A STUDY ON DEVELOPMENT OF PROSPECTIVE MIDDLE SCHOOL TEACHERS' COMPETENCE OF POSING MATHEMATICAL LITERACY PROBLEMS

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

ANİŞ BÜŞRA BARAN SARAÇ

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF MASTER OF SCIENCE
IN
MATHEMATICS EDUCATION IN MATHEMATICS AND SCIENCE EDUCATION

JANUARY 2021

Approval of the thesis:

A STUDY ON DEVELOPMENT OF PROSPECTIVE MIDDLE SCHOOL TEACHERS' COMPETENCE OF POSING MATHEMATICAL LITERACY PROBLEMS

submitted by **ANİŞ BÜŞRA BARAN SARAÇ** in partial fulfillment of the requirements for the degree of **Master of Science in Mathematics Education in Mathematics and Science Education, Middle East Technical University** by,

Prof. Dr. Halil Kalıpçılar
Dean, Graduate School of **Natural and Applied Sciences** ________________

Prof. Dr. Erdinç Çakıroğlu
Head of the Department, **Math. and Sci. Edu.** ________________

Prof. Dr. Mine Işıksal Bostan
Supervisor, **Math. And Sci. Edu., METU** ________________

Examining Committee Members:

Prof. Dr. Erdinç Çakıroğlu
Math. and Sci. Edu., METU ________________

Prof. Dr. Mine Işıksal Bostan
Math. and Sci. Edu., METU ________________

Assist. Prof. Dr. Şerife Sevinç
Math. and Sci. Edu., METU ________________

Assoc. Prof. Dr. Aslıhan Osmanoğlu
Math. and Sci. Edu., Ordu Uni. ________________

Assist. Prof. Dr. Merve Koştur
Math. and Sci. Edu., Başkent Uni. ________________

Date: 27.01.2021

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Last name : Aniş Büşra Baran Saraç

Signature :

ABSTRACT

A STUDY ON DEVELOPMENT OF PROSPECTIVE MIDDLE SCHOOL TEACHERS' COMPETENCE OF POSING MATHEMATICAL LITERACY PROBLEMS

Baran Saraç, Aniş Büşra
Master of Science, Mathematics Education in Mathematics and Science Education
Supervisor : Prof. Dr. Mine Işıksal Bostan

January 2021, 199 Pages

The current study aims to investigate competence of prospective middle school mathematics teachers to generate mathematical literacy problems throughout a designed instruction. Participants of the study, who are three prospective teachers from a public university in Ankara, were selected via purposive sampling. Data of the study were collected in the spring semester of 2019-2020 academic year. Semi-structured interviews, mathematical literacy problems generated by the prospective teachers, open-ended questionnaire and field notes were used to collect the data throughout instructional units. The data which indicate the changes or developments in competence of prospective teachers to generate mathematical literacy problems were analyzed according to three mathematical literacy dimensions (mathematical process, mathematical content and contexts) defined in PISA 2012 mathematics framework (OECD, 2013). The findings of the study demonstrated that prospective teachers used more mathematical processes and mathematical contents in their mathematical literacy problems at the end of the study with respect to their initial problems written at the beginning of the study. Moreover, they constructed cognitively more complex mathematical literacy problems that require students to use multiple mathematical competencies in higher degrees at the end of the study. In addition to that, they constructed more proper real-life contexts in terms of the level of context use and authenticity in their final problems.

Keywords: Mathematical Literacy, Prospective Teachers, Teaching Experiment

ÖZ

ORTAOKUL MATEMATİK ÖĞRETMEN ADAYLARININ MATEMATİK OKURYAZARLIK PROBLEMLERİ ÜRETME BECERİLERİNİN GELİŞTİRİLMESİ ÜZERİNE BİR ÇALIŞMA

Baran Saraç, Aniş Büşra
Yüksek Lisans, Matematik Eğitimi, Fen ve Matematik Bilimleri Eğitimi
Tez Yöneticisi: Prof. Dr. Mine Işıksal Bostan

Ocak 2021, 199 Sayfa

Bu çalışma, ortaokul matematik öğretmen adaylarının matematik okuryazarlığı problemleri üretme becerilerini, tasarlanmış bir eğitim boyunca incelemeyi amaçlamaktadır. Araştırmaya katılmaları için Ankara ilinde bir devlet üniversitesinde okuyan 3 öğretmen adayı amaçlı örneklem yöntemiyle seçilmiştir. Araştırmanın verileri 2019-2020 öğretim yılı bahar döneminde toplanmıştır. Veriler eğitim birimleri boyunca yarı yapılandırılmış görüşmeler, öğretmen adaylarının ürettiği matematik okuryazarlık problemleri, bir açık uçlu anket ve alan notları aracılığıyla toplanmıştır. Öğretmen adaylarının matematik okuryazarlığı problemleri üretme becerilerindeki değişim veya gelişmeleri gösteren veriler, PISA 2012 Matematik Çerçevesi'nde (OECD, 2013) tanımlanan üç matematik okuryazarlığı boyutuna (matematiksel süreç, matematiksel içerik ve bağlam) göre analiz edilmiştir. Araştırmanın bulguları, öğretmen adaylarının araştırma sonunda yazdıkları matematik okuryazarlık problemlerinde, araştırmanın başında yazdıkları problemlere göre daha çok çeşitte matematiksel süreç ve matematiksel içerik kullandıklarını göstermiştir. Dahası, öğretmen adaylarının çalışmanın sonunda daha üst düzey matematiksel yeterlilikler içeren, bilişsel olarak daha karmaşık matematik okuryazarlığı problemleri oluşturdukları tespit edilmiştir. Buna ek olarak, öğretmen adaylarının, araştırma sonunda yazdıkları problemlerinde, günlük hayat bağlamı kullanım düzeyi ve özgünlük açısından daha uygun gerçek yaşam bağlamları oluşturdukları gözlenmiştir.

Anahtar Kelimeler: Matematik Okuryazarlığı, Öğretmen Adayları, Öğretim Deneyi

To all my loved ones

ACKNOWLEDGMENTS

I wish to express my deepest gratitude to my supervisor Prof. Dr. Mine Işıksal Bostan for their guidance, advice, criticism, encouragement, and insight throughout the research. Without her scientific experience and vision, it was impossible to compete this research.

Special thanks to my committee members Prof. Dr. Erdinç Çakıroğlu, Assist. Prof. Dr. Şerife Sevinç, Assoc. Prof. Dr. Aslıhan Osmanoğlu and Assist. Prof. Dr. Merve Koştur for their feedback, contributions, and suggestions for my study.

I would like to thank my colleagues and my friends Zeynep Özel and Emine Çatman Aksoy for their technical assistance for my research and their valuable support.

I would like to thank my lovely family, who always supported me during my study. My father Bayram Baran always believed me and encouraged me; my mother Yaşasın Baran always listened my complains and my sister Eda Nur Baran always cheer me up hundreds of miles away.

In addition, I would thank all my friends. Thank you for not leaving me alone in this challenging process. I am very lucky to have friends like you. Also, I am grateful to the preservice teachers who voluntarily participated in my study.

My deep gratitude is for my darling Uğur Berkay Saraç, who always the source of cheer through this challenging journey. Thanks so much for being there on my side and supporting me all through.

TABLE OF CONTENTS

LIST OF TABLES

TABLES

LIST OF FIGURES

FIGURES

LIST OF ABBREVIATIONS

ABBREVIATIONS

ML	Mathematical Literacy
PST	Prospective Teacher
METU	Middle East Technical University
MoNE	Ministry of National Education
OECD	The Organization for Economic Corporation and Development
NCTM	National Council of Teachers of Mathematics
FMC	Fundamental Mathematical Capabilities

CHAPTER 1

INTRODUCTION

Individuals in every country are increasingly encountering with countless quantitative, probabilistic, statistical and spatial reasoning issues (OECD, 2010). Reading tables, charts or graphs shown in media, using maps, interpreting bus schedules are only a few examples. To meet the needs of modern life, individuals need an ability which they can use and apply mathematical knowledge they learned in schools. This ability is called mathematical literacy. In a broad sense, mathematical literacy is an ability to understand, apply, use and communicate mathematics in everyday contexts to solve the problems that arise in real-world (Hope, 2007; Jablonka, 2003; NIACE, 2011; Tout, 2001). As clearly illustrated in the preceding description, mathematical literacy does not only involve knowing mathematical procedures and executing them, it also involves competence to apply mathematics in a real-world situation (Ojose, 2011). In other words, a mathematical literate person is expected to solve problems that may arise in real life by estimating, communicating and reasoning numerically, statistically and spatially.

According to Ewell (2001), mathematical literacy includes functionality of mathematics and thence it is an important ability to function in a society. Further it is an essential skill for an individual who lives in society (Sumirattana et al., 2017) and is a necessity both in daily and professional life (Ojose, 2011). Therefore, considering the needs and expectations of individuals, training individuals as mathematical literate who can integrate mathematics in real life contexts gained much more importance (Woodbury, 1998). For this reason, understanding of mathematical literacy has started to be seen as an important aspect of the mathematics education from many countries. After mathematical literacy had been accepted as one of the educational policies in many countries, PISA (Programs of International Student Achievement) was developed to assess 15 years-old students' mathematical literacy as well as their science and reading literacy. Accordingly, many countries have shaped their mathematics curricula in order to produce mathematical literate students (Breakspear, 2012). For example, mathematical literacy has been a main objective of mathematics education in countries such as the USA, the UK, Australia, and many other countries (NCTM, 1988; QCA, 2009; South Africa Department of Education, 2003). Turkey also altered its mathematics education with educational reform in 2005. With this reform, solving real life problems, critical thinking, problem solving and emphasis on mathematics and its value have become the goals of mathematics education system in Turkey (MoNE, 2005). In addition to this, mathematical literacy is viewed as one of the general aims that Turkish national mathematical education program tries to achieve (MoNE, 2018).

Although mathematical literacy has become a goal of Turkish mathematics education curricula, PISA results in recent years have showed that Turkish students fail to understand and apply mathematics in real-life applications (OECD, 2014, 2019). It is obvious that educational reforms have not achieved the desired outcome. One of the main reasons for this is that contrasting with general aim of national curriculum, school mathematics primarily continues to give major importance to expressing and conveying basic concepts of geometry, algebra and calculus (Steen et al., 2007; Treffers, 1991). In addition, Ellis (2001) shows that the instructions provided by algebra teachers tighten students' thoughts rather than extending it. He observes that students who leave an elementary algebra class are less able to solve real-life problems after the class than before (Ellis, 2001).

Contrary to the mathematics education taught in the schools, it is important to help students develop their mathematical literacy skills. In order to improve students' mathematical literacy, teaching basic concepts or mathematical formulas is not always enough, teachers should also teach how to connect the mathematics with real-life contexts (Steen et al., 2007). At this point, it is of great importance, therefore, to examine whether the teachers are competent to provide students with such an environment.

It is known that teachers and instructional practices play a significant role for students to understand and apply mathematical knowledge in real world (Hope, 2007). According to National Numeracy Review Report in 2008, teachers' practices, educational resources they bring into classroom and their assessment methods determine the quality of education and success. In addition, students' experience in solving mathematics problems which formulated in different contexts and situations help them to improve their mathematical literacy (De Lange, 1987; Steen et al., 2007). These studies show that it is important for teachers to have the competence to prepare and provide educational resources in various contexts. Taking into consideration the PISA results in Turkey and several other studies explaining teachers' effects on students' achievement (Aksu et al., 2017; De Lange, 1987; Steen et al., 2007), it can be claimed that teachers in Turkey lack competence to provide students an environment which supports their mathematical literacy (Altun & Akkaya, 2014; Ozgen, 2019). To have this competence, teachers should receive training during their undergraduate education, which will eventually help them develop their students' mathematical literacy.

In a national teacher educational research in Australia, which involved 303 secondary school teachers in their first and second years, teachers were asked to evaluate the effectiveness of their preservice preparation for teaching mathematical literacy. They expressed that they did not feel prepared to teach mathematical literacy when they graduated from their teacher education programs (Milton et al., 2007). Besides, by the time the current study is conducted, in many teacher education programs in Turkey, there are only a limited number of courses that teachers can take to learn how to improve the mathematical literacy of their students. This also supports the idea that teachers in Turkey lack the competence to provide students an environment which supports their mathematical literacy because they do not receive the required training. Because of this, training of pre-service teachers about creating such environments is a topic worth researching.

As explained above, an important aspect of the teaching environments generated by teachers to improve students' mathematical literacy is mathematical problems posed in classrooms (De Lange, 1987; Steen et al., 2007). Accordingly, in this study, creation of learning environments for students in order to make them mathematically literate individuals is investigated by focusing on the ML problems posed by future teachers in the classroom.

In order to examine the ML problems posed in learning environments, the mathematics framework of PISA published by OECD (2013) is used in this study. In this framework, a good mathematical literacy problem includes three overarching aspects; mathematical process and the mathematical capabilities that underlie those processes, mathematical content and context. Mathematical processes are defined as what students do to make connections between the context of the problem and mathematics. Individuals solve the mathematical problems which arise in the real-life with these processes. In order to do this, individuals need to transform the real-life problem into a mathematical problem, then they need to apply mathematical rules and operations to solve it and lastly interpret how this mathematical solution works in a real-life context. Moreover, PISA establishes several fundamental mathematical capabilities that underlie these processes, which are required for students to solve the problems and apply the mathematical processes. They also help to understand the cognitive demand and the difficulty of the problems. The second aspect of ML problem, which is mathematical content, is the mathematical phenomenon underlying the problem. Context, which is the last aspect of the ML problem, is defined as a part of a person's life in which the problem arises. Under the context aspect, PISA is concerned with the type of contexts in which ML problems are established, the level of context use while solving the problems, and the authenticity of the contexts (OECD, 2013).

In the light of the given framework, the ML problems posed by the PSTs are investigated under three aspects in this study. First, the problems are examined under which mathematical processes they include and which capabilities they require to be solved. As mentioned, individuals solve real-life problems that involve mathematics with these processes. Therefore, it is crucial for teachers to prepare activities and problems which involve all mathematical processes to teach students how to deal with real-life problems and enrich mathematics teaching (Stacey & Turner, 2015; Turner, 2007). This is why, this study aims to investigate which mathematical processes PSTs use while generating ML problems and how this change. On the other hand, fundamental mathematical capabilities are also very important parts of ML problems. They illustrate which cognitive actions need to take place and at what degrees while ML problems are being solved. Therefore, it is significant for teachers to understand which cognitive actions students use while solving the problems and design teaching and learning activities to improve their students' mathematical proficiency (Stacey & Turner, 2015). This is why, the current study investigates which mathematical capabilities PSTs use while posing ML problems at what degrees and how and why these capabilities and their degrees change.

Secondly, the contents of the problems are investigated because a mathematical learning environment should involve learning materials and problems covering all content areas. As a result, the present study investigates which and how many mathematical content areas PSTs use while posing ML problems and how these content areas change.

Lastly, the problems are examined under the contexts they include, their level of context use and whether they are authentic or not. Context is one of the most important aspects of ML since appropriate mathematical strategies for solving problems are chosen with respect to the contexts of the problems (OECD, 2013). In teaching environments, it is important for students to encounter problems posed in a variety of contexts to be able to solve mathematical problems arising in several areas. Consequently, in the current study, which contexts are used by PSTs while generating ML problems and how this changes are investigated. Level of context use is another issue of concern related to the context of ML problems. The role of context is an essential feature for ML problems, and it is supported to use contexts while formulating or solving the problems (OECD, 2013). This is why, the present study examines to what extend contexts are used in the problems posed by PSTs and how this change. Authenticity of context is also an essential feature of ML problems since asking authentic problems increases the chance of getting realistic answers from students (Palm, 2008; Stacey & Turner, 2015). As a result, the current study examines the authenticity scores of the ML problems posed by PSTs and how these levels change.

By using this framework, the ML problems that PSTs pose can be assessed. Therefore, it is possible to investigate whether a teaching training program could develop PSTs competence to pose efficient ML problems. This may give an answer to the question of how to improve classroom environments in which students raise to be mathematical literate individuals. With this aim, in the current study, a teaching experiment is designed and PSTs are given training in order to search for answers to the research questions which are provided in the next section.

1.1 Purpose of the Study and Research Questions

The purpose of the study is to investigate prospective middle school mathematics teachers' capability to generating mathematical literacy problems throughout a classroom teaching experiment. The following key areas will be addressed for this purpose:

To what extend do prospective middle school mathematics teachers' capability of generate mathematical literacy problems change or develop as they attend to teaching experiment of posing effective mathematical literacy problems?

1. How do mathematical processes and the underlying mathematical capabilities in the mathematical literacy problems generated by PST change/develop during their enrollment in Posing Effective Mathematical Literacy Problems (PEMLP) training?

    1.1 How do mathematical process categories generated by PSTs change or develop?

1.2 How do the level of fundamental mathematical capabilities that underlie mathematical processes in the mathematical literacy problems generated by PST change or develop?

2. How do mathematical contents in the mathematical literacy problems generated by PST change or develop during Posing Effective Mathematical Literacy Problems (PEMLP) training?

3. How does real-life context in the mathematical literacy problems generated by PST change or develop during Posing Effective Mathematical Literacy Problems (PEMLP) training?

   3.1. How do context categories of mathematical literacy problems generated by PSTs change or develop?

   3.2. How does the level of context use (mathematical relevance of contexts) in the mathematical literacy problems generated by PST change or develop?

   3.3. How does the level of authenticity of mathematical literacy problems generated by PST change or develop?

1.2 Rationale and Significance of Study

Literacy is an important tool for meeting the needs of social, political and cultural life (İş, 2003). Contrary to popular belief, being literate is not only individuals' ability to read and write but also understand a specific area and use it in different real-world situations (OECD, 2001). Considering that individuals use mathematical knowledge to make decisions in almost all aspects of real life, it can be argued that being mathematically literate is essential for every citizen, and thus is an ability everyone should have. Therefore, it is crucial to help everyone in society to gain a certain degree of mathematical literacy. This is the duty of educational institutions, that aim to prepare individuals for life. This can be achieved through educational reforms that center on students' mathematical literacy development.

In recent years, mathematical literacy is considered as an important part of the national mathematical education program and is viewed as one of the general aims that Turkish students try to achieve (MoNE, 2018). However, the current results of PISA show that these educational reforms have not been successful for raising mathematically literate students. For example, PISA 2018 results show that mathematical literacy levels of Turkish students are lower than the students in many OECD countries. Besides, 64% of Turkish students' mathematics performance is seen to be at Level 2 and below, which means that they can only solve relatively low-level real-life problems (MoNE, 2019). These observations illustrate why Turkish students' mathematical literacy levels are low. Therefore, examining how to increase this level is very significant.

Many studies investigate the reasons for Turkish students' failure in PISA (Çetin & Gök, 2017; Özberk et al., 2017; Yıldırım et al., 2017) and what factors affect students' mathematical literacy (Aksu et al., 2017; İlbağı Azapağası & Akgün, 2013; Koğar, 2015). Several studies on this issue indicate the valuable effects of teacher's style and preferences in the class (e.g. materials, representation, tasks) on students' mathematical literacy (Höfer & Beckmann, 2009; Kramarski & Mizrachi, 2006; Roth et al., 2015) and some studies show that solving PISA-like context-based problems can help increasing students' mathematical literacy success (Dewantara et al., 2015; Hermawan et al., 2019; Meaney, 2007; Yılmazer & Masal, 2014). Therefore, it is important to investigate teachers' and prospective teachers' capability to select and pose PISA-like context-based problems. From this point of view, the findings of this study are significant since this study has potential to contribute Turkish literature by giving important information about the capability of Turkish middle school teachers of posing PISA-like problems.

In addition to that, as applied in this study, the trainings and instructions prepared to improve the pre-service teachers' understanding about realistic real-life mathematics problems proved to be useful for teacher education (Bonotto, 2007; Verschaffel et al., 1997). In this sense, the contribution of this study to the literature is important because it provides an environment for prospective teachers to develop their views and competences. As PSTs in this study generated their own realistic real-life problems and revised them, they became organizers of learning materials rather than being only curriculum implementers (Even & Tirosh, 1995). Therefore, as also embodied in this study, trainings designed for teaching how to prepare learning materials help PSTs to use their creative and critical thoughts to design effective learning environments (Sevinc & Lesh, 2018). When the previous studies related to teachers' capability to generate mathematical literacy problems are investigated, it is seen that findings of several studies indicate that teachers have difficulty in managing contextual elements of mathematical literacy problems (Aydın & Özgeldi, 2016; Pillay & Bansilal, 2019; Wijaya et al., 2015) and posing effective and high level mathematical literacy problems (Ozgen, 2019; Siswono et al., 2018). Although the deficiencies in teachers and teacher candidates' capability in generating mathematical literacy questions have been identified, there are very few studies on how to overcome these deficiencies. The current study has been designed to fill this gap in the literature. It may provide detailed investigation about how prospective teachers change to improve their capability to pose mathematical literacy problems throughout a course. This study's findings can give teacher educators valuable information about how prospective teachers deal with their difficulties in generating mathematical literacy problems.

The context of the current study is constructed with regard to related literature on context-based problems, teachers' difficulties in constructing context-based problems and PISA Mathematics Framework (OECD, 2013). Previous studies indicated that teachers consider context based problems as plain word problems (Wijaya et al., 2015) and they have difficulties in constructing suitable contexts for mathematics problems (Siswono et al., 2018). Therefore, Siswono and his colleagues (2018) gave training to teachers about the level of context use in PISA-like problems and they found that while the level of context use in PISA-like problems generated by teachers were at the lowest level at the beginning of the study, most of the problems generated after the study required the context to be used while being solved. For this reason, Siswono and his colleagues (2018) suggested future studies to support teachers' understanding of real-life context as well as the structure of PISA problems. They suggested covering other aspects of contexts in mathematical literacy problems, like authenticity which is stated by Palm (2006). Therefore, the context of the current study covers the quality of real-life contexts as well as the mathematical literacy structure of the problems. By this way, prospective teachers' capability of generating mathematical literacy problems is aimed to be investigated in a broader perspective. Hence, this study could contribute to related literature.

Moreover, according to Sfrand (2013), teaching mathematical literacy should be covered in research studies to lead teachers. Therefore, the context of the study could give teachers information about which types of problems support students' mathematical literacy and how they should implement their lessons to improve their students' mathematical literacy.

Lastly, this study is significant for teacher educators since in most universities in Turkey, no specialized course is offered to improve prospective teachers' capability to support students' mathematical literacy. Therefore, this study's learning environment could be an example for teacher educators while constructing a course related to mathematical literacy.

1.3 Definition of Important Terms

In this section, definition of important terms which are used in this study are provided below:

Mathematical Literacy: Mathematics literacy is the capacity of an individual to respond efficiently to the mathematical demands of personal, social, and work-life and adapt to new demands and requirements in a continually changing society to make reasonable decisions (OECD, 2013). Therefore, it requires the ability to formulate, employ, and interpret problem situations presented in various contexts by analyzing, reasoning, and communicating mathematical ideas accurately and satisfactorily as a constructive, committed, and thoughtful citizen (OECD, 2013).

Mathematical Literacy Problems: Mathematical literacy problems are defined as problems located in real-world environments that contain elements or provide information that must be organized and mathematically modeled to arrive at a solution (Freudenthal, 1983).

Mathematical Processes: The mathematical processes are mathematical modeling steps that explain what people do to link mathematics to the context of a problem and thereby solve the problem (OECD, 2013).

Real-life Contexts: The context is the area where the problems take place in a person's life (OECD, 2013).

Level of Context Use: The degree of relevance of context while solving mathematical problem or judging the answer (De Lange, 1987).

Authenticity: Authenticity is one of the characteristics of mathematical tasks or activities that specify the degree of alignment between the mathematical task context and the corresponding real-life situation (Palm, 2008)

CHAPTER 2

LITERATURE REVIEW

This study aims to investigate development of prospective teachers' capability to generate mathematical literacy problems throughout the given course.

The first part begins with the meaning and development of mathematical literacy. Then, it is followed by international perspectives of mathematical literacy, PISA mathematical literacy and, it finally continues with mathematical literacy studies. At the end of this chapter, the summary of literature is given.

2.1 Mathematical Literacy

The first section will include (i) the research studies related to the meaning of mathematical literacy, (ii) international perspectives of mathematical literacy, (iii) the definition of mathematical literacy in view of the PISA mathematics framework.

2.1.1 Meaning and Development of ML

Mathematical literacy (ML) has become a popular term recently. The term mathematical literacy was firstly mentioned in USA in 1944 by National Council of Teaching Mathematics (NCTM) on Post War Plans (Coxford & Jones, 1970, p. 244). In this report, it is noticed that schools are supposed to guarantee students' development as mathematical literate individuals. In recent years, the term mathematical literacy has gained popularity with NCTM 1988 Standards which acknowledged five general aims that support students' inquiry of mathematical literacy; “learn to value mathematics, become confident with their ability to do mathematics, become mathematical problem solvers learn to communicate mathematically, and learn to reason mathematically” (Research Advisory Committee of the National Council of Teachers of Mathematics, 1988, p. 5). However, there is no definition offered for mathematical literacy in this document.

Mathematical literacy has been defined variously by many researchers for many years. Researchers and educators agree that it is not a distinct concept and the arguments about this situation have been still continuing today (Coben et al., 2003; Jablonka, 2003; Withnall, 1995). According to Jablonka (2003), mathematical literacy is inseparable from its context; therefore, the differences among conceptions about mathematical literacy are formed. In other words, mathematical literacy is a culturally and socially embedded practice, and it may have different meanings in every society because of cultural differences (Jablonka, 2003). De Lange (2003) supports this view by claiming that the context of mathematical literacy should be special to cultures as every society has its own culture and social orientations; however, it should be accessible for all people as a part of that culture.

According to Jablonka (2003), there are three possible perspectives for the definition of mathematical literacy depending on the cultural and contextual backgrounds of the people who do the definition. They are;

> The ability to use basic computational and geometrical skills in everyday contexts. The knowledge and understanding of fundamental mathematical notions. The ability to develop sophisticated mathematical models, the capacity for understanding and evaluating another’s use of numbers and mathematical models (Jablonka, 2003, p. 76)

Steen (2001) gives a simple definition about mathematical literacy which is similar to the first definition by Jablonka (2003) mentioned. He claims that mathematical literacy is “capacity to deal effectively with the quantitative aspects of life” (Steen, 2001). On the other hand, de Lange (2003) argues that mathematical literacy should not be limited to quantitative aspects and it should cover mathematics more extensively. According to him, an individual can use his mathematical skills not only for operating with numbers and equations but also for understanding geometry, spatial concepts, etc. For instance, while giving directions to people, individuals use their spatial skills (De Lange, 2003). Although their opinions about the dimensions of knowledge of mathematics conflict with each other, both agree that it should be used while solving problems in real life.

The idea which supports that mathematical literacy is an ability to deal with mathematical aspects of the life is supported by many researchers (Hope, 2007; Jablonka, 2003; McCrone & Dossey, 2007; Ojose, 2011). McCrone and Dossey (2007) states that mathematical literacy is "making math relevant and empowering for anyone" rather than owning higher level of knowledge of mathematics. In other words, people who have mathematical literacy should connect mathematics with real life and they can use mathematics to help them for their work. Likewise, Hope (2007) emphasizes that to know only rules or algorithms are not enough to make an individual mathematical literate. In order to become mathematical literate, a person should have an ability "to reason, analyze, formulate, and solve problems in a real-world setting" (Hope, 2007, p. 29). In addition to that, Hope (2007) defines mathematical literate people as "informed citizens and intelligent customers" (p.29). They can explicitly understand, interpret and analyze the information given in the news, articles etc. and use this information to become conscious customers. In the same way, the view of mathematical literacy, which was mentioned in Australian Association of Mathematics Teachers report (1997) as cited in Forgasz and Hall (2019), claims that in order to become mathematical literate, individuals are required to fulfill the general needs of personal life, occupational life and social life by using mathematics effectively.

Furthermore, there are some researchers who support the idea that mathematical literate person should have a capacity to develop and understand mathematical models (Brown & Schäfer, 2006; Kaiser & Willander, 2005) as well as Jablonka (2003). According to Kaiser and Willander, "the theoretical approach of mathematical literacy relies strongly on applications and modelling" (2005, p. 48). Mathematical literacy aims functional use of mathematics and its applications in real life (De Lange, 2003). This means that individuals should use mathematics for better understanding, analyzing and synthesizing the problems they encounter in real life and making efficient decisions to solve them. This idea constructs the basis of mathematical modeling. Like Kaiser and Willander (2005), Brown and Schäfer (2006) claim that the process of selecting and using mathematics while analyzing and solving the real-life problems is very similar to mathematical modeling. In their study, they demonstrate as an evidence to their claim that in Programme for International Student Assessment (PISA), it is also aimed to define mathematical literacy levels of 15-year-old individuals by checking their solutions for basic mathematical modeling problems.

Definitions above are the results of mathematics educators' seeking for a common definition and a model for evaluating mathematical literacy levels of people (De Lange, 2003; Hope, 2007; Jablonka, 2003; Kilpatrick, 1965; Pugalee, 1999; Steen et al., 2007). Although several researchers define mathematical literacy differently, most of them agree that it is "an ability to deal with real-life problems and challenges with the help of the mathematics" (Hope, 2007; Jablonka, 2003; Kaiser & Willander, 2005; Ojose, 2011). From this point of view, the most accurate definition was offered by OECD in the initial framework for PISA in 1999 and it became the most accepted definition in the literature. Moreover, definition of mathematical literacy, as conceptualized in the PISA, is a comprehensive definition that covers important aspects of other definitions discussed above. The definition has been slightly changed until this year. The recent version of definition was given for PISA 2012 (OECD, 2013, p. 25):

> Mathematical literacy is an individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena. It assists individuals to recognise the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective citizens.

As mentioned above, the definition has been developed throughout the times with different opinions of several researchers. Firstly, some researchers have claimed that it is cultural practice; therefore, it should serve for requirements of several real-life conditions and it should connect mathematics with individuals' requirements of life. Then, some researchers defend that individuals requires mathematical capabilities to understand, analyze, use mathematical concepts, predict a phenomena. Furthermore, several researchers have believed that using mathematical capabilities to solve requirements of real life situations is a cyclic process involving formulating, employing and interpreting; so, it requires to construct mathematical model. When we analyze these definitions of mathematical literacy, we can observe that PISA definition is the most improved version since it includes all the ideas above. According to this definition, mathematical literacy is a cyclic process of using mathematics for variety of real-life contexts involving formulating, employing and interpreting processes. Individuals need to use several mathematical capabilities (reasoning, analyzing, explaining…) and mathematical topics (algebra, numbers, geometry …) to complete these processes. Furthermore, these processes help them to make reasonable decisions and judgements in real life (OECD, 2013). PISA 2012 mathematical literacy definition was used for the reasons stated above in this study.

2.1.2 International Perspective of ML

The notion of mathematical literacy has been documented in various ways in the international literature. There is no agreement on the term mathematical literacy. "Numeracy" and "quantitative literacy" terms are used synonymously with the term "mathematical literacy" by some researchers. In recent years, "numeracy" has been found in PISAAC (Programme for the International Assessment of Adult Competencies) framework of OECD and "quantitative literacy" has been used in Steen's studies (see, e.g., Madison & Steen, 2003). In other words, all of terms, which represent mathematical literacy, maintain their existence. In this part, the definitions and comments about numeracy, quantitative literacy and mathematical literacy are given in order to cover the history and the meaning of the notion more precisely. In addition to that, the similarities and differences between these terms and then, the conclusion drawn about them are demonstrated. This can provide insights about the current international discussions about which ways people could be supported to deal with real-life situations which involve mathematics.

2.1.2.1 Numeracy

There has been growing number of literature disposals for “numeracy” and “mathematical literacy” synonymously (Jablonka, 2003). The notion of numeracy is characterized in multiple ways beginning from a basic ability that must to be gained at the end of the basic education to an ability to deal with quantitative perspectives of real-life situations. It was first used in Crowther Report (1959). The report recommends new ways to define how to teach students to prepare them to deal with mathematical demands of the life. In the report, numeracy is defined as improving communications among groups as well as a reflection of being literate. Later, the term was used in the Cockcroft and his Committee of Inquiry (1982). Cockcroft report mentions about the foundation list of mathematical topics which is related to numeracy. It is used for practical applications of adult life in “Foundation list of mathematical topics” which provide basis for national curriculum and core curriculum of adult numeracy (Coben et al., 2003; Steeds, 2001). The mathematics topics are included in the list are money, representations and measurement. Therefore, notion of numeracy is broadly used in mathematics education programs for adults (De Lange, 2003; Jablonka, 2003) Cockcroft (1982) argues whether numeracy is considered as an ability to deal with mathematical needs of adults’ life. He suggests to cover the ability to use basic operations confidentially in experienced situations as well as the ability to perform basic operations. Then, numeracy is defined by Cockcroft and his Committee of Inquiry (1982) as a competence to use mathematical skills that help individuals to supply mathematical requirements of real-life and understanding the information that is presented under graphs and charts.

The development of the concept has continued with the emphasis on the importance of reasoning skills and making valuable interpretations from real life issues (Coben et al., 2003; South Africa Department of Education, 2003; The British Department for Education and Employment, 1998). For example, Coben (2003) defined the numeracy as follows:

> [Being] numerate means to be competent, confident, and comfortable with one's judgments on whether to use mathematics in a particular situation and if so, what mathematics to use, how to do it, what degree of accuracy is appropriate, and what the answer means in relation to the context (p. 10).

The description above highlights the importance of reasoning and interpretation skills of an individual while selecting and applying necessary mathematics. It also implies the tendency to know which mathematical tool or argument is useful for a particular case.

2.1.2.2 Quantitative Literacy

While the term of numeracy is commonly used in adults' mathematics education programs, the term of "quantitative literacy is preferred by the National Council on Education and the Disciplines rather than mathematical literacy (Steen, 2001). The term also can be seen on The Assessment of National Adult Literacy (NAAL) study having been carried out by Educational Testing Service in 1995. They claim that literacy includes "using printed and written information to function in society, to achieve one's goals, and to develop one's knowledge and potential" (Educational Testing Service, 1995, p. 17). They divide the literacy into three scales; Document Literacy, Pros Literacy and Quantitative literacy. According to their definition of Quantitative Literacy, it refers to knowledge and skills that is necessary to solve arithmetic operations on numbers printed formats (Educational Testing Service, 1995).

Another definition is given by Steen as "Quantitative literacy is the capacity to deal effectively with the quantitative aspects of life" (2001, p. 6). According to de Lange

(2003), these quantitative aspects of life concern with certainties, uncertainties and relations. These correspond to quantity, change and relationships categories in mathematics. In other words, quantitative literacy only covers arithmetic perspectives of real life and it does not involve spatial literacy like numeracy. For this reason, mathematical literacy is considered as an overarching concept which covers quantitative literacy and numeracy by de Lange (2003).

When the international perspectives mentioned above are examined, it can be observed that the notion of mathematical literacy includes other notions (De Lange, 2003) and the term is generally used for assessing students' capacity of using mathematics in real-life (e.g. PISA, TIMMS) unlike the other terms which are commonly used for assessing adults' capacity (e.g. NAAL). For this reason, in this study, it is decided to use the notion of mathematical literacy.

2.2 PISA Mathematical Literacy

This study aims to investigate the development of prospective teachers' competence to pose ML problems throughout the given course. Within the context of exploring prospective teachers' competence of posing problems for mathematical literacy, the specific focus has been on teachers' working related to how to pose problems that have a connection with mathematical contents and authentic real-life situations within the given course. Therefore, in this study, it needs to have a framework which defines mathematical literacy explicitly and in detail, and which describes how to organize mathematical content knowledge in relation to mathematical processes and real life in a problem. When all discussions in section 2.1 are considered, the definitions of the mathematical literacy depend on the requirements, purposes and contextual backgrounds of studies in which it is explained. When viewed from this aspect, the mathematical literacy definition of PISA is the most comprehensive definition since it covers all fundamental aspects of other definitions. It is also considered as global one since PISA is an international exam dealing with assessing mathematical literacy levels of 15-year-old students in all countries around the world. In addition to this, PISA mathematics framework offers a model explaining the connection between mathematical processes, mathematical contents and real-world contexts (OECD, 2013). This model helps prospective teachers to present how to pose effective problems for mathematical literacy.

In this section, mathematics framework which was published by OECD in PISA 2012 Assessment and Analytical Framework Report is examined in detail. Although latest PISA framework was published in 2018, PISA 2012 framework is selected for this study and viewed under this section. As mentioned before, the PISA study is organized once every three years, and during the years it is organized, a subject area (mathematics, science and reading) is determined as the main subject, and major changes in the related framework are done. As 2012 was the last year in which mathematics was chosen as the main subject in the PISA study, the framework of PISA 2012 will be examined in this study.

2.2.1 Definition of ML in the view of PISA Mathematics Framework

Mathematics is a critical tool for individuals, preparedness, life and the society. For this reason, it is important to ask which mathematical capabilities are important for young people' preparedness for her current and future life e.g. occupational or personal life. (OECD, 2013) This is the main question with which PISA concerns. In order to answer this question, the construct of mathematical literacy plays an important role.

The domain of mathematical literacy of PISA is established upon the individuals' capacity to analyze, reason and communicate with mathematical ideas, and use and make sense of mathematics in a variety of contexts. For instance, PISA (2012) mathematical literacy is defined as:

> Mathematical literacy is an individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena. It assists individuals to

> recognize the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective citizens (OECD, 2013, p.25).

According to the definition, an individual is required to play an active role by formulating, employing and interpreting in the situations that involve mathematics to prepare for her life and to become an effective citizen.

