1)

$$\gamma_n = i \sum_i \oint \langle \psi_n | \frac{\partial}{\partial \lambda_i} | \psi_n \rangle \, d\lambda_i$$

$$\lambda_i = \lambda_i(\delta_j)$$

$$\frac{\partial}{\partial \delta_i} = \frac{\partial \lambda_j}{\partial \delta_i} \frac{\partial}{\partial \lambda_j}$$

$$\gamma_n = i \sum_i \oint \langle \psi_n | \frac{\partial}{\partial \delta_i} | \psi_n \rangle \, d\delta_i$$

$$= i \sum_i \oint \langle \psi_n | \frac{\partial}{\partial \lambda_i} | \psi_n \rangle \underbrace{\frac{\partial \lambda_i}{\partial \delta_i} \, d\delta_i}_{d\lambda_i}$$

$$\boxed{\gamma_n = i \sum_i \oint \langle \psi_n | \frac{\partial}{\partial \lambda_i} | \psi_n \rangle \, d\lambda_i}$$

2) $$V(r) = \begin{cases} V_0 & \text{if } r < a \\ 0 & \text{if } r > a \end{cases}$$

$$-\frac{\hbar^2}{2m}\left(\frac{1}{r^2}\frac{d}{dr} r^2 \frac{d}{dr} - \frac{\overset{l(l+1)}{2}}{r^2}\right) R(r) + V(r)R(r) = ER(r)$$

$$\boxed{r = ax}$$

$$\underline{R(r \gg a)} = \frac{e^{ikr}}{r} + e^{2i\delta_1} \frac{e^{-ikr}}{r}$$

$\hookrightarrow \pm e^{2i\delta_1}$

$$\psi(\vec{r}) = R(r) Y_{lm}$$

3) $V(r) = V_0 e^{-ar}$ $\quad f(0) = \frac{m}{2\pi\hbar^2} \int V d^3r$

$\rightarrow f = ()V(q)$

$$\rightarrow \frac{d\sigma}{d\Omega} = |f|^2 \qquad \sigma = \int \frac{d\sigma}{d\Omega} d\Omega$$

$$V(q) = V_0 \int e^{\frac{i\vec{q}\cdot\vec{r}}{\hbar}} e^{-ar} \frac{d^3q}{(2\pi\hbar)^3} \Leftarrow$$

integral

$$= V_0 \frac{2\pi}{(2\pi\hbar)^3} \int_0^\infty r^2 dr \int_{-1}^{1} d(\cos\theta)\, e^{\frac{iqr\cos\theta}{\hbar}} e^{-ar}$$

$$= V_0 \frac{2\pi}{(2\pi\hbar)^3} \int_0^\infty r^2 dr\, e^{-ar} \frac{\hbar}{iqr} \left(e^{iqr} - e^{-iqr}\right)$$

$$= V_0 \frac{4\pi\hbar}{q(2\pi\hbar)^3} \operatorname{Im} \int_0 \underbrace{r\, dr\, e^{-ar}}_{\left(-\frac{\partial}{\partial a}\right)} e^{iqr}$$

$$= V_0 \frac{4\pi k}{(2\pi k)^3} \frac{1}{q} \,\mathrm{Im} \left(-\frac{d}{da}\right) \frac{1}{a-iq}$$

$$= V_0 \frac{4\pi k}{(2\pi k)^3} \frac{1}{q} \,\mathrm{Im} \frac{1}{(a-iq)^2}$$

$$= V_0 \frac{4\pi k}{(2\pi k)^3} \frac{1}{q} \,\mathrm{Im} \frac{(a+iq)^2}{(a^2+q^2)^2}$$

$$= V_0 \frac{4\pi k}{(2\pi k)^3} \frac{1}{\cancel{q}} \frac{2a\cancel{q}}{(a^2+q^2)^2}$$

$$\boxed{f = \frac{4\pi k}{(2\pi k)^3} V_0 \frac{2a}{(a^2+q^2)^2}}$$

$$q^2 = (\vec{k} - \vec{k}')^2 = k^2 + k'^2 - 2kk' \cos\theta$$

$$\boxed{q^2 = 2k^2(1-\cos\theta)}$$

$$dq^2 = 2k^2 \sin\theta \, d\theta$$

$$= \frac{2k^2}{2\pi} \underbrace{(2\pi \sin\theta \, d\theta)}_{d\Omega}$$

$$d\Omega = \frac{\pi}{k^2} dq^2$$

$$\sigma = \int_{q^2_{min}=0}^{q^2_{max}=4k^2} \frac{\pi}{k^2}\, d(q^2) \left(\frac{4\pi\hbar}{(2\pi\hbar)^3} V_0 \frac{2a}{(a^2+q^2)^2} \right)^2$$

4) $\psi(r) = \psi_0(r) + \int d^3r'\, G(r,r') V(r') \psi(r')$

$G \sim e^{ik|r-r'|}$ (in 1D)

$G \sim \frac{e^{ik|r-r'|}}{|r-r'|}$ (in 3D)

$V(r') = \delta^3(\vec{r}')$

$$\boxed{\psi(r) = \psi_0(r) + \frac{e^{ikr}}{r} \psi(0)}$$

$\psi(0) \sim \infty \leftarrow$ not Bound state

or

$\psi(0) = 0 \leftarrow$ not Bound state

$\delta^3(\vec{r}) = \delta(x)\delta(y)\delta(z)$

$\psi = XYZ$

$$\frac{H\psi}{\psi} = E \Rightarrow \frac{-\hbar^2}{2m}\left(\frac{X''}{X} + \frac{Y''}{Y} + \frac{Z''}{Z} \right) + \delta(x)\delta(y)\delta(z) = E$$

$$\delta^3(\vec{r}) = \delta(r) = \lim_{a\to 0^+} \delta(r-a)$$

5) $$H_0 = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}$$

$$V(t) = \gamma e^{-\alpha t}\,\Theta(t)\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$|\psi(t=0)\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$|\psi(t)\rangle = c_1(t)\, e^{-\frac{i}{\hbar}E_1 t}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_2(t)\, e^{-\frac{i}{\hbar}E_2 t}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}(t) = T e^{-\frac{i}{\hbar}\int_0^t V^I(t')\,dt'}\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}(t=0)$$

$$c_1(t=0) = 1$$
$$c_2(t=0) = 0$$

$$|c_2(t\to\infty)|^2 = ?$$

$$c_2(t) = -\frac{i}{\hbar}\int_0^t e^{\frac{i}{\hbar}(E_1-E_2)t}\,\gamma e^{-\alpha t}$$

$$c_2(t\to\infty) = -\frac{i}{\hbar}\gamma\,\frac{1}{\alpha - i\frac{E_1-E_2}{\hbar}}$$