You will receive your graded midterm papers by <u>e-mail</u>.

$\underline{2^{nd} \text{ Midterm Date:}}$ Sat 14, 2019

Start 10:00

Duration 3+ hrs.

$\varepsilon = cp^n$

$\underline{n = 2}$ quantum # of particles $= \{\alpha_1, \alpha_2, \alpha_3, \dots\}$ ⇐ microstate

$$Q_N = \frac{1}{N!} \sum_{\text{micro states}} e^{-\beta E_r}$$

$$\underline{E = \sum_i \varepsilon_i}$$ ⇐ particle index

$\varepsilon_1 = \varepsilon_3$

$$Q_N = \frac{1}{N!} Q_1^N$$

$$E = \sum_i n_i \varepsilon_i (g_i)$$

⇐ single particle states

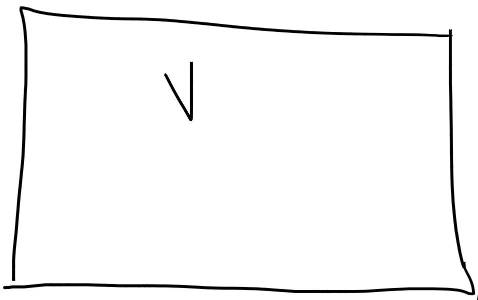
$\boxed{\{n_i\}}$: microstate

- sum over microstates of the system
- sum over the energies of the system
- sum over single particle states
- sum over energies of single particle states

$$\mathcal{Q} = \sum e^{-\beta(E-\mu N)} = \sum_{N E} \underbrace{g_N(E)\, e^{-\beta(E-\mu N)}}_{}$$

N particle density of states

$$E = \sum n_i \varepsilon_i = \sum_{\varepsilon} \underbrace{g(\varepsilon)}_{\text{single particle density of states}} n(\varepsilon)\, \varepsilon$$



$$Q_N = \sum_{\text{microstates}} e^{-\beta E}$$

$$E = \sum_i \varepsilon_i n_i$$

$$\boxed{Q_N = \sum_{\{n_i\}} \left(e^{-\beta \varepsilon_i}\right)^{n_i} \qquad \sum_i n_i = N \Leftarrow}$$

$$\Rightarrow \mathcal{Q} = \sum_{N=0}^{N_\infty} z^N Q_N$$

$$N_\infty = \begin{cases} 1 & \text{for fermions} \\ \infty & \text{for bosons} \end{cases}$$

$\boxed{V \quad (N, \mu) \quad T}$

the particles in the single particle state r.

$$Q_N = \sum e^{-\beta N \varepsilon_r} = e^{-\beta N \varepsilon_r}$$

$$\mathcal{Q} = \sum_{N=0}^{N\infty} z^N \left(e^{-\beta \varepsilon_r}\right)^N =$$

$$\mathcal{Q} = \begin{cases} 1 + z e^{-\beta \varepsilon_r} & \text{for Fermions} \\ \dfrac{1}{1 - z e^{-\beta \varepsilon_r}} & \text{for Bosons} \end{cases}$$

$$\langle n \rangle = \begin{cases} n^{F.D} & \text{for fermions} \\ n^{B.E} & \text{for bosons} \end{cases}$$

$$n^{F.D.} = \frac{1}{e^{\beta(\varepsilon - \mu)} + 1} \qquad n^{B.E} = \frac{1}{e^{\beta(\varepsilon - \mu)} - 1}$$

$$n^{MB} = e^{-\beta(\varepsilon - \mu)}$$

$$n^{F.D} \simeq n^{B.E.} \simeq n^{M.B} \quad \text{if} \quad e^{\beta(\varepsilon - \mu)} \gg 1$$

high temperature $\beta \to 0 \quad e^{\beta(\varepsilon - \mu)} \overset{?}{\to} 1$

low temperature $\beta \to \infty \quad e^{\beta(\varepsilon - \mu)} \overset{?}{\to} \begin{cases} 0 & \varepsilon < \mu \\ \infty & \varepsilon > \mu \end{cases}$

$$N = \sum_{\substack{\text{single} \\ \text{particle} \\ \text{states}}} n(\varepsilon, T, \mu) \Rightarrow \mu = \mu(T, V, N)$$

