EXAMINING 10TH GRADE STUDENTS' PROBLEM-SOLVING PROCESSES IN GEOMETRY USING EYE TRACKING TECHNOLOGY

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submitted by **EMİNE MALCI** in partial fulfillment of the requirements for the degree of **Master of Science in Computer Education and Instructional Technology, Middle East Technical University** by,

Prof. Dr. Halil Kalıpçılar
Dean, Graduate School of **Natural and Applied Sciences** _________________

Dr. Hasan Karaaslan
Head of the Department, **Comp. Edu. and Inst. Tech.** _________________

Assoc. Prof. Dr. Tuğba Tokel
Supervisor, **Comp. Edu. and Inst. Tech., METU** _________________

Examining Committee Members:

Prof. Dr. Yüksel Göktaş
Comp. Edu. and Inst. Tech., Atatürk University _________________

Assoc. Prof. Dr. Tuğba Tokel
Comp. Edu. and Inst. Tech., METU _________________

Prof. Dr. Kürşat Çağıltay
Comp. Edu. and Inst. Tech., METU _________________

Date: 15.02.2021

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name Last name : Emine Malcı

Signature :

ABSTRACT

EXAMINING 10TH GRADE STUDENTS' PROBLEM SOLVING PROCESSES IN GEOMETRY USING EYE TRACKING TECHNOLOGY

Malcı, Emine
Master of Science, Computer Education and Instructional Technology
Supervisor : Assoc. Prof. Dr. Tuğba Tokel

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This study examines the problem-solving strategies of 10th-grade high school students and changes in their eye movements during problem-solving in geometry. Eight 10th-grade students participated in this study. Spatial Ability Test (SAT) was applied to evaluate participants' spatial abilities. Participants were asked to solve six geometry problems on the computer and think aloud while solving problems. During problem-solving, participants' eye movements were recorded by an eye tracker. After the problem-solving process was completed, an interview session was conducted by each participant. The findings showed that participants used different strategies categorized as holistic and analytic in different problems. Moreover, the sub-categories of holistic and analytic strategies were specified. The findings demonstrated how the visual attention of participants was distributed across problems. This study integrating eye-tracking technology, think-aloud, and interview methods revealed detailed information about participants' problem-solving processes.

Keywords: Problem-Solving, Geometry, Eye Tracking Technology

ÖZ

10. SINIF ÖĞRENCİLERİNİN GEOMETRİ PROBLEM ÇÖZME SÜREÇLERİNİN GÖZ İZLEME TEKNOLOJİSİ İLE İNCELENMESİ

Malcı, Emine
Yüksek Lisans, Bilgisayar ve Öğretim Teknolojileri Eğitimi
Tez Yöneticisi: Doç. Dr. Tuğba Tokel

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Bu çalışmanın amacı onuncu sınıfa devam eden lise öğrencilerinin geometri problemleri çözerken kullandıkları stratejileri ve göz hareketlerinde meydana gelen değişimleri araştırmaktadır. Çalışmaya sekiz onuncu sınıf öğrencisi katılmıştır. Katılımcıların uzamsal yeteneklerini değerlendirmek için Uzamsal Yetenek Testi (SAT) uygulanmıştır. Katılımcılardan bilgisayar ekranına yansıtılan altı geometri sorusuna yanıt vermeleri ve problemleri çözerken sesli düşünmeleri istenilmiştir. Problem çözümü esnasında katılımcıların göz hareketleri göz izleme cihazı ile kaydedilmiştir. Problem çözümü sonrasında katılımcılar ile görüşmeler yapılmıştır. Çalışmanın bulguları katılımcıların farklı problemlerde farklı stratejiler kullandıklarını ve bu stratejilerin bütünsel ve analitik olarak kategorize edilebileceğini göstermiştir. Ayrıca, alt kategoriler tanımlanmıştır. Bulgular katılımcıların görsel dikkatlerinin problemler üzerinde nasıl dağıldığını göstermektedir. Bu çalışmada göz izleme verileri sesli düşünme ve görüşme yöntemleriyle desteklenerek katılımcıların problem çözme süreçleri ile ilgili detaylı bilgiler verilmektedir.

Anahtar Kelimeler: Problem Çözme, Geometri, Göz İzleme Teknolojisi

To My Family who have believed in me and supported me throughout every journey of my life

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ABBREVIATIONS

CCT : Cube Comparison Test

CRT : Card Rotation Test

NCTM : National Council of Teacher of Mathematics

PFT : Paper Folding Test

SAT : Spatial Ability Test

SDT : Surface Development Test

CHAPTER 1

INTRODUCTION

This chapter presents an introduction to the study, and includes the background of the study, the purpose of the study, the significance of the study, the study's research questions, as well as the assumptions, limitations, delimitations, and definitions of the terms used throughout the study.

1.1 Background of the Study

Problem solving consists of actions performed for a goal (Anderson, 1985). This ability forms the core of mathematics education since it reflects the ability to cope with problems both within and outside of the school environment (National Council for Teachers of Mathematics [NCTM], 2000; Simsek et al., 2020). The actions taken to resolve problems are related to cognitive processes (Metallidou, 2009), and researchers have proposed various explanations for these processes and stages of problem solving. Lin and Lin (2014a) listed these processes that a problem solver needs to follow as; comprehending the initial text and diagram, searching for information that overlaps with existing knowledge, making deductions, and performing calculations in order to reach a solution. Correspondingly, Gal and Linchevski (2010) indicated that geometry tasks include consecutive phases from neural processes through higher cognitive processes such as reasoning and deduction. Epelboim and Suppes (2001) stated these stages respectively as reading the text, creating a diagram, looking for familiar patterns and recalling relevant information, and reaching a solution. Students might experience difficulties whilst activating the required cognitive processes when problem solving. Representations such as diagrams, graphs, tables, and charts are frequently used in order to assist students in understanding mathematical ideas within the problem-solving context

(Bolden et al., 2015). It has been shown that diagrams presented in geometry problems aid students' understanding of mathematical concepts and problem statements (Brenner et al., 1997), and in leading them towards a viable a solution (Greeno & Hall, 1997). They play a role in representing and explaining the mathematical ideas required to solve problems. Students should, therefore, be able to gather the necessary properties and relevant information from a diagram and to utilize them whilst solving problems of geometry (Laborde, 2005). However, it is uncertain whether or not they are able to obtain the necessary knowledge (Pape & Tchoshanov, 2001), and might face difficulties in reaching the correct inferences during this process. In this regard, strategies appear as methods for performing cognitive tasks in order to facilitate this process (Saczynski et al., 2002). However, they vary depending on the characteristics of the task in hand (Gluck & Fitting, 2003) and thereby include different descriptions and classifications. Mayer (1983) defined a problem-solving strategy as a guideline that does not always lead to a solution (as cited in Gick, 1986, p. 100). Dividing problems into parts is a common technique used in problem solving, regardless of the domain. Newell and Simon (1972) used means-ends analysis as a general strategy, which includes filling the gap between the initial state and the aim of the problem. According to Norqvist et al. (2019), the use of algorithmic and creative reasoning strategies affects learning outcomes. Algorithmic strategies were defined as applying algorithms and rules during problem-solving, while creative strategies were identified as generating original solutions to tasks that were rarely seen in textbooks. Boulter and Kirby (1994) characterized strategies used to identify and illustrate geometric figures' transformations as holistic and analytic. According to their definition, the analytic strategy focuses on figures with a list of features, whereas the holistic strategy targets figures as objects. Gluck and Fitting (2003) also classified strategies as a continuum including holistic and analytic strategies. While they defined holistic strategies as imagining a whole figure's rotation, analytic strategies do not mention any visualized manipulation of the figure.

The processes of problem solving, including selecting and implementing a strategy have the potential to reveal information about a student's views regarding the context, difficulties, understanding, thinking methods, and strategies (Lin & Lin, 2014a; Simsek et al., 2020). In this regard, assessing and evaluating problem-solving processes can be highly remarkable. However, it is critical to select the appropriate assessment tools in order to reveal this information. Prior studies have frequently employed self-reports and interview methods in the assessment of problem-solving processes and have analyzed strategies that students used during these processes (Huang, 2017). Studies have also considered solution time and accuracy as problem-solving measures which generate weak information about the internal cognitive process (Knoblich et al., 2001).

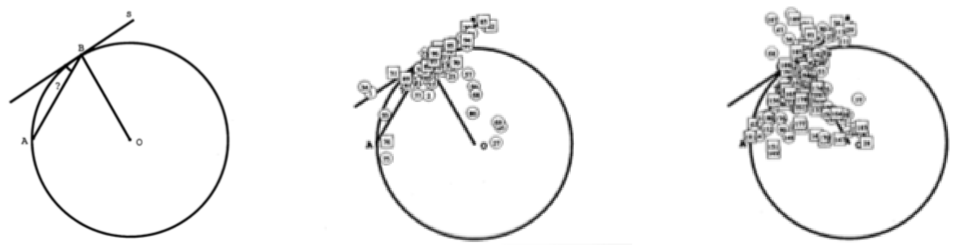
However, whilst the number of studies concerning the cognitive processes of problem solving is quite remarkable, the design of the studies makes a considerable difference. Lin and Lin (2018) highlighted an obstacle related to assessing students' performance; that, although test-oriented strategies may have assessed students' performance during problem solving for decades, they seldom reveal the difficulties experienced by the students. Therefore, teachers have not been able to ensure adequate assistance is provided to students in need of further assistance. More influential techniques need to be used in order to gain insights into problem-solving processes so as to provide students with a more effective level of assistance. Moreover, Simon and Schindler (2020) stated that it is essential to maintain a focus on students' processes and strategies that cab lead them towards a solution, instead of only considering their solutions and outcomes. In this regard, the interview procedure and the think-aloud protocol are considered as the most significant and frequently used techniques in examining all the disparate parts of cognitive activities (Charters, 2003; Mintzes et al., 1999).

Eye-tracking technology can also be a potential source of data related to students' attention focus and the aspects of a problem that the students focus the most upon and face the most difficulties with whilst interpreting (Bolden et al., 2015; Susac et al., 2014). This technology provides the means to reach the mind where problems are presented visually because eye movements form an inseparable part of these processes (Epelboim & Suppes, 2001). Many scholars agree that eye tracking provides fast, objective, direct, and reliable data about the cognitive processes of problem solving (Epelboim & Suppes, 2001; Huang, 2017; Lai et al., 2013; Susac et al., 2014). However, explanations based on this data might differ according to the context of the study. As an example, Duchowski (2007) specified that as the complexity of a problem increases, the fixation number and duration increase in that vein. However, problem difficulty may not be the reason for higher duration in all conditions. In a study conducted by Lin and Lin (2014b), problem solvers who were skilled in geometry spent more time in an output area since they sought more plausible answers. Moreover, students may also fixate more due to boredom whilst looking at a problem (Schindler & Lilienthal, 2019). Therefore, it is important to investigate eye movement changes and to target reaching specific explanations for these changes.

Eye tracking is quite popular in mathematics education, and also in its subdomains. Many research studies have aimed at revealing students' problem-solving strategies in mathematics through the use of eye-tracking method. These studies have included investigations of students' problem solving strategies in transformational geometry (Boulter & Kirby, 1994), complex addition (Green et al., 2007), mental subtraction (Huebner & LeFevre, 2018), ratios (Merkley & Ansari, 2010), fractions (Ischebeck et al., 2016; Miller Singley & Bunge, 2018; Obersteiner & Tumpek, 2016), simple equations (Susac et al., 2014), computation estimation (Ganor-Stern & Weiss, 2016), addition (Green et al., 2007), and function representations (Andrà et al., 2009). Moreover, studies have the potential to extract specific inferences based on eye-tracking data. Galili et al. (2020) stated that participants focus more on the points of shapes that make the shapes different from the others and the locations that require mental operations. Schindler and Lilienthal (2018) explained longer durations by using various strategies, the same strategies more than one, and strategies requiring more time. Schindler and Lilienthal (2019) also associated eye movement measures of the participant with double-checking, the use of symmetry, and mentally drawing a line in another study. Even though the literature is extensive in terms of research studies that have focused on problem-solving strategies in mathematics, students' solution processes still require further investigation within the context of geometry through the utilization of eye movement measures. Problem-solving processes also need to be investigated since there is a potential to reveal specific explanations to eye movements based on the studies' problems and designs.

As previously mentioned, eye-tracking technology is seen as highly applicable to the educational context. The study conducted by Epelboim and Suppes (2001) may be considered as a starting point for the current study. As can be seen in Table 1.1., the task requires the problem solver to find the missing angle. As shown in their study with eye movement patterns of both a novice and an expert, the expert was able to draw an imaginary triangle which allows for the use of angle property of an equilateral triangle and finally leads the problem solver to the solution. The patterns indicated that the novice problem solver reviewed the diagram and focused on the tangent line; however, they were unable to find any information related to the missing angle. This opportunity to find fixation patterns and examine the explanations behind the patterns allows researchers to obtain valuable information about how students think during problem solving in geometry. This data can then be useful in structuring geometry instruction in order to provide more appropriate and valuable assistance to students.

Table 1.1. Experiment conducted by Epelboim and Suppes (2001)



a. the problem	b. fixations of a novice	c. fixations of an expert

1.2 Purpose of the Study

The main purpose of the current study is to examine the processes used in solving geometry problems presented as diagrams with the help of eye tracking techniques. This study investigated how eye-tracking measures differ according to variations in problems and parts of a problem. It also aims to explain how and why such changes occur. Eye-tracking data is compared with Spatial Ability Test (SAT) so as to assess participants' eye movements and supported by think-aloud protocol and interviews to determine the study participants' problem-solving strategies of geometry. The study aims to provide an overall analysis of the participants' problem-solving processes.

1.3 Significance of the Study

There are many research studies to be found in the literature on problem solving. The methods used by these various studies to evaluate problem solving have been based on interviews, observations, and also surveys. In contrast, eye-tracking technology can be implemented in addition to these methods since it offers reliable, fast, and objective data about cognitive processes (Hu et al., 2017). In addition to evaluation methods, measures are also crucial in problem-solving studies. As Acartürk and

Habel (2012) stated, online comprehension measures depending on psychophysical aspects such as eye tracking provide more comprehensive and richer data than offline measures such as response time and accuracy. Within this context, the current study integrated eye-tracking technology along with interview and think-aloud methods in order to obtain robust data about problem-solving processes.

The Spatial Ability Test (SAT) was used to determine the participants students' spatial abilities. The results of this test were then correlated with other data sources in order to examine how this ability affects students' problem-solving processes in geometry. On the other hand, the strategy classification and other findings related to problem-solving processes might help educators understand students' behavior, and eventually to support instructional methods. Therefore, the current study might considerably contribute to geometry problem-solving research studies since it targets the detailed investigation of overall problem-solving processes.

In the case of investigating problem solving, there may be the possibility of simplifying the output phase of the problem-solving process by utilizing multiple-choice questions. In the current study, students answered open-ended geometry problems. A blank sheet was also provided to each student for their calculations. As such, the writing phase of the problem-solving processes was aimed to be highlighted.

1.4 Research Questions

In this study, the following research questions were targeted in order to investigate the problem-solving strategies and eye movement measures according to different problems, and with each problem having two distinct parts.

1. What are the problem-solving strategies that 10th-grade students use during problem solving in geometry?

2. How do eye movement measures change during problem-solving processes at different problems and different parts of the problems?

1.5 Assumptions

For this study, the following assumptions were made:

1. Participants voluntarily took part in the study.
2. Participants fully completed their problem-solving session.
3. Participants answered the interview questions honestly and accurately.
4. The eye tracking data were accurately recorded and analyzed.
5. The Spatial Ability Test was applied and analyzed accurately.
6. The measurements of the instruments were reliable and valid.

1.6 Limitations

The current study was limited to one city in Turkey, and only high school students who volunteered to participate in the study were targeted. The eye tracker used may have affected the participant students' actual performances since they were aware that they were being recorded.

1.7 Delimitations

The current study was restricted to eight high school students from Ankara, Turkey. The study focused on problem-solving strategies and eye movement variations during problem solving in geometry. Only 10th-grade students were included in the study. The data from each participant were collected from a single session, which lasted for approximately 1 hour.

1.8 Definitions of Terms

Eye tracking: a technique which records people's eye movements and determines their eye-fixation patterns.

Fixations: "eye movements that stabilize the retina over a stationary object of interest" (Duchowski, 2007, p. 46).

Total Fixation Duration: total amount of time that a participant spends looking at a certain area.

Area of Interest (AOI): an area of a screen or visual field arranged by researchers based on the participants' interest (Jacob & Karn, 2003).

1.9 Summary

In Chapter One, the importance of problem-solving processes and eye-tracking technology were discussed. Next, what the current study aimed to achieve and why it is deemed of importance were discussed. Then, the study's research questions, assumptions, limitations, and delimitations were stated before lastly, defining the terms used throughout the study. Chapter Two presents a review of problem solving in geometry and eye tracking technology literature.

CHAPTER 2

LITERATURE REVIEW

Problem-solving ability is highly valued and as Anderson (1985) stated, it involves a series of cognitive operations performed for a goal. Within this activity, solvers mentally create representations of a problem as well as its context, and then actively manipulate and test models of these representations. On the other hand, eye tracking is regarded as a technique which records people’s eye movements and determines their eye patterns. It is used in research studies to obtain attentional resources during reasoning, decision making, and other aspects of problem-solving processes. Main eye movements, fixations, and saccades are recorded in order to explore the relationship between eye movements and cognitive processes.

The purpose of this chapter is to review the literature investigating eye movements and processes during problem-solving under five main sections. In the first section, the nature of mathematical knowledge is briefly reviewed in order to make sense of mathematical comprehension and problem-solving. In the second section, spatial ability, its components, and research studies are stated. In the third section, problem-solving processes and problem-solving strategies are presented. In the fourth section, eye tracking technology is presented. In the fifth section, studies are reviewed that utilized eye-tracking technology to gain insights regarding problem-solving in geometry. Lastly, the chapter concludes with a summary and implications.

2.1 Mathematical Comprehension

From an epistemological perspective, the character of mathematical knowledge forms the means of accessing knowledge. Mathematical objects cannot be obtained or perceived without the use of signs and semiotic representations unlike other domains such as geology and astronomy (Duval, 1999). Therefore, the use of semiotic systems like drawings, graphs, sentences, and diagrams either mentally or externally is essential in mathematical activity. Representations help individuals to solve problems more easily (Epelboim & Suppes, 2001). However, the domain of mathematics includes many different kinds of representations, and various combinations and transformations may be needed in order to solve different tasks and problem statements (Duval, 2006). For example, one problem context might require only figural conversion; whereas students, may need to convert one representation to another in a different situation, such as converting an equation to a graph.

In addition to representations, visualization is highly significant to the comprehension of geometry. Visualization makes mathematical objects visible by demonstrating relations, and in helping students to understand these relations and objects which are not attainable from vision alone (Duval, 1999). This requires to realize and comprehend what is intended to be represented, as simply looking at it is deemed insufficient. Gal and Linchevski (2010) explained that the visual process includes obtaining perceptual data from what happens and exists around us from our sensory system, processing complex data, and the brain's period of exposition.

2.2 Spatial Abilities

Spatial ability is regarded as a necessary ability for individuals to understand daily life, and is a characteristic of being a professional in domains such as technology, mathematics, engineering, and science (Wai et al., 2009). Spatial ability can be considered as complementary to the abilities to rotating, altering, binding, and reversing a figure in the mind, according to McGee (1979).

Of this ability, spatial visualization and relations were specified as the two major components (Clements & Battista, 1992; McGee, 1979). To obtain mathematical knowledge and objects, representations such as diagrams, tables, spatial figures, and symbols are frequently employed; therefore, spatial visualization is a fundamental ability in the learning of mathematics (Presmeg, 2006). Improving spatial ability is regarded as one of the essential targets of mathematics and geometry curricula. In this regard, gathering data about students' difficulties in spatial problem-solving processes is deemed to be of significant value to developing these abilities effectively (Kayhan, 2012).

It should be stated that numerous geometry problems are presented as diagrams. Since the solving geometry tasks is a "seeing" process as Gal and Linchevski (2010) specifically described, having effective spatial abilities is therefore essential in this sense. Numerous research studies have been conducted that have shown the relationship between spatial abilities and mathematics. The longitudinal study conducted by Sherman (1979) concluded that spatial visualization significantly predicts geometry performance and mathematical problem solving. Correspondingly, Del Grande (1990) stated that spatial abilities are necessary to understand geometry and to become an expert in that field. Del Grande claimed that disregarding these abilities and focusing instead on deductive states of a subject are among the reasons why geometry is considered by students to be so difficult.

Cheng and Mix (2014) conducted a study with young children to investigate the effects of spatial training on mathematics performance. By using mental rotation task, they concluded that children who received spatial ability training showed a significant improvement in missing-term problems among three problem types; number fact problems, multi-digit calculation, and missing-terms problems. On the contrary, there was no improvement seen in the control group for any of three problem types.

In light of these studies, it may be summarized that spatial ability is one of the factors which has an influence on mathematical problem solving. These abilities may play a role in the diagramming of mathematical problem statements prior to solving it.

Spatial thinking can also be helpful in manipulating mathematical objects and generating and using diagrams flexibly. Students can arrange and transform mathematical elements easily such as manipulating terms through replacement in equations. Therefore, spatial ability should be taken into consideration while examining problem-solving processes in geometry.

2.3 Problem-Solving in Geometry

As the National Council for Teachers of Mathematics (NCTM) stated, problem-solving is regarded as an inseparable aspect of mathematics learning (NCTM, 2000). Correspondingly, the Turkish Ministry of National Education (2018) added a goal to the mathematics curriculum of raising individuals who can express their thoughts and reasoning easily in the problem-solving process for special purposes. Problem-solving ability is included in the process standards which are mentioned in the Principles and Standards for School Mathematics presented by the NCTM (2000). This means that the ability is seen as playing a role in obtaining content knowledge and also in using it within learning processes. In order to find a solution to a problem, students activate many cognitive operations. Therefore, problem-solving processes help them to gain new perspectives, determination, and desire to explore unfamiliar conditions, etc. On the other hand, geometry takes place in the content standards which refers to the content that students learn. Geometric thought is essential to representing and solving problems as geometry is a field which supports proving, reasoning, visualizing, and justifying.

Due to the high cognitive load, solving geometry problems is accepted as a challenging activity, hence learners do not always exactly understand the problem statement or its concepts (Wong et al., 2007). It involves cognitive processes that students produce, but which vary in their responses depending on the problem expression (Simon, 1978). Gal and Linchevski (2010) stated that every geometry task includes visualizing the process whether it is explicit or implicit. This process is regarded as a combination of successive phases, beginning with the physical-neural through to higher cognitive processes such as reasoning, conceptual thinking, deduction, and conclusion. Epelboim and Suppes (2001) indicated that through these phases, students read the text, generate a diagram when nonexistent, search for familiar patterns, recall related information from memory, and draw a conclusion.

Geometry problems include an initial and a goal state. Problem-solvers must therefore understand the initial descriptions of the problem and read the diagram introduced with the problem in order to reach the goal state. In filling the gap between these two states, solvers employ several mathematical concepts that they have already learned. Familiar patterns, relevant connections, effective inferences, and numerical computations all lead solvers towards reaching a viable solution.

The importance of problem-solving ability in mathematics education has been acknowledged by the NCTM (2000) and also by the Turkish Ministry of National Education (2018). In order to improve upon this skillset, the examination of students' strategies and approaches to problems is considered essential so as to better understand their ideas, reasoning process, and the difficulties that they may face during problem solving. This provides teachers and educationalists with the opportunity to design instructions and materials in a way that takes into account this knowledge. For this reason, in the current study, research studies including general and mostly domain-specific problem-solving strategies were reviewed.

Simon and Newell (1971) mentioned a general strategy, namely means-ends analysis, which is a goal-based strategy that focuses on reducing the difference between the actual state and the goal state. As it is a non-task-dependent strategy, it can be applied in a variety of domains. Gick (1986) made a distinction between schema-driven and search-based problem-solving strategies. Problem-solving strategies including schema activation are regarded as schema-driven. However, if there is no activated schema for a particular problem, the solver needs to search and gather information in order to eventually reach a pattern. In this case, the solver uses search-based problem-solving strategies. The researcher also highlighted the difference among these two strategies that since search-based strategies require a search prior to the solution phase, schema-driven strategies directly elicit a strategy-based implementation and solution process.

Concerning the problem-solving procedure, Kuzle (2013) compiled the studies of Watson and Rayner (1920/2013) and Schoenfeld (1981) in order to present a framework. This framework consists of actions, whereby solvers read the problem, understand the problem, notice requirements, find divergent possibilities, schedule the best solution, put the solution into practice, and validate the answer by establishing connections with all the actions. This compilation also specifies the recurrent, cyclic, and dynamic characteristics of these processes.

Specific to the geometry domain, Duval (1999) stated three kinds of operations which allow for the manipulation of a geometrical figure without changes to the features of the original figure. These operations can be used as solving strategies in geometry in order to make understanding problem and interpreting representation that much easier. The first operation is mereologic way. It depends on breaking the figure into parts, or creating another figure through the combination of these parts. For example, a parallelogram can be divided into triangles. The second operation is optic way. It allows problem solvers to change the size of a shape without altering its initial properties. The shape can appear larger or narrower. The third operation is place way; whereby, there is an opportunity to replace the figure in the plane. It is regarded as the smallest change. Together, these three operations may provide insight to the solution of a problem.

Battista (1990) aimed to determine geometric problem-solving strategies used by high school students. The participant students were asked to solve a problem and specify solving strategies either by choosing from those listed or by describing their own strategies. The listed strategies were generated according to whether the students employed visualization or nonspatial strategies. By applying spatial visualization, logical reasoning and geometry achievement tests in addition to geometric problem-solving/strategies test, Battista concluded that if a student has a high spatial ability and low verbal-logical ability, they may often be inclined to employ spatial strategies. They may tend to use a variety of strategies where there is no discrepancy among these abilities.

Boulter and Kirby (1994) conducted a study with middle school students in order to investigate whether students' problem-solving strategies could be described as holistic or analytic. They regarded these two strategies as broad categories in strategy classification. Holistic strategies refer to when students do not divide a question into pieces, but instead approach a problem as a whole. On the other hand, in the analytic strategy, students focus on the properties or parts of the problem. As an example, in the current study, the strategy can be regarded as holistic if students manipulate a complete triangle. However, students should transform the parts of the triangle such as its sides and vertices in order to consider the strategy as analytic in the case of transformation.

Besides the strategy classifications that are used in either general research studies or geometry-specific studies, proportional reasoning strategies should also be mentioned because there are certain problems relative also to geometry tasks. Fisher (1988) conducted a study with 20 secondary school mathematics teachers in order to investigate their solution strategies for proportion problems. A nine-category scheme was used to classify the strategies, and included two main categories, as incorrect and correct. No answer, intuitive, additive, proportion attempt and other were considered as the incorrect strategies, whilst roportion formula, proportional reasoning, algebra, and other were stated as being correct strategies. Under the incorrect category, if a problem solver does not reach an answer, their attempts can be placed within the "no answer" category. Intuitive implies strategies that were derived from a guess or some unreasonable computation. Additive targets strategies when a solver focuses on the difference between the quantities. Proportion attempt might reveal when a solver is aware of the proportion, but does not correctly state the relationship. Other category is valid for all incorrect solutions that cannot be linked to the aforementioned categories. Under the correct category, proportion formula appears when a solver uses direct proportion or inverse proportion formulae. When they do not use the formula, each correct proportion strategy is classified as proportional reasoning. Algebra requires a correct equation that implies proportion over formula. Similar to the incorrect category, the other category includes strategies which cannot be placed within the aforementioned categories.

Another study by Lamon (1993) targeted sixth-grade students who had not been taught ratio or proportion previous to the study in order to examine their informal strategies during problem solving. The researcher generated a sixth category for solutions that are not matching with the hierarchical arrangement from no correct solutions to at least one correct solution each type of problem. The researcher assigned strategies to two fundamental categories, which are nonconstructive and constructive strategies. Nonconstructive strategies consist of avoiding, visual or additive, and pattern building. Constructive strategies consist of preproportional reasoning, qualitative proportional reasoning, and quantitative proportional reasoning. Under the first main category, avoiding describes not forming an interaction with a problem. Visual or additive refers to making visual judgments, responding without causes, or using trial and error methods. Pattern building involves the use of patterns that are independent of numerical relations. Under the second main category, preproportional reasoning requires thinking relatively and intuitively and making sense. Qualitative proportional reasoning also indicates relative thinking and notice of ratio as a unit as well as some numerical relations, while quantitative proportional reasoning requires algebraic symbols to represent proportions with a complete understanding of relationships.

Through the studying of 21 students aged of 12-13 years old, Cox (2013) aimed to examine their strategies while scaling geometric figures and their reasonings behind these strategies by way of clinical interviews. In order to analyze the students' strategies, the researcher created seven categories for scaling geometric figures with the aid of Lamon's framework. These strategies are Avoidance, Additive, Functional Scaling, Visual, Unitizing, Pattern Building, and Blending. The descriptions of Avoidance, Additive, and Visual were indicated, as previously mentioned, by Lamon (1993). Functional Scaling implies using scale factor to find corresponding lengths through multiplication. Unitizing refers to considering a figure as a unit and extending it within some given condition. Pattern building appears when students have no understanding of the function of the scale factor and use verbal or written patterns instead. Lastly, blending includes both numeric and visual reasonings.

