Conductors

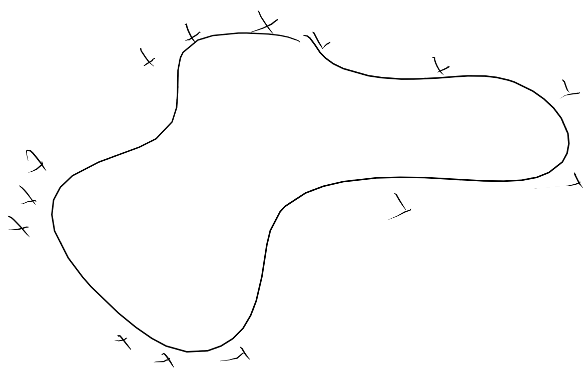
Statics

i) $\vec{E} = 0$ inside a conductor

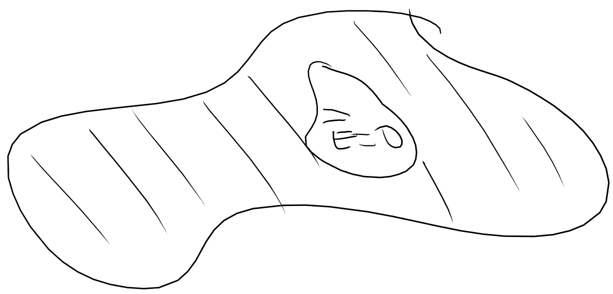
ii) $\vec{\nabla} \cdot \vec{E} = 0 = \frac{\rho}{\varepsilon_0} \Rightarrow$ no charges inside a conductor

iii) all the charge is on the surface of the conductor!

