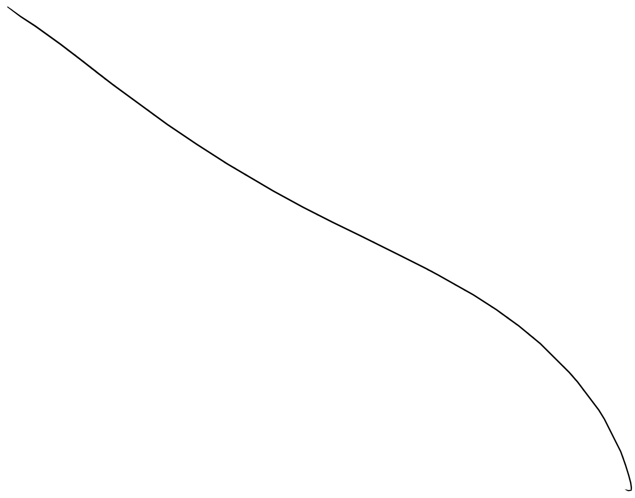
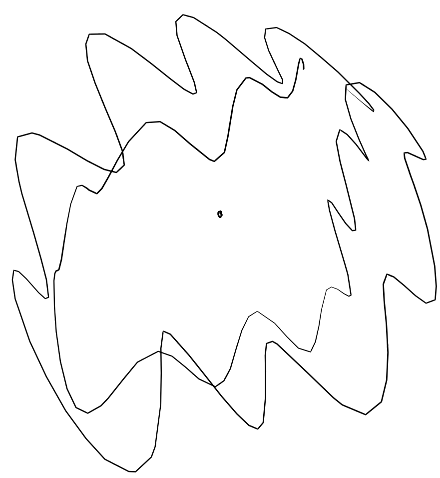
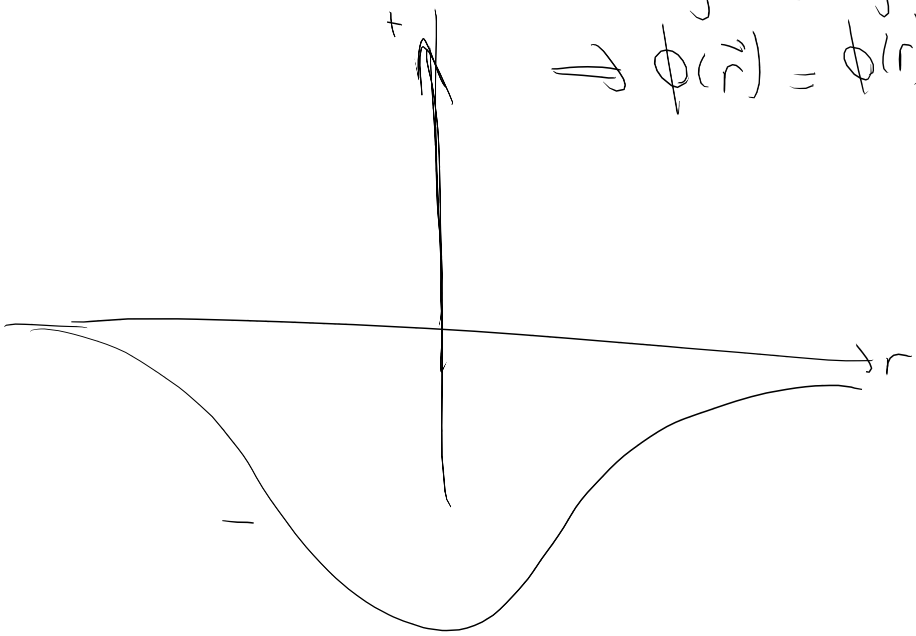


