~~1) 1D $\delta()$ potential~~

2) Finite potential well $V(\vec{r}) = \begin{cases} V_0 < 0 & r < a \\ 0 & r > a \end{cases}$

3) spherical shell $V(\vec{r}) = \lambda \delta(r-a)$

<u>Example</u>

$$V(x) = \lambda \delta(x) \qquad \lambda < 0$$

$$-\frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} + \lambda \delta(x)\psi = E\psi$$

$$E = -\frac{\hbar^2 K^2}{2m}$$

$$\psi = N e^{-K|x|}$$

$$\int_{0^-}^{0^+} \left(-\frac{\hbar^2}{2m} \underline{\frac{d}{dx}\psi'} + \lambda \delta(x)\psi \right) dx = \int_{0^-}^{0^+} E\psi \, dx$$

$$-\frac{\hbar^2}{2m}\left(\psi'(0^+) - \psi'(0^-)\right) + \lambda \psi(0) = 0$$

$$\psi'(0^+) = \frac{d}{dx}\left(N e^{-Kx}\right)\Big|_{x=0^+} = -NK$$

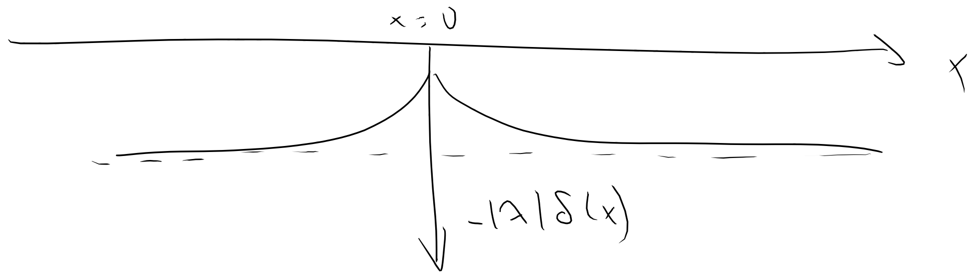
$$\psi'(0^-) = \frac{d}{dx}\left(N e^{-K(-x)}\right)\Big|_{x=0^-} = NK$$

$$-\frac{\hbar^2}{2m}\left(-\cancel{N}K - \cancel{N}K\right) + \lambda \cancel{N} = 0$$

$$\frac{\hbar^2}{m}K + \lambda = 0 \Rightarrow K = -\lambda \frac{m}{\hbar^2} > 0$$

$$\psi = N e^{-|\lambda| \frac{m}{\hbar^2}|x|}$$

$$E = -\frac{\hbar^2}{2m}\frac{\lambda^2 m^2}{\hbar^4} = -\frac{\lambda^2}{2\hbar^2}$$



$$\psi(x) = \psi_0(x) + \int_{-\infty}^{\infty} G(x-x')V(x')\psi(x')$$

$$\left(-\frac{\hbar^2}{2m}\nabla^2 - E\right) G = \delta(x-x')$$

$$\psi(x) = \psi_0(x) - \frac{im}{k}\int_{-\infty}^{\infty} dx'\, e^{ik|x-x'|}\psi(x')V(x')$$

$$V(x') = \lambda\delta(x')$$

$$\psi(x) = \psi_0(x) - \frac{im\lambda}{k}e^{ik|x|}\psi(0)$$

$$\psi(0) = \psi_0(0) - \frac{im\lambda}{k}\psi(0)$$

$$\psi(0) = \frac{1}{1+\frac{im\lambda}{k}}\psi_0(0)$$

$$\Rightarrow \psi(x) = \psi_0(x) - \frac{im\lambda}{k}e^{ik|x|}\frac{1}{1+\frac{im\lambda}{k}}\psi_0(0)$$