2.4 Eye Tracking Technology

Eye-tracking technology provides educational researchers with insights into critical aspects of problem-solving processes that are lacking in traditional measurement methods such as correctness or solving time (Knoblich et al., 2001). New advances in this technology helps researchers to gather data about learners' visual focus of attention, and upon what aspects they stay focused, which provides data that is considered more objective and has increased accuracy (Bolden et al., 2015). In order to understand human cognitive processes in a problem-solving context with eye tracking technology, what measures are actually available first require highlighting as well as the reasons as to why the recording of eye movements is considered important.

People often move their eyes in order to place the entity occupying their central direction of gaze in the greatest visual resolution area to see it more clearly (Bolden et al., 2015). They then pay increased attention to this entity in their sight so as to focus their concentration upon it. Therefore, tracking human eye movements provides a path as to where a person's attention is directed (Duchowski, 2007). This presents an opportunity to gain insight into what people find most interesting within their visual spectrum, to where their attention is drawn, and even how they perceive their views. Duchowski (2007) indicated that fixations, smooth pursuits, and saccades are three types of eye movements that are in need of elaboration. Fixation is defined as the relative stabilization of the human eye upon an object of interest, while saccade is the rapid movement of the eyes between two successive fixations. Similar to fixation, pursuits are regarded as the smooth motion of the eyes.

Eye-tracking technology may be considered a feasible way to analyze human cognitive, physiological reactions as an index of mental processes that can be converted to statistical data, thus can be analyzed quantifiably. Wang et al. (2014) stated that collecting and recording information about eye movement behaviors can provide an opportunity to directly observe a subject's cognitive processes through visualization. In this direction, fixation count, fixation duration, time spent on an area of interest (AOI), fixation count on AOI, and fixation rate are measures frequently used in research studies, and which provide crucial information about a subject's cognitive processing. Within this context, fixation count and dwell time are important factors in the area of problem-solving. Longer dwell time and higher fixation count are considered as students needing a greater cognitive capacity to process information on a specific area (Just & Carpenter, 1976). These measures may therefore provide clues about whether or not learners are focused on their studying or to where their attention has become focused. Associated with mathematics, the more complex aspects of problems can result in a greater number of and fixations of a longer duration (Duchowski, 2007). Similarly, the study conducted by Hegarty et al. (1992) found that students having low levels of accuracy fixated longer on problem statements than students with higher levels of accuracy. This technology has also been used to identify different eye movement profiles of experts and novices while solving the same mathematical problems (Andrà et al., 2009; Epelboim & Suppes, 2001).

2.5 Review of Eye Tracking Technology and Problem-Solving in Geometry Studies

Problem-solving activities in geometry is considered complex in terms of the required mental processing, and; therefore, require an influential diagnosis technique. Eye tracking is regarded as a beneficial channel for educational researchers to link learning outcomes to cognitive processes (Lai et al., 2013). By integrating eye-tracking technology to observe students' problem-solving processes, their visual attention can be recorded (Rayner, 1998). Moreover, students' eye movements have the potential to offer insight into the students' mind in the case of introducing mathematical problems as diagrams (Epelboim & Suppes, 2001). In this regard, several research studies benefiting from this technology have examined the problem-solving processes in the field of geometry.

Epelboim and Suppes (2001) conducted a study with the aim of examining eye movements and visual working memory during problem solving in geometry. Two experts and one non-expert answered problems which required them to find the value of an unknown angle on a diagram. The questions were presented on a high-quality LCD screen, and the participants were not allowed to write or draw anything, but were asked to think aloud instead. Eye-tracking records showed that the experts made many fixations on an imaginary triangle which was not completely shown in the diagram, but was assessed as important to being able to solve the problem. However, the non-expert neither referred to this triangle nor scanned it. It was stated that the determination of the triangle was more than a perceptual process. Indeed, it was considered as an indication of higher-level reasoning regarding the assessment of the the problem.

Wang et al. (2014) used eye-tracking technology to investigate students' spatial abilities. The eye tracking system employed in the study monitored participants' gazes and saccades, and eye fixations, saccades, and heat maps were three dimensions measured in the study. As in the previous study, the participants were not allowed to use a pen or paper to write their answers. As a result of this study, Wang et al. (2014) deduced two key factors that caused the participants to expertly select the correct answers that were specified according to measurements: “ability to fold and move a series of complicated images in their mind and the ability to rapidly rotate these images by comparing the position, proximity and distance of reference points” (p. 348). Spatial ability training was designed based on these factors. The findings of two spatial ability tests applied prior to and following the training stated that participants’ scores were higher in the second test than in the first. It may therefore be inferred that eye-tracking technology can reveal the prerequisite abilities necessary for solving problems, so that instructions can then be designed based according to these findings. With respect to spatial ability, Roach et al. (2016) examined the relationship between mental rotation ability (MRA) and eye movement indices as fixation duration, question, response time, and number of fixations. The results showed that there was no significant correlation between mental rotation ability and eye movement measurements.

In addition to prerequisite skills, eye-tracking technology may help to assess students’ difficulties during problem solving process and the effects that question formats may have on students on reaching a viable solution. Lin and Lin (2018) employed eye tracking and a handwriting device in order to investigate potential sources of difficulty whilst solving geometry problems. In their study, high school students were tasked with answering five geometry problems which required skills in applying similarity properties and the manipulation of figures. Using three eye-tracking measures; dwell time (DT), fixation count (FC), and run count (RC), the study’s results indicated that unsuccessful problem solvers had longer DT, higher FC, and higher RC than those who were successful. The findings also stated that when a pair of adjacent triangles were introduced in a similarity question, the unsuccessful solvers had frequent fixation and re-fixation on the known and unknown variables, and experienced difficulties whilst comprehending triangles.

Andrà et al. (2009) used eye tracking to evaluate the period of sense-making in mathematical representations during problem solving. In their study, participants were asked multiple-choice questions without paper and pencil when seated in front of an eye tracker. The study concluded that experts focused on the input more than novices. In other words, experts looked for hints to finding instead of searching among alternatives. The findings suggested that students need to learn how to read a task and to direct their attention to gather information related to the task. Regarding the exploration of students' interpretations of mathematical representations, in a study conducted by Bolden et al. (2015), participants were asked whether or not symbolic and picture representations matched given calculations. Quantitative data captured using an eye tracker and qualitative analysis of the accompanying video recordings concluded that time for the number lines, one of the mathematical representations, differed significantly from the groups and array representations. In other words, the participants found the number line representation more difficult to interpret, achieving a low average of correct answers. The researchers explained the reason behing this finding as many participants having paid attention only to the last number seen.

A recent article by Schindler and Lilienthal (2019) presents an example of research that has focused on describing eye movement patterns during problem-solving in geometry. They conducted their study with one participant in which geometry problems were given on paper and the participant was allowed to draw and write without restriction. Their findings indicated that the participant's eye movements substantially supported the eye-mind hypothesis, which implies a close link exists between what is fixated and what is processed (Just & Carpenter, 1980). This means that eye movement patterns reflect the subject's attention. The researchers revealed certain information regarding what the participant was drawn to with their eyes, where they focused, why they checked twice; and therefore, several mental processes were specified. There were also some instances which did not support the eye-mind hypothesis. However, their study concluded that the pattern ensured that the participant performed mental processing.

Two research studies were conducted with high school students by Lin and Lin (2014a, 2014b), which examined geometry problem-solving processes in different contexts. In one study, Lin and Lin (2014b) used handwriting tablets to examine differences among eye movement measures of successful and unsuccessful problem solvers. Their findings indicated that the successful solvers concentrated more on the calculation area, while the unsuccessful problem solvers had more FC, RC and longer DT on the problem area. In their other study, Lin and Lin (2014a) investigated the causes of cognitive load through the use of a remote eye tracker. Based on heat maps and eye tracking measures, they concluded that students needed more time and attention to comprehend difficult problems than easier ones. Consequently, the participants had higher FC, RC, and longer DT on the more difficult questions.

In another study, Lee and Wu (2018) aimed to determine whether or not college students were text- or figure-directed. Their findings from eye movement measures indicated that 86% of their participants showed text-directed patterns. This means that most of the participants first reviewed the text before shifting their focus to the figure in order to look for corresponding factors. In another major finding of the study, where antecedent geometrical information was given prior to new information, the participants' reaction time, fixation duration on figure, and saccade number from text to figure were shown to decrease. The results of this study are considered essential to the efficient reading of geometry questions.

2.6 Summary

Problem-solving ability lies at the core of mathematics education and challenging due to the complexity of required cognitive processes. Problem solvers need to understand the problem description, activate familiar patterns, recall information from memory, develop strategies, and draw conclusions in order to resolve the problem. These complex cognitive processes shape the way of evaluating and assessing problem-solving. Traditional measures such as the number of correct answers and the time required to solve a problem are not enough (Knoblich, Ohlsson, & Raney, 2001). To enhance the understanding of problem-solving processes, spontaneous and precise data through eye-tracking should be used since it provides information about students' understandings, interpretations, or difficulties in problem-solving. However, there might be various explanations of eye-tracking data depending on the context of the study. The higher fixation-based measures may imply problem complication (Duchowski, 2007), the desire for more plausible answers (Lin & Lin, 2014a), or being bored (Schindler & Lilienthal, 2019). Therefore, specific explanations using various scenarios should be added to the literature to contribute to understanding problem-solving processes.

The literature results showed that many eye-tracking studies were conducted within algebra, such as number sense, operations, estimation, and equivalence. However, there was less emphasis on problem-solving in geometry. Thus, there is a gap to investigate problem-solving strategies and processes within geometry and spatial sense using eye-tracking technology. When the eye-tracking measures are enriched with other data collection methods like an interview, video recording, or think aloud, researchers can obtain detailed data about problem-solving processes. It can provide data about how students begin to comprehend the problem statement, how many times they need to focus on the diagram, which concepts or properties help them begin to solve the question and proceed, where they have difficulties, and their attention is directed. Research studies about eye-tracking and problem-solving used similar eye-tracking equipment; however, procedure and types of questions differed as expected. Several studies (Lin & Lin, 2014a; Lin & Lin, 2018; Schindler & Lilienthal, 2019) allowed students to write and draw while other studies expected them to answer without doing operations on paper or devices. This situation requires selecting simple problems enough to solve mentally. Andrà et al. (2009) used multiple-choice items; therefore, their conclusions highlighted how students selected among alternatives. It arises an idea that open-ended questions and questions appropriate to students' grade level should be used to reveal detailed information about problem-solving.

2.7 Implications

It is well accepted that problem solving as an activity plays a crucial role in the school learning context. By employing eye-tracking and then supporting the collected data through interviews and think-aloud protocols, the current study aims to address obtaining data about students' cognitive processes during problem solving in mathematics, and specifically in geometry. This technique could also be applied to investigate problem-solving processes in different disciplines such as physics. On the other hand, it may be used in various educational scenarios that have a focus on reading or writing. With the help of findings established from the application of this technique, instructors may be better equipped to provide students with more accurate and beneficial assistance to enhance their learning. Besides the instructors, policy makers may also make inferences from the cognitive perspective of such findings.

CHAPTER 3

METHODOLOGY

The purpose of the study is to examine problem-solving strategies and eye movement changes during problem solving in geometry. This chapter covers research questions, research design, participants of the study, data sources, data analysis, and overall procedure of the study, respectively.

3.1 Research Questions

1. What are the problem-solving strategies that 10th-grade students use during problem solving in geometry?
2. How do eye movement measures change during problem-solving processes at different problems and different parts of the problems?

3.2 Research Design

In the current study, a mixed-methods research design was applied in which both quantitative and qualitative methods were used to investigate the study's research questions. As stated by Creswell and Plano Clark (2011), a mixed-methods design consists of procedures to collect, analyze, and mix both quantitative and qualitative data within a single study. They also stated that the mixing of quantitative and qualitative data has the potential to produce better explanations to complex research problems and questions rather than relying solely on either method. The rationale behind the use of a mixed-method research design in the current study is that using only the eye-tracking technique as quantitative data is not considered sufficient to explain problem-solving processes. A broader set of data is therefore required in order to explore and adequately explain the research questions. There is a need to elaborate upon the participants’ description of their problem-solving strategies, and in general to address problem-solving processes through the interpretation of their eye movements. For the quantitative part of the study, data were gathered from an eye tracker providing total fixation durations, and scores of the Spatial Ability Test were regarded as quantitative data. For the qualitative part of the study, open-ended geometry problems and audio recordings of semi-structured interviews held with the participants, plus think-aloud protocol formed the qualitative data of the current study.

3.3 Participants of the Study

The subject matter related to questions used in the current study is taught during the second semester of the ninth grade in Turkey. Since the data was collected during the first semester of the academic year, the study’s participants consist of 10th-grade students who had learned the subject matter during their previous year. The sample was chosen by convenience sampling from a private high school located in Ankara, the capital of Turkey. Convenience sampling is a common technique employed due often to issues of accessibility, and refers to a preference to recruit certain individuals who are available to participate in a study (Fraenkel et al., 2012). In the current study, the participants were invited via e-mail, and those who responded and volunteered to contribute to the study constitute the sample. Detailed information about the participants is presented in Table 3.1. All of the participants signed a consent form approved by the university’s Human Subjects Ethics Committee to indicate their voluntary participation in the study. Moreover, a consent form was also collected from each participant student’s parent.

Table 3.1. Participants' Demographic Information

Participants	Age (years)	Grade Level	Gender
P1	16	10	Female
P2	16	10	Male
P3	16	10	Male
P4	16	10	Male
P5	16	10	Female
P6	16	10	Male
P7	16	10	Female
P8	16	10	Female

3.4 Study Setting

A pilot study was undertaken with one participant in order to specify the study's tasks. After an introduction to the software was given, the participant solved six geometry questions, and the eye movements of the participant were recorded by using the Tobii X2-60 eye tracker. The same questions were used in the pilot study. After that, the pilot study data were analyzed by the researcher. The size of the geometry questions was enlarged to ensure visual acuity. Then, the task was specified in detail to collect the actual data.

In this study, the Tobii X2-60 Eye Tracker device (see Figure 3.1) and a computer compatible with Tobii Studio Software (see Figure 3.2) were used. The sampling rate of the eye tracker device is 60 Hz, and is considered appropriate for eye-tracking on a screen sized up to 25-inches and with an aspect ratio of 16:9. The computer used in the current study was compatible with the Tobii Studio Software that was to be used, had a 15.6-inch screen size, a 16:9 aspect ratio, and 3840x2160 pixels UHD screen resolution.

The Tobii Studio Software was utilized in performing an analysis of the participants' eye movements captured by the eye tracker. The participants sat in front of the computer and the eye tracker, with the distance between the screen/eye tracker and the participants being approximately 60 centimeters. The device tracked whereabouts users looked on the screen, how long they looked for, and how many times they looked at a certain point on the screen. In the current study, the eye-tracking data was captured at the university’s Design Factory. Geometry problems were divided into two areas of interests, as textual and diagrammatic. The textual part indicated certain information about the problem, while the diagram pointed to a geometric figure with certain information also given.

Figure 3.1. Tobii X2-60 Eye Tracker (as shown in https://www.tobiipro.com/)

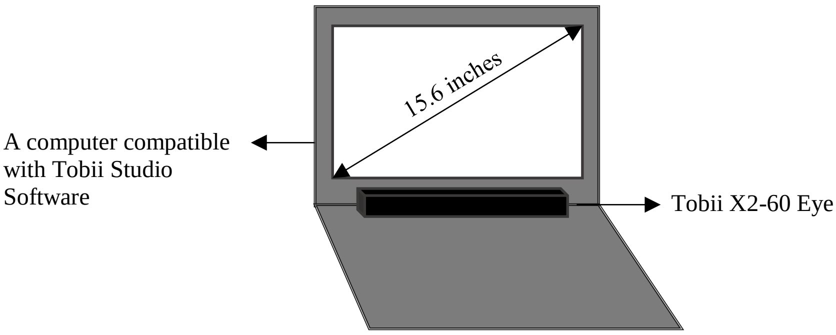


Figure 3.2. Eye Tracker and the Computer Setup

As shown in Figure 3.3, the Areas of Interest were clarified by drawing rectangles. The blue-colored rectangle indicated the AOI diagram, whilst the purple-colored rectangle specified the text AOI.

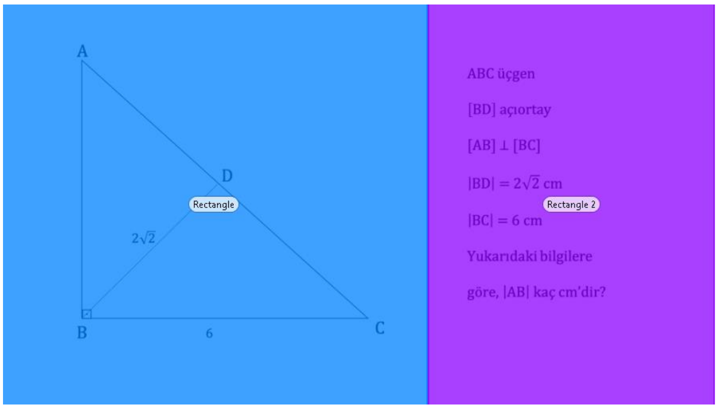


Figure 3.3. Areas of Interests

In order to gain significant information and interpret the captured eye-tracking data, Tobii Studio Software was used. This software allows for the visual presentation of raw data and records it. In the current study's analysis, Tobii Studio Software (version 3.4.8) was used. As can be seen in Figure 3.4, a test including a timeline demonstrated an introduction and the six questions presented in order.

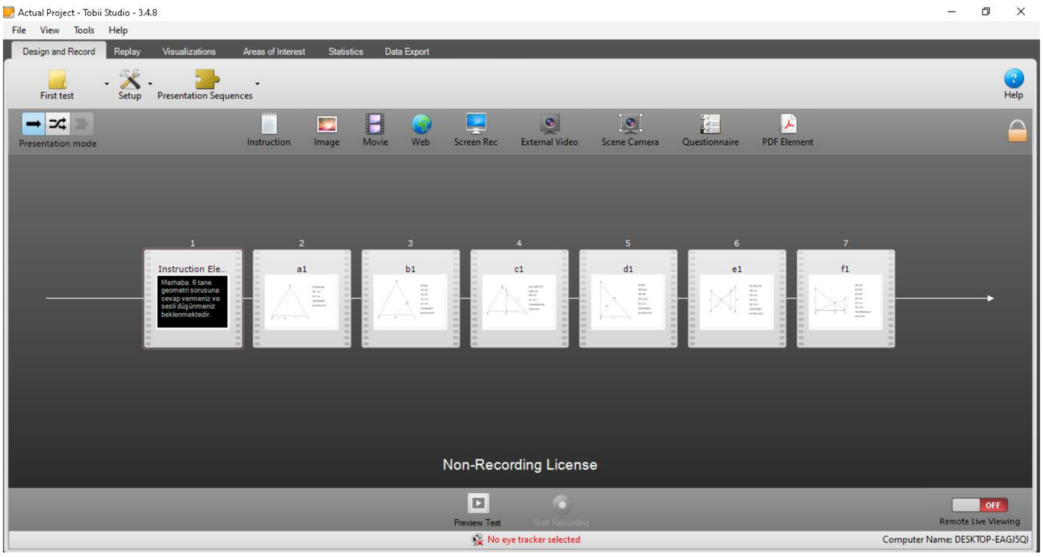


Figure 3.4. Tobii Studio Software

3.5 Materials

A total of six geometry problems were selected in the current study from a textbook used in Turkey for preparation for university entrance, and which is considered appropriate for the grade level according to the Turkish Ministry of Education's high school curriculum. Each problem involved a brief statement and a diagram, and the problem solvers needed to obtain important information from both of these. In order to reflect a true problem-solving context, a blank page was also provided to each student so that they could write, calculate, and finally reach the required solution. To solve the problems, the participants needed to apply the fundamental properties of triangles. Although the participants of the current study were all 10th-grade students, the subject matter of the given geometry problems was actually from the 9th-grade curriculum. The reason for this was that the data was collected during the first semester of the 2019-2020 academic year, and the participant 10th-grade students would not have tackled geometry topics at that point in the semester. The following objectives relate to the geometry problems used in the study, and are sourced from the 9th-grade Mathematics Curriculum in Turkey (see also Table 3.2).

The geometry problems used in the current study are classified as an equilateral triangle, an isosceles triangle, right triangles, and similarity problems. The first problem was based on an equilateral triangle, while the second was an isosceles triangle problem. The third and fourth problems were right triangle problems, while the fifth and sixth were similarity problems. These classifications established an ordered analysis of the results. However, they did not indicate any distinct differentiation between the problems due to overlapping geometric relations. When the prerequisite information for the problems was investigated, the participants were known to have been studying triangle-based concepts since their primary education (except for in Grade 9). During the fifth grade, as students they would have classified triangles based on side and angle, and also learned the sum of the angles that form a triangle. In their eighth grade, the students would have encountered Pythagorean

Theorem and similarity in a triangle. In detail, Question 1 included the properties of an equilateral triangle, 30-60-90 triangle, and Pythagorean Theorem. Question 2 contained the properties of isosceles triangle, and Euclidean Theorem. Question 3 was an example of a right triangle problem, and included 30-60-90 triangle and Pythagorean Theorem. Question 4 implied the properties of a right triangle, interior angle bisector, and also the similarity of triangles. In Questions 5 and 6, knowledge of the similarity rules of triangles was necessary in order to solve the problems.

Table 3.2. Objectives related to geometry problems from ninth-grade Mathematics Curriculum

9.4. Triangles
9.4.1. Basic Concepts of Triangles
9.4.1.1. Students will be able to perform operations on the angle properties of triangles.
9.4.2. Similarity and Congruency in Triangles
9.4.2.2. Students will be able to determine the minimum conditions of two triangles being classed as similar.
9.4.3. Secondary Elements of a Triangle
9.4.3.1. Students will be able to determine the properties of interior and exterior angle bisectors.
9.4.4. Right Triangle and Trigonometry
9.4.4.1. Students will be able to identify Pythagorean Theorem in a right triangle and solve related problems.
9.4.4.2. Students will be able to identify Euclidean Theorem and solve related problems.

3.6 Study Procedure

This study was conducted during the fall semester of the 2019-2020 academic year with eight high school students enrolled to a private high school in Ankara, Turkey. Prior to the data collection phase, an e-mail was sent to students from this school in order to reach potential participants. The study's participants were each student who volunteered to participate in the study. The researcher informed the participants about the aims of the study, how they would contribute to the study, and for what purpose the collected data would be used. At the beginning of the study, the participant students each signed a consent form to confirm their voluntary participation.

Additionally, the students' parents signed a consent form to indicate their approval for the students to participate in the study. In the laboratory environment, Spatial Ability Test (SAT) was applied to measure the students' spatial abilities. Instructions for the test were explained to help the students to understand how to answer each part of the test. Detailed information about the test is provided in Section 3.7.1. The students were expected to complete the SAT within a duration of 30 minutes.
The participant students were asked to solve six geometry problems presented on the eye tracker screen. The eye tracker then recorded their eye movements during the period when the students worked on solving the given geometry problems. The device collected eye movement measurements and total fixation durations as the study's quantitative data. Throughout each geometry task, the participant students were also requested to "think aloud." When the students had completed all of the geometry tasks, the researcher held interviews separately with each participant.

By supporting the collected eye-tracking data with both interviews and the think-aloud protocol, the study aimed to investigate how students answered the assigned geometry problems, which strategies they employed, in which parts they experienced difficulties, if any, and why such difficulties occurred during their problem solving. Correspondingly, the aim of combining both quantitative and qualitative data elements was in order to describe an overall picture of the students' problem-solving processes. At the end of the process, an information form was distributed to each participant so as to report back to them detailed information about the study.

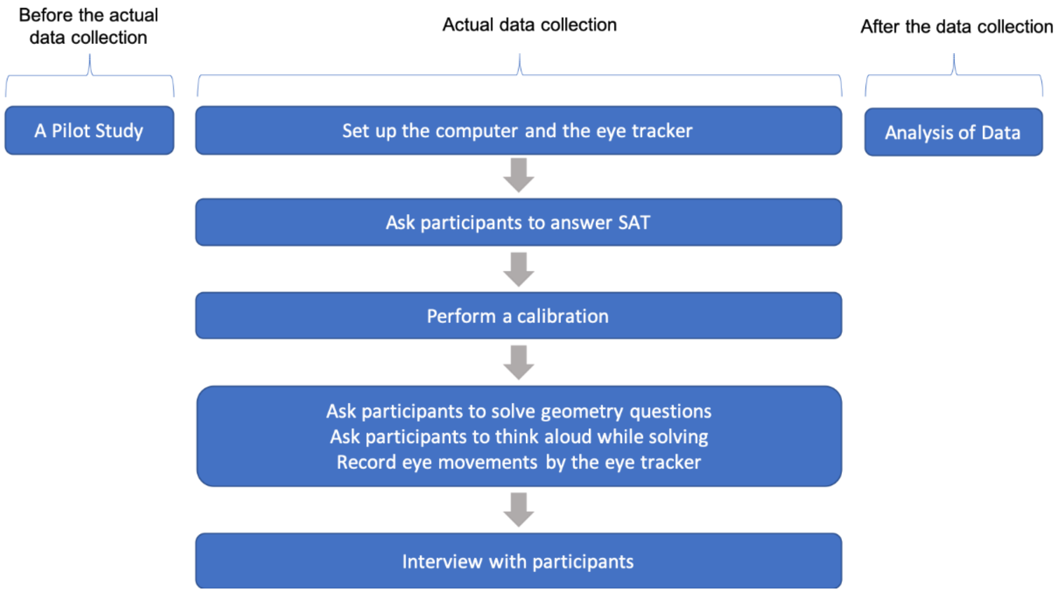


Figure 3.5. Procedure of the Study

3.7 Data Sources

This study examined two areas; i) the participants' geometry problem-solving strategies, and ii) changes in the participants' eye movement measures during problem solving. Three forms of data collection were employed; 1) Spatial Ability Test (SAT), 2) Semi-Structured Interview, and 3) Think-Aloud Protocol. In order to conduct research with these data collection instruments, ethics committee approval was received from the Middle East Technical University Human Subjects Ethics Committee (see Appendix E). Detailed information about each data source is specified as follows.

3.7.1 Spatial Ability Test (SAT)

The Spatial Ability Test was designed by Ekstrom et al. (1976) and subsequently adapted to the Turkish context by Delialioğlu (1996). This Turkish version was applied in the current study to assess the participants' spatial abilities. The SAT includes four different tests, which are Card Rotation Test (CRT), Cube Comparison

Test (CCT), Paper Folding Test (PFT), and Surface Development Test (SDT). The first two tests are used to assess spatial orientation ability, while the second two tests assess spatial visualization. The sum of these four tests determines the participant's SAT score. The sum of CRT and CCT represents the score for spatial orientation, whilst the sum of PFT and SDT represents the score for spatial visualization. Sample questions are provided in Appendix F.

Card Rotation Test (CRT): This test was designed to assess the ability to notice the differences between shapes. Therefore, the participants were asked to decide whether or not the rotated shapes were in fact the same as the initial shape. There were 20 different questions, each with eight sub-questions. Thus, this test consists of 160 questions and sub-questions in total. Since each correct answer is awarded 1 point, the maximum total score for the test is 160. The total time allocated for the test is 6 minutes.

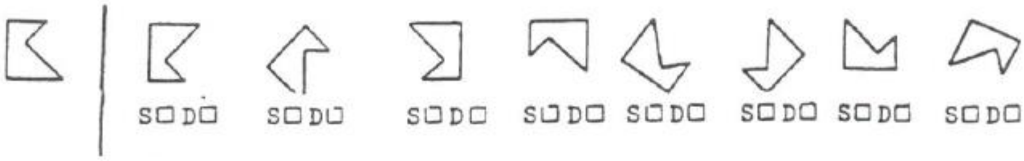


Figure 3.6. CRT sample question (Ekstrom et al., 1976)

Cube Comparison Test (CCT): This test requires deciding whether or not the two cubes presented in each question are the same. Each cube has six surfaces, which include a letter, number, or symbol. Only three surfaces of the cubes are shown in each question, and the participants should decide according to these surfaces. The number of questions and the total score for the test is 42, with 6 minutes allocated for completion of the test.

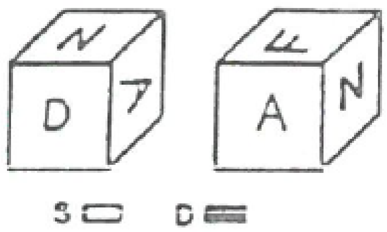


Figure 3.7. CCT sample question (Ekstrom et al., 1976)

Paper Folding Test (PFT): This test requires imagining the folding and unfolding of a piece of paper. Participants have to decide which one is the unfolded state of the initially folded state. Since there are 20 questions and each correct answer is awarded 1 point, the maximum total score for the test is 20, with the time allocated for this test limited to 6 minutes.

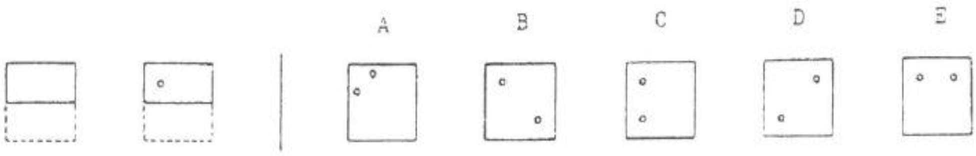


Figure 3.8. PFT sample question (Ekstrom et al., 1976)

Surface Development Test (SDT): This test requires imagining different objects by folding a piece of paper. Then, numbers shown on the sides of the unfolded object and letters on the sides of the folded object should be matched. The test contains 12 questions, and each question includes five matchings. Since each correct match is awarded 1 point, the total score for this test is 60. Subjects are tasked with completing this test within a period of 12 minutes.