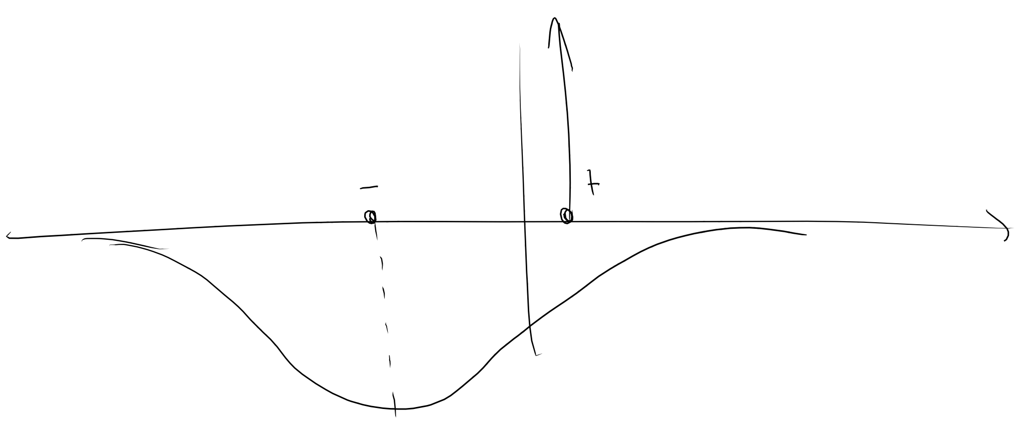


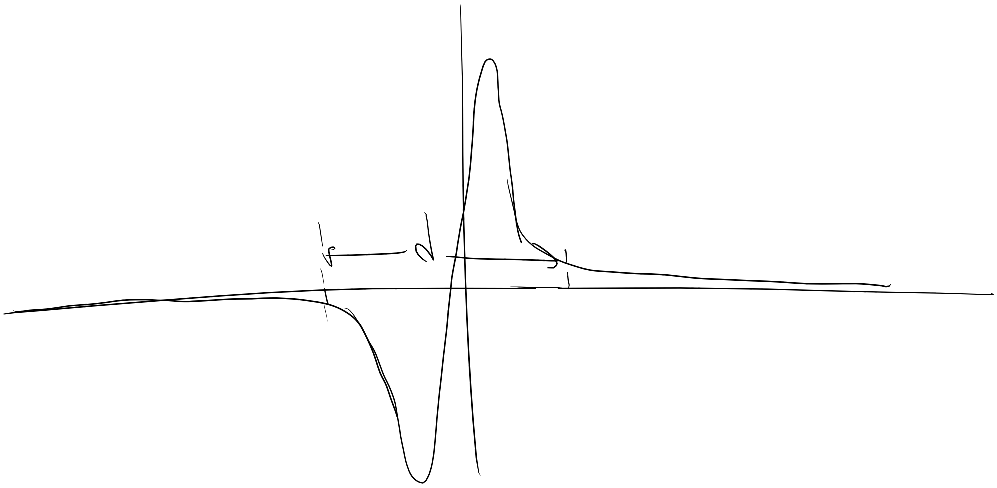
$\rho(\vec{r}) = \rho(r)$

$\Rightarrow \phi(\vec{r}) = \phi(r)$



$\vec{E} \neq 0$





$$\phi(r) = \frac{e^{-\alpha r}}{r}(1-\alpha r) = \frac{e^{-\alpha r} - 1}{r} - \alpha e^{-\alpha r} + \frac{1}{r}$$

$$\nabla^2 \phi = -4\pi\rho$$

$$\nabla^2\left(\frac{1}{r}\right) = \overbrace{-4\pi\,\delta^{(3)}(\vec{r})}$$

$$\nabla^2 \phi = \frac{1}{r^2}\frac{d}{dr} r^2 \frac{d\phi}{dr} \qquad \text{since } \phi(\vec{r}) = \phi(r)$$

$$\frac{d\phi}{dr} = -\alpha \frac{e^{-\alpha r}}{r}(1-\alpha r) - \frac{e^{-\alpha r}}{r^2}(1-\alpha r) + \frac{e^{-\alpha r}}{r}(-\alpha)$$

$$r^2 \frac{d\phi}{dr} = \frac{e^{-\alpha r}}{\cancel{r}}\left[-\alpha + \alpha^2 r - \frac{1}{r} + \cancel{\alpha} + \cancel{\alpha}\right] r^{\cancel{2}}$$

$$\frac{d}{dr}\left(r^2 \frac{d\phi}{dr}\right) = -\alpha e^{-\alpha r}\left[-\alpha + \alpha^2 r - \frac{1}{r}\right] r + e^{-\alpha r}\left[-\alpha + \alpha^2 r - \frac{1}{r}\right]$$

$$+ e^{-\alpha r} r \left[\alpha^2 + \frac{1}{r^2} \right]$$

$$\rho = -\frac{1}{4\pi} \frac{1}{r^2} \underbrace{\frac{d}{dr}\left(r^2 \frac{d\phi}{dr} \right)}$$

$$= -\frac{1}{4\pi} \frac{e^{-\alpha r}}{r^2} \left\{ \alpha^2 r - \alpha^3 r^2 + \cancel{\alpha} - \cancel{\alpha} + \alpha^2 r - \cancel{\frac{1}{r}} + \alpha^2 r + \cancel{\frac{1}{r}} \right\}$$

$$= -\frac{1}{4\pi} \frac{e^{-\alpha r}}{r^2} \left\{ 3\alpha^2 r - \alpha^3 r^2 \right\}$$

$$\rho(r) = -\frac{1}{4\pi} \frac{e^{-\alpha r}}{r^3} \left\{ 3(\alpha r)^2 - (\alpha r)^3 \right\}$$

alternative

$$4\pi Q_{enc} = \oint \vec{E} \cdot d\vec{S} = E_r(r)\, 4\pi r^2 = -\frac{d\phi}{dr} 4\pi r^2$$