2.2.2 Organizing the Domain

PISA assesses that mathematical literacy level of 15-year-old students, which are the degree to students, can manage with mathematics efficiently when they deal with real life situations, problems or challenges including mathematics. PISA 2012 defines three interrelated aspects to analyze mathematical literacy of students namely mathematical processes and the underlying mathematical capabilities, mathematical contents and contexts. Mathematical processes are defined as “what individuals do to connect the content of the problem with mathematics”, mathematical contents are defined as which contents are aimed to use and context is defined as in which location used for mathematics problems (OECD, 2013, p. 27). These aspects are elaborated in following sections.

2.2.2.1 Mathematical Processes and the Underlying Mathematical Capabilities

Mathematical processes which are defined as what students do to connect content of the problem and mathematics, include three categories. These are formulating, employing and interpreting. They constitute a cyclic mathematical modeling process which is given in Figure 2.1. In brief, formulating is defined as determining essentials for analyzing, setting up and solving the problem. In other words, it is the process of translating the real world setting into mathematics by using mathematical structure, representations and specificity. The process of formulating requires to recognize the mathematical structures and variables in real-life context, define constraints and assumptions behind any mathematical model, represent the situations through using appropriate representations and translate to each other, understand the mathematical relationships between context and mathematical symbolism and represent them with mathematical language.

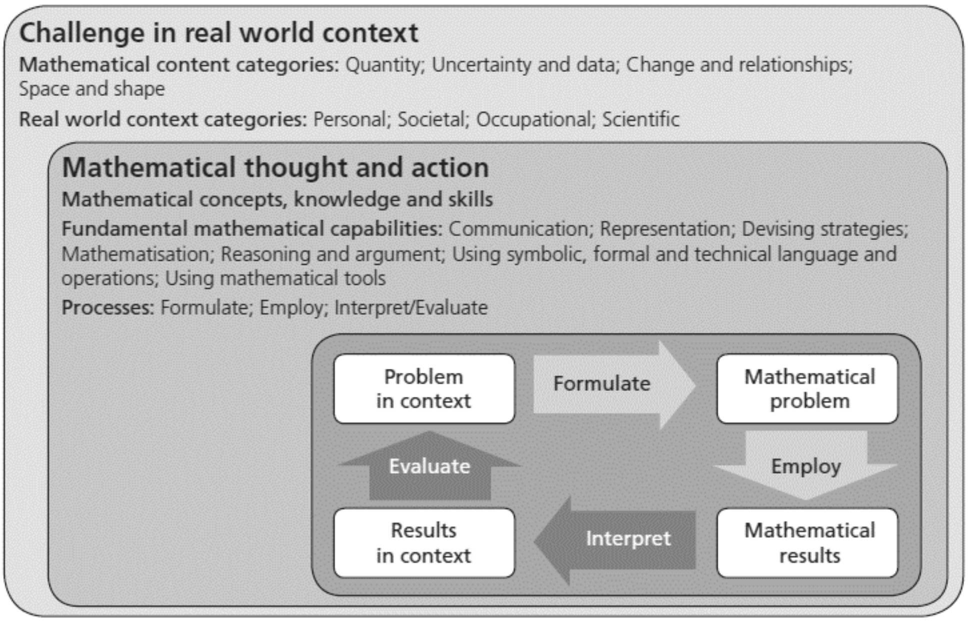


Figure 2.1 Model of Mathematical Literacy in Practice (OECD, 2013, p.26)

Employing is described as "individuals being able to apply mathematical concepts, facts, procedures, and reasoning to solve mathematically-formulated problems to obtain mathematical conclusions." (OECD, 2013, p. 29). Employing is the process of finding mathematical solution. It may require some procedures for finding the solution such as creating and applying strategies, using useful mathematical tools like technology, applying mathematical procedures, using different representations and switching among them and making generalizations based on findings.

The process of interpreting is to turn mathematical content into real life situation, evaluate whether results are reasonable in the context and reflect them. This mathematical process category involves both interpreting and evaluating processes.

Interpreting process might include interpreting mathematical results in the real-world context, judging and explaining the reasonability of the result, understanding the effects of real world on mathematical model, analyzing the limitations of the mathematical model.

As mentioned before, Figure 2.1 illustrates that modeling cycle constitutes the central form of representation of students as active problem solvers in PISA. Most of the cases in real life and some processes of this cycle were covered by others and individuals need to complete the rest of the cycle. For instance, some charts demonstrate final results of the process and it only needs to be interpreted by individuals.

Moreover, PISA 2012 establishes that the mathematical capabilities underlie these processes and mathematical literacy. These capabilities stem from Niss and his colleagues' KOM project (Stacey & Turner, 2015). According to Niss (2003), mathematical competency is an ability to understand, judge and apply mathematics in a certain extent of intra- and extra- world contexts. Based on this project, Niss (2003, pp. 7–8) categorizes eight competencies which are i) thinking mathematically, (ii) posing and solving mathematical problems, (iii) modelling mathematically, (iv) reasoning mathematically, (v) representing mathematical entities, (vi) handling mathematical symbols and formalism, (vii) communicating in, with, and about mathematics, (viii) making use of aids and tools. PISA 2012 Assessment and Analytical framework for mathematics gives a specific attention to these competences (referring as capabilities) that underlie a solid base for mathematical processes stated above and mathematical literacy. These fundamental capabilities used in PISA 2012 mathematics framework are (i) communication capability which includes understanding the challenges by reading, decoding and analyzing the statements, tasks or a question in the formulation process, and summarizing, presenting the results and giving explanations and justifications for the solution in the solution and interpretation process, (ii) mathematising capability which comprises of converting a real-life problem to mathematical world by organizing, conceptualizing, and/or formulating a mathematical model, or interpretation or evaluation of this model in relation to real world situation, (iii) representation capability which involve selecting, using, interpreting, converting among a range of representations (tables, graphs, pictures, diagrams, equations …) to save the situations and relate them with the problem, (iv) reasoning and argument capability which consist of selecting and linking the elements of the problems to make deductions from them, to check the assumptions, or give a rationale to solutions of problems, (v) devising strategies for solving problems capability which involve a set of control process for selecting, implementing a strategy or a plan to use mathematics while solving problems occurring in real-life context, (vi) using symbolic, formal and technical language and operations capability which includes understanding, employing, interpreting the use of mathematical symbolic expressions in mathematical structures and, understanding and employing formal structures based on definitions or rules, (vii) using mathematical tools capability which consists of information required to be able to use mathematical tools such as physical tools, computer based tools, calculators and, knowing limitations of these tools (OECD, 2013). Moreover, PISA 2012 framework extends explanation of these capabilities by defining each of them under mathematical processes in Figure 2.2 below. This figure gives explanations about how to use each fundamental mathematical capability while formulating, employing and interpreting the problem. For example, while formulating situations to benefit from representation capability, students need to generate a mathematical representation of real-world situation in the problem. Representation capability could be used in employing process by understanding, relating and using several representations while interacting with a problem. Lastly, the capability of interpreting representation helps students to compare or evaluate between two different representations or interpret mathematical solutions from several representations.

The mathematical processes, which are what people do to connect mathematics with the real-life contents, are explained above. In PISA framework, mathematical literacy definition is analyzed under three interrelated aspects, which are mathematical processes, mathematical content knowledge and context in which

	Formulating situations mathematically	*Employing* mathematical concepts, facts, procedures and reasoning	*Interpreting*, applying and evaluating mathematical outcomes
Communicating	Read, decode, and make sense of statements, questions, tasks, objects, images, or animations (in computer-based assessment) in order to form a mental model of the situation	Articulate a solution, show the work involved in reaching a solution and/or summarise and present intermediate mathematical results	Construct and communicate explanations and arguments in the context of the problem
Mathematising	Identify the underlying mathematical variables and structures in the real world problem, and make assumptions so that they can be used	Use an understanding of the context to guide or expedite the mathematical solving process, e.g. working to a context-appropriate level of accuracy	Understand the extent and limits of a mathematical solution that are a consequence of the mathematical model employed
Representation	Create a mathematical representation of real-world information	Make sense of, relate and use a variety of representations when interacting with a problem	Interpret mathematical outcomes in a variety of formats in relation to a situation or use; compare or evaluate two or more representations in relation to a situation
Reasoning and argument	Explain, defend or provide a justification for the identified or devised representation of a real-world situation	Explain, defend or provide a justification for the processes and procedures used to determine a mathematical result or solution Connect pieces of information to arrive at a mathematical solution, make generalisations or create a multi-step argument	Reflect on mathematical solutions and create explanations and arguments that support, refute or qualify a mathematical solution to a contextualised problem
Devising strategies for solving problems	Select or devise a plan or strategy to mathematically reframe contextualised problems	Activate effective and sustained control mechanisms across a multi-step procedure leading to a mathematical solution, conclusion, or generalisation	Devise and implement a strategy in order to interpret, evaluate and validate a mathematical solution to a contextualised problem
Using symbolic, formal and technical language and operations	Use appropriate variables, symbols, diagrams and standard models in order to represent a real-world problem using symbolic/formal language	Understand and utilise formal constructs based on definitions, rules and formal systems as well as employing algorithms	Understand the relationship between the context of the problem and representation of the mathematical solution. Use this understanding to help interpret the solution in context and gauge the feasibility and possible limitations of the solution
Using mathematical tools	Use mathematical tools in order to recognise mathematical structures or to portray mathematical relationships	Know about and be able to make appropriate use of various tools that may assist in implementing processes and procedures for determining mathematical solutions	Use mathematical tools to ascertain the reasonableness of a mathematical solution and any limits and constraints on that solution, given the context of the problem

Figure 2.2 The Relationship Between Mathematical Processes and Fundamental Mathematical Capabilities (OECD, 2013, p.32)

mathematical literacy problems are placed. In the following section, the second aspect, mathematical content knowledge is going to be examined.

2.2.2.2 Mathematical Content Knowledge

In the previous section, mathematical processes, which PISA 2012 Mathematics framework uses to analyze mathematical literacy, are described. PISA 2012 Mathematics framework also adopts mathematical content for analyzing mathematical literacy. The frameworks' definitions and division of contents depend on the curriculums applied in schools. The reason for this difference is that contents in school curriculums are usually organized around content strands like algebra, numbers, geometry and historically well-known branches; however, in real life, we cannot solve problems or challenges by using only one set of rules. In other words, problems or challenges in real life can be solved by creative combination of different mathematical concept, rules or tools (OECD, 2013). For this reason, PISA 2012 Assessment and Analytical framework for mathematics define mathematical contents more broadly than school curriculums. They are (i) change and relationships, (ii) space and shape, (iii) quantity, (iv) uncertainty and data. Change and relationship content area encompass creating symbolic and graphical representations of mathematical relationships, interpreting them and transforming them into different forms, as well as modeling change and relationships with appropriate functions. The growth of organisms, the cycle of the seasons, changes in unemployment rates and weather changes can be given as examples of the situations where change and relationships are seen in everyday life. Space and shape content area is a phenomenon encountered in most areas in our world (patterns, properties of objects, positions, orientations and representations of objects etc.). It includes activities such as understanding perspective, reading and drawing maps, transforming shapes by using technology. The subject of quantity includes understanding relative sizes, recognizing numerical patterns, and displaying quantities and measurement-related properties of objects in the real-world using numbers. The most important components of quantity are to perceive numbers, show numbers in different ways, understand the meaning of operations, have an idea about the magnitude of numbers, and make mathematically perfect mental calculations and predictions. Finally, the uncertainty and data content include identifying the role of variation in processes, understanding the quantification of that variation, recognizing uncertainty and error in measurement, and being aware of chance. It also consists of shaping, interpreting, and evaluating assumptions obtained in conditions where uncertainty is essential. Thanks to this categorization, mathematical items in PISA has a considerable variability across the mathematical domain. (OECD, 2013).

Until now, mathematical process and mathematical content category of PISA mathematical literacy definition has been covered. In the following section, the third and last part of PISA mathematical literacy definition, contexts are going to be examined.

2.2.2.3 Contexts

One of the important parts of mathematical literacy definition of PISA is the context. Context is an important aspect of the mathematical literacy. It is defined as “the aspect of an individual’s world in which the problems are placed” (OECD, 2013, p. 37). The choice of mathematical strategies, procedures or tools often depends on the context that problem comes up. The contexts that are used by PISA attempt to be relevant with 15-year-old students’ interests, and the demands of society that they are going to be placed as reflective, active and engaged citizens since the aim of the assessment is to evaluate the ability to use mathematical process and content knowledge in real life. For this reason, the context used in PISA has a high variety of range. Four context categories for mathematical literacy are divided by PISA survey, they are (i) personal, (ii) occupational, (iii) societal and (iv) scientific context categories. Problems labelled in personal context category are related to individual’s himself, his family or his interests. Shopping, personal scheduling, games, sports could be given as an example of these context categories. Problems in occupational context category are related to work life like productivity, scheduling, design and profit. Problems categorized in societal context category concern with social issues, community and citizenship e.g. public transport, taxes. Problems in scientific context category are associated with natural world issues, mathematical applications, science and technology. Climate change, medicine, world of mathematics could be the examples of this context category (OECD, 2013).

Considering all interrelated aspects of mathematical literacy, problems that aim to assess mathematical literacy can be stated as a special type of mathematical modeling problems since the definition involves mathematical modeling and real-world contexts. Although PISA 2012 Assessment and Analytical Framework covers modeling cycle in the part of mathematical processes in detail, the framework covers limited information about real-life contexts and how to set a mathematical problem which contains modeling cycle in real-life context. To obtain this information, we need a further investigation about what the definition of real-life context for PISA survey is and which methodologies or frameworks they use while setting problems including modeling cycle in real life. For this reason, following sections are going to interrogate what the meaning of context for PISA framework is and how PISA establishes authentic mathematics problems in real-world context.

2.2.3 Setting PISA Items in Real World Contexts

In order to understand how to set context of mathematical literacy problems, we need to know the meaning of context according to PISA mathematical literacy framework. The word of "context" usually refers to situations in which students engage while making operations although it has various meanings in different educational assessments (Stacey & Turner, 2015). However, PISA survey uses the word of "context" in the meaning of real-world used in problems. While setting mathematics problems for mathematical literacy in real-world context, PISA dwells on two important issues which are relevance and the role of context in solution process and achieving authenticity of context. In the following sections, detailed information given related to these terms and their frameworks are going to be expressed.

2.2.3.1 Relevance and Role of Context

Knowledge of context could be used in problem solving process in many ways. About the related issue, PISA follows de Lange's approach (1987). According to de Lange (1987), context can appear in problems to present them as real-life problems or can be used while solving the problem or judging the answer. Thus, the degree of importance of context in PISA items varies. At the lowest level, context is not used for the solving process of the problem. It is called as fake context, camouflage context or "zero-order" context. A real-life problem is categorized under "zero-order" use of context when real-world elements are not required to understand or solve the problem. However, this does not mean that context must be completely irrelevant to the question for the question be considered as zero-order. At the middle level, which is called as "first-order" use of context, specific features of context are required in the process of understanding, solving, interpreting or evaluating the problem (OECD, 2010). At the highest level, "one really needs to move backwards and forwards between the mathematical problem and its context in order to solve the problem or to reflect on the answer within the context to judge the correctness of the answer (OECD, 2010, p. 31). As stated differently, students need to deduce new information from existing knowledge of the context to solve the problem. According to PISA second-level, problems include higher level of mathematical literacy (OECD, 2010) since purposeful interpretation of contexts are essential to real-life while dealing with problems or challenges. All information required to solve problems are not usually given in real-life. Furthermore, PISA tries to avoid from using "zero-order" use of context.

As mentioned before, the role of context is one of the essential parts of the PISA mathematical literacy context. In the next section, another important component of

PISA mathematical literacy while setting real-life mathematics problems is authenticity.

2.2.3.2 Authenticity of Context

Authenticity is defined as “the concordance between mathematical school tasks and situations in the real world beyond the mathematics classroom” (Palm, 2008, p. 43). It is mostly conceived as the level of how realistic a problem situation is (Bonotto, 2005). According to Stacey and Turner (2015), PISA should include authentic items as a result of the definition of mathematical literacy. This is one of the reasons why PISA mathematics item developers pay attention to achieve authenticity of each item. The other reason is that asking authentic mathematical problems increases the chance of getting realistic answers from students (Palm, 2008; Stacey & Turner, 2015).

To achieve the authenticity for mathematical problems is not a simple task. Palm (2006) proposes a framework, which is also followed by PISA, to make sure that stimulations of real world are faithful to reality. In this framework, he defines eight aspects to make school task authentic, which are (i) event, (ii) question, (iii) information/data, (iv) presentation, (v) solution strategies, (vi) solution requirements, (vii) circumstances, (viii) purpose. Table 2.1 includes definitions of aspects of the framework.

The framework includes different aspects of real-world context to be considered when stimulating real world. Palm (2006) claims that this framework is necessary to develop real world mathematics tasks. This framework reflects the aspect of problems which are important for supporting mathematics use of students (Wernet, 2017). In order to explain the aspects more clearly, a problem shown in Palm’s (2008) study will be given as an example and aspects will be explained on the problem.

Table 2.1 Description of Palm's (2006) aspect of authentic mathematics tasks

Task Aspects	**Description**
Event	This aspect is the event happening in the school task. It is preferred that the event has actually happened or can happen in real-life.
Question	This aspect reflects that questions posed in the task should be suitable for being asked in real-life situation.
Information/data	This aspect is the information and data supplied in the task. The information and data in the task must match with the information and data in real life. Moreover, the numerical values should be equal or very close to values in real-life.
Presentation	The language structure (difficulty in items, word count, sentence structure) should match with real-life situation and it should not prevent students from understanding the problem.
Solution Strategies	All solution strategies in the corresponding real-life situation should be available for students while solving the school task.
Solution Requirements	The validity of solution methods and answers to a task should be assessed by considering corresponding real-word criteria.
Circumstances	The all circumstances (availability of tools, guidance, consultation) while solving the task should be similar with the real-world situation.
Purpose	The purpose of the solving school task is required to be clear for students. It should align with the purpose of solving real life situation so that students make appropriate assumption while solving.

> All students in the school will go on a school trip together on the 15th of May. You and the other organizing students have decided that everyone will go by bus, and that you will order the buses. You have seen in the student rosters that there are 360 students in the school. Your teacher said that you can order the buses from Swebus, and that each bus can hold 48 students(Palm, 2008, p. 43).

The event in the problem is planning a trip which can happen in real life. The question asked in the problem is how many busses I should order, which is suitable for real-life situation. Information/data given in the problem is that each bus have a place for 48 people which is reasonable for a real bus. Presentation of the problem is language use in the problem which is clear. Solution strategies is to divide number of people into bus capacity then to decide how many buses is needed which is an available and reasonable strategy for people in real-life. There are no requirements available for this problem. The problem will be authentic for circumstances aspect if students have accessed the tools, they could use in real life such as calculator. Lastly, purpose of the problem, which is ordering suitable number of buses, is reasonable for students.

Up to this point, we summarized the meaning of mathematical literacy and the conceptual framework of the study, which is PISA 2012 Assessment and Analytical framework. We examined three interrelated aspects (mathematical process, content, context) of mathematical literacy definition according to PISA framework and necessary components of context (the role of context, authenticity) aspect for establishing mathematical literacy problems.

In this study, our purpose is to contribute to the improvements of Turkish students' mathematical literacy levels through overcoming the deficiencies of prospective mathematics teachers about mathematical literacy. For this purpose, literature search is conducted about national and international studies related to mathematical literacy improvement in general. In this section, we are going to examine some of the related literature. Then, studies specifically related to prospective teachers for the development of mathematical literacy are going to be examined in a separate section.

2.3 Works Related to ML Improvement

When the result of PISA are published, the need for how to improve students' mathematical literacy become one of the most important issues in mathematics education research. Sfand (2013) focuses on the importance of the issue with these words: "The question of how to teach for mathematical literacy must be theoretically and empirically studied. When we consider the urgency of the issue, we should make sure that such research is given high priority." (p. 141).

Many studies investigate what factors could affect students' mathematical literacy. These researches enable to understand what the current situation in classrooms when considering mathematical literacy is and what can be done to improve them when it comes to how to teach mathematical literacy. Several studies on this issue indicate the valuable effects of teacher's style and preferences in the class (e.g. materials, representation, tasks) on students' mathematical literacy (Höfer & Beckmann, 2009; Kramarski & Mizrachi, 2006; Roth et al., 2015). For example, Höfer and Beckman (2009) examined the effects of linking science into mathematics lessons to develop their mathematical literacy and functional thinking. In order to conduct the study, two mathematical lessons enacted with science was carried out in a German high school with students with 14 to 17-year-old. The data was collected via observation and teaching techniques were analyzed through audio and video analysis. Based on the interpretation of the data analysis, Höfer and Beckman (2009) claim that teachers should prepare lessons which includes applied and conventional knowledge. In this study, students' understanding of subject matter is combined with mathematical activities through the science experiments. Likewise, Sawatzki and Sullivan (2018) investigated the effects of teachers' presentation of real-world financial context on students' activation of mathematical knowledge, reasoning and skills. The framework of the study was PISA mathematical literacy assessment framework of OECD. 10 to 12-year-old students from Australian primary school participated to the study. The findings of the study indicated that students seemed that they were engaged with the task because it involved authentic real-life elements which was supported with reasonable arguments and it demonstrated that mathematics could be useful in real life. Moreover, the study suggested teachers to generate lessons and support students to interpret the real-life situation in the context before they make a decision.

The studies mentioned above are mainly what teachers do or can do to enhance students' mathematical literacy. When such studies are examined, it can be seen that there are many works in the literature related to the specific idea that teachers' conducting real-life or PISA-like contextual problem solving activities in class could help to increase students' level in mathematics literacy; therefore, teachers should generate lessons to involve these kinds of problems (Dewantara et al., 2015; Hermawan et al., 2019; Meaney, 2007; Yılmazer & Masal, 2014). For instance, Dewantara, Zulkardi and Darmawijoyo (2015) explore the potential effects of PISA-like problems arising during the study on students' development of mathematical literacy in terms of activating fundamental mathematical capabilities. The data from 28 seventh grade students in Indonesia were gathered by students' answers to problems and interviews. The study revealed that 10 practicals, valid for PISA-like mathematics problems solved in classroom, had potential to increase students' achievement in mathematical literacy. In the same way, in their study on understanding the relationship between 7th grade academic performance of students and their level of mathematical literacy, Yılmazer and Masal (2014) suggest that in order to develop students' mathematical literacy, real-life, open ended problems involving interpretation should be provided for students. Moreover, a study was conducted by Hermawan and his colleagues (2019) to explore how students' reasoning and argumentation skills could be supported with mathematical literacy problems about function and relation topic offered by teachers. Junior high school mathematics lessons held in scientific approach in the function, relation topic and mathematical literacy problem activity sheets of students was collected as a data source. The results of the study were demonstrated since mathematical literacy problems helped students to understand context of the problem better and motivate them. Students' reasoning and argumentation skills could be supported with mathematical literacy problems in a lesson designed by scientific, discovery or problem-based approach. The studies mentioned above indicate that there are lots of works in the literature about positive effects of teachers' implementation of PISA like context-based problems as a classroom activity for the development of students' mathematical literacy.

The studies in this section investigate the factors affecting students' mathematical literacy improvement. As mentioned, most of the studies demonstrate that teachers' knowledge, teaching style and preferences play an active role on improving students' mathematical literacy. Thus, teachers need to improve their knowledge and skills about mathematical literacy and generate lessons where students actively participate in real-life situations and interpret the conditions. In recent years, several studies have been conducted to investigate teachers' knowledge and skills about mathematical literacy, their teaching style and preferences. In the following sections, international and national studies about the related topic are going to be examined separately.

2.4 ML Studies related to Teachers in International Context

Some studies in the international context examine teachers' capability of mathematical literacy while solving PISA-like problems (Ahmad et al., 2018; Suharta & Suarjana, 2018; Vilakazi, 2010). There are also studies that aim at teachers' capability or practices that support students' mathematical literacy (Pillay & Bansilal, 2019; Siswono et al., 2018; Wijaya et al., 2015).

To illustrate, a case study was carried by Suharta and Suarjana (2018) with 12 elementary prospective teachers from Indonesia to investigate prospective teachers' mathematical literacy. Moreover, they aimed to investigate whether there is any significant difference in prospective teachers' mathematical level based on gender and mathematical skills which they were assigned during basic mathematics course. The data was collected from a mathematical literacy test which was prepared by referring 2006, 2009, 2012 PISA tests and interviews, and was analyzed through using benchmarks norms from Education Handbook of Ganesha University. According to the results of this study, Suharta and Suarjana (2018) stated that mathematical literacy scores categorized as low based on gender and mathematical skill differences. In addition to that, in terms of content, prospective teachers had the highest average score in Probability and Statistics, and the lowest average score in Algebra and Function. In terms of competence, the highest average score was in knowledge process and the lowest average score was in reasoning competence. This means that although prospective teachers have basic mathematics content knowledge, they have problems with reasoning, problem solving and reflection skills.

As parallel to Suharta and Suarjana's study (2018), Ahmad and his collegues (2018) conducted a case study to investigate mathematical literacy level of junior high school teachers while solving PISA-like problems (mathematical literacy base problems). The data was collected from 24 junior high school teachers in a local area in Indonesia through test and semi-structured interviews. Ahmad and his colleagues stated that teachers had difficulty in solving the mathematical literacy problems with level 4 or above. Similarly to the study of Suharta and Suarhana (2018), findings of Ahmad and his colleagues' research indicated that teachers failed to solve high level problems for which they need to develop mathematical models, identify constraints in the context, select appropriate problem-solving strategies, solve the problems by using reasoning skills and proper mathematical representations and interpret the mathematical solution in the context. This result from the information and courses given during higher education did not help to improve mathematical literacy abilities of teachers (Ahmad et al., 2018; Suharta & Suarjana, 2018).

There are also international studies focusing on teachers' capability or practices that support students' mathematical literacy. To illustrate, Pillay and Bansilal (2019) investigated the approaches used by mathematics teachers who are specialized in the area of mathematical literacy to support participation of students in contextual settings. For this purpose, they carried out a case study with a mathematics teacher who was teaching mathematics literacy for two years. The data was obtained by lesson observations and interviews. Pilay and Bansilal's study (2019) demonstrated that she mostly gave incorrect or irrelevant explanations for contextual terms. Moreover, the teacher's discussion did not help students to focus on contextual terms that give necessary information about the situation, mathematical topics and procedures that will be used. According to researchers, this result indicates that many teachers have difficulty in managing contextual elements in the problems or activity sheets. This means that the approaches used by teachers to support students' mathematical literacy are misleading and ineffective.

In addition to that, Wijaya and his colleagues (2015) restricted the topic that they were interested in and they aimed to explore teachers' practices and beliefs about context-based tasks and their explanation of students' difficulties while solving these tasks. The questionnaire was collected to obtain reactions of teachers about context-based tasks from 27 teachers and 4 teachers' lessons was observed and recorded during the study. The results of questionnaire demonstrated that even though teachers believed that context-based tasks were helpful for students' learning mathematical literacy, they considered context-based tasks as plain word problems, which gives the necessary information to solve the problems. Moreover, teachers stated that they barely used context-based tasks including redundant information. The data from observations revealed that teachers did not support students while solving these tasks and they mostly used teacher-centered approach.

Parallel to the results of previous studies, Siswono and his colleagues (2018) conducted a qualitative study to show secondary mathematics teachers' concept of mathematical literacy while constructing PISA-like authentic mathematical problems. Researchers gave an instruction during the study which includes the nature and structure of PISA and problem posing techniques. Teachers asked to generate two groups and two individual assignments for generating PISA-like problems. An interview was conducted to understand participant's concept of mathematical literacy and their experiences during the instructions. An adopted framework from PISA mathematics framework was used to analyze the problems. Analysis of data indicated that teachers had difficulty while choosing context for problems and analyzing them by using PISA framework which includes content, context and processes. In addition to that, researchers claimed that teachers' capability of generating PISA-like problems increased gradually. This was observed from decreasing usage of camouflage contexts throughout the lessons. However, teachers still had problems with constructing problems which is appropriate for PISA framework regarding authenticity, language structure. When the findings of the studies mentioned above were investigated, it was observed that teachers were lack of ability to solve higher order mathematical literacy problems (Ahmad et al., 2018; Suharta & Suarjana, 2018; Vilakazi, 2010) and support students' mathematical literacy (Pillay & Bansilal, 2019; Siswono et al., 2018; Wijaya et al., 2015). Therefore, they need to improve their mathematical literacy and to develop useful strategies and methodologies to support students' mathematical literacy.

2.5 ML Studies related to Teachers in National Context

Besides researches conducted in international context, some studies were conducted in order to explore teachers' knowledge and abilities related to mathematical literacy in the national context. Similar to the international studies, national studies in the context can be divided into studies aimed to investigate teachers' ability of mathematical literacy while solving PISA-like problems (Aydın & Özgeldi, 2016; Kabael & Barak, 2016) and studies aimed to investigate teachers' capability or practices that support students' mathematical literacy (Demir & Altun, 2018; Ozgen, 2019; Şahin & Başgül, 2018). To illustrate, Kabael and Barak (2016) focused on changes on pre-service teachers' mathematical literacy achievement scores throughout teacher education program. The data was collected from 22 pre-service middle school teachers who were studying their 4th or 6th semester in a public university in Turkey. For this research, some selected mathematical problems from PISA 2003 and PISA 2012 were used as a research instrument. At the end of the data analysis, Kabael and Barak (2016) concluded that there was not a significant difference between the mathematical literacy scores of pre-service teachers throughout the program and both scores were lower than what was expected. In addition to that, many pre-service teachers had difficulty in formulating and mathematizing the problems because they had difficulty in comprehending and implementing the real-life components of the problems while solving problems (Kabael & Barak, 2016).

In the same way, Aydın and Özgeldi (2016) constructed a study to investigate elementary middle school teachers' achievement and difficulties in PISA 2012 released items. 56 prospective teachers asked to solve 26 item-written test. The items of the test were categorized according to the type of mathematics knowledge (contextual, conceptual or procedural knowledge) required. The analysis of the test showed that pre-service teachers mostly had difficulty with the problems requiring combinations of all types of mathematical knowledge. Moreover, though teachers could successfully solve the items requiring only procedural knowledge, they correctly challenged to solve and give explanations for their solution while they were solving conceptual knowledge items. In addition to that, they had difficulty in solving problems that involves non-routine contextual information. Similar to the study of Kabael and Barak (2016), Aydın and Özgeldi (2016) concluded that teachers had problems requiring to comprehend and implement the context, interpret and evaluate their solution in the context.

There are also some studies in Turkey to investigate teachers' capability or practices that support students' mathematical literacy (Demir & Altun, 2018; Ozgen, 2019; Şahin & Başgül, 2018). For example, Şahin and Başgül (2018) conducted a case study with 55 mathematics teacher candidates to explore the appropriateness of mathematical problems constructed by the nature of PISA. Participants took an instruction related to PISA while they were asked to prepare 3 PISA problems. Their problems were analyzed under PISA mathematics organization of domain categories and question types. Analysis of data indicated that most of the problems posed by pre-service teachers were suitable for the nature of the PISA.

Likewise, teachers and pre-service teachers' mathematical literacy problem posing skills were investigated by Özgen (2019). The data was obtained from 5 elementary mathematics teachers and 13 pre-service mathematics teachers who participated in an instruction about mathematical literacy. Three problems from each pre-service teacher were analyzed by using PISA mathematical framework. However, Özgen investigated the mathematical literacy levels of problems unlike Şahin and Başgül's study (2018). Findings of the study showed that problems posed by participants were usually at level 4 and mathematics teachers were more successful than pre-service teachers while levels of problems were considered. In addition to that, Özgen (2019) claimed that prospective teachers and teachers rarely posed problems at the highest level. When the findings of the studies mentioned above investigated, it was observed that teachers were lack of ability to solve higher order mathematical literacy problems (Aydın & Özgeldi, 2016; Kabael & Barak, 2016) and support students' mathematical literacy(Ozgen, 2019; Şahin & Başgül, 2018). Therefore, they need to improve their mathematical literacy and develop useful strategies and methodologies to support students' mathematical literacy.

2.6 Summary of the Literature

According to recent arguments in mathematics education, mathematical literacy is an important element in an individual's life as it is necessary for processing, communicating and interpreting mathematical information in a variety of contexts (OECD, 2013; Stacey & Turner, 2015). Although there are different opinions about the meaning and definition of mathematical literacy, many researchers have agreed on that it is "an ability to deal with real-life problems and challenges with the help of the mathematics" (Hope, 2007; Jablonka, 2003; Kaiser & Willander, 2005; Ojose, 2011). Moreover, the most comprehensive and international definition is given by OECD in the PISA 2012 Assessment and Analytical Framework.

International exams that assess students' mathematical literacy level like PISA illustrate that students' level of mathematical literacy is low in almost all nations.

Therefore, improving students’ mathematical literacy has become one of the most important issues in mathematical education research (Sfard, 2013). According to literature review, there are several studies illustrating that teachers’ knowledge, preferences and teaching styles have an important effect on improving students’ mathematical literacy (Höfer & Beckmann, 2009; Kramarski & Mizrachi, 2006; Roth et al., 2015). Consequently, there are some studies intending to investigate teachers’ knowledge about mathematical literacy and their capability to support students’ mathematical literacy. We separately investigate these studies in national and international manners. Our literature review indicate that teachers and prospective teachers have difficulties in solving mathematical literacy problems (Ahmad et al., 2018; Suharta & Suarjana, 2018; Vilakazi, 2010) and they lack the capability to support students’ mathematical literacy, especially in posing PISA-like context-based problems (Ozgen, 2019; Siswono et al., 2018; Wijaya et al., 2015).

The studies mentioned above demonstrate the importance of teachers' knowledge and teaching style about mathematical literacy in increasing students' mathematical literacy level (Höfer & Beckmann, 2009; Kramarski & Mizrachi, 2006; Roth et al., 2015). There are also research studies that consider what teachers could do to enhance students’ mathematical literacy. These studies mainly emphasize on the effects of solving realistic mathematical literacy problems in class (Dewantara et al., 2015; Yılmazer & Masal, 2014). According to the findings of these studies, solving realistic mathematical literacy problems in class helps students to face with real-life elements within school environment and enables teachers to improve students’ mathematical literacy level. Therefore, teachers need to select mathematical literacy problems from textbooks or generate new mathematical literacy problems. However, there are only a few problems which support mathematical literacy in mathematics textbooks (Gatabi et al., 2012). Thus, it is essential for teachers to generate mathematical literacy problems. When teachers’ incompetence in writing context-based mathematical literacy problems is considered, it is clear that teachers and prospective teachers require an education supporting their competence to generate mathematical literacy problems. Nevertheless, there are only few studies which aim to give education to prospective teachers to support their mathematical literacy problem posing skills. Moreover, when these studies are investigated, it is observed that teachers still have problems with constructing mathematical literacy problems after they are educated in both national and international studies. All these studies follow case study methodology which helps them to extensively understand prospective teachers' level of constructing mathematical literacy problems. However, we demand to use teacher experiment methodology in this study since the teacher experiment methodology focuses on not only understanding the prospective teachers' level of constructing mathematical literacy problems but also observing how they develop their knowledge. Therefore, to examine prospective teachers' capability to pose mathematical literacy problems in a teaching experiment is believed to theoretically contribute to the literature. The current study is also assumed to practically contribute to the literature in terms of the given instruction about how to generate effective mathematical literacy problems. As a consequence, the aim of this study is to investigate prospective teachers' capacity to generate mathematical literacy problems throughout the given course.

CHAPTER 3

METHOD

This study aims to investigate prospective middle school mathematics teachers' capability to generate mathematical literacy problems throughout a classroom teaching experiment. This chapter is devoted to the description of the research questions, the research design, sampling and the participants of the study, the context of the study, data collection instruments and procedures, trustworthiness, the researcher role and bias, limitations and ethical considerations of the study.

3.1 Research Design

The aim of the study is to investigate to what extent prospective teachers' capability to generate mathematical literacy problems change or develop throughout the given instruction. To achieve this goal, qualitative research is carried out since its purpose is to grasp what meanings individuals form, how they think and experience in a special setting (Bogdan & Biklen, 2007; Merriam, 2009).

Teaching experiment is employed to investigate research problems of this study. Teaching experiment method evolves from Piaget's clinical interviews which aim to examine students' understanding. It gives researchers an opportunity to observe and experience students' mathematical learning at firsthand (Steffe & Thompson, 2000). While teaching experiment aims to understand how students' existing mathematical learning is influenced by a constructive teaching, clinical interviews only aims to understand students' current understanding and knowledge. To put it in another way, teaching experiment is not only interested in students' mathematical conceptions at the beginning and the end of the teaching, it is also interested in how learners change, restructure and organize their current mathematical schemes in the process (Steffe & Thompson, 2000). Therefore, researcher observes how students behave when they engage in mathematical activity and tries to construct a model for the modifications that they make in their current understanding. To make this observation, researcher should take an active part in teaching experiment. In other words, researcher of the study must be the teacher of the teaching sessions (Steffe & Thompson, 2000).

This study is a teaching experiment since it aims to investigate how PSTs change or develop their competency of generating mathematical literacy problems throughout an instruction as well as their competencies at the beginning and at the end of the instruction. In the study, researcher take active part in the course as a teacher. She makes observations and takes notes about how PSTs engages to the activities during instructions and she makes interviews with PSTs throughout the course to investigate their understanding about mathematical literacy problems and their construction of mathematical literacy problems. Moreover, she investigates how they change throughout the instruction. In this way, we consider teaching experiment methodology as the best methodology to follow.