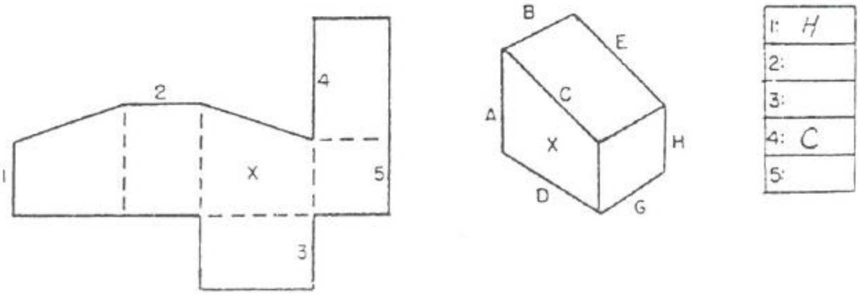


Figure 3.9. SDT sample question (Ekstrom et al., 1976)

3.7.2 Semi-Structured Interview

The study aims to understand the participants' overall problem-solving processes. Specifically, this was the problem-solving strategies employed whilst solving the assigned geometry problems, and the corresponding changes in eye movement measures exhibited during the exercise. In order to explore the reasons behind variations in eye movement, and the strategies employed during problem solving, a semi-structured interview protocol consisting of open-ended questions was designed by the researcher. Interviews were conducted as a separate single session with each participant. In each interview, the following three questions were posed for each geometry problem.

1. Which strategies did you use while solving the problem?
2. How did these strategies help you in solving the problem?
3. In which points of the problem solving did you feel you were lacking?

Based on these questions, the participants were expected to explain the strategies and problem-solving processes that they had employed in solving the given geometry problems. If the participants had difficulty in explaining their strategy, or their explanations were deemed by the researcher to be inadequate, additional questions were asked in order to clarify the problem-solving processes used. Each of the interviews were audio recorded. The interview protocol is presented in Appendix I.

3.7.3 Think-Aloud Protocol

The think-aloud protocol was applied with the aim being to understand the problem-solving processes that the students employed. Each student was asked to verbally explain out loud the problem-solving strategies and processes they employed whilst they were working on solving the problems. Data obtained through this protocol was considered essential to obtaining a greater depth of information regarding each students' problem solving. It was also expected that the data would complement the other means of data collection. Even if the students were unable to find the correct solution to each problem, there was still an opportunity to learn their starting points, prior knowledge, and/or their realizations during their problem solving. The think-aloud protocols were conducted at the university's Human Computer Interaction Laboratory while the students were solving their assigned geometry problems.

3.8 Data Analysis

In the mixed-method research design, using quantitative and qualitative methods together makes the research problem more understandable than relying on just a single method (Creswell & Plano Clark, 2011). SAT, eye movements, think-aloud, and the interviews were the data sources used in the current study.

Spatial Ability Test (SAT), which consists of four different parts, was applied to assess the participants' spatial abilities at the beginning of the study. The four tests of the SAT are Paper Folding Test (PFT), Surface Development Test (SDT), Cube Comparison Test (CCT), and Card Rotation Test (CRT). The maximum scores achievable for each part, PFT, SDT, CCT, and CRT, are 20, 60, 42, and 160, respectively. The participants' scores were analyzed according to the answers given in the study of Ekstrom et al. (1976). To analyze the data gathered by the eye tracker, Tobii Studio Software was used. Tobii Studio generates data based on total fixation durations within AOIs, and for the visualization of this data, heat maps were created to reveal the participants' levels of visual attention.

The audio recordings of each participant's interview and think-aloud protocol were transcribed verbatim, and are presented in Appendices J and K, respectively. These transcriptions were then analyzed by the researcher to assess the strategies employed in solving the given geometry problems. The analysis of the participants' responses was achieved through content analysis. The content analysis code list used to assess the participants' problem-solving strategies was formed according to the literature, and is presented in Appendix H. After all the direct quotations from the interviews and think-aloud protocols had been coded, a mathematics teacher unconnected to the study acted as a second rater to check the validity of the participants' codes that the researcher had initially determined. Afterwards, any necessary changes were applied in order to reach a consensus between the researcher and the second rater.

3.9 Role of the Researcher

The role of the researcher in the current study was setting up the computer and eye tracker. The researcher then prepared a brief introduction and the six tasks/questions using the Tobii Studio Software to record the participants' eye movements. Prior to collecting the data, the researcher conducted a pilot study with one other participant so as to check the proposed questions' viability in terms of scale and appropriateness, and also to assess the study process itself in application. The researcher contacted both the participants and their parents in order to gather signed consent forms to confirm each students' voluntary participation. During the data collection process, the researcher conducted the Spatial Ability Test (SAT), the eye tracking session, and an interview for each participant. The researcher encouraged the participants to explain their problem-solving strategies and thoughts regarding each task during their interview session. The researcher then analyzed the collected data and interpreted the findings of the study accordingly.

3.10 Validity and Reliability

Validity and reliability are two significant factors to consider when designing a study. Validity has been defined as “appropriateness, correctness, meaningfulness, and usefulness of the inferences” (Fraenkel et al., 2012, p. 148). Triangulation is one of the well-known strategies used for internal validity. Denzin (1978) proposed four types of triangulation: employing multiple methods, multiple data sources, multiple investigator and multiple theories for findings. In the current study, multiple methods, data sources and investigator procedure were applied to ensure the internal validity. This study acquired both qualitative and quantitative data through an eye tracker, interviews, think-aloud protocol and SAT from eight participants. Investigator triangulation occurred when a mathematics teacher independent of the study who had more than 10 years’ experience analyzed the same qualitative data and checked the codes formed from the interviews and think-aloud protocols. The participants’ direct quotations and problem-solving strategies were also reviewed by this second-rater, and a consensus reached where any differences existed between the two raters’ thoughts. Moreover, the six geometry questions used in the current study were assessed by two mathematics teachers, who each possess more than 10 years’ experience. The suitability and the subject matter covered in each question were also examined by them.

3.11 Summary

In this chapter, the study’s research questions and research design were presented. In addition, the study’s participants were described, as was the study’s setting, materials, procedure, data sources, and methods of data analysis.

CHAPTER 4

RESULTS

In this chapter, the results of the study are presented according to the research questions.

1. What are the problem-solving strategies that 10th-grade students use during problem solving in geometry?
2. How do eye movement measures change during problem-solving processes at different problems and different parts of the problems?

This chapter presents the findings of the study under three main sections, together with relevant subsections. The first section presents the spatial ability test scores of the participants, whilst the second section presents the eye movement measurement findings, including total fixation duration analysis and heat maps for each question. The third section presents the participants' problem-solving strategies, and is divided into two subsections, as holistic and analytic strategies. Each subsection also has some related subcategories.

4.1 Spatial Ability Test (SAT) Scores

In this section, the participants' correct, incorrect, and unanswered responses to the SAT are analyzed in detail. As previously mentioned in Section 3.7.1, the SAT consists of four individual tests; Card Rotation Test (CRT), Paper Folding Test (PFT), Cube Comparison Test (CCT), and Surface Development Test (SDT). Participants' correct, incorrect, and unanswered responses are presented in tabular form in Appendix G. In Table 4.1, both the total scores for each test and the whole SAT, plus each participants' scores for each test and again the whole SAT.

Table 4.1. Participants' Spatial Ability Test (SAT) Scores

	Spatial Ability Test				
Participants	Paper Folding Test (PFT)	Surface Development Test (SDT)	Cube Comparison Test (CCT)	Card Rotation Test (CRT)	Total Score of SAT
P1	11	50	29	153	243
P2	19	58	32	158	267
P3	14	39	28	107	188
P4	17	56	26	105	204
P5	11	27	29	38	105
P6	12	26	30	88	156
P7	14	55	33	89	191
P8	16	49	24	107	196
Total Score	20	60	42	160	282

From Table 4.1, it can be seen that Participant P1's SAT results were the second highest score overall, having been the only participant to answer all of the items (i.e., zero unanswered questions). In the PFT, Participant P1 achieved 11 correct and nine incorrect answers, with 11 being the lowest score of all the participants for this test. In the SDT, Participant P1 achieved 50 correct and 10 incorrect answers, whilst in the CCT, she achieved 29 correct and 13 incorrect answers. In the CRT, Participant P1 achieved a score of 153 with 153 correct and seven incorrect answers, giving her the second highest score of all participants. Overall, Participant P1's SAT score was 243, representing 243 correct, 39 incorrect, and zero unanswered questions.

Participant P2 achieved the highest scores in three of the tests, the second highest score in CCT, and the highest overall total score at 267. In the PFT, all of the questions were answered correctly except for one, resulting in the highest PFT score. In the SDT, Participant P2 achieved an almost perfect score, but was unable to match the lettered edges with the numbered ones in two examples, resulting in a SDT score of 58. In the CCT, he achieved 32 correct, six incorrect, and four unanswered questions, and in the CRT, Participant P2 answered all of the questions with just two errors.

Participant P3 correctly answered more than half of the items. In the PFT, no questions were left unanswered; however, he had six incorrect answers. In the SDT, Participant P3 had 39 correct, 12 incorrect, and nine unanswered questions, whilst in the CCT, he had 28 correct, eight incorrect, and six unanswered questions. In the CRT, the number of unanswered questions was quite high at 45, and he also had eight incorrect answers.

Participant P4 answered more than the half of the questions in each test. He had the second highest score in the PFT at 17, with one incorrect and two unanswered questions. In the SDT, P4 completed all of the items, but with four incorrect matchings. Same as for the PFT, Participant P4's SDT score was the second highest after P2. In CCT, Participant P4 had 26 correct, three incorrect, and 13 unanswered questions. Lastly, P4 had a high number of unanswered questions in the CRT, at 53, and with two incorrect answers, his score was 105.

Participant P5 had the lowest total SAT score, and the lowest for both the PFT and CRT tests. She achieved 11 correct, five incorrect, and four unanswered questions in the PFT, resulting in a score of 11, which the same as Participant P1. In the SDT, Participant P5 achieved 27 correct and nine incorrect answers. She left nine questions unanswered in the first part of the SDT, and 15 unanswered in the second part at the end of the test; having been unable to finish all of the items within the 6 minute limit. In the CCT, she answered all of the items except for one, but answered 12 questions incorrectly, resulting in a CCT score of 29. In the CRT, her score was 38 which was a relatively low score. In the first part of the CRT, she achieved 38 correct, six incorrect, and 38 unanswered questions. In the second part, P5 did not mark any of the two options; therefore all 80 items were regarded as unanswered; hence, her CRT score was based only upon her score for the first part.

Participant P6 received the second lowest total SAT score. In the PFT, his score was 12, having answered 12 questions correctly, five incorrectly, and three questions remained unanswered. In the SDT, P6 answered correctly to less than half of the items, as did Participant P5, with 21 incorrect and 13 unanswered questions, giving him the lowest SDT score of 26. In the CCT, Participant P6 made decisions as to whether or not the two drawings of a cube were the same for all items except one. From this, 11 of the answers were incorrect, resulting in a CCT score of 30. He had the second lowest score for the CRT, having completed all of the items correctly except for one in the first part, but left 70 items unanswered in the second part.

Participant P7 answered all of the items in the PFT, although six were incorrect. In the SDT, she had 55 correct, three incorrect, and two unanswered questions. She had the highest score for this part after P2 and P4. In the CCT, she answered all of the items; however, nine of them were answered incorrectly. Lastly, in the CRT, she did not mark 28 of the items in the first part, nor 39 in the second part, totaling 67 unanswered questions for the CRT. Combined with had four incorrectly answered questions, her score for the CRT was 89.

Participant P8 correctly answered more than the half of the items for each part, which was the same for all participants except for P5 and P6. In the PFT, Participant P8 achieved 16 correct and four incorrect answers, whilst she had 49 correct, nine incorrect, and two unanswered questions in the SDT. In the CCT, she answered 24 of the items correctly, with 12 unanswered questions in the second part and none in the first part, and six incorrect answers. In the CRT, she had 107 correct decisions, which was the same as P3. Participant P8 had 30 questions left unanswered for the first part of the CRT and 21 unanswered for the second. With two incorrect and 51 unanswered questions, her CRT score was 107.

4.2 Eye Movement Measures

The participants were tasked with solving six geometry questions, and their eye movements were recorded using an eye tracker, and later analyzed by the aforementioned bespoke software. Each of the six questions were divided into two areas, with both a diagram part and a textual part. These areas are also referred to as Areas of Interests (AOIs). The diagram AOI implies the problem figures, while the

Text AOI refers to the written information related to the problem. Total fixation duration was used as a measure to calculate the total time that each participant fixated on the AOIs. Heat maps that represent the visual attention were also created in order to verify the data. The heat maps use different colors to indicate fixations made in certain AOIs, with the color red highlighting the longest fixation time and green being the least. In this section, the eye movements analysis for each participant is presented.

4.2.1 Eye Movement Analysis of Participant 1

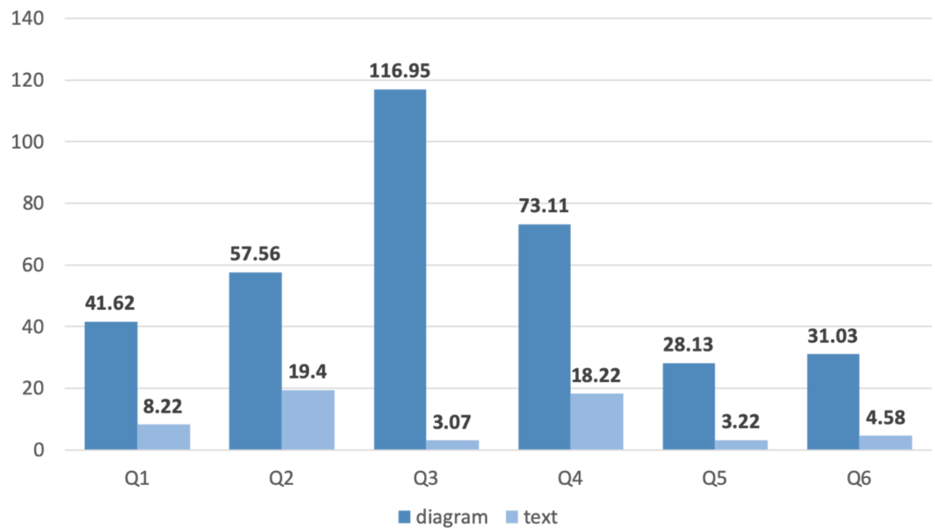


Figure 4.1. Total Fixation Durations of P1 on AOIs (seconds)

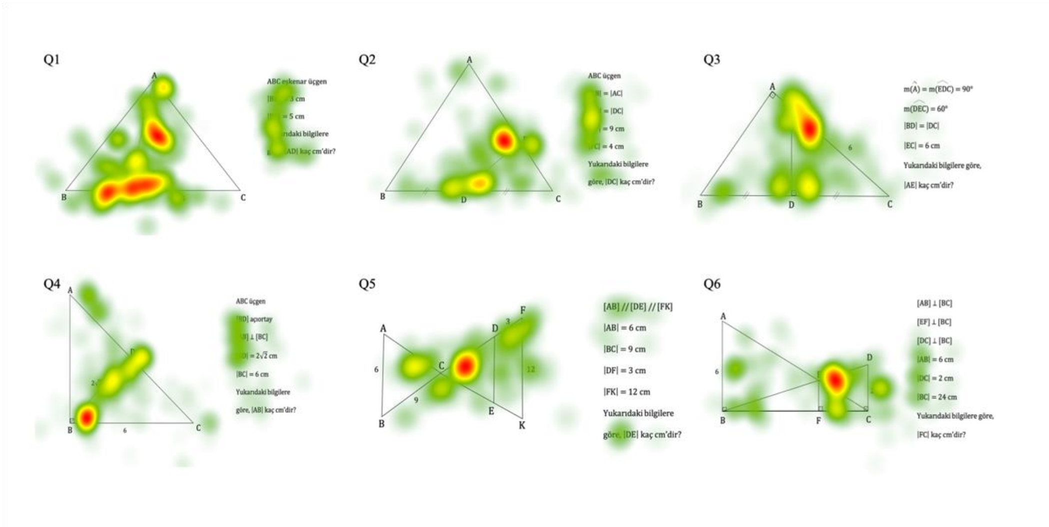


Figure 4.2. Heat maps of P1's Visual Attention

For all six questions, the total fixation duration was shown to be higher for the diagram than for the textual part. For Question 1 (Q1), P1 spent 41.62 seconds looking at the diagram and 8.22 seconds viewing the text. According to the heat map shown in Figure 4.2, P1 focused more on Point A, the center of the triangle, and the base, especially Point D. During the interview, P1's statement whilst explaining how she solved this question supported the heat map, in that she modified the diagram by drawing a line from Vertex A, and also performed mental calculations on these areas.

For Q2, Participant P1 spent 57.56 seconds on the diagram and 19.40 seconds on the textual part. The heat map indicated that she looked at the equalities and values in the textual part, and that her focus was on Point F and Point D. Especially, she spent time on the right angles in the diagram. During her interview, P1 mentioned experiencing difficulty whilst realizing Euclid's theorem.

For Q3, P1 spent almost all of the time looking at the diagram part (116.95 seconds), and only 3.07 seconds in viewing the text. This duration was the least among other results for the textual parts. When the heat map was checked, it can be seen that P1 focused on the line segment ED, and especially Point E. The reason for this focus and this highest duration may be inferred from the think-aloud and interview sessions. P1 attempted to draw a line from Point A to Point D in order to apply a theorem and then found the line's length. However, she did not use these two pieces of information to reach a solution. She said, “I guess I am moving away” in the think-aloud session. During her interview, she stated, “I might extend the question because I looked at small and big triangles frequently, and I am confused since I did not perform any operations on it,” hence, she focused much more on the diagram.

For Q4, she spent 73.11 seconds on the diagram and 18.22 seconds on the textual part. Line segment BD and its length were the most focused upon areas of the diagram. Information regarding angle bisector, perpendicular line segments, and line segment BD’s length in the textual part also held her attention. Although she focused more on Point B, she explained that she could not use the angle bisector information. She also stated her expectation of there being equal sides, given angles, or perpendicularity in order to solve the question.

For Q5, P1 spent 28.13 seconds looking at the diagram and 3.22 seconds on the textual part. According to the heat map, P1’s focus was gathered around Point C, the right-hand side. In the textual part, she looked at the information about parallel line segments and the line segment asked in the question. She associated the diagram with an hourglass, and applied similarity rules twice with two different pairs of triangles. Point C was the vertex of all the triangles; therefore, the focus was expected to gather on this point.

For Q6, Participant P1 spent 31.03 seconds viewing the diagram and 4.58 seconds viewing the textual part. The heat map indicated that her focus was on the area around Point E and the small triangle EDC in the diagram. In the textual part, P1 looked at the given values, and, similar to the previous question, she linked the diagram to an hourglass and applied similarity rules. Point E was in the middle of similar triangles; therefore, she fixated mostly upon this point.

4.2.2 Eye Movement Analysis of Participant 2

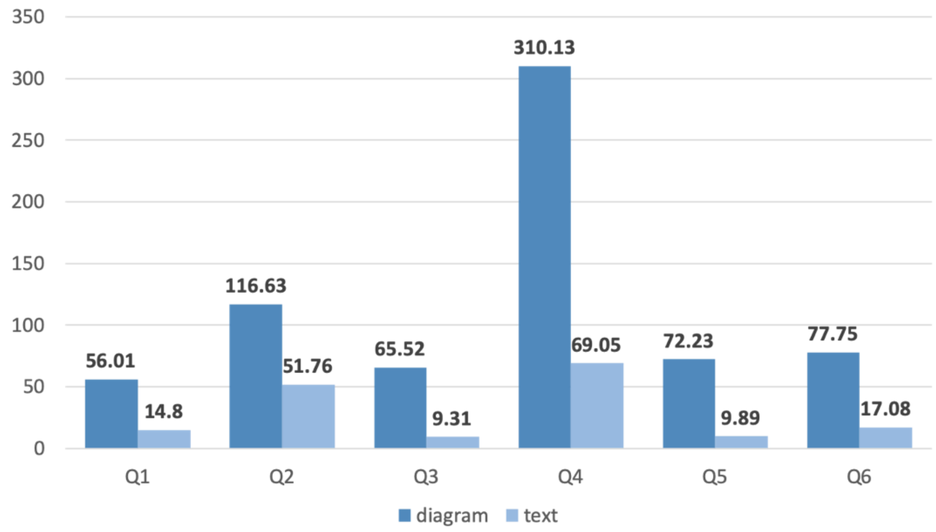


Figure 4.3. Total Fixation Durations of P2 on AOIs (seconds)

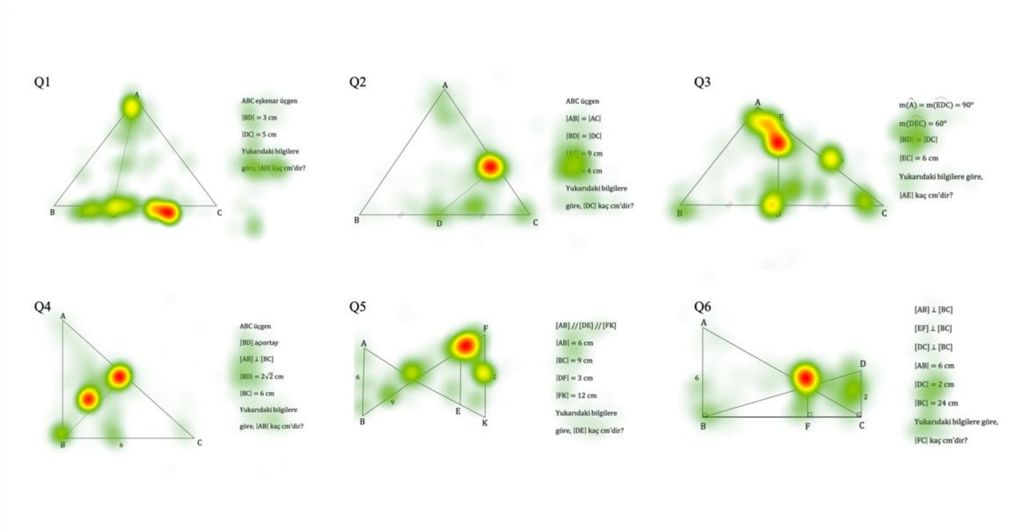


Figure 4.4. Heat maps of P2's Visual Attention

For Participant P2, the diagrams were fixated upon more than the textual part for all six questions (see Figure 4.3). For Q1, he spent 56.01 seconds on the diagram and 14.80 seconds looking at the textual part. When the heat maps presented in Figure 4.4 were investigated, it was noted that he focused on the area between the Point D and Point C. The attention was also distributed across Point D, the area close to Point D, and Point A. As P2 mentioned during the think-aloud session, he drew a height from Point A, then named the endpoint of the height on the base as Point H, which was located close to the area he had been most fixated upon.

For Q2, he spent 116.63 seconds looking at the diagram, and 51.76 seconds on the textual part. The heat map showed that he had a tendency to focus on the vertices of triangles A, B, C, and F; however, he spent most of the time on Point F. In both the think-aloud and interview sessions, he said, “We apply Euclid’s theorem here, but I do not remember the operations,” and Point F was at a crucial place to apply this theorem.

For Q3, 65.52 seconds passed while P2 looked at the diagram, and 9.32 seconds for the textual part. The heat map indicated that the most focused upon area was a place that included Point E and 60°. He also focused on Points A, D, C, and B, as well as Item 6. In the think-aloud session Participant P2 specified that there was a 30-60-90 triangle. He mentioned that he applied trigonometry, sin30, and sin60 to solve this problem in the interview. Therefore, it was expected he would visually check all of the vertices and angles.

For Q4, he spent most of the time (310.13 seconds) looking at the diagram, and 69.05 seconds on the textual part. The heat map showed that he focused mostly on Point D and $2\sqrt{2}$. He also looked at Point B and Item 6. While trying to solve this question, he assumed that a right angle existed even where there was none. He also tried to draw a line from Point D to the line segment BC; however, he could not figure out the rest of the question.

For Q5, he spent 72.23 seconds looking at the diagram, and 9.89 seconds looking at the textual part. In the heat map, the most focused upon area was Point D. He also focused on Point C and Items 12 and 9. He stated similar triangles while solving the question and in the think-aloud session. In the interview, he mentioned that he twice applied similarity rules. He used all of the lengths given in the question; however, he applied the rules around the triangles of CDE and CFK.

For Q6, he spent 77.75 seconds focused on the diagram, and 17.08 seconds on the textual part. As seen in the heat map, his attention was mostly gathered on Point E. He also focused on Item 2 and Point B. In the think-aloud session, he specified triangles AEB and EDC as being similar. Therefore, Point E was located in a critical position to apply similarity rules.

4.2.3 Eye Movement Analysis of Participant 3

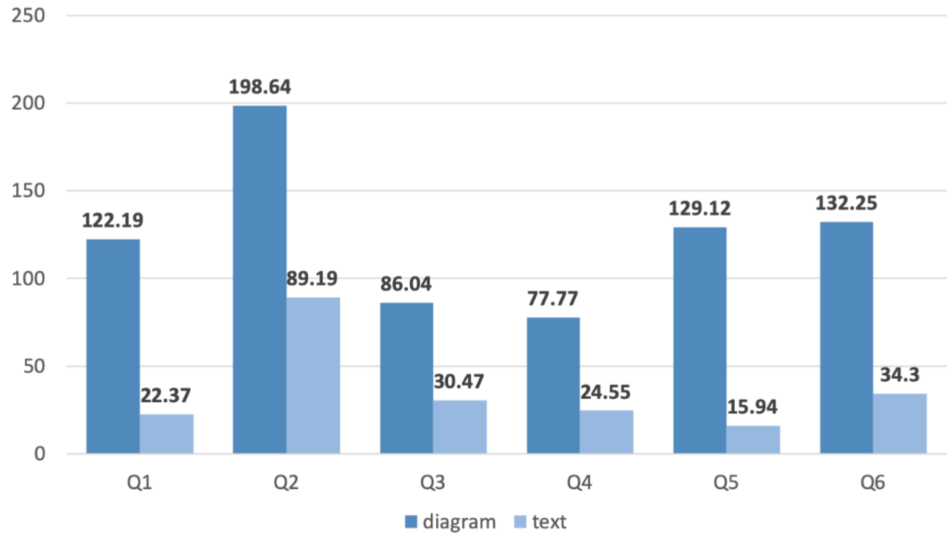


Figure 4.5. Total Fixation Durations of P3 on AOIs (seconds)

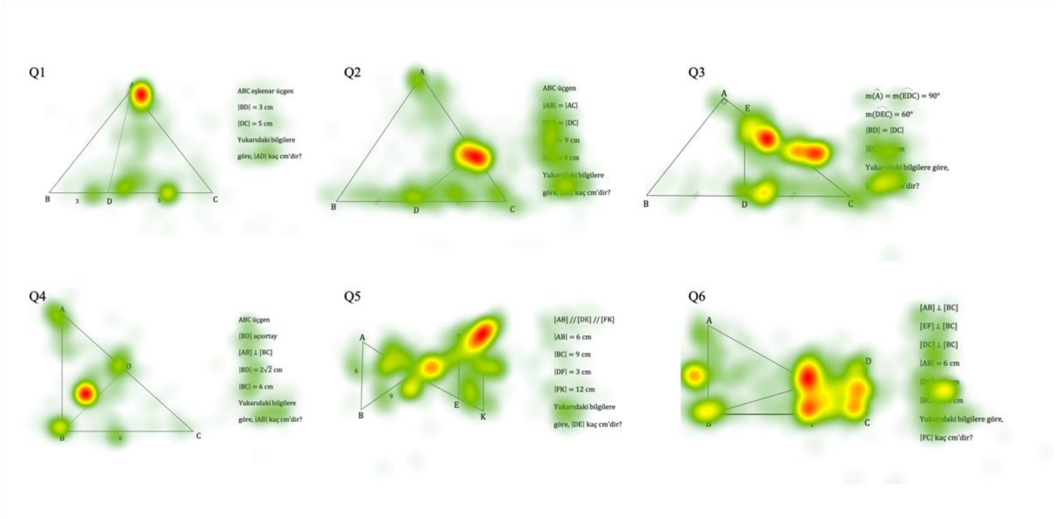


Figure 4.6. Heat maps of P3's Visual Attention

As shown in Figure 4.5, Participant P3 spent a higher duration focused on the diagrams than on the textual parts in each of the six questions. For Q1, he spent 122.19 seconds analyzing the diagram, and 22.37 seconds on the textual part. The heat map showed that he spent most of the time looking at Point A, and also focused on areas around Items 3 and 5, and on Point D. In the think-aloud session, he explained a solution strategy whereby he drew a perpendicular line from Point A to the line segment BC. The heat map serves as an illustration of the strategy he applied.

For Q2, he fixated more on both the diagram and the textual parts compared to the other questions, with 198.64 and 89.19 seconds, respectively. From the heat map, it can be concluded that the area most focused upon was Point F and the right angle; however, his attention was in fact distributed to many areas of the question. In the interview, he explained that he found the question to be difficult. In the think-aloud session, he stated that he assumed that the line segment BC was 10 cm, and there was a special right triangle 3-4-5, but without any justification.

