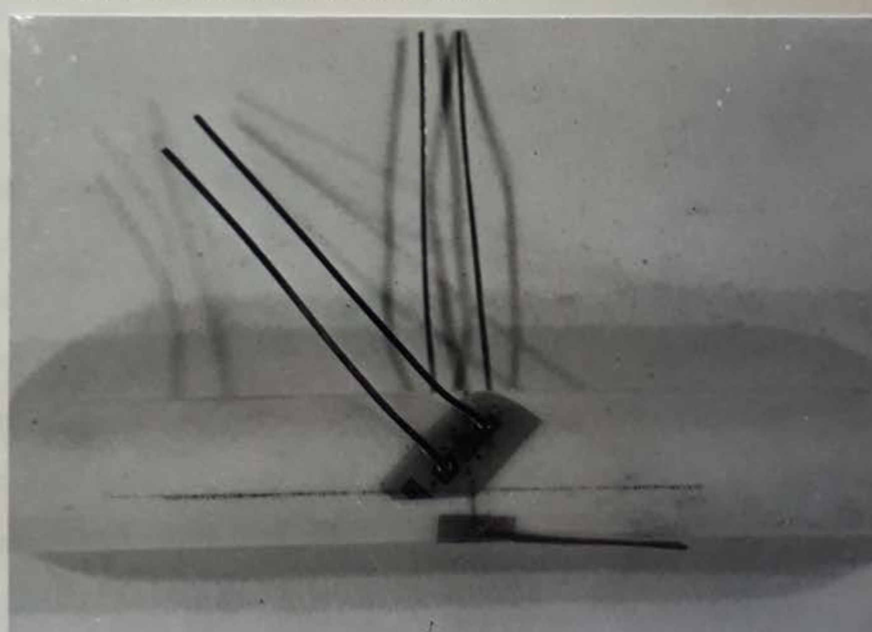
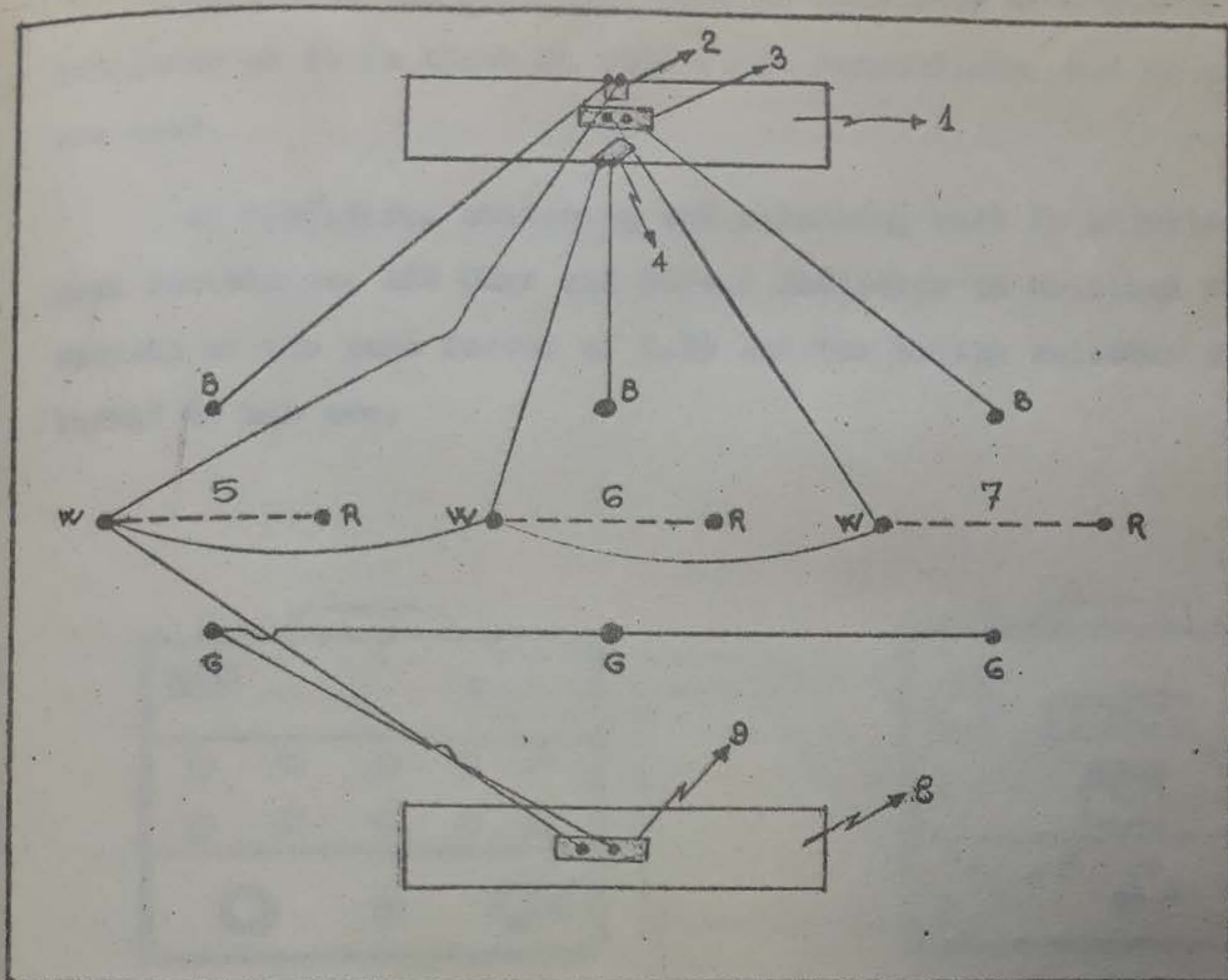


FIGURE 4 Strain gages bonded on the specimen in circumferential direction and in the direction making an angle of 45° with longitudinal axis.