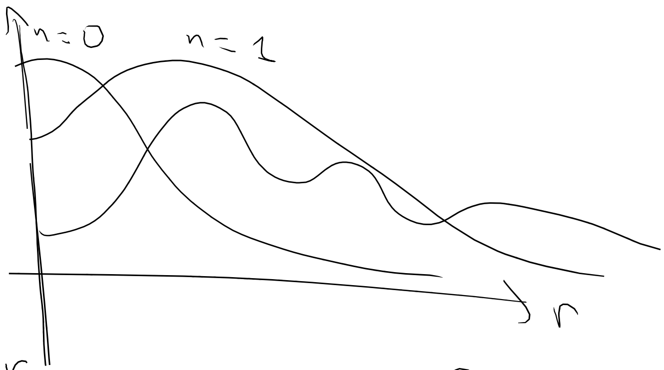
$$\boxed{Q_{enc} = -r^2 \frac{d\phi}{dr}}$$

charge in a sphere of volume r

$$\frac{d\phi}{dr} = -\alpha \frac{e^{-\alpha r}}{r}(1-\alpha r) - \frac{e^{-\alpha r}}{r^2}(1-\alpha r)$$

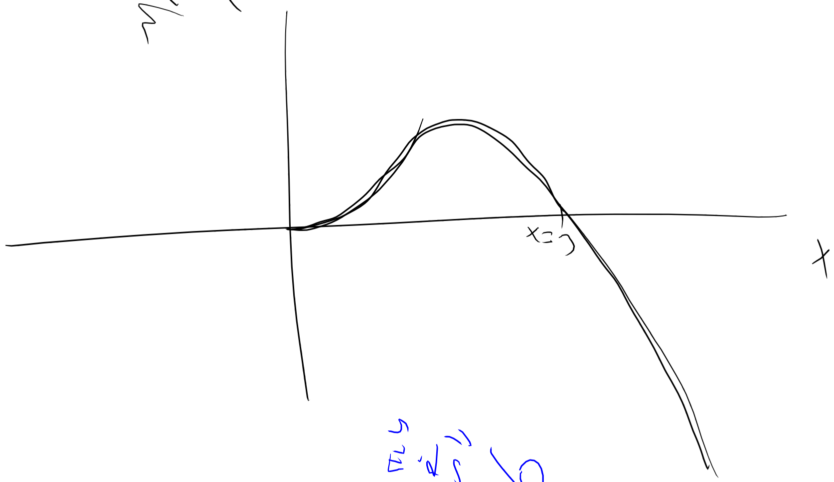
$$+ \frac{e^{-\alpha r}}{r}(-\alpha)$$

atoms in excited s-states

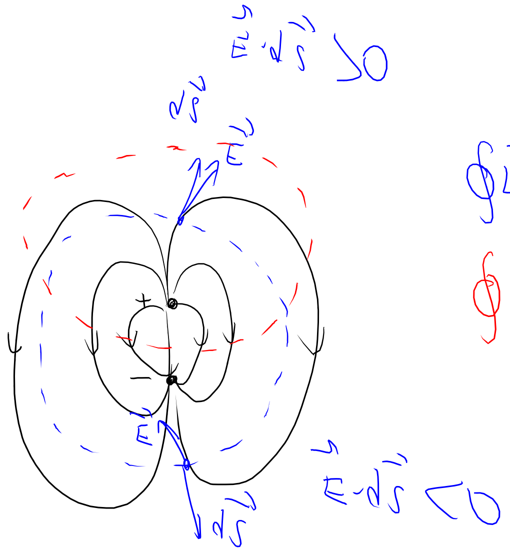


$$\rho^{e}(r) = -\frac{1}{4\pi}\frac{e^{-\alpha r}}{r^3}\left\{3(\alpha r)^2-(\alpha r)^3\right\}$$