For Q3, he spent 86.04 seconds on the diagram, and 30.47 seconds on the textual part. The heat map showed that his focus was mainly on the 60° and Item 6. He also concentrated on the right angle at Point D in the diagram, and line segment AE and the given length of EC in the textual part. In the think-aloud session, he only stated that there was a 30-60-90 triangle; whereas, in the interview, he mentioned that he tried to remember the properties of a 30-60-90 triangle.

For Q4, he spent 77.77 seconds viewing the diagram, and 24.55 seconds checking the textual part. The heat map indicated that he spent most of the time on $2\sqrt{2}$. He also focused on Points A, B, and D, and also on Item 6. In the textual part, he viewed all parts of the text. In the think-aloud session, he stated having found a 45-45-90 triangle and assumed that angle D was 90°. This assumption led him directly to a solution; however, there was no expression related to 90°.

For Q5, he spent 129.12 seconds on the diagram, and 15.94 seconds on the textual part. When the heat map was analyzed, his visual attention was shown to be on different parts of the diagram, especially line segment DF and Point C. He associated the diagram with a bow tie. Since the unknown lengths and asked length in the question were on the right-hand side of the diagram, he focused on the proportionality on this part.

For Q6, he spent 132.25 seconds and 34.30 seconds for the diagram and textual parts, respectively. The heat map showed that he mainly focused on the right-hand part of the diagram. The most focused upon areas were Points E and F. He also spent time on all the vertices of triangle BDC and on Item 6. In the think-aloud and interview sessions, he mentioned proportionality; however, he could not find how to apply it. He also stated in the interview that if EF's length was given, he could have solved the task comfortably. This statement might be the reason he focused on the line segment.

4.2.4 Eye Movement Analysis of Participant 4

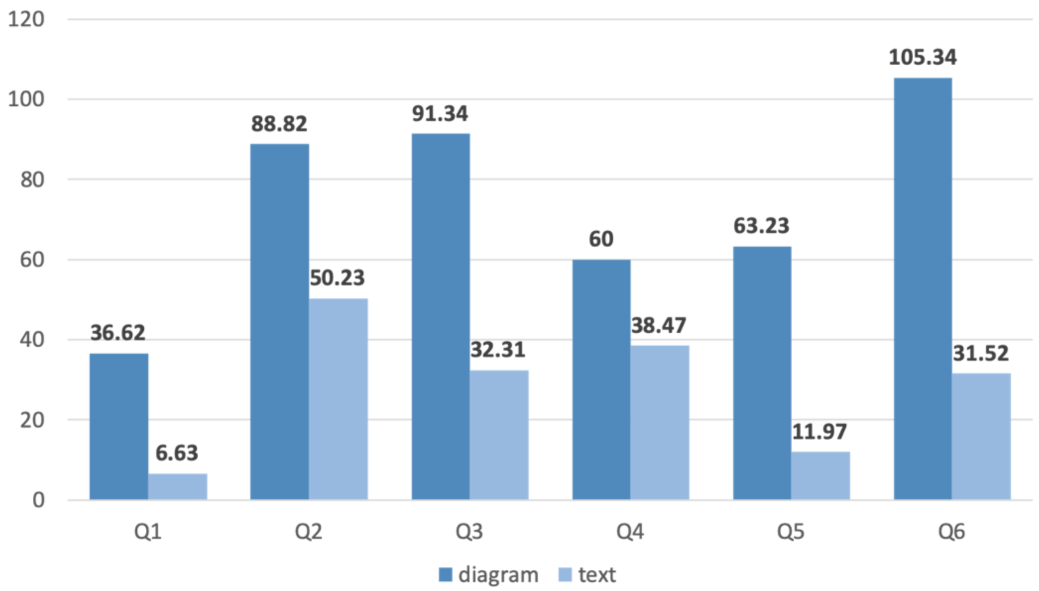


Figure 4.7. Total Fixation Durations of P4 on AOIs (seconds)

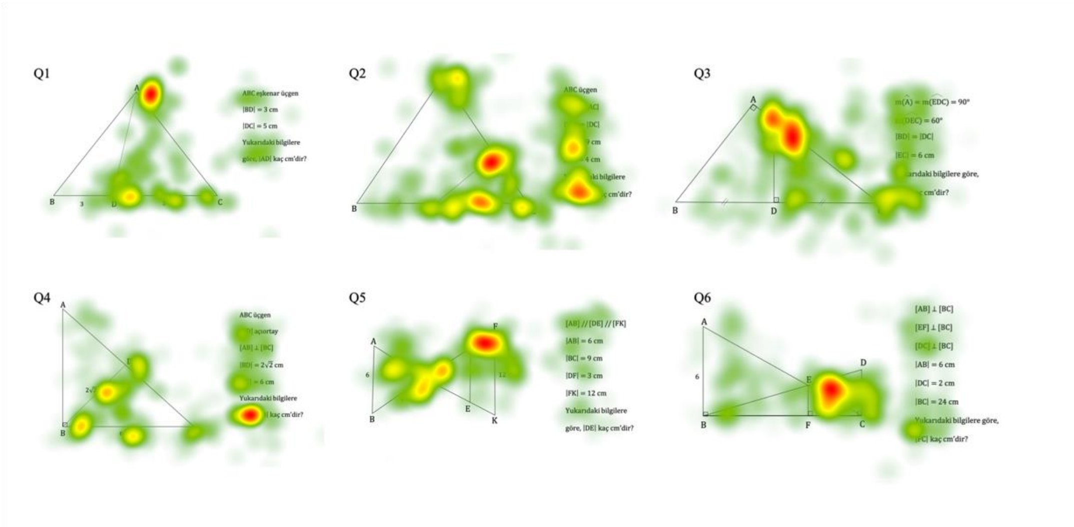


Figure 4.8. Heat maps of P4's Visual Attention

According to Figure 4.7, Participant P4 focused longer on the diagrams than the textual parts in all six questions. For Q1, the time spent on the diagram was 36.62 seconds, and 6.63 seconds on the textual part. According to the heat map, the place most fixated upon was the area near to Point A, followed by the area near to Point D. The accumulated eye movements constituted a line from Point A to the line segment DC. Correspondingly, he stated in the think-aloud session that he drew a perpendicular line from Point A.

For Q2, he took 88.82 seconds to look at the diagram, and 50.23 seconds for the textual part. The duration for the textual part in Q2 was the highest fixation time of all the texts. As seen in the heat map, he fixated more on the asked line segment than the given equalities in the textual part. He spent more time on the diagram, and his attention was mostly on Point F. The small triangle FDC and Vertex A drew his attention. As he mentioned in the interview, he drew a line from Point A to Point D; however, he could not proceed beyond that since the angles were not given.

For Q3, he spent 91.34 seconds on the diagram, and 32.31 seconds on the textual part. In this question, he spent the highest amount of time on the diagram among other parts of the question. The heat map showed that he fixated more on Point E and 60°. He also spent time looking at Point C, the area near to Point D, and the hypotenuse right across Point D in the diagram, and also the asked line segment in the textual part. According to the think-aloud and interview statements, he realized the 30-60-90 triangle, CED, and was able to find the side lengths of this triangle.

For Q4, he spent 60 seconds looking at the diagram, and 38.47 seconds looking at the textual part. According to the heat map, the focusing areas were Point B, $2\sqrt{2}$, Point D, Item 6, and Point C. He also fixated more on the asked line segment AB in the textual part. During the interview, he specified that he tried to use the line segment BD and an angle bisector. Moreover, he looked at Point D to search for the right angle.

For Q5, he distributed his attention with 63.23 seconds looking at the diagram, and 11.97 seconds at the textual part. The area most fixated upon was line segment DF, as can be seen in the heat map. Later, he fixated more on Point C and its surrounding area. As stated in the interview, he applied similarity rules in order to solve the task.

For Q6, he spent 105.34 seconds and 31.52 seconds looking at the diagram and textual parts, respectively. The heat map indicated that his attention was centered on the area of the quadrilateral EFCD. In both the think-aloud and interview sessions, he explained the similarity of triangles DCB and EFB, and the parallel lines EF and DC and also EF and AB. Therefore, it was expected that he would have maintained his focus on these areas.

4.2.5 Eye Movement Analysis of Participant 5

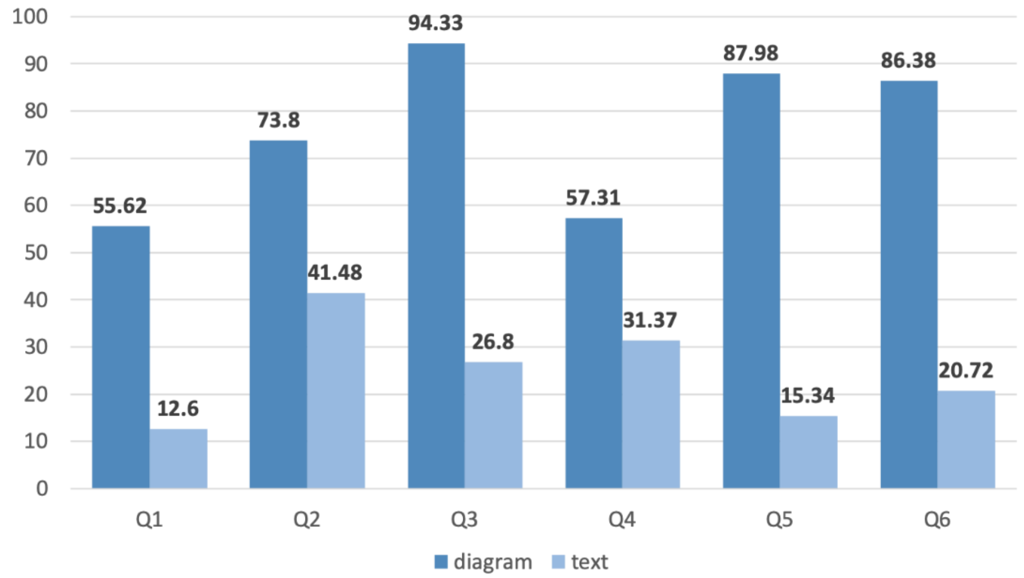


Figure 4.9. Total Fixation Durations of P5 on AOIs (seconds)

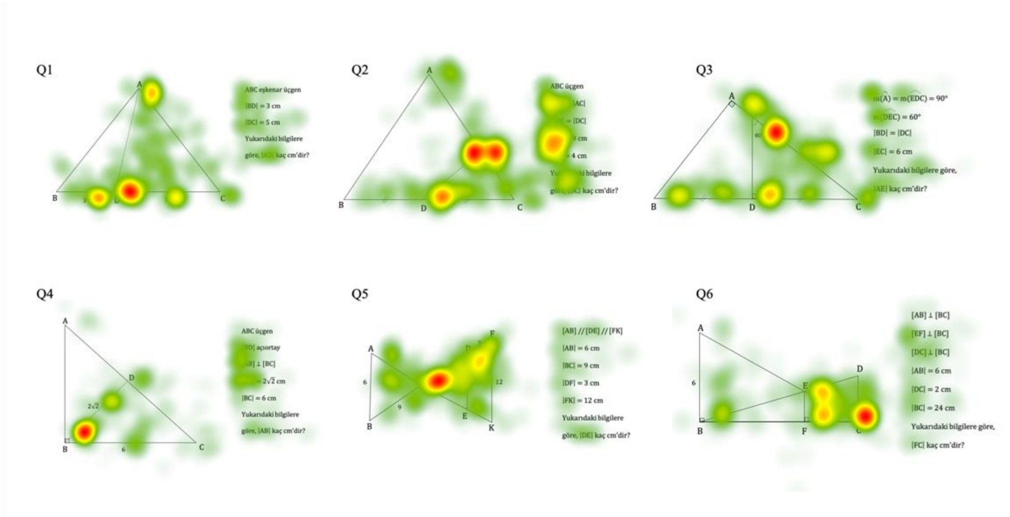


Figure 4.10. Heat maps of P5’s Visual Attention

As shown in Figure 4.9, Participant P5 fixated more on the diagrams in all six questions. For Q1, she spent 55.62 seconds on the diagram, and 12.60 seconds on the textual part, which was the minimum time spent looking at the textual part of any of the questions. According to the heat map, the most fixated place was Point D on the diagram. She also focused on Items 3 and 5, and also on Point A, and reviewed all of the textual part. In both the think-aloud and interview sessions, she mentioned the hat rule. However, she also remarked that she did not remember the rule, and that while trying to remember it, she said it was about adding or multiplying some side lengths. This, therefore, may be the reason for her having focused on the lengths.

For Q2, she spent 73.80 seconds on the diagram, and 41.48 seconds on the textual part. The heat map indicated that the most focused upon area was Point F. She looked at all points on the triangle, but especially Point F and then Point D. In the textual part, she looked at the given lengths. In the think-aloud session, she explained how to analyze the isosceles triangle and 90° at Point F. However, she later assumed that it was an equilateral triangle without any proof.

For Q3, a total of 94.33 seconds passed while she was focused on the diagram, and she spent a further 26.8 seconds looking at the textual part. This duration was the maximum that she spent analyzing any of the diagrams in any of the questions. According to the heat map, she reviewed most parts of the sides BC and AC of the triangle. The place that she most focused upon was the area near to Point E and 60°. She gathered her attention on Point D. While thinking aloud, she explained the 30-60-90 triangles in the diagram; however, she could not remember their side lengths. The heat map supported that she looked at the angles in the diagram.

For Q4, she spent 57.31 seconds on the diagram, and 32.37 seconds on the textual part. The heat map revealed her viewing behavior, and that her attention was mostly on Point B. She focused on $2\sqrt{2}$, Item 6, and also Point D as well. She stated that the angle bisector divided the 90° into two equal parts of 45° during her think-aloud and interview sessions. Therefore, it was expected that she had focused on Point B. However, she could not progress the task since she could not remember further information about the angle bisector.

For Q5, she allocated 87.98 seconds to looking at the diagram, and 15.34 seconds at the textual part. As the heat map showed, she spent time on almost all parts of the diagram except line segment AB. She fixated more on line segment CF, and especially on Point C. In the think-aloud session, she utilized a strategy by drawing Z. In the interview, she stated that Angle C was the same whilst drawing Z.

For Q6, 86.38 and 20.72 seconds passed while she maintained her attention on the diagram and textual parts, respectively. The heat map demonstrated that Point C was the area that drew her attention the most on the diagram. She also looked at the line segment EF and at Point B. In her interview, she specified how she proceeded and surmised that there was a 30-60-90 triangle. Since she tried to find the angles and the assumed Point C was 30°, she mostly focused on this point.

4.2.6 Eye Movement Analysis of Participant 6

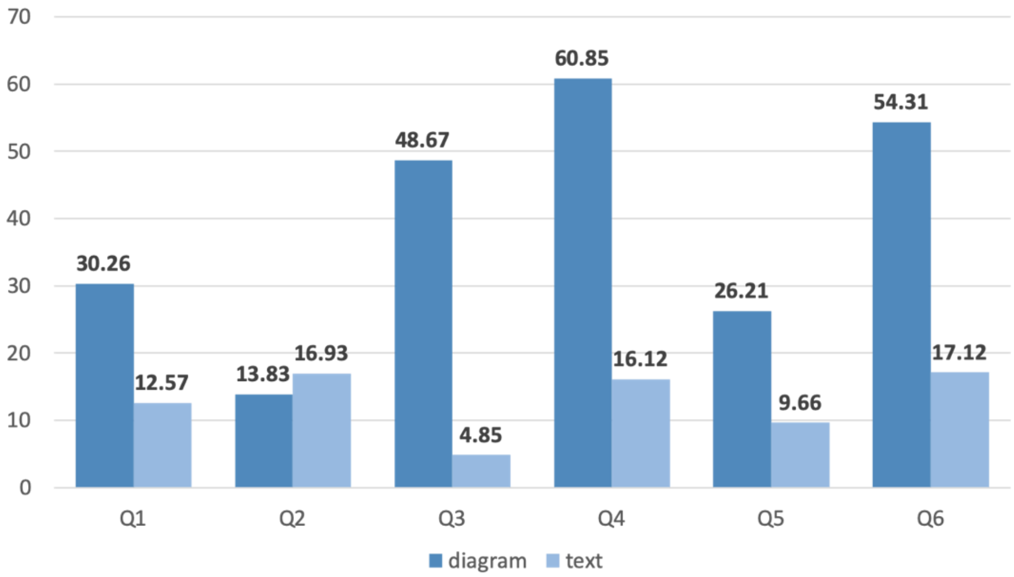


Figure 4.11. Total Fixation Durations of P6 on AOIs (seconds)

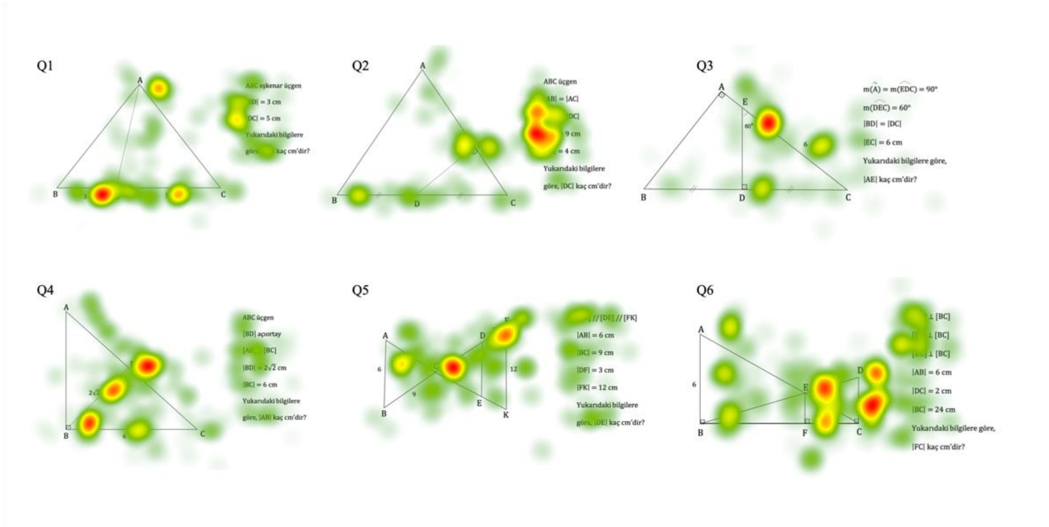


Figure 4.12. Heat maps of P6’s Visual Attention

In all questions except for Q2, Participant P6 spent most of the time looking at the diagrams. However, he spent a greater amount of time looking at the textual part than the diagram for Q2. For Q1, he spent 30.26 seconds on the diagram, and 12.57 seconds on the textual part. The heat map showed that the focus was on the area between Item 3 and Point D. He also looked at Item 5 and the place near to Point A in the diagram. As he stated during the think-aloud session, he assumed that the triangle was an equilateral; therefore, he added Items 3 and 5 in order to reach a solution.

For Q2, he spent more time on the textual part, with 16.93 seconds compared to 13.83 seconds for the diagram. The most focused upon area were the equalities given in the textual part. In the diagram, Points F and D, and the area near to Point B drew his attention. He extracted the information of an isosceles triangle from the text, as he stated in both think-aloud and interview sessions. He also found that both sides of Point F were 90°.

For Q3, he spared 48.67 seconds to look at the diagram, and 4.85 seconds to look at the text. The textual part’s fixation duration was the minimum amount of time spent among any of the textual parts. He spent most of his time looking at the area near the 60° angle, and Point D and Item 6 were other areas of his focus. In the think-aloud session, he placed angles at Points B and C according to 30-60-90 triangles.

However, he did not specify any further comments to allow for inferences to be made.

For Q4, 60.85 seconds was spent looking at the diagram, which was the highest duration for all the diagrams. He also spent 16.12 seconds looking at the textual part. According to the heat map, his attention was distributed mostly across Point D, $2\sqrt{2}$, and Point B. He also looked at line segment AC and Item 6. He mentioned the given information during the think-aloud session, and used the angle bisector to divide the 90° angle into two of equal 45°. Moreover, he regarded Point F, the most fixated area, as 90°, even though there was no statement related to it.

For Q5, he spent 26.21 seconds on the diagram, and 9.66 seconds on the textual part. The heat map indicated that the most focused upon place was Point C. Other areas of focus were Point F and the area near to Point A. As he stated during the think-aloud session, he linked the diagram with a reflection of Point C, and realized that there were some proportional line segments.

For Q6, he spent 54.31 seconds while focusing on the diagram, and 17.12 seconds on the textual part. As shown in the heat map, the areas of focus were near Points D, E, and F, and especially Point C. He also looked at the left-hand side of the diagram and at the beginning of the textual part. He stated in both the think-aloud and interview sessions that even if he mentioned the proportionality in the question, he could not make any progress because of not being able to remember the rules.

4.2.7 Eye Movement Analysis of Participant 7

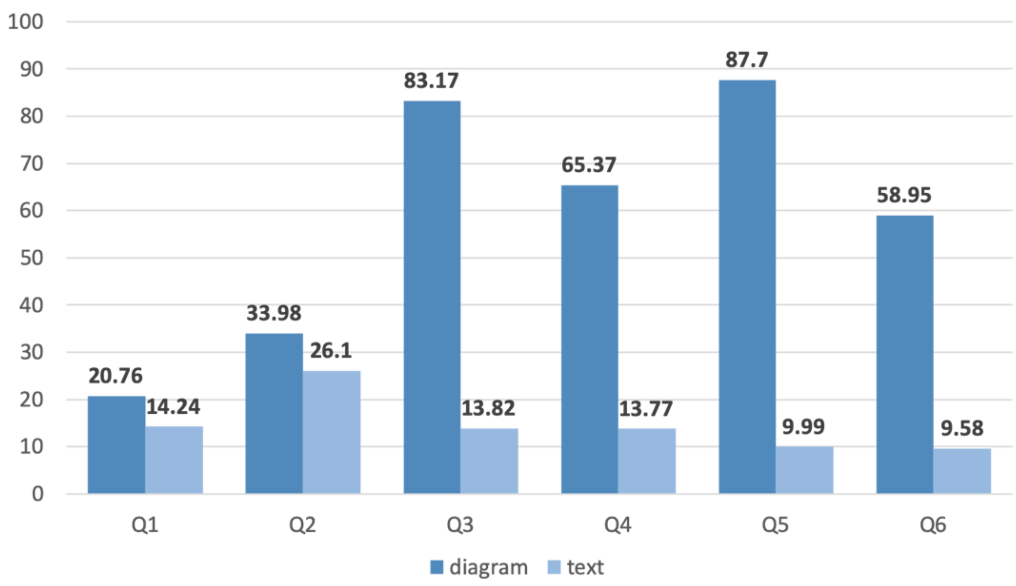


Figure 4.13. Total Fixation Durations of P7 on AOIs (seconds)

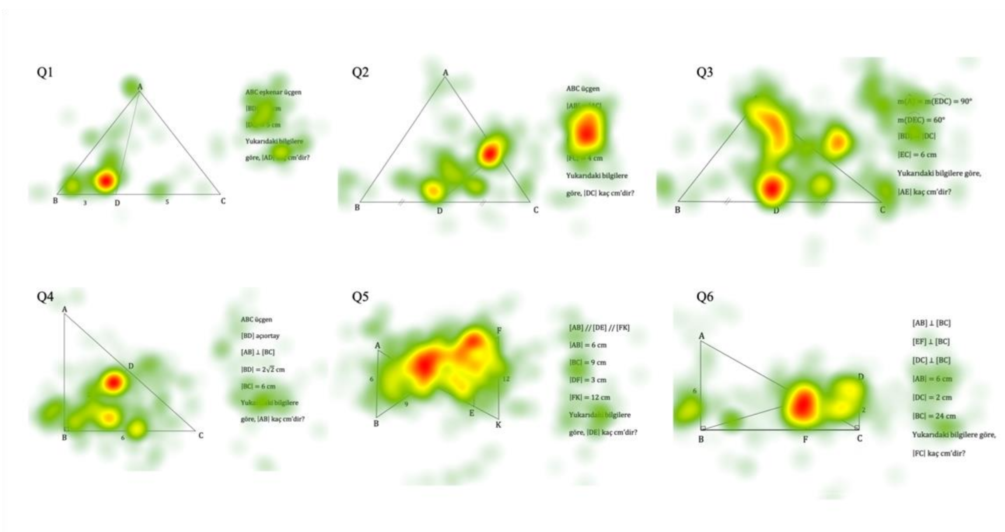


Figure 4.14. Heat maps of P7's Visual Attention

As can be seen in Figure 4.13, Participant P7 spent more time on the diagram than the textual part in all six questions. For Q1, her visual attention was distributed as

20.76 seconds on the diagram, and 14.24 seconds on textual part. When the heat map was investigated, the most focused upon area was Point D and the associated angle. She also scanned all of the textual part, and focused on Point B. She only stated that she tried to find the similarity.

For Q2, she took 33.98 seconds to look over the diagram, whilst taking 26.10 seconds to view the textual part. The heat map showed that she mainly focused upon areas around Point F in the diagram, and upon the equalities given in the textual part. The line segment DF drew her attention. As she mentioned during the think-aloud session, she tried to draw a line; whilst in the interview session, she indicated a rule that included a line segment and right angle; however, she could not remember the rule.

For Q3, she spent 83.17 seconds looking at the diagram, and 13.82 seconds looking at the textual part. When the heat map was investigated, she had focused mostly on Point D on the diagram. The line segment ED and Point A attracted her attention as well as Item 6. She explained during the think-aloud session that she looked at the right angles at Points A and D. She also placed angles of 30-60-90 triangles. She could not remember this triangle's side lengths; however, she clarified what she would do if she had remembered them.

For Q4, she spared 65.37 seconds to examine the diagram, and 13.77 seconds to read the textual part. As the heat map indicated, she fixated significantly on the area between $2\sqrt{2}$ and Point D. Mainly, her focus was on triangle DBC. When the think-aloud session was examined, she realized the angle bisector, which divided 90° into two equal 45° parts. She proceeded by assuming that Angle D was a right angle, and then applied Pythagorean theorem even though there was no right angle. She explained this assumption that she extracted it from the types of questions from the previous year.

For Q5, she spent most of her time on the diagram, with 87.70 seconds, which was the highest duration for any of the questions. She also spent 9.99 seconds looking over the textual part. The heat map showed that she spent most of her time looking at almost all parts of the diagram. The most focused upon areas were Point C and line segment CF. In both the think-aloud and interview sessions, she explained that there were similar triangles, ABC and CED, plus CED and CKF. She also stated the angles and proportional line segments; however, she could not express the proportionality numerically.

For Q6, 58.95 seconds passed while she examined the diagram, and 9.58 seconds whilst viewing the textual part. When the heat map was examined, the right-hand part of the diagram drew her attention, and mostly to line segment EF. She defined the similar triangles and ratio of similarity whilst solving the question and thinking aloud.

4.2.8 Eye Movement Analysis of Participant 8

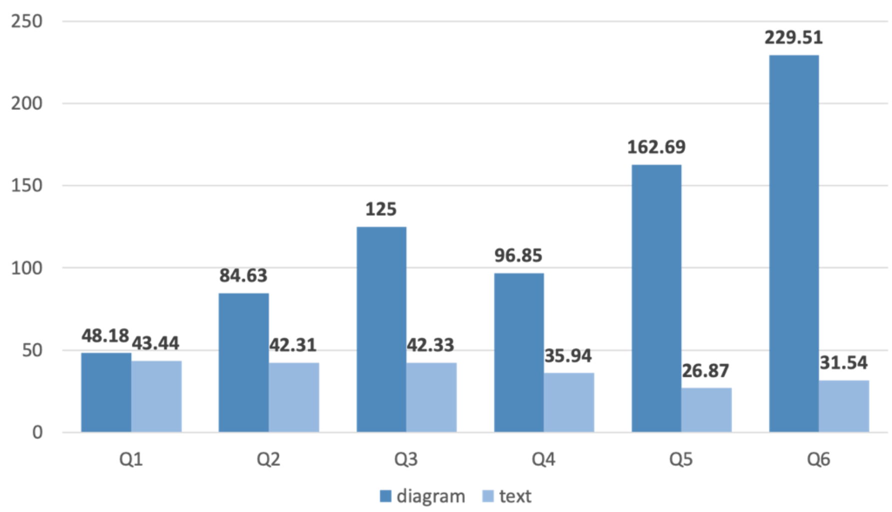


Figure 4.15. Total Fixation Durations of P8 on AOIs (seconds)

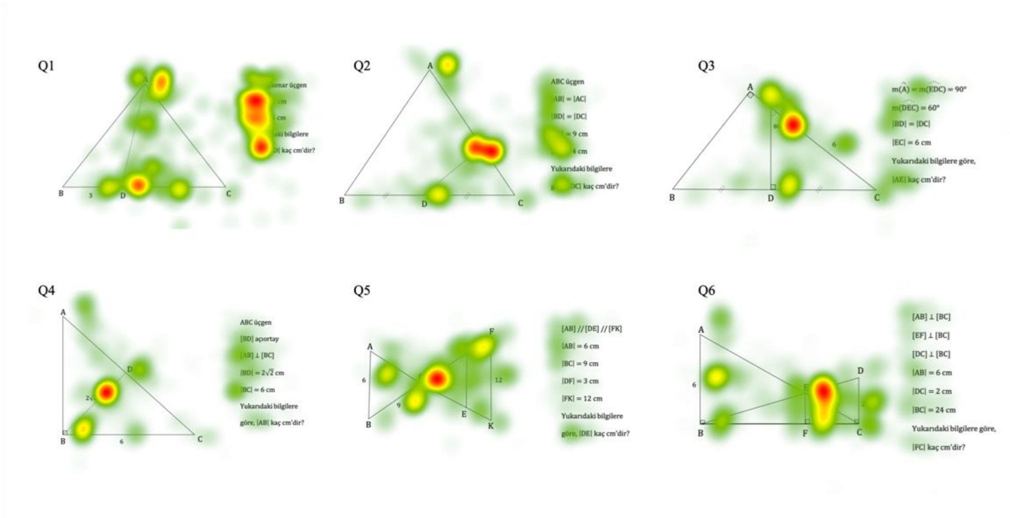


Figure 4.16. Heat maps of P8’s Visual Attention

All six questions showed higher fixation durations for the diagrams over the textual parts. For Q1, the durations for Participant P8 were closer to each other when compared to the other questions. For Q1 she spent 48.18 seconds looking at the diagram, and 43.44 seconds looking at the textual part. When the heat map was investigated, she focused on the information in the textual part. On the diagram, Points D and A drew her attention. As she stated during the think-aloud session, she tried to apply Pythagorean theorem; however, she paid no attention to where the right angle was. In the interview session, she mentioned an attempt to draw a line.

