$$\Rightarrow |\psi(t)\rangle = c_a(t) e^{-\frac{i}{\hbar}E_a t}|a\rangle + c_b(t)|b\rangle e^{-\frac{i}{\hbar}E_b t}$$

$$\frac{d}{dt}\begin{pmatrix} c_a \\ c_b \end{pmatrix} = -\frac{i}{\hbar}\underbrace{\left(e^{\frac{i}{\hbar}H_0 t} H' e^{-\frac{i}{\hbar}H_0 t}\right)_{2\times 2}}_{H'^I(t)} \begin{pmatrix} c_a \\ c_b \end{pmatrix}$$

$$\int_0^t dt \, \boxed{\frac{d}{dt} C = \int_0^t -\frac{i}{\hbar} H'^I(t) \, C \, dt} \Leftarrow$$

$$C(t) = T \exp\left\{-\frac{i}{\hbar}\int_0^t dt' \, H'^I(t')\right\} C_0$$

$$C^\dagger(t)\, C(t) = C_0^\dagger C_0 = \sum_a |c_a|^2 = 1$$

$$\langle a|H'^I(t)|b\rangle = \langle a| e^{\frac{i}{\hbar}H_0 t} H' e^{-\frac{i}{\hbar}H_0 t}|b\rangle$$

$$= e^{\frac{i}{\hbar}(E_a - E_b)t}\langle a|H'|b\rangle \equiv e^{i\omega_0 t} H'_{ab}$$

$$\langle a|H'^I(t)|a\rangle = \langle a|H'|a\rangle = H'_{aa}$$

$$\boxed{\omega_0 = \frac{E_a - E_b}{\hbar}}$$

$$H'_{aa} = H'_{bb} = 0$$

$$\frac{d}{dt}\begin{pmatrix} c_a \\ c_b \end{pmatrix} = \begin{pmatrix} 0 & e^{i\omega_0 t} H'_{ab} \\ e^{-i\omega_0 t} H'_{ba} & 0 \end{pmatrix}\begin{pmatrix} c_a \\ c_b \end{pmatrix}$$

$$\dot{c}_a = e^{i\omega_0 t} H'_{ab} c_b$$
$$\dot{c}_b = e^{-i\omega_0 t} H'_{ba} c_a$$

$$\frac{d}{dt}\left(|c_a|^2 + |c_b|^2\right) = 0$$

$$H' = V\cos\omega t$$

$$\left.\begin{aligned} \dot{c}_a &= e^{i\omega_0 t} V_{ab} \cos\omega t \, c_b \\ \dot{c}_b &= e^{-i\omega_0 t} V_{ba} \cos\omega t \, c_a \end{aligned}\right\}$$

at $t=0$; $|\psi\rangle = |a\rangle \Rightarrow c_a(t=0) = 1$

$c_b(t=0) = 0$

$$\dot{c}_a = e^{i\omega_0 t} V_{ab} \cos\omega t \cdot 0 = 0 \Rightarrow c_a = 1$$
$$\dot{c}_b = e^{-i\omega_0 t} V_{ba} \cos\omega t \Rightarrow c_b(t) = \int_0^t e^{-i\omega_0 t'} \cos\omega t' \, V_{ba} \, dt'$$

$$\cos\omega t = \frac{e^{i\omega t} + e^{-i\omega t}}{2}$$

$$c_b(t) = \int_0^t \left[e^{i(\omega-\omega_0)t'} + e^{-i(\omega+\omega_0)t'}\right] dt' \frac{V_{ba}}{2}$$

$$c_b(t) = \left[\frac{e^{i(\omega-\omega_0)t} - 1}{\omega-\omega_0} + \frac{e^{-i(\omega+\omega_0)t} - 1}{\omega+\omega_0}\right]\frac{V_{ba}}{2}$$

$$\frac{c_b(t)}{\frac{V_{ba}}{2}} = \left[\frac{2i \sin\left(\frac{(w-w_0)t}{2}\right) e^{i(w-w_0)t/2}}{w-w_0} + \frac{2i \sin\left(\frac{(w+w_0)t}{2}\right) e^{i(w+w_0)t/2}}{w+w_0} \right]$$

$$\boxed{c_b(t) \simeq \frac{i V_{ba} \sin\left(\frac{(w-w_0)t}{2}\right) e^{i(w-w_0)t/2}}{w-w_0}}$$