$3x^2-x^3$



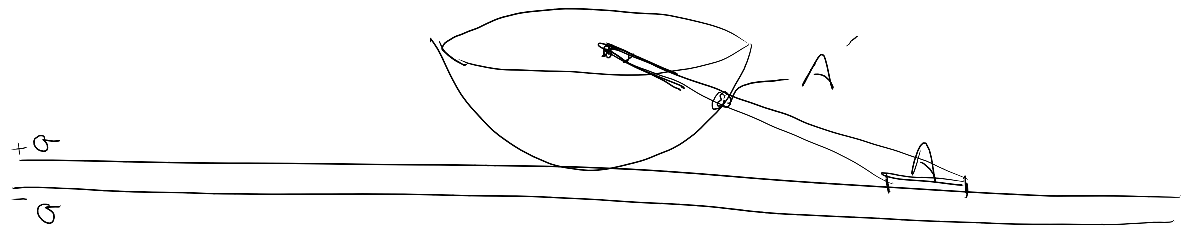
$$\rho^{e}(\vec r) = -e|\psi(\vec r)|^2$$



$$\oint \vec E\cdot d\vec s = 0$$

$$\oint \vec E\cdot d\vec s \neq 0$$

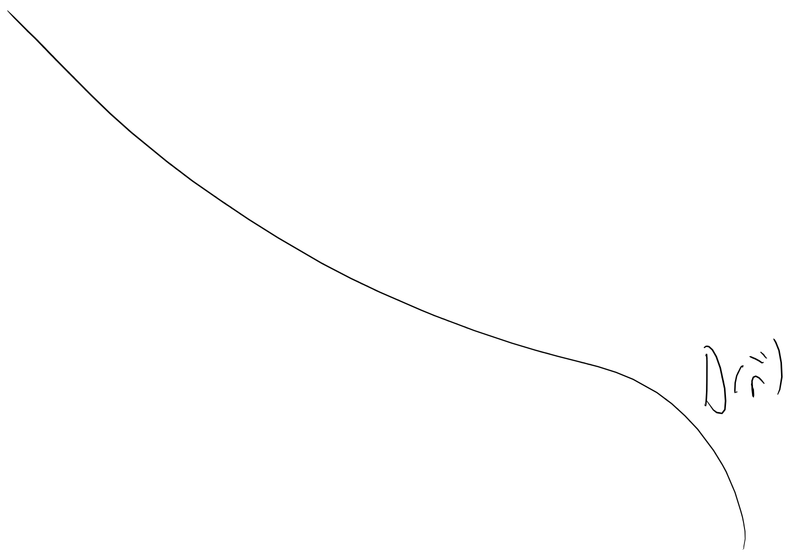
$$\phi(\vec r) = \int D(\vec r')\, \frac{\hat n \cdot (\vec r - \vec r')}{|\vec r - \vec r'|^3}\, da = \int_S D(\vec r')\, d\Omega$$



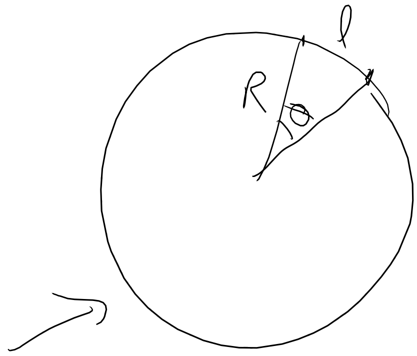
$$\phi \propto \int d\Omega = 2\pi \neq 0$$

$$E_{in} = 4\pi\sigma$$

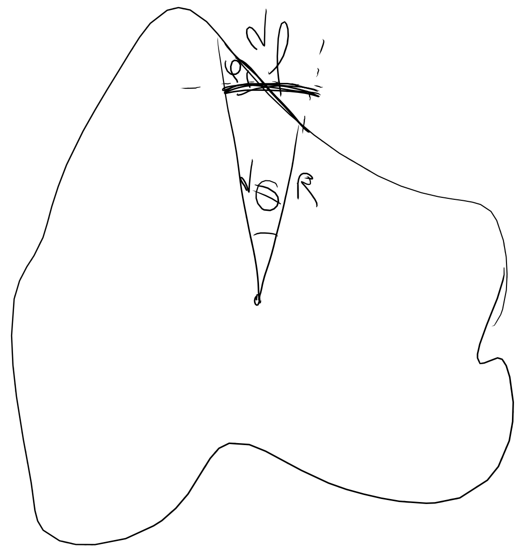
$$\Delta\phi = 4\pi \underbrace{(\sigma d)}_{D(\vec r')}$$