For Q2, her attention was gathered with 84.64 seconds on the diagram, and 42.31 seconds on the textual part. As seen in the heat map, the most focused upon area was Point F. She also focused on Points A and D on the diagram, and upon the given lengths in the textual part. In the think-aloud session, she only expressed the given information from the text and diagram.

For Q3, she took 125 seconds to look over the diagram part and 42.33 seconds for the textual part. The heat map showed that she fixated mostly on the 60° angle. She also focused on the areas near to Points A and D. In the think-aloud session, she highlighted the right angles and placed angles 30° and 60° on the diagram; however, she could not remember the side lengths of the 30-60-90 triangle. Even if she expressed that the opposite side of 90° was the longest side and the opposite side of 30° was the shortest, she could not find the exact ratios.

For Q4, she spent 96.85 seconds looking at the diagram, and 35.94 seconds on the textual part. The heat map demonstrated that the most focused upon area was $2\sqrt{2}$. She also looked at Points B and D. As stated during the think-aloud session, she used the angle bisector and divided Angle B into two equal 45° parts. She also described her attempt to apply Pythagorean theorem; however, she did not realize that there was no right angle.

For Q5, she spent 162.69 seconds looking at the diagram, and 26.87 seconds on the textual part. As seen in the heat map, she tended to focus on the diagram, especially Point C. The line between Item 6 and Point F also drew her attention. She mentioned the parallel line segments during the think-aloud session; however, she could not progress with the numerical relations in proportionality.

For Q6, she allocated most of her time to analyzing the diagram, with 229.51 seconds, whilst she took 31.54 seconds to look at the textual part. When the heat map was examined, the focus was mostly on Point E and line segment EF. In the think-aloud session, she stated the equal ratios between line segments and the proportionality; however, she could not express this proportionality numerically.

4.3 Problem-Solving Strategies

As previously stated, the current study aimed to reveal problem-solving strategies used during geometrical problem solving. Therefore, the transcripts of the think-aloud protocol and participant interviews were analyzed in order to classify and describe the applied strategies. Table 4.2 and Table 4.3 show the number of strategies used by the participants and in each of the six questions. Participant statements from the think-aloud and interview sessions regarding the strategies employed were clarified according to explicit strategy categories.

Table 4.2. Number of Strategies Used by Participants

	Holistic Strategies		Analytic Strategies		
	Mereologic Way	Visual Strategy	Preproportional Reasoning	Qualitative Proportional Reasoning	Quantitative Proportional Reasoning
Participant					
P1	4	2			2
P2	3		1		3
P3	1	1	1		
P4	3		2		1
P5		2	2		
P6		1	2		
P7	1		2		1
P8	1		1	2	
Total	13	6	11	2	7

Note: Total number of questions = 6. More than one strategy may be used in one question.

Table 4.3. Number of Strategies used in Questions

	Holistic Strategies		Analytic Strategies		
	Mereologic Way	Visual Strategy	Preproportional Reasoning	Qualitative Proportional Reasoning	Quantitative Proportional Reasoning
Question					
Q1	5	1			
Q2	4				
Q3	2		3		1
Q4	2		2		
Q5		4	3	1	3
Q6		1	3	1	3
Total	13	6	11	2	7

4.3.1 Holistic Strategies

As Gluck and Fitting (2003) defined, holistic strategies include “representing and manipulating spatial information in a spatial way” (p. 297). Visualization is mostly attached to these strategies along with other spatial relations. Holistic strategies have two subcategories, which are mereologic and visual in this study. Therefore, students’ holistic strategies were analyzed according to these subcategories.

4.3.1.1 Mereologic Way

The mereologic way is one of three operations that Duval (1999) specified in order to modify a figure. It allows for dividing a whole figure into parts, and also to gather them together in order to construct a new figure. It is also known as "reconfiguration." In the current study, the mereologic way appears as dividing a triangle into two parts by drawing a height.

For Question 1,

P1 explained her strategy during the interview as;

> [P1]: There is an equilateral triangle. A line that I drew from Vertex A must be height, median, and also angle bisector.

Similarly, P2 and P3 explained their strategies during their interviews and think-aloud protocols, respectively as;

> [P2]: Equilateral triangle. I drew a perpendicular line. A right triangle occurred there.

> [P2]: I drew a height. When I drew the height, it divided into two 30-60-90 triangles.

> [P3]: I tried to draw a perpendicular line.

> [P3]: If we draw a perpendicular line, this is 8 and this is 30 from 30-60-90 triangles.

P4 specified his solution strategies the same way in both the interview and think-aloud processes as;

> [P4]: I drew a perpendicular line from Point A to the base.

In her interview, P8 expressed how she was thinking of dividing the triangle by a line as;

> [P8]: Since it is equilateral, as I remember when we draw a line, 3 and 5 have the same side.

For Question 2,

P1 and P2 mentioned their solution strategies in both the interview and think aloud sessions as;

> [P1]: The sides AB and AC are equal, so this is an isosceles triangle. Again, in this isosceles triangle, the line I drew gave me the height, median, and angle bisector.

> [P1]: $\overline{AF}$ 9 cm, $\overline{FC}$ 4 cm, $\overline{AC}$ 13 cm. AB and AC isosceles triangle, so AB is also 13 cm. 13 and 13. Then I can draw a line perpendicularly. It is height, median, and angle bisector.

> [P2]: Here, I drew a perpendicular line again.

> [P2]: What is $\overline{DC}$? $\overline{BD}$ is equal to $\overline{DC}$. $\overline{AB}$ is equal to $\overline{AC}$, so this is an isosceles triangle. If we draw a perpendicular line, then $\overline{AF}$ is 9 and $\overline{FC}$ is 4.

P4 stated his way of thinking in the interview as;

> [P4]: In this question, I drew a perpendicular line once again.

While problem solving, P7 expressed her attempt to solve this question as;

> [P7]: I drew a perpendicular line, but I wasn’t sure.

For Question 3,

During the think-aloud process, P1 mentioned a line that she drew from Vertex A to Point D. She also found the length of this line while analyzing the question; however, she did not use this information whilst reaching her solution. Her other attempt also related to this strategy in drawing a line from Vertex B to Point E in order to complete triangle BEC.

> [P1]: If I drop a line from Vertex A to Point D, I will extract a theorem. If I didn't find it incorrectly, it is $3\sqrt{3}$. Does it bring something if I complete this triangle? When I draw a line from Vertex B to Point E, I found it to be 6 cm; but I think I'm getting off the point.

P1 also made a statement about the solution to this question during her interview;

> [P1]: Here, if I draw a line from the right angle, it extracts a theorem.

In P4's interview, the strategy was explained as;

> [P4]: First, I tried to draw a perpendicular line for this question.

For Question 4,

While solving the problem and thinking aloud, P1 mentioned drawing a line to divide the triangle into parts; however, she didn't continue since this line wouldn't divide the side of the triangle into two equal parts.

> [P1]: If I drew a perpendicular line, it wouldn't divide the triangle equally.

During the think-aloud process, P2 tried to solve this question in a different way. At the beginning of the question, he was in a dilemma about whether or not the figure was a square, stating that;

> [P2]: Angle bisector divides the angle as 45°/45°. If we draw a line from here and call it as height, in the middle part, does it create a square when these are

perpendicular? Yes, it creates a square. Let's solve it differently. No, it does not create a square.

During the interview of P2, an additional question was asked by the researcher, "Did you analyze the question well?", to which P2 responded;

[P2]: I thought I did my best. I had a lot of thoughts, but I couldn't explain them exactly or talk enough about it. Actually, I started to interpret myself because I thought like this; If a draw a line here, a quadrilateral appears.

4.3.1.2 Visual

Lamon (1993) considered visual strategy as a kind of trial or error process. If students give answers without clarification about how they reached that conclusion or drew an analogy according to the visual properties of a figure, the presence of this strategy can be mentioned. In the current study, the students may have made judgements based on what a figure looked like.

For Question 1,

P5 associated the solving strategy with a comma in the interview, stating;

[P5]: When I looked at the diagram, I remembered that we were doing something like the comma rule in the questions that we solved last year.

While solving problems and expressing her thoughts during the think-aloud session, P5 used the comma association again, and she also used the word "hat" while referring to her solution attempt.

[P5]: There is something similar to the comma rule. I remember we drew a hat.

For Question 5,

P1 associated similar triangles with an hourglass, and stated that;

> [P1]: I looked at the small hourglass.

P3 linked similar triangles to a bowtie, and stated his thoughts respectively in both the interview and the think-aloud processes as;

> [P3]: I clearly saw a bowtie.

> [P3]: There is a bowtie rule.

P5 related triangles to the letter Z, and mentioned as such in the interview and solving processes as;

> [P5]: I first saw the letter Z.

> [P5]: We can draw a Z here.

P6 indicated the use of visual judgment as;

> [P6]: This is a kind of reflection of C.

For Question 6,

Again, P1 used the hourglass to describe the combination of similar triangles, and stated in the interview that;

> [P1]: At the same time, when I looked at the big hourglass, there is actually a right triangle.

4.3.2 Analytic Strategies

Analytic strategies, on the other hand, are defined as to “represent and manipulate spatial information by reducing it to an essentially nonspatial, list-like format”

(Gluck & Fitting, 2003, p. 297). They also added that analytic strategies decrease the difficulty of spatial information; therefore, they require more time and less effort compared to holistic strategies.

4.3.2.1 Preproportional Reasoning

Preproportional reasoning relies upon intuition and sense-making activities.

For Question 3,

P4 indicated the sense of proportionality during the interview as;

> [P4]: I saw that these two have a certain proportion.

P5 stated the presence of proportionality during the interview, stating that;

> [P5]: In this question, 90 and 60 were given. So, the opposite, Point C is 30. There is something like the opposite side of 90 is 2x or the opposite side of 60 $2\sqrt{x}$.

P8 expressed her thoughts about the solving process and mentioned about the 30-60-90 triangle. She answered an additional question posed by the researcher, "Did you remember the side property of a 30-60-90 triangle?" as;

> [P8]: The property that I remember is related to size. The opposite side of 90 is the biggest side. The opposite side of 30 is the smallest. Also, these two sides are equal. I thought that I might use a proportion from there.

For Question 4,

P2 expressed the way of solving during the interview as;

> [P2]: Actually, I probably would use similarity. This is a right triangle.

P7 was unable to solve the question because she could not remember the side lengths of a 30-60-90 triangle. However, she mentioned that she could have resolved the question if she had remembered that information;

> [P7]: I would use similarity between the triangles ACB and DCE. Then, I would find the long side of the ABC triangle and the side AC by using the similarity ratio.

For Question 5,

P5 tried to analyze why she could not solve the question, and in doing so, answered an additional question asked by the researcher during the interview, “Did you feel inadequate because you could not apply the rule?” as;

> [P5]: Yes, there is again a proportionality rule here.

P6 stated in the interview that;

> [P6]: $\overline{\mathrm{BC}}$ and $\overline{\mathrm{CF}}$ are proportional to each other.

P7 showed a strong ability in analyzing the similarity of triangles and proportionality; however, she was unable to demonstrate an understanding of the numerical relations of a proportion during the think-aloud process, saying;

> [P7]: There is a similarity between the sides, so I will equalize the angles. I will use the ratio. There are similar triangles between the triangles ABC and CDE. The angles BAC and DEC are equal. The other angles in the triangles are equal. The triangles ACB and ECD are equal. I found this equality.

For Question 6,

P3 stated while thinking aloud that he did not have too many things to say; however, he mentioned about his guess, saying;

[P3]: Something related to proportionality might be applied here, but I could not remember how to do it.

P4 used the language of ratio and illustrated his thinking during the interview and the think-aloud sessions as;

[P4]: I tried to use similarity because there are specific ratios, and these are parallel, DC and EF. Since AB and EF are parallel, I saw that these are also parallel. But, I couldn't see where to find the ratio. So, I couldn't do the rest.

[P4]: DCB triangle is similar to EFB. At the same time, ABC triangle is similar to EFC. But I couldn't solve this question.

P6 answered an additional question posed by the researcher, "Did you notice any information?", stating that;

[P6]: I supposed that the Angle A and Angle D were proportional or something happens.

4.3.2.2 Qualitative Proportional Reasoning

Qualitative proportional reasoning can be mentioned if students use ratio as a unit and relative thinking, and understand some numerical relations (Lamon, 1993).

For Question 5,

P8 used ratio as a unit and stated some numerical relations during the think-aloud session, saying

[P8]: All lines are parallel. $\overline{AB}$ is 6 and $\overline{FK}$ is 12, so it is twice the length. If the ratio of $\overline{CB}$ and $\overline{CK}$ is $\frac{1}{2}$, $\overline{CK}$ might be 18 and $\overline{CE}$ might be 15.

For Question 6,

P8 tried to use some numerical reasoning during the think-aloud process, as;

> [P8]: The ratio between $\overline{AB}$ and $\overline{EF}$ is the same as the ratio between $\overline{BF}$ and $\overline{FC}$. The ratio between $\overline{DC}$ and $\overline{EF}$ is equal to the ratio between $\overline{CB}$ and $\overline{FB}$. 24 and 6 is proportional to $\overline{FC}$ and $\overline{EF}$. Both 6 and 2 will be proportional to 24. I don't know.

4.3.2.3 Quantitative Proprotional Reasoning

Quantitative proportional reasoning strategy implies a full understanding of functional and scalar relationships to use algebraic expressions standing for proportion (Lamon, 1993). This is included in the code list of the strategies because almost half of the questions are related to similarity, which thereby requires proportional reasoning.

For Question 3,

During the interview, P2 stated a way of using similarity, saying that;

> [P2]: Here, I mostly used trigonometry, but also similarity. First, this is 30° because of similarity. Because it says 30-60-90. Since this is 90° and a right triangle, this is 60°. Since it says that these two are equal, by using trigonometry the hypotenuse is 6. Opposite side of 30° is 3 because sin30 is $\frac{1}{2}$. Since sin60 is $\frac{\sqrt{3}}{2}$, we divided it into 2 and multiplied by $\sqrt{3}$. It is $3\sqrt{3}$. This is $6\sqrt{3}$. Since the hypotenuse is $6\sqrt{3}$, I used trigonometry. The opposite side of 60° is $6\sqrt{3}$. $6\sqrt{3}$ times sin60 equals 9. I subtracted 6 from 9 and found 3.

For Question 5,

P1 used an algebraic expression to state the proportion. She applied similarity rules twice with two different pairs of similar triangles and expressed them in the interview and also in the think-aloud session as;

[P1]: I applied similarity rule here. The ratio of 6 to 12 would give me the ratio of $\overline{BC}$ to $\overline{CF}$. It equals to $\frac{1}{2}$. From the ratio $\frac{9}{x}$, I found $\overline{CF}$ as 18 cm, so $\overline{CD}$ equals to 15 cm. Then $\overline{DE}$ was asked. I looked at the small hourglass. The ratio of 9 to 15 must equal to the ratio of 6 to $\overline{DE}$. Then I found that it is 10.

[P1]: These are parallel to each other. I would probably proceed with this similarity. The ratio of 6 to 12, $\frac{1}{2}$ should be equal to the ratio of $\overline{BC}$ to $\overline{CF}$. So, $\overline{CF}$ must be 18. So, $\overline{CD}$ is 15. When $\overline{CD}$ is 15, the ratio of 6 to 15 gives me the ratio of 9 to $\overline{DE}$.

During the think-aloud session, P1 realized that proportion $\frac{6}{15} = \frac{9}{\overline{DE}}$ was incorrect because the product of 15 and 9 is not divisible by 6. Therefore, she checked her findings and wrote the true proportion as $\frac{9}{15} = \frac{6}{\overline{DE}}$.

P2 explained the use of SAS Similarity Theorem twice in the interview and think-aloud sessions, saying that;

[P2]: First, there is a similarity between ΔABC and ΔCFK. The ratio is 2. Then from the similarity this is 18 (referring to $\overline{CF}$). From the similarity of ΔCDE and ΔCFK, this is 15 by subtracting 3 from 18. The ratio of similarity is $\frac{18}{15}$ and it equals $\frac{6}{5}$. We said it equals to $\frac{12}{x}$. We saw it is twice so *x* is 10.

[P2]: Similarity. Now these two are similar triangles. ΔABC and ΔCKF and ΔCED. On similarity, ΔABC and ΔCKF is 2 times of each other. So, the length of the side CF will be 18 because of 9 times 2. So, $\overline{CD}$ will be 15. If it will be 15, $\frac{18}{15}$ is our ratio of similarity, ΔCKF and ΔCDE. When we put this ratio $\frac{18}{15}$ to $\frac{12}{x}$, $\frac{18}{15}$ equals to $\frac{6}{5}$ so $\overline{DE}$ is 10.

P4 specified his thinking process during the interview and think-aloud sessions, saying that;

> [P4]: I used similarity here. It gave parallel angles. I first marked them. Then, I saw that it was between 6 and 12. So, I found $\frac{1}{2}$ as a ratio. I found these sides depending on the ratio. 9 times 2 equals 18. I saw that this was 15. The ratio of 9 to 15 equals the ratio of 6 to *x*. I found the answer.

> [P4]: Since $\overline{\mathrm{AB}}$, $\overline{\mathrm{DE}}$ and $\overline{\mathrm{FK}}$ lines are parallel, there is a ratio between $\overline{\mathrm{AB}}$ and $\overline{\mathrm{FK}}$, $\frac{1}{2}$. To find the line CD, we should calculate twice the BC side. $\overline{\mathrm{CD}}$ equals 15. The ratio of 9 to 6 equals the ratio of 15 to *x*, and *x* gives DE the side. I cross-multiplied and found the result 10. $\overline{\mathrm{DE}}$ equals 10.

For Question 6,

P1 used variables to apply the ratio and stated her opinions in the interview and think-aloud sessions, stating that;

> [P1]: I applied the same ratio here, but this time I gave symbols such as *m* since I didn't know the numbers. This is *m* and this is 3*m* (referring to $\overline{\mathrm{EC}}$ and $\overline{\mathrm{EA}}$, respectively). There actually is a right triangle and there is a similarity too. Since they are perpendicular, they will give the similarity. I named them as 3*a* and *a* (referring to $\overline{\mathrm{BF}}$ and $\overline{\mathrm{FC}}$, respectively). 4*a* equals 24. I equilazed and found *a* as 6. This is 6 so this is 18.

> [P1]: Now, I will name $\overline{\mathrm{ED}}$ as *m* and $\overline{\mathrm{BE}}$ as 3*m* because the ratio of $\overline{\mathrm{DC}}$ to $\overline{\mathrm{AB}}$ gives me $\frac{1}{3}$. It says $\overline{\mathrm{BC}}$. $\overline{\mathrm{AB}}$ is 6 and $\overline{\mathrm{BC}}$ is 24. There will be a similarity. I named $\overline{\mathrm{BE}}$ [3*m*] and $\overline{\mathrm{ED}}$ as *m*. These are also equal to the ratio of $\overline{\mathrm{BF}}$ to $\overline{\mathrm{FC}}$, so if we say *a* and 3*a*, 4*a* equals 24. *a* is equal to 6.

P2 stated the application of similarity theorem twice during the interview and the think-aloud session, stating that;

> [P2]: It says this is 24. ΔABC and ΔEDC are similar. The ratio is 3 so this is three times that. Then this is divided into two, as 3*x* and *x*. At the same time, ΔDBC and ΔEBF are also similar. Instead of applying similarity directly, since the ratios are the same, $\frac{\mathrm{BF}}{\mathrm{FC}}$ equals $\frac{\mathrm{BE}}{\mathrm{ED}}$. Since these are 3*x* and *x*, I called these sides as 3*k* and *k*. I divided 24 by 4 because the coefficient is 4. It is 6. Since the coefficient of *k* is 1, *k* equals 6.

> [P2]: It says $\overline{\mathrm{BC}}$ is 24. Now, first, we can form a similarity from there. ΔABC and ΔEDC are similar triangles. The ratio of similarity is 3 times each other. Then, we call the side BE as 3*x* and the side ED as *x*. If the length of BC is 24, since it is 3*x* and *x*, BF will be 3*k* and FC will be *k*. So, we divide 24 into 4. It is 6. Since *k* is equal to *k*, the answer is 6. $\overline{\mathrm{FC}}$ equals 6.

P7 had an attempt to use ratio as a unit and apply arithmetic to the unit; therefore, her strategy can be regarded as an example of quantitative proportional reasoning. However, since she could not apply ratio properly and find the correct side lengths, her solution is considered to be a lower-level strategy. She stated while thinking aloud that;

> [P7]: $\overline{\mathrm{BC}}$ is 24. There is a similarity between triangles ABC and DCB. They are similar triangles. There is also a similarity between the triangles ABC and EFC. The ratio of similarity is $\frac{2}{6}$ so $\frac{1}{3}$. We don't know anyting else. We know $\overline{\mathrm{BC}}$ is 24. $\frac{1}{3}$ equals $\frac{\mathrm{x}}{24}$. So, we established that *x* is 8. It is $\overline{\mathrm{BD}}$.

4.4 Summary of the Results

The results of Spatial Ability Test (SAT) are summarized below.

- P1 took the second highest score from the SAT. She is the one who answered all of the items. Her PFT score was the lowest score within all PFT scores.
- P2 had highest scores in all parts of the test except CCT.
- P3 gave correct answers to more than half of the items.
- P4 answered more than the half of all parts. He had the second highest score in PFT.
- P5 had the lowest total score of SAT as well as PFT and CRT.
- P6 had the second lowest scores in total compared to other results. He had the lowest score for SDT.
- P7 got the highest score for CCT.
- P8 answered correctly more than the half of the items for each part same as the all participants except P5 and P6.

The results of problem-solving strategies are summarized below.

- One participant focused more on the text part than diagram part in the second question.
- Participants preferred the merelogic way mostly and used preproportioanl reasoning at least once.
- Participants with higher SAT scores mostly used the mereologic way, three times and more.
- Participants who did not use the mereologic way took lower scores of spatial ability test. They used visual and preproportional reasoning to solve questions.
- Participants with first three highest scores used more strategies than those with lower scores. They used three different types of strategies among all strategies.

- For the equilateral triangle, the isosceles triangle, and right triangle problems, participants tended to use mereologic way and preproportional reasoning.
- For similarity problems, they preferred visual strategy, preproportional reasoning, and quantitative proportional reasoning.
- Some participants tried to use preproportional reasoning because they could not express the proportionality with numbers and ratios.
- Only one participant preferred qualitative proportional reasoning. She had a sense of proportionality and understanding of some numerical relations; however, she could not ensure the overall proportionality and use numbers to express it.
- Some participants used appropriate strategies; however, they could not apply them since they did not remember the crucial information.
- Some participants had an attempt to memorize the question type and solution ways of these questions; therefore, they did not focus on analysis of the question or use of geometric knowledge.
- Some participants tended to look for familiar patterns and figures such as a right angle, equal lengths, isosceles triangle and equilateral triangle. When they were not obviously stated in the question, they attributed some label to geometric figures without proof or justification and continued to solve questions.

The results of eye-movements measures are summarized below.

- Almost all participants focused more on the diagram AOI than the text AOI.
- Participants fixated more on diagram part to find geometrical and numerical relations.
- Mental manipulation and proportional reasoning are related to longer fixation durations in the diagram part.
- One participant spent more time on text part while reading aloud the given equalities since he could not solve this question.

- Some participants did not spend considerable time in some questions since they could not create a strategy and solution idea; therefore, they skipped these questions.
- Participants mostly focused on angles, numbers, angle bisector, and points crucial to solve problems in the diagram part.
- Participants mostly focused on given information such as equal, parallel, perpendicular, and asked line segments in the text part.
- Participants referenced the prerequisite theorems and rules depending on the order and spent more time on these properties.
- Participants had higher fixation durations on the diagram parts due to modifying the diagram, remembering the rules and theorems, mental calculations, trying alternative methods, crosschecking, and finding crucial properties.

CHAPTER 5

DISCUSSION AND CONCLUSION

The purpose of this study was to examine the problem-solving strategies of participants and changes in their eye movement measures based on six different geometry problems they were presented with, each with different parts (i.e., diagrammatical and textual). This chapter presents a discussion of the results, as well as the implications and limitations of the study, plus recommendations for future studies in this area of research.

The first section discusses the problem-solving strategies employed by the participants, whilst the second discusses the participants' eye movement changes during the problem-solving process. A conclusion is then offered, along with suggested implications, the study's potential limitations, and also recommendations for future studies.

5.1 Discussion

Problem-solving ability is an essential skill in both mathematics and geometry, as well as a number of other domains. Increased efforts to assess this ability in learners, and the acquisition of knowledge about the components and processes involved is suggested to result in greater understanding and new opportunities to support the instruction of students.

Eye-tracking technology is a widely used method in the mathematics domain, since the eyes play a fundamental role in the comprehension of visual information (Strohmaier et al., 2019). Accordingly, the current study has targeted the examination of geometrical problem-solving processes applied by high school students.

Overall, the study's results showed that the participants focused more on the diagrammatic element of the problems they were presented with, rather than the textual part, for several specific reasons. The participants employed various strategies for different problems and in different ways. Data collected from eye tracking was supported by integrating think-aloud and interview processes in order to reveal possible explanations, and to elicit and add further understanding regarding additional eye movement inclinations of problem solvers. As Hyönä (2010) suggested, complementing eye tracking data with other methods contributes to the potential to reveal more detailed explanations. These specific and detailed explanations regarding problem-solving processes may be applied to enhance geometry learning and instruction in the future.

5.1.1 Participants' Problem-Solving Strategies

The study investigated the strategies employed by the participants whilst solving geometry problems through data gathered from interview and think-aloud sessions. Problems having more than one potential solution and requiring the application of mental processes were selected for the study. The problems could be classified according to four categories; as related to equilateral triangles, isosceles triangles, right triangles, or similarity problems. The first problem task presented to the study's participants was an example of an equilateral triangle problem, while the second was an isosceles triangle problem. The third and fourth problems were based on right triangles, while fifth and sixth were similarity problems. However, there was no strong distinction between the problem types due to the overlapping geometric properties involved.

The participants employed different strategies in different ways for different questions. In the study, the strategies used during problem solving were categorized as being either holistic or analytical. Under holistic strategy, both mereologic way and visual strategy were examined. Under analytical strategy, preproportional, qualitative proportional, and quantitative proportional strategies were investigated.

The results gathered through the think-aloud and interview sessions indicated that the mereologic way was the most preferred strategy employed by the participants during their problem solving. Participants who had higher spatial ability scores from their SAT mostly employed the mereologic way as a holistic strategy. This finding is in line with the results presented by Gages (1994), in that holistic strategies were preferred by participants who achieved higher spatial ability scores. The participants with the top three highest SAT scores also employed more strategies than those with lower SAT scores. The participants with higher spatial ability scores employed three different types of strategies. This finding is in accordance with the result that participants good at spatial skills have an access to a wide range of spatially based strategies (Casey et al., 2017). They also employed the mereologic way on three or more occasions. The mereologic way was seen when the participants mentally manipulated the given geometric figure, and participants with higher SAT scores employed this strategy the most. Participants who did not opt to use the mereologic way generally achieved lower spatial ability test scores, and instead used visual and preproportional reasoning in their problem solving.

According to the findings of the participants' think-aloud and interview sessions, different strategies were employed in solving the six geometry questions. For the equilateral, isosceles, and right triangle problems, the participants tended to use the mereologic way and preproportional reasoning. However, for the similarity problems, the participants preferred visual strategy, preoperational reasoning, and quantitative proportional reasoning. Some of the participants' problem solving were not at the qualitative and quantitative proportional reasoning level; therefore, they tried to use preproportional reasoning since they could not express the proportionality with numbers and ratios. The results of the study indicate that preproportional reasoning was used at least once by seven of the eight participants; however, qualitative proportional reasoning was the least preferred strategy, except for one participant who preferred it as they could not use quantitative proportional reasoning. This means that the participant had a sense of proportionality and understanding of some numerical relations; however, they were unable to ensure the overall proportionality or use numbers to express it.