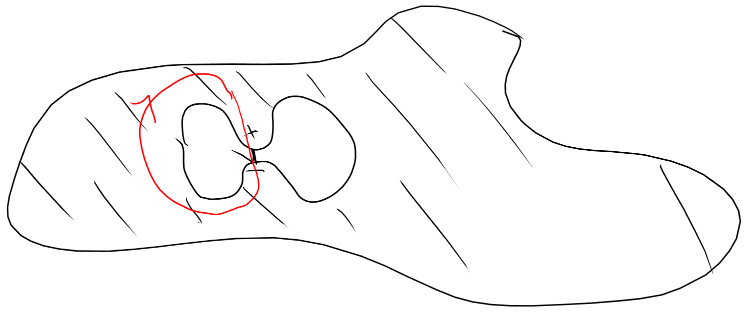
Ex



Ex



Ex

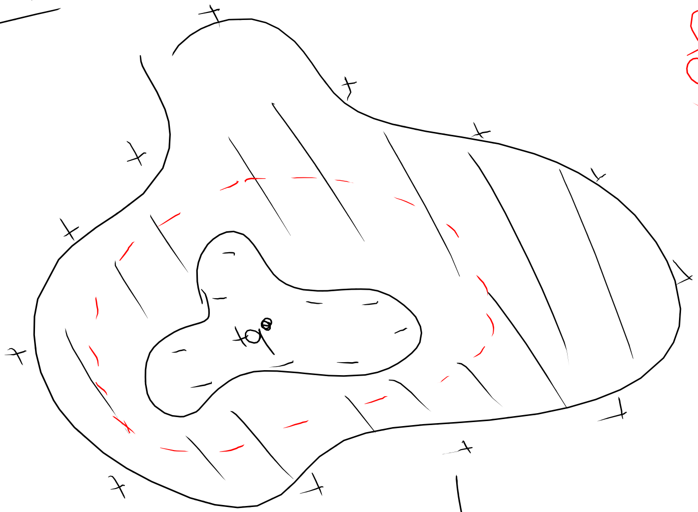


$\oint \vec{E} \cdot d\vec{l} \neq 0$

but

$\vec{\nabla} \times \vec{E} = 0$

Ex



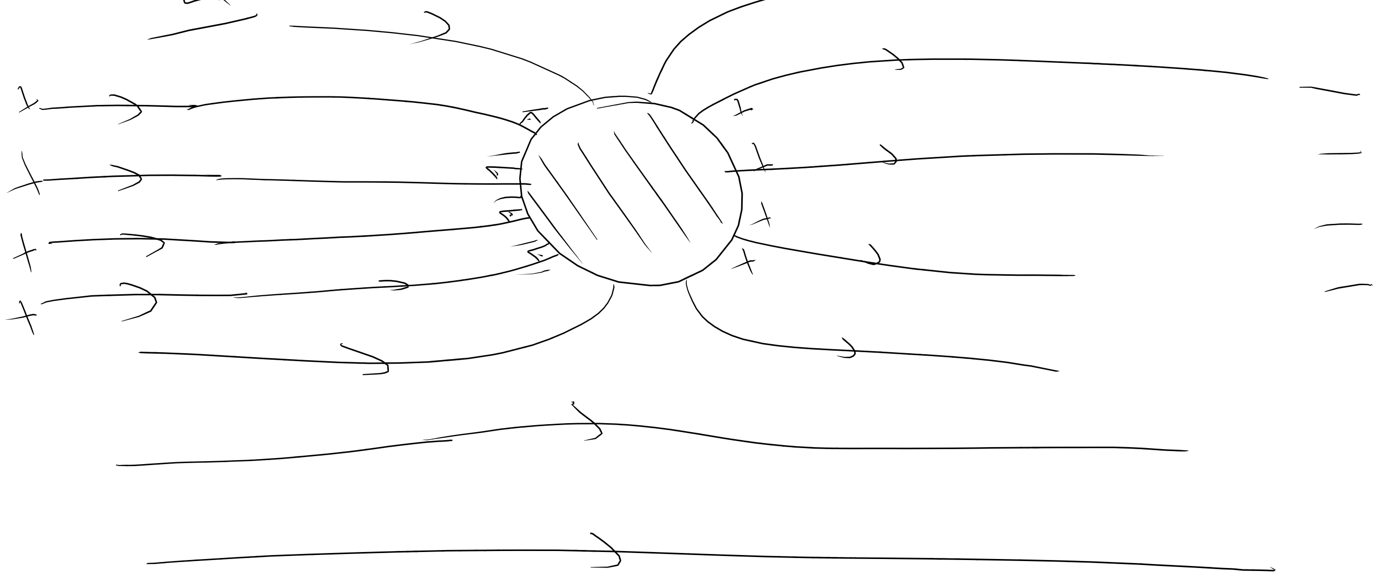
$$\oint \vec{E} \cdot d\vec{S} = 0 = \frac{Q_{enc}}{\varepsilon_0}$$

$q_{\text{inner surface}} = -q$

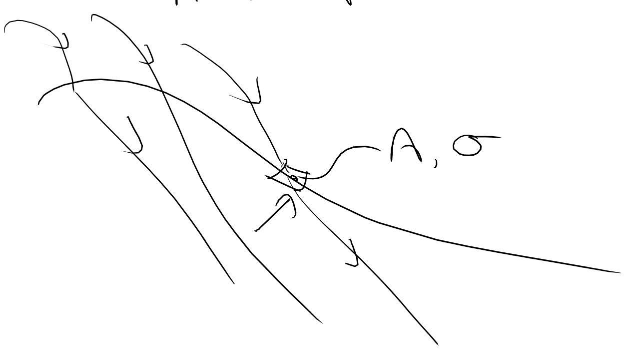
Ex

$\vec{E}$ is always perpendicular to the surface of the conductor.

Ex



$$P = \lim_{A\to 0}\frac{F}{A} = \lim_{A\to 0}\frac{qE}{A} = \lim_{A\to 0}\frac{\sigma \cancel{A} E}{\cancel{A}} = \sigma E$$



$P = \sigma E$

$E = ?$

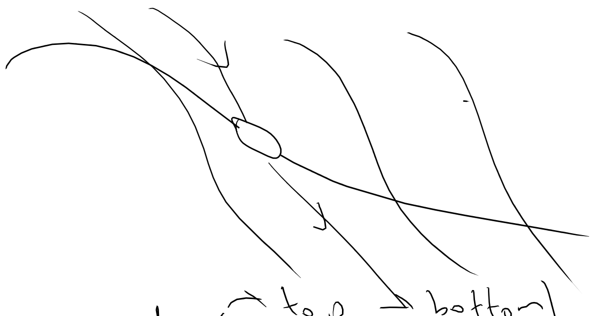
$$E = \frac{E_{top} + E_{bottom}}{2}$$

$$\boxed{\vec{E}_{top} - \vec{E}_{bottom} = \frac{\sigma}{\varepsilon_0}\hat{n}}$$

$$\vec{E} = \vec{E}_A + \vec{E}_{other}$$

E_{other} is continuous

$$\vec{E}_A^{\,top/bottom} = \pm\frac{\sigma}{2\varepsilon_0}\hat{n} \Leftarrow$$