1 Bone specimen

2 Circumferential gage

3 Axial gage

4 Gage making an angle of 45° with longitudinal axis of the specimen

5 Channel I

6 Channel II

7 Channel III

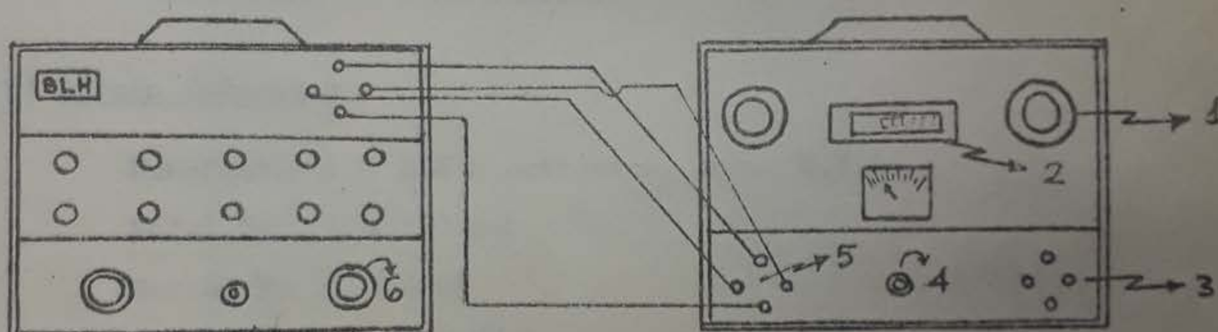
8 Dummy specimen

9 Compansating gage

FIGURE 5 Connections between gage terminals, compansating gage and the channels of the switching and balancing unit.

Switching and Balancing Unit is connected to a strain indicator as it is shown in Fig. 6. In connections, 0.4 mm cables are used.

In operation, switching and balancing unit is adjusted to gage resistance, 120 Ohms and Strain Indicator is switched to operate at the gage factor of 1.99 and the bridge selecter is turned to two arm.



SWITCHING AND BALANCING UNIT STRAIN INDICATOR

1 Gage Factor
2 Strain Indicator
3 4 Arm Bridge
4 Bridge Selecter
5 2 Arm Bridge
6 Gage Resistance.

FIGURE 6 Connections between switching and balancing unit and strain indicator.

Description of the instruments used for the set-up in Fig.6 are

a) Switching and Balancing Unit

Manufacturer : BLH Electronics, Inc., U.S.A.

Model No : 225

Serial No : 2761

Ass'y No : 203791-4

Calibration : 0-10 channels

b) Strain Indicator

Manufacturer : BLH Electronics, Inc., U.S.A.

Model No : 120 C-B

Serial No : 4065

Ass'y No : 279466-1

3.5 Set-ups for Pure Tension, Hydrostatic Pressure and Pure Torsion Tests

a) Pure Tension Test

For pure tension test, a universal joint is suspended to a frame and its lower end is mounted to the upper end of the specimen by means of a screw mechanism. The lower end of the specimen is mounted, by the same mechanism, to another universal joint so that bending deformations in simple tension test can be eleminated. The loading mechanism, used in simple tension test, is shown in Fig. 7.

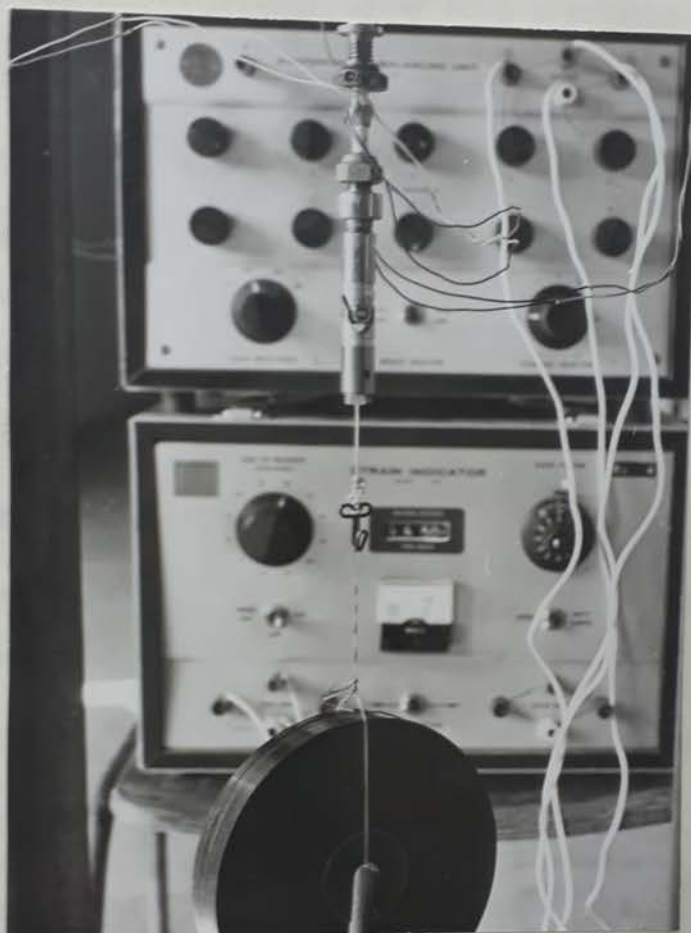


FIGURE 7 Pure tension test set-up.

b) Hydrostatic Pressure Test

Hydrostatic pressure is applied to the specimen by Armthor Dead Weight Pressure Gage having the specifications;

Manufacturer : Armthor Testing Instrument Co., Inc.
Brooklyn, New York, U.S.A.
Accuracy : Within 1/10 of 1 % of the indicated reading.
Capacity p.s.i: 10 000
Compressing Fluid: SAE 10 Oil

A tube is designed for putting the specimen in it as it is shown in Fig. 8. The gage terminals, soldered to connecting wires, are taken out through the 4 holes of the tube. The inner and outer ends of the holes are closed by the adhessive 404. The complete set-up for hydrostatic pressure test is shown in Fig.9.

c) Pure Torsion Test

The specimen is mounted to a Biaxial Testing Machine having the specifications

Manufacturer : Carl Schenk Machinen Fabrik Gmb. H.
Model : PWO
Oscillating Moment, kgm: $\mp$0.04 to $\mp$1.50
Maximum Moment, kgm : 3.00
Alternating angle of drive (degrees) : 18°
Frequency of testing, rpm : 1400
Dimensions, mm : 490x450x490
Weight, kg : 85

and required torques are given by hand (See Figs. 10,11).