angle



$$\theta = \frac{l}{R}$$

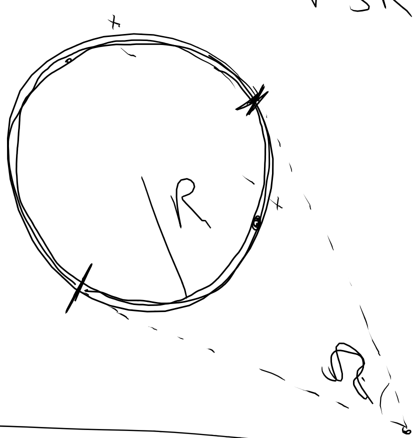


$$d\Theta = \frac{dl\cos\varphi}{R}$$

$$\boxed{\Omega = \frac{A}{R^2}}$$

Ex

ϕ

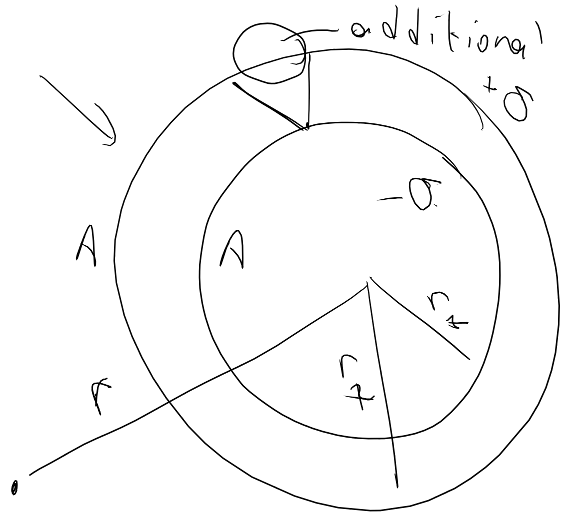


$$\lim_{\substack{\sigma_+ \to \sigma_- \to \infty \\ d \to 0 \\ \sigma d \to D}}$$

$r > R$ $$\phi(r) = \frac{Q_+}{r} + \frac{Q_-}{r}$$ ✓

$$\phi(r) = \frac{(Q_+ + Q_-)}{r} = 0$$

$$\boxed{\phi = \int D(\vec{r}')\,d\Omega} = D_+\Omega + D_-\Omega = 0$$



additional charge $= \sigma \Delta A = \sigma\, 4\pi(r_+^2 - r_-^2)$

$= 8\pi D R$

$$D = \sigma d$$

$$Q_+ = \sigma r_+^2 \, 4\pi$$

$$Q_- = -\sigma r_-^2 \, 4\pi$$

$$Q_+ + Q_- = \sigma \underline{(r_+^2 - r_-^2)}\, 4\pi$$

$$= \sigma \underbrace{(r_+ - r_-)}_{d}\, 4\pi (r_+ + r_-)$$

$$\boxed{Q_+ + Q_- = D\, 8\pi R}$$

$r_+ - r_- = d$

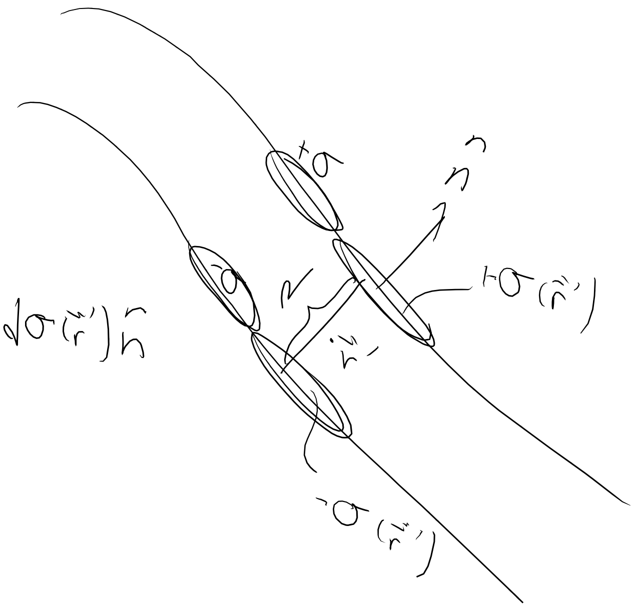
$\lim_{d \to 0}$ $(r_- \to R,\ r_+ \to R)$

$\sigma \to \infty$

$\sigma d \to D$

$Q_+ \to Q_- \to \infty$

$$\phi(r) = \frac{Q_+ + Q_-}{r} = \boxed{\frac{8\pi R D}{r} = \phi(r)}$$



$$\vec{D}(\vec{r}') = \lim_{\substack{d \to 0 \\ \sigma \to \infty \\ d\sigma \to D}} d\sigma(\vec{r}')\, \hat{n}$$