There are several elements of teaching experiment methodology. According to Steffe and Thomson (2000), teaching experiment consists of a group of teaching episodes lined up sequentially. Every one of them includes a teaching agent (teacher-researcher), a student or students, a witness of teaching episode, and a way of recording to save what happened in these episodes. In addition to that, the researcher needs to set the research hypotheses to guide the selection of participants and determine the general objective of researcher (Steffe & Thompson, 2000).

In this teaching experiment, there are five teaching episodes which are constructed sequentially. Each teaching episodes or units involves a teacher-researcher (myself), a witness (a teacher educator studying in the field of mathematics education) to help teacher-researcher to understand and interpret PSTs' actions and mathematical conceptions and a way of recording (extensive observation notes) to save what happened in teaching units.

Another important point in the teaching experiments research hypotheses are generally formulated between teaching episodes. After reviewing the teaching episodes, researcher could generate a new hypothesis to be tested in the next episode. Therefore, teaching episodes help to organize subsequent teaching episodes and to conduct a retrospective teaching analysis (Steffe & Thompson, 2000).

In this teaching experiment, in order to generate an environment where prospective teachers can actively learn, useful knowledge was gathered from related literature. First, contexts of the teaching units are prepared by teacher-researcher based on related literature and major research hypothesis (an instruction including discussions related to mathematical literacy and components of mathematical literacy problems and several tasks to select and pose effective mathematical literacy problems help PSTs to enrich their opinions about mathematical literacy problems and help them to generate various types of authentic mathematical literacy problems) are formulated to test and to guide overall intentions. Then, research hypothesis of each lesson had some revisions and refinements during the teaching period based on students' misconceptions and unexpected reactions in order for the teaching to become more efficient for PSTs. The preparation and implementation of the teaching experiment based on these theoretical principles are going to be examined in detail in the following parts.

Athough teaching experiment method is conducted to carry out the study, the current study presents the findings of the teaching experiment. In other words, this study is one phase of the teaching experiment which presents how participants of the study gainedfrom the teaching experiment. However, how participants of the study developed througout the teaching sessions will not be presented in this study.

3.2 Participants of the Study

Participants of this study is selected through purposeful sampling strategy since the study aims to selecting individuals who provide a rich data and are more helpful to explore the issue in-depth rather than generalizing the results of the study (Creswell, 2012; Patton, 2002).

The most frequently used nonprobability sampling strategy among all is purposive sampling (Merriam, 2009). It can be defined as intentionally selecting individuals who provide the most information about a phenomenon when a researcher wants to discover and gain insight about it. As stated above, the aim of this study is to investigate prospective mathematics teachers' capability to generate mathematical literacy problems throughout classroom teaching experiment. This aim and some other reasons to be explained requires that only a specific group of prospective teachers are chosen as sample of this study, which means that purposeful sampling strategy is the sampling method of this study.

In order to understand the selection of participants of this study, a brief explanation was given about the context of the study. Detailed information about the context of the study is going to be explained in the following chapters. This study was carried out within the context of an elective undergraduate course with the name of "Projects in elementary science and mathematics education". Therefore, prospective teachers who selected the elective course were also participants of the study.

The sample selection of the study was carried out in two processes. Setting the selection criteria for the elective course was the first process of sample selection. We preferred to admit prospective teachers who are 4^{th} grade students and who completed teaching method course and most of the pedagogical courses. This serves the aim of the research for several reasons. First, they have completed most of the courses in the teacher education program so that they are qualified in mathematics education courses and pedagogical courses. Therefore, they are familiar with pedagogical and developmental needs of middle school students and they have knowledge of developmental limitations of middle school students. This enables them to write effective mathematics problems. Moreover, they have revised mathematics curriculum at least once so it can be assumed that they have knowledge about curriculum needs like mathematics literacy and students' preliminary knowledge. Because of these reasons, 10 senior prospective mathematics teachers were enrolled to the course.

In order to investigate prospective teachers' capability to generate mathematical literacy problems in detail, three prospective teachers were selected from the study group as a second process of the sample selection. The second sample selection was done in order to investigate effects of different capabilities to generate mathematical literacy problems. Therefore, three participants were selected based on their initial understanding and knowledge about mathematical literacy and their initial capability to generate mathematical literacy problems. In order to decide prospective teachers' initial capability to generate mathematical literacy problems, prospective teachers' first draft of mathematical literacy problems and an open-ended questionnaire which was collected at the beginning of the teaching period were used. 10 prospective teachers' capability to generate mathematical literacy problems were categorized as under low, middle and high level according to this data. For this classification, PISA mathematics framework and design issues concerning the use of real-world contexts in PISA problems were used. Moreover, analysis of the questionnaire revealed their awareness about mathematical literacy. At the end of this analysis, one prospective teacher from high (Melike), middle (Tuğçe) and low (Ayşe) categories was chosen purposefully. All names of prospective teachers are changed with pseudonyms. All participants are regular students who completed all mathematics courses and all must mathematics education courses in the department, which include Methods of Teaching Mathematics I-II courses. Moreover, they have completed School Experience course in which they need to observe a classroom environment for a semester. In addition to this, in the same semester as this study, they enrolled Teaching Practice course. This course requires preperation of activities and lesson plans which is highly related to the aims of the current study. They are academically successful students who completed most mathematics courses and mathematics education methodology courses with higher grades. The cumulative GPA's of Tuğçe, Melike and Ayşe are 3.29, 3.12 and 3.12, respectively. Therefore, they have all the necessary preliminary work to pose an efficient mathematical problem.

After providing information related to prospective teachers' academic success, an attention was given to their actions and their interests. Melike is 24 years and a quiet person, but she is willing to express her opinions during classroom discussion. She wants to improve herself in teaching mathematics, and for this reason, she considers continuing her education after her graduation. She has completed Mathematical Modelling for Teachers course which could help her constructing mathematical processes of mathematical literacy problems. She is also interested in technology in mathematics education. Her cumulative GPA was 3,12 so she was also an honor student.

Tuğçe is 22 years old and very expressive student. She is willing to participate in discussions during lesson. She is passionate to become a successful mathematics teacher. Moreover, she has completed some elective courses which could help her to benefit from different contexts while posing a word problem. They are Child and Media, Climate Change Education for Sustainability. Her cumulative GPA was 3,29 so she was an honor student.

Ayşe is 22 years old and very expressive student. She is very passionate about mathematics. She has completed almost all elective courses given by mathematics education department including problem solving. She has interest in algebra, especially pattern generalization topic. She took a variety of elective courses which could help her using different contexts e.g. Child and Media, Food Myths and Facts. Her cumulative GPA was 3,12 so she was also an honor student.

3.3 Context of the Study

In this section, context of the study will be examined under two subsections. Firstly, context of the study will be examined from a broader perspective. This study is related to prospective middle school mathematics teachers who enroll in teacher education program. Therefore, detailed information about the program will be given in order to describe general context of the study. Then, content of the course in which the teaching sessions were carried out and the content of the teaching episodes will be examined.

3.3.1 Broad Context of the Study

The context of the study is middle school mathematics teacher education program in a public university in Ankara. It is a four-year undergraduate degree program, training prospective teachers to become mathematics teachers for middle school. The middle schools in Turkey covers the grades from 5 to 8. The program consists of 43 courses (37 must, 6 elective) (See Appendix A). Must courses includes history, English language, Turkish language, physics, pure mathematics, elementary mathematics education and educational sciences courses.

Prospective middle school teachers have opportunity to deepen their mathematical knowledge with the help of pure mathematics courses and learn basic principles and methodologies about science of education with the help of educational science courses. Moreover, they have opportunity to learn how middle school students learn mathematics effectively and how they can design the learning and teaching process, with the help of mathematics teaching methods courses and teaching practice courses.

In the past, there was no course offered in the middle school mathematics teaching program where prospective teachers learn theoretical and practical necessities of mathematics classrooms such as mathematical literacy. This absence in the curriculum made it necessary to have a course where prospective teachers learn theoretical and practical issues in mathematics education. Therefore, "Projects in elementary science and mathematics education" course was redesigned to fill the gap in the program and mathematical literacy was chosen as one of the theoretical issues to be considered in the course in 2018-2019 spring semester.

In the next chapter, detailed information about the course will be given where teaching sessions were carried out and the characteristics of these teaching sessions.

3.3.2 Content and General Characteristics of Teaching Episodes

The general purpose of the teaching episodes is to ensure that the prospective teachers have a mathematical background and knowledge in mathematical literacy, PISA mathematics framework, and properties of a real-life problem for constructing and revising mathematical literacy problems. The second purpose is to develop prospective teachers' capability to generate mathematical literacy problems.

The course is designed with the aim of helping prospective teachers to study practical and theoretical necessities of elementary mathematics and giving them an opportunity to practice these necessities in a classroom environment. At the end of the course, students are expected to develop an individual project related to topics of the course. While constructing learning objectives and teaching units of the study, the description of the course was put into consideration.

Considering the description of the course, we prepared a course where prospective teachers learn mathematical literacy and the main principles to prepare mathematical literacy problems in a constructive environment. In this process, we asked them to generate and revise their own mathematical literacy problems as an individual project. To achieve these purposes, we prepared 5 weeks of instruction. Each teaching unit lasted for 3 hours. To construct instructions and to set learning objectives, we benefited from published articles and theses related to mathematical literacy and context-based mathematical tasks, PISA framework, PISA technical reports, other PISA related sources published by OECD, books related to the assessment of mathematical literacy, and feedbacks of an experienced researcher. Learning objectives were constructed based on the principles and frameworks which are used in the construction of the PISA mathematics items.

The topics which are intended to be covered each week are given in Table 3.1 Detailed information about the context of teaching units will be given in following sections.

Table 3.1 Content of Teaching Episodes and Their Brief Descriptions

Week	Topic	Brief Description
1	Introduction to ML and PISA	Mathematical literacy definition of PISA and components of ML problems
2	Mathematical Relevance	Level of context use
3	Authenticity	The role of authenticity in mathematical literacy
4	Mathematical Processes	Details of mathematical processes and their underlying mathematical capabilities
5	Peer Review	Whole class discussions related to quality of mathematical literacy problems generated by PSTs

The language of the course was English. Each teaching sessions covered three parts. They began with the brief explanation of the context of the week. In the second part, prospective teachers were given instructional tasks which they work on the problems on their own. In this part, they had chance to construct their own understanding by exploring the principles of mathematical literacy problem while analyzing the mathematical literacy problems. In the third part, prospective teachers asked to discuss and exchange their ideas with their peers. Teacher-researcher benefitted from PISA mathematics framework to guide the discussion and supported prospective teachers to gain necessary information and conceptual understanding while constructing their own problem. Moreover, she had chance to observe and take notes about discussions to revise the further instructions and analyze the data. Prospective teachers were used to involved in discussions since most of the education courses in which they have participated includes discussion as educational method. Therefore, they were open to share their ideas and discuss them in a respectful manner.

The teaching units were placed in a conference room in the Faculty of Education building. Desks and chairs were arranged in a half circle to encourage all participants to actively participate in discussions. There was a computer and a projector in the room which was used to reflect teaching materials and activity sheets. In addition to that, one replica of each activity sheets that examined in the classroom were distributed to each participant so that they could work individually while analyzing the problems and take necessary notes on the papers.

In this study, researcher benefitted from aim of the study and the general research hypothesis to set each week's learning objectives. Then she revised research hypothesis of each week based on gap and misconceptions about the previous weeks. Therefore, learning objectives of the study were constructed by researcher between teaching episodes as suggested.

The general characteristics of the teaching sessions were mentioned in this section to define the learning environment more clearly. Implementation procedures of teaching sessions is going to be mentioned detailly in next chapter.

3.4 Implementation Procedures of Teaching Experiment

3.4.1 Week 1

The aim of the first teaching session was to collect participants' opinions related to mathematical literacy. This would help us to observe how their opinions will change throughout the process, enable us help them to construct and improve their definitions, and provide us the opportunity to indicate components of mathematical literacy problems (mathematical processes, content and context) which will be the basis for their problems afterward. For this purpose, an open-ended questionnaire which asks PSTs' opinions and their awareness about mathematical literacy and PISA was implemented at the beginning of teaching sessions. Then, the meaning and different types of literacy, and the PSTs' answers to questionnaire that reflect their opinions about mathematical literacy were discussed. This discussion and the questionnaires showed us that most of prospective teachers had lacked knowledge about the meaning of the mathematical literacy notion, PISA and PISA assessment items. This was an expected situation because prospective teachers had not taken any courses related to mathematical literacy. Moreover, the result of a previous study of the teacher-researcher with different prospective teachers at the same faculty had also showed that prospective teachers had lacked knowledge about PISA and mathematical literacy (Baran & Işıksal Bostan, 2018). Therefore, the first teaching episode continued with informing prospective teachers about the meaning and definition of mathematical literacy, historical development of the notion, assessment of mathematical literacy via PISA and Turkish students' success level in PISA, as it planned. To help prospective teachers to construct their own definition of mathematical literacy, different definitions from the literature were demonstrated and the similarities and differences among the definitions were discussed. Finally, the assessment of mathematical literacy, PISA, international exam aiming to assess mathematical literacy, Turkish students' failure in the PISA and its reasons, the components of the problems and the PISA's mathematical literacy assessment framework were introduced. At the end of the first teaching session, prospective teachers were asked to generate a mathematical literacy problem considering the components of the PISA problems (process, context, content), which would be discussed in a more detailed way in the following teaching sessions.

The learning objectives of the first week shown in Table 3.2 was constructed based on literature review since there was no data available about prospective teachers' understanding or conceptions about mathematical literacy. They are related to context of first lesson.

3.4.2 Week 2

Expanding the prospective teachers' knowledge and capability to pose effective mathematical literacy problems and capturing prospective teachers' understanding of mathematical literacy problems and their current capability to generate mathematical literacy problems are main aims of the study. For this purpose, in the second and third weeks, the focus was on the topic of how to generate context component of the mathematical literacy problems efficiently. The reason for this was related to the teacher-researcher's initial analysis of the initial mathematical literacy problems posed by prospective teachers. This analysis showed that prospective teachers had problems with reflecting the real-life context authentically and their context was mostly fake (zero level) contexts defined in de Lange's study (1987) that students do not use their components while solving the problems.

The main topic of interest in the second week was the role of context in writing mathematical literacy problems. The second week started with a brief discussion about last week's topic to refresh the PSTs' information. Then, another discussion about the importance of real-life context in mathematical problems and using variety of real-life contexts while teaching started. The learning environment was generally constructed on the basis of de Lange's framework about mathematics relevance of contexts (De Lange, 1987) and PISA definition of real-life context. Detailed information about frameworks was given in Chapter 2. On the other hand, learning objectives related to frameworks were determined as in the Table 3.2.

To achieve learning objectives, instructional task 1 was designed. It is shown in Appendix B1. Task 1 consists of two parts. The first part of the instructional task consists of three mathematical literacy problems with zero-order context use, first-order context use, and second-order context use, respectively. This part was aimed to make participants explore that the level of context use is different in the three problems covered in the task. To achieve this purpose, prospective teachers were asked to discuss the differences among these three problems considering their contexts briefly. The first part of the instructional task and the discussion about the differences were aimed to realize learning objective 2.1 shown in Table 3.2. The second part of the task consists of PISA mathematical items which include different contexts and require a different level of context usage to be solved. For each problem, prospective teachers were asked to find the components of the problem according to the PISA framework (process, content and context), and the level of context use in the problem according to de Lange's framework (1987). They worked individually while solving the questions. Then, teacher-researcher facilitated discussions, asked prospective teachers to share their ideas and solutions to their friends and defend their solutions. The teacher-researcher guided the discussions by posing necessary questions to prospective teachers in order for them to explore the level of context use in the problems. After all participants had agreed on the right answer, the discussions ended. If they had not agreed on the right answer, the teacher-researcher would have given the answer and asked what the reason for this answer could have been. This part of the instructional task helped us to observe learning objectives 2.1 and 2.2 shown in Table 3.2. At the end of the lesson, prospective teachers were asked to revise their mathematical literacy problems to increase the level of context use of the problem, by using the knowledge they had gained through the teaching session. This assignment was aimed to realize learning objective 2.3.

3.4.3 Week 3

As it was mentioned before, prospective teachers have problems with reflecting the real-life context authentically. Therefore, the main topic of this week was to achieve authenticity. For this purpose, the importance of achieving authenticity and what makes a mathematical literacy problem authentic were discussed. In order to emphasize the importance of the authenticity, prospective teachers were informed about a study conducted by a of Verschaffel and his colleagues (1994). In this study, students were asked to solve a mathematics problem where they need to use the context to find a solution, and only 20% of students who participated in the study gave realistic answers. The problem was: "Steve has bought 4 planks of 2.5 m each. How many planks of 1 m can he get out of these planks" (Verschaffel et al., 1994, p. 276).

The prospective teachers were also told that a study constructed to examine the reasons of the failure had showed that solving unrealistic problems in schools is very long-standing feature of mathematics classrooms; therefore, students do not consider the context of the problem while solving the problem (Verschaffel, 2009). Then, prospective teachers were asked to express their opinions about what makes a mathematical literacy problem authentic for them. After this, Palm's authenticity framework, which includes six criteria for authentic mathematics task, was examined. Detailed information about Palm's framework (2006) was given in Chapter 2. In order to foster the understanding, teacher-researcher prepared a summary table involving of six criteria of Palm's authenticity framework in Turkish.

The learning objectives of this week are constructed based on PISA mathematical literacy framework and Palm's authenticity framework which is illustrated in Table 2.1.

To achieve each learning objective shown in the Table 3.2 under third week, instructional task 2 were utilized in a teaching session. Instructional task 2, consists of two parts. The first part of the task was similar to the second part of the first task as it includes different PISA mathematics problems. Prospective teachers were asked to determine whether the problem was authentic or not by analyzing the characteristics defined by Palm's authenticity framework, as well as finding the components of the problems according to PISA framework (process, content and context), and the level of context use in the problem according to de Lange's framework (1967). This part was designed specially achieve learning objectives 1.2, 2.2 and 3.2 which are shown in Table 3.2. Learning objective 1.2 and 2.2 was the previous week's objectives. However, it is obtained that PSTs had difficulty in categorizing ML problems into their components and differentiating the problems according to whether the context is needed to solve the problem or not via lecture notes. Therefore, these objectives are generated to test in this episode.

The second part of the task 2 consists of some word problems which were taken from the literature (Palm, 2008). Prospective teachers were asked to find which characteristics of authentic tasks are included or not included in the questions and what the better versions of these problems according to authenticity could be. Prospective teachers were expected to work on the task individually and then share and discuss their ideas with the class. This part of the activity was aimed to achieve learning objectives 3.1 and 3.2, which are shown in Table 3.2. At the end of the activity, participants were assigned to improve authenticity level of some problems by making necessary changes.

3.4.4 Week 4 and 5

At the beginning of the teaching session, a brief discussion about the previous sessions' topics (components of mathematical literacy problems defined by PISA (OECD, 2013) and de Lange's (1987) level of context use framework) was carried out. In the previous teaching session, context component of mathematical literacy problems had been examined deeply. The fourth teaching session was focused on process component of mathematical literacy problems. The general aim of the week was to expand prospective teachers' knowledge about mathematical processes (formulating, employing and interpreting) and their underlying fundamental mathematical capabilities defined by PISA (OECD, 2013, pp. 30–31) (communicating, mathematising, representation, reasoning and argument, devising strategies for solving the problem, using symbolic, formal and technical language and operations and using mathematical tools). When the first mathematical literacy problems constructed by PSTs at the beginning of teachers are investigated, it is observed that most of them does not involve the processes of formulating and interpreting. Therefore, in this session, firstly increasing PSTs knowledge about all processes and encourage them to pose problems which involves multiple processes is aimed. Moreover, increasing PSTs awareness of which fundamental capabilities are demanded from students and their effects on problem difficulty was another aim of this lesson.

Participants were informed about the mathematical processes of mathematical literacy problems in detail and the underlying mathematical capabilities were mentioned in this session. Detailed examples of mathematical literacy problems which consist of different mathematical processes were illustrated. In order to foster the understanding, teacher-researcher prepared a summary table consisting of information about mathematical processes and their underlying principles in Turkish. In order to inform prospective teachers about the frameworks to pose mathematical literacy problems, direct teaching methods were used. However, the remaining part of the session was designed for helping prospective teachers to construct their own understanding. Therefore, instructional task 3 and discussions were utilized for this aim. They also enabled us to realize learning objectives of fourth teaching session, which is given in Table 3.2.

Instructional task 3 consists of mathematical literacy problems taken from PISA. For each one, prospective teachers were asked to solve the problem and then analyze it according to the components of PISA mathematics framework, and to define which processes were required to use while solving the problem and how. In addition to that, they were asked to find the fundamental mathematical capabilities for solving the problem. After they work on each problem individually, they were asked to express their solutions in a discussion with whole class and defend their solutions while giving reasonable explanations. This task helps us to observe learning objectives 4.1, 4.2 ,4.3, 4.4 and 4.5 which shown in Table 3.2. When discussions were completed, teacher researcher gave the last assignment which is peer review. In this assignment, PSTs are asked to revise their problems based on the topics they learned throughout units and asked them to bring revised version to the class to present and get feedback from their classmates. This assignment would be used to achieve learning objectives 5.1, 5.2 and 5.3 which are shown in Table 3.2. A prospective teacher had been chosen and informed that she was going to present her problem in the 4th weeks' teaching session. This was done in order to test the process and discussion beforehand so that the data gathered from this process could be used to revise the process and make the discussion more effective in the final teaching session. The reason for selecting this particular prospective teacher is that she was willing to participate in the lessons, she was successful than most of her peers in general, and she was the teacher candidate who was closest to finalizing her problem.

After making sure that each prospective teacher understood the final assignment, chosen prospective teacher shared her mathematical literacy problem to whole class. She presented her problem and stated her opinions according to the criteria in the final task. Then, a discussion was set to discuss the problem according to the criteria.

Table 3.2 Lesson Objectives of Teaching Episodes

Week	*Topics of Episodes*	*Objectives*
1	Introduction to ML and PISA	1.1 Construct their own mathematical literacy definition. 1.2 Categorize the mathematical literacy problems into their components according to PISA Framework. 1.3 Develop mathematical problems which carry the components of mathematical literacy definitions.
2	Mathematical Relevance	2.1 Understand the importance of using a variety of contexts while teaching mathematics. 2.2 Differentiate the mathematical problems according to where context is needed to solve the problem or context is not needed to solve the problem 2.3 Generate mathematical literacy problems where context is relevant and required to solve the problem or interpret the results.
3	Authenticity	3.1 Determine whether the characteristics of the mathematics problem is realistic or not. 3.2 Explain the characteristics of authentic mathematics problem according to Palm' authenticity framework 3.3 Modify a mathematical problem to make it more authentic.
4	Mathematical Processes	4.1 Understand the importance of using a variety of processes while teaching mathematics. 4.2 Define each process in modeling cycle of mathematical literacy problems and their role in the modelling process. 4.3 Determine major cognitive demand to solve the problem to define mathematical process of the problem. 4.4 Define required mathematical capabilities to solve the mathematical literacy problem.
5	Peer Review	5.1 Categorize the mathematical literacy problems into their components according to PISA framework. 5.2 Revise level context of use of the problem so that students could use the context while solving the problem. 5.3 Modify the mathematical problem according to Palm's authenticity framework.

At the end of the discussion, prospective teachers reached a consensus on which criteria were satisfied and which is not and made suggestions about revising the problem according to the criteria. It was observed that prospective teachers had difficulty remembering the related frameworks about the criteria. It was decided to prepare a work sheet including the related frameworks and descriptions about them.

As it was mentioned above, in the fifth and last teaching session, each prospective teacher was asked to present her or his revised mathematical literacy problem to the whole class and explain how she or he met the criteria. Then a discussion was started to receive the opinions of other prospective teachers about each mathematical literacy problem and these discussions were followed by some suggestions about how the problem could satisfy the criteria. The work sheet given to the prospective teachers was observed to be so beneficial that they were able to take notes for themselves easily and they managed to follow the discussion.

This final task given was an opportunity for us to observe what the level of their mathematical literacy problem was after they had received all instructions.

3.5 Data Collection Procedure

Data was collected throughout the teaching sessions. In teaching experiments, qualitative data was preferred because a huge set of data comes from teaching episodes and clinical interviews (Cobb & Steffe, 2011). At the same time, all data was obtained qualitatively in this study by using multiple data sources. This gives an opportunity for researcher to generate an organization of data analysis and helps her to appropriately analyze the data (Fraenkel, Wallen, & Hyun, 2012). The multiple data sources were individual pre- and post- interviews, initial and revised mathematical literacy problems, artifacts collected during the lessons and field notes which were taken from teacher-researcher and observer-researcher. A chain of evidence was provided by data sources to understand the nature and development of middle school mathematics teachers' mathematical literacy capability to generate mathematical literacy problems. Detailed information about data collection tools and the data provided by each source were given in the following sections.

3.5.1 Initial and Revised Versions of Mathematical Literacy Problems

This study investigates prospective teachers' capability to generate mathematical literacy problems and to what extent it changes with teaching experiment. For this reason, mathematical literacy problems that are posed by prospective teachers through the teaching sessions are one of the most relevant and important data for this study. They are the major data source of the study. Participants' initial and revised versions of mathematical literacy problems were collected throughout the process.

Mathematical literacy problems posed by prospective teachers were collected for three times throughout the all process. This process was shown in Figure 3.1. In order to gather first version of the mathematical literacy problem, prospective teachers were assigned to generate a mathematical literacy problem after the first teaching unit which covers introduction to mathematical literacy, PISA and the structure of PISA problems. They were assigned at the end of the first teaching sessions since in order to compare the problems posed by prospective teachers through all instructions, the structure of the problems was desired to be similar.

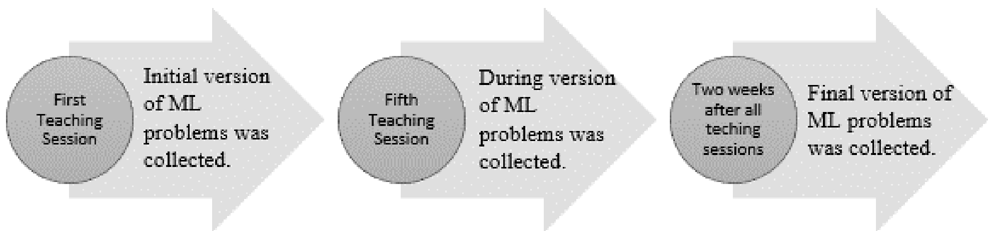


Figure 3.1 Process of Collecting PST's ML Problems

Secondly, the during version of mathematical literacy problems was collected at the fifth teaching sessions. Although prospective teachers were assigned to revise their problem with the knowledge they gained from teaching sessions, second version was collected in the fifth week. The reason for why we did not prefer to collect mathematical problems was that each week's topic was interrelated to each other. In other words, if prospective teachers changed one component of mathematical literacy, it could affect all components of the problem. Therefore, we wanted to investigate all changes and its effects holistically.

Lastly, prospective teachers were assigned to bring final version of mathematical literacy problems at the end of the teaching experiment.

3.5.2 Personal Interviews

Our purpose in teaching experiment is to construct a model of students' actions to engage in activities and looking behind the reasons why they do such an action (Glasersfeld, 1995). This model enables us to represent students' understanding of mathematics. According to Clement (2000), students' way of understanding and thinking about a situation or concept can be learned from researcher by conducting interviews with them. Therefore, interview is used to understand the students' actions to engage in activities and collect and analyze participants' reasoning in this study.

In this study interviews with PSTs is used to bring out each prospective teachers' reflections, attitudes and understanding about mathematical literacy problems and the reasoning laying behind the revisions they made in the mathematical literacy problems.

There were two interviews conducted during the study. First interview was conducted before the second teaching session after they posed the first draft of their mathematical literacy problem. The aim of the interview was to obtain their opinions, beliefs about mathematical literacy problems, and which criteria they consider while constructing their first draft. Interview protocol of the first interview is given in Appendix D1. Some of the example questions asked in this interview are “Can you explain the question you wrote?” and “What did you pay attention to while generating the problem?”.

The second interview were carried out after the teaching units where PST completed their final draft of mathematical literacy problem. This interview was constructed to investigate the contribution of teaching units to their skills of posing mathematical literacy problems and their considerations, ideas and beliefs while readjusting their mathematical literacy problems. The aim of it is to gather new data that corroborate the evidences about participants’ capability to pose mathematical literacy problems that we obtained from final draft of the mathematical literacy problems. These interviews have a semi-structured nature, it takes about an hour and it is more likely to pursue certain group of questions that the research seeks for. Interview protocol of the second interview is given in Appendix D2. Some of the example questions asked in this interview are “Could you compare your problems in terms of mathematical processes? What changes did you make? Why?” and “Could you compare your problems in terms of level of context use? What changes did you make? Why?”.

In an interview, as well as its nature, interview questions are important to follow research questions. For this purpose, an expert mathematics educator examined the interview protocol and she recommended to delete some questions that might not serve for its purpose and might influence the interviewee with the researcher’s perspective. Also, the interview with one of the prospective teachers was carried out as a pilot interview. Some probing questions were added or deleted according to the protocol. Lastly, in order to obtain relatively true account of prospective teachers, the interviews were constructed by researcher.

In the interview process, they were given to initial and final version of their mathematical literacy problem, and they were reminded to interrelated aspects of mathematical literacy (content, context and process) that were covered during the teaching units in order to have a closer look into the process of how they think and design while constructing and revising their problem. Each interview lasted within thirty minutes to an hour.

3.6 Data Analysis

In the current study, the data gathered from the semi-structured interviews, prospective teachers' initial and final mathematical literacy problems and observation notes were analyzed in order to clarify in-depth description of prospective middle school mathematics teachers' capability to generate mathematical literacy problems and how their capability changed through of the teaching sessions. As it was mentioned as sampling and participants of the study part, we will focus on only three participants for data analysis whose names are Tuğçe, Melike and Ayşe.

In order to examine data, I initially arranged the data chronologically which is illustrated in Figure 3.2. Then, data related to the initial and final interviews of Tuğçe, Melike and Ayşe were transcribed, and transcribed form of interviews, initial, during and final versions of mathematical literacy problems and observation notes were examined.

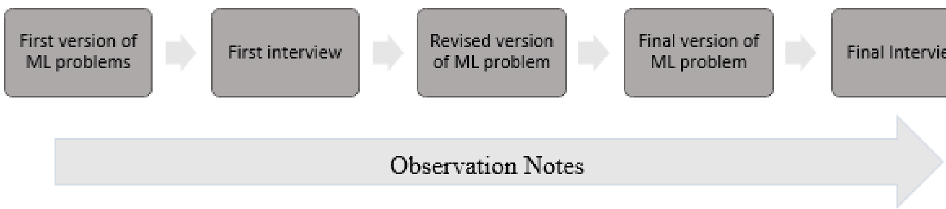


Figure 3.2 Sequence of Data Collection Procedures

Main goal of the study is to investigate the prospective teachers' capability to pose mathematical literacy problems. For this aim, how to design a mathematical problem where students' use their mathematical literacy capability was investigated by teacher-researcher. According to mathematical literacy definition generated by PISA 2012 (OECD, 2013), mathematical literacy consists of three interrelated aspects (mathematical content, context and mathematical processes). This means that a mathematical literacy problem should include a real-life problem and it should be solved with the help of a mathematical content after completing some mathematical processes. Therefore, research questions of the research are formed under three dimensions (components). Data are groups into three parts to answer the research problems.

In order to answer first research question, "How does mathematical processes and the underlying mathematical capabilities in the mathematical literacy problems generated by PST change/develop during the teaching experiment?" changes in PSTs' mathematical literacy problems regarding to mathematical processes was noted. Since the first dimension which is mathematical processes involves two categories which are mathematical process categories and fundamental mathematical capabilities underlying mathematical processes, the changes were grouped into these two categories.

The first category of mathematical processes is mathematical process categories. PISA Mathematics Framework (2013) was used to determine the mathematical process categories of mathematical literacy problems generated by PSTs. Table 3.3 presents the coding of mathematical process categories. Similar to PISA, if a problem involves more than one process, its process category is decided based on which mathematical process is most demanding to solve the problem.

The second category of mathematical processes is fundamental mathematical capabilities. This classification help us to define which mathematical capabilities is required to solve the problem in which degrees. PISA Mathematics Framework (2013) is used to data analysis. There are seven fundamental capabilities are defined in the framework; communication, devising strategies, mathematising, representation, using symbols, operations and formal language and reasoning and argument and mathematical tools. Using mathematical tool capability is omitted in this study since it depends on the practice of the problems in a classroom setting but the problems constructed in the study was not taken place in an actual classroom. Thus, it is not suitable for coding research data. Therefore, six fundamental capabilities defined in PISA Mathematics Framework (2013) is used to code the data. Each fundamental capability is classified into four level from level 0 to level 3 according to PISA framework. Level 0 was given to lowest level of demand for activating any competency and level 3 is given to highest level.

Table 3.3 The Coding of Mathematical Processes (OECD, 2013)

Mathematical Process Categories	Coding
Formulating	The cognitive challenge lies in recognizing the mathematical structures in the context and translating it to mathematical structure.
Employing	The cognitive challenge lies in applying mathematical facts, procedures, and reasoning to solve pre-mathematized problems.
Interpreting	The cognitive challenge lies in translating mathematical results to back into the context and determining whether the solution make sense.

Second domain of mathematical literacy problems is mathematical content. In order to answer second research question, how does mathematical contents in the mathematical literacy problems generated by PST change/develop during the teaching experiment? The changes in PSTs' mathematical literacy problems regarding to mathematical contents was noted. Table 3.4 presents the coding of mathematical content categories. Similar to PISA, if a problem involves more than one content, its content category is decided based on which mathematical content is most demanding to solve the problem.

Third and last domain of mathematical literacy problems is mathematical content. In order to answer third research question, how does real-life context in the mathematical literacy problems generated by PST change/develop during the teaching experiment? The changes in PSTs' mathematical literacy problems regarding to context was noted. Since the third domain which is real-life context involves three categories which are context categories, level of context uses and authenticity, the changes were grouped into these three categories.

Table 3.4 The Coding of Mathematical Contents (OECD, 2013)

Mathematical Content Categories	Coding
Change and Relationship	The primary demand of problems comes from forming and operating with algebraic equations and the problem mainly concerns changes.
Space and Shape	The primary demand of problems comes from geometric operations.
Quantity	The primary demand of problems comes from understanding measurements, units, counts, magnitudes, indicators, relative size, and numerical trends and patterns.
Uncertanity and Data	The primary demand of problems comes from dealing with the data, finding trends or calculating probability.

The first category of context is context categories. PISA Mathematics Framework (2013) was used to determine context categories of mathematical literacy problems generated by PSTs. Table 3.5 presents the coding of context categories.

The second category of context of ML problems is called as level of context use. This category is used to determine in what degrees students engage with context of the problem while solving it. The dimension classified into three levels which is also taken from de Lange's study. They are zero level of context use, first level of context use and second level of context use. Table 3.6 represents the coding categories of level of context use of ML problems.

Table 3.5 The Coding of Real-Life Context Categories

Context	Coding
Personal	Context of the problems are related to individual's himself, his family or his interests.
Social	Context of the problems are related to social issues, community and citizenship.
Occupational	Context of the problems are related to work life.
Scientific	Context of the problems are related to natural world issues, mathematical applications, science and technology.

While solving the lowest level, which is called as zero level of context use, students do not require to consider the aspects of the problem. Context can be found as consisting only mathematical terms or just to camouflage the mathematical terms. Increased level of context uses which is called as first level context use appears when context is "needed for solving the problem and judging the answer" (OECD, 2010, p. 31). Lastly, highest level context use appears when "students need to move backwards and forwards between the mathematical problem and its context in order to solve the problem or to reflect on the answer within the context to judge the correctness of the answer" (OECD, 2010, p. 31).

Table 3.6 The Coding of Level of Context Use (De Lange, 1987)

Level of Context Use	Coding
Zero Order of Context Use	Context is not needed for solving the problem and judging the answer.
First Order of Context Use	Context needed for solving the problem and judging the answer
Second Order of Context Use	Students need to move backwards and forwards between the mathematical problem and its context in order to solve the problem or to reflect on the answer within the context to judge the correctness of the answer.

The third category of context of ML problems is authenticity. Authenticity establish the alignment between mathematical literacy problems and corresponding real-world situation. According to Palm (2006, pp. 44–46) authenticity of mathematical tasks or problems could be divided into eight categories. They are (i) event, (ii) question, (iii) information/data, (iv) presentation, (v) solution strategies, (vi) solution requirements, (vii) circumstances, (viii) purpose of the context. In this study, seven categories of authenticity were investigated since these mathematical literacy problems designed in the study did not applied in a classroom environment, so we omitted the environment category. Each seven aspects used to analyze mathematical literacy problems are described in Table 2.1 In this study, we will also use the same categorization. Each category is divided into three levels; full alignment, partial alignment and no alignment. The coding scheme was constructed by researcher based on data analysis given in the

Table 3.7.

3.7 Trustworthiness of Study

In order to judge and validate the quality, adequacy, or goodness of any quantitative research, validity and reliability are important areas. While "validity is defined as the appropriateness, meaningfulness, and usefulness of the inferences researchers make based specifically on the data they collect", reliability can be defined as "the consistency of these inferences over time, location, and circumstances" (Fraenkel et al., 2012, p. 458). Although, reliability and validity terms are used only for quantitative researches, different terms are used in qualitative researches to judge validity and reliability issues. According to Lincoln and Guba (1985), credibility, transferability and dependability, confirmability terms are used in qualitative research instead of internal validity, reliability, external validity, and objectivity terms. In the following sections, I will explain how I have benefitted from these terms to judge trustworthiness issues of this research.