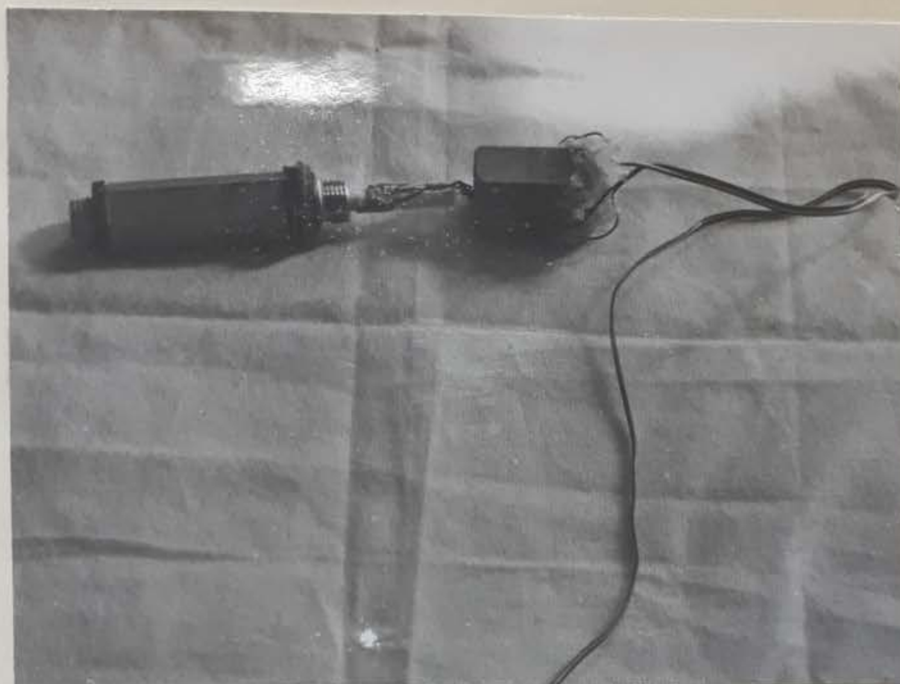


FIGURE 8 Pressure tube with a specimen.

FIGURE 9 Armthor dead weight pressure set-up.

FIGURE 10 A specimen placed in Biaxial Testing Machine.

FIGURE 11 Complete set-up for Pure Torsion Test.

3.6 Procedure for Experiments

In each of the three tests mentioned previously instantaneous loads or pressures are applied for minimizing the creep effect.

In pure tension test, axial and circumferential strains are measured for the loads of 2, 4, 6, 8, 10 Ibs. Before loading a reading for the free state of specimen is taken. By subtracting the reading measured for a specific load from the one measured in free state, the strain corresponding to this applied load is found.

In hydrostatic pressure test the same procedure is used in measuring instantaneous axial and circumferential strains for the applied pressures of 100, 200, 300, 400 and 500 p.s.i.

In pure torsion test, similarly, five values of strains are measured for five different torques applied in Biaxial Testing Machine.

EXPERIMENTAL DATA AND RESULTS

4.1. Experimental Data

Two tests for each of the two specimens are performed in pure tension and hydrostatic pressure tests where axial and circumferential strains are measured.

Similarly, performing two tests for each of the two specimens, the strains in the direction making an angle of 45° with longitudinal axis of the specimen, are measured in pure torsion test.

Experimental results are tabulated in Tables 1-6.

	F, lb	σ , kg/cm^2	$\varepsilon^T_{\theta\theta}$, $\mu\varepsilon$	ε^T_{zz}, $\mu\varepsilon$
TEST I	2.	3.20	-5	15
	4.	6.40	-11	29
	6.	9.60	-16	45
	8.	12.80	-22	59
	10.	16.00	-28	74
TEST II	2.	3.20	-5	14
	4.	6.40	-10	29
	6.	9.60	-16	44
	8.	12.80	-22	60
	10.	16.00	-28	74

Table 1 Data obtained in pure tension tests for the specimen I

	PP, p.s.i.	P, kg/cm^2	$\varepsilon^H_{\theta\theta}$, $\mu\varepsilon$	ε^H_{zz}, $\mu\varepsilon$
TEST I	100	7	-22	-8
	200	14	-41	-16
	300	21	-63	-24
	400	28	-81	-32
	500	35	-99	-40
TEST II	100	7	-21	-8
	200	14	-42	-16
	300	21	-62	-24
	400	28	-80	-32
	500	35	-99	-40

Table 2 Data obtained in hydrostatic pressure tests for the specimen I

	F, lb	σ, kg/cm^2	$\varepsilon^T_{\theta\theta}$, $\mu\varepsilon$	ε^T_{zz}, $\mu\varepsilon$
TEST I	2	3.20	-5	14
	4	6.40	-11	28
	6	9.60	-16	44
	8	12.80	-21	59
	10	16.00	-28	74
	2	3.20	-6	15
	4	6.40	-11	30
	6	9.60	-16	45
	8	12.80	-21	59
	10	16.00	-28	74

Table 3 Data obtained in pure tension tests for the specimen II

	PP, p.s.i.	P, kg/cm^2	$\varepsilon_{\theta\theta}^{H}$, $\mu\varepsilon$	ε_{zz}^{H}, $\mu\varepsilon$
TEST I	100	7	-20	-8
	200	14	-42	-16
	300	21	-62	-24
	400	28	-81	-32
	500	35	-99	-40
TEST II	100	7	-22	-8
	200	14	-42	-16
	300	21	-62	-24
	400	28	-81	-32
	500	35	-100	-40

Table 4 Data obtained in hydrostatic pressure tests for the specimen II

	M_T, kg.cm	τ_0 , kg/cm^2	$\varepsilon_{\theta z}$, $\mu\varepsilon$
TEST I	1.453	34.28	580.
	2.907	68.58	975.
	4.361	102.88	1335.
	6.079	143.41	1680.
	7.268	171.46	1955.
TEST II	0.661	15.59	440.
	2.114	49.88	865.
	3.304	77.94	1130.
	4.758	112.24	1400.
	6.476	152.77	1645.

