$$P_{a\to b}(t) = |c_b(t)|^2 = \frac{|V_{ab}|^3}{\hbar^2}\,\frac{\sin^2[(\omega_0-\omega)t/2]}{(\omega_0-\omega)^2}$$

$$H' = V(\vec r)\cos(\omega t)$$

$$\hbar\omega_0 = E_b - E_a$$

$$P_{a\to b}(t) \longrightarrow \int P_{a\to b}(t)\,\rho(\omega)\,d\omega \propto t$$

$$R = \frac{dP}{dt} = \frac{\pi}{\varepsilon_0\hbar^2}|V_{ab}|^2\rho(\omega_0)$$

$$\int \frac{\sin^2\left(\frac{\omega-\omega_0}{2}t\right)}{(\omega-\omega_0)}\,d\omega = \int \frac{\sin^2 x}{\frac{4}{t^2}x^2}\,\frac{2}{t}\,dx = t \text{ (const)}$$

$$x = \frac{\omega-\omega_0}{2}t \Rightarrow (\omega-\omega_0) = \frac{2}{t}x$$

$$H' = -qE_0 z\cos(\omega t - \vec k\cdot\vec r)$$

$$k \propto \frac{1}{\lambda}$$

$$\langle\psi_b|H'|\psi_a\rangle = \int d^3r\,\psi_b^*(\vec r)\,\psi_a(\vec r)\,\underbrace{\cos(\omega t-\vec k\cdot\vec r)}_{\approx 1}$$

inside the atom.

$$V(r) = -qE_0 z$$

$$V_{ab} = -E_0\,\mathcal{P} \qquad \mathcal{P} = \hat n\cdot\vec{\wp}$$

$$\mathcal{P} \equiv \langle\psi_b|\underbrace{qz}_{\text{dipole moment of the electron}}|\psi_a\rangle \qquad \vec{\wp} = \langle\psi_b|q\vec r|\psi_a\rangle$$

"electric dipole transition"

$$\frac{dN_b}{dt} = -AN_b - \underbrace{B_{ab}N_b\rho(\omega_0)} + \underbrace{B_{ba}N_a\rho(\omega_0)} = 0$$

$$B_{ab} = B_{ba}$$

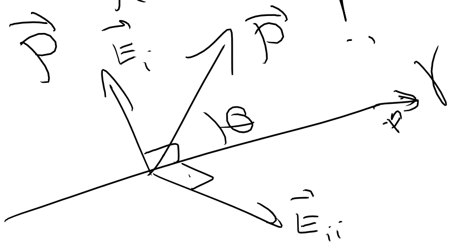
$$A = \frac{\omega^3|\vec{\wp}|^2}{3\varepsilon_0\hbar^2}$$

$\psi(\vec{r}) = R_{nl}(r)\, Y_{lm}(\Omega)$

$\langle \psi_a | \vec{r} | \psi_a \rangle = 0 \qquad \langle \psi_b | \vec{r} | \psi_a \rangle \neq 0$

i) average over polarizations
ii) average over directions.

i) average over polarizations



$P^2 \to (\hat{n}\cdot\vec{P})^2$

$\to \frac{1}{2}(\hat{y}\cdot\vec{P})^2 + \frac{1}{2}(\hat{z}\cdot\vec{P})^2$

$\vec{P} = P\cos\theta\,\hat{x} + \overline{P\sin\theta\cos\phi\,\hat{y}} + \overline{P\sin\phi\sin\phi\,\hat{z}}$

$\frac{1}{2}(\hat{y}\cdot\vec{P})^2 + \frac{1}{2}(\hat{z}\cdot\vec{P})^2 = \frac{\sin^2\theta}{2} P^2$

ii) average over directions

$\frac{1}{4\pi}\int d\theta\, d\phi\, \sin\theta\, \frac{\sin^2\theta}{2} P^2$

$= \frac{1}{2} P^2 \frac{1}{4\pi} 2\pi \int_0^\pi d\theta\, \sin\theta\,(1-\cos^2\theta)$