$$\boxed{\mu = \mu(T, N/V)}$$

For bosons

$$\frac{1}{e^{\beta(\varepsilon-\mu)} - 1} < \infty \Rightarrow e^{\beta(\varepsilon-\mu)} \neq 1$$

$$\Rightarrow \varepsilon \neq \mu$$

$$\Rightarrow \mu \neq \varepsilon \quad \text{for any single particle energy}$$

$$\mu < 0$$

<u>microcanonical ensemble</u>

$$N, V, E = U$$

$$\frac{1}{T} = \left(\frac{\partial S}{\partial U}\right)_{N,V} \quad \left(\frac{\partial S}{\partial V}\right)_{U,N} = \frac{P}{T}$$

$$\left(\frac{\partial S}{\partial N}\right)_{U,V} = -\frac{\mu}{T}$$

<u>canonical ensemble</u>

$$N, V, T \Rightarrow A = E - TS$$

$$\left(\frac{\partial A}{\partial T}\right)_{N,V} = -S$$

$$\left(\frac{\partial A}{\partial V}\right)_{T,N} = -P$$

$$\left(\frac{\partial A}{\partial N}\right)_{T,V} = \mu$$

grand canonical ensemble

$$\phi = -kT \ln \mathcal{Q} \qquad \mu, V, T$$

$$\left(\frac{\partial \phi}{\partial \mu}\right)_{V,T} = -N \quad , \; - \ldots$$

$$\left(\frac{\partial \mu}{\partial T}\right)_{N,V} \overset{?}{=} -\frac{S}{N}$$