$$\vec{E}_{other} = \frac{1}{2}\left(\vec{E}^{top} + \vec{E}^{bottom}\right)$$

$$P = \sigma E_{other}$$

For a conductor, $\hat{n}$ pointing out of the conductor

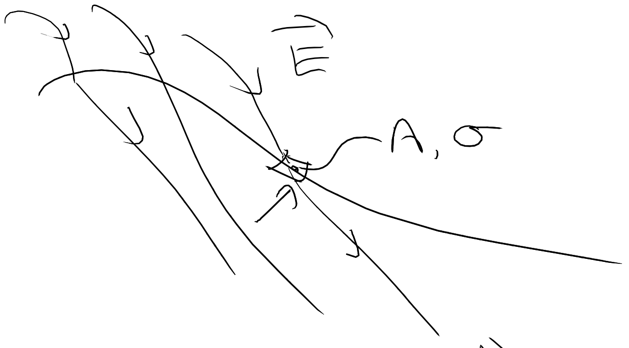
$$\vec{E}^{bottom} = 0$$

$$\vec{E}^{top} = \frac{\sigma}{\varepsilon_0}\hat{n}$$

$$P = \sigma\frac{1}{2}|\vec{E}^{top}| = \frac{\sigma^2}{2\varepsilon_0} = \boxed{\frac{1}{2}\varepsilon_0 \vec{E}^2 = P}$$

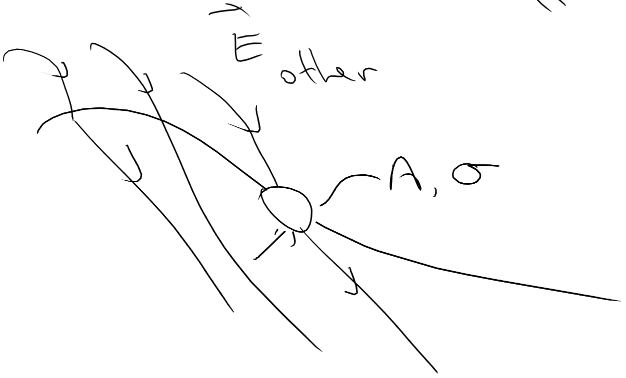
$$[P] = \frac{N}{m^2} \qquad [u] = \frac{J}{m^3} = \frac{Nm}{m^3} = \frac{N}{m^2}$$

$$\boxed{P = \frac{1}{2}\varepsilon_0 \vec{E}^2}$$



$\vec{E} = \vec{E}_{other} + \vec{E}_A$

$\parallel$



$+$

$\vec{E}_A$

Uniqueness Thm

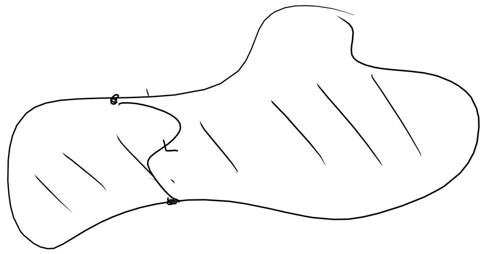
if $\left\{ \begin{array}{l} \vec{\nabla} \cdot \vec{E},\ \vec{\nabla} \times \vec{E} \text{ and B.C on } \vec{E} \\ \nabla^2 \phi \text{ and B.C on } \phi \end{array} \right\}$ are known,

$\vec{E}$ (ϕ) is unique!

Question

a) $\frac{Q}{4}$, $\frac{3Q}{4}$?

b) $> \frac{Q}{4}$, $< \frac{3Q}{4}$?



$$\rho_Q(\vec{r}) = Q\tilde{\rho}(\vec{r})$$

$$\int d^3r\, \rho_Q(\vec{r}) = Q \Rightarrow \int \tilde{\rho}(\vec{r})\, d^3\vec{r} = 1$$

