$\bar{O}$, $\Delta O = \sqrt{\langle (O-\bar{O})^2 \rangle}$

$$\boxed{\frac{\Delta O}{\bar{O}} \propto \frac{1}{\sqrt{N}}} \Leftarrow (\Delta O)^2 \sim N$$

$$\bar{O} \sim N$$

$\underline{S(E,V,N)}$

$E = \langle E \rangle = E^*$

$$S(\underbrace{E^*}_{},V,N) = k \ln \Omega(E,V,N)$$

$$\frac{1}{T} = \frac{\partial S}{\partial E^*} \qquad \frac{1}{T} = \frac{\partial S}{\partial E}$$

10^{27} $\qquad$ 10^{9}

$$\Rightarrow \boxed{\Omega(E,V,N) \propto \sum_{E(p,q) \le E} 1}$$

$$\Rightarrow dU = T\,dS - P\,\underbrace{dV} - \vec{m}\cdot d\vec{B}$$

$$U_B = -\vec{m}\cdot\vec{B} \qquad \Rightarrow \frac{\partial S(E,V,N,B)}{\partial B} = \frac{m}{T}$$

3N dimensional $\Rightarrow$ 6N?

Ω = # of states of a given energy

$\Sigma(E)$ = # of states having energy less than E

$$\boxed{\Sigma(E) \sim \Omega(E)}$$

$$\Sigma(E) = \sum_{\substack{\{n_x^i, n_y^i, n_z^i\} \\ \frac{\hbar^2 \pi^2}{2mL^2} \sum (n_x^{i2} + n_y^{i2} + n_z^{i2}) \leq E}} 1 = \int_{\substack{q_i \in V \\ \sum_{i=1}^N \frac{p_i^2}{2m} \leq E}} \frac{\prod_{i=1}^N dq_i dp_i}{h^{3N}}$$

$$\Delta x \Delta p \geq \frac{\hbar}{2} = \frac{h}{4\pi}$$

$$\int_{\sum_{i=1}^N \vec{n}_i^2 \leq \varepsilon} \prod_{i=1}^N d^3 n_i \sim V \int_{\sum_{i=1}^N p_i^2 \leq E'} \prod_{i=1}^N d^3 p_i$$

$$\int_{|p_i| \leq R} d^{3N} p_i = \int_0^R p^{3N-1} dp \underbrace{\int d\Omega_{3N}}_{f(N)}$$

$$\boxed{\sqrt{\pi} = \int_{-\infty}^{\infty} dx\, e^{-x^2}} \Rightarrow (\sqrt{\pi})^{3N} = \int \prod_{i=1}^{3N} dx_i\, e^{-\sum x_i^2} = \int_0^\infty r^{3N-1} dr\, d\Omega_{3N}\, e^{-r^2}$$

$$I = \int e^{-x^2} dx$$

$$I^2 = \int e^{-(x^2+y^2)} dx\,dy = \underbrace{\int_0^\infty e^{-r^2} r\,dr}\int_0^{2\pi} d\phi$$

$$= \underbrace{\left.\frac{-e^{-r^2}}{2}\right|_{r=0}^{\infty}}_{1/2} 2\pi$$

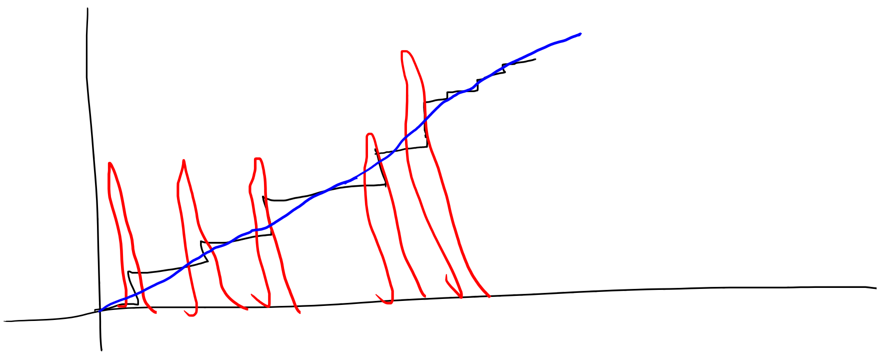
$$I^2 = \pi$$

$$I = \sqrt{\pi}$$

$\Sigma(E) =$ # of states of energy $\varepsilon < E$

$\Omega(E) =$ # of states of energy $\varepsilon = E$

$$\Sigma(E) = \sum_{E} \Omega(E)$$



$$\Omega(E) = \Sigma\left(E+\frac{\Delta}{2}\right) - \Sigma\left(E-\frac{\Delta}{2}\right) = \underbrace{\frac{\partial \Sigma}{\partial E}}_{\text{surface area of the hypersurface}} \Delta$$

$$S = k\ln\Omega(E) = k\ln \frac{\partial\Sigma}{\partial E} E\left(\frac{\Delta}{E}\right)$$

$$= k\ln\left(\frac{\partial\Sigma}{\partial E}E\right) + k\ln\frac{\Delta}{E}$$

$0.01\}$

$\frac{1}{\sqrt{N}}\}$

$$\Sigma \propto E^N \qquad E\frac{\partial\Sigma}{\partial E} \propto N E^N$$

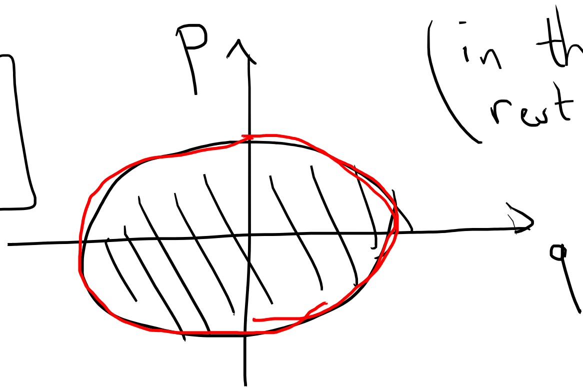
$$\Rightarrow E\frac{\partial\Sigma}{\partial E} \sim \Sigma$$

$$\Sigma(E) = \sum_{\varepsilon^* < E}\Omega(\varepsilon^*) \simeq \Omega(\varepsilon^*_{max})$$

$\frac{\Delta}{E}$ is nonzero?