$$\psi(x) = e^{ikx} + f e^{ik|x|}$$

$$\boxed{f = \frac{i m \lambda}{k + i m \lambda}}$$

f diverges when $k = -i m \lambda$

$$\psi(x) = \psi_0(x) + f e^{m\lambda|x|}$$

normalized $\longrightarrow$ $e^{2m\lambda|x|}$

$$\psi = N e^{-|\lambda| \frac{m}{\hbar^2} |x|}$$

$$K = \frac{k^2}{2m} = \frac{m\lambda^2}{2}$$

$$\psi(\vec{r}) = \psi_0(\vec{r}) + f_k(\Omega) \frac{e^{ikr}}{r}$$

$k = iK \; ; \; K > 0$

$f_{iK} \longrightarrow \infty$

$N \propto |\psi|^2$

$$\frac{dN}{dt} = -\Gamma N \Rightarrow \psi(t) \propto e^{i(Et - \frac{\Gamma}{2}t} = e^{i(E + i\frac{\Gamma}{2})t}$$

$e^{2i\delta_l}$: s-matrix element

$$\langle \text{final state} | \underbrace{S(t_f, t_i)} | \text{initial state} \rangle$$

$S = 1 + iT$: t-matrix

$$(H^0 + V)|\psi\rangle = E|\psi\rangle$$

$$(H^0 - E)|\psi\rangle = -V|\psi\rangle$$

$$|\psi\rangle = |\psi^0\rangle + \frac{1}{E - H^0} V|\psi\rangle$$

$$(E - H^0)|\psi^0\rangle = 0$$

$$G(\vec{r}, \vec{r}\,') = \langle \vec{r} | \frac{1}{E - H^0} | \vec{r}\,' \rangle$$

$$V|\psi\rangle \equiv T|\psi^0\rangle$$

$$|\psi\rangle = |\psi^0\rangle + \frac{1}{E - H^0} T|\psi^0\rangle$$

$$\underbrace{V|\psi\rangle}_{T|\psi^0\rangle} = V|\psi^0\rangle + V\frac{1}{E - H^0} T|\psi^0\rangle$$

$$\Rightarrow \boxed{T = V + V\frac{1}{E - H^0} T}$$

$$T - V \frac{1}{E - H^0} T = V$$

$$\left(1 - V \frac{1}{E - H^0}\right) T = V$$

$$(E - H^0 - V) \frac{1}{E - H^0} T = V$$

$$(E - H) \frac{1}{E - H^0} T = V$$

$$\boxed{T = (E - H^0) \frac{1}{(E - H)} V}$$

$$T = V + V \frac{1}{E - H^0} T$$

$$\langle x' | T | x \rangle = \langle x' | V | x \rangle + \langle x' | V \frac{1}{E - H^0} T | x \rangle$$

$$= \lambda \delta(x) \delta(x - x')$$

$$+ \lambda \delta(x') \langle x' | \frac{1}{E - H^0} T | x \rangle$$

$$= \lambda \delta(x) \delta(x') + \lambda \delta(x') \langle x' | \frac{1}{E - H^0} T | x \rangle$$

$$\langle x' | T | x \rangle = \lambda \delta(x) \delta(x') + \lambda \delta(x') \int dx'' \langle x' | \frac{1}{E - H^0} | x'' \rangle \underbrace{\langle x'' | T | x \rangle}_{\delta(x'') f(x)}$$

$$\langle x' | T | x \rangle = \delta(x') f(x)$$

$$f(x) = \lambda \delta(x) + \lambda \langle 0 | \frac{1}{E - H^0} | 0 \rangle f(x)$$

$$f(x) = \frac{\lambda \delta(x)}{1 - \lambda \langle 0 | \frac{1}{E-H^0} | 0 \rangle}$$

$$\boxed{\langle x' | T | x \rangle = \frac{\lambda \delta(x) \delta(x')}{1 - \lambda \langle 0 | \frac{1}{E-H^0} | 0 \rangle}}$$

$$\langle p' | T | p \rangle = \frac{\lambda \delta(p - p')}{1 - \lambda \langle 0 | \frac{1}{E-H^0} | 0 \rangle} \quad f$$