$\nabla^2 \Phi_Q = 0$ everywhere except the surface of the conductor

$\Phi_Q = \text{const}$ on the surface

$$\oint dS \frac{\partial \Phi_Q}{\partial n} = -\frac{Q}{\varepsilon_0}$$

Φ_Q is known. What is the soln. of

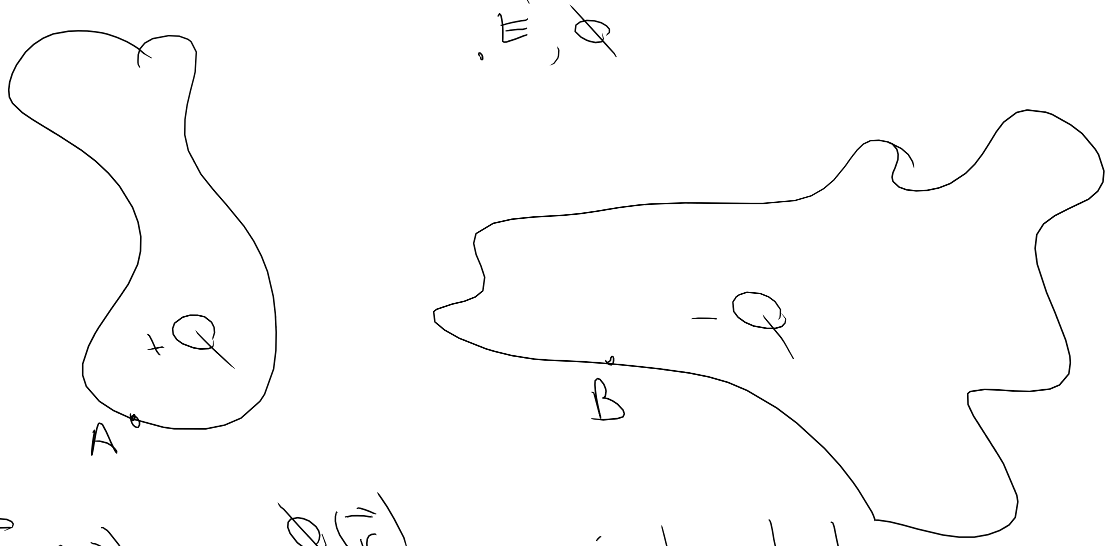
$\nabla^2 \Phi_{2Q} = 0$ outside

$\Phi_{2Q} = \text{const}$ on the surface

$$\oint dS \frac{\partial \Phi_{2Q}}{\partial n} = -\frac{2Q}{\varepsilon_0}$$

$$\Phi_{2Q} = 2\Phi_Q$$

$\vec{E}, \Phi$



$$\frac{\vec{E}(\vec{r})}{Q}, \quad \frac{\Phi(\vec{r})}{Q}$$ are independent of Q

$$\frac{V_A - V_B}{Q} = \frac{\Phi(\vec{r}_A) - \Phi(\vec{r}_B)}{Q}$$: independent of Q

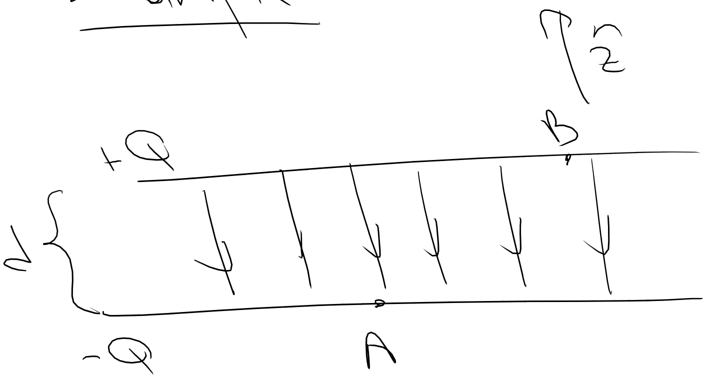
$$\frac{\Delta V}{Q} \equiv \frac{1}{C}$$ depends only on the geometry of the conductors

C : capacitance

Electrical Breakdown!

By definition $\Delta V, Q, C > 0$

Example



$$\vec{E} = -\frac{\sigma}{\varepsilon_0}\hat{z}$$

$$\Delta V = \int_A^B \vec{E}\cdot d\vec{l}$$

$$= \vec{E}\cdot\Delta\vec{l}$$

$$\Delta V = \frac{\sigma}{\varepsilon_0} d$$

S: surface area of plates

$$Q = \sigma S$$

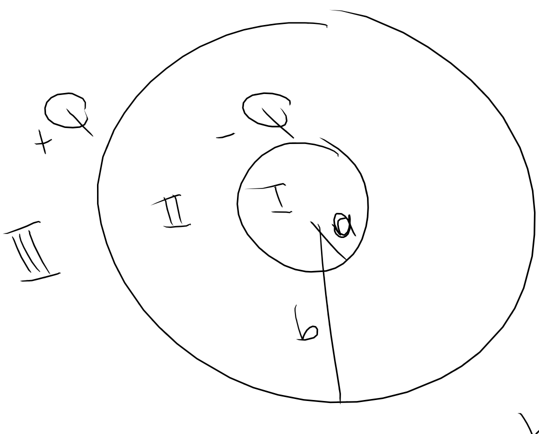
$$\frac{1}{C} = \frac{\Delta V}{Q} = \frac{\cancel{\sigma}}{\varepsilon_0} d \, \frac{1}{\cancel{\sigma} S}$$