Table 3.7 The Coding of Authenticity (Palm, 2006)

Task Characteristic	**Full alignment (2)**	**Partial alignment (1)**	**No alignment (0)**
Event	The event described in problem has a fair chance of taking place	The event is likely to be occur in real life without some elements	The event has no change to take place in real life.
Question	All questions of task might be posed in a real-life situation.	Some questions in the task might not be asked in real life or very unlikely to be asked.	None of the questions of the task could be asked in real-life situation.
Information /Data	The mathematical information given in the task is realistic, accessible and it is specific enough to solve the problem.	Some information given in the task is not realistic or not accessible or not specific enough to solve the problem.	Solution strategy could be used while solving the problem is not plausible in real-life.
Solution Strategies	Mathematical representations used in the problem is clear and, reading level and amount of text is suitable for target population.	Some of mathematical representations used in the problem is not clear but the reading level and amount of text is suitable for target population.	The problem is not clear enough to solve it.
Presentation	All types of assumptions allowed or followed in the task is consistent with corresponding real-life situation.	Some assumptions allowed or followed in the task is not consistent with corresponding real-life situation.	All assumptions allowed or followed in the task is not consistent with corresponding real-life situation.
Solution Requirements	All types of assumptions allowed or followed in the task is consistent with corresponding real-life situation.	Some assumptions allowed or followed in the task is not consistent with corresponding real-life situation.	All assumptions allowed or followed in the task is not consistent with corresponding real-life situation.
Purpose	The purpose of the problem is clear for students.	Although purposes of some questions are clear to students some are not.	The purposes of all questions are not clear to students.

3.7.1 Credibility

According to Lincoln and Guba (1985), credibility, which is parallel to internal validity, is one of the important factors in building the trustworthiness of the study. Credibility tries to answer the questions "How congruent are the findings with reality? Do the findings capture what is really there? Are investigators observing or measuring what they think they are measuring?" (Merriam, 2009, p. 213). In order to ensure credibility, six strategies are suggested by Merriam (2009) which are triangulation, member checks (respondent validation), long-term observation, peer-debriefing or peer-examination, participatory or collaborative modes of research and researcher's role (reflexivity).

In this research, first of all, triangulation method is used to increase the credibility. Four modes of triangulation exist in the literature (Denzin, 2017; Patton, 2002) which are; data triangulation (the use of multiple and variety sources of data), method triangulation (the use of multiple and variety methods), investigators triangulation (the use of multiple researchers) and theory triangulation (the use of multiple theory or perspectives to clarify single set of data). Data triangulation is the most type of triangulation which is also used in this study. According to Lincoln and Guba, multiple source of data may entail "multiple copies of one type of source" or "different sources of the same information" (1985, p. 305). the data was collected with multiple sources such as prospective teachers' initial, during and final mathematical literacy problems, semi-structured interviews and observation notes. Therefore, data triangulation is used in in this study. Moreover, the investigators triangulation was used since all teaching sessions was observed by the professor in the field of mathematics education. She provided important ideas about teaching experiment, process and the way of prospective teachers' reasoning which she captured. Also, the data of study was coded by researcher-teacher and co-coder.

Member-checking is another technique that is used in the study to increase the credibility of the study. Member-check is one of most crucial techniques where data, implementations and conclusions are checked by the participants of the study from whom data were collected (Lincoln & Guba, 1985). To enable this technique, PSTs final version of ML problems and their evaluations on mathematical processes, contexts, contents defined by PISA framework and the level of context use and authenticity during the final interviews are discussed with them.

In this way, I identified whether I interpreted their level correctly or not. Furthermore, long term observation was last strategy that I used to ensure the credibility. I spent 6 weeks with the participants, so this enabled me to build thrust into my relationships with the participants.

3.7.2 Transferability

Transferability is the second criteria to build trustworthiness in qualitative studies which is parallel to external validity in quantitative research. External validity deals with to what extend research findings are generalizable to other situations (Merriam, 2009). However, qualitative studies, a single participant or small number participants are selected purposefully to understand their context in depth, results of the study probably do not fit for someone else (Merriam, 2009). Therefore, external validity of the study cannot be provided for a qualitative research. Instead, transferability is used in qualitative research by giving thick description about the implementation process. In this study, thick descriptions regarding the study were given by providing the context of the study, selection criteria for participants, the purpose and context of data instruments, the duration and length of teaching sessions and data collection sessions, the context of teaching sessions.

3.7.3 Dependability

Another criterion which establishes trustworthiness of the study is dependability, which refers to reliability in quantitative study. According to Merriam (Merriam, 1998), reliability is "… the extent to which research findings can be replicated" (p.220). However, replication of a qualitative study does not generate to same result, so researchers who conduct a qualitative study consider whether the results consistent and dependable (Merriam, 2009). This means that whether any researcher who wants to analyze the same data, the findings make sense to him/her (Merriam, 2009). The first strategy that is used to ensure dependability is triangulation. Triangulation is a strategy which is used to increase both validity and reliability (Lincoln & Guba, 1985). Data triangulation and investigator's triangulation was used in the current study as discussed above. A second strategy that is used to establish dependability is inquiry audit (audit trial) which is suggested by Lincoln and Guba (1985). According to strategy, researcher should describe how she collected the data, how she established the categories for data analysis and how she made decisions throughout the study in detail (Merriam, 2009). In the present study, all details of data collection, teaching sessions and the nature of data collection tools are explained. In addition to that, it is discussed how the data in methodology part is analyzed. Also, a second coder analyze the data by using the same categories. This is also expected to increase the dependability of the study.

3.7.4 Confirmability

The last criteria to ensure trustworthiness of the study is confirmability. Confirmability tries to answer, "How can one establish the degree to which findings of an inquiry are determined by subjects and the conditions of inquiry and not by biases, motivations, interests or perspectives of inquirer?" (Lincoln & Guba, 1985, p. 290). According to Shenton (2004) triangulation, description of methodology, ensures confirmability of the study and reduces the effects of investigator's bias. These strategies were used the present study as it was mentioned previous chapters.

3.8 The Role of Researcher

The teacher-researcher participated in the study as a participant-observer. This enabled me to control the flow of the teaching units, organize them and observe participants as an instructor as well as a researcher. This also brings a challenge to this study which makes me qualified for understanding the program as a research participant and describing the program as observer (Patton, 2002).

I was a graduate student of their university's department. Some of the prospective teachers met me before the study. Moreover, me and participants of the study share the same university culture. This helped to foster build confidence to share their ideas more freely during teaching experiment and interviews. This also decreased the time that participants spent to get used to the teaching experiment. In order to that, I spent extra time with them individually to increase communication among us. By this way, I learned more about participants' characteristics. I used this information while leading discussions and while selecting who would present her/his problem first. For example, one participant was very outgoing, and another participant was very silent, I tried to give similar time to both participants during discussions to express their solution or opinion.

As being researcher-teacher of the study, I implemented my tasks and my plans. I was studying more than two years on mathematical literacy, so I felt very knowledgeable on the topic. This helped me to become comfortable during teaching experiments and about answering the students' answers on the topic.

One of the disadvantages of being teacher-researcher was that I had limited experience in teaching, so I sometimes had difficulty while managing the classroom environment. At this point, observer-teacher, who is an experienced teacher and researcher, helped me. She also knew the participant from other courses. Thus, she gave very helpful tips and suggestions to manage the classroom environment.

3.9 The Role of Observer Researcher

An experienced mathematics education researcher also participated in the teaching units. She was familiar with the participants since she was an instructor of the several courses in which they attended. It is observed that this enabled to generate a warm classroom environment where prospective teachers express their opinions freely while discussing the topics and problems. Moreover, she controlled the plans of the teaching units before they were implemented, observed the students and teacher-researcher throughout the units and gave feedbacks to teacher-researcher about the plans of the units and instruction.

3.10 Ethical Considerations

In order to conduct the research, necessary permissions were taken from the Ethics Committee at METU (see Appendix D) to apply interviews and questionnaire. They approved that this study does not cause harm to prospective middle school teachers who participate in the study. As stated before, this was elective course, so all participants selected the course voluntarily. In addition to that, the participants are informed about the coverage of teaching sessions (context of the course, discussions) and data collection process (observations, interviews and assignments). As a result, 10 prospective teachers decided to select the course.

Another ethical consideration that is taken into consideration was to ensure confidentiality of research data. For this, I ensured that no one except for me and observer my advisor the data collected during study and I informed the participants about their responses and personal information would be remain confidentially. In order to ensure confidentiality, participants' names are changed with pseudonyms names such as Ayşe, Melike and Tuğçe.

3.11 Limitations of the Study

There are some limitations of the study in terms of: (i) data recording, and (ii) researchers' role. All limitations and how I try to handle them and eliminate their effects will be explained in the following.

The first limitation of the study was data recording. According to Steffe and Thomson (2000), a way of recording is an important element of the teaching experiment methodology. Also, video recording is common preferred method of study. However, in this study video-recording was not used as a recording tool. Instead, me and observer-teacher took notes related to improvements of prospective teachers and significant communication among them which could serve the aim of the study. The reason for this is that prospective teachers could act differently and avoid expressing their opinions while they were recorded. In this method, it is possible to lose data related to classroom discussion. However, in order to lower the change of data loss, we try to write thick descriptions related to important discussions where prospective teachers show their knowledge and understanding. Moreover, we conduct two interviews during teaching experiment where they showed their reasoning and understanding.

The second limitation of the study was my role as teacher-researcher. I was a master student who finished her undergraduate education three years ago. Therefore, I had lack of experience to conduct research studies and being a teacher. However, in order to eliminate this limitation, I participated each teaching sessions with a researcher who experiences teaching and researching for many years. Also, she knew the prospective teachers before starting to teach the experiment. I took suggestions before each teaching session related to how I should act in the classroom as teacher-researcher and I get feedbacks from her after each teaching session. Also, I read many books related to role of teacher-researcher in order to prepare myself.

CHAPTER 4

FINDINGS

This section provides the findings of prospective middle school teachers' capability to generate mathematical literacy problems. Findings which are gathered from the Melike, Tuğçe, and Ayşe' mathematical literacy problems and their interviews and open-ended are presented and discussed under this section. Moreover, findings collected from each prospective teacher are presented in three dimensions based on research questions. They are (i) mathematical processes and the underlying capabilities of ML problems generated by PSTs, (ii) mathematical content of ML problems generated by PSTs. and (iii) the context of ML problems generated by PSTs. The Figure 4.1 illustrates the correspondence between the dimensions and research questions.

4.1 Tuğçe's Capability to Generate ML Problems

Tuğçe's capability to generate mathematical literacy problem was classified as the middle level at the beginning of the teaching experiment. Therefore, she is one of the prospective teachers whose progress we want to analyze throughout the experience.

4.1.1 Mathematical Processes and the Underlying Capabilities of ML Problems

The first dimension of mathematical literacy problems is mathematical processes and the capabilities that underlie those processes refer to the processes in which a person engages into connecting problem context with mathematics and the required capabilities to do that. This section includes two parts. First part covers the mathematical processes of Tuğçe's initial and final mathematical literacy problems and how they have changed throughout the teaching experiment. The categorization of mathematical processes of ML problems generated by PSTs was given in the previous chapter in Table 3.3. The second part covers the mathematical capabilities of Tuğçe's initial and final mathematical literacy problems and how they changed throughout the teaching experiment. The categorization and level descriptions of fundamental mathematical capabilities that underlie mathematical processes were given in the previous chapter in Table 3.3.

The initial problem that was generated by Tuğçe is shown in Figure 4.2. The original (Turkish) version of this problem is given in the Appendix F1.

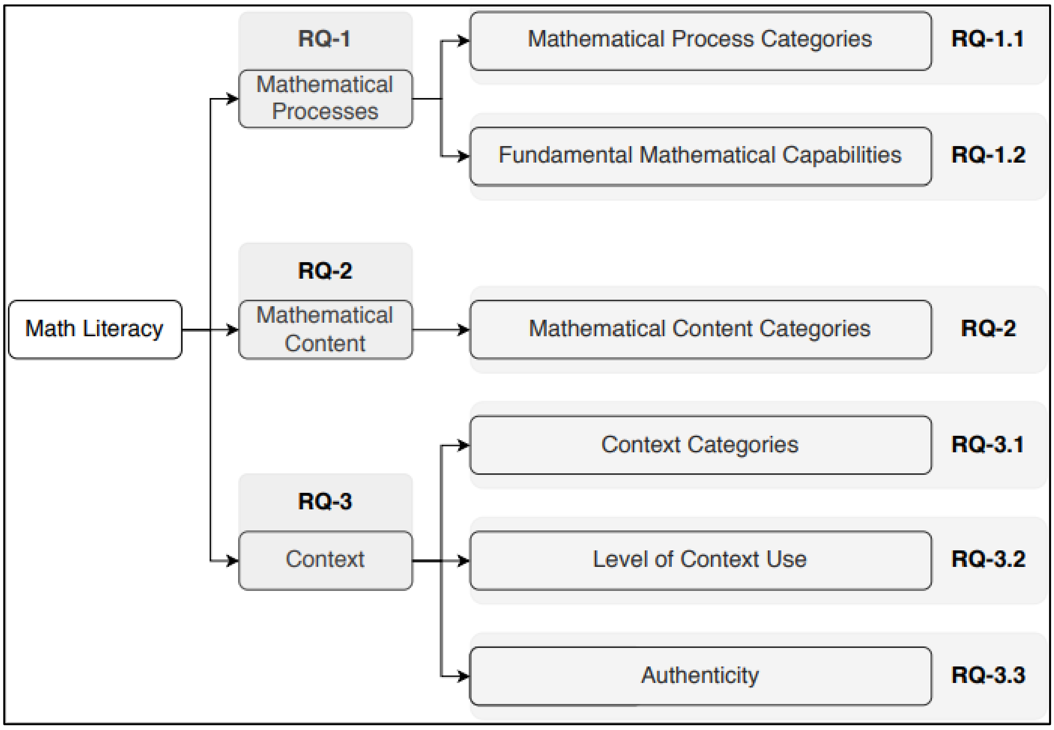
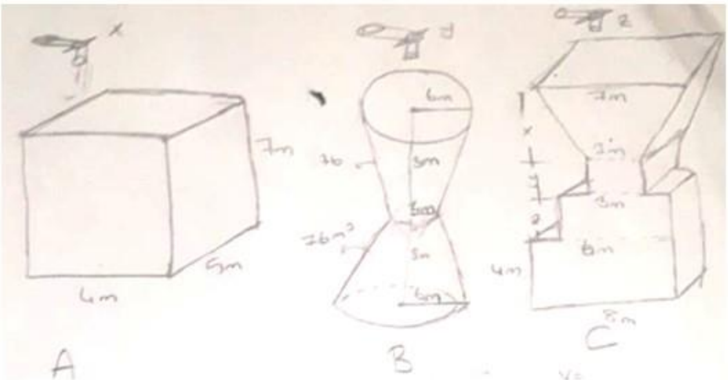


Figure 4.1 The correspondence between the research questions and mathematical literacy dimensions

In the initial question that Tuğçe wrote, Aunt Ayşe was looking for an answer to how to place her barrels under the taps in the most efficient way. To solve it, students should figure out that Aunt Ayşe needs to put the barrel with the highest volume under the fastest tap (the tap with highest flow rate) and the second largest barrel under the second-fastest tap and so on. The solution strategy for Tuğçe's initial problem is given in Figure 4.3.

Tuğçe constructed her final problem with the information she gained from the lectures and her classmates' feedback. The final problem is given in Figure 4.4. The original Turkish version of Tuğçe's final problem is also given in the Appendix F2.



Aunt Ayşe lives in a village in which some houses lack water installations and require the water be carried from the village fountain. Aunt Ayşe wants to fill her three different barrels, named A, B and C, whose dimensions are given in the figure above. The village fountain has three different taps and each tap has a different flow rate:

1st tap: 10 m3 per minute,

2nd tap: 20 m3 per minute,

3rd tap: 15 m3 per minute

There is a queue for water around the village fountain. Aunt Ayşe wants to fill her barrels as soon as possible so that she will not make the other people wait. In order for Aunt Ayşe to fill her barrels the most efficient way, which barrel must she put under which tap?

Figure 4.2 Tuğçe's initial ML problem

The time spent on filling the barrels is equal to the total volume of the barrels divided by the total flow rate of the taps. In order to minimize the time, one must maximize the total flow rate of the taps since the total volume of the barrels are constant. This goal is achieved by utilizing the most "powerful" tap as long as possible. On the other hand, switching the bottles in real life would cost some water and time. therefore, one must prefer the solution which requires the least amount of bottle movement. Therefore, the largest barrel must be put under the most "powerful" tap at first. Using the same logic, the second largest barrel must be put under the second "powerful" tap, and so on. When any of the barrels are filled with water, that barrel must be removed, and the above configuration must be satisfied with the remaining bottles.

$$V_A = 4m.5m.7m = 140\ m^3$$

$$V_B = \frac{\pi}{3}.6m.[(6m)^2 + (3m)^2 + 6m.3m] = 152\ m^3$$

$$V_C = 4m.8m.2m + 6m.2m.2m + 3m.2m.2m + (8m - 3m).3m.2m = 130m^3$$

$$So, V_B > V_A > V_C \ and\ t_2 > t_3 > t_1$$

$$Therefore, Barrel\ A\ should\ be\ placed\ under\ tap\ 3$$

$$Barrel\ B\ should\ be\ placed\ under\ tap\ 2$$

$$Barrel\ C\ should\ be\ placed\ under\ tap\ 1$$

Figure 4.3 Solution Strategy for Tuğçe's initial problem

In her final problem, Tuğçe asked again how Aunt Ayşe places her bottles under the taps in the most efficient way, but this time she asked students to calculate the exact time. In addition to that, in the final version of the problem, she added one more question. The additional one questioned the way and how the height of the water level vs volume of water would be for each bottle as it is filled. The solution strategy for Tuğçe’s final problem is given in Figure 4.5.

Tuğçe’s initial and final mathematical literacy problems, their solution strategies were shown here. In the following sections, mathematical processes and underlying capabilities of Tuğçe’s initial and final problems and how they have changed through teaching experiment will be examined.

Bucket **Pitcher** **Vessel**

According to problem, aunt Ayşe lives in a village in which some houses lack water installations and require the water be carried from the village fountain. Aunt Ayşe wants to fill her three different barrels, named bucket, pitcher and vessel, whose volumes are 28 m3, 15 m3 and 23 m3, respectively. The village fountain has four different taps and each tap has a different flow rate:

1st tap: 10 m3 per minute,

2nd tap: 20 m3 per minute,

3rd tap: 15 m3 per minute

4th tap: 12 m3 per minute

a. There is a queue for water around the village fountain. Aunt Ayşe wants to fill her barrels as soon as possible so that she will not make the other people wait. In order for Aunt Ayşe to fill her barrels the most efficient way, which barrel must she put under which tap? Approximately how many minutes it takes to fill all barrels?
b. Find out which of the graphics given below belongs to which barrel and explain briefly why.

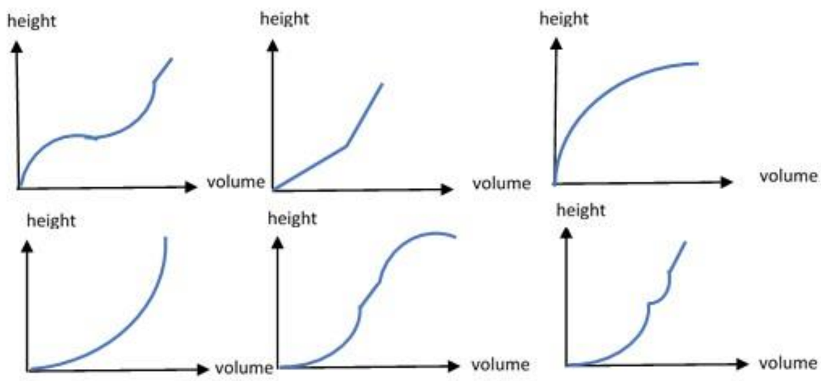


Figure 4.4 Tuğçe's final ML problem

a. The time spent on filling the bottles is equal to the total volume of the bottles divided by the total flow rate of the taps. In order to minimize the time, one must maximize the total flow rate of the taps since the total volume of the bottles are constant. This goal is achieved by utilizing the most "powerful" tap as long as possible. On the other hand, switching the bottles in real life would cost some water and time. therefore, one must prefer the solution which requires the least amount of bottle movement. Therefore, the largest bottle must be put under the most "powerful" tap at first. Using the same logic, the second largest bottle must be put under the second "powerful" tap, and so on. When any of the bottles are filled with water, that bottle must be removed, and the above configuration must be satisfied with the remaining bottles.

In order to find the least time for filling the bottles, we should find how long time required to fill each bottle. Then, the longest time required to fill bottle give us the least time to fill all of them.

$$t_A = \frac{V_A}{b_2} = \frac{28\ m^3}{20\ m^3/h} = 1{,}4\ min$$

$$t_B = \frac{V_B}{b_4} = \frac{15\ m^3}{12\ m^3/h} = 1{,}25\ min$$

$$t_C = \frac{V_C}{b_3} = \frac{23\ m^3}{15\ m^3/h} = 1{,}53\ min$$

After 1,4 min later we could put the bottle 3 under tap A to decrease the total time.

b. As the height of bottle, A increases, the water filling rate will gradually decrease as its diameter increases. Therefore, the third graph is most suitable for bottle A. As the height of bottle B increases, firstly its diameter expands, then it decreases by narrowing and lastly it remains constant. For this reason, at first the water filling rate will gradually decrease then it will gradually increase, and finally it will be constant.

Figure 4.5 The solution strategy for Tuğçe's final ML problem

4.1.1.1 Mathematical Processes of ML Problems

As shown in Figure 4.2 and Figure 4.4, while Tuğçe’s initial problem consists of one sub-question (item), Tuğçe’s final problem consists of two sub-questions (items). Each item is analyzed separately since each item requires different kinds of procedures to solve. The analysis of Tuğçe’s ML problems according to their mathematical process categories are given in Table 4.1. Tuğçe’s initial problem asks the most efficient way to fill the barrels from taps. The item relies most heavily on students’ abilities for employing mathematical concepts, facts, procedures, and reasoning. The primary cognitive challenge is to devise a strategy to match bottles in the most efficient way. In addition to that, students need to calculate the volumes of barrels the mathematical figures of which are given.

The first item of Tuğçe’s final problem similarly asks the most efficient way to fill the barrels from taps, but it also asks how many minutes are required to load all bottles. It also relies heavily on the employing process since devising a strategy to fill all bottles efficiently is a major cognitive challenge in the item. However, the second item of final problem calls most heavily on students’ abilities to formulate a situation mathematically. This item's real cognitive challenge is to translate real figures of bottles to a mathematical representation to draw height-volume graph of each bottle.

Table 4.1 Mathematical Process Categories of Tuğçe's ML Problems

Tuğçe’s Problems	Mathematical Processes
Initial Problem	Employing
Final Problem	a. Employing
	b. Formulating

4.1.1.2 Fundamental Mathematical Capabilities underlying Mathematical Processes

The second element of the first dimension is fundamental mathematical capabilities underlying mathematical processes. These mathematical capabilities are necessary to understand the world mathematically and solve mathematical problems in life. In this study, mathematical literacy problems' fundamental mathematical capabilities were classified under four levels depending on demands for activating each capability. Level 0 is given to the lowest level of demand for activating any competency, and level 3 is given to the highest level. This classification is taken from PISA 2012 Mathematics Framework (OECD, 2013). Table 3.3, given in Chapter 3, includes detailed descriptions of each capability and their coding. For Tuğçe's problem, levels for each competency is shown in Table 4.2. The codes are given by researcher.

Table 4.2 Fundamental Mathematical Capability Levels of Tuğçe's ML Problems

Mathematical Competencies	Tuğçe's Initial Problem	Tuğçe's Final Problem	
		a	b
Communication	1	1	2
Devising Strategies	2	2	2
Mathematising	2	1	2
Representation	1	0	2
Using Symbols and Operations	0	2	0
Reasoning and Argument	1	1	2

Tuğçe's initial problem shown in Figure 4.2, calls for calculating the volumes of barrels and ordering barrels and taps most efficiently. For the *communication competency*, the problem text's information must be read, and some are needed to be ignored (for example, the water line is crowded). The aim of the problem is expressed clearly and simply. The problem involves linking two separate elements in the question statement (volume of barrels and flow rate of taps). The constructive aspect includes writing a short sentence that explains which barrel should be placed under which tap. Therefore, level 1 seems appropriate for *communication competency*. Tuğçe's final problem, shown in Figure 4.4, involves two sub-questions, and each is analyzed differently for which fundamental mathematical competencies demand from students. Although the first sub-question is a revised version of the initial problem, the second sub-question is an additional question for which height-volume graphs of bottles is needed. The *communication* level of the first sub-question is proposed as level 1, which is the same as the initial problem because the question requires combining two separate elements (volume of bottles and flow rates of taps), extraneous information is given in the problem text (crowdedness of the line and extra tap), and constructive communication involves writing a short statement. However, for the second sub-question, the *communication* level of the problem is level 2. Similarly, the question statement is clear and aim directed in the second item. Different than the initial problem and the first item of the final problem, there are components that are needed to be associated with each other while making a solution. More specifically, the solution process, as the height of the bottles increases, and the information on how the dimensions of the cross-sectional areas change are investigated to solve the question. In addition to that, the expressive communication component of the second item requires an explanation.

For *devising a strategy* for the initial problem, the solution strategy involves two steps: finding the volume of barrels and ordering volumes and flow rates of barrels individually to determine the most efficient way to fill the taps. Level 2 description fits for devising a strategy competency since the strategy is straightforward, but it involves two steps. For *devising the strategy* of the first item of the final problem, as seen in Table 4.2, students need to devise a straightforward strategy which is similar to the initial problem about filling the bottles in the most efficient way. The strategy has multi-stages involving matching suitable bottles and taps, then calculating the required time for filling all bottles. Therefore, level 2 is suitable. *Devising strategy* competency of the second item of the final problem is also proposed level 2. The strategy to solve the question is to observe how the cross-sectional area of each bottle changes as its height increases. Although there is a straightforward strategy to solve the problem, it is a multi-stage strategy since it requires to use an identified strategy repeatedly to compare the changes in cross-sectional areas.

Two distinct modeling steps occur in the initial problem. The first is to formulate an equation for finding volumes of barrels, and the second is to design a model for matching the barrels and the taps to minimize the filling time. Having two modeling steps leads to level 2 for *mathematizing*. However, the *mathematizing capability* of the first sub-question is level 1 as there is only one model in the question which must be constructed for maximizing the flow rate for filling the bottles in the most effective way. Moreover, the variables and constraints are given in the problem. The *mathematizing* level of the second sub-question is level 2. The reason for this is that the students need to describe the relationship between height and volume of the bottles while defining the variables that affect this relationship and constraints to model the second sub-question. This leads to a level 2 definition for mathematizing.

The initial problem text includes some standard representations, which are the figures of barrels. In order to solve the problem, it is necessary to interpret the shapes to write the volume equation and read the dimensions on it. Therefore, the problem requires students to interpret some standard representations to solve the problem, which fits the level 1 definition of *representation*. However, in the final problem, Tuğçe changed the figures of barrels with the image of bottles, and she gave the volumes of these bottles directly instead of expecting students to calculate it. Thus, in order to solve the first sub-question, students do not have to interpret the shapes or graphs; they are only required to read numbers from the text. This corresponds to level 0 for *representation* competency of the first sub-question of the final problem. In the second sub-question of the final problem, students are expected to understand the images to express the changes in their velocity and show the relationship between their velocity and their height. Therefore, understanding a complex three-dimensional shape and constructing a graph based on shapes' dimensions are necessary to solve the problem which fits level 2 for representation competency.

For *using symbols, operations, and formal language* competency, level 0 is proposed for the initial problem. This is why operations to solve the problem involves short arithmetic operations (calculating the volumes of barrels) with integers by using elementary facts (volume formula of regular shapes). Then, they need to interpret the amount of water flowing per minute in the context of daily life and how they will contribute to the filling of the container. As a result, students will explain and use mathematical definitions and perform a short arithmetic operation. Therefore, level 0 is appropriate for the initial problem. However, level 2 is proposed for *using symbols, operations, and formal language* competency of the first sub-question of the final problem since employing multiple rules, definitions, etc. are necessary for solving the first item of the final problem. To put it in a more specific way, the solution process outlined includes writing down and employing the formula connecting flow rate, volume and time, using it three times, calculating how much empty space remains for bottle three, and then putting bottle three under the tap with highest flow rate to calculate the remaining time. When the second sub-question is investigated, it is seen that no calculation or application of mathematical rules is required to solve it. Therefore, *using symbols, operations, and formal language* competency of the second item of the final problem are proposed as level 0.

The last competency that defines mathematical item demand is *reasoning and argumen*t. For solving the initial problem, the reasoning is necessary while deciding to maximize the flow rate and lower the filling time as much as possible, putting the largest barrel under the tap with the highest flow rate is necessary. This fits the level 1 definition since reasoning steps are related to only one aspect of the problem. Similarly, initial problem *reasoning and argument* competency of the first sub-question requires students to make a small inference using reasoning about the same aspect of the problem. Therefore, the level 1 description of *reasoning and argument* competency also fits for the first item of the final problem. However, for solving the second item of the final problem, students are required to draw inferences about how the volume and height of water in bottles changes over time. Therefore, students need to draw inferences by joining pieces of information from separate aspects of the problem leads to level 2 for *reasoning and argumen*t competency.

Moreover, the excerpt taken from Tuğçe’s second interview illustrated that why Tuğçe changed demand levels of some competencies while she was revising her problem.

> ***Researcher:*** *Did you change the levels of fundamental mathematical capacities while revising your problem? Why?*
>
> ***Tuğçe:*** *Yes, I made some changes to increase their levels because I like more challenging problems. And I aimed that level 1 and level 2 rather*

than level 0 for each competency since I think mathematical literacy problems are challenging.

4.1.2 Mathematical Content of ML Problems generated by Tuğçe

The second dimension of mathematical literacy problems- mathematical content- refers to mathematical knowledge and understanding in mathematical literacy problems. In this study, based on PISA mathematics framework categorization, mathematical contents are classified into four categories: change and relationship, space and shape, quantity and uncertainty, and data. Detailed information about the coding is given in Chapter 3 in Table 3.4. The codes are given by researcher.

Table 4.3 Mathematical Content Categories of Tuğçe's ML Problems

Tuğçe's Problems		Mathematical Contents
Initial Problem		Quantity
Final Problem	a.	Quantity
	b.	Change and Relationship

As seen in Table 4.3, Tuğçe's initial problem (see in Figure 4.2) lies on the quantity content category since students are expected to calculate the volumes of barrels rather than ordering their size to match with suitable taps. Similarly, Tuğçe's first item of the final problem (see in Figure 4.4) also lies on the quantity content category since students are expected to calculate filling time for each bottle. However, the second item of the final problem (see in Figure 4.4) lies on the change and relationship content category since students are expected to examine the height and volume changes over time.

Tuğçe's initial problem lies in the quantity content category since students are expected to calculate the volumes of barrels than order their size to match with suitable taps. Similarly, Tuğçe's first item of the final problem also lies in the quantity content category since students are expected to calculate filling time for each bottle. However, the second item of the final problem lies in the change and relationship content category since students are expected to examine the height and volume changes over time.

4.1.3 Real-life Context of ML Problems generated by Tuğçe

The third dimension of mathematical literacy problems -context- refers to the real-life situation where the problem has occurred. This section includes three parts. The first part covers the content categories of Tuğçe's initial and final mathematical literacy problems, how they have changed throughout the teaching experiment. The second part covers the level of context use of Tuğçe's initial and final mathematical literacy problems, how they have changed throughout the teaching experiment. The third part covers the authenticity level of Tuğçe's initial and final mathematical literacy problems, how they have changed throughout the teaching experiment.

4.1.3.1 Real-life Context Categories of ML problems generated by Tuğçe

In this study, based on the PISA mathematics framework categorization, context categories of ML problems are investigated under four categories: personal, occupational, social, and scientific content category. Detailed information about the coding is given in Chapter 3 in Table 3.5.

Table 4.4 illustrates the context categories of Tuğçe's initial and final problems.

Table 4.4 Real-life Context Categories of Tuğçe's Initial Problem

Tığçe's Problems	Real-life Contexts
Initial Problem	Personal
Final Problem	a. Personal
	b. Scientific

As it is illustrated in Table 4.4, the initial problem and the first sub-question of the final problem are personal because the problem is related to an individual (Aunt Ayşe). However, the second sub-question of the final problem lies in the scientific context since it includes data related to the volume and heights of bottles, and it asks students to find the mathematical representations of the relationship between them.

4.1.3.2 The Level of Context Use in ML problems generated by Tuğçe

In this study, the contexts of ML problems are investigated under three categories based on the level of context use appears in the context: zero-order, first order, second order context use. Detailed information about the coding is given in Chapter 3 in Table 3.6. The codes are given by researcher and some excepts taken from interviews are given to support the claim.

As it is seen in Figure 4.3, which shows the solution strategy of Tuğçe's initial problem, zero-order context use appears in the problem since the context used in the problem, which carries water with different bottles, is not relevant and not needed for solving the problem and judging the answer. While solving the problem, students need to find the volumes of different bottles and order them, so; students do not need to use contextual information (shapes of bottles, etc.). In addition to that, the following excerpt, which is taken from the second interview of Tuğçe, is also an example of the lowest level of context use appearing in the problem.

> ***Researcher:*** *Do students need to use context when solving your first question? If so, at what level do you think they use it?*
>
> ***Tuğçe:*** *No, they do not use it.*
>
> ***Researcher:*** *Why?*

Tuğçe: *The question just gives the volume, it doesn't matter what the shape of the container is here. This question will be solved even if I tell them another object with the same volume with the container. These containers don't do much. The only important things are the volume of the barrels and speed of flow from the taps, so this happens to be level 0, context does not help much.*

In this description, Tuğçe explained that students do not use the context elements (shape of bottles, …), and even if she changes the context of the problem, it does not affect the solution since students only care about mathematical properties (volume of bottles and tap rates). Therefore, this problem belongs to the level 1 context use category.

The level of context use of Tuğçe’s initial and final problems are given in a tabular form in Table 4.5.

Table 4.5 Level of Context Use of Tuğçe's ML Problems

Tuğçe’s Problems	Level of Context Use
Initial Problem	Zero-order
Final Problem	a. Zero-order
	b. First-order

The final problem, which is given in Figure 4.4, involves two sub-questions. Therefore, we need to analyze the level of context use categories of each sub-question one by one. According to the first sub-question of the revised problem, Aunt Ayşe wants to carry water from four village taps with her three different bottles (bucket, pitcher, and vessel). She would like to match taps and bottles in the most effective way, and she would like to know how many minutes she would spend for filling all bottles since village taps have different flow rates and the bottles have different volumes. Similar to the initial problem, the context of the problem does not require to solve the problem of judging the answer for this sub-question. Therefore, the level of context use of the first sub-question of Tuğçe's final problem is level zero. The second sub-question is about the height-volume graph of the bottles while they are being filled. The question asks students to match the bottles with the correct graphs. In order to solve the problem, students need to consider the shapes of the bottles. In other words, the context of the problem is needed for solving the problem. For this reason, the level of context use of Tuğçe's second sub-question is level 1.

When the initial and final problems are compared, it is seen that Tuğçe increased the level of context use of her problem from zero-order to first order. She increased the level by adding sub-questions, which requires to consider the context elements of the problem while solving the problem. Moreover, the following excerpt, which is taken from Tuğçe's second interview, supports the view.

> ***Researcher:*** *You told me in our first meeting that you didn't think you could raise the level of the problem, so you wrote a new question.*
>
> ***Tuğçe:*** *I told that. I thought about the level of the problem. My friends told me that the level of context use in my problem is zero-order and I agreed with them. This is the reason to add this graph to raise the level a little more. Because here, the graphic is related to the shape, so it should be used in the question as well. My purpose in adding this graph was this, raising the level a little more.*
>
> ***Researcher:*** *So what about the level here?*
>
> ***Tuğçe:*** *So this is level 1, the one including the chart. In the first question, it becomes the zeroth level. It does not use containers; it does not use the given context; it solves the question using only the numbers in the question.*

4.1.3.3 The Level Authenticity of ML Problems Generated by Tuğçe

In this study, the last criteria for the context of ML is authenticity. Seven components of authenticity will be investigated, and each category is divided into three levels;

full alignment, partial alignment, and no alignment based on data analysis. Detailed information about the coding is given in Chapter 3 in

Table 3.7.

Authenticity aspects of Tuğçe’s initial and final problem, their scores, and rationales for scores are shown in the Table 4.6.

Table 4.6 Authenticity Levels of Tuğçe's ML Problems

Aspect of Authenticity	Tuğçe’s Initial Problem	Tuğçe’s Final Problem
Event	1	2
Question	2	0
Information/data	0	1
Presentation	1	1
Solution Strategies	1	2
Solution Requirements	0	0
Purpose	2	1

According to the Table 4.6, event authenticity of Tuğçe’s initial problem is considered as partial alignment. Carrying water from a village tap is a likely event. People from various villages in Turkey still carry their drinking water from village taps since they do not have water taps in their houses, or it is not safe to drink city water. Therefore, the event is a common thing for many people in Turkey. However, these people do not use barrels for carrying water since they are too big. This is why this problem is considered under partially alignment category for event authenticity. Furthermore, the following excerpt, which is taken from Tuğçe’s second interview, shows that Tuğçe was inspired by a real-life situation while writing the problem; hence this event is aligned with real-life although some elements of the event are not realistic.