Table 5 Data obtained in pure torsion tests for the specimen I

	M_T, kg.cm	τ_o, kg/cm^2	$\varepsilon_{\theta z}$, $\mu\varepsilon$
TEST I	1.718	40.53	790.
	3.172	74.83	1055.
	5.022	118.47	1340.
	7.004	165.23	1650.
	8.458	199.53	2010.
TEST II	0.661	15.59	490.
	2.114	49.87	705.
	3.833	90.42	1120.
	5.418	127.81	1430.
	7.533	177.71	1865.

Table 6 Data obtained in pure torsion for the specimen II

4.2. Determination of Elastic Coefficients and Technical Constant

Two specimens are extracted from the same calf femur and two sets of experimental data are tabulated for each of the bone specimens in three tests, namely, pure tension, hydrostatic pressure and pure torsion tests.

In determining the five elastic coefficients of fresh animal bone, a computer program is developed. The coefficient matrices $\underline{A}^{(m)}$ and load vectors $\underline{C}^{(m)}$ of Eqn. (2.25) are generated by the strain measurements at known stresses of hydrostatic pressure and pure tension tests. The experimental data of the two tests are linearly approximated by the Eqn. (2.25), i.e.,

$$[\sum_{m=1}^{n} \underline{A}^{(m)^T} \underline{A}^{(m)}] \underline{X} = \sum_{m=1}^{n} \underline{A}^{(m)^T} \underline{C}^{(m)} \quad . \tag{4.1}$$

Upon solving these equations for x_i (i= 1-4), the optimum values of the material constants C_{11}, C_{12}, C_{13} and C_{33} can immediately be be determined from Eqns. (2.12) while the optimum value of the shear modulus, C_{44}, on the plane parallel to longitudinal axis can be found by using Eqn. (2.27), i.e.,

$$C_{44} = \frac{\sum_{m=1}^{n} \tau_{0}^{(m)} \varepsilon_{\theta z}^{(m)}}{2 \sum_{m=1}^{n} \varepsilon_{\theta z}^{(m)^2}} \tag{4.2}$$

In determination of elastic coefficients of each of the two specimens, two sets of the experimental data are combined and the results for each specimen are shown in the columns a, b of the Table 7. As it is mentioned previously, the two specimens prepared for this study, are extracted from the same calf femur. For this reason, experimental results of both specimens, tabulated in Tables 1-6 , are combined and more realistic experimental values for the elastic coefficients and technical constants are obtained (see Table 7, column c).

Elastic Coefficients and Technical Constants of Calf Femur	Using the data obtained for the ;		
	Specimen I	Specimen II	Both Specimens
C_{11} $(^{GN}/_{m^2})$	19.00	20.20	19.50
C_{12} $(^{GN}/_{m^2})$	10.60	9.37	10.00
C_{13} $(^{GN}/_{m^2})$	11.50	11.40	11.50
C_{33} $(^{GN}/_{m^2})$	30.10	30.10	30.10
C_{44} $(^{GN}/_{m^2})$	3.98	4.37	4.18
E_{zz} $(^{GN}/_{m^2})$	21.20	21.30	21.20
E_{rr} $(^{GN}/_{m^2})$	11.90	14.20	13.10
$G_{\perp}$ $(^{GN}/_{m^2})$	3.98	4.37	4.18
$\nu_{z\theta}=\nu_{zr}$	0.388	0.387	0.388
$\nu_{r\theta}$	0.428	0.317	0.374
ν_{rz}	0.218	0.259	0.239
	(a)	(b)	(c)

Table 7 Elastic coefficients and technical constants of calf femur computed by using the data for each and both of the two specimens.

Chapter 5

DISCUSSIONS AND CONCLUSIONS

Static tests, namely, pure tension, hydrostatic pressure and pure torsion tests are performed to determine linear elastic constants of animal bone. In each of these tests, strain-stress relationships of bone should be in linear region, as long as its linear elastic properties are concerned. Strain gages which are very sensitive in strain measurements even for small quantites of loads are very useful for determination of linear elastic deformation of calf femur compact section. Bonfield and Datta (16) suggest that the linear elastic strain limit is about 230 $\mu\varepsilon$ in pure tension test. On the other hand, the findings of the present study have shown that the upper bound for linear elastic strain is about 2000 $\mu\varepsilon$ in pure torsion test and infinite in hydrostatic pressure test.

In reality the fresh bone is a viscoelastic material, therefore strain readings may change with time at constant loads. Since in this study only instantaneous linear elastic behaviour of the bone is investigated, an instantaneous load-unload technique is used in each of the three tests mentioned previously.

Study of data tabulated in Table 1 indicates that the readings have the deviations of maximum 3 $\mu\varepsilon$ from the linearly approximated strains. These deviations are generally due to the errors which may arise in the preparation of the specimen. In the machining process the force acting on the cutting edge bends the specimen so that its diameter may not be uniform along its longitudinal axis. For this reason, it is machined at very small feeds which will lessen bending of the specimen. Also, it is machined at very high speeds since bone is very brittle material and it is required to have a very good surface finish on it for bonding process. Water is used as coolant in order not to change the properties of the fresh bone.

The diameter of the bone specimen was large enough for bonding three strain gages on the middle portion of the bone specimen. By the aid of these strain gages the strains in axial and circumferential directions and in the direction making an angle of 45^{o} with longitudinal axis of the bone specimen are measured in pure tension, hydrostatic pressure and pure torsion tests.

The gages were bonded almost within an accuracy of 0.1 mm of their true positions due to slipping of the gages during compression for bonding. Bonded gages and their terminals are coated by coating material for protecting them from the effects of oil which is used in hydrostatic pressure test.

In pure tension test, eccentric application of axial load generates bending moment on the specimen and that yields bending strains in addition to pure tension strains. The probability of this error is minimized by two universal joints which are attached to the both ends of the specimen by means of a screw mechanism. The specimens are long enough so that the test sections are not affected by the stresses due to the tightening, generated on the clamped ends of the specimen.