$$G = \mu N = E - TS + PV$$

$$dG = -S\,dT + V\,dP + \mu\,dN$$

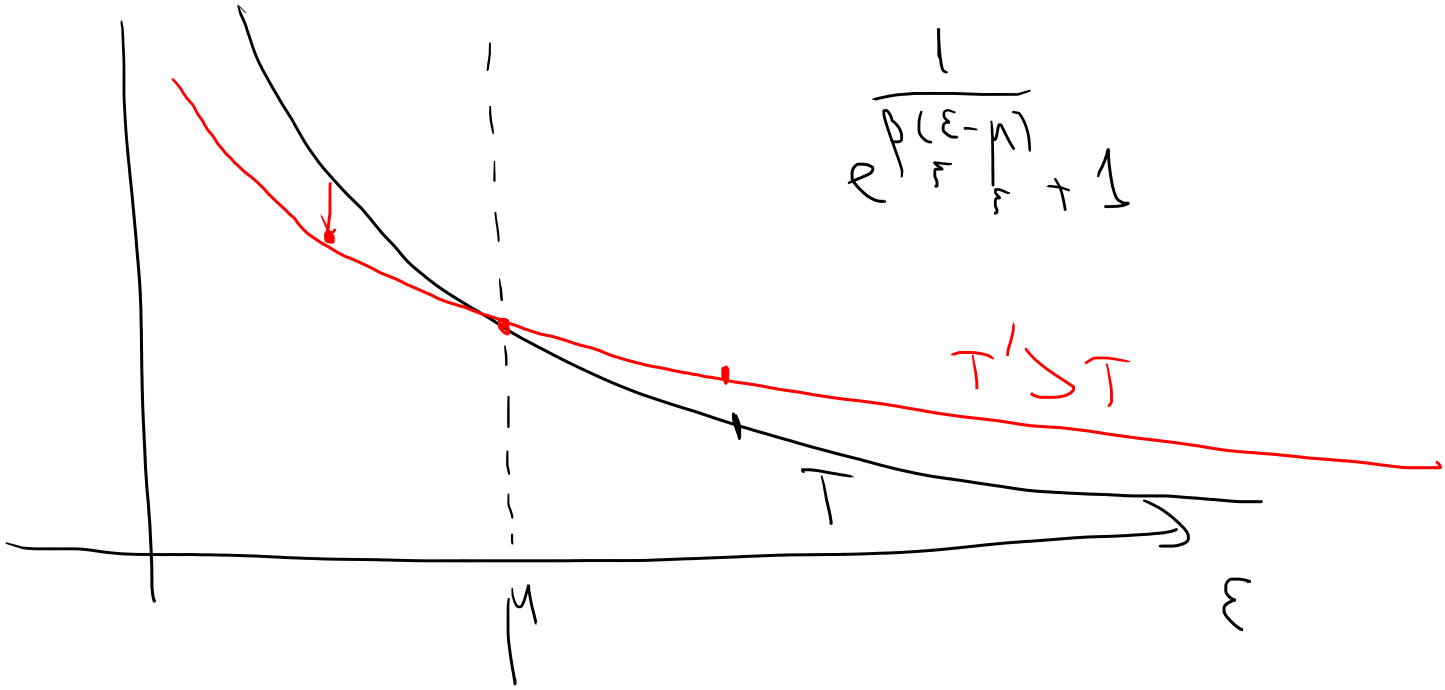
$$= (d\mu)N + (dN)\mu$$

$$d\mu = -s\,dT + v\,dP$$

$$\left(\frac{\partial \mu}{\partial T}\right)_{N,V} = \underbrace{-s} + \underbrace{v}\left(\frac{\partial P}{\partial T}\right)_{N,V}$$

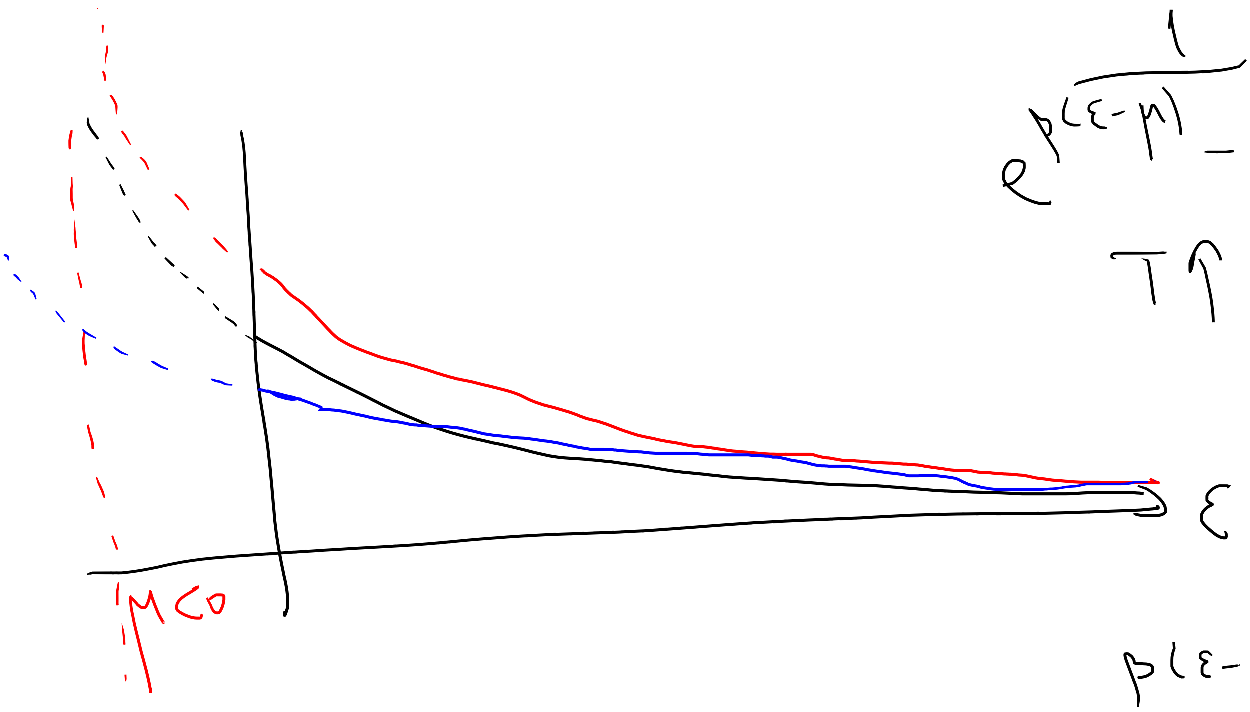
$$\frac{\partial}{\partial T}_{N,V}\left[N = \sum \frac{1}{e^{\beta(\varepsilon-\mu)} + a}\right]$$