> ***Researcher:*** *Let's evaluate it in terms of the event. Does this question have an event that we may encounter in daily life?*

> ***Tuğçe:*** *Yes. This question came to my mind from here: My parents stay in the village. There is water in the village now, but there was no water until about five years ago. For example, we used to go fill it from the fountain and come home. In fact, this problem still exists in some villages. In other words, it is a context that we can really face in daily life. In other words, such problems may arise even in the province or district, let alone the village.*

According to Table 4.6, Tuğçe increased the event authenticity of her problem from partial alignment to full alignment. The reason for this improvement is that she used realistic traditional bottles (bucket, pitcher, and vessel) instead of unusual big barrels.

The second aspect of authenticity is question. The question aspect of Tuğçe's initial problem is considered as full alignment since the question asked in the problem, which is "In order to fill the barrels in a most efficient way, which barrel should be placed under which tap?", is a reasonable question for aunt Ayşe. People would consider time efficiency in many situations in real-life. Aunt Ayşe could consider it since she has limited time, or she does not want people to wait in line too long. When the changes between the initial problem and the final problem are investigated, it can be observed that Tuğçe changed her initial problem by revising the question of the initial problem and by asking one more question. It is claimed that these changes decreased the question authenticity aspect of Tuğçe's problem. For example, Aunt Ayşe would not require the information of the exact time for filling the bottles. She probably would not use this information in her real life. Furthermore, she probably does not need to select the right height-volume graphs for bottles. This is very scientific knowledge, and it could be used by mathematicians, scientists or engineers. Therefore, the sub-questions of problem are not realistic, and the question aspect of authenticity is coded as no alignment. In addition to that, the excerpt taken from Tuğçe' s second interview supports this view.

> ***Researcher:*** *Let's look at the questions you asked now. Is the problem here a problem that aunt Ayşe may face?*

> ***Tuğçe:*** *The question I am asking here… OK, she does not come across with the exact values, with cubic meters, but ultimately she may encounter such kind of a question. So when she goes to the fountain, she can think about which barrel to put in which tap. She cannot measure millimeters by millimeters, but it may be a situation that should be thought and commented as "this tap is flowing more, this tap is flowing less, it will fill faster if I put it under this, but if I put it under this, it will fill more slowly.*

For information or data aspect, Tuğçe's initial problem context is not aligned with real life context. The mathematical information which is given in the problem is the flow rates of the taps and the sizes and shapes of the barrels. Existence of this information is available in real life situations and specificity of the information is specific enough to solve the problem; however, mathematical information in the problem does not match with real-life values. More specifically, the flow rates of the tap and the sizes and the shapes of the barrels do not match with real-life. The second and the third barrels are interesting and deviant considering their sides and shapes. Therefore, the information aspect of authenticity is coded with the value 0. The following excerpt is an example of information aspect of the Tuğçe's initial problem is not aligned.

> ***Researcher:*** *Let's look at information and data. Have you looked at the real life equivalents of the values you gave here?*
>
> ***Tuğçe:*** *Yes, I looked on the internet. I looked at how much a tap flows per minute, and I gave these values accordingly.*
>
> ***Researcher:*** *You said tank here. Have you looked at their volumes?*
>
> ***Tuğçe:*** *No, I don't remember looking at them. But in terms of likelihood of shapes, the first shape is a bit absurd, but others may happen in real life.*

This response reveals that although Tuğçe set the information related to flow rates of taps, she claims that the information related to barrels is unrealistic and odd. On the other hand, she probably confused liters with meter-cubes and as a consequence, the flow rates of the taps in the question are actually unrealistic. Therefore, her initial problem's information aspect is not aligned with realistic values.

In order to investigate the final problem's information aspect and observe the changes and improvements between initial and final problem, the mathematical information in the final problem and how it changed should be determined. The mathematical information in the initial problem is the flow rates of the taps and the sizes and shapes of the barrels. Although the information in the final problem is very similar to initial problem, prospective teacher added one more tap to the problem and she used bucket, pitcher and vessel which are water bottles instead of using barrels and she gave the volume of these bottles instead of sizes. Existence and specificity of the problem does not change because existing information are still flow rates of taps and sizes of bottles and they are still specific enough to solve the problem. Therefore, existence and specificity of information does not change so they still match with information in an authentic situation. For the realism, instead of using unusual and odd barrels, she used water bottles which are known and realistic objects, but the volumes of bottles are too big and not in the acceptable range for a realistic situation. In other words, although there is an improvement related to information aspect, there are still some values that do not match with realistic values. Therefore, the information aspect of Tuğçe's final problem is still coded as partially aligned. In the excerpt which is taken from Tuğçe's second interview, Tuğçe mentioned that she changed the barrels to bottles in order to increase the authenticity of the problem.

> ***Tuğçe:*** *I researched the taps and shapes. I pulled out the shape that was weird when I revised it. I changed it for a more possible shape.*

The next aspect of the authenticity of the context is the presentation. When we examine Tuğçe's initial problem according to presentation aspect, it is observed that the shape which represents the third barrel is not clear and some of the dimensions of the barrels could not be read. However, the language of the problem is clear, and the context includes suitable amount of text. Therefore, the presentation aspect of authenticity is coded with the value 1.

The presentation aspect of the final problem is partially aligned with real life situations, so it is coded with the value one since although the language of the problem is clear and the amount of text is suitable for an eight-grade student, but the mathematical presentation (volume height graph of bottles) are not suitable with 8th grade students. This is because 8th grade students only learn graphs of linear equations.

Solution strategies are another aspect of the authenticity. In order to decide the level of solution strategies aspect of initial problem of Tuğçe, we need to investigate the solution strategies of the problem which is given in Figure 4.3. In order to solve the problem, students need to find the volumes of the barrels and sort them from smaller to higher. Then, they need to put the barrel with highest volume under the tap which has highest flow rate and the barrel with lowest volume under the tap which has lowest flow rate. In real life, people usually buy containers or barrels considering their volumes, so, they would probably know their barrels' volumes. Therefore, calculating the volume of barrels is not a common solving strategy for a person who owns the barrel. However, putting the barrels under taps considering barrels' volumes and taps' flow rates is a strategy which people in real life would use. Therefore, solution strategy aspect is coded as partially aligned. Tuğçe's following description of alignment between solution strategies in the problem and real life can be given as an example of the idea that solution strategies in the problem are partially aligned with real life.

> ***Researcher:*** *Well, does aunt Ayşe solves your question in daily life as you direct it?*
>
> ***Tuğçe:*** *Yes, I think she solves it that way.*
>
> ***Researcher:*** *So what kind of solution should the student use here?*

> ***Tuğçe:*** *In the first version of the problem, I wanted the student to find the volumes of solids, compare them according to the velocity of the water flowing from the taps, and comment. But this would be a bit of a high level for Aunt Ayşe, so Aunt Ayşe will ultimately calculate the volume of these containers, it may be a little strange. At the same time, when the containers are sold, their volume is written. She doesn't have to know the volume of this because it is written on it.*

Final problem has two sub-questions as it is stated before, so we need to investigate each one separately. The first sub-question of Tuğçe's final problem has similar solution strategies with the initial problem. In order to solve the sub-question, students need to leave the tap which has the least flow rate since aunt Ayşe wants to fill the bottles as soon as possible. Then, they should put the bottle with higher capability under the tap with higher flow rate and so on. However, students do not need to calculate the volumes of the bottles in the final version since they are already given in the problem. As a consequence, it is observed that Tuğçe increased the solution strategy aspect of the authenticity since students do not need to calculate the volumes of the bottles which are available in real life.

The second sub-question of Tuğçe's final problem is "finding the suitable height-volume graph of the bottles while they are being filled". For this problem, the solution strategies are available and plausible with the solutions in real life. Therefore, the final problem is coded as fully aligned with an authentic situation in terms of solution strategies authenticity. This shows that the solution strategy authenticity of Tuğçe's problem increased from partially aligned to fully aligned.

The next aspect of authenticity is solution requirements. In Tuğçe's initial problem, changing in flow rates of village taps could be one assumption which individuals take into consideration while solving the problem. However, this assumption is not considered while writing the problem. Therefore, there is no alignment between the real-life situation and school task.

At the same time, there is no alignment between the solution requirements of Tuğçe's final problem and the real-life situation. The reason is that flow rate changes depending on seasonal changes and other possible assumptions in the situation are not considered while writing the final problem. Therefore, no changes or increments related to solution requirement aspect of Tuğçe's mathematical literacy problem are observed.

The final aspect of the authenticity is purpose of the context. The purpose of solving Tuğçe's initial problem is clear to students since students know that aunt Ayşe does not want people to wait in line. The following excerpt is another example that the purpose of the initial problem is clear to students.

> ***Researcher:*** *In terms of purpose, is Aunt Ayşe's purpose of solving the question clear?*
>
> ***Tuğçe:*** *I think it's clear. After all, she wants to finish her job as soon as possible. So she wants a fast process, so she needs to solve this question.*

In this response, Tuğçe claims that the purpose of this question is clear for students since aunt Ayşe wants to finish her job as soon as possible in order not to make the people in the line wait, which is given in the problem. Thus, as indicated in Table 4.6, the purpose of initial problem is clear and fully aligned with corresponding real-life situation.

When we investigate the purpose of the final problem, since the first question is the same as the initial problem, we could claim that the purpose of the first question is clear for students. However, the purpose of the second question is not clear for students. In the second question, students are asked to find the appropriate graph for the bottles. However, there is no reason for students or Aunt Ayşe to find such a graph. Therefore, the purpose of the final problem is partially aligned with corresponding real-life situation. This means that Tuğçe decreased the purpose aspect of authenticity of her problem from fully aligned to partially aligned.

Besides to what extend authenticity ML problems posed by PSTs which was shown above, it is important to investigate how they changed authenticity of ML problems to answer research question. Following except taken from second interview with Tuba shows that how she decided to change information aspect of authenticity while revising her problem.

> ***Researcher:*** *Well, there was a revision in the classroom, or are there any parts you added or removed there? In terms of these processes.*
>
> ***Tuğçe:*** *It was said that the terms I used in the classroom might not be realistic. For example, it was said that instead of barrels, there were various bottles that we use in daily life, such as the jug pitcher. It was also said that it would be better to write them under the visuals in order to be understandable and clear. I revised it.*

The above excerpt shows that she revised authenticity of her problem with the feedbacks she received from classroom discussions.

4.1.4 Summary of Tuğçe's Findings

This section offers an overview of Tuğçe's findings in order to evaluate her findings holistically. The following table presents how ML components (mathematical process, mathematical content and context) of Tuğçe's problems changed or developed.

The first category of mathematical process component is mathematical process categories. As seen in Table 4.7, Tuğçe's initial problem lies in employing, and her final problem involves two sub-questions: one of them lies in employing, and the other one lies in formulating process category. The second category is fundamental mathematical capabilities. Table 4.7 presents how level of demand for each fundamental mathematical capability of Tuğçe's problems changed.

Table 4.7 Overview of Developments in Tuğçe's ML Problems

ML Components	Dimensions of Each Components		Initial Problem	Final Problem a	Final Problem b
Mathematical Processes	Mathematical Process Categories		Employing	Employing	Formulating
	Fundamental Mathematical Capabilities	Communication	1	1	2
		Devising Strategies	2	2	2
		Mathematising	2	1	2
		Representation	1	0	2
		Using Symbols and Operations	0	2	0
		Reasoning and Argument	1	1	2
Content	Mathematical Content Categories		Quantity	Quantity	Change and Rel.
Context	Real-life Context Categories		Personal	Personal	Scientific
	Level of Context Use		Zero-order	Zero-order	First-order
	Authenticity	Event	1	2	
		Question	2	0	
		Information/data	0	1	
		Presentation	1	1	
		Solution Strategies	1	2	
		Solution Requirements	0	0	
		Purpose	2	1	

As it is observed in the table, level of demand for communication, representation, and reasoning and argument capabilities increased from level 1 to level 2, using symbols and operations capability increased from level 0 to level 2. Moreover, no changes are observed in level of demand for devising strategies and mathematising capabilities.

Second component of ML problems (mathematical content) has only one category which is mathematical content categories. Table 4.7 shows that Tuğçe's initial problem lies in quantity content category, her final problem involves two sub-questions: one lies in quantity, and the other one lies in change and relationship category.

Third component of ML problems (context) has three categories (real-life context categories, level of context use and authenticity). As it is seen in Table 4.7, Tuğçe's initial problem lies in personal context category, her final problem involves two-sub-questions: one lies in personal, and the other one lies in scientific context category. For second context category, Table 4.7 illustrates that how level of context use of Tuğçe's ML problems changed. It is seen that level of context use of Tuğçe's initial problem is zero-order, and her final problem involves two sub-questions: one lies in zero order and the other one lies in first-order context use. Lastly, for third context category, Table 4.7 shows how authenticity scores of Tuğçe' ML problems changed. As it is seen in the table, Tuğçe increased information/data category from level 0 to level 1 and event, solution strategies categories from level 1 to level 2. However, she decreased the question category from level 2 to level 0 and purpose category from level 1 to level 0.

4.2 Melike's Capability to Generate ML Problem

Melike's capability to generate mathematical literacy level was classified high at the beginning of the teaching experiment. Therefore, she is one of prospective teachers whose progress we want to analyze throughout the experience.

4.2.1 Mathematical Processes and the Underlying Capabilities of ML Problems

The initial problem that Melike generated at the very beginning of the teaching experiment is given in the Figure 4.6. Original Turkish version of Melike’s initial problem is also given in the Appendix F3.

> Depending on the annual motion of the Earth and axis inclination, the angle of sun rays falling on the earth varies. This change causes the length of the shadows to change constantly throughout the day.
>
> A male person with an average height wants to measure the height of an aspen tree by taking advantage of the shade length during the day. When the angle between the surface of earth and the sun's rays falling to the ground is 60°, the shadow of this person, who stands approximately 3 meters from the tree, and the shadow of the aspen tree ends in the same place. How tall can the tree be?

Figure 4.6 Melike's initial ML Problem

As seen in the Figure 4.6, the initial problem developed by Melike is about a person who would like to measure a tree’s height by benefitting from an ancient method. A student who wants to solve the problem should benefit from the angle of sun rays falling on the earth, his height and his shadow’s height. One of the solution strategies is given in the Figure 4.7.

In order to find the solution of Melike’s initial problem, students need to estimate average height of a man. Then they need to visualize the real-life problem and mathematize it by drawing realistic figures. Also, they need to place the angles with considering important situations. For example, the angle between the tree and the ground should be right angle like the angle between the man and the ground. After placing the angles, students should notice the special 30°-60°-90° triangle and the triangle similarity. These properties would lead to solutions with correct operations. Moreover, this is also the solution of sub-question 1 of Melike’s final problem.

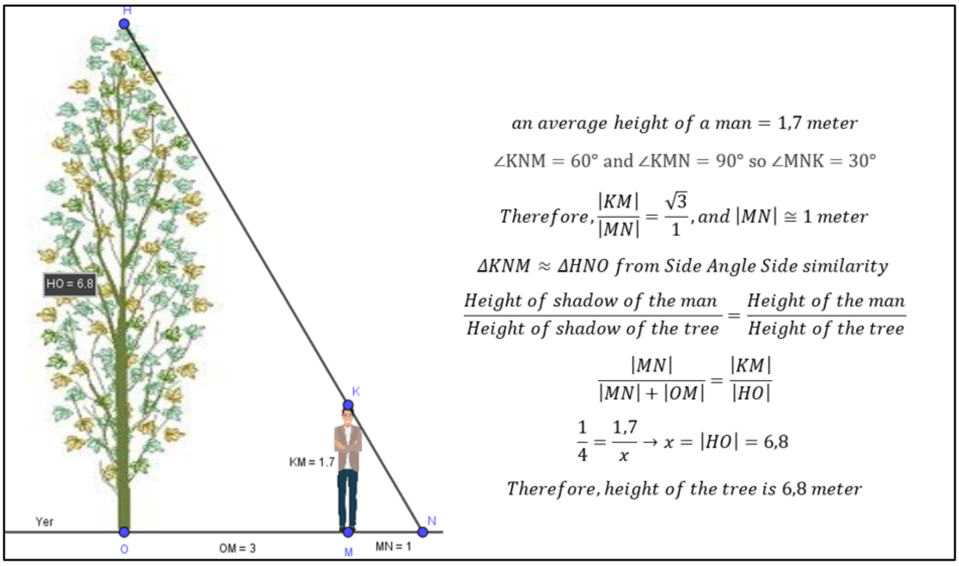


Figure 4.7 Solution Strategy of Melike's Initial ML Problem

After she completed the lectures and take the feedbacks from her classmates, she revised her problem. The final problem is given in the Figure 4.8. Original Turkish version of Melike's final problem is also given in the Appendix F4.

Depending on the annual motion of the Earth and axis inclination, the angle of sun rays falling on the earth varies. This change causes the length of the shadows to change constantly throughout the day.

a. A person with an average man height wants to measure the height of an aspen tree by taking advantage of the shade length during the day. When the angle between the surface of earth and the sun's rays falling to the ground is 60°, the shadow of this person, who stands approximately 3 meters from the tree, and the shadow of the aspen tree ends in the same place. How tall can the tree be?
b. The angle between the sun's rays and the surface of earth constantly changes during the day. When this angle decreases or increases, how does the position of the person who wants to measure the height of the tree using his shadow height must change?

Figure 4.8 Melike's Final ML Problem

As it is seen Figure 4.8, Melike did not change the context of the problem and the question she asked in initial version, but she added one sub-question to the problem. The second sub-question asks how a person who wants to measure a tree's height should change her position depending onto the angle of sun rays falling on the earth. The additional question asks what the relationship between the position of a man and the angle of sun rays falling on the earth is. It is an essay type question and seeks for reasoning instead of algebraic or numerical answer. As the answer to the first sub-question of the final problem is same as the answer of initial problem, solution strategy to solve second sub-question is given in Figure 4.9.

As the angle between the sun's rays and the surface of earth increases, our shadow length will decrease. Therefore, the person making the measurement should reduce the distance between the tree and itself. It needs to approach the tree and measure accordingly.

Figure 4.9 Solution Strategy of Melike's Second Sub-question of Final Problem

The second sub-question of Melike's final problem aims students to reach a generalization for the relationship between a man's position and angle of sun rays falling on the earth. In order to solve the question, students need to consider many situations where an angle of sun rays falling on the earth in different degrees and they need to reach a generalization.

Melike's first, second and final mathematical literacy problems, their solution strategies and why she wanted to change the context of her first problem was shown here. In the following sections, mathematical processes and underlying capabilities of Melike's the initial and final problems and how they have changed through teaching experiment will be examined.

4.2.1.1 Mathematical Processes of ML Problems

PISA mathematics framework categorization of Melike’s first, initial and final problem was given in the Table 4.8. As shown in Figure 4.6 and Figure 4.8, while Melike’s initial problem consists of one sub-question(item), Melike’s final problem consists of two sub-questions (items). Since each item requires different kinds of procedures to solve, each item is analyzed separately. In Melike’s initial problem, individuals mostly engage formulating process. When the solution strategy is investigated, it is seen that the most challenging part of the problem is modeling the real-world situation by drawing the representations of real-world objects.

Table 4.8 Mathematical Process Categories of Melike's ML Problems

Melike’s Problems		Mathematical Processes
Initial Problem		Formulating
Final Problem	a.	Formulating
	b.	Interpreting

First sub-question of Melike’s final problem is also relies heavily on formulating process. As the initial problem and first sub-question of final problem is the same, additional explanation is not given separately here and throughout the section. Unlike the first sub-question, interpreting is a category in which individuals engage mostly while solving the second sub-question. The reason for is that the formulating process of the problem is already followed in first question so: interpreting how the length of shadow of a person changes depending on sun’s position and how it effects the place where a person needs to stand to measure the tree’s height are the main processes of this question.

Moreover, the excerpt taken from Melike’ second interview illustrated that why Melike added a new problem which lies on interpreting process while she was revising her problem.

Researcher: *Could you compare the mathematical processes of your initial and final problem? What kinds of changes you did?*

Melike: *After the feedback, I also gave importance to the processes because my question was on the formulation step. I thought if I could add something to the problem that would allow students interpret so that my problem become more high level.*

4.2.1.2 Fundamental Mathematical Capabilities Underlying Mathematical processes

Fundamental mathematical capability levels of Melike's ML problems are given in Table 4.9. Sub-questions (items) of mathematical literacy problems similarly requires different kinds of procedures to solve and they have different demands for activating each competency; therefore, each sub-question is analyzed differently.

Table 4.9 Fundamental Mathematical Capability Levels of Melike's ML Problems

Mathematical Competencies	Melike's Initial Problem	Melike's Final Problem	
		a	b
Communication	2	2	3
Devising Strategies	3	3	3
Mathematising	2	2	3
Representation	3	3	3
Using Symbols and Operations	1	1	2
Reasoning and Argument	0	0	1

For Melike's initial problem, students need to read and understand the text (e.g. understanding a man is standing next to a tree and there are three meters between them, both have shadows) and decide which information is relevant and which information is not (e.g. the angle of sun rays falling on the earth varies depending on the annual motion of the Earth and axis inclination is an irrelevant information) in order to understand what is exactly required (finding the tree's height by using man's height, shadow and angle of sun's rays). There are several components that are needed to be associated with each other in the problem (an average man's height, the man's shadow, sun's ray, height of the tree and shadow of a tree). For finding the solution the corresponding real-life situation should be investigated specifically and find the geometric shape is being constructed. Many elements are used in the problem to find another element. However, students do not need to understand complex situations like conditional statements logically. Therefore, the *communication capability* level of the initial problem is level 2. As it stated before, the second sub-question of Melike's final problem will be investigated since the first sub-question of the final problem is the same with the initial problem and it has been investigated under the initial problem. In order to solve the second problem, students need to visualize the corresponding real-life situation similarly to the solution of initial problem. However instead of using significant values which are given in the problem, students need to try lots of values to generalize the solution. Therefore, students need to understand all the communicative properties to solve the sub-question 2. For solving the second sub-question, students should understand the complex situation like conditional statements logically (how should the man change his position if the angle of sun's rays falling on the earth increases) so the *communication capability* level of the final problem is level 3.

The strategy needed to solve Melike's initial problem involves devising a complex multi-step strategy. When the solution strategy of the problem is investigated, students need to draw mathematical correct figures depending on the problem context at first, then uses special angle properties to find height of the man's shadow and the tree's shadow. Finally, they need to find the tree's height by using ratings from similarity. Therefore, the *devising strategies capability* of the problem fits to level 3 description of the devising strategy competency. When we investigate Melike's final problem, the first sub-question again fits to the level 3 description of devising strategies competency. Moreover, since the similar solving strategies need to be carried on solving the second sub-question, it fits the level 3 description of devising strategies competency. In other words, students need to solve the first sub-question for different angles of sun rays falling on the earth, the strategy to solve the second sub-question is similar to first sub-question. Therefore, second sub-question of Melike's final problem is level 3 for devising strategies competency. When the initial and final sub-question of Melike's problems are compared considering the devising strategies competency, it is seen that both initial and final problem demands highest level of devising strategies competency from students.

Mathematising competency in Melike's initial problem includes determining the elements (height of man, the angle of incidence of sun rays, the distance between the man and the tree) of the context which help to find the height of the tree. Then, context should be visualized by drawing its figure. In addition to that students need to establish the relationship between the lengths and construct a mathematical model (similar triangles and similarity equations) where all constraints are already defined in the problem. Furthermore, the possible outputs of the model should be interpreted in the context of the problem. As students generate a model on the basis of certain variables, relationships and assumptions specified in the question and interpret this model in context, the mathematising level of the initial problem is level 2. The second sub-question of final problem will similarly be considered since the first problem of the final problem is same with the initial problem. As mentioned before, in order to solve the second problem, students need to solve the first sub-question with different variables (angle of sun rays falling on the earth). As a consequence, students need to do same mathematising for second question repeatedly. However, in the second sub-question, they also need to make different assumptions about sun's position to decide how the shadow of tree changes. Therefore, the mathematising level of the final problem of Melike is level 3.

In Melike's initial problem, students should generate a complex visual representation with many variables by using verbal representations. In other words, it is necessary to transform the tree position, shadow dimensions, person size and angle of incidence of sunlight into a complex representation given with verbal representations. Since devising a representation that captures a complex mathematical entity indicates the highest level for representation competency, the representation level of Melike's initial problem is 3. Moreover, the representation competency level for Melike's second sub-question of final problem is level 3. The reason is that, as it is stated, students need to solve the first sub-question (the initial problem) with many variables to obtain a generalization which is answer for sub-question 2. Therefore, students need to generate multiple representations that captures a complex mathematical entity, which makes representation competency of second sub-question level 3.

In order to solve Melike's initial problem, students should generate a figure by visualizing the information given to interpret the expression "angle of sun rays falling on the earth" in the context of real life and then write a proportional statement based on triangle similarity. Since the average human height will be around 1.7 meters, they need to operate with rational numbers. Since the question includes creating a simple proportional expression containing variables and performing operations with rational numbers, for using symbols, operations and formal language level 1 is proposed for Melike's initial problem. However, for solving second sub-question of final question, students need to manipulate the expressions. For example, after constructing the similarity equations, students change the value of angle of the sun rays falling on the earth to investigate how the man's position should change. Also, using repeated calculations from level 1 reflects the level 2 competence for using symbols, operations and formal language. Therefore, for Melike's mathematical literacy problem proposed level increased from level 1 to level 2 for using symbols, operations and formal language.

Reasoning and argumentation competence is the least required competence for solving Melike's initial problem. In other words, students are only required to produce a logical argument for an average human size in solving the problem, so reasoning and argument competence is proposed as level 0. There are no reasoning steps required to find this information. The *reasoning and argument* competence of Melike's second sub-question of final problem is higher than the reasoning and argument competence of Melike's initial problem. The reason for this is that students need to draw inferences to answer the second sub-question of final problem. Firstly, students should consider the situations where the angle of sun rays falling on the earth decreases or increases and they should make conjectures to understand the location of the man. Therefore, students need to draw inferences to understand one aspect of the problem which corresponds to first level of reasoning and argument competence.

Moreover, the excerpt taken from Melike's second interview illustrated that why Melike changed demand levels of some competencies while she was revising her problem.

> ***Researcher:*** *Did you change the levels of fundamental mathematical capacities while revising your problem?*
>
> ***Melike:*** *Yes, I tried to increase the levels.*
>
> ***Researcher:*** *Why?*
>
> ***Melike:*** *I think that I construct more higher level mathematical literacy problem when I increase the demand of mathematical competencies. How many different competencies it addresses, ie visualization etc. I also thought that it takes the problem to a higher level.*

4.2.2 Mathematical Content of Mathematical Literacy Problems generated by Melike

Table 4.10 illustrates mathematical content categories of Melike's initial and final problems. Melike's initial problem (see in Figure 4.6) and the first item of final problem (see in Figure 4.8) lie in shapes and spaces content category since interpreting the positions and heights of visual objects, drawing them on a paper and solving the problem by using triangle similarity are the main demands of the problem. Melike's second item of final problem (see in Figure 4.8) also lies in shapes and spaces content category since students need to interpret the positions of shapes.

Table 4.10 Mathematical Content Categories of Melike's ML Problems

Melike's Problems		Mathematical Contents
Initial Problem		Shapes and Spaces
Final Problem	a.	Shapes and Spaces
	b.	Shapes and Spaces

4.2.3 Real-life Context of ML Problems generated by Melike

This section includes three parts. First part covers the content categories of Melike's initial and final mathematical literacy problems, how they have changed throughout the teaching experiment. The second part covers level of context use of Melike's initial and final mathematical literacy problems, how they have changed throughout the teaching experiment. Third part covers authenticity level of Melike's initial and final mathematical literacy problems, how they have changed throughout the teaching experiment.

4.2.3.1 Real-life content Categories of ML problems generated by Melike

Table 4.11 illustrates context categories of Melike's initial and final problems.

Table 4.11 Context Categories of Melike's ML Problems

Melike's Problems		Real-life Contexts
Initial Problem		Personal
Final Problem	a.	Personal
	b.	Scientific

The initial problem and the first sub-question of final problem is personal because the problem related to an individual who wants to measure a tree's height in his garden. However, second sub-question of final problem lies in the scientific context since context of the problem relies on mathematics applications in real-world.

4.2.4 The Level of Context Use in ML problems generated by Melike

Table 4.12 Level of Context Use of Melike’s ML Problems

Melike’s Problems		Level of Context Use
Initial Problem		Second-order
Final Problem	a.	Second-order
	b.	First-order

As mentioned before, level of context use is the degree to determine which students use the context of the problem while solving mathematical problems. In order to determine the level of context use of Melike’s mathematical literacy problems, which are shown in Table 4.12, and specify the changes between the initial and final version, we need to check the solution way of the problems. According to solutions shown in Figure 4.7 and Figure 4.9, students firstly need to estimate the average man’s height from their special real-world knowledge. Then students need to use this knowledge and given pre-mathematized knowledge (the distance, the angle of sun rays, …) to present it in a mathematical form to solve the problem. Since students use their own knowledge to estimate an information in the problem, it can be claimed that Melike’ initial problem and the first sub-question of final problem have second level context use.

Final problem involves two sub-questions. As we analysed the initial problem which is same with the first sub-question of final problem, we will only analyze the second sub-question of the final problem. Unlike the initial problem, the second sub-question has first order context use. The reason is that students use the context elements in the problem to solve it, but they do not need to use their own special knowledge. In other words, students need to mathematize the context elements which are position of three and the angle of sun rays falling to the earth to find the position of the man. Therefore, students use contextual knowledge to solve the problem, but they do not need to use their own knowledge since all necessary knowledge is given in the problem.

4.2.5 The Level Authenticity of ML Problems Generated by Melike

As it was stated the last criteria that we will investigate in this study is authenticity levels of mathematical literacy problems. Authenticity in mathematics problems has eight dimension which are event, question, information/data, solution strategies, presentation, solution requirements, purpose and circumstances.

The event aspect, which is the first aspect of authenticity, claims that the event in the question should be likely to happen in real life. The event described in Melike's initial problem is that a person would like to measure a tree's height by benefiting from the sun's position. This method is used in ancient times to estimate the size of objects. For example, ancient mathematician Thales used this method to measure the Keops pyramid (Bilim ve Çocuk, 2020). Therefore, this event is possible to happen in real life. Thus level 2 is proposed for the event aspect of the authenticity of Melike's initial problem. Melike did not change the event of the problem while revising it; therefore, the event aspect of the final problem is also level 2. Melike maintained the event characteristics of the authenticity of the problem at the highest level.

Table 4.13 Authenticity Levels of Melike's ML Problems

Aspect of Authenticity	Melike’s Initial Problem	Melike’s Final Problem
Event	2	2
Question	2	2
Information/data	1	1
Presentation	2	2
Solution Strategies	1	1
Solution Requirements	2	2
Purpose	2	2

The second characteristic of authenticity is the question. There is one question asked in the initial problem which Melike posed. The question asks what the height of a tree in front of which a man stands is. It is a reasonable question since a person may wonder a tree's height for many reasons, such as cutting the tree without harming. For this reason, the person in a corresponding real-life situation can encounter with a similar question. Therefore, the question aspect of the initial problem's authenticity is fully aligned with the corresponding real-life situation. It is proposed as level 2. The final problem also has the same question, and it also has a second question that asks the relationship between the position of a man and the angle of sun rays falling on the earth. It is also a reasonable question for a person who wanders how she should change her position to measure the tree's height at different times of the day. In other words, this question can also be asked by a person who is in a corresponding real-life situation. Therefore, two questions in the final task are reasonable and related to a corresponding real-life problem. For this reason, level 2 is proposed for question aspect of authenticity in the question. The level of the question was maintained at the highest level.

The third aspect of the authenticity is information and data. The information provided in Melike's initial problem is the reason for the change in shadow lengths of objects during the day and the angle of sun rays' falling on the earth, and the information asked in the question are the height of an average man and the height of a tree. First of all, it is known in real-life that the change in shadow length of objects in the world is due to the earth's annual motion and axis tilt. Therefore, this information is aligned with scientific data. Secondly, the angle of sun rays' falling on the earth could be between 0° and 90°. Therefore 60° , which is the given angle, is in acceptable range. However, this information is difficult to obtain for a person in real life. Finally, an average height for a man is 1.70 meters for Turkey. This value could change from 1.50 m to 1.75 m in different countries the world. This information is not given in the problem so it may seem unimportant. However, students will find the height of the tree depending to this information. Therefore, it is important that students find the tree's height in acceptable range when they accept an average height of a man 1.5 to 1,7 meter. When the necessary calculations are made, the tree's height would be found between 6 to 6,8 meters. The tree's height in the problem is in an acceptable range although some trees in our country can be as tall as 40 meters. Therefore, all information and data in the initial problem has realistic values but some of them is not available in real life so the information level of initial problem is 1. As stated before, the first question in the final task is the same as the initial problem. Therefore, although the information in the question has realistic values, some information is not available in real life. In addition to that, there is no additional information given in the second question in the final problem. Thus, the information in the final problem is also partially aligned with corresponding real-life situation which implies level 1.

The following aspect of authenticity is presentation. For Melike's initial problem, the prospective teacher used a suitable number of words for 15 years old or 8^{th} grade student. Students are also used to technical terms such as angle and sun's position since they have covered these topics before. Therefore, the presentation of the initial problem does not affect the solution process negatively. Thus, level 2 is proposed for the presentation aspect of the initial problem. This also means that the first question of the final problem is fully aligned with the corresponding out of school situation. Moreover, the second question of the final problem is fully aligned with the real-life situation since the question clear. Therefore, level 2 is proposed for the presentation aspect of the final problem.

The fifth aspect of authenticity is the solution strategies. When we evaluate the solution strategies of Melike's initial problem, we observe that the strategy available in school task is originated from a scientific approach in real life. This strategy is plausible to use in corresponding real-life situation since many people do not have specialized materials to measure high materials like trees, they would probably use the sun's position to measure tree height. However, some strategies available in real-life are not available in the problem situation. For example, using some technological materials to measure the height of a tree is not a possible strategy could be used in the task, but it could be used in real-life. Therefore, Melike's initial problem leads to level 1 for the solution strategies aspect of authenticity. In order to evaluate the solution strategy aspect of the final problem, we will only investigate the solution strategies of second sub-question since the first sub-question is same with the initial problem. Students need to find solutions for many situations where the angle of sun rays' falling on the earth is different to find the solution for second sub-question of final problem or they need to use their reasoning to understand how a person should move depending on sun's position. In the corresponding real-life situation, people who wonder the same problem probably use the same method for solution since their situation is very specific to search on the internet. For this reason, the solution strategy available in the second-sub question is also an available strategy in real life and it is also plausible to use. It is observed that although newly added problem's solution strategies are fully aligned, Melike has not revised possible solution strategies available in first sub-question. Therefore, the first and second questions of final problem leads to level 1 for solution strategies competency.

Solution requirements are the following aspect of the authenticity. The solution required by Melike's initial problem requires estimates based on the given information, which is reasonable for the real life situation. One of the estimates is height of an average man which is not given in the problem directly. This requirement is consistent with the corresponding real-life situation since if a person who would like to use his height to measure size of another object, he should estimate his height. Therefore, solution requirements of the initial problem and the corresponding real word situation are fully aligned. The initial problem is the same with first sub-question of the final problem so same situation is also valid for it. For second sub-question of final problem, there is no requirement for students need to consider while solving the problem. Therefore, final problem is also fully aligned with corresponding real-life situation.

Purpose is one of the salient elements of authenticity since students usually wonder whether the purpose of the problem is clear or not. In the initial problem, the purpose of the problem is to find the height of a tree. This aim is important for a person who does not have technological materials for measuring the height of big objects. For this reason, why students need to make the necessary operations are clear to students. In other words, they know the aim of finding the measure of a tree. Therefore, the purpose competency of initial problem and first sub-question of final problem is proposed at level 2. Moreover, the aim of second sub-question of final problem is important for a person who want to make a generalization. Therefore, level 2 is proposed for Melike's initial and final problem.

4.2.6 Summary of Melike's Findings

This section offers an overview of Melike's findings in order to evaluate her findings holistically. The following table presents how ML components (mathematical process, mathematical content and context) of Melike's problems changed or developed.

For the first category of mathematical process component (mathematical process categories), Table 4.14 represent that Melike's initial problem lies in formulating process category, and her final problem involves two sub-questions: one lies in formulating and the other one lies in interpreting process category. For the second category of the component (fundamental mathematical capabilities), level of demand for reasoning and argument capability increased from level 0 to level 1, for using symbols and operation capability increased from level 1 to level 2, for communication and mathematising capabilities increased from level 2 to level 3. On the other hand, level of demand for devising strategies and representation capabilities did not change.

For mathematical content categories component of ML problems, Table 4.14 shows that all problems generated by Melike lie in shapes and spaces content category.