The coefficient matrix $\underline{A}$ and the load vector $\underline{C}$ appearing in Eqn. (2.15) are generated with the strain measurements at known stresses in pure tension and hydrostatic pressure tests. Using the method of least squares explained in Chapter 2, the experimental data may be linearly approximated by solving the system of four equations

$$[\sum_{m=1}^{n} \underline{A}^{(m)^T} \cdot \underline{A}^{(m)}]\underline{X} = \sum_{m=1}^{n} \underline{A}^{(m)^T} \underline{C}^{(m)} , \qquad (5.1)$$

for the optimum values of x_i (i= 1-4), which are related to the optimum values of linear elastic constants by Eqns. (2.12).

In pure torsion test, the experimental data is linearized by the least squares equation;

$$C_{44} = \frac{\sum_{m=1}^{n} \tau_o^{(m)} \epsilon_{\theta z}^{(m)}}{2 \sum_{m=1}^{n} \epsilon_{\theta z}^{(m)^2}} , \qquad (5.2)$$

where C_{44} is the shear modulus on the plane which is paralel to the longitudinal axis. The symbols appearing in Eqns. (5.1) and (5.2) are defined in Chapter 2.

In all experiments, almost a linear stress-strain relationship is observed. Indeed the experimental data obtained in pure torsion test, does not deviate from its linear approximation significantly as it is seen in Fig. (12).

The necessary conditions for the positive definitness of the strain energy function for a transversly isotropic linear elastic solid are (18)

$$\begin{aligned} C_{11} &> |C_{12}| \\ (C_{11}-C_{12})\, C_{13} &> 2\, C_{13}^{\,2} \\ C_{44} &> 0 \end{aligned} \qquad (5.3)$$

It should be observed that the experimental values of elastic constants obtained in the present study, satisfy these conditions, Eqns. (5.3). The present results and the results of several previous studies are tabulated in Table 8. (See page 49)

The elastic constants of the fresh bone determined in this study can reliably used in the future studies of bioengineering. In particular, this information may be useful in the analysis of the musculo-skeletal systems and in the design of prosthetic devices.

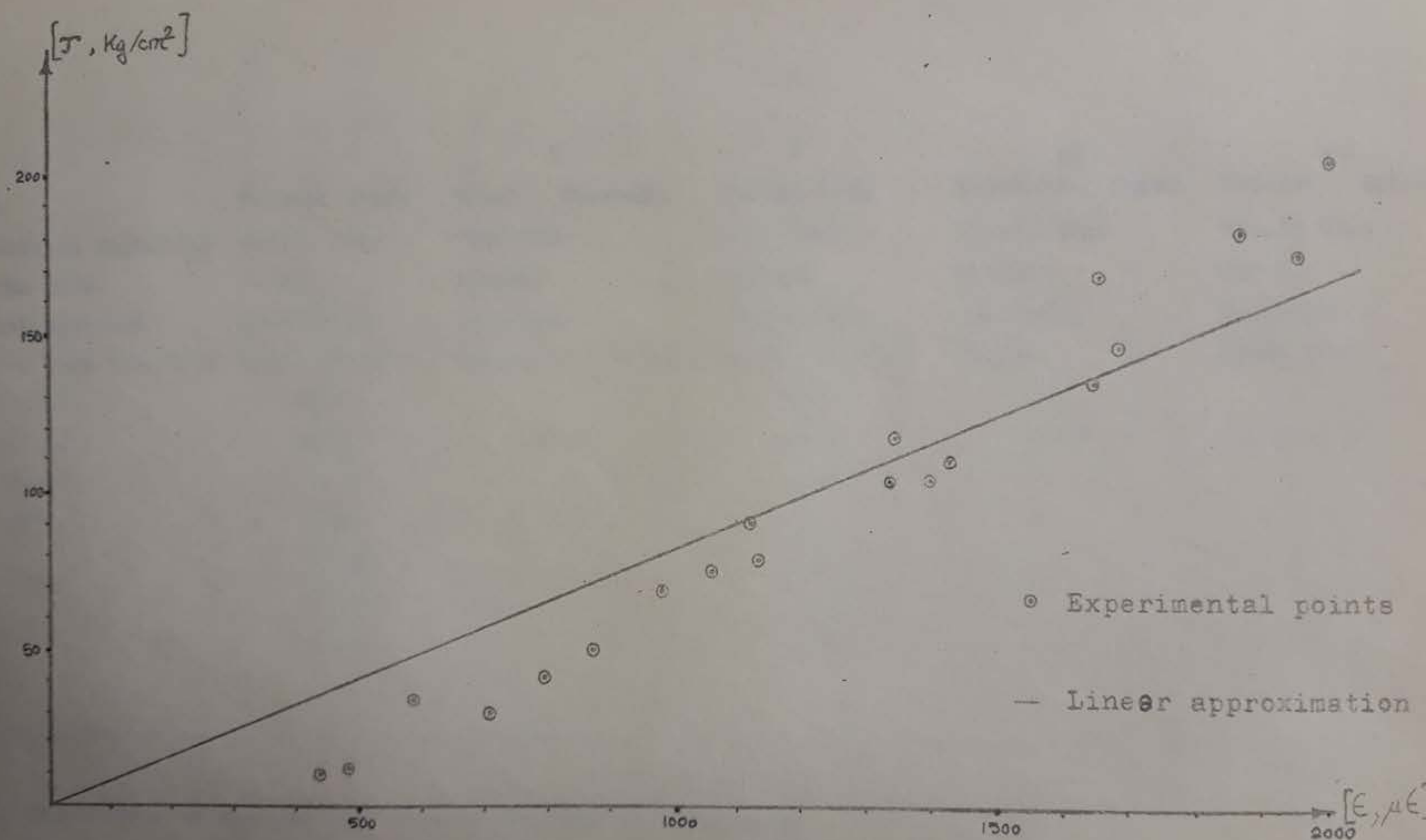


FIGURE 12. Linear approximation of the experimental data in pure torsion test

Referance	Present Study	5 Brash - Skorecki	4 Sydney - Lang	16 Bonfield - Datta	17 McLeish - Haboobi
Experimentation Technique	Strain Gage	Vibration	Ultrasonic	Strain Gage	Strain Gage
Type of the Test	Static	Dynamic	Dynamic	Static	Static
Theoretical Approach	Anisotropic	Isotropic	Anisotropic	Isotropic	Isotropic
Type of the Bone Specimen	Femur of Calf	Tibiae of Old Beef	Bovine Phalanx	Tibiae	Human Bone
E_{rr} (GN/m^2)	13.1	-	11.3	-	-
E_{zz} (GN/m^2)	21.2	23.4	22.0	27.3	1[illegible].7-19.8
C_{44} (GN/m^2)	4.18	-	5.40	-	-
$G_{\perp}$ (GN/m^2)	4.75	-	-	-	-
ν_{zr}	0.388	-	0.487	-	-
$\nu_{r\theta}$	0.374	-	0.397	-	-
ν_{rz}	0.238	-	0.204	-	-

Table 8 Comparision of the present results with the previous ones.

