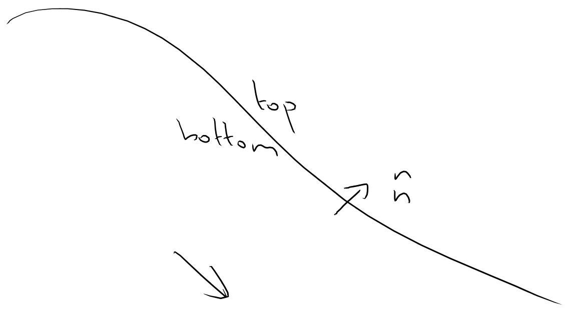
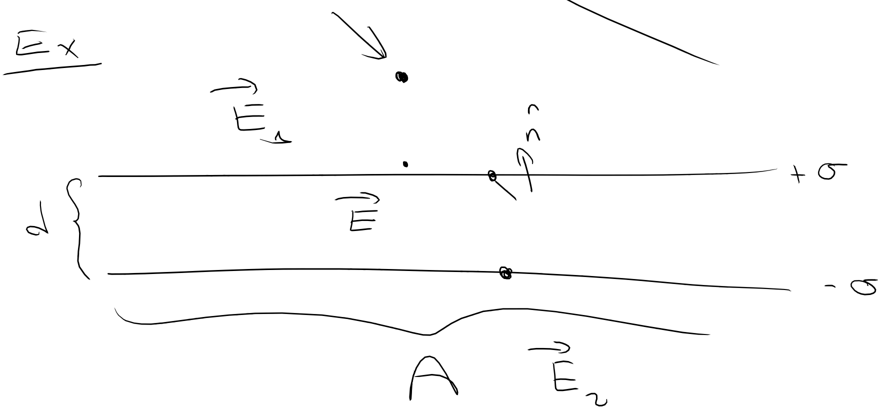
$$\left(\vec{E}_{top} - \vec{E}_{bottom}\right)\cdot\hat{n} = \frac{\sigma}{\varepsilon_0}$$



Ex



$d \ll \sqrt{A}$

$$\left.\begin{aligned}\left(\vec{E}_1 - \vec{E}\right)\cdot\hat{n} &= \frac{\sigma}{\varepsilon_0}\\ \left(\vec{E} - \vec{E}_2\right)\cdot\hat{n} &= -\frac{\sigma}{\varepsilon_0}\end{aligned}\right\} \quad \vec{E}_1 = \vec{E}_2 \text{ + additional information} \Rightarrow \vec{E}_1 = \vec{E}_2 = 0$$

$$-\vec{E}\cdot\hat{n} = \frac{\sigma}{\varepsilon_0} \qquad \vec{E} = E\,\hat{n}$$

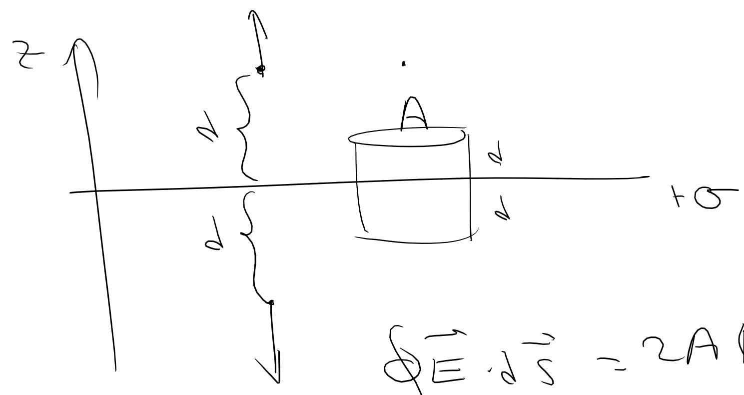
$$-E = \frac{\sigma}{\varepsilon_0} \Rightarrow E = -\frac{\sigma}{\varepsilon_0} \Rightarrow \vec{E} = -\frac{\sigma}{\varepsilon_0}\hat{n}$$