In some cases, the participant students could not achieve the correct answers since they missed crucial geometric information in applying their strategies, as was also reported by Lin and Lin (2014a). In their study, Lin and Lin specified hidden elements and the mental rotation of a triangle as two major reasons for participants experiencing difficulties whilst solving geometry questions. In the current study, the reasons behind some participants' inability to apply a strategy and solve a question may be drawn from the interview or think-aloud transcripts, as some participants repeated being unable to remember the question type or how to solve them. This means that they focused on the solution instead of analyzing the question or the use of geometric knowledge. Moreover, some of the participants tended to look for familiar patterns and figures such as a right angle, equal lengths, or an isosceles or equilateral triangle. When they were not stated obviously in the question, they attributed some labels to geometric figures without proof or justification, and then continued to attempt to solve the question. These findings may be related as some students paid attention to recognizing and remembering the question type while looking at the diagram; therefore, they may not have noticed the more important aspects of the questions or fully analyzed the question that well or even correctly. This result is consistent with the finding that the more experienced students had certain expectations about the placement of values and labels (Goldberg & Helfman, 2011). These expectations can be twofold. While they might help in the reading of the diagram, and also the text in some cases, the participants may have been misdirected by such information, as was the case in the current study.

While the use of strategies led the participants to a solution, they sometimes had difficulty with the solving process itself. Although some solution strategies that the students employed were appropriate to the problem task in hand, the students missed some of the important geometrical information that was available for the application of these strategies in some cases. In problems that required a line to be drawn, some of the participants thought about it; however, they could not figure out the rest.

The participants frequently expressed that they did not remember crucial information. Critical properties and rules related to problems used in the study were the Pythagorean Theorem, the side lengths of 30-60-90 and 45-45-90 triangles, and Euclid's theorem. Moreover, attributing labels to geometric figures without proof or justification misdirected some of the students whilst attempting to solve some of the problems. For example, assuming a line was perpendicular or that a triangle was equilateral or isosceles without adequate analysis resulted in an incorrect solution being put forward.

5.1.2 Participants' Eye Movement Measures

The current research investigated the participants' visual attention distributed across different problems and parts of the problems. As an expected result of the study, all participants except for one, and in only one question, focused more upon the diagram AOI than the textual AOI. This finding is in accordance with the result of study conducted by Lin and Lin (2014b). Regarding the mathematics, as Duchowski (2007) stated, the more complex aspects of a problem will result in increased fixation durations and counts. Similarly, the parts of problems that required cognitive processes caused greater fixation counts and longer dwell time (Just & Carpenter, 1976). In the current study, the longer duration finding for the diagram part of the problems may be interpreted as participants having spent time looking for geometrical or numerical relations. This is in line with the findings that participants had higher dwell time on the illustration of the problem since they searched for a relationship between the task and solution method (Norqvist et al., 2019). Some problems used in the current study required manipulation of the given diagram in the equilateral, isosceles, and right triangle problems, and the use of proportional reasoning in the similarity problems. This result may also be interpreted as mental manipulation and proportional reasoning being related to longer fixation durations in the diagram part of the tasks. This finding is in accordance with the result that participants fixated more on the area, requiring mental manipulation to solve a task (Galili et al., 2020).

According to the results of the eye tracking data related to AOIs, only one participant focused more upon the textual part than the diagram in the second question. The interview and think-aloud results went on to support this finding, that whilst solving the question, the participant read aloud the given equalities in the text. One possible reason is that inability to extract crucial information from a diagram can lead to the search for useful alternative information in the text. This finding is in line with the results of a study by Lin and Lin (2014b), in that participants who were unsuccessful in solving problems tended to look more at the text AOI for some questions.

As Lin and Lin (2014b) proposed, problem solvers are inclined to seek essential information generally placed in the diagram when they experience difficulty, as shown through solvers' fixations and references gathered in the diagram area. Lin and Lin also revealed that fixation counts, dwell time, and run counts were positively correlated with perceived difficulty in the figure area. Andrà et al. (2015) also associated longer fixation durations with the difficulty. Parallel to these findings, the current study concluded that when some participants experienced difficulty in certain problems, they focused more upon the diagram in their search for a solution. On the contrary, skipping some questions resulted in very low fixation durations because some participants had no idea about creating a solution in the current study. Therefore, longer fixation duration cannot always be explained as evidence of difficulty experienced by a participant, or by the complexity of the question. The current study's findings indicated that participants had higher fixation durations on the diagrams due to their modifying the diagram, remembering the rules and theorems, mental calculations, trying alternative methods, crosschecking, and finding crucial properties according to the think-aloud and interview results. It can be concluded, therefore, that there may be several reasons for participants' viewing behaviors, and that these depend on the problem characteristics and individual differences.

With respect to visual attention, Just and Carpenter (1980) considered the time spent looking at an object as the time spent thinking about it. Separating relevant areas from the irrelevant is important in problem solving. Knoblich et al. (2001) indicated that when essential task-related areas were referenced, problem solvers, and even novices, could probably complete a task successfully. The main areas of focus on the diagrams were angles, numbers, angle bisector, and points crucial to solve the given problems. The participants mostly focused on the given information, such as equal, parallel, perpendicular, and asked line segments in the textual part. This finding is in accordance with the study by Hegarty et al. (1992), which pointed out that key information and concepts such as variables and numbers required longer fixations. Therefore, problem solvers should pay attention to learning to read a given task properly, and to distribute their attention in order to extract relevant information, as stated by Andrà et al. (2009). As an explanation to longer scan paths, Goldberg and Helfman (2011) stated that longer durations operations may be required, and that the places and orders of the values are important. Prerequisite theorems and rules to solve questions used in the current study depend on the order and specific proportion; hence, the structural properties of the required information were seen in the heat maps.

The findings of the current study indicated that visual attention distributed across different areas might have various explanations related to mental processes, as put forwards by Schindler and Lilienthal (2019). In the case of the current study's P1, the reason for spending most of the time on the third question was that she tried to manipulate the diagram by drawing lines. When she drew the first one, she found additional information. In the second attempt, she acquired a solution; however, this question could also be solved without modification. Another finding was that no opportunity to perform operations on the questions resulted in more time spent on them. Just and Carpenter (1976) suggested that a higher fixation count was related to cognitive processes. In the current study, the mental calculations used to solve questions on a computer required longer fixations.

It may be concluded, therefore, that the current study's participants experienced certain difficulties that may offer an explanation for the longer fixation times, as was also similarly by Jacob and Karn (2003). The total time that P2 spent on the diagram AOI was remarkable by comparison with all other results. The reason behind this high duration was the experiencing of significant difficulty in solving the given problem. Although P2 analyzed the question appropriately, he could not realize a triangle or its angles. Therefore, he tried to solve the question based on a false assumption, and was aware of not reaching a solution when he crosschecked. There were other participants who also attempted to solve questions based on incorrect assumptions. P3 found familiar numbers and attached them with a special right triangle, but without any proof. He also assumed another triangle as being a right, whilst P8 attempted to apply a theorem without realization of a right angle location. This is in line with the result that some students focused on searching for familiar information since they lack problem-solving skills (Norqvist et al., 2019).

As Ostad and Sorensen (2007) stated, speech based on a task has positive relevance to metacognition and success in task completion. In the current study, P4 demonstrated a lack of being able to express his thoughts whilst problem solving. Similarly, P6 experienced difficulty thinking aloud. In the subsequent interview, while explaining the solution strategy, P6 demonstrated self-corrective behaviors and indicated having not remembering the rules.

In P5's problem-solving process, the reasons for not being able to complete the task successfully were not remembering the rules in two questions and not interpreting diagrams appropriately having used incorrect assumptions. In the final question (Q6), P5 stated, "Now, I realized that I did not remember any rules, anything." Moreover, she tended to reactivate information from questions solved in previous years. Correspondingly, P7 could not recall the rules or significant properties, although she approached the problems appropriately and selected useful strategies.

She specifically stated in her interview that, “I did not know the rules or the correct solutions. I forgot all of them. I could not generate a solution without any idea about how to do it.” The same condition could be seen in the solution process of P4, as previously mentioned. Participant P8 experienced a similar situation; nevertheless; she demonstrated the necessary geometrical reasoning to a certain degree. She could not recall one theorem related to a right triangle; however, she specified how side lengths were in proportion.

Eye-tracking provided indications on how much time students spent time on the diagram or text parts and in which points they fixated more. Heat maps also showed the attention distribution and overall visual tendency while solving questions. Eye-tracking data was explained through the use of participants’ statements during interviews and think-aloud. Most participants gave hints about the reasons for higher fixation durations in specific points and the problem-solving approaches during interviews and think-aloud. Eye-tracking data demonstrated which information or point participants noticed; however, noticing does not always imply interpreting successfully. Although eye-tracking could not entirely clarify participants’ problem-solving strategies and processes in the current study, it helped understand participants’ processing time and visual attention.

5.2 Conclusion

The current study demonstrated that think-aloud and interview sessions provide beneficial data for the studying of participants’ detailed thoughts regarding problem-solving as a process. The participants who scored higher in their spatial ability test tended to use a higher number of strategies than those with lower scores. The high scorers appeared more inclined to manipulate the geometric figure using the mereologic way. Conversely, the participants with lower SAT scores demonstrated a tendency to employ visual and preproportional strategies. Although the participants successfully used strategies to solve some of the problems, there were certain reasons that clearly inhibited their problem-solving processes. Properly analyzing the geometric figure and the problem statement, finding geometric relationships with justification, recalling the requisite information, implementing strategies, and applying theorems or rules are key aspects of solving problems. Some of the participants were unable to remember the rules or discover geometric connections; therefore, they could not complete the given tasks. In some instances, result-oriented behaviors were observed, whereby participants concentrated on recalling question types and solution approaches from the previous year. This finding might be interpreted as there being a tendency to memorize types of questions and figures and also solution paths in learning processes. Understanding problem solvers' tendencies is essential to revealing strategy choices, approaches to problems, difficulties, and specific information related to the solving process. Therefore, task instructions could be better designed and revised based on the findings of the current study.

Visual perception and geometry are closely interconnected; therefore, eye-tracking technology is considered to be suited to the analysis of geometrical problem-solving processes. It may be accepted that appropriately relating eye movement data with cognitive processes presents a significant challenge, because there may be noteworthy specific indications that depend on factors related to individuals and to each specific task. In the current study, the strategies and approaches to the problems were targeted rather than the final outcomes of the problems in order to acquire possible explanations to eye movements. Solution strategies and approaches varied from one participant to another, and also between problems as well as eye movement measures. Mental calculations, mental manipulations, attempts to remember rules and theorems, and approaches to finding familiar properties are offered as the reasons for the participants' visual fixations. As a result, eye-tracking technology supported by think-aloud and interview techniques has been shown to provide objective and rich data regarding problem-solving processes; however, the explanations vary depending on the research design.

5.3 Implementation/Guidelines for Teachers

During problem solving, the current study's participants employed various strategies in order to solve the given problems. Some of the students applied these strategies and retrieved the necessary information to solve the problems. However, one notable tendency was that the participants might focus on recalling previously used solutions and approaches for certain question types. The reason for this behavior might be derived from geometry instruction in schools. Guiding students to categorize questions and teaching them how to solve certain types of problems prevents students from considering each question as a unique event or from fully analyzing it. This situation may also affect a students' ability to create and change problem-solving strategies as necessary, and to bring new perspectives to solving problems. Therefore, teachers should be advised not to lead students to categorize problems, nor to encourage rote memorization of solution paths according to question type.

Based on the direction of the current study's findings, the problem solvers' spatial abilities resulted in diverse strategy selection and implementation. Geometry instruction should therefore target the improvement of these abilities in areas such as mental manipulation and spatial relations. Moreover, instructors can provide accurate assistance to promote geometry learning with the help of analyzing problem-solving processes.

According to the eye-tracking results, participants who fixated on essential aspects of the problems and remembered required geometrical knowledge to solve these problems completed problem-solving processes successfully. Therefore, teachers might design the geometry instruction to discuss geometry problems especially posed by diagrams and reveal significant parts of these diagrams. Therefore, students might have an opportunity to visualize the geometrical knowledge.

The suggestions for mathematics teachers are summarized below.

- Teachers can prepare their students to develop and use a variety of effective strategies while solving problems. When students recognize that there are plenty of strategies and their teachers encourage them to use different solution ways, they develop confidence and independence in problem-solving.
- Teachers can not direct students to categorize the problems and assign solution ways to these problems instead of analyzing problems correctly and exploring properties or concepts. Students might focus on memorizing these solution ways in problem-solving.
- Teachers can provide students with the opportunity to explore problems by allowing enough time.
- Teachers can allocate time to analyze students' strategies and solutions.
- They can ask students to express their thoughts about the problem-solving process; therefore, the use of terminology, verbal explanations, and symbols promotes performance on using mathematical language.

5.4 Limitations and Recommendations for Future Research

The current study has several limitations. First, the number of participants was low at eight, and the target level was restricted to the tenth grade. Therefore, this study could be repeated with participants from other K-12 levels, or from more diverse backgrounds such as students with different prerequisite knowledge levels or learning disabilities.

Moreover, the eye tracking data was collected using the Tobii X2-60 Eye Tracker, which is suitable for on-screen studies, and was selected based on availability and usability. The current study could therefore be repeated with different eye tracking equipment types that allow participants to write and draw according to the context of the study. Possible explanations of eye movements have been seen to vary according to the type of problems presented, and to the design of the research application; therefore, the current study could be repeated using an alternative set of problems, and with eye movement measures to examine problem-solving processes in detail.

Lastly, the current study's participants solved problems outside of the classroom environment, and were aware of their being recorded; therefore, these circumstances may in some way affect their actual performance. Participants may experience difficulty in the think-aloud session, since they were also solving geometry problems at the same time. The simultaneous cognitive activities might affect both the think-aloud and problem-solving performances. Therefore, the current study could be repeated by using Retrospective Think Aloud (RTA) sessions; thereby allowing the participants' eye movements to be interpreted after the problem-solving process.

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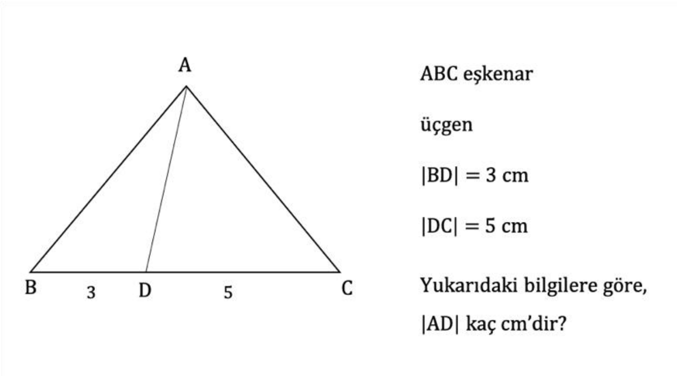
APPENDICES

A. Geometry Questions

1)



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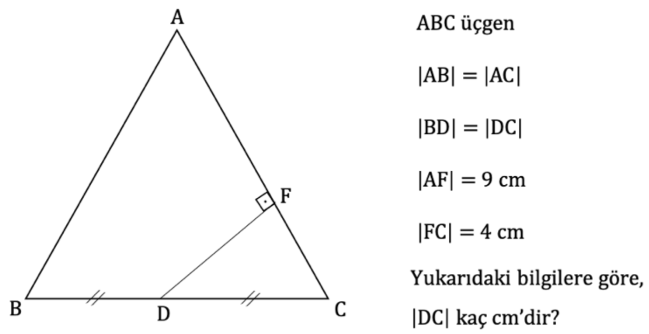

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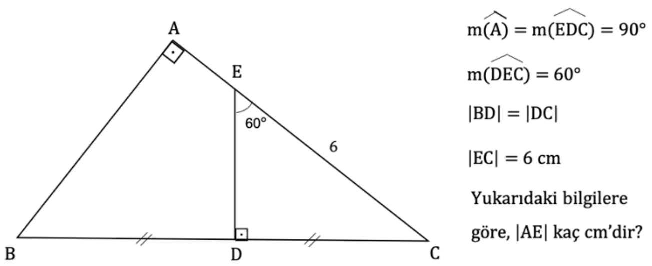

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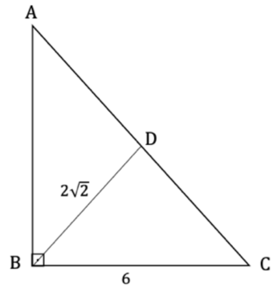


ABC üçgen

[BD] açıortay

[AB] ⊥ [BC]

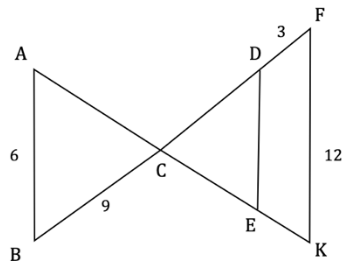
$|BD| = 2\sqrt{2}$ cm

$|BC| = 6$ cm

Yukarıdaki bilgilere göre,

|AB| kaç cm'dir?

5)



[AB] // [DE] // [FK]

$|AB| = 6$ cm

$|BC| = 9$ cm

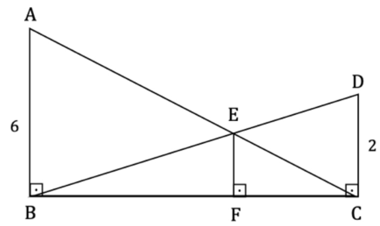
$|DF| = 3$ cm

$|FK| = 12$ cm

Yukarıdaki bilgilere göre,

|DE| kaç cm'dir?

6)



[AB] ⊥ [BC]

[EF] ⊥ [BC]

[DC] ⊥ [BC]

$|AB| = 6$ cm

$|DC| = 2$ cm

$|BC| = 24$ cm

Yukarıdaki bilgilere göre,

|FC| kaç cm'dir?

B. Parent Approval Form (Turkish)

Sevgili Veliler,

Bu araştırma, ODTÜ Bilgisayar ve Öğretim Teknolojileri Yüksek Lisans öğrencisi Emine Malcı tarafından Doç. Dr. Tuğba Tokel danışmanlığındaki yüksek lisans tezi kapsamında yürütülmektedir.

Bu çalışmanın amacı nedir? Çalışmanın amacı, katılımcıların geometri problemlerini çözme süreçlerini göz izleme teknolojisini kullanarak incelemektir.

Çocuğunuzun katılımcı olarak ne yapmasını istiyoruz?: Bu amaç doğrultusunda, çocuğunuzdan altı tane geometri sorusuna yanıt vermesini isteyeceğiz ve göz hareketleri verilerini göz izleme cihazıyla toplayacağız. Süreç boyunca sesli düşünmesini ve soruları cevapladıktan sonra bir görüşmeye katılmasını isteyeceğiz. Öğrencinin cevaplarını ses kaydı biçiminde toplayacağız. Katılmasına izin verdiğiniz takdirde çocuğunuz Orta Doğu Teknik Üniversitesi Bilgisayar ve Öğretim Teknolojileri Bölümü Laboratuvarı'nda çalışmaya katılacaktır. Sizden çocuğunuzun katılımcı olmasıyla ilgili izin istediğimiz gibi, çalışmaya başlamadan çocuğunuzdan da sözlü olarak katılımıyla ilgili rızası mutlaka alınacak.

Çocuğunuzdan alınan bilgiler ne amaçla ve nasıl kullanılacak?: Çocuğunuzdan alacağımız cevaplar tamamen gizli tutulacak ve sadece araştırmacılar tarafından değerlendirilecektir. Elde edilecek bilgiler sadece bilimsel amaçla kullanılacak, çocuğunuzun ya da sizin ismi ve kimlik bilgileriniz, hiçbir şekilde kimseyle paylaşılmayacaktır. Çocuğunuzun verdiği yanıtlar geometri problem çözme süreçleriyle ilgili bilgilerin saptanmasına önemli bir katkıda bulunacaktır.

Çocuğunuz ya da siz çalışmayı yarıda kesmek isterseniz ne yapmalısınız?: Çocuğunuzun cevaplayacağı soruların onun gelişimine herhangi bir olumsuz etkisi olmayacağından emin olabilirsiniz. Yine de, bu formu imzaladıktan sonra hem siz hem de çocuğunuz katılımcılıktan ayrılma hakkına sahipsiniz. Katılım sırasında sorulan sorulardan ya da herhangi bir uygulama ile ilgili başka bir nedenden ötürü çocuğunuz kendisini rahatsız hissettiğini belirtirse, ya da kendi belirtmese de araştırmacı çocuğun rahatsız olduğunu öngörürse, çalışmaya sorular tamamlanmadan ve derhal son verilecektir.

Bu çalışmayla ilgili daha fazla bilgi almak isterseniz: Çalışmaya katılımınızın sonrasında, bu çalışmayla ilgili sorularınız yazılı biçimde cevaplandırılacaktır. Çalışma hakkında daha fazla bilgi almak için Bilgisayar ve Öğretim Teknolojileri Bölümü öğretim üyelerinden Doç. Dr. Tuğba Tokel (E-posta: stugba@metu.edu.tr) ya da yüksek lisans öğrencisi Emine Malcı (E-posta: emine.malci@metu.edu.tr) ile iletişim kurabilirsiniz. Bu çalışmaya katılımınız için şimdiden teşekkür ederiz.

Yukarıdaki bilgileri okudum ve çocuğumun bu çalışmada yer almasını onaylıyorum (Lütfen alttaki iki seçenekten birini işaretleyiniz.

Evet onaylıyorum___ ***Hayır, onaylamıyorum___***

Velinin adı-soyadı: _______________ Bugünün Tarihi:_____________

Çocuğun adı soyadı ve doğum tarihi:_________________

(Formu doldurup imzaladıktan sonra araştırmacıya ulaştırınız).

C. Informed Consent Form (Turkish)

Bu araştırma, ODTÜ Bilgisayar ve Öğretim Teknolojileri Yüksek Lisans öğrencisi Emine Malcı tarafından Doç. Dr. Tuğba Tokel danışmanlığındaki yüksek lisans tezi kapsamında yürütülmektedir. Bu form sizi araştırma koşulları hakkında bilgilendirmek için hazırlanmıştır.

Çalışmanın Amacı Nedir?

Araştırmanın amacı, katılımcıların geometri problemlerini çözme süreçlerini göz izleme teknolojisini kullanarak incelemektir.

Bize Nasıl Yardımcı Olmanızı İsteyeceğiz?

Araştırmaya katılmayı kabul ederseniz, sizden 6 tane geometri sorusuna cevap vermeniz, bu süreçte sesli düşünmeniz ve sorulara cevap verdikten sonra bir görüşmeye katılmanız beklenmektedir. Bu çalışmaya katılım ortalama olarak 1 saat sürmektedir.

Sizden Topladığımız Bilgileri Nasıl Kullanacağız?

Araştırmaya katılımınız tamamen gönüllülük temelinde olmalıdır. Çalışmada sizden kimlik veya kurum belirleyici hiçbir bilgi istenmemektedir. Cevaplarınız tamamıyla gizli tutulacak ve sadece araştırmacılar tarafından değerlendirilecektir. Katılımcılardan elde edilecek bilgiler toplu halde değerlendirilecek ve bilimsel yayımlarda kullanılacaktır. Sağladığınız veriler gönüllü katılım formlarında toplanan kimlik bilgileri ile eşleştirilmeyecektir.

Katılımınızla ilgili bilmeniz gerekenler: Çalışma genel olarak kişisel rahatsızlık verecek sorular veya uygulamalar içermemektedir. Ancak, katılım sırasında sorulardan ya da herhangi başka bir nedenden ötürü kendinizi rahatsız hissederseniz cevaplama işini yarıda bırakıp çıkmakta serbestsiniz. Böyle bir durumda çalışmayı uygulayan kişiye, çalışmadan çıkmak istediğinizi söylemek yeterli olacaktır. Çalışma sonunda, bu araştırmayla ilgili sorularınız cevaplanacaktır. Katılımcılar geometri problemlerini çözme süreçleriyle ilgili bilgi alacaklardır.

Araştırmayla ilgili daha fazla bilgi almak isterseniz: Bu çalışmaya katıldığınız için şimdiden teşekkür ederiz. Çalışma hakkında daha fazla bilgi almak için Bilgisayar ve Öğretim Teknolojileri Bölümü öğretim üyelerinden Doç. Dr. Tuğba Tokel (E-posta: stugba@metu.edu.tr) ya da yüksek lisans öğrencisi Emine Malcı (E-posta: emine.malci@metu.edu.tr) ile iletişim kurabilirsiniz.

Yukarıdaki bilgileri okudum ve bu çalışmaya tamamen gönüllü olarak katılıyorum.

(Formu doldurup imzaladıktan sonra uygulayıcıya geri veriniz).

İsim Soyad Tarih İmza

---/----/-----

D. Debriefing Form (Turkish)

Bu araştırma daha önce de belirtildiği gibi, ODTÜ Bilgisayar ve Öğretim Teknolojileri Yüksek Lisans öğrencisi Emine Malcı tarafından Doç. Dr. Tuğba Tokel danışmanlığındaki yüksek lisans tezi kapsamında yürütülen bir çalışmadır. Çalışmanın amacı katılımcıların geometri problemlerini çözme süreçlerini göz izleme teknolojisini kullanarak incelemektir.

Çalışmada 6 tane geometri sorusuna cevap vermiş, bu süreçte sesli düşünmüş ve sorulara cevap verdikten sonra bir ankete katılmış bulunmaktasınız. Farklı geometri problemlerinde ve bir problemin farklı kısımlarında göz hareketi ölçümlerinde nasıl bir değişme meydana geldiğini saptamak amacıyla göz hareketleriniz göz izleme teknolojisi kullanılarak kaydedilmiştir. Bu teknoloji bir kişinin görsel bir ekranla etkileşime geçerek nereye, ne kadar baktığı ve odaklandığıyla ilgili nicel ve objektif verileri sunmada etkili bir yöntemdir. Göz hareketi ölçümleriniz sizlerin geometri problemine ne kadar baktığınız, hangi kısma baktığınız, baktığınız halde bir kavramanın gerçekleşip gerçekleşmediği yönündeki sorulara yanıt vermeyi kolaylaştırmaktadır. Ölçümler sesli düşünme yöntemiyle ve anket sonuçlarıyla desteklenerek sizlerin geometri problemlerini çözerken kullandığınız yöntemleri saptanmaya katkı sağlayacaktır.

Bu çalışmadan alınacak ilk verilerin Eylül 2019 sonunda elde edilmesi amaçlanmaktadır. Elde edilen bilgiler <u>sadece</u> bilimsel araştırma ve yazılarda kullanılacaktır. Bu araştırmaya katıldığınız için tekrar çok teşekkür ederiz.
Araştırmanın sonuçlarını öğrenmek ya da daha fazla bilgi almak için aşağıdaki isimlere başvurabilirsiniz.

Doç. Dr. Tuğba Tokel (E-posta: stugba@metu.edu.tr)
Emine Malcı (E-posta: emine.malci@metu.edu.tr)

Çalışmaya katkıda bulunan bir gönüllü olarak katılımcı haklarınızla ilgili veya etik ilkelerle ilgi soru veya görüşlerinizi ODTÜ Uygulamalı Etik Araştırma Merkezi'ne iletebilirsiniz.
e-posta: ueam@metu.edu.tr

E. Ethics Approval Form (Turkish)

UYGULAMALI ETİK ARAŞTIRMA MERKEZİ
APPLIED ETHICS RESEARCH CENTER

ORTA DOĞU TEKNİK ÜNİVERSİTESİ
MIDDLE EAST TECHNICAL UNIVERSITY

DUMLUPINAR BULVARI 06800
ÇANKAYA ANKARA/TURKEY
T: +90 312 210 22 91
F: +90 312 210 79 59
ueam@metu.edu.tr
www.ueam.metu.edu.tr

Sayı: 28620816 / 281

22 EKİM 2019

Konu: Değerlendirme Sonucu

Gönderen: ODTÜ İnsan Araştırmaları Etik Kurulu (İAEK)

İlgi: İnsan Araştırmaları Etik Kurulu Başvurusu

Sayın Doç.Dr. Tuğba TOKEL

Danışmanlığını yaptığınız **Emine MALCI'nın "10. Sınıf Öğrencilerinin Geometri Problem Çözme Süreçlerinin Göz İzleme Teknolojisi İle İncelenmesi"** başlıklı araştırması İnsan Araştırmaları Etik Kurulu tarafından uygun görülmüş ve **263 ODTU 2019** protokol numarası ile onaylanmıştır.

Saygılarımızla bilgilerinize sunarız.

Prof. Dr. Tülin GENÇÖZ
Başkan

Prof. Dr. Tolga CAN
Üye

Doç.Dr. Pınar KAYGAN
Üye

Dr. Öğr. Üyesi Ali Emre TURGUT
Üye

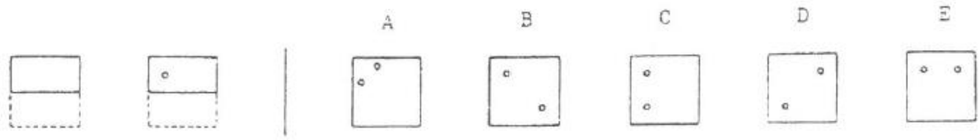
Dr. Öğr. Üyesi Şerife SEVİNÇ
Üye

Dr. Öğr. Üyesi Müge GÜNDÜZ
Üye

Dr. Öğr. Üyesi Süreyya Özcan KABASAKAL
Üye

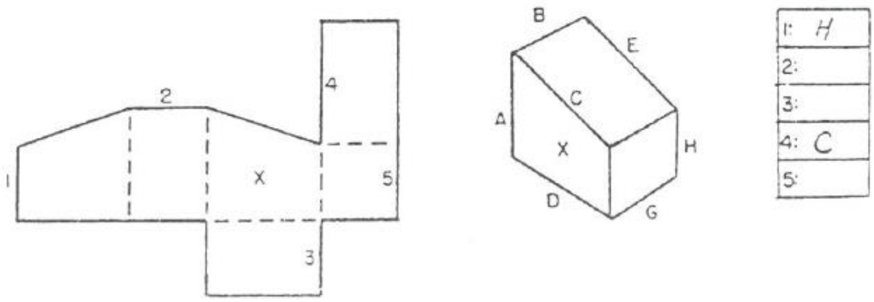
F. Sample Questions of Spatial Ability Test (SAT)

Paper Folding Test (PFT)



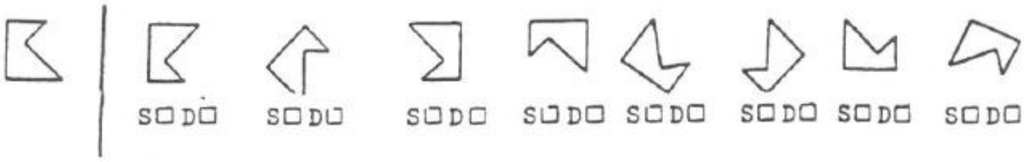
The square paper on the left side of the vertical line was folded and one hole was punched. After unfolding the paper, which one of these figures on the right of the line will occur?