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APPENDICES

Appendix A

DETERMINATION OF YOUNG'S MODULUS BY TRANSVERSE VIBRATION TECHNIQUE

The equation which governs transverse vibrations of a beam is

$$EI \frac{\partial^4 y}{\partial x^4} + m \frac{\partial^2 y}{\partial t^2} = q(x,t) \quad , \qquad (A.1)$$

where x the axial distance measured along the beam, t is time, E is Young's modulus in axial direction, I is moment of inertia of the cross section of the beam, y is transverse deflection, m is mass per unit length and q is applied distrubeted load per unit length.

The governing equation for free flexural vibration problem can be obtained by taking q 0, which yields

$$\frac{\partial^4 y}{\partial x^4} + \frac{m}{EI} \frac{\partial^2 y}{\partial t^2} = 0 \quad . \qquad (A.2)$$

For a beam of length L which is clamped at the left end and free at the right, the boundary conditions take forms

$$\begin{aligned} &\text{at} \quad x=0 \quad , \quad \frac{\partial y}{\partial x} = 0 \quad , \quad y=0 \\ &\text{at} \quad x=L \quad , \quad \frac{\partial^2 y}{\partial x^2} = 0 \quad , \quad \frac{\partial^3 y}{\partial x^3} = 0 \quad , \end{aligned} \qquad (A.3)$$

if the origin is assumed to coincide with the clamped end.

The solution of Eqn. (A.2) is assumed to have the form,

$$y = f(x) \, . \, \mathrm{Sin}(wt+\alpha) \quad , \qquad (\ .4)$$

where f(x) is amplitude function, w is angular frequency and is phase angle. Eqn. (A.4) yields the following relations for the space and time derivatives;

$$\frac{\partial^4 y}{\partial x^4} = \frac{\partial^4 f}{\partial x^4} \, \mathrm{Sin}\,(wt+\alpha)$$

$$\qquad (A.5)$$

$$\frac{\partial^2 y}{\partial t^2} = - w^2 \, f(x) \, \mathrm{Sin}\,(wt+\alpha) \quad .$$

If Eqns. (A.5) are replaced in the governing Eqn. (A.2), one gets

$$\frac{\partial^4 f}{\partial x^4} - k^4 \, f(x) = 0 \quad , \qquad (A.6)$$

where $k^4 = \frac{mw^2}{EI}$.

The solution of Eqn. (A.6) is,

$$f(x) = m_1 \mathrm{Cos}\,kx + m_2 \,\mathrm{Sin}\,kx + m_3 \,\mathrm{Cosh}\,kx + m_4 \,\mathrm{Sinh}\,kx, \quad (A.7)$$

where m_i (i= 1-4) are integration constants. If Eqns. (A.4) and (A.7) are substituted in the boundary conditions Eqns. (A.3), one obtains

$$m_3 = -m_1$$
$$m_4 = -m_2 \qquad (A.8)$$

$$-m_1 \text{ Cos } kL - m_2 \text{ Sin } kL + m_3 \text{ Cosh } kL + m_4 \text{ Sinh } kL = 0$$
$$m_1 \text{ Sin } kL - m_2 \text{ Cos } kL + m_3 \text{ Sinh } kL - m_4 \text{ Cosh } kL = 0 ,$$

which is a system of four linear algebraic equations governing the values of m_1, m_2, m_3 and m_4. When in Eqns. (A.8) m_1 and m_2 are eleminated, one gets a system of two equations

$$\begin{bmatrix} (\text{Cos } kL + \text{Cosh } kL) & (\text{Sin } kL + \text{Sinh } kL) \\ (\text{Sin } kL - \text{Sinh } kL) & -(\text{Cos } kL + \text{Cosh } kL) \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \qquad (A.9)$$

which is an eigen value problem in which kL may be regarded as the eigen value. For non-trivial solution, determinant of the coefficient matrix of Eqn. (A.9) should vanish. This yields

$$\text{Cos } kL \ \text{Cosh } kL = -1 \qquad (A.10)$$

Eqn. (A.10) governs the natural frequencies, each of which corresponds to a different mode. For example, the values of kL for first five modes are found to be as

$$(kL)_{1,2,3,4,5} = 1.8751,\ 4.694,\ 7.855,\ 10.996,\ 14.137. \qquad (A.11)$$

If the beam is assumed to be of rectangular cross section of height h, from the second of Eqn. (A.6), it follows that

$$E = 48 \frac{\pi^2 \rho f^2 L^4}{h^2 (kL)^4} \quad , \tag{A.12}$$

which relates the Young's modulus to the natural frequency f. In Eqn.(A.12) ρ is the mass density and the frequency f is related to angular frequency w by

$$w = 2\pi f \quad . \tag{A.13}$$

Using Eqn. (A.12), Young's modulus E may be determined if the mass density, length and height of the beam are known and if the value of kL, corresponding to the mode for which the frequency f is measured, is computed by solving Eqn. (A.10).

Appendix B

DETERMINATION OF YOUNG'S MODULUS BY LONGITUDINAL VIBRATION TECHNIQUE

The equation which governs longitudinal vibration of a beam is

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2} , \quad (B.1)$$

where $c = \frac{E}{\rho}$ is the bar velocity, ρ is the mass per unit volume, u is the displacement in axial direction, t is the time and x is the axial distance measured along the beam.

