

$$|S_m = +\frac{1}{2}\rangle = ?$$

$$\langle S_m = \pm \frac{1}{2} | +z \rangle = ?$$

$$|S_m = +\frac{1}{2}\rangle = |\alpha_1| + z\rangle + |\alpha_2| - z\rangle$$

$$|\alpha_1|^2 + |\alpha_2|^2 = 1$$

$$\langle S_m = \pm \frac{1}{2} | +z \rangle = \mp \alpha_{\pm}$$

### Hyperfine Splitting

$$\vec{\mu}_p = g_p \frac{e}{2m_p} \vec{S}_p$$

$$\vec{\mu}_e = g_e \frac{(-e)}{2m_e} \vec{S}_e$$

$$\vec{B}_{\mu} = \frac{\mu_0}{4\pi r^3} \frac{1}{r^3} \left[ 3(\vec{\mu} \cdot \vec{r}) \vec{r} - \vec{\mu} \right] + 2 \frac{\mu_0}{r^3} \vec{\mu} \vec{S}(r)$$

$$\nabla^2 \frac{1}{r^3} \times \vec{S}(r)$$

$$H' = - \frac{\vec{\mu}_e \cdot \vec{B}_p}{r^3}$$

$$= - \frac{\mu_0}{4\pi r^3} \frac{1}{r^3} \left[ 3(\vec{\mu}_p \cdot \vec{r})(\vec{\mu}_e \cdot \vec{r}) - \vec{\mu}_p \cdot \vec{\mu}_e \right]$$

$$+ 2 \frac{\mu_0}{r^3} \vec{\mu}_p \cdot \vec{\mu}_e \vec{S}(r)$$

$$\text{Tensor interaction} = \left\{ \frac{\mu_0}{4\pi r^3} \frac{1}{r^3} g_p \frac{e}{2m_p m_e} \left[ 3(\vec{S}_p \cdot \vec{r})(\vec{S}_e \cdot \vec{r}) - \vec{S}_p \cdot \vec{S}_e \right] \right\}$$

$$\text{scalar spin-spin interaction} = \left\{ -2 \frac{\mu_0}{r^3} g_p \frac{e}{2m_p m_e} \vec{S}_p \cdot \vec{S}_e \vec{S}(r) \right\}$$

$$\begin{aligned}\vec{s}_p \cdot \vec{s}_e &= \frac{1}{2} \left\{ (\vec{s}_p + \vec{s}_e)^2 - \vec{s}_p^2 - \vec{s}_e^2 \right\} \\ (\vec{s}_p \cdot \vec{r})(\vec{s}_e \cdot \vec{r}) &= \frac{1}{2} \left[ (\vec{s}_p \cdot \vec{r} + \vec{s}_e \cdot \vec{r})^2 - (\vec{s}_p \cdot \vec{r})^2 - (\vec{s}_e \cdot \vec{r})^2 \right] \\ (\vec{s}_e \cdot \vec{r})^2 &= \frac{\hbar^2}{4} \quad ; \quad (\vec{s}_p \cdot \vec{r})^2 = \frac{\hbar^2}{4} \\ \vec{s}_e^2 &= \vec{s}_p^2 = \frac{3\hbar^2}{4}\end{aligned}$$

$$\begin{aligned}|nlm\rangle &\otimes |s_e m_s\rangle \otimes |s_p m_{sp}\rangle \\ \Rightarrow |nlm\rangle &\otimes |s_e s_p s_T m_s\rangle\end{aligned}$$

$$s_T = 0, 1$$

$$m_s = -s_T, \dots, s_T$$

$$\begin{aligned}s_T = 0 &\Rightarrow \text{singlet state} \quad (l=0) \\ s_T = 1 &\Rightarrow \text{triplet state}\end{aligned}$$

$$\langle l=0 | 3(\vec{s}_e \cdot \vec{r})(\vec{s}_p \cdot \vec{r}) - \vec{s}_e \cdot \vec{s}_p | l=0 \rangle = 0$$

$$\begin{aligned}E_{n=0}^{(1)} &= \langle 000s_1 \left| -2 \frac{M_0}{\gamma n_p m_e} g_p \frac{e^2}{\gamma n_p m_e} \vec{s}_p \cdot \vec{s}_e \delta'(\vec{r}) \right| 000s_1 \rangle \\ &= -2 \frac{M_0}{\gamma n_p m_e} g_p \frac{e^2}{\gamma n_p m_e} |\psi_{000}(0)|^2 \left( s_T^2 - \frac{3\hbar^2}{2} \right)\end{aligned}$$

$\frac{\Delta E}{\hbar} = 1420 \text{ MHz}$

$\rightarrow 21 \text{ cm}$   
 (microwave)

$$\Delta E \sim 10^{-6} \text{ eV}$$

## Stark Effect

$$\vec{E}_{ext} = E_{ext} \hat{z}$$

$$\vec{E} = -\vec{\nabla}V$$

$$H' = (-e) \underbrace{E_{ext}}_V (-z)$$

$E_{ext}$  is uniform  $\rightarrow U = ?$

$$H' = e E_{ext} z$$

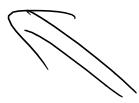
$$\langle nlm | H' | nlm \rangle = e E_{ext} \langle nlm | z | nlm \rangle = 0$$

$$P Y_{lm} = (-1)^l Y_{lm}$$

$$P(z | nlm) = (-z)(-1)^l \langle nlm | z | nlm \rangle = (-1)^{l+1} \langle z | nlm \rangle$$

$$P(nlm) = (-1)^l \langle nlm |$$

$$\langle nlm | z | nlm \rangle \neq 0 \text{ only if } l'-l \text{ is odd}$$



If  $n=1$ , there is no correction to the energies since  $\not\#$  degeneracy.

If  $n=2$ ,  $|200\rangle, |211\rangle, |210\rangle, |21-1\rangle$  are degenerate.

$$\langle 200 | z | 200 \rangle = 0$$

$$[L_z, z] = 0$$

$$\langle 200 | z | 211 \rangle = 0$$

$$0 = \langle 200 | [L_z, z] | 211 \rangle$$

$$= \langle 200 | L_z z - z L_z | 211 \rangle$$

$$= -\hbar \langle 200 | z | 211 \rangle$$

$$\langle 200|z|210 \rangle \neq 0$$

$$H' = \begin{pmatrix} 100 & 111 & 110 & 11-1 \\ 0 & 0 & \delta & 0 \\ 0 & 0 & 0 & 0 \\ \delta^* & 0 & 0 & 0 \\ 0 & 0 & 0 & -\delta \end{pmatrix}$$

$$\delta = \langle 210 | H' | 200 \rangle$$

$$\text{eigenvektoren} \left\{ |211\rangle, |21-1\rangle, \frac{1}{\sqrt{2}}(|210\rangle + |200\rangle) \right. \\ \left. \frac{1}{\sqrt{2}}(|210\rangle - |200\rangle) \right\}$$

$$\text{eigenwerte} \left\{ 0, 0, \delta, -\delta \right\}$$