Surface Development Test (SDT)



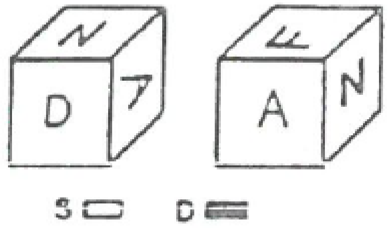
The drawing on the left was folded along the dashed lines and the object on the right was constructed. Which of the numbered edges will match to the lettered edges? Write the letters in the numbered places.
Note: Surface marked as X implies same surface of the drawing and the object.

Card Rotation Test (CRT)



Decide whether the figures on the right side of the vertical line can be formed by rotating the figure on the left side of the line. If the figures can be formed, the figures are the same, so mark as S (Same), else mark as D (Different).

Cube Comparison Test (CCT)



Each cube consists of six faces, each with a different letter, number, or symbol. Decide whether the cubes on the right and the left are the same. If they are the same, mark as S (Same), else mark as D (Different).

G. Spatial Ability Test Scores of Participants

Table G. 1. SAT Scores of P1

Test	Part	Correct	Incorrect	Unanswered	Total
Paper Folding	I	5	5	0	11
	II	6	4	0	
Surface Development	I	24	6	0	50
	II	26	4	0	
Cube Comparison	I	15	6	0	29
	II	14	7	0	
Card Rotation	I	78	2	0	153
	II	75	5	0	

Table G. 2. SAT Scores of P2

Test	Part	Correct	Incorrect	Unanswered	Total
Paper Folding	I	10	0	0	19
	II	9	1	0	
Surface Development	I	30	0	0	58
	II	28	2	0	
Cube Comparison	I	15	3	3	32
	II	17	3	1	
Card Rotation	I	80	0	0	158
	II	78	2	0	

Table G. 3. SAT Scores of P3

Test	Part	Correct	Incorrect	Unanswered	Total
Paper Folding	I	7	3	0	14
	II	7	3	0	
Surface Development	I	18	9	3	39
	II	21	3	6	
Cube Comparison	I	11	5	5	28
	II	17	3	1	
Card Rotation	I	54	1	25	107
	II	53	7	20	

Table G. 4. SAT Scores of P4

Test	Part	Correct	Incorrect	Unanswered	Total
Paper	I	8	0	2	17
Folding	II	9	1	0	
Surface	I	29	1	0	56
Development	II	27	3	0	
Cube	I	10	1	10	26
Comparison	II	16	2	3	
Card	I	49	2	29	105
Rotation	II	56	0	24	

Table G. 5. SAT Scores of P5

Test	Part	Correct	Incorrect	Unanswered	Total
Paper	I	3	4	3	11
Folding	II	8	1	1	
Surface	I	14	7	9	27
Development	II	13	2	15	
Cube	I	16	5	0	29
Comparison	II	13	7	1	
Card	I	38	6	36	38
Rotation	II	0	0	80	

Table G. 6. SAT Scores of P6

Test	Part	Correct	Incorrect	Unanswered	Total
Paper	I	6	1	3	12
Folding	II	6	4	0	
Surface	I	12	13	5	26
Development	II	14	8	8	
Cube	I	14	6	1	30
Comparison	II	16	5	0	
Card	I	79	1	0	88
Rotation	II	9	1	70	

Table G. 7. SAT Scores of P7

Test	Part	Correct	Incorrect	Unanswered	Total
Paper	I	6	4	0	14
Folding	II	8	2	0	
Surface	I	25	3	2	55
Development	II	30	0	0	
Cube	I	15	6	0	33
Comparison	II	18	3	0	
Card	I	51	1	28	89
Rotation	II	38	3	39	

Table G. 8. SAT Scores of P8

Test	Part	Correct	Incorrect	Unanswered	Total
Paper	I	9	1	0	16
Folding	II	7	3	0	
Surface	I	23	7	0	49
Development	II	26	2	2	
Cube	I	16	5	0	24
Comparison	II	8	1	12	
Card	I	50	0	30	107
Rotation	II	57	2	21	

H. Codes of Think Aloud Protocol and Interviews

Problem Solving Strategies

1. Holistic Strategies

    1.1. Mereologic Way

    1.2. Visual

2. Analytic Strategies

    2.1.Preproportional Reasoning

    2.2.Qualitative Proportional Reasoning

    2.3.Quantitative Proportional Reasoning

İ. Interview Questions (Turkish)

a) Bu soruyu hangi yöntemleri kullanarak yanıtlamaya çalıştınız?

b) Kullandığınız yöntemler problem çözme aşamasında size nasıl yardımcı oldu?

c) Bu soruyu çözerken eksik hissettiğiniz noktalar nelerdi?

J. Transcriptions of the Interviews (Turkish)

P1

1) BC eşkenar üçgen. Demek ki AB ve AC de 8'er cm. AD kaç cm'dir? Burası 8 burası 8. Hıı tamam. 4 1 4 Dik üçgen değil. 60 60. 30 60 90 yapsam. Burası 4. 60ın yo 30un karşısı 4se şurası 8 tamam orayı bulduk zaten. $4\sqrt{3}$ bir dakika. Şurası 60 şurası 30. Şurası 4. Şurası 8. 4 8 $4\sqrt{3}$ olacak. 1e $4\sqrt{3}$. $4\sqrt{3}$ ün karesi artı 1in karesi şu 1 şu 48.49.7 buldum.
2) DC kaç cm'dir? Şurdan. AF 9 cm. FC 4 cm AC 13 cm. AB ile AC ikizkenar üçgen. AB de 13 cm demek ki. 13 13 şunu dik olarak indirebilirim o zaman. Hem yükseklik hem kenarortay hem açıortay olur. Bunu indirdiğimde FC 4 cm. Burası a a olsa şu açı ile şu açı aynı alfa alfa. FC 4 cm. Şey yapabilirim bunu dik indirdiğimde şurası 4 şurası 9du. h kare eşittir p çarpı k'dan 36. h 6. Yani FD 6 buldum. Burası 4tü.6nın karesi artı 4ün karesi 16 artı 36dan $\sqrt{52}$ buldum.
3) Şurası 30. Dik açının karşısı 6ysa 30un karşısı 3 olacak. 60 yani DC $3\sqrt{3}$ olacak. DC $3\sqrt{3}$ BD de DC $3\sqrt{3}$ olacak bu sebeple. Şu A köşesinden D ye bir tane doğru indirdiğimde buradan muhteşem üçlüyü çıkarmış olacağım. $3\sqrt{3}$ bulmuştum yanlış bulmadıysam. Bir dakika burası 6 aynen $3\sqrt{3}$ bulmuştum. O zaman A ile D arasındaki doğru da $3\sqrt{3}$ olacak. Benden AEyi istiyor. C açısı 30 dereceydi o zaman B açısını da 60 derece yapar bu. Burdan şu üçgeni tamamlasam bir şey kazandırır mı? BE den bir tane doğru çizsem üçgeni tamamlasam 6 orayı buldum. Uzaklaşıyorum galiba. 6 şurayı bulsam. $3\sqrt{3}$ $3\sqrt{3}$ şurası 3. Şurdan üçgeni tamamladığımda $3\sqrt{3}$ 3 $3\sqrt{3}$ün karesi 27 3ün karesi 9 36. BE den gelen doğru da şey olacak. 36. 6 olacak BE. Dikin karşısı 6 olmuş olacak. Orası 30. $3\sqrt{3}$ $3\sqrt{3}$ $3\sqrt{3}$ de orası. Muhteşem üçlüden onu buldum. Orayı da 60 yapar. 90ın karşısı $6\sqrt{3}$se 30un karşısını bu $3\sqrt{3}$ yapar. 60ın karşısı da $\sqrt{3}$ katı olacağı için 9 yapar. AE 3 olur.
4) BD açıortaymış. 45 45 böldü. AB BD ye dik tamam. AB kaç cm'dir? BD açıortay 45 45. İkizkenar falan filan bir şey dememiş. AB BCye dik. Alfa beta desem. 45

artı alfa pardon 45 artı beta C ye beta dedim Aya alfa dedim. 45 45 bölüyor. ABD ile DBC 45 45 oluyor. O zaman buradan bir şey çıkar mı? 45orası beta orası 90 edecek. Alfa artı beta 90. 45 artı alfa BDC açısını verecek bana. 45 artı beta da ADByi vermesi lazım. Bunların toplamı da 180 etmesi lazım. Yine aynı yere vardım. Bir şey çıkmadı. Alfa artı betayı 90 buldum. $2\sqrt{2}$ oradan dik indirsem eşit bölmüyor. Açıortay. Ben bunu geçiyorum. Yapamadım.

5) Bunlar birbirine paralelmiş. Burada benzerlikten ilerleyeceğim büyük ihtimalle. 6nın 12ye olan oranı $\frac{1}{2}$ BCnin de CFye olan oranına eşit olması lazım. O yüzden CFnin 18 olması lazım. O yüzden CD 15 olur. CD 15 olursa 6nın 15e olan oranı 9un DEye olan oranını verir bana. 15 çarpı 9 bölü 6 dan kesirli çıktı ya bi dakika. 6nın 12ye $\frac{1}{2}$. 9 18 olması lazım CD 18 pardon CF 18 CD 15 oluyor. 9un 15e olan oranı yanlış oranlamışım 9un 15e olan oranı 6nın DEye olan oranına eşit olur. 15 çarpı 6 bölü 9 dan 90 bölü 9 dan 10 olması lazım yanlış çarpmadıysam. Aynen.

6) Şimdi ED ye m BE ye de 3m diyeceğim çünkü DC ile AB nin oranı $\frac{1}{3}$ ü veriyor bana. Bana BCyi demiş. AB 6 BC 24 tamam. Orada benzerlik de olacak. BEye 3m demiştim. EDye de m demiştim. Bunlar aynı zamanda BF ile FCnin oranına da eşittir yani onlara da a a desek 4a 24 olur. a 6 olur. FCyi 6 BFyi de 18 bulmuş olurum. BFyi 18 FCyi de 6 bulmuş olurum. FC 6.

P2

1) Yüksekliğini indirdim. Yüksekliğini indirdiğim zaman iki tane 30-60-90 üçgenine böldüğü zaman şurası 4 oldu. Burası 8 oldu. Bundan sonra yükseklik de aslında bi tane nokta koyalım H noktası olsun. H noktası ile D noktası arasında 1 cm kalıyor çünkü ikiye bölmüş oluyor tam olarak. 4 4 oluyor 1 tane de 1cm oluyor. O h'ın yüksekliği $4\sqrt{3}$ oluyor. Geri kalan da işte Pisagor teoreminden AD 7 çıkıyor.

2) DC kaç? AB DCye eşit ay pardon BD DCye eşit.AB ACye eşit yani bu ikizkenar üçgen. Dik indirirsek AF 9 FC 4. Neyi soruyor? DC yi soruyor. AF. AF nerde? Bunu unuttum ya nasıldı bu teorem. Öklid'ti sanırım. Nasıl yapılıyordu? Neyse başka şekilde yaparız. AF 9 FC 4. Yani bu 13e 13 ikizkenar. Ya bunu

hatırlamıyorum aslında. Ya burda Öklid uyguluyorduk ama nasıl işlemlerini hatırlamıyorum. Sanırım FC ve ACnin çarpımıydı DCnin karesini veriyordu. O zaman öyle deneyelim bir. AC 13 FC 4 olduğuna göre 4 çarpı 13 kaç eder? 26 52 eder. $\sqrt{52}$ olacak sanırım DC. $\sqrt{52}$ de evet $\sqrt{52}$ diyorum ben.

3) BD DCye eşit. EC 6 diyorsa o zaman ED 3 oluyor. DC $3\sqrt{3}$. BD de $3\sqrt{3}$ oluyor. O zaman $3\sqrt{3}$ $3\sqrt{3}$ $6\sqrt{3}$ oluyor. BC $6\sqrt{3}$ olduğuna göre başka bir şey bilmemiz gerekiyor mu? AByi bilmemiz gerek. Aslında burası da 30-60-90 üçgeni çünkü ACB açısı 30. $6\sqrt{3}$ olduğuna göre BCnin o zaman 30un karşısı AB $3\sqrt{3}$ olacak. Öbürü de 9 olacak. Yani AC 9. 9dan 6yı çıkartıyoruz. AE 3 ediyor.

4) Hem açıortay hem de kenaortay mı? Evet kenarortay. Dik yok. BD $2\sqrt{2}$. BC ne kadar oluyor? 6. 45 olarak ayırıyor. Açıortay 45 45 diye ayırıyor. Desek ki şimdi buradan bir dik indirsek ve buna yüksekliği desek. Ortadaki kısımda eğer şimdi bunlar dik dik olacağından dolayı bir kare oluşturacak mı? Evet oluşturacak kare. Biraz daha farklı bir yoldan gidelim o zaman. Yok hayır kare oluşturmayacak. Yok neyse ya çok zorlamayayım şimdi aklıma gelmedi bir şey. Oranın dik olduğunu söylese kenarortay olacağından eşit olacak. Ama aynı zamanda dik üçgen olduğu için $2\sqrt{2}$ $2\sqrt{2}$ olacak. Ondan dolayı ACnin uzunluğu $4\sqrt{2}$ olacak. $4\sqrt{2}$ olduğu zaman da bu bir ikizkenar dik üçgen olacak. Dediğim şeylerden dolayı. O yüzden AB de 6 olacak. Bunu kafamdan yaptım ama bir sağlamasını da yapabilirim aslında. Şimdi $4\sqrt{2}$nin karesi kaç ediyor? 32 ediyor. Olmadı. Tutmadı. Onun için olmuyor öyle. Geçen senenin konuları. Keşke çalışsaymışım biraz. Unutmuşum hepsini. Neyse geçiyorum o zaman bunu da.

5) Benzerlik. Şimdi bunlar ikisi benzer üçgenler. ABC ve CKF ve CED. Benzerlik şeyi ABC ve CFKnin ki 2 katı birbirinin. Onun için CF kenarının uzunluğu 18 olacak çünkü 9 çarpı 2den. Onun için CD 15 olacak. Bize neyi soruyor? DEyi soruyor. 15 olacaksa $\frac{18}{15}$ bizim benzerlik oranımız CFK ve CDEnin. O zaman bu oranın $\frac{18}{15}$e $\frac{12}{x}$ koyduğunuz zaman $\frac{18}{15}$ kaç eder $\frac{6}{5}$ eder. O zaman DE 10 olacak.

6) BC 24 diyor. Şimdi ilk başta şuradan bir benzerlik kurabiliriz. AEB ve EDC benzer üçgenler. Benzerlik oranları birbirinin 3 katı. O zaman BE kenarına 3x

deriz. EDye de x deriz. Şimdi bize FCyi soruyor. BCnin uzunluğu 24 ise 3x e x olduğu için orası da BF 3k FC de k olacak. O zaman 24ü 4e böleriz. 4e bölünce de 6 çıkıyor. 6 da k ya eşit olduğu için cevap 6 olacak. FC eşittir 6.

P3

1) AD. Şurası 8 olsa. Eşkenar üçgen. Aklıma bir şey gelmiyor ki. Bir dakika. Dik indirsek. 30-60-90dan burası 8 burası 30sa. Pisagoru kullandım. Dik indirdim. Pisagoru kullandım. 5in karesi artı a kare eşittir 8in karesi dedim. Oradan buldum ADyi.
2) AB eşittir ACye. BD eşittir DCye. İkizkenar olduğunu düşünüyorum üçgenin. Zaten şu an izlediğim bir yöntem olsa çözüme ulaşacağım da aklıma bir şey gelmiyor. BCnin 10 olduğunu düşünüyorum. FDCnin 3-4-5 olabileceğini düşündüm.
3) 30-60-90 üçgeni var. Başka bir şey söyleyemem bunun hakkında o yüzden sesli düşünemiyorum.
4) 90-45-45 olduğunu düşünüyorum. AByi 4 buldum.
5) Papyon kuralı var. Belki CKF üçgeninin içinde Tales Teoremi olabilir emin değilim. FK ABnin 2 katı. Bu yüzden FC de BCnin 2 katı olması lazım. DEyi 10 buldum.
6) 6. soruiçin çok söyleyecek bir şeyim yok sadece tahminde bulunabilirim çünkü aklıma bir şey gelmedi. Oranlı katlar ile ilgili bir şey uygulanabilir ama benim aklıma şu an nasıl uygulayacağım gelmedi.

P4

1) Adan aşağıya yükseklik indirdim. O zaman iki taraf da 4 cm oluyor. 60-90-30 açıları olduğu için 3-4-5 üçgeni olur. 3ün karesi artı 1in karesinden ADnin karesini buluruz. Cevap $\sqrt{10}$ olur.
2) 9. AC 13 cm. İkizkenar üçgen. Bulamadım galiba.
3) C açışı 30. 90 derece 6ysa 30un karşısı 3. 60ın karşısı $3\sqrt{3}$ olur. BD de $3\sqrt{3}$ olur.
4) DBC açısı ve ABD açısı 45 45 derece açılar olur. Bulamadım.

5) AB,DE ve FK doğruları paralel oldukları için AB ve FK arasında oran var $\frac{1}{2}$ oranı. CF doğrusunu bulabilmek için BC kenarının 2 katını almamız gerekiyor. CD doğrusu 15 olur. 9un 6ya oranını 15in xe oranı yaptım. x de DE kenarını verdim ve içler dışlar yaptım. Sonucu da 10 buldum. DE 10 çıktı.
6) DCB üçgeni EFB üçgenine eştir. Aynı zamanda ABC üçgeni de EFC üçgenine eştir, benzerdir. Bu soruyu bulamadım geçiyorum.

P5

1) ABC eşkenar üçgen BveD 3 cmmiş. DveC 5 cm. AD kaç cm diyor. Bunda böyle bir virgül kuralı gibi bir şey vardı. Şapka çiziyorduk diye hatırlıyorum ama. Tam bilmiyorum ne yapacağımı. Belki topluyor ya da çarpıyor olabilir miyim? ADC üçgeni eşkenar olabilir mi ya da ikizkenar en azından dik değil çünkü AD dik inmiyor.
2) ABC üçgen eşitmiş AveB AveC. BD DCye eşit. AF de 9 cm. FC 4 cm. Tamam şimdi AveC AveB ye eşitmiş. AF 9 cm. DFC üçgeni dik kenar. AF 9 cm. FC de 4 cm. E o zaman 13 oluyor 13 cmmiş AC. AB de 13 cm oluyor. O zaman bu ikizkenar yani eşkenar olabilir mi? DFC üçgeninin iç açılarının toplamı 180 olması lazım o zaman D 45 C de 45 oluyor. Ama bu etkiliyor mu? O zaman şey olabilir 13ü 2ye bölüp bulabilirim yani 6,5 yok evet 6,5.
3) 60-30-90 üçgeni var burda. AE kaç cm diyor. Bunun bir kuralı vardı. Ama hatırlamıyorum. Şey miydi 90nın karşısı 2x 60ın karşısı bir tanesi köklü bir şeyler vardı hatırlamıyorum ama. C 30 oluyor açısı. B köşesinin de 60 oluyor o zaman. Yine orada da 90-60-30 üçgeni var. 90 6ya bakıyorsa o zaman.
4) Şimdi yine bu soruyu da çözemedim ama açıortaysa 45 45 bölüyor B köşeşini ve acaba açıortaylar dik mi iniyordu? AB 6 cm mi? 6 cm diyeyim.
5) Burada Z çizebiliyoruz. O zaman B ve Dnin açıları aynı oluyor. A ve Enin de açıları aynı oluyor. C 6 kenarına bakıyor o zaman DE 6 olabilir mi ama şekil öyle göstermiyor çok. FCK üçgeni eşkenar olabilir. Çünkü A dokuza bakıyorsa o zaman E de 9a bakıyor. 9 artı 3 12. O zaman DE 6 diyeyim.
6) Şu an hiçbir kuralı hatırlamadığımı fark ettim. Hiçbir şeyi. Geçen sene gördük bunları aslında ama hatırlamıyorum. Geçiyorum bunu bitti sanırım.

P6

1) BD 3. DC 5. BC 8. Eşkenarsa 8.
2) AB ACye eşit. İkizkenar. 9. AC 11. AC AByе eşitse orası da 11. BC DCye eşitse 11. DC 11.
3) 30-60-90. 90 6ysa C 30sa B de 60tır. a kare artı b kare 6ysa. Cnin 30 olması gerekiyor o zaman B de 60 olmalı ama işlemleri yapamadım.
4) BD 2kök2. BC 6. 90dı 45 45. BDA 90 olacak. 90 45. C ve A o zaman 45. O zaman x kare artı 2kök2nin karesi 6nın karesi olmalı o zaman xkare artı 8 = 36 x kare 28 oluyor. O zaman x kök28. kök28 kök28. x kare artı 6nın karesi eşittir 28. x kare artı 36. Yapamadım.
5) AB DEye paralel. DE FKye paralel. O zaman AB 6ysa BC 9sa DF 3se FK 12yse DE kaç onu soruyor. DE 6 o zaman.
6) AB BCye dik. EF de BCye dik. DC de BCye dik. Tamam AB 6ysa DC 2yse BC de 24 ise FC kaç cmdir? 24 2 6. Bunun bir kuralının olması gerekiyor ama hatırlayamadım o yüzden yapamadım.

P7

1) Eşkenar üçgense iki kenarı eşitledim. Adını unuttuğum bir kural yapacağız. Ama kuralı bilmiyorum nasıl yapacağım unuttum tamamen. Bilmiyorum ki.
2) AB eşittir ACye. Büyük bir eşkenar üçgen var ay ikizkenar üçgen var. 4 cmse. Hiçbir şey bilmiyorum ki. Dik bir çizgi çekeriz. Bilmiyorum ama geçiyorum bunu.
3) A da 90 EDC de 90 tamam. 90-60-30 üçgeni var. C kısmı 30 oluyor. 90nın karşısı 6ysa 30un karşısı 3 oluyor diye hatırlıyorum. 60ın karşısı da 3kök2 mi oluyordu hiçbir fikrim yok. Sonra diğerini BD kenarıyla eşitliyorum. Şuanda yapamıyorum ama bulsam ne yapacağımı anlatayım mı?

   E) Tabii tabii lütfen.

   Tamam. DC kenarını bulunca BD kenarıyla eşitleyecektim. Ve tüm o büyük kenarı toplayacaktım. Sonra oradan 30-60 üçgeni yapacaktım. Çünkü B açışı da 60 oluyor. Sonra 30-60-90 üçgeni orantısıyla AC kenarının tüm uzunluğunu bulup 6 çıkaracaktım. Ve AEyi bulacaktım. Ama DC kenarının uzunluğunu bulamadığım için çünkü 90-60-30 üçgeninin orantısını bilmiyorum. Yapamadım.

4) 45 oluyor açıortaysa B açısı 45 45 bölünüyor. BC 6ymış. Açıortay dik mi bölüyordu? AB BCye dikmiş. Bilmiyorum. ABnin kaç cm olduğunu soruyor bana. ABD açısı 45se ve o açıortayın dik kestiğini düşünürsek diğeri de 45 oluyor. 45in karşısı 2kök2yse 90nın karşısı 2 oluyor. AB kenarı 2yse BC kenarı da 6ysa 2kare artı 6kare eşittir xkare yapıyoruz. 4 artı 36 x kare yapıyor. 40 da xkareye eşit oluyor. x kök40 oluyor. Ben direkt AByi 2 bulmuşum ama.
5) Paralellik var kenarlar arasında o yüzden açı eşitleyeceğim. Belki oradan yapabilirim. DEyi soruyor. Oran yapacağım. Biraz düşünebilir miyim? Benzer üçgenlik var. ABC ve CDE üçgenleri arasında. Bir de CDE ve CFK arasında benzer üçgenlik var. Yani ACB ve DCE açıları eşit. BAC ve DEC açıları eşit. Diğer açılar eşit üçgendeki. ACB üçgeni ECD üçgeni eşit onun eşitliğini buldum. Yani CD kenarını 9 bulabilirim. Sonra orayı toplayıp 12 bulurum. Bunu da yapamadım. Hiçbirini yapamadım.
6) BCnin tamamı 24. ABC ve DCB üçgenleri arasında benzerlik var benzer üçgenler. Bir de ABC ve EFC üçgenleri arasında benzerlik var. Benzerlik oranı $\frac{2}{6}$ yani $\frac{1}{3}$. Başka bir şey bilmediğimize göre. Bir de BCnin 24 cm olduğunu biliyoruz. $\frac{1}{3}$ten $\frac{x}{24}$ yapıyoruz. Yani x'i 8 buluyoruz. Orası da BD kenarı. Hipotenüsten bulacağız hepsinin kenarlarını. Aralarında bir oran bulamadığım için yapamıyorum. Sadece benzerlik. Bunu da geçiyorum.

P8

1) ABC eşkenar üçgen. AD. Burada eşkenar üçgen olduğu için. BD 3 ve CD 5miş. Eşkenar üçgen olduğu için de her kenarı yani 8. O zaman AD 5 8 ve AD kenarı olan bir üçgenin bir kenarı. O zaman $5^2 + 8^2$ nin karekökünden sanırım bulabiliyoruz. Şey bunun hesaplamasını tam hatırlamıyorum ama.
2) İkizkenarmış ABC üçgeni o zaman BC 4se AF de 9 cm ise o kenar 13 cm. DCyle de BD aynı uzunlukta o zaman ABC üçgeninden DFC ye dik bir çizgi çekmişler ACye. 13ü 4 ve 9a ayırmış.

3) ABC BCA dikmiş. EDC de dik o zaman C açısı 30. ABC üçgeninde de o zaman B açısı 60. DC kenarı 60lı açının karşısında o zaman BC de 90ın karşısında belki burada bir orantı vardır. EDC de 90 60 ve 30. Bunu da yapamadım.

4) BD açıortaysa o zaman 90ı 45 45 tam ortadan ayırmış. Aynı zamanda da BD $2\sqrt{2}$ o zaman AByi bulmak için BDye sadece açıortaymış. $2\sqrt{2}$nin karesiyle 6nın karesini toplasak bunun karekökünü alsak AByi bulmak için.

5) Çizgilerin hepsi paralel şimdi AB 6 FK da 12ymiş o zaman 2 katı uzunlukta. CK şey DF 3se EK de 3. CB ile CKnın orantısı da $\frac{1}{2}$ ise CK 18 olabilir belki. CE de 15 olabilir. 15 olamaz. AB 6 FK da 12 ama bilmiyorum.

6) BC 24müş. FCyi bulmak için ABC üçgeninin 6 24 varmış kenarda. BDC üçgeninde de 2 ve 24 var. FCyi bulmak için. Hepsi dik o zaman 6 24. ABC üçgeni var dik üçgen. EFC üçgeni de dik üçgen. O zaman AB ile EF arasındaki oran BF ile FC arasındakiyle aynı. DC ile EF arasındaki oran da CB ile FB arasındakiyle aynı. 24 ve 6 FC ve ve EFye oranlı buradan hem 6 hem 2 24 ile aynı oranlı olacak. Bilmiyorum.

K. Transcriptions of the Think-Aloud Protocol (Turkish)

Interview of P1

1) a) Burada eşkenar dediği için bu üçünün de eşit olduğunu anladım. BC 8 ise diğerleri de 8er santimetre olması lazım AB ile AC. Ve eşkenar üçgende şöyle bir kural var. Tepeden yani A açısından indirdiğim bir doğru aynı zamanda yükseklik aynı zamanda kenarortay aynı zamanda da açıortay olmak zorunda. Bunu indirdiğimde 8 cm 4-4 ayıracaktır. O yüzden şuradan indirdiğimde şurası 1 cm olacaktır. Sonra ben nereden ilerledim şu an hatırlayamıyorum. Şurası 8 cmdir. Eşkenar üçgen olduğu için 60-60-60 olacaktır. 30-60-90 üçgeninden ilerledim. 90'nın karşısı 8'di. Şurası da 30 oluyor şurası 60 olduğu için burası da 4 demekki sağlıyor dedim. 60'da 4ten 4 kök üç buldum şu indirdiğim doğrunun uzunluğunu şurası da 1'di. $1^2 + (4\sqrt{3})^2 = AD^2$ ne eşit oradan denklemi çözüp.

b) Evet

c) Eşkenar üçgende kuralı hatırlarken zorlandım.