The solution of Eqn. (B.1) is assumed to be in the form

$$u = f(x) . \text{Sin } (wt + \alpha) , \quad (B.2)$$

where f(x) is amplitude function, w is angular frequency and is phase angle. If the trial solution of Eqn. (B.2) is substituted in Eqn. (B.1), one gets

$$\frac{\partial^2 f}{\partial^2 x} + \frac{w^2}{c^2} f = 0 . \quad (B.3)$$

The solution of Eqn. (B.3) is

$$f = b_1 \text{ Sin } \frac{w}{c} x + b_2 \text{ Cos } \frac{w}{c} x , \quad (B.4)$$

where b_1 and b_2 are integration constants.

For a beam of length L which is clamped at the left end and free at the right, the boundry conditions take the forms

$$\text{at} \quad x=0 \quad , \quad u=0$$

$$\text{at} \quad x=L \quad , \quad \frac{\partial u}{\partial x}=0 \quad , \qquad \text{(B.5)}$$

if the origin of x axis coincides with the left end.

When the solution Eqn. (B.4) is substituted in boundary conditions, for having a non-trivial solution, one gets

$$\text{Cos} \frac{wL}{c} = 0 \quad , \qquad \text{(B 6)}$$

which yields

$$\frac{wL}{c} = \frac{2n+1}{2} \pi \quad . \qquad \text{(B.7)}$$

For the lowest mode (n=0)

$$k = \frac{w}{c} = \frac{\pi}{2L} \quad , \qquad \text{(B.8)}$$

can be obtained. In Eqn. (B.8), k is the wave number which is related to the wave length λ by

$$\lambda = \frac{2\pi}{k} \quad . \qquad \text{(B.9)}$$

From Eqns. (B.8) and (B.9), it follows that in the lowest mode of the longitudinal vibration, the length of the beam is equal to one quarter of the wave length, i.e,

$$L = \frac{\lambda}{4} \quad \text{(for the lowest mode).} \tag{B.10}$$

On the other hand, the bar velocity is related to the frequency f and wave length by