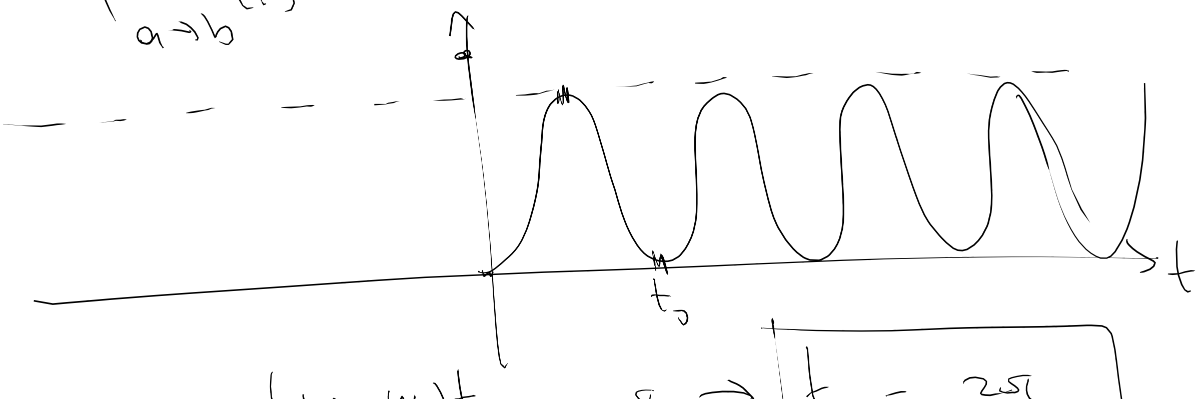
$\sin\theta \simeq \theta$

$$P_{a\to b}(t) \equiv |c_b(t)|^2 = \frac{|V_{ba}|^2 \sin^2\left(\frac{(w-w_0)t}{2}\right)}{(w-w_0)^2}$$

$$|c_b(t)|^2 + |c_a(t)|^2 = 1 + O\left(|V_{ba}|^2\right)$$

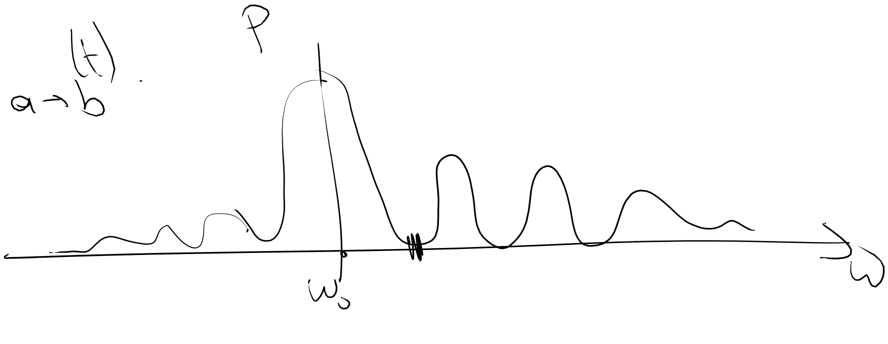
$$\lim_{w\to w_0} P_{a\to b}(t) = |V_{ba}|^2 \frac{t^2}{4}$$

$P_{a\to b}(t)$



$$\frac{(w-w_0)t_0}{2} = \pi \Rightarrow \boxed{t_0 = \frac{2\pi}{w-w_0}}$$

$P_{a\to b}(t)$



$$\frac{(\omega-\omega_0)t}{2} = \pi \Rightarrow \boxed{\omega-\omega_0 = \frac{2\pi}{t}}$$

$$\hbar\omega + E_a = E_b \Rightarrow \hbar\omega = E_b - E_a = \hbar\omega_0 \Rightarrow \omega = \omega_0$$

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

Stimulated emission, Spontaneous Emission, Absorption.

