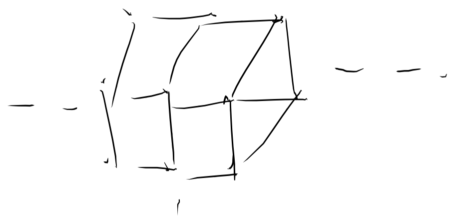
$$\boxed{\psi(x) = \psi_0(x) - \frac{im}{\hbar}\int_{-\infty}^{\infty} dx'\, e^{ik|x-x'|}\, \psi(x')V(x')}$$



$$V(x) = \lambda\,\delta(x-a) + \lambda\,\delta(x+a) \qquad a \geq 0$$

$$\psi(x) = \psi_0(x) - \frac{im}{k}\lambda\left(e^{ik|x-a|}\psi(a) + e^{ik|x+a|}\psi(-a)\right)$$

$$\psi(a) = \psi_0(a) - \frac{im}{k}\lambda\left(\psi(a) + e^{2ika}\psi(-a)\right)$$

$$\psi(-a) = \psi_0(-a) - \frac{im}{k}\lambda\left(\psi(a)e^{2ika} + \psi(-a)\right)$$

$$\psi_0(x) = e^{ikx}$$

$$\psi_0(a) = e^{ika}$$

$$\psi_0(-a) = e^{-ika}$$

$$\underbrace{\begin{pmatrix} 1 + \frac{im}{\hbar}\lambda & \lambda\frac{im}{\hbar}e^{2ika} \\ \lambda\frac{im}{\hbar}e^{2ika} & 1 + \frac{im\lambda}{\hbar} \end{pmatrix}}_{M} \begin{pmatrix} \psi(a) \\ \psi(-a) \end{pmatrix} = \begin{pmatrix} \psi_0(a) \\ \psi_0(-a) \end{pmatrix}$$

$$\begin{pmatrix} \psi(a) \\ \psi(-a) \end{pmatrix} = M^{-1} \begin{pmatrix} \psi_0(a) \\ \psi_0(-a) \end{pmatrix}$$

$$\psi(x) = \psi_0(x) - \frac{im}{\hbar} \lambda \left(e^{ik|x-a|} \psi(a) + e^{ik|x+a|} \psi(-a) \right)$$

$x \gg a$

$$\psi(x) = \psi_0(x) - \frac{im}{k} \lambda e^{ikx} \left(\psi(a) + \psi(-a) \right)$$

$k \to iK$

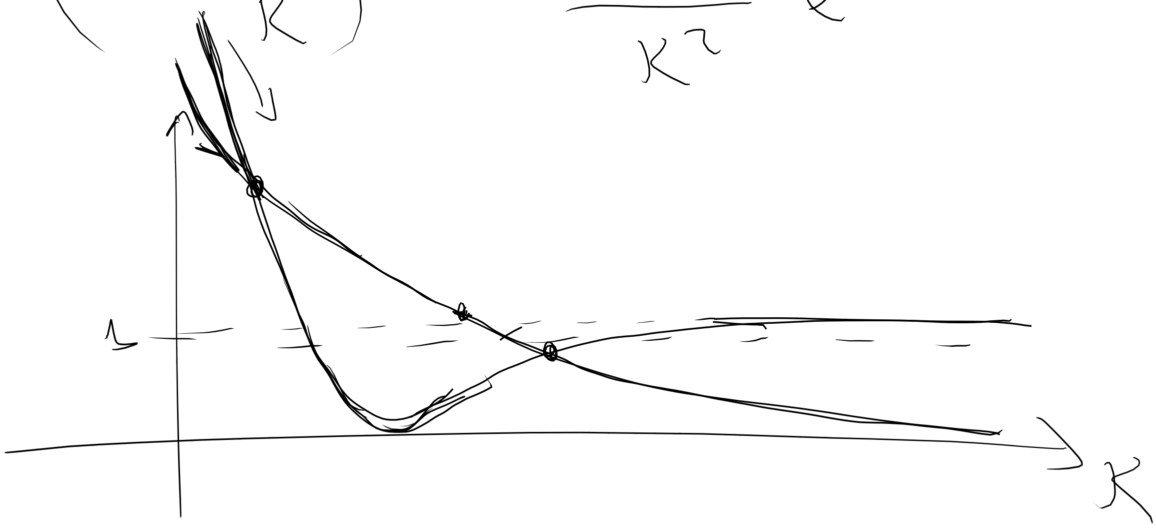
$$f = -\frac{im}{k} \lambda \left(\psi(a) + \psi(-a) \right)$$