$$c = \frac{E}{\rho} = f.\ \lambda \quad . \tag{B.11}$$

From Eqns. (B.10) and (B.11), it follows that

$$E = 16\ \rho\ f^2\ L^2 \tag{B.12}$$

Young's modulus may be computed if the mass density and length of the bar, are known and the frequency f corresponding to the first mode is measured.

```
APPENDIX C
MAIN PROGRAM
PROGRAM:ELASTIC COOFICIENTS OF ANIMAL BONE
SUPERVISED BY: ASST. PROF. DR. YALCIN MENGI
PREPARED BY: ERK INGER
MECHANICAL ENGINEERING DEPARTMENT
M.E.T.U.
PART 1: TENSION TEST AND HYDROSTATIC TEST RESULTS
PART 2: TORSION TEST RESULTS


*****************************************************************************
   S Y M B O L S :
UNITS OF ELONGATION:(MICRO INCHES PER INCHES)
TET=CIRCUMFERENTIAL ELONGATION IN TENSION TEST
TEZ=AXIAL ELONGATION IN TENSION TEST
TER=RADIAL ELONGATION IN TENSION TEST
HEZ=AXIAL ELONGATION IN HYDROSTATIC PRESSURE TEST
HER=RADIAL ELONGATION IN HYDROSTATIC PRESSURE TEST
SIG=NORMAL STRESS APPLIED IN TENSION TEST(KG/CM**2)
PP=APPLIED HYDROSTATIC PRESSURE (P.S.I.)
P=APPLIED HYDROSTATIC PRESSURE (KG/CM**2)
TORST=TORSIONAL ELONGATION
TORF=APPLIED TORQUES IN TORSION TEST (KG.CM)
TORSG=TORSIONAL SHEAR STRESS(KG/CM**2)
F=APPLIED NORMAL FORCE IN TENSION TEST (LB)
E=DIAMETER OF BONE SPECIMEN(CM)
PI=CONVERSION FACTOR OF INCH INTO CM.
PL=CONVERSION FACTOR OF POUND INTO KG.
DD=CONVERSION FACTOR OF KG/CM**2 INTO DYNE/CM**2
PZT=POISONS RATIO IN ZET-TETA DIRECTION
PZR=POISONS RATIO IN AR-ZET DIRECTION
DEZ=MODULUS OF ELASTICITY IN ZET DIRECTION(DYNE/CM**2)
PRT=POISONS RATIO IN AR-TETA DIRECTION
PRZ=POISONS RATIO IN AR ZET DIRECTION
DER=MODULUS OF ELASTICTY IN AR DIRECTION(DYNE/CM**2)
*****************************************************************************
 DIMENSION TET(20),TEZ(20),HET(20),HEZ(20),SIG(20),P(20),A(20,4,4)
*Y(20,4),B(20,4,4),C(20,4,4),D(20,4),AA(4,4),YY(4),R(4,1),X(4),
*TORST(20),TORF(20),TORSG(20),TER(20),HER(20),PP(20),F(20)
 N=20
 READ(5,91) (TET(I),I=1,N)
 READ(5,91) (TEZ(I),I=1,N)
 READ(5,91) (HET(I),I=1,N)
 READ(5,91) (HEZ(I),I=1,N)
 READ(5,91) (F(I),I=1,N)
 READ(5,91)(PP(I),I=1,N)
 READ(5,91) (TORST(I),I=1,N)
 READ(5,91) (TORF (I),I=1,N)
 E=0.6
 DD=7.5
 DO 21 I=1,N
 TORSG(I)=16.*TORF(I)/(3.14*E**3)
21 CONTINUE
 PL=0.452
 PI=2.54
 CP=PL/(PI**2)
 DO 26 I=1,N
 SIG(I)=(4.*F(I)*PL)/(3.14*E**2)
 P(I)=PP(I)*CP
```

```
20 CONTINUE
   DO 2 M=1,N
   DO 33 I=1,4
   DO 33 J=1,4
33 A(M,I,J)=0.0
   A(M,1,1)=TET(M)
   A(M,1,2)=TEZ(M)
   A(M,2,2)=TET(M)
   A(M,2,3)=TEZ(M)
   A(M,3,1)=HET(M)
   A(M,3,2)=HEZ(M)
   A(M,3,4)=-P(M)
   A(M,4,2)=HET(M)
   A(M,4,3)=HEZ(M)
   A(M,4,4)=P(M)*TET(M)/TEZ(M)
 2 CONTINUE
   DO 4M=1,N
   Y(M,1)=0.0
   Y(M,2)=SIG(M)
   Y(M,3)=-P(M)
   Y(M,4)=-P(M)*(1.+TET(M)/TEZ(M))
 4 CONTINUE
   DO 5M=1,N
   DO 6I=1,4
   DO 6J=1,4
 6 B(M,I,J)=A(M,J,I)
 5 CONTINUE
   DO 7 M=1,N
   DO 8I=1,4
   DO 8 J=1,4
   C(M,I,J)=0.0
   DO 9K=1,4
 9 C(M,I,J)=C(M,I,J)+B(M,I,K)*A(M,K,J)
 8 CONTINUE
 7 CONTINUE
   DO 10 M=1,N
   DO11 I=1,4
   D(M,I)=0.0
   DO 12 K=1,4
12 D(M,I)=D(M,I)+B(M,I,K)*Y(M,K)
11 CONTINUE
10 CONTINUE
   DO 13 I=1,4
   DO 13J=1,4
   AA(I,J)=0.0
   DO 14 M=1,N
14 AA(I,J)=AA(I,J)+C(M,I,J)
13 CONTINUE
   DO 15 I=1,4
   YY(I)=0.0
   DO 16 M=1,N
16 YY(I)=YY(I)+D(M,I)
15 CONTINUE
   DO 17 I=1,4
17 R(I,1)=YY(I)
   EPS=1.E-6
   CALL GELG(R,AA,4,1,EPS,IER)
   DO 18 I=1,4
18 X(I)=R(I,1)
   C11=X(1)/(1.-X(4)**[illegible]
```

```
      C13=X(2)/(1.-X(4))
      C12=C11*X(4)
      C33=X(3)+(C13**2)/C11
      DO 23 I=1,N
      TER(I)=-1./C11*(C12*TET(I)+C13*TEZ(I))
      HER(I)=-1./C11*(C12*HET(I)+C13*HEZ(I)+P(I))
   23 CONTINUE
      SUMA=0.0
      SUMB=0.0
      DO 22 I=1,N
      SUMA=SUMA+TORST(I)**2
      SUMB=SUMB+TORST(I)*TORSG(I)
   22 CONTINUE
      C44=0.5*SUMB/SUMA
      DD=981000
      DC11=DD*C11
      DC12=DD*C12
      DC13=DD*C13
      DC33=DD*C33
      DC44=DD*C44
      PZT=C13/(C12+C11)
      PZR=C13/(C12+C11)
      DEZ=(C33-2.*C13**2/(C12+C11))*DD
      PRT=(C33*C12-C13**2)/(C11*C33-C13**2)
      PRZ=(C13*(C12-C11))/(C13**2-C33*C11)
      DER=(((C11-C12)*(C33*C11+C33*C12-2.*C13**2))/(C11*C33-C13**2))*DD
      WRITE(6,41)(TET(I),I=1,N)
      WRITE(6,42)(TEZ(I),I=1,N)
      WRITE(6,45)(HET(I),I=1,N)
      WRITE(6,46)(HEZ(I),I=1,N)
      WRITE(6,47)(F(I),I=1,N)
      WRITE(6,48)(SIG(I),I=1,N)
      WRITE(6,49)(PP(I),I=1,N)
      WRITE(6,50)(P(I),I=1,N)
      WRITE(6,51)(TORST(I),I=1,N)
      WRITE(6,52)(TORF(I),I=1,N)
      WRITE(6,53)(TORSG(I),I=1,N)
      WRITE(6,20) (X(I),I=1,4)
      WRITE(6,43) C11,C12,C13,C33
      WRITE(6,24) (TER(I),I=1,N)
      WRITE(6,25) (HER(I),I=1,N)
      WRITE(6,44) C44
      WRITE(6,27)DC11,DC12,DC13,DC33,DC44
      WRITE(6,54)PZT,PZR,DEZ,PRT,PRZ,DER
   91 FORMAT(5F10.6)
   41 FORMAT(4X,'TET(I):',4X,5(E15.6,3X))
   42 FORMAT(4X,'TEZ(I):',4X,5(E15.6,3X))
   45 FORMAT(4X,'HET(I):',4X,5(E15.6,3X))
   46 FORMAT(4X,'HEZ(I):',4X,5(E15.6,3X))
   47 FORMAT(6X,'F(I):',4X,5(E15.6,3X))
   48 FORMAT(4X,'SIG(I):',4X,5(E15.6,3X))
   49 FORMAT(5X,'PP(I):',4X,5(E15.6,3X))
   50 FORMAT(6X,'P(I):',4X,5(E15.6,3X))
   51 FORMAT(2X,'TORST(I):',4X,5(E15.6,3X))
   52 FORMAT(3X,'TORF(I):',4X,5(E15.6,3X))
   53 FORMAT(2X,'TORSG(I):',4X,5(E15.6,3X)),//,5X,'A=',E10.3,2X,
   20 FORMAT(///,6X,' R E S U L T S :','X=',E10.3)
     *'B=',E10.3,2X,'C=',E10.3,2X,'C12=',E10.3,2X,'C13=',E10.3,
   43 FORMAT(//,6X,'C11=',E10.3,2X,'C12=',
     *2X,'C33=',E10.3)
   24 FORMAT(///,6X,'RADIAL ELONGATION IN TENSION TEST:',//,5(
     *6X,E10.3,2X))
   25 FORMAT(///,6X,'RADIAL ELONGATION IN HYDROSTATIC TEST:',
     *//,5(6X,E10.3,2X))
   44 FORMAT(//,6X,'C44=',E10.3)
   27 FORMAT(///,6X,'DC11=',E10.3,2X,'DC12=',E10.3,2X,'DC13=',
     *'DC33=',E10.3,2X,'DC44=',E10.3)
   54 FORMAT(///,6X,'PZT=',E10.3,2X,'PZR=',E10.3,2X,'DEZ=',
     *E10.3,2X,'PRT=',E10.3,2X,'PRZ=',E10.3,2X,'DER=',E10.3)
      STOP
```