2) AB ile AC eşitmiş demekki bu ikizkenar üçgendir dedim. Bu ikizkenar üçgende de yine indirdiğim aynı zamanda yükseklik, kenarortay ve açıortayı verecekti bana. İndirdim. Kenarortaydı zaten. Aynı zamanda yükseklik olduğunu fark ettim. Burası 4e 9du. 13 bulmuştum. Sonra burdan şeyi fark ettim. Öklid buldum. Dikten dik indirmişler. Hkare yani DF nin karesi eşittir AF çarpı FCden buldum. 9 x 4 =36 o da karesini verdiği için DFyi 6 buldum. Benden DCyi istiyordu. 6nın karesi artı 4ün karesi eşittir DCnin karesi.

b) Evet

c) Öklid olduğunu anlarken zorlandım.

E) Soruyabaşlarken bir strateji geliştiriyor musun? Burada şunu yapmalıyım gibi?

Kuralları hatırlayıp sonra kurallardan sorunun çözümüne ulaşmaya çalışıyorum.

3) Burada 30-60-90 üçgenini fark ettim. Burası o yüzden 30 olması lazım dedim. Dikliğin karşısı 6 ise, 30un karşısının 3 olması gerektiğini fark ettim. Ayrıca burda dikten şöyle bir şey atarsan bunun muhteşem üçlüyü getireceğini fark ettim. O yüzden burası da eşit oluyor. Bu da $3\sqrt{3}$ oluyordu burdan buna 3 dediğim için. 30 un karşısı 3 ise 60 ın karşısı $3\sqrt{3}$ oluyordu. $3\sqrt{3}, 3\sqrt{3}$ şuradan indirdiğim doğru da $3\sqrt{3}$. Galiba orada biraz uzattım soruyu.

Soruyu uzattım çünkü bir küçük üçgene bir büyük üçgene baktım biraz karıştırdım. Üzerine de işlem yapamadığım için

4) Bu benim boş bıraktığım soruydu galiba.

E) Burada ne seni zorladı? Hangi yöntemi denedin olmadı?

Ya burada ben şunların eşit olmasını filan beklerdim. En azından bana bir çözüm verirdi. Ya da açı beklerdim. Hiçbir şey yoktu burada. Hani şurası dik olsaydı en azından bir şeyler üretirdim.

E) Bu açıortay bilgisini kullandın mı?

Açıortay bilgisini kullanamadım. Aklıma bir şey çağrıştırmadı öyle diyeyim.

E) Yani burada aslında bilgileri kullanmada ve bir bütün oluşturmada sıkıntı yaşadın.

Aynen aynen.

5) a) Burda da benzerlik yaptım direkt. 6nın 12ye olan oranı BC nin CF ye olan oranını vericekti bana. Bu da$\frac{1}{2}$ yaptı. $\frac{9}{x}$ ten CF yi 18 buldum. CD de 15 geliyordu. Sonra DE kaç cmdir diye sormuştu. Sonra da küçük kum saatine baktım. 9 un 15 e olan oranı 6nın DE ye olan oranına eşit olması gerekiyordu. Oradan da 10 bulmuştum.

c) Hayır. Benzerlik daha kolaydı çünkü.

6) a) Burada da yine aynı oranı uyguladım ama bu sefer sayıları bilmediğim için semboller verdim m gibi. Bu m burası da 3mi verdi bana. Aynı zamanda şu büyük kum saatine baktığım zaman pardon. Şurada aslında bir tane dik üçgen var. Ve burada da bir benzerlik var. Çünkü bunların da şey olması lazım dik oldukları için bunlar da bana benzerliği verir. Bunlar da 3a ya a olsun dedim. 4a 24müş bunu eşitledim. a yı 6 buldum. Burası 6 burası da 18 yaptı.

b) Üçgenleri bir an karıştırırım diye düşündüm ama halletim çok sıkıntı yaşamadım.

Interview of P2

1) **a)** Eşkenar üçgen. Şuradan dik indirdim. Şurada bir dik üçgen oluştu. Şurayı biliyoruz 1 cm. Şu dikten 30-60-90dan $4\sqrt{3}$ oluyor. Şurası 8 çünkü burası da 8. Oraya ikisinin de karesini alıp topladığımızda da 49 çıkıyor. 7 oluyor burası.

b) Ya aslında problem çözme aşamalarının direkt kendisi gibiydi yöntemler.

E) Burada bir dikme çizdin problemde var olmayan.

Evet yükseklik oluyor zaten o da.

E) Hangi yöntemi kulandın? Pisagoru mu kullandın?

Evet. Orayı hesaplamamda yardımcı oldu.

c) Burada pek yoktu.

2) Burada yine ilk başta bir dik indirdim. Dik indirdikten sonra zaten burası ikizkenar oluyor öyle diyor çünkü AB ve AC. AF 9 diyor. FC de 4 diyor. 13 13 oluyor. Bir kural işte o kuralı unuttum. Biraz daha çalışmam gerekiyor. Öklid kuralı diye geçiyordu sanırım. Şu kenarla şu kenarın çarpımı şunun karesiydi sanki. Yani aklımda kalanla hesaplamaya çalıştım. Şu 4 ile 13ü çarptım. Yani benim aklımda kalan bu kadardı.

E) Biraz sayı ilişkilerinden gittin o zaman tam hatırlamadığın için yöntemi.

Evet.

b) Aslında eksik eksik parçalar verilmiş. Ben bunu birleştirerek üzerine yöntem koyabileceğim başka yöntemler de kullanabileceğim bir şekil oluşturdum.

c) Dediğim gibi o geçen seneden kalma konuları tekrar etmem gerekiyor.

3) Burada daha çok trigonometri kullandım. Bir de benzerlik kullandım. İlk olarak zaten şuradan bunun benzer olduğu şurası 30 derece oluyor. Çünkü burada 30-60-90 olduğunu söylüyor. O zaman burası da 90 derece olduğu için dik üçgenin özelliklerinden burası da 60 derece oluyor. Şunların birbirine eşit olduğunu söylediği için zaten ilk başta şurada trigonometri kullanarak hipotenüs 6 30un karşısı 3 olacak çünkü sin30 $\frac{1}{2}$ oluyor. Sin 60 da $\frac{\sqrt{3}}{2}$ olduğu için bunu 2ye bölüp $\sqrt{3}$ ile çarpıyoruz. $3\sqrt{3}$ oluyor burası. Burası da $6\sqrt{3}$ oluyor. Ondan sonra yine buradan hipotenüsü $6\sqrt{3}$ olduğu için yine trigonometri kullandım. Dedim ki işte 60ın karşısı $6\sqrt{3}$ çarpı sin60 dedim oradan da 9 çıkıyor burası. Ondan da 6 yı çıkartıp 3 buldum.

b) İşimi kolaylaştırdı aslında üçgeni yorumlamamı sağladı.

c) Yani burda da pek sıkıntı olmadı.

Ya aslında burada benzerlik kullanacaktım muhtemelen. Burası dik üçgen. Şimdi hatırladım aslında bu kuralı. Şurası açıortay olduğu zaman şurası ve şurasının ilişkisi şunun şurasına ilişkisine benziyordu. Sonra sanırım şu ikisinin çarpımı ve şunların çarpımı şunun kare kökü şu değeri veriyordu. Bunlardan da aslında işte işin içine sayıları koyup denklem kurup çözebilirdim ama atladım.

b) Çözüm olmadığı için boş bırakıldı.

c) E) Bu soruyu iyi analiz edebildin mi sence?

Elimden geleni yapabildiğimi düşünüyorum şu an. Ya aslında kafamdakini tam anlatamadım çok fazla şey geçti de. Yeterince konuşmadım. Aslında burda biraz kendimi yorumlamaya başladım çünkü şöyle bir şey düşündüm. Eğer şimdi şuraya dik indirirsem burası dörtgen olacak. Bu ikisinin yükseklikleri buraya x buraya y diyelim. O zaman diyeceğim ki $\sqrt{x^2+y^2} = 2\sqrt{2}$. Ondan sonra işte buradan birbiri cinsinden yazacağım.

E) Şu an tekrar çözmene gerek yok.

Kafamdan geçenleri anlattım.

4) Burada üçgen benzerliği vardı.

E) Kaç kere benzerlik uyguladın?

İki defa. İlk başta şu ABC ve CFK nin benzerliği vardı. 2ydi onunki. Ondan sonra burası şuna benzer olduğu için burası 18 oluyordu. Bu ikisi de benzer olduğu için CDE ve CFK burası da 15 oluyordu 18den 3 çıkarttığım zaman. Benzerlik oranı 18 bölü 15 oluyordu eşittir 6 bölü 5. Buraya da eşittir 12 bölü x diyorduk. İşte burdan iki katı olduğunu görüyorduk. X de 10 çıkıyordu.

5) Yine benzerlik vardı.

E) Kaç kere benzerlik uyguladın?

Yine iki defa sanırım. Bize buranın 24 olduğunu söylüyor. Şu ABC ve EDC üçgenleri benzer. Benzerlik oranları 3. Yani bu bunun üç katı. O zaman burası 3x e x diye ayrılır. Aynı zamanda DBC ve EBF de benzer üçgenler. Ama işte orada da direkt benzerlik uygulamak yerine şu oranlar aynı olduğu için yani aslında BF bölü FC eşittir BE bölü ED oluyor. Burası 3x e x ken burası 3k ya k dedim. 24ü dörde böldüm çünkü kat sayıları 4. Oradan 6 çıktı. K nın da kat sayısı bir olduğu için k eşittir 6 oldu.

b) Olmadı.

Interview of P3

1) a) Dik indirmeye çalıştım. Aslında bir şey unuttum yanlış buldum.Dik indirdim.8 5 burayı buldum, diki buldum 7 olarak. Büyük ihtimalle yanlış buldum daha sonra burayı bulmayı unuttum.

E : AD yi 7cm yazmışsın.

Yanlış büyük ihtimalle evet.

E: Nasıl yaptığını bana gösterir misin ?

Dik indirdim. Pisagor yaptım.8 dedim.Burdan gittim.

E : Burada pisagoru 5 olarak mi aldın ?

Evet yanlışlıkla 5 olarak aldım. Burayı düşünmedim.O yüzden yanlış buldum.

E: Sonucun doğru da o yüzden baktım ben. Aslında şunların 4 4 ayrılması gerekiyordu.4 e 1 eşkenar üçgen dik indirince.

b) Sonuca gitmemde yardımcı oluyor. Daha kolay gitmemi sağlıyor.Ancak kullandığım yöntemlerde bazı şeyleri atlayarak kullandığım için.Daha sonradan sıkıntı çıkarabiliyor bana sonuç bulma konusunda.Genel olarak bütün geometri soruları için de söylüyorum.Bunun için de dahil.

c) Eksik hissettiğim bi kısım yoktu. Aslında şurayı kaçırmasam 4 4 olduğunu çok rahat bulunabilir.Yine bulmuşum ama parçalı düşündüğüm için bulmuşum.Kafamda oranın 7 olduğunu zaten en başından tahmin ettiğim için 4 4 olduğunu sadece yazmamışım veya düşünememişim.

2) a) Bu soru zorladı. Bu soru bence kolay değildi.Ben burada tahmin yürüttüm.Şurda pisagor olduğunu 3 4 5 olduğunu düşünerek yaptım. 13 13 olduğunu biliyordum ve eşkenar olmadığını düşündüğümden buranın 10 olabileceğini düşündüm. Daha küçük olması gerektiğinen açılardan dolayı.Bu yüzden acaba burası 10 sa da burada 3 4 5 var mıdır diye düşündüm.O yüzden 5 dedim.

c)Evet var. Kendimde eksik hissettiğim nokta soruyu çözerken doğru düzgün bir yöntem kullanamıyorum.Çünkü bu soru zorladı beni.

E : Sence problemi güzel analiz edebildin mi ? Diagramı güzel okuyabildin mi verilenleri ?

Diagramı okusam bile bence ben büyük ihtimalle bu soruyu çözebilme yetisinde değilim. Çünkü çok üzerinde durmuyorum.Geometriyi çok sevmediğimden böyle bir soruyu da çözmüşümdür mutlaka önceden iyi de çözüyürumdur ancak unutmuşum çözmeyi.Bu yüzden diagram yeterliydi ben büyük ihtimalle analiz edemedim.

3) a) Bu soruda 30 60 90 üçgeni vardı. Ancak ben 30 60 90 üçgeninin kurallarını hatırlamak için bir daha uğraştım.2kök3 dedim. Ama dediğim gibi 30 60 90 üçgenini kullanmaya çalıştım. Çok yardımcı oldu ancak hatırlasaydım daha yardımcı olacaktı.

c) Yok. 30 60 90 ın kurallarını hatırlasaydım.

4) a) 45 45 çünkü açıortay olduğu için.45 45 90 vardı. O yüzden burası kök2 ise burasının 4 olacağını düşündüm.

b) Buraya yerleştirdim ADB nin olduğu yere. O kuralı burada uygulamaya çalıştım.

c) Zorlandığımı hissetmedim.

5) a) Net papyon gördüm. Direkt içler dışlarla bu üçgenin CFK üçgeninin 2 katı olduğunu gördüm. Buranın toplamının 18 olduğunu gördüm. 9 18. Buraya da 15 kaldığını gördüm. D ye de direk 10 dedim.

E: Benzerlik yöntemini kullandın. Eksik hissettiğin nokta yok.

6) a) Bu soruyu çözemedim zaten aklıma bir şey gelmedi.8 dedim ama 16 8 olabileceğinden düşündüm.6 2 3 katı durumu var. 24 ü 3e bölüp oradan bulmaya çalıştım. Katlı olarak buraya 2k buraya k deyip.

c)Soruda EF verilmiş olsa belki daha rahat çözebilirdim.

Interview of P4

1) a) A’dan aşağıya dik indirdim. Ondan dolayı 30 60 90’dan 4 e 4 oluyor. Ondan dolayı nasıl yapmıştım?

E) 30 60 90 üçgeninden bahsettin.

Yani burası 30 burası 60 olduğu için buranın karşısı 8 oluyordu. 60ın karşısı $4\sqrt{3}$tü galiba.

E) Şurası da 1’di küçük yer.

Aa evet burada tekrar yaptım.

E) Ne bulmuştun cevabı?

$\sqrt{10}$ bulmuştum ama doğru mu yapmıştım hatırlamıyorum.

E) Yani burada yöntem olara bir dikme indirdin. İki eş parçaya ayırıp Pisagor yöntemini kullanmaya çalıştın.

Evet.

b) Sonucu bulmak için.

c) Burayı bulmakta ilk başta kafam karıştı. Ondan sonra zaten AD kenarı da bulmada zorlandım ilk başta. görmek için.

E) Şimdi ben şu iki kağıda baktığımda 30 60 90 üçgeninin kenar özelliklerini bulmada biraz zorlanmışsın. Aslında burada yapabiliyorsun. Demekki özelliği biliyorsun ama değil mi orada bir karışıklık olmuş.

Evet evet.

2) a) Bu soruda da yine dik indirdim. İki kenarın iki olduğunu buldum.

E) Verilenleri kullanabildin mi? 9 ve 4 cmler verilmiş.

Evet buralara yazdım. Buranın da aynı olduğunu buldum ikizkenar olduğu için. Ama açıları veremediğim için tam bulamadım.

E) Açı vermeden sence bu soruyu çözebilir miydik?

Çözebiliriz ama şey hangi açı hangisiyle eş onu bulmaya çalıştığım için galiba.

c) Üçgenin özelliğini göremedim.

3) Bu soruda şuradan bir dikme indirmeyi denedim başta. burayı bulabilmek için aradaki farkı. Burayla da bunun belli bir orana sahip olduğunu gördüm. Buranın 60 olduğunu söyledim. Burayı da 90 verdim.

E) Bir sonuca varabildin mi?

Şuralara değerler vermiştim. 30un karşısı 3 demiştim. 60ın karşısına da $3\sqrt{3}$ dedim. Ama sonuç bulamadım.

c) Şurayı nasıl gelebileceğim konusunda.

E) Sence iyi analiz edebildin mi problemi?

Edemedim galiba.

4) Bunda da yani ikizkenarlık ya da bu kenarlarda eşlik bulmaya çalıştım. Çünkü buralar dik olması için bir şey var mı diye.

E) Gerekiyor mu?

Yokmuş zaten.

E) Açıortay bilgisini kullandın mı?

Kullandım. Buraya 45 45 verdim ama yine de sonuca gelemedim.

c) E) Bir bilgi eksikliğinden mi çözemedin sence yoksa verilenleri mi kullanamadın?

Kullanamadım verilenleri.

5) Burada benzerlik kullandım. Paralel açıları vermişti. Oraları işaretledim en başta. Ondan sonra 6 ile 12 arasında olduğunu gördüm. Bu yüzden de işte $\frac{1}{2}$ oranı olduğunu buldum. Ona göre de bu kenarları buldum. 9un 2 katı 18dir diye. Ondan sonra buranın 15 olduğunu gördüm. Ve 9da 15 ise 6 da x dir diyip buldum sonucu.

c) Yoktu.

6) Bu soruda da benzerlik kullanmaya çalıştım çünkü belirli oranlar ve buralar paraleldi DC ve EF. Aynı zamanda da AB ve EF paralel olduğu için bunların da benzer olduğunu gördüm. Ama oranı nerden bulabileceğimi göremedim. Ondan dolayı devamını yapamadım.

c) Aslında ne yapacağını biliyordun ama hangi üçgenler arasında benzerlik kurulacağını mı bulamadın?

Evet.

Interview of P5

1) a) Şekle ilk bakınca geçen sene çözdüğümüz sorularda böyle şey yaptığımız aklıma geldi virgül kuralı gibi bir şey yapıyorduk. Bunları sanırım çarpıp şu kenarı filan buluyorduk. Ama hatırlayamadığım için çözemedim zaten.

E) Başka yöntem aklına geldi mi?

Hani açılara bakayım dedim ama hiçbir açı verilmediği için ordan da.

E) Eşkenar üçgen verilmiş.

60 60

E) 60ı çıkarabilirdin.

b) Bu soruda o kural uygulanıyor mu bilmiyorum ama eğer kuralı hatırlasaydım büyük ihtimal hani o kuralı uygulayıp bulacaktım.

c) Eğer bir kural uygulanıyorsa onu hatırlayamamak.

2) a) FC 4 AF 9 olduğu için orayı 13 buldum. Sonra AC ile AB eşit olduğu için AB de 13 olacak. İkizkenar. Sonrası yani 90ın karşısı 90 olabilir diye düşündüm F noktasında. DCyi bulmam gerekiyordu. Orada da şey dedim. Acaba eşkenar olabilir mi dedim sanırım. AC 13se AB 13se o zaman BC de 13tür diye düşünüp ikiye böldüm.

   b) Doğru bulup bulmadığımı bilmiyorum. Emin değilim soruda.

3) a) Bu soruda 90 ve 60 verilmiş o zaman karşısı da 30 olacak C noktası. Onun şeyi vardı 90ın baktığı yer 2x gibi bir şey mi oluyordu ya da 60ın baktığı yer 2kök gibi bir şey oluyordu x.

   E) 30-60-90 üçgeninin özelliklerini hatırlayamadın.

   Evet ama en azından onu görebildim. Sonra şey dedim C 30sa o zaman Bde 60tır. O zaman büyük üçgen de 30-60-90dır dedim ama ilerleyemedim.

   c) E) Eksik hissettiğin nokta o zaman 30-60-90 üçgeninin özelliklerini hatırlayamamak mı?

   Evet.

4) a) Açıortay diyor. Açıyı ikiye böldüğünü düşündüm. 45 45 oluyor. Şey dedim acaba açıortay dik mi iniyordu diye hatırlamaya çalıştım ama orayı şey yapamadım. Sonra, sonra ne varmış? Tanıdık sayılar var. $\sqrt{2}$ 6 ama devamını getiremedim yine bunda da.

   c) E) Açıortayın kuralını hatırlayamamak sanırım.

   Bir de görememek.

5) E) Bu ne sorusuydu?

   Bunda ilk Z'yi gördüm. O zaman Cler aynı. A Eye denk geliyor. B de Dye denk geliyor. Sonra orda şeyi de düşündüm. Acaba açının baktığı kenar aynı olabilir mi uzunluk açısından diye düşündüm. O yüzden CD 9 olabilir. 12 oluyordu CF. FK de 12 o zaman CK de 12dir gibi düşündüm. C nin olduğu açı 6ya bakıyorsa o zaman DE de 6 olabilir diye düşündüm. 6 da kaldım.

   c) Kuralı tam olarak uygulayamamak mı?

   Evet ama burda yine bir oranlama şeyi vardı.

6) a) Burada şimdi çok garip olacak ama ilk başta şeyi vermişsiniz AByi vermişsiniz DCyi vermişsiniz ama BCnin 24 olduğunun verilmemesini şey yaptım bir gördüm ilk. Sonra kafamda onu oraya yerleştirdim. Sonra naptım? Hatırlamıyorum. Yine 30-60-90 uygulayabilir miyim ya da bulabilir miyim en azından açıları diye düşündüm. Şurada Cnin küçük açı olması lazım 30 olabilir mi acaba orası dedim.

   E) Ama bize bununla ilgili bir bilgi verilmemiş.

   Evet devam ettirmedim o yüzden.

   c) Görememek direkt görselden.

Interview of P6

1) a) Bu soruyu eşkenar üçgen diyordu o yüzden 5 ile 3ü toplayıp 8 bulup eşkenar üçgen olduğu için direkt 8 dedim.

   E) Peki AD bir eşkenar üçgenin kenarı mıydı?

   Aa değilmiş.

   O zaman şey yapardım. Bunun için de bir kural vardı aslında kuralları hiç hatırlamadığım için.

   c) Soruyu okumamışım tam. Bir de kuralları çok iyi hatırlayamadım.

2) a) DCyi soruyordu. Buna yanıt vermiştim bir dakika. Bu ikizkenardı o zaman BD DCye eşit. AF 9. Hıı. AF 9du FC 4tü o zaman AC ile AB eşit olduğu için 9 artı 4 11 11. Burada ikizkenarlıktan gitmeye çalışmıştım. Şurası 90sa şurası Fnin diğer tarafı da 90dır diye düşündüm. Oradan a kare artı b kare yapmaya çalıştım.

   c) Yoktu galiba.

3) a) İç açıların toplamı 180se şurası 30dur şurası da 60dır diye yaptım. Sonrasında buna da sin cos tan falan onları hatırlamak zorladı işte onlar eksikti bende yapamadım o yüzden.

4) a) Önce a kare artı b kare eşittir c kare yapmaya çalıştım.

   E) Hangi sayılarla yapmaya çalıştın onu?

   $2\sqrt{2}$nin karesi artı DCnin uzunluğunun karesi eşittir 6nın karesi yapmaya çalıştım.

E) Bunu uygulamak için Dnin dik olması gerekir miydi?

Evet gerekirdi.

E) Bize vermiş mi öyle bir bilgi?

Şöyle yapmaya çalışmıştım onu açıortay diyorsa 45 45 bölmesi gerekiyor. Kenarortay olması gerekiyordu onun için.

c) Yine matematiksel olarak.

5) a) Bunda şey şu Cnin yansıması gibi oluyordu. Bunun için hatta BC ile CF filan birbirine oranlı oluyordu. Oradan buldum.

   b) Soruyu kısa bir şekilde çözmemde yardımcı oldu.

   c) Yoktu.

6) a) Bunda eminim bunun bir kuralı vardı ama hiç hatırlamadığım için çözemedim bunu.

   E) Fark ettiğin bir şeyler var mıydı, bir bilgi?

   A ile D nin açıları filan galiba birbirine oranlıydı veya bir şey oluyordu. Onları hatırlıyordum. Bir de 90lardan şurası işte ne bileyim dik üçgen filan oluyordu. Büyük ihtimal orada da a kare artı b kareyi filan kullanacaktık.

   c) Kurallar.

Interview of P7

1) a) Eşkenar üçgen demişti. Buradan benzerlik bulmaya çalıştığımı hatırlıyorum. Doğru yol o mu bilmiyorum ama en yapamadığım soru buydu. Hiçbir fikrim yoktu.

   c) Çünkü geçen seneki kuralları ve yapım türlerini yani şekillerini bilmiyordum unutmuşum hepsini. Ve hani önceden nasıl yapıldığını bilmeden kendim oluşturamadım çözüm yolu.

2) a) Burada da yine benzerlik ve dik üçgeni kullanmaya çalıştım. Evet ikisini. Bir de adını bilmediğim bir kural vardı.

   E) Nasıl bir kuraldı o?

   İşte hiçbir fikrim yok. Yani bu AC kenarından dik inince ve eşit şeye bölünce belli oranlar kuruyorduk. Ama yine dediğim gibi geçen seneki konuları unuttuğum için tamamen o yöntemin tam olarak nasıl olduğunu da unuttum.

c) E) Eksik olduğun noktaları ifade ettin zaten burada.

Evet.

3) a) Burada 90-60-30 üçgeni buldum. Sonra onun oranını kurup DC kenarını bulmayı denedim. Yapamadım. Ama eğer bulsaydım buradan benzerlik yapacaktım. Yani kenarları eşitleyecektim. Büyük kenarı bulacaktım.

E) Benzerliği nasıl kuracaktın?

ACB üçgenini DCE üçgenine benzetecektim. Sonra oradan ABC üçgeninin uzun kenarını bulup benzerlik orantısıyla AC kenarının tamamını bulacaktım sonra oradan 6yı çıkarıp istenilen yer AEyi bulacaktım.

c) 90-60-30 üçgeninin orantılarını bilmemek.

4) a) İkizkenar üçgen var.

E) İkizkenar üçgeni nereden anladın?

Hiçbir fikrim yok. Açıkcası soruda verildi diye otomatik saymışımdır muhtemelen dikkatsizlik olarak. Ama onun dışında 45-45-90 üçgeni buldum. Sonra bu sanırım açı dik iniyor olacaktı bir şekilde hani geçen seneki soru tiplerini hatırladığım kadarıyla yoksa onu bulamazdım nasıl dik indiğini. Ve oradan da şeyi yapacaktık sanırım BC ve CAyı BD ve DAya oranlayacaktık.

c) Yine oran yapacağımız konuyu bilmemek.

5) a) Burada da benzerlik kuracaktım yine. ABC üçgeni ve CED üçgeni arasında sonra yine CED üçgeni ile CKF üçgeni arasında kuracaktım. Açıları buldum. Açıların ne kadar aynı olduğunu buldum hangilerinin aynı olduğunu. Oradan benzerlik kuracaktım. Ama yine belli bir şeydeki o yöntemdeki belli bir yeri hatırlayamadığım için CD kenarını bulamadım. Eğer bulsaydım CD ile CFyi oranlayıp sonra DE ile FK yi o orana göre ayarlayacaktım ve DEyi bulacaktım.

c) Eksik hissettiğin noktayı yine söyledin zaten değil mi?

Evet.

6) a) Burada yine benzerlik kurdum DCB ile ABC üçgeni arasında sonra bir de ABC üçgeni ile EFC üçgeni arasında kurdum. Burda da BCyi uzun kenar

olarak vermiş hipotenüs yapacaktım oradan ACyi bulacaktım. Ama orada da bir şeyler ters gitti tam hatırlamıyorum.

Interview of P8

1) a) Eşkenar olduğu için hatırladığım kadarıyla yani bir çizgi çektiğimizde 3 ve 5 yani ikisinin de bir kenarı aynı oradan bir yöntem.
   c) Yöntemi hatırlayamadım.
2) a) Burada BD ile DCnin eşit olmasını kullanmaya çalıştım. Çizdiği dikin de FC 4müş ve AF 9muş onu kullanamaya çalıştım.
   E) Bir sonuca ulaşabildin mi?
   Ulaşamadım.
   c) Bilmiyorum bir formül.
3) a) ABC dik üçgen. İçinde de bir dik üçgen var oradan açıları kullanarak 90-60-30 açısı vardı onu kullanmaya çalıştım. Diğeri de öyle.
   E) 30-60-90'ın özelliğini hatırlayabildin mi? Kenarları arasındaki ilişkiyi.
   Hatırladığım ilişki sadece büyüklükleriyle ilgili. Yani 90ın karşısındaki en büyük kenar oluyor. 30 daki en küçük. Bir de şu iki kenar eşitmiş. Oradan belki bir orantı yaparım diye düşündüm ama.
   c) 30-60-90 üçgeni ile ilgili bir özel noktayı kaçırdım.
4) a) Açıortay vermiş onu kullanmaya çalıştım. Açıortayın uzunluğu 2kök2 oradan bir şey bulmaya çalıştım. Bulamadım ama.
   E) Burada soruyu sence iyi yorumlayabildin mi? Eksik kalan şey neydi soruyu çözmek için?
   Yani açıortayın bir özelliğini kaçırdım eksik kaldı.
5) a) Burada paralellik verilmiş onu kullanmaya çalıştım. Paralel olan kenarların iki tanesinin uzunluğu verilmiş onları kullanarak ve de 9 ve 3ü kullanarak diğer paralel kenarın uzunluğunu bulmaya çalıştım.
   E) Bulabildin mi DEnin uzunluğunu?
   Bulamadım.
   E) CDyi buldun sanırım onu 15 olarak duydum ben.
   Sanırım 15 olarak düşündüm.

E) DCyi bulduktan sonra DEye geçiş yapabildin mi peki?

Geçiş yapamadım.

c) Orantılarıyla ilgili bir noktada eksik kaldı.

6) a) Diklikleri kullanarak bir şey yapmaya çalıştım. ABC üçgeni ile DBC üçgeninin ikisi de dik ve kenarlarının değerlerini kullanarak bir şey bulmaya çalıştım.

c) E) Eksik hissettiğin nokta neydi? Şunu bilseydim bu soruyu çözebilirdim diyebiliyor musun?

EFyi bilseydim belki