Table 4.14 Overview of Developments in Melike's ML Problems

ML Components	Dimensions of Each Components		Initial Problem	Final Problem a	Final Problem b
Mathematical Processes	Mathematical Process Categories		Formulating	Formulating	Interpreting
	Fundamental Mathematical Capabilities	Communication	2	2	3
		Devising Strategies	3	3	3
		Mathematising	2	2	3
		Representation	3	3	3
		Using Symbols and Operations	1	1	2
		Reasoning and Argument	0	0	1
Content	Mathematical Content Categories		Shapes & Spa.	Shapes & Spa.	Shapes & Spa.
Context	Real-life Context Categories		Personal	Personal	Scientific
	Level of Context Use		Second-order	Second-order	First-order
	Authenticity	Event	2	2	
		Question	2	2	
		Information/data	1	1	
		Presentation	2	2	
		Solution Strategies	1	1	
		Solution Requirements	2	2	
		Purpose	2	2	

The third component of ML problems (context) has three categories (real-life context categories, level of context use and authenticity). As it is seen in Table 4.14, Melike's initial problem lies in personal context category, her final problem involves two-sub-questions: one lies in personal, and the other one lies in scientific context category. For the second category of context, Table 4.14 illustrates that the level of context use of Melike's initial problem is second order, and her final problem involves two sub-questions: one lies in second order, and the other one lies in first order context use. Lastly, for third context category, Table 4.14 shows that authenticity scores of Melike's problem did not change in any category.

4.3 Ayşe's Capability to Generate ML Problem

Ayşe's capability level of creating mathematical literacy problems was classified as low at the beginning of the teaching experiment. Therefore, she is one of the prospective teachers that we want to analyze her progress throughout the experience.

4.3.1 Mathematical Processes and the Underlying Capabilities of ML Problems

The initial mathematical literacy problem that she generated at the very beginning of the teaching experiment (first lesson) is given in the Figure 4.10. Original Turkish version of Ayşe's initial problem is also given in the Appendix F5.

The monthly payments of a house for electricity, water and gas are as in the table.

Electricity	200 TL
Water	150 TL
Gas	450 TL

How many years it takes for someone who pay exactly the same prices each month after moving into this house to pay 78000 TL in total?

Figure 4.10 Ayşe's Initial ML Problem

The problem scenario involves a person who would like to know for how many years he could pay exactly the same money for his expenses. Expenses of electricity, water and gas are given in the problem. To solve the problem, students need to calculate the total amount of expenses in a month by adding all expenses and dividing the total money which the house owner has to do this value. The calculation can be completed by converting 97,5 months into years by dividing months by 12 since there are 12 months in a year. The solution method of the problem is shown in the Figure 4.11.

$$Sum\ of\ monthly\ bills{:}\ Electricity + Water + Gas = 200\ TL + 150\ TL + 450\ TL = 800\ TL$$

$$\frac{Total\ Paid\ Money}{Sum\ of\ monthly\ bills} = time(month) \rightarrow \frac{78000\ TL}{800\ TL} = 97{,}5\ which\ is\ 97\ month\ 15\ days$$

$$\frac{month}{12} = year \rightarrow \frac{97{,}5}{12} = 8{,}125\ years,\ which\ is\ 8\ year\ 1\ month\ 15\ days$$

Figure 4.11 Solution Strategy for Ayşe’s Initial Problem

Ayşe wanted to continue with her initial problem, and she has revised it throughout the teaching experiment. Then, she revised her problem based on the information she gained from the lectures. The revised problem that she posed is given in the

Figure 4.12. Turkish version of Ayşe’s revised problem is also given in the Appendix F6. As seen in the Figure 4.12, Ayşe did not change the context of the problem which is paying expenses. However, instead of saying that there is a certain value for electricity, water and gas expenses each month, she gave a table containing the lowest and highest values for these expenses within three months. She also defined some conditions among expenses and asked the maximum amount payable for expenses at the end of the 3rd month. A possible solution strategy for Ayşe’s revised problem is given in the Figure 4.13.

	Price
Electricity	70-250
Water	72-110
Gas	240-600

The minimum and maximum values of electricity, water and natural gas prices paid as house expenses are given in the table. It is known that electricity prices have been less than water prices for 3 months and if you pay more than 300 TL for natural gas in the first two months, the natural gas expenses of third month becomes less than 300 TL. According to this information, at the end of the 3rd month, what is the maximum amount of money that can be paid for total expenses?

Figure 4.12 Ayşe's Revised ML Problem

Ayşe reached the final version of her problem after she benefited from classroom discussions related to her problem. The final version of Ayşe's problem is given in the Figure 4.14. Original Turkish version of Ayşe's final problem is also given in the Appendix F7. In the final problem, Ayşe asked what could be the highest price that can be paid for total expenses in three months. Moreover, she added one sub-question to the problem. The second question asks the lowest amount the person in the question can pay in the next month, according to the past expenses given in the question.

Ayşe's initial, revised and final mathematical literacy problems, their solution strategies and why she wanted to change the context of her first problem were shown here. In the following sections, mathematical processes and underlying capabilities of Ayşe's the initial and final problems and how they have changed throughout the teaching experiment will be examined.

Each bill will be represented with their first letter, i.e. W: water bill, E: electricity bill, G: gas bill. The water and the electricity expenses are independent from the gas expenses. Therefore, the gas expenses can be calculated separately. Let the gas payments in the first, second and third months be called as G1, G2 and G3, respectively.

Maximum value could be paid for water expense in a month is 110 TL so upper limit of water expenses in 3 months are 330 TL.

We know from the question statement that electricity must be lower than water since maximum value for water expense in a month is 110 TL so maximum value for electricity expense in a month must be less than 110. Then, upper limit of electricity expenses in 3 months must be less than 330 TL so E<330 TL

We know from the question statement that G3 < 300 TL if G1 > 300 TL and G2 > 300 TL. We also know from the question statement that G1, G2 and G3 are all smaller than 600 TL. Therefore, G3 is either smaller from 300 TL or smaller from 600 TL, depending on the values of G1 and G2. To calculate the maximum amount of gas expenses, both possibilities must be evaluated.

Let's first consider the case when G3 < 300 TL. Then, both G1 and G2 are higher than 300 TL, therefore G1 < 600 TL and G2 < 600 TL. In this case, G1 + G2 + G3 < 300 + 600 + 600 = 1500 TL. Therefore, the upper limit of the gas expenses in 3 months are calculated as 1500 TL.

In the second case, G3 < 600 TL. This means that either G1 < 300 TL or G2 < 300 TL. No matter which one is smaller than 300 TL, the other one must be maximized in order to get the maximum amount of expenses. Therefore, without losing generality, let G1 < 300 TL. Then, G2 < 600 TL and G3 < 600 TL. In this case, G1 + G2 + G3 < 300 + 600 + 600 = 1500 TL. Therefore, the upper limit of the gas expenses in 3 months are calculated as 1500 TL.

Since the upper limit of gas expenses in 3 months are the same in both cases, 1500 TL is the actual maximum value.

Therefore, we know that $(\mathrm{W})_{max} = 330\,TL$, $(\mathrm{E})_{max}$<330TL and $(\mathrm{G})_{max} < 1500TL$

so $(\mathrm{G})_{max}$+$(\mathrm{W})_{max} + (\mathrm{E})_{max} < 2160TL$

Figure 4.13 Solution Strategy for Ayşe's Revised Problem

Expenses	Price (TL)
Electricity	70,23 - 250,41
Water	72,06 - 110,98
Gas	240,45 - 603,85

When Ahu examines the bills she has paid to her house for a year, she makes a list of the range of electricity, water and natural gas prices paid each month.

- Ahu, who increased the use of water during the summer months, realizes that the amount she paid for her electricity bill during the summer months does not exceed the amount she paid for the water bill.

- Ahu observes that the natural gas bills in this season are less than 286.01 liras, as she turns off the boiler in summer.

a. Accordingly, how many liras could Ahu have paid at most for all the expenses in the table during the summer months?
b. Ahu pays every month for internet bills and phone bills, in addition to the bills given in the table. She forgets to record the amounts she paid for the internet and telephone bills in July, but remembers that the amount she paid for these two bills in July was between 50 liras and 125 liras. With this information, Ahu wants to estimate the minimum amount she will pay in August. What should the budget Ahu will allocate for all payments for August be at least?

Figure 4.14 Ayşe’s Final ML Problem

a. Each bill will be represented with their first letter, i.e. W: water bill, E: electricity bill, G: gas bill. The water and the electricity expenses are independent from the gas expenses. Therefore, the gas expenses can be calculated separately.

Maximum value could be paid for water expense in a month is 110,98 TL so upper limit of water expenses in 3 months are 332,94 TL.

We know from the question statement that electricity must be lower than water. Thus, maximum value for electricity expense in a month must be less than 110,98 since maximum value for water expense in a month is 110,98 TL. Then, upper limit of electricity expenses in 3 months must be less than 332,94 TL so E<332,94 TL

We know from the question statement that gas expense in a month must be lower than 286,01 so the maximum value for gas expenses in 3 months must be less than 858,03TL so G<858,03 TL

Therefore, we know that $(\mathrm{W})_{max} = 332{,}94\ TL$, $(\mathrm{E})_{max}$<332,94 TL and $(\mathrm{G})_{max} < 858{,}03\ TL$

so $(\mathrm{G})_{max}+(\mathrm{W})_{max} + (\mathrm{E})_{max} < 1523{,}71\ TL$

b. Let we denote the internet expenses for a month and telephone expenses for a month as I_1 and T_1 respectively. We know from the question statement that the expenses for the internet and the expenses for telephone in a month must be between 50 TL and 125 TL so $50\ TL < I_1 < 125\ TL\ and\ 50\ TL < T_1 < 125\ TL.$

Also, we know that 70,23 TL $< E_1 <$ 250,41 TL

72,06 TL $< W_1 <$ 110,98 TL,

240,45 TL $< G_1 <$ 603,85 TL

Therefore, 482,74 TL < $I_1 + T_1 + E_1 + W_1 + G_1$ < 1215,24

Figure 4.15 Solution Strategy for Ayşe's final problem

4.3.1.1 Mathematical Processes of ML Problems

As Ayşe's initial, revised and final problems are investigated, it is observed that while Ayşe's initial and revised problems involve one item, her final problem involves two items. As each item requires different kinds of procedures to solve, each item is analyzed separately. The mathematical processes of each items in Ayşe's ML problems are given in the Table 4.15.

The solution strategies of Ayşe's initial problem is investigated, it is seen that individuals mostly engage employing process since the mathematical demand here is to calculate total expenses and diving the total money to sum of monthly expenses.

Table 4.15 Mathematical Process Categories of Ayşe's ML Problems

Ayşe's Problems		Mathematical Processes
Initial Problem		Employing
Revised Problem		Formulating
Final Problem	a.	Formulating
	b.	Employing

The revised problem and the first sub-question of final problem are an example of formulating category, since the main demand of these problems is to express statements in the problems like mathematical expressions. In addition to this, second sub-question of final problem is placed on the employing mathematical facts, concepts, procedures and reasoning category since devising a strategy for solving the question is the main demand of the problem.

4.3.1.2 Fundamental Mathematical Capabilities underlying mathematical processes

In this section, we are interested in how six mathematical competencies in which degrees are used in Ayşe's initial, revised and final problem and how they have changed from initial problem to final problem through the teaching experiment.

Fundamental mathematical capability levels of Ayşe's ML problems are given in Table 4.16.

Communication competency is the first competency that is established by PISA and it helps to define how students communicate with the problem while understanding answering to the problem. The communication competency of the initial problem is proposed as level 0 since receptive aspect involves information (water, electricity and gas expenses for a month) which is directly related to problem and there is no additional information given in the problem. Moreover, the constructive aspect includes only presenting a numeric result. On the other hand, communication competency of Ayşe's revised problem is much higher than initial one. The reason why is that in the revised problem, instead of giving exact numbers for each expense (water, electricity etc.), Ayşe gave a range for them and she added a conditional statement which involves complicated relationships in the receptive aspect. In addition to that, Ayşe linked some elements of the problem (water expense and electricity expense) so that students need to understand these elements and their connection to solve the problem. Besides this, students need to present a sequence of calculations and make some explanations while answering to revised problem, instead of showing a numeric result like in the initial problem. These changes increased the communication capability of Ayşe's problem from level 0 to level 3 since conditional statements made the receptive aspect of the problem highly complicated to understand, and were required explanations and calculation steps while presenting the solution made the constructive aspect more challenging.

Table 4.16 Fundamental Mathematical Capability Levels of Ayşe's ML Problems

Mathematical Competencies	Ayşe's Initial Problem	Ayşe's Revised Problem	Ayşe's Final Problem	
			a	b
Communication	0	3	2	3
Devising Strategies	2	2	2	2
Mathematising	0	1	1	2
Representation	0	0	0	1
Using Symbols and Operations	1	2	2	2
Reasoning and Argument	0	1	0	1

Ayşe's final problem involves two sub-questions, so the mathematical competencies of the questions were examined separately. Since the first problem, revised problem and the first sub-question of the final problem seek for answers to the same question, the first sub-question of final problem will be evaluated as the revised version of the initial problem, and the second sub-question will be evaluated as a newly added question. In her final problem, Ayşe changed her revised problem by removing the conditional statements and she provided some extraneous sentences to the problem (e.g. reason for giving ranges for expenses). In addition to that there are multiple elements in the problem (water, gas and electricity expenses) that need to be linked and students are asked to express a brief explanation and solution steps while answering the question, similarly to the revised problem. These changes, mainly removing conditional statement, decreased the communication competency of the first sub-question of final problem to level 2. The second sub-question of final problem is a newly added question, so we analyze it independently. The question asks students to examine the current situation to estimate a linked future situation (estimating minimum saving that Ayşe should have considering the previous expenses). While answering the question, students should produce an argument about whether Ayşe should make the budget to be prepared according to the upper limit or the lower limit of the expenditures, and while producing this argument, they should consider all expenses together. Therefore, level 3 looks appropriate for communication.

Devising strategies is another competency that we analyze while determining the item demand of participants' mathematical literacy problems. The strategy was needed for solving Ayşe's initial problem (calculating how many years are required to pay 78000 TL with the knowledge of monthly expenses) involves two distinct but straightforward steps: find the total expenses in a month, and combine that with the budget to calculate how many years are required. The strategy applied to solve the problem involves straightforward two stages hence it fits to the level 2 description. The revised problem requires to form multi-stage strategy similarly to initial problem. In other words, in order to find the maximum value for total expenses, it is required to find the maximum value for each expense. Each expense will be calculated separately by using the relationships between the expenses. Although these steps are distinct, they are straightforward so there is only one strategy to solve this problem. Therefore, devising strategy competency of Ayşe's revised problem is level 2. The solution strategy of the first sub-problem of final problem also requires straightforward multi-stage steps to calculate the maximum expenses that Ayşe paid in summer months. Although Ayşe removed the conditional statement in the final version of the problem, students still need to calculate each expense separately by using the relationship between them, similarly to initial and revised problems. Thus, the first sub-question of final problem requires straightforward multi-stage strategy which indicates level 2 for devising strategy. The second sub-question of final problem includes two sub-goals (calculating range for total expenses in July and allocating the minimum budget for total expenses in August). This sub-question requires students to form several straightforward strategies. Therefore, devising strategy competency for second sub-question of final problem is proposed as level 2.

The following fundamental mathematical competency is mathematising. As it is mentioned in Chapter 3, mathematising involves translating a real-life problem into a mathematical model and using this model to interpret the possible outcomes and evaluating the sufficiency of the model. For Ayşe's initial problem, students are expected to calculate the total expenses in a month by adding each expense, and then divide the money she saved for expenses this value in order to find how many years she would pay. Formulating a model for solving this problem is not needed since the expenses are already given in the table and students do not expected to write a formula or equation. Thus, level 0 is appropriate for mathematising. For Ayşe's revised problem, a modeling must be constructed to find the total maximum expenses for three months where the required assumptions, variables and relationships are given in the problem. This fits the level 1 for mathematising. Similarly, for Ayşe's first sub question of final problem, students need to construct a model for the total maximum expenses for three months. Although Ayşe changed some of assumptions and relationships in the problem, required assumptions and relationships are still given in the question. For this reason, the first sub-question of final problem fits level 1 for mathematising. The second sub-question of final problem requires two modeling steps. The first is formulating a model for estimating the telephone and internet expenses in August (here the constraints, relationships and assumptions are not clear and need to be defined) and the second is formulating a model for determining the minimum budget for August (constraints for other expenses are clear and given in the problem). The second sub-question of final problem leads to level 2 for mathematising because of the first modeling step. As a result, Ayşe increased the mathematising competency of her problem by adding constraints and relationships to her initial problem that require students to construct a model for solving the problem, and by adding a second sub-question to final problem that requires students to determine the constraints to construct a model for determining the budget for the following month.

Representation is another competency under fundamental mathematical capabilities of mathematical literacy problems. It involves decoding the given representation in the problem text, using or translating between the representations to solve the problem or selecting suitable representation to represent the solution. Ayşe's initial problem involves a table showing electricity, water and gas expenses. This table is an example of simple representation which illustrates isolated values. In addition to this, while solving or presenting the solution, students do not require usage of any representation. In this case, level 0 is suitable for representation of Ayşe's initial problem. Similarly, Ayşe's revised problem includes a simple representation which illustrates the range for each expense in a year. Reading these isolated values from the table is again the only necessity under the representation competency for this problem. Although the revised problem involves relationships between the values as opposed to the initial problem, no representation is used to represent them, and no representation is needed to be constructed to show the solution. Thus, level 0 is also appropriate for representation competency of the revised problem. In addition to that, the first sub-question of Ayşe's final problem uses a simple representation where students read a range of values for each expense in a year and no representation is required to solve or illustrate the solution for this question. Thus, level 0 is also appropriate for first sub-question of final problem. Differently than revised problem and first sub-question of final problem, second sub-question involves numeric values of telephone and the internet expenses which students must interpret to find a trend for expenses in August. Thus, level 1 fits for representation competency of second sub-question of final problem. As it can be seen, Ayşe increased the representation competency level of her questions from 0 to 1.

Another competency is using symbols, operations and formal language. This competency deals with comprehending and using mathematical procedures and language. Ayşe's initial problem involves some level 0 calculation (adding 3-digit numbers to find total expenses) along with a division calculation which might lead to decimal results (calculating how many years required to pay 78000 TL with all expenses). Arithmetic level 0 calculations which involves fractions or decimals leads to level 1 for using symbols, operations and formal language competency. However, for revised problem, level 2 is proposed for this competency. The solution process defined involves writing down then manipulating the statements by using the maximum values of expenses from the table; then computing all of them to find the maximum cost for expenses. Therefore, students need to employ multiple rules and definitions while solving this problem and this seems to fit level 2 definition. Here, we understand that statements given in revised problem, which define the relationship between the expenses, increased the using symbols, operations and formal language competency of Ayşe's problem from level 1 to level 2. Similarly, for Ayşe's first sub-question of final problem, level 2 is proposed for this competency because, although Ayşe changed some of statements of revised problem, students still need to employ multiple rules or definitions for repeated calculations from level 1.For second sub-question of final question students need to make a prediction about the expenses in August based on their knowledge about previous expenses. While doing it, they need to consider the statements in problem text which requires employing multiple rules or definitions for repeated calculations from level 1. Therefore, level 2 is also appropriate for second sub-question of final problem.

Reasoning and argument is the last capability that we would analyze under this section. This competency includes drawing valid inferences after a mental process. More detailed information about the capability and the level definitions are given in

Chapter 3. The reasoning and argument competency of Ayşe's initial problem is proposed to be at level 0, since the problem requires only direct inferences from the information given in the text. No additional reasoning steps are required to calculate the expenses to find how many years a person could pay for the same monthly expenses. For revised problem, a small inference related to one aspect of the problem is made when deciding when more money would be paid based on the conditional statement. Therefore, level 1 is appropriate for reasoning and argument capability of Ayşe's revised problem. As it is stated earlier Ayşe decided to replace the conditional statement in revised problem with non-conditional statement while forming the final version so students do not have to draw inferences to solve the first sub-question of final problem. In other words, drawing direct inferences from the text is needed to solve first sub-question of final problem which leads to level 0 for reasoning and argument competency. Finally, the second sub-question of final problem which Ayşe added to final version of her problem requires students to draw inferences while predicting the expenses in the August. For making this inference students need to investigate telephone and internet expenses in July, which is a simple mathematical entity which covers only one aspect of the problem. Therefore, level 1 is appropriate for reasoning and argument capability of second sub-question of final problem.

4.3.2 Mathematical Content of ML Problems generated by Ayşe

Table 4.17 illustrates mathematical content categories of Ayşe's initial, revised and final problems.

Table 4.17 Mathematical Content Categories of Ayşe's ML Problems

Ayşe's Problems	Mathematical Contents
Initial Problem	Quantity
Revised Problem	Quantity
Final Problem	a. Quantity
	b. Uncertanity and Data

Ayşe's initial, revised problems and first item of final problem lie in quantity content category while second item of final problem lies in uncertainty and data content category. The reason why Ayşe's initial problem lies in quantity content is students are expected to find how much year a person could pay same bill by using some number operations. Similarly, students are required to find the maximum amount of money that could be paid for total expenses by using the numerical relationships between the expenses in revised problem and first problem of final problem.

Lastly, students are expected to predict amount of expenses in august by investigating the trends in previous months in second problem of final problem so the central concept in the question is interpreting data which implies the question should be categorized under uncertainty and data content category.

4.3.3 Real-life Context of ML Problems generated by Ayşe

This section includes three parts. First part covers the content categories of Ayşe's initial, revised and final mathematical literacy problems, how they have changed throughout the teaching experiment. The second part covers level of context use of Ayşe's initial, revised and final mathematical literacy problems, how they have changed throughout the teaching experiment. Third part covers authenticity level of Ayşe's initial, revised and final mathematical literacy problems, how they have changed throughout the teaching experiment.

4.3.3.1 Real-life Context Categories of ML problems generated by Ayşe

Table 4.18 illustrates context categories of Ayşe's initial, revised and final problems.

Table 4.18 Real-life Context Categories of Ayşe's ML Problems

Ayşe's Problems		Real-life Contexts
Initial Problem		Personal
Revised Problem		Personal
Final Problem	a.	Personal
	b.	Personal

All problems generated by Ayşe lie in personal context category because the problems are related to an individual who wants to plan her budget based on some calculations she made on monthly expenses.

4.3.3.2 The Level of Context Use in ML problems generated by Ayşe

The level of context use of Ayşe's initial and final problems are given in a tabular form in the Table 4.19.

Table 4.19 Level of Context Use of Ayşe's ML Problems

Ayşe's Problems		Level of Context Use
Initial Problem		Zero-order
Revised Problem		First-order
Final Problem	a.	First-order
	b.	First-order

As it seen in the Table 4.19, zero-order context use appears in Ayşe's initial problem since the context used in the problem, which is how many years it takes to pay a certain amount of money with monthly bills, is not relevant and needed for solving the problem. In other words, nothing about the specific features of expenses is needed to understand or solve the problem. Context is given to camouflage the arithmetic problem. In addition to that, the following excerpt which is taken from the second interview of Ayşe is also an example of the idea that zero level of context use appears in the problem.

> ***Ayşe:*** *In the discussions we had in the classroom, I realized that the level of my first problem remained at zero because the context doesn't help students solve the question. I can demonstrate this by changing the context in a comfortable way. Let there be a man who deposits different amounts of money in the bank each month. If he deposits a certain amount each month, in how many years can he save that money? Context has changed so here context is used as camouflage.*

In this description, Ayşe explained that she gained idea about the level of context use of her problem in classroom discussions. She claimed that her problem is zero order since students do not use the context elements while solving the problem and even if she changes the context of the problem, it does not affect the solution since mathematical operations are maintained.

As we stated earlier, Ayşe decided to maintain the context while she is revising her problem and she tried to increase the level of context use of the problem by enabling students to use the existing context. For this purpose, instead of giving prices of expenses for every week, she gave the highest and lowest prices of them and she gave some statements. As these statements are used for identifying variables and relationships between the expenses, students must use the context of the problem while solving it. Therefore, first level of context use appears in the revised problem. Moreover, following excerpt which is taken from Ayşe’s second interview supports this view.

> ***Ayşe:*** *In my second question, there are expressions that will help students while solving the question. I might not be able to replace them with anything else. These help students solve the question and make the context even more specific. So I think my question is level one.*

Final problem involves two sub-questions. Therefore, we need to analyse the level of context use category of each sub-question individually. According to first sub-question of the final problem, Ayşe wants to calculate the maximum value of expenses in summer months and for doing this, she benefits from the notes she took that express her claims. In this sub-question, students are expected to use the statements given to identify the variables and relationships between the expenses to find the highest value for expenses. Therefore, students must use specific features of the context while solving the problem. This indicates level 1 context use. Second sub-question of final problem is also an example of first order context use since in this problem students are required to predict the telephone and the internet expenses on August based on their range on July. This shows that students need to use the specific features of context to predict the desired value. Predicting a value may seem like as an example of second order context use but students are not required to use their specific real-world knowledge while doing it. Therefore, level 1 is given for second sub-question of final problem.

4.3.3.3 The Level Authenticity of ML Problems Generated by PSTs

Authenticity is the last criteria that we investigate in this study. Our purpose is to define the degree of suitability of mathematical literacy problems posed by prospective teachers to a real-life situation. As we stated earlier there are eight categories defined by Palm (2006) to evaluate authenticity of the mathematical tasks. A three-point scale which is taken from Wernet's study (2017) is used for evaluating each category. The overall authenticity score of the problems are decided according to median score of each authenticity of the problem.

Authenticity aspects of Ayşe's initial, revised and final problems, their scores, and rationales for scores are shown in the Table 4.20.

Table 4.20 Authenticity Levels of Ayşe's ML Problems

Aspect of Authenticity	Ayşe's Initial Problem	Ayşe's Revised Problem	Ayşe's Final Problem
Event	2	2	2
Question	1	2	2
Information/data	1	2	2
Presentation	2	2	2
Solution Strategies	1	1	1
Solution Requirements	0	1	2
Purpose	0	0	1

Event is the first category to evaluate the authenticity of mathematical tasks, which is also a characteristic for mathematical literacy problems. The event in the initial problem, which is calculating all expenses to plan your budget, is a realistic and a person has high chance to see or do it himself/herself in real life. Therefore, the event in the initial problem is considered as fully aligned with real-life. In addition to that, the following excerpt which is taken from Ayşe's second interview supports this view.

> ***Ayşe:*** *Yes, invoice is something that can be encountered in our daily life, or losing the bill and trying to guess how much it will be paid next month is something that can be experienced.*

Ayşe preferred to use a similar context while revising her problem. While revising the context of her problem, she only changed the duration for calculating the expenses. Thus, context is evolved to figuring out how much money is spent for expenses in summer. Similar to the initial problem, context of revised problem is fully aligned with corresponding real-world situation.

There are two sub-questions for final problem that Ayşe stated. Differently than other components of mathematical literacy problem, authenticity of sub-questions of a problem will be analyzed together to decide the authenticity level of a problem. The context of first sub-question of final problem is the same with revised problem so it is a possible real-world situation as we stated. For second sub-question, planning budget for next month is the context of the problem. This context is very similar to context of the initial problem. As we stated earlier, planning budget is a regular event for almost every family. Therefore, first and second problem's context is common occurrence for a person, and they are fully aligned with corresponding real-life situation. Table 4.20 indicates that Ayşe constructed a problem which has an authentic event and she maintained this property while she was revising her problem.

Question is the second aspect of authenticity. The question in Ayşe's initial problem is considered as partial alignment. The reason why is that a person may ask himself or herself for how many months his or her money can be enough to pay bill, but it is not clear for students. In the first revision process, Ayşe revised her initial problem from "how many years it takes to pay 78000 TL, paying the same bills each month" to "what can the maximum price a person paid for total expenses for three months be". The question of revised problem is fully aligned with a corresponding real-life situation because the question could be asked in real-life. Final version of Ayşe's problem involves two sub-questions. One of them is the same with the question of the initial problem so it is also considered as likely to be asked in real life. Moreover, second sub-question of final problem is also considered as likely to be asked in real-life since a person may ask herself to estimate the next month's total expenses to plan her budget. When the changes between the initial problem and the final problem are investigated, it can be observed that although the question in initial less likely to be asked in real-life, the questions in revised and final problem are very likely to be asked in real life. In addition to that the excerpt taken from Ayşe' s second interview supports this view.

> ***Ayşe:*** *In my last question, he guesses by himself, how much will I pay next month, I lost my bill from the previous month ... What is it called home economy? He is doing economy, calculating, asking how much he can raise his money next month? Actually, we always do this. I spent that much*

this month, how much could I have spent on what? No, I don't do it but people do. How much will I spend next month?

Third aspect of authenticity is information which refers to all information and data in the problem as we stated earlier. For information or data aspect, Ayşe's initial problem context is classified as partially aligned with real life context. The mathematical information which is given in the problem is the prices for bills. They are realistic values and the information given is accessible in real-life but a person cannot spend same prices for each bill every month. Therefore, the information given in the problem is partially authentic. For revised problem, the given limits for expenses in revised problem are acceptable in real life. Moreover, similar to the initial problem, the existing information given in the problem is also accessible in real-life and, it is specific enough to solve the problem. Therefore, the information and data aspect of the authenticity for revised problem is also fully aligned with corresponding real-life situation. Although there are slight changes in the numbers that shows the prices for bills, the information and data aspect of authenticity of the final problem is also fully aligned with corresponding real-life situation. There are two changes about the mathematical information given in the final problem compared to revised one. One of them is that Ayşe preferred to use decimal numbers while giving the prices of bills in the first sub-question. Although we usually receive an expense as a decimal number in real life, this does not increase the authenticity of the problem since both are acceptable. The other one is that Ayşe gave an additional mathematical information in the second sub-question which is the person in the question received her telephone and internet bill between 50 TL and 125 TL. This information is also having realistic values so we can claim that the information and data aspect of the problem is fully aligned with a corresponding real-life situation.

The following aspect of the authenticity of the context is the presentation. The language, tables, graphs and pictures of the problem are parts of presentation aspect in mathematical literacy problem. It tries to determine whether presentation of the problem stand in front of interpreting the problem or not. When we examine Ayşe's initial problem according to presentation aspect, it is observed that the language is clear, there is no information blocking the ways of understanding the question. Moreover, the amount of text is suitable for 8^{th} grade student and table is understandable. Therefore, the presentation aspect of the initial problem is fully aligned with real life situation, so it is coded with the value 2. Similarly, for revised and final problems, the presentation way of the problem does not block understanding the problem, the amount of text is suitable for students and the language is clear. Therefore, presentation aspect of revised and final problem is fully aligned with real life situation, so it is coded with the value 2.

Solution strategies is another aspect of the authenticity. It concerns with the alignment between the availability and plausibility of strategies in the problem or task and strategies in the real life. In order to decide the level of solution strategies aspect of initial problem of Ayşe, we need to investigate the solution strategies of the problem. In order to solve the problem, students need to calculate the total expenses in a month and divide total money the person have by this number. Although this strategy can be used in corresponding real-life situation, there are many other strategies to solve the problem. In other words, a person who would like to estimate how many years his money is enough for paying the bills can use many strategies to solve the problem. Therefore, the solution strategies aspect of initial problem is partially aligned with real-life situation and it is coded with the value 1. In the revised problem the person in the problem would like to find the maximum expenses he paid for summer months. He also took some notes about the bills and based on these notes he would like to solve his problem. In a corresponding real-life situation, this person can ask the institution he paid bills to for the exact values of bills. Like the initial problem, there are other ways to solve the problem in a corresponding real-life situation. Therefore, the solution strategy of revised problem is also partially aligned with a real-life situation. First sub-question of final problem is the same with the revised problem so there are other strategies to solve it. However, a person who face with second sub-question of final problem in a real-life need to estimate the next month's total expenses based on the knowledge he has. Therefore, although this person would reach much more information in real-life, this does not affect his solution strategies to solve the second sub-question. The strategy used in second sub-question of final problem is fully aligned with corresponding real-life situation. In summary, solution strategy aspect of authenticity of final problem is also proposed as partially aligned since the first sub-question could be solved in different ways in real-life, there is no other ways to solve the second sub-question differently in real-life.

The next aspect of the authenticity is solution requirements. This aspect draw attention on the requirements that individuals take into consideration while solving a real-life problem and claims that these assumptions should be considered acceptable for students who solve the task. In Ayşe's initial problem, paying exactly the same prices each month is one requirement given in the problem. This requirement is not consisting with real-life since it is almost impossible to spend exactly the same amount of water, gas and electricity for several months, let alone the inflation affects the prices yearly. For this reason, there is no alignment between the solution requirement of initial problem and requirements that a person should consider in a corresponding real-life situation. Also, there are several requirements that students need to take into consideration while solving the revised problem. They are that the electricity expense is lower than water expense for three months and that if this person pays more than 300 TL for two months for gas expense, in third mouth he would pay less than 300 TL for it. The first requirement is reasonable in real-life, but the second requirement is less likely to be considered in real-life. Therefore, solution requirement of revised problem is partially aligned with corresponding real-life situation. Lastly, there are also two requirements in the final problem that students need to consider while solving the problem. They are that water expense is lower than electricity expense for three months and that gas expense does not exceed 286,01 TL for summer months. Both requirements are reasonable and could be encountered in real-life, so the solution requirement of final problem is fully aligned with corresponding real-life situation. It could be observed that Ayşe increased the solution requirement competency of her problem from level 0 to level 2 by revising it.

The final aspect of the authenticity is purpose of the context. In a realistic situation, the purpose of solving the problem is clear for individuals, therefore the purpose of authentic mathematics problems should be clear to students. Students should have a reason to solve the problem. The purpose of solving Ayşe's initial problem is not clear to students since students do not know why they are calculating how many years a person can use his money, paying exactly the same amount of money each month for expenses, if this man has 78000 TL. No reason is given to students to solve the problem, so the purpose of initial problem is not clear and it is not aligned with real-life situation. Similarly, in revised problem, students do not know why a person wants to calculate the highest value he paid for expenses in last summer. It is not clear for students so there is no match between the purpose of revised problem and corresponding real-life situation. When we investigate the purpose of the final problem, since the first question remained the same, we could claim that the purpose of the first question is also not clear for students. However, the purpose of second sub-question is clear for students. In the second question, students are informed about Ayşe wants to make her budget for next month, so why they need to calculate the next month's expenses is clear for students. Therefore, the purpose of final problem is partially aligned with corresponding real-life situation.

Besides to what extend authenticity ML problems posed by PSTs which was shown above, it is important to investigate how they changed authenticity of ML problems to answer research question. Following except taken from second interview with Ayşe shows that how she decided to change information aspect of authenticity while revising her problem.

> ***Researcher:*** *How should a daily life question be? What elements do you think a question that can be encountered in daily life should include?*
>
> ***Tuğçe:*** *For example, values should be realistic. For example, the first numbers I wrote were without commas. In the discussions we made in the*

classroom, it was said that it cannot be encountered in daily life. That's right because there are pennies. Now I added the pennies, it became more realistic. I'm throwing that this problem may arise in our lives.

4.3.4 Summary of Ayşe's Findings

This section offers an overview of Ayşe's findings in order to evaluate her findings holistically. The following table presents how ML components (mathematical process, mathematical content and context) of Ayşe's problems changed or developed.

For the first category of mathematical process component (mathematical process categories), Table 4.21 represents that Ayşe's initial problem lies in employing, her revised problem lies in formulating and her final problem involves two sub-questions: one lies in formulating and the other one lies in employing process category. For second category of the component (fundamental mathematical capabilities), level of demand for representation and reasoning and argument capabilities increased from level 0 to level 1, for mathematising capability increased from level 0 to level 2, for communication capability increased from level 0 to level 3 and for using symbols and operation capability increased from level 1 to level 2. On the other hand, level of demand for devising strategies capability did not change.

For mathematical content categories component of ML problems, Table 4.21 shows that Ayşe's initial and revised problems lie in quantity category and her final problem involves two sub-questions: one lies in quantity, and the other one lies in uncertainty and data category.

Table 4.21 Overview of Developments in Ayşe's ML Problems

ML Components	Dimensions of Each Components		Initial Problem	Revised Problem	Final Problem a	Final Problem b
Mathematical Processes	Mathematical Process Cat.		Employing	Formulating	Formulating	Employing
	Fundamental Math. Capabilities	Communication	0	3	2	3
		Devising Strategies	2	2	2	2
		Mathematising	0	1	1	2
		Representation	0	0	0	1
		Using Symbols and Op.	1	2	2	2
		Reasoning and Argument	0	1	0	1
Content	Mathematical Content Cat.		Quantity	Quantity	Quantity	Uncert. & Data
Context	Real-life Context Cat.		Personal	Personal	Personal	Personal
	Level of Context Use		Zero-order	First-order	First-order	First-order
	Authenticity	Event	2	2	2	
		Question	1	2	2	
		Information/data	1	2	2	
		Presentation	2	2	2	
		Solution Strategies	1	1	1	
		Solution Requirements	0	1	2	
		Purpose	0	0	1	

Third component of ML problems (context) has three categories (real-life context categories, level of context use and authenticity). As it is seen in Table 4.21, all problems generated by Ayşe lie in personal context category. For the second category of context, Table 4.21 illustrates that the level of context use of Ayşe's initial problem is zero order, and her revised and final problems are first order. Lastly, for third context category, Table 4.21 shows how authenticity scores of Ayşe's problem changed. As it is seen from the table, Ayşe increased purpose category from level 0 to level 1, solution strategies category from level 0 to level 2, and question and information/data categories from level 1 to level 2. On the other hand, she did not change the authenticity scores of event and presentation categories.

CHAPTER 5

DISCUSSION, IMPLICATIONS AND RECCOMMENDATIONS

This study aims to investigate prospective middle school mathematics teachers' capability to generate mathematical literacy problems throughout a classroom teaching experiment. In the light of this aim, the findings of the study are discussed in this section. Furthermore, the recommendations for upcoming studies and educational implications are presented.