---

uniform time dependent electric field

$$\vec{E}(\vec{x},t) = \vec{E}_0 \cos(\omega t)$$

$$V = -q\vec{E}_0 \cdot \vec{r} \cos(\omega t)$$

assume $\vec{E}_0 = E_0 \hat{z}$

$$\boxed{V = -qE_0 z \cos(\omega t)}$$

$$\langle nlm| V |nlm\rangle = -qE_0 \cos(\omega t) \langle nlm|z|nlm\rangle$$

$$[L_z, z] = 0$$

$$0 = \langle nlm| [L_z, z] |nlm\rangle = \langle nlm| L_z z - z L_z |nlm\rangle$$

$$= 0$$

$$P^\dagger z P = -z$$

$$\langle nlm|z|nlm\rangle = -\langle nlm| P^\dagger z P |nlm\rangle$$

$$= -(-1)^l (-1)^l \langle nlm|z|nlm\rangle$$

$$= -\langle nlm|z|nlm\rangle$$

$$\Rightarrow \langle nlm|z|nlm\rangle = 0$$

$$x+iy \longrightarrow e^{i\phi\cdot 1}(x+iy) \qquad \underbrace{e^{i\phi L_z}|nlm\rangle} = e^{i\phi m}|nlm\rangle$$

$$x-iy \longrightarrow e^{+i\phi(-1)}(x-iy)$$

$$z \longrightarrow e^{i\cdot 0\cdot\phi}\, z$$

$$\frac{1}{r}(x+iy) \propto Y_1^{1}(\Omega)$$

$$\frac{1}{r}(x-iy) \propto Y_1^{-1}(\Omega)$$

$$\frac{1}{r}z \propto Y_1^{0}$$

$$z|nlm\rangle \propto Y_1^0 \otimes Y_l^m \qquad \begin{matrix} l+1 \\ l \\ l-1 \end{matrix}$$

$$\underbrace{\langle n l' m|}\underbrace{z|nlm\rangle} = 0 \quad \text{if } l'-l \neq \pm 1, 0$$

$$[L^2,[L^2,z]]$$

$$\langle n l' m|z|nlm\rangle = 0 \quad \text{if } l'-l \neq \pm 1$$

$$\langle \underbrace{n'l'm'}_{a}|V|\underbrace{nlm}_{b}\rangle = -E_0\, \wp_{ab} \cos\omega t$$

$$\wp_{ab} = \langle n'l'm'|qz|nlm\rangle$$

$p = qd$ electric dipole

$$\{|100\rangle, |110\rangle\}$$

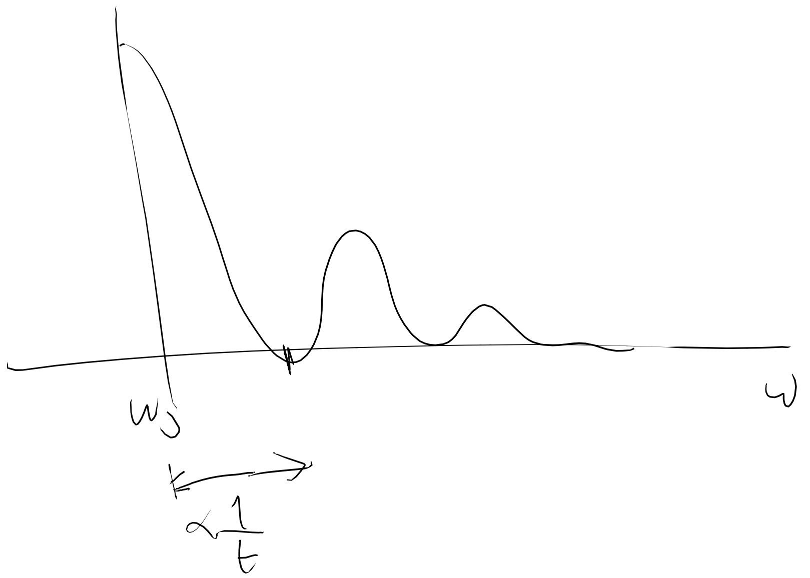
$$P_{absorption} = \frac{|V_{ba}|^2 \sin^2\left(\frac{(\omega-\omega_0)t}{2}\right)}{(\omega-\omega_0)^2}$$

$$|V_{ba}|^2 = |\mathcal{P}_{ba} E_0|^2$$

$$P_{absorption} = \mathcal{P}_{ba}^2 E_a^2 \frac{\sin^2\left(\frac{(\omega-\omega_0)t}{2}\right)}{(\omega-\omega_0)^2}$$