$$\boxed{C = \varepsilon_0 \frac{S}{d}}$$

$$\boxed{Q = C\,\Delta V} \qquad \boxed{Q = \text{cevat}}$$

Example

Spherical Capacitor



$$\vec{E}_{I} = 0$$

$$\vec{E}_{III} = 0$$

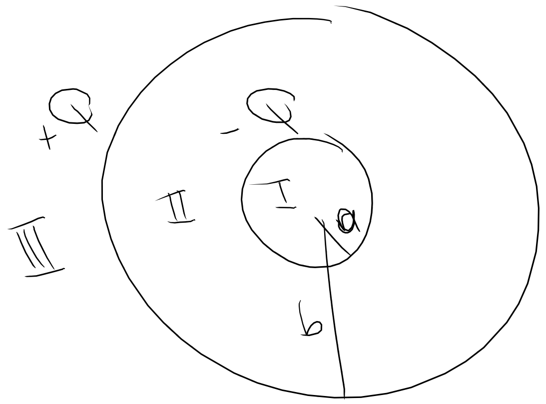
$$\vec{E}_{II} = \vec{E}_{outer} + \vec{E}_{inner}$$

$$\vec{E}_{II} = 0 + \frac{1}{4\pi\varepsilon_0}\frac{-Q}{r^2}\hat{r}$$

$$\Delta V = \int \vec{E}\cdot d\vec{l} = \int_a^b \frac{1}{4\pi\varepsilon_0}\frac{-Q}{r^2}\,dr = \frac{1}{4\pi\varepsilon_0} Q\left(\frac{1}{b} - \frac{1}{a}\right)$$

$$|\Delta V| = \left| Q \frac{1}{4\pi\varepsilon_0} \frac{a-b}{ab} \right| \equiv \frac{Q}{C}$$

$$\boxed{C = 4\pi\varepsilon_0 \frac{ab}{b-a}}$$

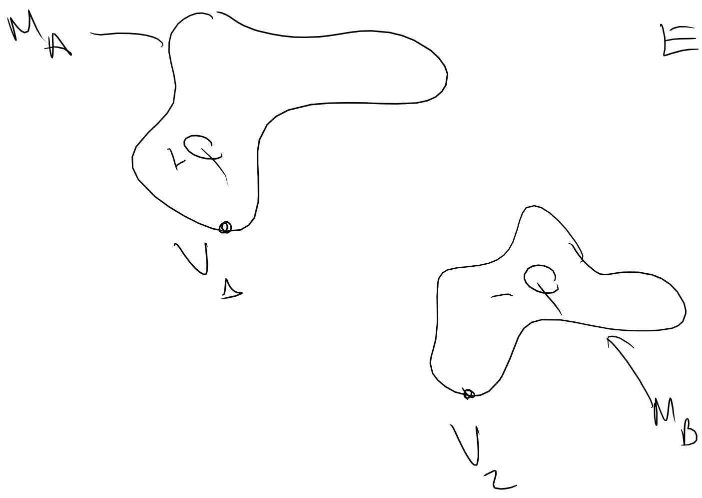


$$\vec{E}_{III} = \vec{E}_{outer} + \vec{E}_{inner}$$

$$= \left(\frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \hat{r}\right) + \left(\frac{1}{4\pi\varepsilon_0} \frac{-Q}{r^2} \hat{r}\right)$$

$$= 0$$

Ex energy stored in a capacitor.



$$E = \frac{1}{2} \sum q_i V_i$$

$$= \frac{1}{2} \sum_{q_i \in M_A} q_i V_i + \frac{1}{2} \sum_{q_i \in M_B} q_i V_i$$

$$= \frac{1}{2} \sum_{q_i \in M_A} q_i V_1 + \frac{1}{2} \sum_{q_i \in M_B} q_i V_2$$

$$= \frac{1}{2}(+Q)V_1 + \frac{1}{2}(-Q)V_2$$

$$E = \frac{1}{2} Q (V_1 - V_2)$$

$$= \frac{1}{2} Q \frac{Q}{C} = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} C (\Delta V)^2$$

$$E = \frac{1}{2} C (\Delta V)^2 = \int_{\text{all space}} d^3r \left(\frac{1}{2} \varepsilon_0 \vec{E}^2\right)$$