Example

$$A = (2\pi R^2)\,\frac{\Theta}{2\pi} = \Theta R^2$$

$$\Theta = \frac{A}{R^2} \equiv \Omega$$

$$\frac{\Theta}{A} = \frac{1}{R^2} = \text{curvature of sphere}$$

$$H(p,q;\lambda(t)) \equiv H(t)$$

$$H(p,q;\lambda)\,\psi_n(x,\lambda) = E_n(\lambda)\,\psi_n(x,\lambda)$$

$$H(p,q;\lambda(t))\,\psi_n(x,\lambda(t)) = E_n(\lambda(t))\,\psi_n(x,\lambda(t))$$

$$\psi_n(x,\lambda(t)) \equiv \psi_n(x,t)$$

$$\Psi(x,t=0) = \psi_n(x,t=0)$$

$$\Psi(x,t) = \psi_n(x,t)\,e^{-\frac{i}{\hbar}\int_0^{t} E_n(t')\,dt'}\;e^{-\frac{i}{\hbar}\gamma_n(t)}$$

$$\gamma_n(t) = ?$$

$$H(t)\Psi = i\hbar\frac{\partial\Psi}{\partial t}$$

$$\cancel{E_n\psi_n(x,t)\,e^{-\frac{i}{\hbar}\int()}\,e^{-i\gamma_n(t)}}$$

$$= i\hbar\,\frac{\partial\psi_n(x,t)}{\partial t}\,\cancel{e^{(\,)}}\,\cancel{e^{(\,)}} + \cancel{E_n(t)\,\psi_n(x,t)\,e^{(\,)}\,e^{(\,)}}$$

$$+\,\psi_n\,\cancel{e^{(\,)}}\cdot\frac{d\gamma_n}{dt}\,\cancel{e^{(\,)}}$$

$$\int d^3x \left(i\hbar \frac{\partial \psi_n(x,t)}{\partial t} + \psi_n \frac{d\gamma_n}{dt} = 0 \right) \psi_n^*(x,t)$$

$$\frac{d\gamma_n}{dt} = -i\hbar \langle \psi_n | \frac{\partial}{\partial t} | \psi_n \rangle$$

$$= -i\hbar \langle \psi_n | \frac{\partial}{\partial \lambda_i} | \psi_n \rangle \frac{d\lambda_i}{dt}$$

$$\boxed{d\gamma_n = -i\hbar \langle \psi_n | \frac{\partial}{\partial \lambda_i} | \psi_n \rangle \, d\lambda_i}$$

$$\gamma_n(t) = -i\hbar \int_{\vec{\lambda}(t=0)}^{\vec{\lambda}(t)} \langle \psi_n | \vec{\nabla}_{\lambda_i} | \psi_n \rangle \cdot d\vec{\lambda}$$

$$\Psi(x,t) = \psi_n(x,t)\, e^{-\frac{i}{\hbar}\int_0^t E_n(t')dt'}\, e^{-\frac{i}{\hbar}\gamma_n(t)}$$

$$= \psi_n(x,t)\, e^{-\frac{i}{\hbar}\int^t E_n(t')dt'}\, e^{i\left[\frac{i}{\hbar}\int \langle \psi_n | \vec{\nabla}_{\lambda_i} | \psi_n \rangle \cdot d\vec{\lambda}\right]}$$

$$\frac{d}{d\lambda_i}\left(\langle \psi_n | \psi_n \rangle = 1 \right)$$

$$\underbrace{\left(\frac{d}{d\lambda_i} \langle \psi_n | \right) | \psi_n \rangle}_{} + \langle \psi_n | \frac{d}{d\lambda_i} | \psi_n \rangle = 0$$

$$\underbrace{\left(\langle \psi_n | \frac{d}{d\lambda_i} | \psi_n \rangle \right)^* + \langle \psi_n | \frac{d}{d\lambda_i} | \psi_n \rangle}_{} = 0$$

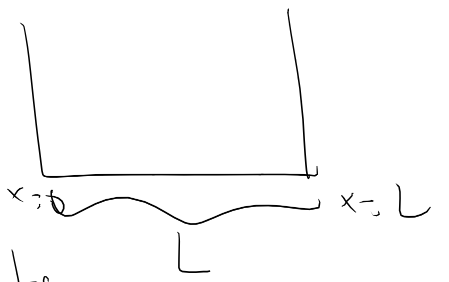
$$2\,\mathrm{Re}\left(\langle \psi_n | \frac{d}{d\lambda_i} | \psi_n \rangle \right) = 0$$

$\Rightarrow \langle \psi_n | \frac{d}{d\lambda_i} | \psi_n \rangle$ is pure imaginary

$$\underbrace{i\oint \langle \psi_n | \vec{\nabla}_\lambda | \psi_n \rangle \cdot d\vec{\lambda}}_{\text{Berry Phase}} \neq 0$$

$$= i\int \left(\vec{\nabla}_\lambda \times \langle \psi_n | \vec{\nabla}_\lambda | \psi_n \rangle\right) \cdot d\vec{A}$$