$$0 = \left(\frac{\partial}{\partial T}\sum \frac{1}{e^{\beta(\varepsilon-\mu)} + a}\right)_{N,}$$



$$\frac{1}{e^{\beta(\varepsilon-\mu)}+1}$$

$$\sum \frac{1}{e^{\beta(\varepsilon-\mu)}+a} = N \qquad T \Uparrow \qquad \mu \Downarrow$$



$$\frac{1}{e^{\beta(\varepsilon-\mu)}-1}$$

$T \Uparrow \qquad \beta \Downarrow$

$\varepsilon - \mu > 0$

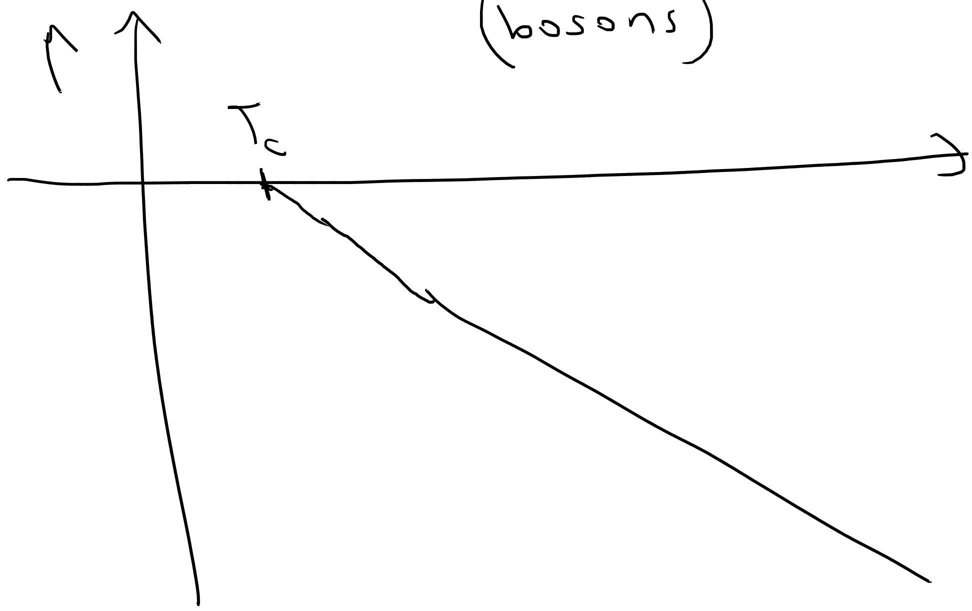
$\beta(\varepsilon-\mu) \Downarrow$

$e^{\beta(\varepsilon-\mu)} \Downarrow$

$\frac{1}{e^{\beta(\varepsilon-\mu)}-1} \Uparrow$ (for fixed ε & μ)

$\Rightarrow \mu \Downarrow$ as $T \Uparrow$

(bosons)



$T_c = 0$ or $T_c > 0$

$e^{\beta(\varepsilon-\mu)} \xrightarrow[T\to\infty]{} \infty$

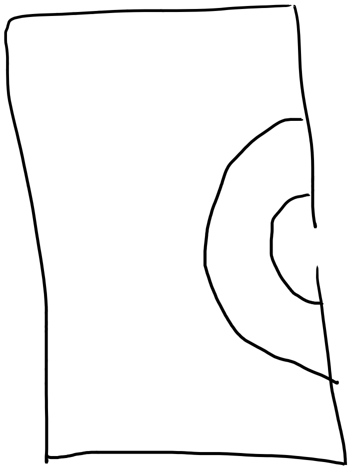
$U = \frac{3}{2}NkT$ violates third law of thermodynamics!

$$S(T,V) \longrightarrow 0 \Rightarrow S(T,V) \simeq T^{\delta} f(V) \quad \delta > 0$$

$$C_V = \left(\frac{dQ}{dT}\right)_V = T\left(\frac{\partial S}{\partial T}\right)_V \xrightarrow{T\to 0} \delta f(V) T^{\delta}$$

$$C_V \xrightarrow{T\to 0} \delta f(V) T^{\delta} \xrightarrow[T\to 0]{} 0$$

$$\boxed{U = \frac{3}{2}NkT} \Rightarrow C_V = \frac{3}{2}Nk$$



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