5.1 Discussion

This section covers the discussions related to the current study's findings, which are gathered under three main sections based on the research questions. Moreover, the first, second, and third sections include discussions related to mathematical processes, mathematical content categories, and real-life contexts of mathematical literacy problems generated by PSTs and how they changed or developed throughout the teaching experiment, respectively. Moreover, in this section, previous studies' findings are compared and contrasted with the findings of the present research.

5.1.1 Mathematical Processes of ML Problems Generated by PST

The first dimension of mathematical literacy is the mathematical processes and capabilities underlying these processes. This study's first overarching aim is to determine which mathematical processes are used in mathematical literacy problems and how they have differed throughout the teaching units. The analysis part of this study revealed that PSTs generated new questions whose mathematical processes were different from their first questions since they gained information about mathematical processes in mathematical literacy problems. For example, Tuğçe generated her initial problem lying mostly in employing process category at the beginning of the teaching experiment. Then, she revised her initial problem without changing its process category and added one more sub-question, which mostly lies in the formulating process. Therefore, at the end of the teaching units, she had a final problem with two sub-questions, one of which lies mostly in the employing process, and the other mainly lies in the formulating process. Moreover, Ayşe generated her initial problem in employing the category at the beginning of the study. Through the teaching experiment, she revised her problem and she generated her final problem with two sub-questions, one of which lies in the employing process and the other lies in the formulating process. In addition to that, Melike generated her initial problem in the formulating process. Then, she revised her initial problem without changing its process category and added one more sub-question which mostly lies in interpreting process. These findings demonstrate that prospective teachers diversified the mathematical processes of their mathematical literacy problems, and their newly generated problems cover more than one process at the end of teaching units. This finding can be considered as an intended outcome of this study. According to Blum and his colleagues (2007), students' engagement in extended tasks or problems which involve several connected mathematical processes is a necessity for teaching and assessment. Moreover, Turner (2007) pointed out that presentations of PISA problems involving multiple cycles between classroom processes enrich mathematics teaching. Thus, these studies show that the outcome of the teaching experiment is beneficial for PSTs. The reason why similar results are observed in the present study may be related to the fact that in the teaching units, teachers are presented with various types of PISA problems that involve formulating, employing and interpreting processes, and the importance of each process is discussed in the lessons. In other words, having experience with different types of problems involving various processes might be useful for PSTs to generate problems using multiple processes. This might be the reason why they diversify their problems' mathematical processes and construct their final problems involving multiple processes.

The second inclusive aim held in this study is to determine to what extent fundamental mathematical capabilities are demanded in mathematical literacy problems generated by PSTs and how these values differed throughout the teaching units. As PSTs gained more information about mathematical literacy problems and the mathematical competencies requested from students in PISA items, they increased mathematical competency demands in their problems. In other words, they constructed cognitively more complex mathematical literacy problems that require students to use multiple mathematical competencies in higher degrees. For example, Tuğçe increased the communication, representation, symbols and formalism, and reasoning and argument levels of her problem throughout the teaching experiment. Similarly, Melike increased the communication, mathematising, symbols and formalism, and reasoning and argument levels of her problem. Lastly, Ayşe increased communication, mathematising, representation, symbols and formalism, and reasoning and argument levels of her problem. These findings are consistent with Crespo's study (2003) in which preservice teachers posed cognitively more complex problems after attending serious seminars and receiving feedbacks related to problems they posed. Similarly, in the current study, preservice teachers posed cognitively more challenging mathematical literacy problems after they participated in several teaching units involving discussions and received feedback on their problems. The reason for this change in the present study may be that example problems shown in teaching units were non-traditional and often compelling to attract PSTs, which changed the PSTs' perception of mathematical literacy problems in the direction that such problems should be challenging. As evidence, the interview data revealed that PSTs believed that to pose higher-level mathematical literacy problems, they needed to increase the cognitive demand of problems.

5.1.2 Mathematical Content Categories of ML Problems Generated by PST

The second aspect of the findings is the mathematical content of mathematical literacy problems. The current study's overarching aim related to the mathematical contents of ML problems is to determine which mathematical content categories are used in mathematical literacy problems and how they have differed throughout the teaching units. The data analysis illustrated that the initial problems of Tuğçe and Ayşe lie in the quantity content category and the initial problem of Melike lies in the shapes and spaces content category. These findings are consistent with the findings reported by several research studies in the literature indicating that preservice teachers mostly prefer quantity and shapes and spaces content areas while posing mathematical literacy problems (Şahin & Başgül, 2018; Siswono et al., 2018). Similar to the studies conducted by Şahin and Başgül (2018) and Siswono and his colleagues (2018), two of the three teachers chose quantity and shapes and spaces contents in the present study. This might be because there are a higher number of objectives in the elementary and middle school mathematics curriculum in quantity and shapes and spaces content areas (MoNE, 2018).

As discussed in the previous parts of this study, all prospective teachers added an item to their mathematical literacy problems while revising the initial items. Melike generated her new item in space and shape content, Tuğçe generated hers in change and relationship content, and Ayşe generated hers in uncertainty and data content. It can be observed that the newly generated items are in various different content categories. The reason why these outcomes are observed may be related to the fact that teachers are presented with various types of problems that involve different types of content throughout the teaching units. This might have given prospective teachers an idea about how they can construct mathematical problems in different contents.

5.1.3 Real-life Context of ML Problems Generated by PST

The third aspect of the findings is real-life contexts of mathematical literacy problems. The first overarching aim of this study related to contexts of mathematical literacy problems is to determine which context categories are used in mathematical literacy problems and how they differ throughout the teaching units. The results of this study showed that all prospective teachers generated their initial problems in the personal context category. This is a consistent finding with those reported by many other research studies in the literature (Kohar et al., 2019; Şahin & Başgül, 2018). For example, the results of the study carried by Şahin and Başgül (2018) reported that most of the mathematical literacy problems constructed by pre-service teachers lie in the personal context. These findings are also consistent with the study of Kohar and his colleagues (2019) where the personal context category is one of the most used context categories among student teachers while they are posing PISA-like problems. Similar to what Kohar and his colleagues stated, the most preferred context at the beginning of the teaching experiment in the present study is personal context. The reason for this could be that it is easier to connect mathematics with personal context, like shopping, for teachers or students. As mentioned before, a study conducted by Suharta and Suarjana (2018) supports this claim by providing evidence that prospective teachers can more easily connect mathematics with the problems that arise in the personal context. This evidence can be connected to our study by claiming that both solving and posing mathematical problems require PSTs to connect real-life context with mathematics; therefore, if solving problems in a personal context is easier for PSTs, then posing problems in personal context must also be easier for them.

At the end of the teaching units, prospective teachers revised their problems by revising the initial items and adding one new item to their problems. Prospective teachers revised their initial items without changing their context category so Ayşe's, Melike's, and Tuğçe's revised items (first sub-question of final question) lie in the personal context category. For the second-sub question, Ayşe and Melike generated an item in scientific context, and Tuğçe generated an item in personal context. To put it another way, most prospective mathematics teachers diversify the context of their problem by generating sub-questions in several context categories at the end of the teaching units. This finding can be considered as an intended outcome of this study. As mentioned in the literature, using a variety of contexts in mathematical literacy problems increases the possibility of students connecting the problems with the situations in which they deal in the 21st century (OECD, 2013). In addition to that, connecting mathematics with real-life contexts that students operate in their lives increases students' motivation (Blum & Niss, 1989; Pierce and Stacey, 2006). Based on the related literature, prospective teachers' use of various contexts in ML problems they generated at the end of the current study might increase the chance for students to connect mathematics with many different situations and their motivation.

The reason for that prospective teachers' constructed problems include various context categories in the present study might be PSTs' increased experience with ML problems involving different types of contexts. In other words, prospective teachers were presented with various types of PISA problems involving different types of contexts and this might have given prospective teachers an idea about how they could construct mathematical problems in different contexts.

Another inclusive aim held in this study related to contexts of mathematical literacy problems is to investigate changes or developments in the level of context use of the problems generated by PSTs. The role of context is an essential feature for mathematical literacy problems, and it is supported to use first or second-level contexts in mathematical literacy problems (OECD, 2009). The analysis of the data revealed that while zero order of context use (camouflage context) appeared in the initial mathematical literacy problems of Ayşe and Tuğçe, second-order of context use appeared in the initial problem of Melike. Hence it can be said that most of the prospective teachers used context only to make the problem look like a real-life problem. These findings are parallel to that reported in a study of Kohar and his colleagues (2019), which is that zero-order context use appeared in mathematical problems generated by student teachers. Similar to Kohar and his colleagues' study, prospective teachers constructed their mathematical literacy problems with zero-order context use at the beginning of the present study. The reason for this might be that PSTs are more familiar with these problems since most of the contextual problems in textbooks use contexts only to camouflage mathematics inside (Wijaya et al., 2015). Another reason might be that PSTs do not have adequate knowledge about the level of context use of mathematics problems. Therefore, they do not understand which types of contexts are beneficial for solving problems, and which types of contexts they have a higher chance to face with in real-life situations.

At the end of the teaching units, data related to final problems of prospective teachers revealed that Tuğçe posed two items, one lying in zero-order context use and the other lying in first-order context use; Ayşe posed two items, both of them lying in first level context use, and Melike posed two items, one lying in first-order context use and the other lying in second-order context use. This analysis showed that almost all prospective teachers constructed their problems' contexts as suitable for the nature of mathematical literacy problems because, except one sub-question of Ayşe's problem, all problems involve either first or second-order context use. These findings might be considered as consistent with the findings of a study conducted by Siswono and his colleagues (2018), who ask teachers to pose PISA-like problems and revise them after they got feedback from the research team. The findings of this study revealed that the problems posed by teachers improved with the feedback they received, and less than 4% of mathematics problems posed by teachers were categorized under zero level context use at the end of the study (Siswono et al., 2018). Similar to Siswono and his colleagues' study (2018), participants received feedback from their peers related to the level of context use of their problems in the current study. Therefore, the increase in the level of context use of mathematics problems in the current study and Siswono and his colleagues' study (2018) might be due to the feedbacks received by the PSTs. Moreover, interview transcripts taken from Tuğçe and Ayşe's second interviews support this claim. In these interviews, they claimed that they realized via the feedbacks that the level of context use of their problem was zero-order, and then they decided to revise their problems.

The final inclusive aim of the study related to the contexts of mathematical literacy problems is to investigate the changes or developments in the authenticity of mathematical literacy problems generated by PSTs throughout the teaching experiment. Findings of the present study indicated that many authenticity categories of Ayşe and Tuğçe's mathematical literacy problems are coded as no alignment or partial alignment with a corresponding real-life situation, and almost all authenticity categories of Melike's problem are coded as full alignment. This shows that Ayşe and Tuğçe had problems with posing realistic mathematics problems. The findings of the study are consistent with previously mentioned research conducted by Paredes and his colleagues (2020), which revealed that pre-service teachers who participated in study had problems with constructing realistic mathematics tasks. Similar to what Paredes and his colleagues stated, prospective teachers in the current study had difficulty in constructing realistic mathematics problems. There might be several reasons for this difficulty experienced by PSTs in the present study. One of the reasons can be that PSTs are not familiar with realistic mathematics problems (Paredes et al., 2020). Another reason might be that PSTs do not have adequate knowledge about task authenticity since they did not receive any courses or training related to authenticity. In other words, when the courses they had taken during their undergraduate years are considered, it is seen that none of the courses covered mathematical task authenticity as a subject-matter. Therefore, PSTs lack the knowledge about task authenticity. However, data collected at the end of the teaching units showed that whereas authenticity scores of Melike's final problem did not change, some of the authenticity scores of Ayşe and Tuğçe's problems increased. For example, authenticity scores of event and solution strategies categories of Tuğçe's ML problem increased from partial alignment to full alignment, and authenticity score of information and data category of her problem increased from no alignment to partial alignment. Moreover, the authenticity score of the information and data category of Ayşe's ML problem increased from partial alignment to full alignment, and the authenticity score of the solution requirement category of her problem increased from no alignment to full alignment. Therefore, the authenticity scores of Tuğçe and Ayşe’s problems can be observed to increase throughout the teaching experiment. There might be several reasons for the development of the authenticity scores of Tuğçe and Ayşe’s problems. One of the reasons might be that PSTs' knowledge about authenticity increased throughout the teaching experiment since they received instruction about task authenticity, and they had seen plenty of authentic ML problems in instructional activities in the current study. This claim might be supported by the interview transcript taken from Ayşe’s second interview. In the interview, she claimed that she revised the numbers she used in her problem to make them more realistic because the elements of authenticity of her question were criticized in the lessons. Another reason of the development of the authenticity scores might be that PSTs benefited from feedbacks that their peers gave them related to the authenticity of the problems they generated. With the feedbacks, they realized which aspects of their problems were less authentic, and they developed ideas about how to improve them. The data gained from Tuğçe and Ayşe’s interviews support this claim. In the interviews, they claimed that the discussions in the classroom helped them realize the authenticity of their problem. As noted earlier, no development was observed in the authenticity scores of Melike’s ML problem. The reason for this might be that most of the authenticity scores of Melike’s ML problem were in full alignment; therefore, it was already highly authentic.

5.2 Implications for Educational Practices

In light of the findings, this study has several implications that should be taken into consideration by prospective teachers, in-service teachers, teacher educators, and textbook writers. Prospective teachers, as future teachers, should help students to develop their mathematical literacy. Therefore, it is crucial to equip teacher candidates by offering them suitable methods to increase the mathematical literacy of their students. The present study underlines the importance of increasing prospective teachers’ capability to pose mathematical literacy problems in a teaching experiment study. In the content of the study, PSTs were challenged to learn essential elements of mathematical literacy (mathematical processes, content, and context) and how to set mathematical literacy problems, while working collaboratively in an activity supported environment.

The findings of the study showed that while PSTs mostly use employing process, quantity content, and personal context categories to generate their initial ML problems, they used various process, content, and context categories while generating their final problems. In the previous section, it was discussed that the reason for the former situation might be their familiarity with problems including the employing process, quantity content, and personal context categories. It was also stated that increasing PSTs' experience with problems involving various types of processes, contents and contexts can be useful for them to generate different kinds of problems. For this reason, teacher trainers might integrate various types of ML problems into their mathematics education courses, similarly to this study. They might benefit from the context of teaching units in this study. Similarly, in order to increase PSTs' experience with various types of problems, textbook authors may give attention to using mathematical problems involving various processes, contents, and contexts.

Solving a problem requires students to engage with the context of the problems in different degrees (De Lange, 1987). Although the context may be given only to camouflage the mathematical operations (zero-order context use), it is preferred to give relevant contexts which are “needed for solving the problem or judging to answer” in ML problems (OECD, 2009, p.31). The findings of the study showed that zero-order context use usually appears in the initial problems generated by PSTs. In the previous chapter, it was discussed that this might be caused by the PSTs familiarity with the context-based mathematics problems which involve zero-order context use in textbooks. Therefore, textbook writers might consider giving place to problems with first or second-order context use instead of problems with zero-order context use, in order to help PSTs gain experience with such questions.

This study also aimed to increase PSTs' familiarity with using mathematical problems with first or second-order context use by illustrating them these types of problems. The result of the study showed that PSTs increased the level of context use of their ML problems at the end of the teaching units. Therefore, teacher trainers might benefit from the content of teaching units in the present study and offer PSTs a learning environment which emphasizes the importance of the level of context use in mathematics education courses, such as teaching methodology courses.

Authenticity is another important aspect of mathematical literacy examined in the current study. It is shown in literature that unrealistic answers of students to mathematical problems or tasks might result from not realistic or authentic tasks (Palm, 2008). Therefore, it is essential that mathematical problems or tasks used in the classroom is authentic. The findings of the current study showed that while PSTs mostly generate non-authentic tasks at the beginning of the teaching units, they increased the level of authenticity in their ML problems after the teaching units. Some applications held in the teaching experiment are shown as the reasons for this development. More specifically, increasing the PSTs' knowledge about the authenticity of mathematics problems, increasing PSTs' familiarity with authentic problems by illustrating them such examples, and creating an environment where PSTs discuss the authenticity levels of ML problems with their peers can be shown as the possible reasons for this development. Therefore, teacher trainers might benefit from the content of teaching units in the present study and offer PSTs a learning environment which emphasizes the importance of authenticity of mathematics problems in mathematics education courses.

5.3 Recommendations for Further Research Studies

This study investigated the contribution of a teaching experiment, which is on posing mathematical literacy problems, to PSTs’ capability to generate effective mathematical literacy problems. To begin with, the present study was limited to the findings that were received from three participants throughout the five-weeks-long teaching units and following interviews. For further research, a similar study could be conducted with more participants to explore whether the same improvements will be obtained for PSTs with different prior knowledge or not.

Besides, in the current study, the source of the feedbacks received by prospective teachers about their mathematical literacy problems were limited to their peers. In a further study, the problems prepared by the PSTs can be asked to middle school students and their feedback can be obtained on issues such as authenticity, the comprehensibility of the language, and the difficulty level.

Lastly, the participants of the current study were limited to prospective teachers. A similar study can be conducted with in-service teachers to compare in-service and prospective teachers to determine whether or not experience generates a difference between them in terms of their capacity to generate ML problems.

REFERENCES

Ahmad, D., Suherman, S., & Maulana, H. (2018). Teacher Mathematical Literacy: Case Study of Junior High School Teachers in Pasaman. *IOP Conference Series: Materials Science and Engineering*. https://doi.org/10.1088/1757-899X/335/1/012109

Aksu, G., Güzeller, C. O., & Eser, M. T. (2017). PISA 2012 sonuçlarının duyuşsal özelliklere göre kümeleme çalışması. *Hacettepe University Journal of Education*, *32*(4), 838–862. https://doi.org/10.16986/HUJE.2017026754

Altun, M., & Akkaya, R. (2014). Mathematics Teachers' Comments on PISA Math Questions and Our Country's Students' Low Achievement Levels. *Journal of Education*, *29*(1), 19–34.

Aydın, U., & Özgeldi, M. (2016). The PISA Tasks: Unveiling Prospective Elementary Mathematics Teachers ' Contextual , Conceptual , and Procedural Knowledge : *Scandinavian Journal of Educational Research*, *63*(1), 105–123.

Baran, A. B., & Işıksal Bostan, M. (2018). Views of Prospective Middle School Mathematics Teachers on Mathematical Literacy. In M. Peker, G. Ocak, B. Aydoğdu, E. Eğmir, F. Güngör, F. Özdinç, K. Kasapoğlu, & A. M. Uzun (Eds.), *International Congress on Science and Education 2018 (ICSE2018)* (p. 845). Anı Yayıncılık.

Bogdan, R., & Biklen, S. K. (2007). *Qualitative research for education : an introduction to theories and methods.* (5th ed.). Pearson A & B.

Bonotto, C. (2005). How informal out-of-school mathematics can help students make sense of formal in-school mathematics: The case of multiplying by decimal numbers. *Mathematical Thinking and Learning*, *7*(4), 313–344.

Bonotto, C. (2007). How to Replace Word Problems with Activities of Realistic Mathematical Modelling. In *Modelling and applications in mathematics education* (pp. 185–192). Springer.

Breakspear, S. (2012). *The policy impact of PISA: An exploration of the normative effects of international benchmarking in school system performance*. OECD Publishing.

Brown, B., & Schäfer, M. (2006). Teacher education for Mathematical Literacy: A modelling approach. *Pythagoras*, *0*(64), 45–51. https://doi.org/10.4102/pythagoras.v0i64.98

Çetin, S., & Gök, B. (2017). Öğrencilerin matematik okuryazarlık puanlarını etkileyen faktörlerin modellenmesi: PISA 2012 örneği. *Hacettepe University Journal of Education*, *32*(4), 982–998. https://doi.org/10.16986/HUJE.2016023162

Clement, J. (2000). Analysis of clinical interviews: Foundations and model viability. *Handbook of Research Design in Mathematics and Science Education*, 547–589.

Coben, D., Cotwell, D., Macrae, S., Boaler, J., Brown, M., & Rhodes, V. (2003). *Adult numeracy: review of research and related literature*. National Research and Development Centre for adult literacy and numeracy.

Cockcroft, W. H. (1982). *Mathematics Counts: Report of the Committee of Inquiry into the Teaching of Mathematics in Schools in England and Wales*.

Coxford, A., & Jones, P. (1970). A history of mathematics education in the United States and Canada. *Washington, DC: NCTM*.

Creswell, J. W. (2012). *Educational research : planning, conducting, and evaluating quantitative and qualitative research* (4th ed.). Pearson.

Crowther, G. (1959). *The Crowther Report*. A Report of the Central Advisory Council for Education (England). http://www.educationengland.org.uk/documents/crowther/

De Lange, J. (1987). *Mathematics - Insight and Meaning*. OW & OC, Utrecht.

De Lange, J. (2003). Mathematics for Literacy. *Quantitative Literacy: Why Numeracy Matters for Schools and Colleges*, 75–89.

Demir, F., & Altun, M. (2018). Development of mathematical literacy question writing process and skills. *Egitim ve Bilim*, *43*(194), 19–41. https://doi.org/10.15390/EB.2018.7111

Denzin, N. K. (2017). *The research act: A theoretical introduction to sociological methods*. Aldine Pub. Co.

Dewantara, A. H., Zulkardi, & Darmawijoyo. (2015). Assessing seventh graders' mathematical literacy in solving pisa-like tasks. *Indonesian Mathematical Society Journal on Mathematics Education*, *6*(2), 39–49. https://doi.org/10.22342/jme.6.2.2163.117-128

Educational Testing Service. (1995). *National adult literacy study*. Educational Testing Service (ETS).

Ellis, W. J. (2001). Numerical common sense for all. *Mathematics and Democracy: The Case for Quantitative Literacy*, 61–65.

Even, R., & Tirosh, D. (1995). Subject-matter knowledge and knowledge about students as sources of teacher presentations of the subject-matter. *Educational Studies in Mathematics*, *29*(1), 1–20. https://doi.org/10.1007/BF01273897

Forgasz, H. J., & Hall, J. (2019). Learning about numeracy: The impact of a compulsory unit on pre- service teachers' understandings and beliefs. *Australian Journal of Teacher Education*, *44*(2). https://doi.org/10.14221/ajte.2018v44n2.2

Fraenkel, J. R., Wallen, N. E., & Hyun, H. (2012). *How to design and evaluate research in education.* (8th ed.(Mc). McGraw-Hill Companies.

Freudenthal, H. (1983). *Didactical phenomenology of mathematical structures (p. 595)*. Springer.

Gatabi, A. R., Stacey, K., & Gooya, Z. (2012). Investigating grade nine textbook problems for characteristics related to mathematical literacy. *Mathematics Education Research Journal*, *24*(4), 403–421. https://doi.org/10.1007/s13394-012-0052-5

Glasersfeld, E. von. (1995). *Radical constructivism : a way of knowing and learning.* Falmer Press.

Hermawan, L. I., S Lestari, N. D., Rahmawati, A. F., & Suwano. (2019). Supporting students' reasoning and argumentation skills through mathematical literacy problem on relation and function topic. *IOP Conference Series: Earth and Environmental Science*, *243*(1), 1. https://doi.org/10.1088/1755-1315/243/1/012106

Höfer, T., & Beckmann, A. (2009). Supporting mathematical literacy: Examples from a cross-curricular project. *ZDM - International Journal on Mathematics Education*, *41*(1–2), 223–230. https://doi.org/10.1007/s11858-008-0117-9

Hope, M. (2007). Mathematical Literacy. *Principal Leadership*, *7*(5), 28–31.

İlbağı Azapağası, E., & Akgün, L. (2013). An investigation of the mathematical literacy of students aged 15 in terms of pisa 2003 mathematical literacy questions: results from turkey. *International Journal of Progressive Education, 9*(3), 194–217.

İş, Ç. (2003). *A cross-cultural comparison of factors affecting mathematical literacy of students in programme for international student assessment (PISA)*. METU Ankara.

Jablonka, E. (2003). Mathematical Literacy. In *Second International Handbook of Mathematics Education* (pp. 75–102). https://doi.org/10.1007/978-94-010-0273-8_4

Kabael, T., & Barak, B. (2016). Research of Middle School Pre-Service Mathematics Teachers' Mathematical Literacy on PISA Items. *Turkish Journal of Computer and Mathematics Education*, *7*(2), 321. https://doi.org/10.16949/turcomat.73360

Kaiser, G., & Willander, T. (2005). Development of mathematical literacy: Results of an empirical study. *Teaching Mathematics and Its Applications*, *24*(2–3), 48–60. https://doi.org/10.1093/teamat/hri016

Kilpatrick, J. (1965). Understanding Mathematical Literacy: The Contribution of Research. *Educational Studies in Mathematics*, *47*(1), 101–106.

https://doi.org/10.2307/3612329

Koğar, H. (2015). Examination of factors affecting PISA 2012 mathematical literacy through mediation model. *Egitim ve Bilim*, *40*(179), 45–55. https://doi.org/10.15390/EB.2015.4445

Kohar, A. W., Wardani, A. K., & Fachrudin, A. D. (2019). Profiling context-based mathematics tasks developed by novice PISA-like task designers. *Journal of Physics: Conference Series*, *1200*(1). https://doi.org/10.1088/1742-6596/1200/1/012014

Kramarski, B., & Mizrachi, N. (2006). Online discussion and self-regulated learning: effects of instructional methods on mathematical literacy. *The Journal of Educational Research*, *99*(4), 218–230.

Lincoln, Y. S., & Guba, E. G. (1985). *Naturalistic inquiry.* Sage Publications.

Madison, B. L., & Steen, L. A. (Eds.). (2003). *Quantitative literacy: Why numeracy matters for schools and colleges*. Woodrow Wilson National Fellowship Foundation.

McCrone, S. S., & Dossey, J. A. (2007). Mathematical Literacy—It's Become Fundamental. *Principal Leadership: High School Edition*, *7*(5), 32–37.

Meaney, T. (2007). Weighing up the influence of context on judgements of mathematical literacy. *International Journal of Science and Mathematics Education*, *5*(4), 681–704. https://doi.org/10.1007/s10763-007-9093-8

Merriam, S. B. (1998). *Qualitative research and case study applications in education.* (2nd ed.). Jossey-Bass Publishers.

Merriam, S. B. (2009). *Qualitative research : a guide to design and implementation.* Jossey-Bass Publishers.

Milton, M., Rohl, M., & House, H. (2007). Secondary Beginning Teacher's Preparedness to Teach Literacy and Numeracy: A Survey. *Australian Journal of Teacher Education*, *32*(2).

National Institute of Adult Continuing Education. (2011). *Numeracy Counts: NIACE committee of inquiry on adult numeracy learning final report*.

Niss, M. (2003). Mathematical Competencies and the Learning of Mathematics : the Danish Kom Project. *Proceedings of the 3rd Mediterranean Conference on Mathematical Education*, 115-124.

OECD. (2001). *Knowledge and Skills for Life [electronic resource]: First Results from PISA 2000*. OECD Publishing. https://doi.org/10.1787/9789264195905-en

OECD. (2009). *Take the Test. [electronic resource]: Sample Questions from OECD's PISA Assessments*. OECD Publishing. https://doi.org/10.1787/9789264050815-en

OECD. (2010). *Learning Mathematics for Life. [electronic resource]: A Perspective from PISA*. OECD Publishing. https://doi.org/10.1787/9789264075009-en

OECD. (2013). *PISA 2012 Assessment and Analytical Framework : Mathematics, Reading, Science, Problem Solving and Financial Literacy*. OECD Publishing. https://doi.org/10.1787/9789264190511-en

OECD. (2014). *PISA 2012 Results: What Students Know and Can Do (Volume I, Revised edition, February 2014): : Student Performance in Mathematics, Reading and Science*. OECD Publishing. https://doi.org/10.1787/9789264208780-en

OECD. (2019). *PISA 2018 Results (Volume I) : What Students Know and Can Do*. OECD Publishing. https://doi.org/10.1787/5f07c754-en

Ojose, B. (2011). Mathematics literacy : are we able to put the mathematics we learn into everyday use? *Journal of Mathematics Education*, *4*(1), 89–100.

Özberk, E. H., Atalay Kabasakal, K., & Boztunç Öztürk, N. (2017). Investigating the Factors Affecting Turkish Students' PISA 2012 Mathematics Achievement Using Hierarchical Linear Modeling. *Hacettepe University Journal of EducationJournal of Education*, *32*(3), 544–559. https://doi.org/10.16986/HUJE.2017026950

Ozgen, K. (2019). Problem-posing skills for mathematical literacy: The sample of teachers and pre-service teachers. *Eurasian Journal of Educational Research*, *2019*(84), 179–214. https://doi.org/10.14689/ejer.2019.84.9

Palm, T. (2006). Word problems as simulations of real-world situations : A proposed framework. *For the Learning of Mathematics - An International Journal of Mathematics Education*, *26*(1), 42–47.

Palm, T. (2008). Impact of authenticity on sense making in word problem solving. *Educational Studies in Mathematics*, *67*(1), 37–58. https://doi.org/10.1007/s10649-007-9083-3

Paredes, S., Cáceres, M. J., Diego-Mantecón, J. M., Blanco, T. F., & Chamoso, J. M. (2020). Creating realistic mathematics tasks involving authenticity, cognitive domains, and openness characteristics: A study with pre-service teachers. *Sustainability*, *12*(22), 1–17. https://doi.org/10.3390/su12229656

Patton, M. Q. (2002). *Qualitative research and evaluation methods.* (3rd ed.). Sage Publications.

Pillay, E., & Bansilal, S. (2019). Negotiating the Demands of Teaching Mathematical Literacy. *Africa Education Review*, *16*(3), 84–100. https://doi.org/10.1080/18146627.2017.1340807

Pugalee, D. K. (1999). Constructing a Model of Mathematical Literacy. *The Clearing House*, *73*(1), 19–22. https://doi.org/10.1080/00098659909599632

Research Advisory Committee of the National Council of Teachers of Mathematics. (1988). NCTM Curriculum and Evaluation Standards for School Mathematics: Responses from the Research Community. *Journal for Research in Mathematics Education*, *19*(4), 338–344. https://doi.org/10.2307/749544

Roth, W. M., Ercikan, K., Simon, M., & Fola, R. (2015). The assessment of mathematical literacy of linguistic minority students: Results of a multi-method investigation. *Journal of Mathematical Behavior*, *40*(Part A), 88–105. https://doi.org/10.1016/j.jmathb.2015.01.004

Şahin, Ö., & Başgül, M. (2018). Matematik Öğretmeni Adaylarının PISA Problemi Kurma Becerilerinin İncelenmesi. *Uluslararası Alan Eğitimi Dergisi*, *4*(2), 128–148. https://doi.org/10.32570/ijofe.483163

Sawatzki, C., & Sullivan, P. (2018). Shopping for Shoes: Teaching Students to Apply and Interpret Mathematics in the Real World. *International Journal of Science and Mathematics Education*, *16*(7), 1355–1373. https://doi.org/10.1007/s10763-017-9833-3

Sevinc, S., & Lesh, R. (2018). Training mathematics teachers for realistic math problems: A case of modeling-based teacher education courses. *ZDM - The International Journal on Mathematics Education*, *50*(1–2), 301–314. https://doi.org/10.1007/s11858-017-0898-9

Sfard, A. (2013). Why mathematics? What mathematics? *The Best Writing of Mathematics*, 130–142.

Shenton, A. K. (2004). Strategies for ensuring trustworthiness in qualitative research projects. *Education for Information*, *22*(2), 63–75. http://10.0.12.161/EFI-2004-22201

Siswono, T. Y. E., Kohar, A. W., Rosyidi, A. H., Hartono, S., & Masriyah. (2018). Searching for Authentic Context in Designing PISA-like Mathematics Problem: From Indoor to Outdoor Field Experience. *Journal of Physics: Conference Series*, *953*(1). https://doi.org/10.1088/1742-6596/953/1/012197

South Africa Department of Education. (2003). *National curriculum statement, Grades 10-12 (General): Mathematical Literacy*. Pretoria: Department of Education.

Stacey, K., & Turner, R. (2015). Assessing mathematical literacy: The PISA experience. *Assessing Mathematical Literacy: The PISA Experience*, 1–321. https://doi.org/10.1007/978-3-319-10121-7

Steeds, A. (Ed.). (2001). *Adult Numeracy Core Curriculum*. Cambridge Training and Development on behalf of The Basic Skills Agency.

Steen, L. A. (Ed.). (2001). *Mathematics and Democracy: : The Case for Quantitative Literacy*. NCED.

Steen, L. A., Turner, R., & Burkhardt, H. (2007). Developing Mathematical Literacy. In W. Blum, P. L. Galbraith, H.-W. Henn, & M. Niss (Eds.), *Modelling and Applications in Mathematics Education: The 14th ICMI Study* (pp. 285–294). Springer US. https://doi.org/10.1007/978-0-387-29822-1_30

Steffe, L. P., & Thompson, P. W. (2000). Teaching experiment methodology: Underlying principles and essential elements. *Research Design in Mathematics and Science Education*, *February*, 267–307.

Suharta, I. G. P., & Suarjana, I. M. (2018). A case study on mathematical literacy of prospective elementary school teachers. *International Journal of Instruction*, *11*(2), 413–424. https://doi.org/10.12973/iji.2018.11228a

Sumirattana, S., Makanong, A., & Thipkong, S. (2017). Using realistic mathematics education and the DAPIC problem-solving process to enhance secondary school students' mathematical literacy. *Kasetsart Journal of Social Sciences*, *38*(3), 307–315. https://doi.org/10.1016/j.kjss.2016.06.001

T.C. Milli Eğitim Bakanlığı [Ministry of Education] (MoNE). (2005). *PISA 2003 projesi ulusal nihai rapor*.

T.C. Milli Eğitim Bakanlığı [Ministry of Education] (MoNE). (2018). *Matematik dersi öğretim programı (ilkokul ve ortaokul 1, 2, 3, 4, 5, 6, 7, 8. sınıflar) [Middle school mathematics curriculum grades 1 to 8]*.

T.C. Milli Eğitim Bakanlığı [Ministry of Education] (MoNE). (2019). *PISA 2018 Türkiye ön raporu*.

The British Department for Education and Employment. (1998). *The Implementation of the National Numeracy Strategy: The Final Report of the Numeracy Task Force*. DfEE.

Tout, D. (2001). What is numeracy? What is mathematics? In G. E. FitzSimons, J. O'Donoghue, & D. Coben (Eds.), *Adult and life-long education in mathematics: Papers from Working Group for Action 6, 9th International Congress on Mathematical Education, ICME 9* (pp. 31–36). Language Australia in association with Adults Learning Mathematics – A Research Forum (ALM).

Treffers, A. (1991). Meeting innumeracy at primary school. *Educational Studies in Mathematics*, *22*(4), 333–352.

Turner, R. (2007). Modelling and applications in PISA. In *Modelling and applications in mathematics education* (pp. 433–440). Springer.

Verschaffel, L. (2009). *Words and worlds: Modelling verbal descriptions of situations*. Sense Publications.

Verschaffel, L., De Corte, E., & Borghart, I. (1997). Pre-service teachers' conceptions and beliefs about the role of real-world knowledge in mathematical modelling of school word problems. *Learning and Instruction*, *7*(4), 339–359.

Verschaffel, L., De Corte, E., & Lasure, S. (1994). Realistic considerations in mathematical modeling of school arithmetic word problems. *Learning and Instruction*, *4*(4), 273–294.

Vilakazi, A. S. (2010). *An Exploration of Mathematical Literacy Teachers' Perceptions of, and Performance in Mathematical Literacy Tasks Based on Algebra*.

Wernet, J. L. W. (2017). Classroom Interactions Around Problem Contexts and Task Authenticity in Middle School Mathematics. *Mathematical Thinking and Learning*, *19*(2), 69–94. https://doi.org/10.1080/10986065.2017.1295419

Wijaya, A., van den Heuvel-Panhuizen, M., & Doorman, M. (2015). Teachers' teaching practices and beliefs regarding context-based tasks and their relation with students' difficulties in solving these tasks. *Mathematics Education Research Journal*, *27*(4), 637–662.

Withnall, A. (1995). Towards a definition of numeracy. *Adults Learning Maths—a Research Forum ALM-1: Proceedings of the Inaugural Conference of Adults Learning Maths—a Research Forum*, 11–17.