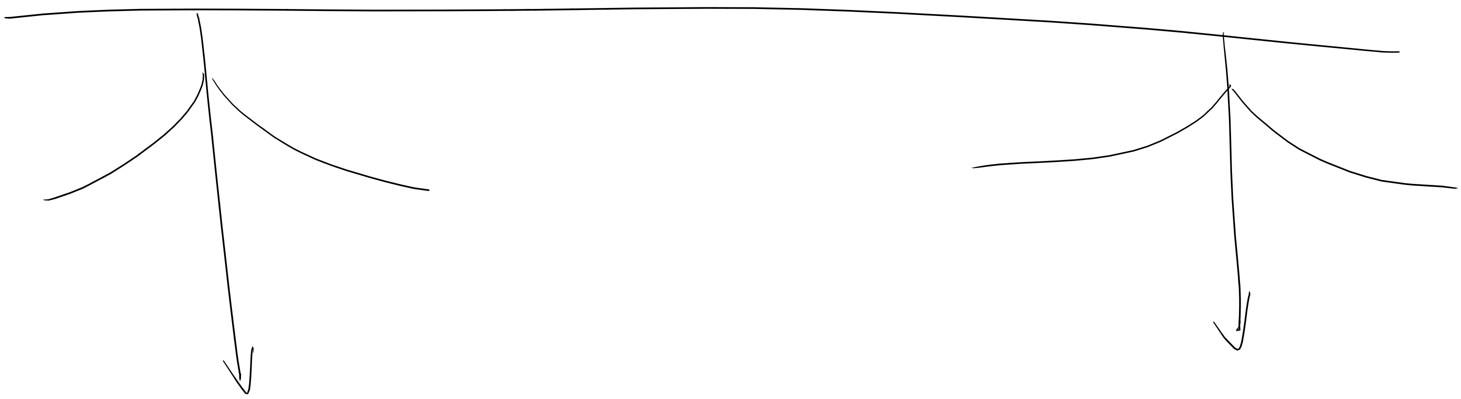
$$f(K) \underset{K=K_b}{=} \infty$$

$$\det \begin{pmatrix} 1 + \frac{m\lambda}{K} & \lambda \frac{m}{K} e^{-2Ka} \\ \lambda \frac{m}{K} e^{-2Ka} & 1 + \frac{m\lambda}{K} \end{pmatrix} = 0$$

$$\left(1 + \frac{m\lambda}{K} \right)^2 - \frac{\lambda^2 m^2}{K^2} e^{-4Ka} = 0$$

$$\left(1 + \frac{m\lambda}{K} \right)^2 = \frac{\lambda^2 m^2}{K^2} e^{-4Ka}$$





$(a \to \infty)$

$$H_0 = \begin{pmatrix} \varepsilon & 0 \\ 0 & \varepsilon \end{pmatrix} \qquad V = \begin{pmatrix} 0 & \delta \\ \delta & 0 \end{pmatrix}$$

$$H = \begin{pmatrix} \varepsilon & \delta \\ \delta & \varepsilon \end{pmatrix}$$

new eigenstates $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ +1 \end{pmatrix}$

eigenvalues $\varepsilon_\pm = \varepsilon \mp \delta$

energy band

Periodic Potential in 1D

$$V(x+a) = V(x)$$

$$H = \frac{p^2}{2m} + V(x)$$

$$T_a f(x) = f(x+a)$$

$$[H, T_{na}] = 0$$

$$T_{na} = (T_a)^n \; ; \; (T_a)^{-1} = T_{-a}$$

$$T_{na} T_{ma} = T_{(n+m)a}$$

$$T_a T_{-a} = T_a (T_a)^{-1} = \mathbb{1} = T_0$$

$$[H, T_a] = 0 \Rightarrow \text{diagonalize } T_a!$$

$$T_a \psi(x) = \lambda \psi(x) = \psi(x+a)$$

$$\int_{-\infty}^{\infty} dx \, |\psi(x)|^2 = 1$$

$$1 = \int_{-\infty}^{\infty} d(y+na) \, |\psi(y+na)|^2 = \int_{-\infty}^{\infty} dy \, |\psi(y+na)|^2$$