<u>Example</u> Infinite Well



$$\psi_n = \sqrt{\frac{2}{L}}\,\sin\left(\frac{n\pi}{L}x\right)$$

$$L = L(t)$$

$$\gamma(t) = i\int_{L_i}^{L_f} \langle \psi_n | \frac{\partial}{\partial L} | \psi_n \rangle \, dL$$

$$= i\int_{L_i}^{L_f} dL \int_0^L dx \left(\frac{2}{L}\right) \sin\left(\frac{n\pi}{L}x\right)\left(-\frac{n\pi}{L^2}x\right)\cos\left(\frac{n\pi}{L}x\right)$$

$$= i\int_{L_i}^{L_f} dL \left(\frac{-2}{L^2}\right) \int_0^L dx \, \sin\left(\frac{n\pi}{L}x\right)\left(\frac{n\pi}{L}x\right)\cos\left(\frac{n\pi}{L}x\right)$$

$$\frac{n\pi}{L}x = y \Rightarrow x = \frac{L}{n\pi}y \Rightarrow dx = \frac{L}{n\pi}dy$$

$$= i\int_{L_i}^{L_f} dL \left(\frac{-2}{L^{\cancel{2}}}\right) \frac{\cancel{L}}{n\pi} \int_0^{n\pi} dy \, \sin(y)\cos(y)\, y$$

$$= \frac{-2i}{n\pi} \ln\left(\frac{L_f}{L_i}\right) \int_0^{n\pi} dy \, \sin(y)\cos(y)\, y$$

$$\int_0^{n\pi} dy\, y \sin y \cos y$$

$$= \int_0^{n\pi} dy\, \frac{y}{2} \sin(2y) = \int_0^{n\pi} dy\, \frac{y}{2} \frac{d}{dy} \frac{\cos 2y}{(-2)}$$

$$= -\frac{1}{4}\left[y \cos 2y \Big|_{y=0}^{n\pi} - \int_0^{n\pi} dy \cos 2y \right]$$

$$= -\frac{1}{4}\left[n\pi - \cancel{\frac{\sin 2y}{2}\Big|_{y=0}^{n\pi}} \right]$$

$$\underset{=0}{}$$

$$= -\frac{n\pi}{4}$$

$$\gamma_n = -2i \ln\left(\frac{L_f}{L_i}\right)\left(\frac{-n\pi}{4}\right)\frac{1}{n\pi}$$

$$\boxed{\gamma_n = \frac{i}{2} \ln\left(\frac{L_f}{L_i}\right)}$$

Correction take the derivative of the normalization

$$\gamma_n(t) = i \int_{L_i}^{L_f} dL \int_0^L dx\, \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi}{L} x\right) \frac{d}{dL}\left(\sqrt{\frac{2}{L}} \sin\left(\frac{n\pi}{L} x\right)\right)$$

$$\gamma_n(t) = i \int_{L_i}^{L_f} dL \int_0^L dx\, \sqrt{\frac{2}{L}}^{\sin\left(\frac{n\pi}{L}x\right)} \left(-\frac{1}{2L}\right) \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi}{L} x\right)$$

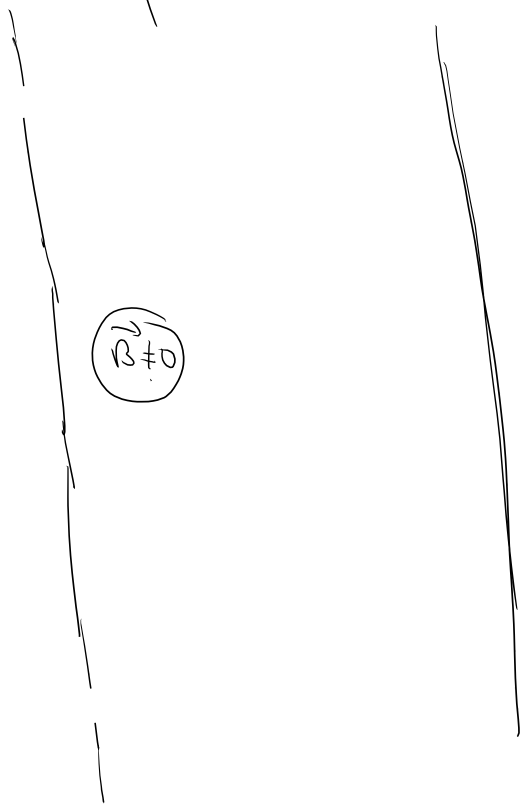
+ previous term

$$= i \int_{L_i}^{L_f} dL \left(\frac{-1}{2L}\right) \quad = -\frac{i}{2} \ln\left(\frac{L_f}{L_i}\right)$$

$$= -\frac{i}{2}\ln\left(\frac{L_f}{L_i}\right) + \frac{i}{2}\ln\left(\frac{L_f}{L_i}\right)$$

$$\boxed{\gamma_n(t) = 0}$$

Can the phase be measured? Yes!



Aharonov-Bohm effect.