Yıldırım, Y., Şahin, M. G., & Sezer, E. (2017). Pisa 2012 türkiye örnekleminde okul özelliklerinin matematik okuryazarlığına etkisi. *Elementary Education Online*, *16*(3), 1092–1100. https://doi.org/10.17051/ilkonline.2017.330244

Yılmazer, G., & Masal, M. (2014). The Relationship between Secondary School Students' Arithmetic Performance and their Mathematical Literacy. *Procedia - Social and Behavioral Sciences*, *152*, 619–623. https://doi.org/10.1016/j.sbspro.2014.09.253

APPENDICES

A. Courses in the Elementary Mathematics Education Program

1st YEAR	MATH111	Fundamentals of Mathematics	MATH112	Discrete Mathematics
	MATH115	Analytic Geometry	MATH116	Basic Algebraic Structures
	MATH119	Calculus with Analytic Geometry	MATH120	Calculus for Functions of Several Variables
	EDS200	Introduction to Education	CEIT100	Computer Applications in Education
	ENG101	English for Academic Purposes I	ENG102	English for Academic Purposes II
	IS100	Introduction to Information Technologies and Applications		
2nd YEAR	PHYS181	Basic Physics I	PHYS182	Basic Physics II
	MATH219	Introduction to Differential Equations	MATH201	Elementary Geometry
	STAT201	Introduction to Probability &Stat. I	STAT202	Introduction to Probability & Stat.II
	ELE221	Instructional Principles and Methods	ELE225	Measurement and Assessment
	EDS220	Educational Psychology	ENG211	Academic Oral Presentation Skills

	HIST2201	Principles of Kemal Atatürk I	HIST2202	Principles of Kemal Atatürk II
	HIST2205	History of The Turkish Revolution I	HIST2206	History of The Turkish Revolution II
	MATH260	Basic Linear Algebra	ELE310	Community Service
3rd YEAR	ELE341	Methods of Teaching Mathematics I	ELE329	Instructional Technology And Material Development
	TURK201	Elementary Turkish	ELE342	Methods of Teaching Mathematics
	TURK305	Oral Communication	EDS304	Classroom Management
		Elective	TURK202	Intermediate Turkish
		Elective	TURK306	Written Expression
4th YEAR	ELE301	Research Methods	ELE420	Practice Teaching in Elementary Education
	ELE435	School Experience	EDS416	Turkish Educational System and School Management
	ELE465	Nature of Mathematical Knowledge for Teaching	EDS424	Guidance
		Restricted Elective		Elective
		Elective		

B. Instructional Tasks

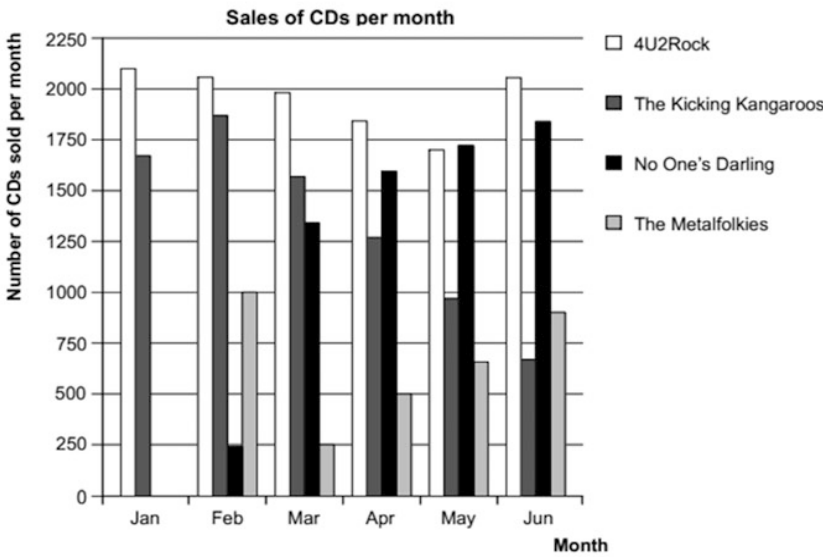
B1. Instructional Task 1

PART A:

Example of Zero-order Context Use Mathematical Literacy Problem

CHARTS

In January, the new CDs of the bands *4U2Rock* and The Kicking Kangaroos were released. In February, the CDs of the bands *No One's Darling* and *The Metalfolkies* followed. The following graph shows the sales of the bands' CDs from January to June.



Question 1:

In which month did the band No One's Darling sell more CDs than the band The Kicking Kangaroos for the first time?

A No month

B March

C April

D May

Question 2:

The manager of The Kicking Kangaroos is worried because the number of their CDs that sold decreased from February to June.

What is the estimate of their sales volume for July if the same negative trend continues?

A 70 CDs

B 370 CDs

C 670 CDs

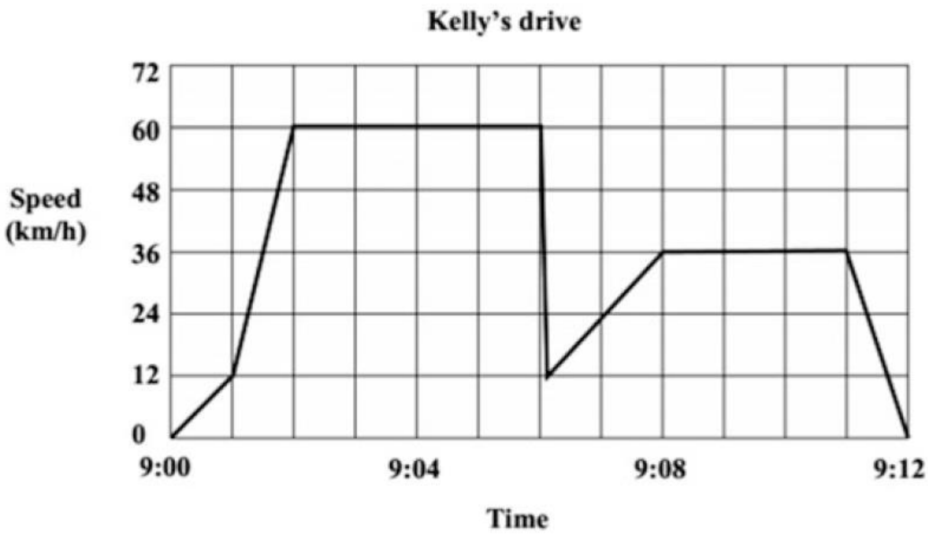
D 1340 CDs

Example of First-order Context Use Mathematical Literacy Problem:

CAR DRIVE

Kelly went for a drive her car. During the drive, a cat ran in front of the car. Kelly slammed on the brakes and missed the cat. Slightly shaken, Kelly decided to return home.

The graph below is simplified record of the car's speed during the drive.



What time was it when Kelly slammed on the brakes to avoid the cat?

Example of First-order Context Use Mathematical Literacy Problem:

ROCK CONCERT

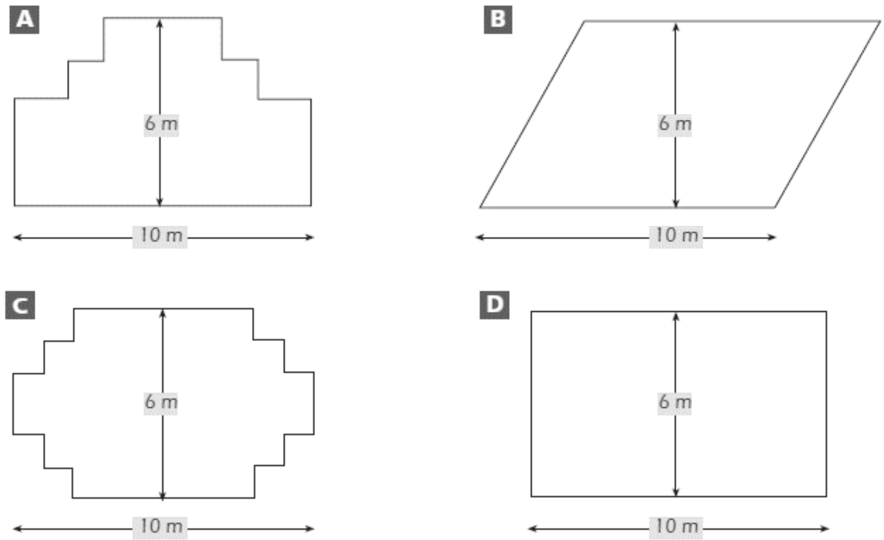
For a rock concert a rectangular field of size 100 m by 50 m was reserved for the audience. The concert was completely sold out and the field was full with all the fans standing.

Which one of the following is likely to be the best estimate of the total number of people attending the concert?

PART B :

CARPENDER (OECD, 2009)

A carpenter has 32 metres of timber and wants to make a border around a garden bed. He is considering the following designs for the garden bed.



Circle either "Yes" or "No" for each design to indicate whether the garden bed can be made with 32 metres of timber.

Garden bed design	Using this design, can the garden bed be made with 32 metres of timber?
Design A	Yes / No
Design B	Yes / No
Design C	Yes / No
Design D	Yes / No

	Question 1
Mathematical Process	
Mathematical Content	
Context	
Level of Context Use	

B2. Instructional Task 2

PART A:

DRIP RATE (OECD, 2009)

Infusions (or intravenous drips) are used to deliver fluids and drugs to patients.

Nurses need to calculate the drip rate, D, in drops per minute for infusions.

They use the formula $D = \frac{dv}{60n}$ where

d is the drop factor measured in drops per millilitre (mL)

v is the volume in mL of the infusion

n is the number of hours the infusion is required to run.

Question 1:

A nurse wants to double the time an infusion runs for.

Describe precisely how D changes if n is doubled but d and v do not change.

Question 2:

Nurses also need to calculate the volume of the infusion, v, from the drip rate, D.

An infusion with a drip rate of 50 drops per minute has to be given to a patient for 3 hours. For this infusion the drop factor is 25 drops per millilitre.

What is the volume in mL of the infusion?

1. **Determine mathematical processes, mathematical contents, contexts and level of context use of each question given above.**

	Question 1	**Question 2**
Mathematical Process		
Mathematical Content		
Context		
Level of Context Use		

2. **Determine whether the problem given above is authentic or not in terms of the criteria given below. Explain your reasoning**

Criteria for Authentic Problems	
Event: Can the event be encountered in daily life?	
Question: Can the question asked in the question come across in daily life?	
Information/data: Are the numbers or information provided compatible with real life?	
Presentation: Does it contain words that would not be encountered in real life? Is the language of the question clear and appropriate for the 8th grade student?	
Solution Strategies: Are all solution strategies available in real-life also available in the task?	
Solution Requirements: Are the determined requirements to solve the problem also necessary in a daily life situation?	
Purpose: Is the purpose of the question clear to the student?	

HEARTBEAT (OECD, 2009)

For health reasons people should limit their efforts, for instance during sports, in order not to exceed a certain heartbeat frequency.

For years the relationship between a person's recommended maximum heart rate and the person's age was described by the following formula:

Recommended maximum heart rate = 220 - age

Recent research showed that this formula should be modified slightly. The new formula is as follows:

Recommended maximum heart rate = 208 – (0.7 × age)

Question:

The formula recommended maximum heart rate = 208 – (0.7 × age) is also used to determine when physical training is most effective. Research has shown that physical training is most effective when the heartbeat is at 80% of the recommended maximum heart rate.

Write down a formula for calculating the heart rate for most effective physical training, expressed in terms of age.

1. **Determine mathematical processes, mathematical contents, contexts and level of context use of each question given above.**

	Question 1
Mathematical Process	
Mathematical Content	
Context	
Level of Context Use	

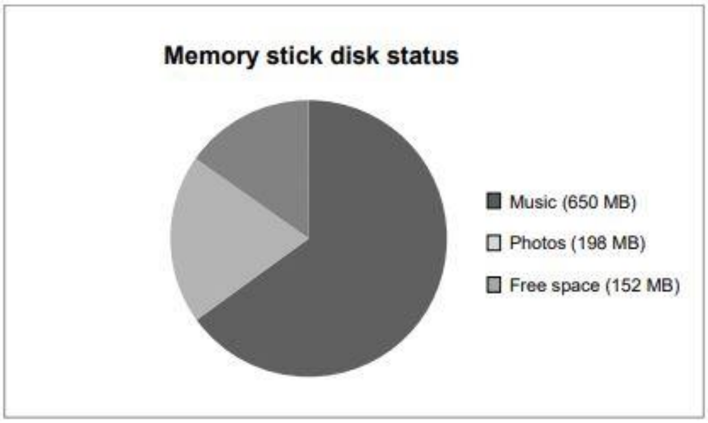
2. **Determine whether the problem given above is authentic or not in terms of the criteria given below. Explain your reasoning**

Criteria for Authentic Problems	
Event: Can the event be encountered in daily life?	
Question: Can the question asked in the question come across in daily life?	
Information/data: Are the numbers or information provided compatible with real life?	
Presentation: Does it contain words that would not be encountered in real life? Is the language of the question clear and appropriate for the 8th grade student?	
Solution Strategies: Are all solution strategies available in real-life also available in the task?	
Solution Requirements: Are the determined requirements to solve the problem also necessary in a daily life situation?	
Purpose: Is the purpose of the question clear to the student?	

MEMORY STICK (OECD, 2009)

A memory stick is a small, portable computer storage device.

Ivan has a memory stick that stores music and photos. The memory stick has a capacity of 1 GB (1000 MB). The graph below shows the current disk status of his memory stick.



Translation Note: Please translate "memory stick" with the commonly used term in your language, for example, "USB key".

Question 1:

Ivan wants to transfer a photo album of 350 MB onto his memory stick, but there is not enough free space on the memory stick. While he does not want to delete any existing photos, he is happy to delete up to two music albums.

Ivan's memory stick has the following size music albums stored on it.

Album	Size
Album 1	100 MB
Album 2	75 MB
Album 3	80 MB
Album 4	55 MB
Album 5	60 MB
Album 6	80 MB
Album 7	75 MB
Album 8	125 MB

By deleting at most two music albums is it possible for Ivan to have enough space on his memory stick to add the photo album? Circle "Yes" or "No" and show calculations to support your answer.

Answer: Yes / No

....

....

....

1. **Determine mathematical processes, mathematical contents, contexts and level of context use of each question given above.**

Mathematical Process	
Mathematical Content	
Context	
Level of Context Use	

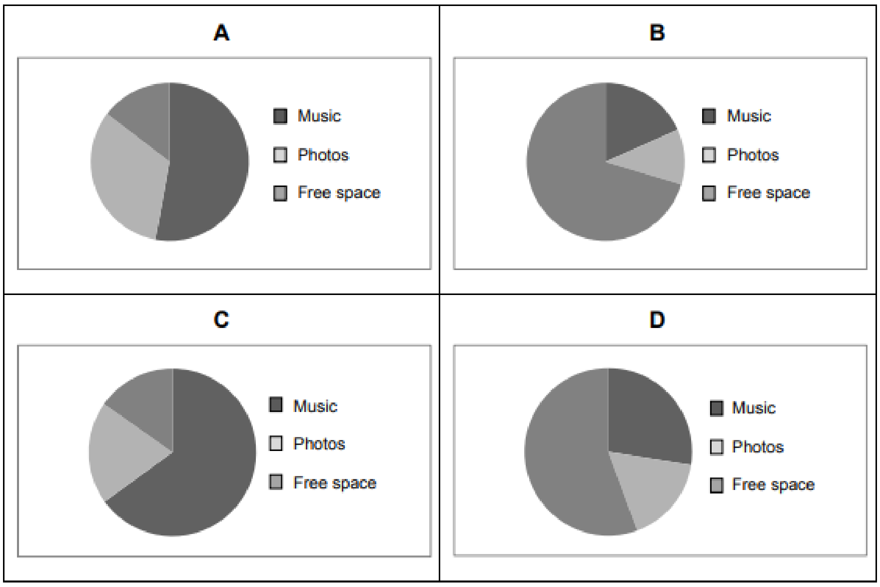
Question 2:

During the following weeks, Ivan deletes some photos and music, but also adds new files of photos and music. The new disk status is shown in the table below:

Music	550 MB
Photos	338 MB
Free space	112 MB

His brother gives him a new memory stick with a capacity of 2GB (2000 MB), which is totally empty. Ivan transfers the content of his old memory stick onto the new one.

Which one of the following graphs represents the new memory stick's disk status? Circle A, B, C or D.



1. **Determine mathematical processes, mathematical contents, contexts and level of context use of each question given above.**

Mathematical Process	
Mathematical Content	
Context	
Level of Context Use	

2. **Determine whether the problem given above is authentic or not in terms of the criteria given below. Explain your reasoning**

Criteria for Authentic Problems	
Event: Can the event be encountered in daily life?	
Question: Can the question asked in the question come across in daily life?	
Information/data: Are the numbers or information provided compatible with real life?	
Presentation: Does it contain words that would not be encountered in real life? Is the language of the question clear and appropriate for the 8th grade student?	
Solution Strategies: Are all solution strategies available in real-life also available in the task?	
Solution Requirements: Are the determined requirements to solve the problem also necessary in a daily life situation?	
Purpose: Is the purpose of the question clear to the student?	

PART B:

Revise the problems below to be more authentic according to the criteria of authenticity.

1. **Birthday Problem**

You have a birthday party, and if you count yourself you are 4 children at the party. You receive a bag with 18 balloons from your grandfather. For no one to be sad he wants you to share them equally so that each of you get the same number of balloons. How many balloons shall you give to each child?

Revised Version

2. **Ride Question**

You are going to a camp for 4 days, but you also want to ride horses. Your dad sees in the camp brouche that you have 45 minutes free time each day, and that horses can be rented for tours on path in the woods that takes 10 minutes. To know how much money you will bring you must calculate how many tours you have time to ride. How many 10-minute tours do you have time to do during these days?

Revised Version

B3. Instructional Task 3

CLIMBING MOUNT FUJI (OECD, 2009)

Mount Fuji is a famous dormant volcano in Japan.

Translation Note: Please do not change the names of locations or people in this unit: retain "Mount Fuji", "Gotemba" and "Toshi".

Question 1:

Mount Fuji is only open to the public for climbing from 1 July to 27 August each year. About 200000 people climb Mount Fuji during this time.

On average, about how many people climb Mount Fuji each day?

A 340

B 710

C 3400

D 7100

E 7400

Question 2:

The Gotemba walking trail up Mount Fuji is about 9 kilometres (km) long. Walkers need to return from the 18 km walk by 8 pm.

Toshi estimates that he can walk up the mountain at 1.5 kilometres per hour on average, and down at twice that speed. These speeds take into account meal breaks and rest times.

Using Toshi's estimated speeds, what is the latest time he can begin his walk so that he can return by 8 pm?

BOOKSHELVES (OECD, 2009)

To complete one set of bookshelves a carpenter needs the following components:

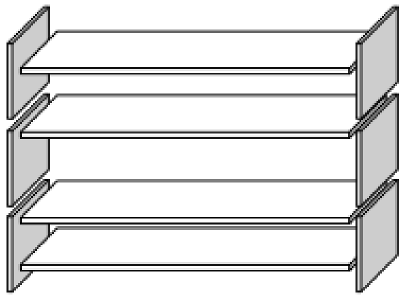
4 long wooden panels,

6 short wooden panels.

12 small clips,

2 large clips and

14 screws

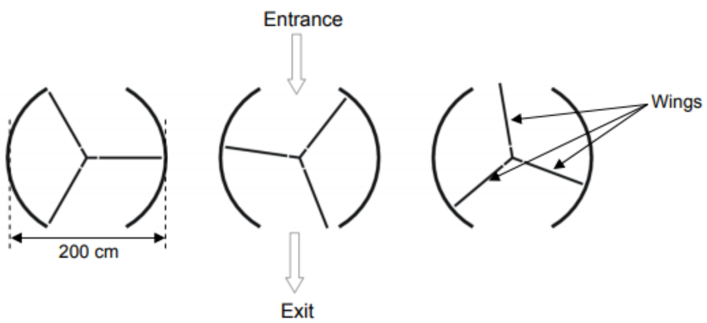


The carpenter has in stock 26 long wooden panels, 33 short wooden panels, 200 small clips, 20 large clips and 510 screws.

How many sets of bookshelves can the carpenter make?

REVOLVING DOOR (OECD, 2009)

A revolving door includes three wings which rotate within a circular-shaped space. The inside diameter of this space is 2 metres (200 centimetres). The three door wings divide the space into three equal sectors. The plan below shows the door wings in three different positions viewed from the top.



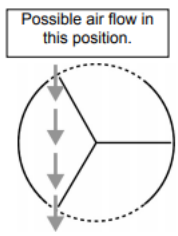
Question 1:

What is the size in degrees of the angle formed by two door wings?

Size of the angle: .... ^{o}

Question 2:

The two door openings (the dotted arcs in the diagram) are the same size. If these openings are too wide the revolving wings cannot provide a sealed space and air could then flow freely between the entrance and the exit, causing unwanted heat loss or gain. This is shown in the diagram opposite.



What is the maximum arc length in centimetres (cm) that each door opening can have, so that air never flows freely between the entrance and the exit?

	CLIMBING MOUNT FUJI		BOOKSHELVES	REVOLVING DOOR	
	Q1	Q2		Q1	Q2
Mathematical Process					
Mathematical Content					
Context					
Level of Context Use					
Fundamental Mathematical Capabilities					
Communication					
Devising Strategies					
Matematising					
Representation					
Using symbols, operations and formal language					
Reasoning and Argument					
Using Mathematical Tool					
Authenticity					
Event					
Question					
Information/data					
Presentation					
Solution Strategies					
Solution Requirements					
Purpose					

B4. Instructional Task 4 (Peer Evaluation)

• Is the mathematics correct?

• Does the content sit well with the PISA Framework?

Content:

Context:

Process:

•What is the level of context use?

• Is the context authentic?

Criteria for Authentic Problems	
Event: Can the event be encountered in daily life?	
Question: Can the question asked in the question come across in daily life?	
Information/data: Are the numbers or information provided compatible with real life?	
Presentation: Does it contain words that would not be encountered in real life? Is the language of the question clear and appropriate for the 8th grade student?	
Solution Strategies: Are all solution strategies available in real-life also available in the task?	
Solution Requirements: Are the determined requirements to solve the problem also necessary in a daily life situation?	
Purpose: Is the purpose of the question clear to the student?	

C. Interview Protocol of Interviews

C1. Interview Protocol of First Interview

1. Can you explain the question you wrote?
2. What did you pay attention to when generating the problem?
3. What process did you follow while generating the question?

C2. Interview Protocol of Second Interview

1. What does mathematical literacy mean to you?
2. What are the elements that a mathematical literacy question must have according to you?
3. What did you pay attention to while revising your question?
4. Let's consider the question in terms of mathematical processes. What mathematical processes do your problems include?
5. Could you compare your problems in terms of mathematical processes? What changes did you make?
6. Let's consider the question in terms of the context it involves. Which type of context do your problems include?
7. Could you compare your problems in terms of type of contexts? What changes did you make?
8. Let's consider the question in terms of authenticity. Do you think the problems you pose are authentic, why?
9. Could you compare your problems in terms of authenticity? What changes did you make?
10. Let's consider the question in terms of level of context use. To what extent do students use context in your problems?
11. Could you compare your problems in terms of level of context use? What changes did you make?

12. Let’s consider the question in terms of mathematical content. What mathematical contents do your problems include?
13. Could you compare your problems in terms of mathematical content? What changes did you make?
14. What kind of processes do you follow if you write a mathematical literacy question now?

D. **Coding of Fundamental Mathematical Capabilities** (Stacey & Turner, 2015, pp. 110–114)

	Communication Capability
0:	Understand short sentences or phrases relating to concepts that give immediate access to the context, where all information is directly relevant to the task, and where the order of information matches the steps of thought required to understand what the task requests. Constructive communication involves only presentation of a single word or numeric result
1:	Identify and link relevant elements of the information provided in the text and other related representation/s, where the material presented is more complex or extensive than short sentences and phrases or where some extraneous information may be present. Any constructive communication required is simple, for example it may involve writing a short statement or calculation, or expressing an interval or a range of values
2:	Identify and select elements to be linked, where repeated cycling within the material presented is needed to understand the task; or understand multiple elements of the context or task or their links. Any constructive communication involves providing a brief description or explanation, or presenting a sequence of calculation steps
3:	Identify, select and understand multiple context or task elements and links between them, involving logically complex relations (such as conditional or nested statements). Any constructive communication would involve presenting argumentation that links multiple elements of the problem or solution

	Devising Strategies Capability
0:	Take direct actions, where the solution process needed is explicitly stated or obvious
1:	Find a straight-forward strategy (usually of a single stage) to combine or use the given information
2:	Devise a straight-forward multi-stage strategy, for example involving a linear sequence of stages, or repeatedly use an identified strategy that requires targeted and controlled processing
3:	Devise a complex multi-stage strategy, for example that involves bringing together multiple sub-goals or where using the strategy involves substantial monitoring and control of the solution process; or evaluate or compare strategies

Mathematising Capability	
0:	Either the situation is purely intra-mathematical, or the relationship between the extra-mathematical situation and the model is not relevant to solving the problem
1:	Construct a model where the required assumptions, variables, relationships and constraints are given; or draw conclusions about the situation directly from a given model or from the mathematical results
2:	Construct a model where the required assumptions, variables, relationships and constraints can be readily identified; or modify a given model to satisfy changed conditions; or interpret a model or mathematical results where consideration of the problem situation is essential
3:	Construct a model in a situation where the assumptions, variables, relationships and constraints need to be defined; or validate or evaluate models in relation to the problem situation; or link or compare different models

Representation Capability	
0:	Either no representation is involved; or read isolated values from a simple representation, for example from a coordinate system, table or bar chart; or plot such values; or read isolated numeric values directly from text
1:	Use a given simple and standard representation to interpret relationships or trends, for example extract data from a table to compare values, or interpret changes over time shown in a graph; or read or plot isolated values within a complex representation; or construct a simple representation
2:	Understand and use a complex representation, or construct such a representation where some of the required structure is provided; or translate between and use different simple representations of a mathematical entity, including modifying a representation
3:	Understand, use, link or translate between multiple complex representations of mathematical entities; or compare or evaluate representations; or devise a representation that captures a complex mathematical entity

	Using Symbols and Formal Language Capability
0:	State and use elementary mathematical facts and definitions; or carry out short arithmetic calculations involving only easily tractable numbers. For example, find the area of a rectangle given the side lengths, or write down the formula for the area of a rectangle
1:	Make direct use of a simple mathematical relationship involving variables (for example, substitute into a linear relationship); use arithmetic calculations involving fractions and decimals; use repeated or sustained calculations from level 0; make use of a mathematical definition, fact, or convention, for example use knowledge of the angle sum of a triangle to find a missing angle
2:	Use and manipulate expressions involving variables and having multiple components (for example, by algebraically rearranging a formula); employ multiple rules, definitions, results, conventions, procedures or formulae together; use repeated or sustained calculations from level 1
3:	Apply multi-step formal mathematical procedures combining a variety of rules, facts, definitions and techniques; work flexibly with complex relationships involving variables, for example use insight to decide which form of algebraic expression would be better for a particular purpose

	Reasoning and Argument Capability
0:	Draw direct inferences from the information and instructions given
1:	Draw inferences from reasoning steps within one aspect of the problem that involves simple mathematical entities
2:	Draw inferences by joining pieces of information from separate aspects of the problem or concerning complex entities within the problem; or make a chain of inferences to follow or create a multi-step argument
3:	Use or create linked chains of inferences; or check or justify complex inferences; or synthesize and evaluate conclusions and inferences, drawing on and combining multiple elements of complex information, in a sustained and directed way

E. Permission From the Ethical Comitte at METU

UYGULAMALI ETİK ARAŞTIRMA MERKEZİ
APPLIED ETHICS RESEARCH CENTER

ORTA DOĞU TEKNİK ÜNİVERSİTESİ
MIDDLE EAST TECHNICAL UNIVERSITY

DUMLUPINAR BULVARI 06800
ÇANKAYA ANKARA/TURKEY
T: +90 312 210 22 91
F: +90 312 210 79 59
ueam@metu.edu.tr
www.ueam.metu.edu.tr

Sayı: 28620816 / 187

03 Nisan 2019

Konu: Değerlendirme Sonucu

Gönderen: ODTÜ İnsan Araştırmaları Etik Kurulu (İAEK)

İlgi: İnsan Araştırmaları Etik Kurulu Başvurusu

Sayın Prof.Dr. Mine IŞIKSAL BOSTAN

Danışmanlığını yaptığınız **Aniş Büşra BARAN'ın "A Study on Development of Prospective Middle School Teachers' Competence of Posing Mathematical Literacy Questions"** başlıklı araştırması İnsan Araştırmaları Etik Kurulu tarafından uygun görülmüş ve **177-ODTÜ-2019** protokol numarası ile onaylanmıştır.

Saygılarımızla bilgilerinize sunarız

Prof. Dr. Tülin GENÇÖZ
Başkan

Prof. Dr. Ayhan SOL
Üye

Prof. Dr. Ayhan Gürbüz DEMİR (4.)
Üye

Prof. Dr. Yaşar KONDAKÇI
Üye

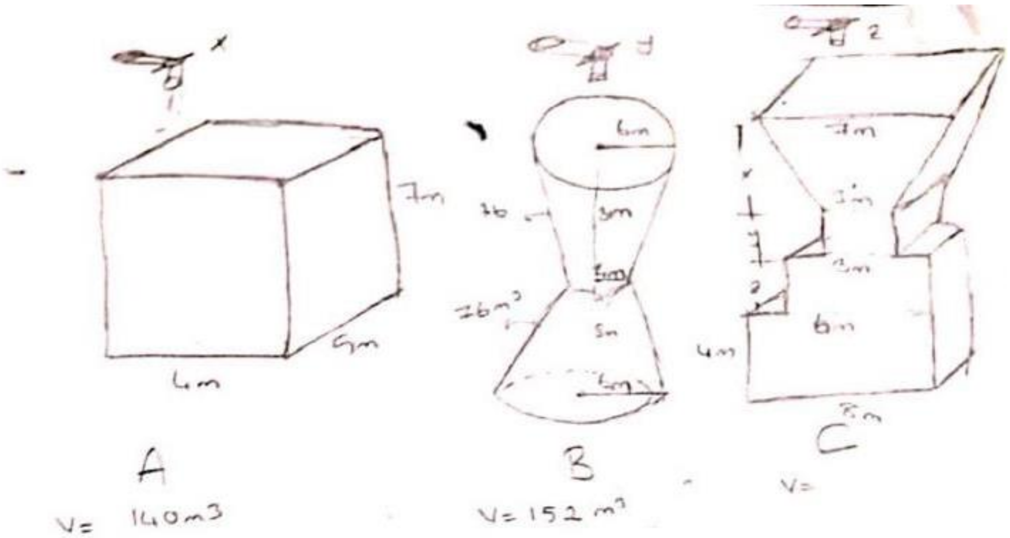
Doç. Dr. Emre SELÇUK
Üye

Doç. Dr. Pınar KAYGAN
Üye

Dr. Öğr. Üyesi Ali Emre TURGUT
Üye

F. Turkish Versions ML Problems generated by PSTs

F1. Tuğçe’s Initial Problem



Ayse Teyze'nin köyünde her evde, su tesisatı olmadığı
için köy meydanındaki cesmelerden su taşıması gerekmektedir.
Yukarıdaki A, B ve C şekillerindeki gibi 3 farklı su varillere
sahip olan Ayse Teyze bunları doldurmak istemektedir.
Köy meydanında 3 farklı musluk bulunmaktadır ve her musluktan
dakikada, akan su miktarı farklıdır

1. musluk : Dakikada 10 m³
2. musluk : Dakikada 20 m³
3. musluk : Dakikada 15 m³

Köy meydanında cok fazla su sırası vardır. Bu yüzden
Ayse Teyze bir an önce varillerini doldurup bekletmek istemiyor.
Ayse Teyze'nin en verimli şekilde su doldurabilmesi için hangi varili
hangi cesmeden doldurmalıdır?

F2. Tuğçe's Final Problem

Kova

İbrik

Güğüm

Ayşe teyzenin köyünde her evde su tesisatı olmadığı için köy meydanından su taşıması gerekmektedir. Ayşe teyze yukarıdaki şekilde görülen kova, ibrik ve güğümünü köy meydanından doldurmak istemektedir. Bu kapların hacimleri sırasıyla 28 m^3, 15 m^3ve 23 m^3 tür. Köy meydanında 4 farklı musluk bulunmaktadır ve her musluktan akan su miktarı farklıdır.

1.musluk; dakikada 10 m^3

2.musluk; dakikada 20 m^3

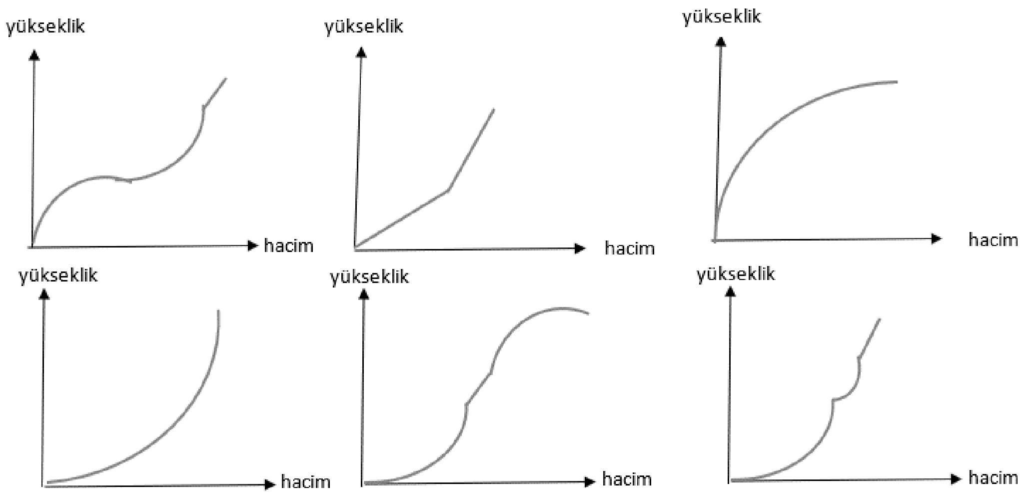
3.musluk; dakikada 15 m^3

4.musluk; dakikada 12 m^3

Köy meydanında çok fazla su sırası vardır. Bu yüzden Ayşe teyze bir an önce kaplarını doldurup sıradakileri bekletmek istememektedir.

Buna göre Ayşe teyzenin en verimli şekilde su doldurabilmesi için hangi kabı hangi çeşmeden doldurmalıdır? Ve kaplar yaklaşık olarak minimum kaç dakikada dolar?

Aşağıda verilen grafiklerden hangisinin hangi şekle ait olabileceğini bulunuz ve nedenini kısaca açıklayınız.



F3. Melike's Initial Problem

Gölge Boyu

Dünya'nın yıllık hareketine ve eksen eğikliğine bağlı olarak Güneş ışınlarının yeryüzüne düşme açısı değişiklik gösterir. Bu değişiklik gün içinde gölge boylarının uzunluğunun sürekli olarak değişmesine yol açar.

Ortalama insan boyuna sahip bir kişi gün içindeki gölge boyundan faydalanarak bir kavak ağacının boyunu ölçmek istemektedir. Güneş ışınlarının yere düşme açısı 60° olduğunda, ağaçtan yaklaşık olarak 3 metre uzakta duran bu kişinin gölgesi ile kavak ağacının gölgesi aynı yerde bitmektedir. Ağacın boyu kaç metre olabilir?

F4. Melike's Final Problem

Gölge Boyu

Dünya'nın yıllık hareketine ve eksen eğikliğine bağlı olarak Güneş ışınlarının yeryüzüne düşme açısı değişiklik gösterir. Bu değişiklik gün içinde gölge boylarının uzunluğunun sürekli olarak değişmesine yol açar.

a) Ortalama insan boyuna sahip bir kişi gün içindeki gölge boyundan faydalanarak bir kavak ağacının boyunu ölçmek istemektedir. Güneş ışınlarının yere düşme açısı 60° olduğunda, ağaçtan yaklaşık olarak 3 metre uzakta duran bu kişinin gölgesi ile kavak ağacının gölgesi aynı yerde bitmektedir. Ağacın boyu kaç metre olabilir?
b) Gün içinde Güneş ışınlarının yeryüzüne düşme açısı sürekli olarak değişiklik gösterir. Bu açının azaldığı ya da arttığı durumlarda, kendi gölge boyunu kullanarak ağacın boyunu ölçmek isteyen kişinin konumu nasıl değişir?

F5. Ayşe's Initial Problem

Bir evin elektrik, su ve doğal gaza aylık ödediği miktarlar tablodaki gibidir.

Elektrik	200 TL
Su	150 TL
Doğalgaz	450 TL

Bu eve taşındığından beri her ay hepsi için de aynı ücreti ödeyen biri kaç yıl sonra 78000 TL

F6. Ayşe's Revised Problem

	Fiyat
Elektrik	70-250
Su	72-110
Doğal gaz	240-600

Bir eve aylık ödenen elektrik, su ve doğal gaz fiyatlarının en az ve en çok değerleri tabloda verilmiştir. 3 ay boyunca elektriğin sudan daha az fiyatta ödendiği ve doğal gaz için eğer ilk iki ay 300 liradan fazla ödenirse 3.ay 300liradan daha az geldiği biliniyor. Buna göre 3.ayın sonunda toplam giderler için en fazla ne kadar ödenmiş olabilir?

F7. Ayşe's Final Problem

Fatura Tipi	Fiyat (TL)
Elektrik	70,23 - 250,41
Su	72,06 - 110,98
Doğal gaz	240,45 - 603,85

FATURALAR

Ahu bir yıl boyunca evine ödediği faturalarını incelediğinde her ay ödenen elektrik, su ve doğal gaz fiyatlarının aralığını bir liste yapıyor.

- Yaz ayları boyunca su kullanımını arttıran Ahu, yaz ayları boyunca elektrik faturası için ödediği miktarın su faturası için ödediği miktarı geçmediğini fark ediyor.
- Ahu, yazın kombiyi kıstığından bu mevsimde gelen doğal gaz faturalarının 286,01 liradan daha az geldiğini gözlemliyor.

a) **Buna göre Ahu yaz ayları boyunca tablodaki bütün giderler için en fazla kaç lira ödemiş olabilir?**

b) Ahu tabloda verilen faturalar haricinde internet faturası ve telefon faturası için de her ay ödeme yapıyor. Temmuz ayında internet ve telefon faturası için ödediği miktarları kaydetmeyi unutuyor ama bu iki fatura için temmuz ayında ödediği miktarın 50 lira ile 125 lira arasında bir miktar olduğunu hatırlıyor. Ahu bu bilgilerle ağustos ayında ödeyeceği minimum miktarı tahmin etmek istiyor. **Bunun için Ahu'nun ağustos ayı için bütün ödemelerine ayıracağı bütçe en az ne kadar olmalıdır?**