$= \frac{P^2}{4}\left(-\cos\theta + \frac{\cos^3\theta}{3}\right)\Bigg|_{\theta=0}^{\pi}$

$= \frac{P^2}{4}\, 2\left(1-\frac{1}{3}\right) = \frac{P^2}{3}$

$P = \frac{\pi}{\varepsilon_0 \hbar^2} E_0^2 P^2 \rho(\omega) \longrightarrow \underbrace{\frac{\pi}{3\varepsilon_0\hbar^2} E_0^2 P^2 \rho(\omega_0)}$

$B_{ab} = B_{ba} = \frac{\pi}{3\varepsilon_0\hbar^2} E_0^2$

$A = \frac{\omega^3 |\vec{P}|^2}{3\pi\varepsilon_0\hbar c^3}$

$\frac{dN_b}{dt} = -AN_b \Rightarrow N_b(t) = N_b(0)e^{-At}$

lifetime $= \tau = \frac{1}{A}$

$$\frac{dN_b}{dt} = -A_1 N_b - A_2 N_b - A_3 N_b \cdots$$

$$\tau = \frac{1}{A_1 + A_2 + \cdots}$$

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

$N_{st} \propto |\psi|^2$ $\qquad N_{st} = N_0 e^{-At}$

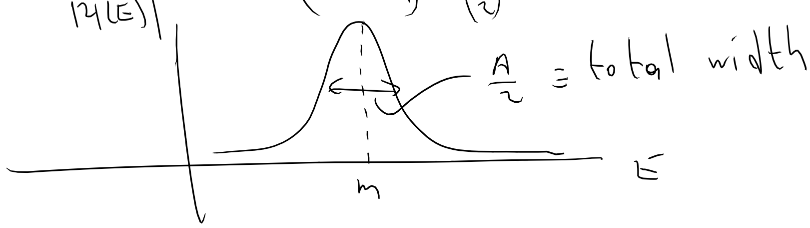
$\cancel{dN|\psi|^2}$

$$\psi(t) = e^{-\frac{A}{2}t + i\frac{Et}{\hbar}} \qquad (\hbar = 1)$$

rest frame $E = m_{s1}$

$$\psi(E) = \frac{1}{E - (m_{s1} + \frac{iA}{2})}$$

$$|\psi(E)|^2 = \frac{1}{(E - m_{s1})^2 + (\frac{A}{2})^2}$$



X(3872)

$B \to J/\psi\, \phi\, K$

$B \to X K$

$\quad \hookrightarrow J/\psi\, \phi$

Example Harmonic oscillator moving in the x direction.

$$P = \langle n' | q x | n \rangle = q \langle n' | x | n \rangle$$

$$= q\sqrt{\frac{\hbar}{2m\omega}} \langle n' | a + a^\dagger | n \rangle$$

$$= q\sqrt{\frac{\hbar}{2m\omega}} \langle n' | \left(\sqrt{n}\,|n-1\rangle + \sqrt{n+1}\,|n+1\rangle\right)$$

$$P = q\sqrt{\frac{\hbar}{2m\omega}} \left(\sqrt{n}\,\delta_{n',n-1} + \sqrt{n+1}\,\delta_{n',n+1}\right)$$

spontaneous emission

$$\mathcal{P}^2 = q\sqrt{\frac{n\hbar}{2m\bar{\omega}}}\,\delta_{n',n-1}$$

$$\frac{E_n - E_{n'}}{\hbar} = \frac{\hbar\bar{\omega}}{\hbar} = \bar{\omega}$$

$$A = \frac{\omega^3}{3\pi\varepsilon_0\hbar c^3}\mathcal{P}^2 = \frac{\cancel{\omega^3}^{\,2}}{3\pi\varepsilon_0\cancel{\hbar} c^3}\, q^2\frac{n\cancel{\hbar}}{2m\cancel{\omega}}$$

$$= \frac{q^2\omega^2 n}{6m\pi\varepsilon_0 c^3}$$

$$P = A\hbar\omega = \frac{q^2\omega^2}{6m\pi\varepsilon_0 c^3}(n\hbar\omega) = \frac{q^2\omega^2}{6m\pi\varepsilon_0 c^3}\left(E - \cancel{\frac{\hbar\omega}{2}}\right)$$