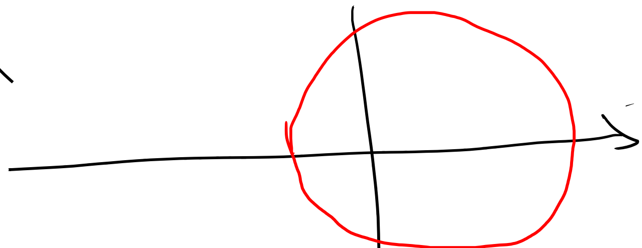
$$\left\{ \frac{\partial\rho(p,q,t)}{\partial t} = 0 \right. \; ?$$

$$\boxed{\frac{1}{2m}p^2 + \frac{1}{2}m\omega^2 q^2 < E}$$



(in the rest frame)

in a moving reference frame at a later time



$$\boxed{\frac{\partial\rho}{\partial t} + \{\rho, H\} = 0}$$

$$\frac{d\rho}{dt} = \frac{\partial\rho}{\partial t} + \{\rho, H\} = 0$$

if $\Rightarrow$ $\boxed{\frac{d\rho}{dt} \neq \frac{\partial \rho}{\partial t}}$ $\quad \{\rho, H\} = 0$

$$\frac{d}{dt}\int_V \rho \, d\omega_{3N} = -\int_{\partial V} \vec{j} \cdot d\vec{A}_{3N-1}$$

$$\Uparrow$$

$$\boxed{\frac{\partial \rho}{\partial t} + \nabla \cdot \vec{j} = 0} \Leftarrow \frac{\partial \rho}{\partial t} = 0 \text{ in equilibrium.}$$

$$\frac{d\rho}{dt} = \frac{\partial \rho}{\partial t} \qquad \frac{\partial \rho}{\partial t} = 0 \qquad \frac{d\rho}{dt}$$

$$\boxed{\frac{d\rho}{dt} = \frac{d}{dt}\rho(p(t), q(t), t)}$$

$$\{\rho, H\} = 0 \Rightarrow \ln \rho = \alpha - \beta H + \vec{\delta} \cdot \underset{0}{\vec{P}} + \vec{\eta} \cdot \underset{0}{\vec{M}}$$

$\rho(p, q)$

$$\prod_{i=1}^{N} d^3 p_i \, d^3 q_i = \prod_{i=1}^{N} d^3 P_i \, d^3 Q_i$$

$$dx\,dy\,dz\,dp_x\,dp_y\,dp_z = dr\,d\theta\,d\phi\,dp_r\,dp_\theta\,dp_\phi$$

$$\rho(x, y, z, p_x, p_y, p_z) = \rho'(r, \theta, \phi, p_r, p_\theta, p_\phi)$$

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}(\vec{j}) = \frac{\partial \rho}{\partial t} + \vec{\nabla}(\rho \vec{v})$$

$$= \frac{\partial \rho}{\partial t} + \frac{\partial(\rho \dot{p})}{\partial p} + \frac{\partial}{\partial q}(\rho \dot{q})$$

$$= \frac{\partial \rho}{\partial t} + \frac{\partial \rho}{\partial p}\dot{p} + \frac{\partial \rho}{\partial q}\dot{q}$$

$$+ \rho \underbrace{\left(\frac{\partial \dot{p}}{\partial p} + \frac{\partial \dot{q}}{\partial q}\right)}_{0}$$

$$= \underline{\frac{d\rho}{dt}} + 0$$

$$\dot{p} = -\frac{\partial H}{\partial q}$$

$$\frac{\partial \dot{p}}{\partial p} = -\frac{\partial^2 H}{\partial p \partial q} = -\frac{\partial^2 H}{\partial q \partial p} = -\frac{\partial \dot{q}}{\partial q}$$

$$\frac{d\rho}{dt} = \frac{d}{dt}\boxed{\rho(p(t), q(t), t)}$$

$$\underline{P_r} = \frac{n_r}{N}$$

$$N = \int \rho \, dw$$

$$n_r \simeq \rho(q_r, p_r)\Delta w$$

sums

- summing over members of the ensemble
- summing over allowed microstates of a given system

– summing over single particle states.

$$f(z) = \sum a_n z^n$$

$n \in$ integers ⇐

$$a_n = \frac{1}{2\pi i} \oint \frac{f(z)}{z^{n+1}}$$

$z^{1/2}$ has a branch cut

$$\sum a_n z^{E_i}$$

$$3.1\ \text{eV} = \underline{31} \left(\frac{1}{10}\text{eV}\right) \qquad 3.4\ \text{eV} = \underline{34} \left(\frac{1}{10}\text{eV}\right)$$

$$\psi(q_i, t) = e^{-\frac{i}{\hbar}Et}\, \psi(q_i)$$