Previous Example

λ

$\vec{E} = E(r)\hat{\rho}$ + Gauss' Law

$$\Rightarrow E(r) = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r}$$

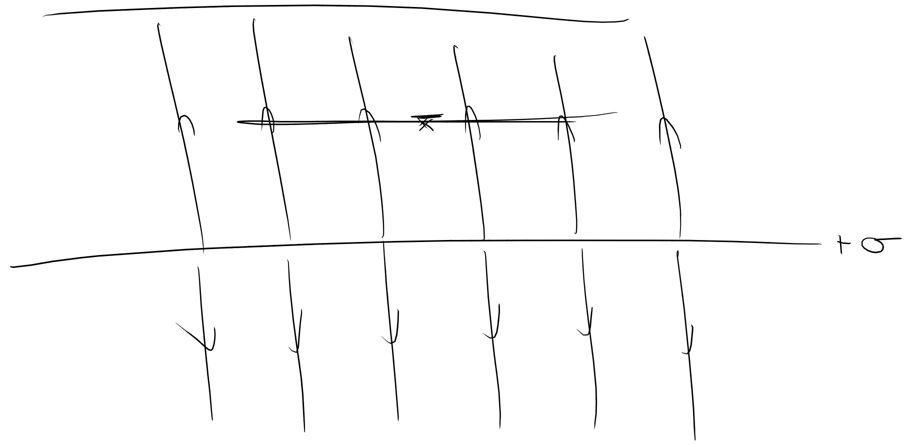


$$\vec{E} = E(z)\begin{cases} \hat{z} & \text{if } z>0 \\ -\hat{z} & \text{if } z<0 \end{cases}$$

$$\oint \vec{E}\cdot d\vec{S} = 2A\,E(d) = \frac{A\sigma}{\varepsilon_0}$$

$$E(d) = \frac{\sigma}{2\varepsilon_0}$$ independent of d!

Without Gauss' law

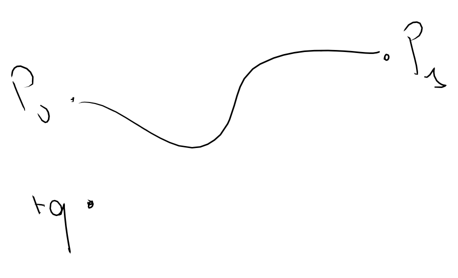


$$\vec{\nabla}\cdot\vec{E} = \cancel{\frac{\rho}{\varepsilon_0}}$$

Curl of $\vec{E}$

$$\int_A (\vec{\nabla}\times\vec{E})\cdot d\vec{A} = \oint_{\partial A} \vec{E}\cdot d\vec{l}$$

$$\int_{P_0}^{P_1} \vec{E}\cdot d\vec{l} = ?$$



method i

$$\vec{E}(\vec{r}) = \frac{q}{4\pi\varepsilon_0}\frac{\hat{r}}{r^2}$$

$$\vec{E}\cdot d\vec{l} \qquad d\vec{l} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$$

$$\vec{E}\cdot d\vec{l} = \frac{q}{4\pi\varepsilon_0}\frac{dr}{r^2}$$

$$\int_{P_0}^{P_1}\vec{E}\cdot d\vec{l} = \int_{r_0}^{r_1}\frac{q}{4\pi\varepsilon_0}\frac{dr}{r^2} = \frac{q}{4\pi\varepsilon_0}\left(-\frac{1}{r_1}+\frac{1}{r_0}\right)$$

$$\text{if } P_0 = P_1 \Rightarrow r_0 = r_1 \Rightarrow \oint\vec{E}\cdot d\vec{l} = 0$$

$$\Rightarrow \int(\vec{\nabla}\times\vec{E})\cdot d\vec{A} = 0$$

$$\Rightarrow \boxed{\vec{\nabla}\times\vec{E} = 0} \Leftarrow$$

Method ii

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int d^3r'\,\rho(r')\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}$$

$$\left[\vec{\nabla}_r\times\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}\right]_x = \frac{\partial}{\partial y}\frac{z-z'}{|\vec{r}-\vec{r}'|^3} - \frac{\partial}{\partial z}\frac{y-y'}{|\vec{r}-\vec{r}'|^3}$$