$$H' = V\cos\omega t = -qE_0 z\cos\omega t$$

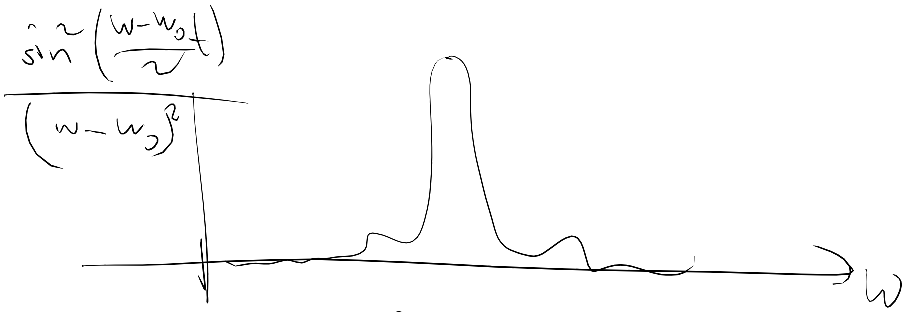
$$\mathcal{P} = \langle qz \rangle$$



$$P_{absorption} = \int d\omega\, \rho(\omega)\, \mathcal{P}_{ba}^2 E_a^2 \frac{\sin^2\left(\frac{(\omega-\omega_0)t}{2}\right)}{(\omega-\omega_0)^2}$$

$$= \rho(\omega_0)\mathcal{P}_{ba}^2 E_0^2 \int_{-\infty}^{\infty} d\omega \frac{\sin^2\left(\frac{\omega-\omega_0}{2} t\right)}{(\omega-\omega_0)^2}$$

$$x = \frac{w-w_0}{2} t \Rightarrow dw = \frac{2}{t} dx$$



$$P_{abs} = \rho(w_0)\, \mathcal{P}_{ba}^2\, E_0^2\, \frac{2}{t}\left(\frac{t}{2}\right)^2 \underbrace{\int_{-\infty}^{\infty} dx\, \frac{\sin^2 x}{x^2}}_{\pi}$$

$$P_{abs} = \mathcal{P}_{ba}^2 \left(\frac{1}{2}E_0^2\right)\rho(w_0)\,\pi t$$
$$= \pi \mathcal{P}_{ba}^2 \left(\frac{1}{2}\varepsilon E_0^2\right)\rho(w_0)\, t$$

$$\boxed{\frac{dP_{abs}}{dt} = \pi \mathcal{P}_{ba}^2\, u\, \rho(w_0)}$$

$$\boxed{\frac{dP_{emis}}{dt} = \pi \mathcal{P}_{ab}^2\, u\, \rho(w_0)}$$

$$V = -q\vec{E}\cdot\vec{r} \cos wt$$
$$= -\frac{q\vec{E}\cdot\vec{r}}{2}\left(e^{iwt} + e^{-iwt}\right)$$

$$V \Rightarrow (\;)\left(a^{\dagger} e^{iwt} + a e^{-iwt}\right)$$

absorption $|n_\gamma = 1\rangle |000\rangle \longrightarrow |n_\gamma = 0\rangle |110\rangle$

st. emission $|n_\gamma = 1\rangle |110\rangle \longrightarrow |n_\gamma = 2\rangle |000\rangle$

spontaneous emission $|n_\gamma = 0\rangle |110\rangle \longrightarrow |n_\gamma = 1\rangle |000\rangle$

$$\frac{dN_b}{dt} = -AN_b - B_{ab} N_b \rho(\omega_0) + B_{ba} N_a \rho(\omega_0)$$

in equilibrium $\frac{dN_b}{dt} = 0$

$$\frac{dN_a}{dt} = -B_{ba} N_a \rho(\omega_0) + B_{ab} N_b \rho(\omega_0) + AN_b = 0$$

$$\frac{d(N_a + N_b)}{dt} = 0$$

$$\rho(\omega_0) = \frac{A N_b}{-B_{ab} N_b + B_{ba} N_a} = \frac{A}{B_{ba}\frac{N_a}{N_b} - B_{ab}}$$

$$N_a \propto e^{-\beta E_a} ; N_b \propto e^{-\beta E_b} \Rightarrow \frac{N_b}{N_A} = e^{-\beta(E_b - E_a)}$$

$$\frac{N_a}{N_b} = e^{-\beta\hbar\omega_0}$$

$$\beta = \frac{1}{k_B T} \qquad \rho(\omega_0) \propto \frac{\omega^3}{e^{\beta\hbar\omega_0} - 1}$$

$$B_{ab} = B_{ba} ; A = \frac{\hbar\omega^3}{\pi^2 c^3} B_{ba}$$