$$= \int_{-\infty}^{\infty} dy\, |\lambda|^{2n}\, |\psi(y)|^2$$

$$1 = |\lambda|^2\, 1 \qquad (n=1)$$

$$\Rightarrow |\lambda| = 1 \Rightarrow \lambda = e^{ika} \qquad e^{i\tilde{k}a^2} = e^{i(\underbrace{\tilde{k}a}_{k})a}$$

$$\psi_k(x+a) = e^{ika}\psi_k(x)$$

1st Briouline zone

$$\left.\begin{array}{l} \psi_k(x) = e^{ikx} u_k(x) \\ \hline u_k(x+a) = u_k(x) \end{array}\right\} \quad -\frac{\pi}{a} < k \leq \frac{\pi}{a}$$

$$\psi_k(x) = e^{ikx} u_k(x) = \underbrace{e^{i(k+\frac{2\pi n}{a})x}} \underbrace{e^{-i\frac{2\pi}{a}nx} u_k(x)}_{\text{periodic with period } a}$$

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi_k(x) + V(x)\psi(x) = E\psi(x)$$

$$\frac{d\psi_k(x)}{dx} = ik e^{ikx} u_k(x) + e^{ikx}\frac{du_k}{dx}$$

$$\frac{d^2\psi}{dx^2} = (ik)^2 e^{ikx} u_k(x) + 2(ik) e^{ikx} u_k' + e^{ikx} u_k''$$

$$-\frac{\hbar^2}{2m}\left[-k^2 u_k + 2ik u_k' + u_k''\right] + V u_k = E u_k$$

$$-\frac{\hbar^2}{2m}\left[-k^2 u_k + 2ik\, u_k' + u_k''\right] + V u_k = E(k) u_k$$

$$\boxed{V = \lambda \sum_{n=-\infty}^{\infty} \delta(x - na)}$$

$$u_k(x) = \sum b_m e^{i p_m x}$$

$$p_m a = 2\pi m \Rightarrow \boxed{p_m = \frac{2\pi}{a} m}$$

$$\psi(x) = \sum b_m e^{i(k+p_m)x}$$

$$\sum_m b_m \frac{(k+p_m)^2}{2m} e^{i(k+p_m)x}$$

$$+ \frac{\lambda}{a} \sum_l e^{i p_l x} \sum_m b_m e^{i(k+p_m)x}$$

$$= E \sum_m b_m e^{i(k+p_m)x}$$

$$0 = \sum_m b_m \left[\frac{(k+p_m)^2}{2m} - E\right] e^{i p_m x} + \frac{\lambda}{a} \sum_{ml} b_m e^{i(p_m + p_l)x}$$

$$p_m + p_l = p_{m+l} \qquad \sum_N \delta_{N, m+l}$$

$$0 = \sum_n b_n \left[\frac{(k+p_n)^2}{2m} - E\right] e^{ip_n x} + \frac{\lambda}{a} \sum_{\cancel{N}_m} \sum_l b_{n-l}\, e^{ip_{N_m} x}$$

$$0 = \sum_n e^{ip_n x} \left\{ \left[b_n \left[\frac{(k+p_n)^2}{2m} - E \right] + \frac{\lambda}{a} \sum_l b_{n-l} \right] \right\}$$

$$\Rightarrow \boxed{b_n \left[\frac{(k+p_n)^2}{2m} - E \right] + \frac{\lambda}{a} \sum_l b_{n-l} = 0}$$

$$\sum_l b_{n-l} = 1 \Leftarrow$$

$$\boxed{p_n = \frac{n\pi}{a}}$$

$$\boxed{b_n = \left(-\frac{\lambda}{a}\right) \frac{1}{\frac{(k+p_n)^2}{2m} - E}}$$

eigenvalue eqn.

$$\boxed{\left(-\frac{\lambda}{a}\right) \sum_l \frac{1}{\frac{(k+p_n)^2}{2m} - E(k)} = 1}$$

$E = ?$