$$= \frac{\partial}{\partial y}\frac{(z-z')}{\left[(\vec{r}-\vec{r}')^2\right]^{3/2}} - \frac{\partial}{\partial z}\frac{y-y'}{\left[(\vec{r}-\vec{r}')^2\right]^{3/2}}$$

$$= (z-z')\left(-\frac{3}{2}\right)\frac{1}{\left[(\vec{r}-\vec{r}')^2\right]^{5/2}}\frac{\partial}{\partial y}\left[(x-x')^2+(y-y')^2+(z-z')^2\right]$$

$$- (y-y')\left(-\frac{3}{2}\right)\frac{1}{\left[(\vec{r}-\vec{r}')^2\right]^{5/2}}\frac{\partial}{\partial z}\left[(x-x')^2+(y-y')^2+(z-z')^2\right]$$

$$= (z-z')\left(-\frac{3}{2}\right)\frac{1}{|\vec{r}-\vec{r}'|^5}\,2(y-y')$$

$$- (y-y')\left(-\frac{3}{2}\right)\frac{1}{|\vec{r}-\vec{r}'|^5}\,2(z-z')$$

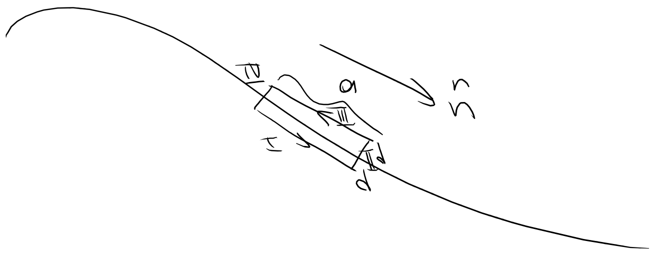
$$= 0$$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int d^3r' \, \rho(r') \, \frac{\vec{r}-\vec{r}\,'}{|\vec{r}-\vec{r}\,'|^3}$$

$$\vec{\nabla}\times\vec{E} = \frac{1}{4\pi\varepsilon_0}\int d^3r' \, \rho(r') \, \underbrace{\vec{\nabla}_r \times \frac{\vec{r}-\vec{r}\,'}{|\vec{r}-\vec{r}\,'|^3}}_{0} = 0$$

$$\left.\begin{aligned} \vec{\nabla}\times\vec{E} &= 0 \\ \vec{\nabla}\cdot\vec{E} &= \frac{\rho(\vec{r})}{\varepsilon_0} \end{aligned}\right\}$$

$$0 = \int(\vec{\nabla}\times\vec{E})\cdot d\vec{A} = \oint \vec{E}\cdot d\vec{l}$$



$$\oint \vec{E}\cdot d\vec{l} = \int_{I}\vec{E}\cdot d\vec{l} + \underbrace{\int_{II}\vec{E}\cdot d\vec{l}}_{\overset{d\to 0}{=}\,0} + \int_{III}\vec{E}\cdot d\vec{l} + \underbrace{\int_{IV}\vec{E}\cdot d\vec{l}}_{\overset{d\to 0}{=}\,0}$$

$$(d\vec{l})_{I} = dl\,\hat{n}$$
$$(d\vec{l})_{III} = dl\,(-\hat{n})$$

$$= (\vec{E}\cdot d\vec{l})_{I} + (\vec{E}\cdot d\vec{l})_{III}$$
$$= (\vec{E}_{I} - \vec{E}_{III})\cdot dl\,\hat{n} = 0$$

$$(\vec{E}_{I} - \vec{E}_{III})\cdot\hat{n} = 0$$

$\Rightarrow E^{\parallel}$ is continuous

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \Rightarrow \Delta(E^{\perp}) = \frac{\sigma}{\varepsilon_0}$$

$$\vec{\nabla} \times \vec{E} = 0 \Rightarrow \Delta(E^{\parallel}) = 0$$

Δ: change across the surface

$\vec{\nabla} \times \vec{E} = 0$ in electrostatics