Classically

$$P = \frac{q^2 a^2}{6\pi\varepsilon_0 c^3}$$

$$x = x_0\cos\omega t$$

$$a = -\omega^2 x$$

$$a^2 = +\omega^4 x^2 \xrightarrow{\text{average}} \frac{\omega^4 x_0^2}{2}$$

$$P = \frac{q^2\omega^4 x_0^2}{12\pi\varepsilon_0 c^3} = \frac{q^2\omega^2}{6\pi\varepsilon_0 c^3 m}\Big(\underbrace{\tfrac{1}{2}m\omega^2 x_0^2}_{E}\Big)$$

$$P = \frac{q^2\omega^2}{6\pi\varepsilon_0 m c^3}E$$

Selection Rule

Transitions are proportional to

$$\langle\psi_a|\vec{r}|\psi_b\rangle$$

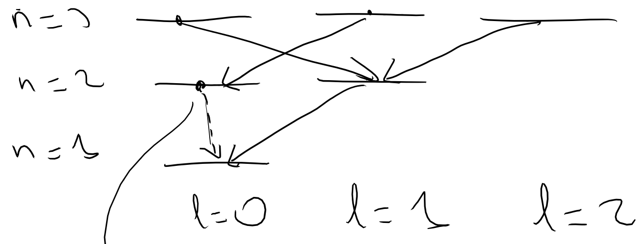
$\langle n'l'm'|z|nlm\rangle = 0$ if $m \neq m'$

$\langle n'l'm'|x,y|nlm\rangle = 0$ if $m - m' \neq \mp 1$

$\langle\psi_a|\vec{r}|\psi_b\rangle = 0$ if $\Delta m \neq \mp 1, 0$

$$\langle n'l'm'|\vec{r}|nlm\rangle = 0 \text{ if } \Delta l = \mp 1$$

$$P|nlm\rangle = (-1)^l |nlm\rangle$$



→ metastable state.

$$C = T\left\{ e^{-i\int_0^t H'^{t}(t')\,dt'} \right\} C_0$$

2^{nd} order $(H')^2 = \left(V(\vec{r})\cos\omega t\right)^2$

$$= V(\vec{r})^2 \cos^2(\omega t)$$

$$= V(\vec{r})^2 \frac{\left[\cos(2\omega)t + 1\right]}{2}$$

$$= q^2 E_0^2 z^2 \; \frac{\cos(2\omega t) + 1}{2}$$

$$H' = -qE_0 z \cos(\omega t - \vec{k}\vec{r})$$

$$\langle \psi_b | H' | \psi_a \rangle = -qE_0 \int d^3r\, \psi_b^*(\vec{r})\psi_a(\vec{r})\, z \cos(\omega t - \vec{k}\vec{r})$$

$$\cos(\omega t - \vec{k}\cdot\vec{r}) \simeq \cos(\omega t) - \sin(\omega t)(-\vec{k}\cdot\vec{r}) + O(\vec{k}\cdot\vec{r})^2$$

$$\int_0^t dt' \langle \psi_b | H^I | \psi_a \rangle = + \langle \psi_b | \overbrace{(-qE_0 z)}\, | \psi_a \rangle \frac{e^{i(\omega_0 - \omega)t} - 1}{(\omega - \omega_0)}$$

$$+ \underbrace{\langle \psi_b | \overbrace{(-qE_0 z)(\vec{k}\cdot\vec{r})} | \psi_a \rangle} \frac{e^{i(\omega_0 - \omega)t} - 1}{\omega - \omega_0}$$

$$\langle \psi_b | \overbrace{z(\vec{k}\cdot\vec{r})} | \psi_a \rangle = ?$$

$$\vec{r} \propto (l = 1)$$