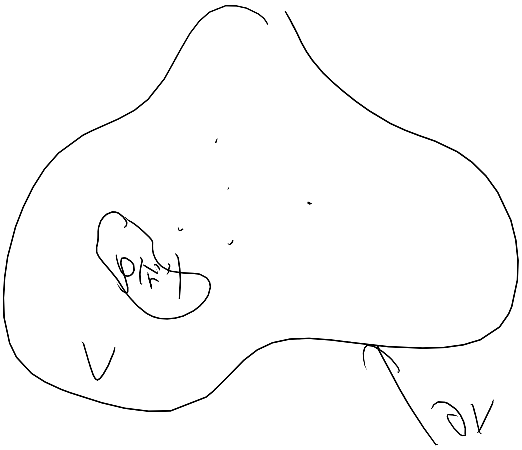
$$\phi(\vec{r}) = \int_V dV' \rho(\vec{r}') G(\vec{r},\vec{r}') + \oint_{\partial V} d\vec{s} \left(\phi(\vec{r}) \frac{\partial G}{\partial n} - \frac{\partial \phi}{\partial n} G \right)$$

$$\Leftarrow \begin{cases} \nabla^2 \phi = -4\pi\rho \\ \phi|_{\partial V} = \phi_B \end{cases}$$

$$\Rightarrow$$



$$\phi(\vec{r}) = \int_{\text{allspace}} dV' \frac{\tilde{\rho}(\vec{r}')}{|\vec{r}-\vec{r}'|}$$

$$\Downarrow$$

$$\rho(\vec{r}') = \tilde{\rho}(\vec{r}') \quad \text{if } \vec{r}' \in V$$

$$\int_{\text{all space}} dV' = \int_V dV + \int_{\text{allspace}\setminus V} dV$$

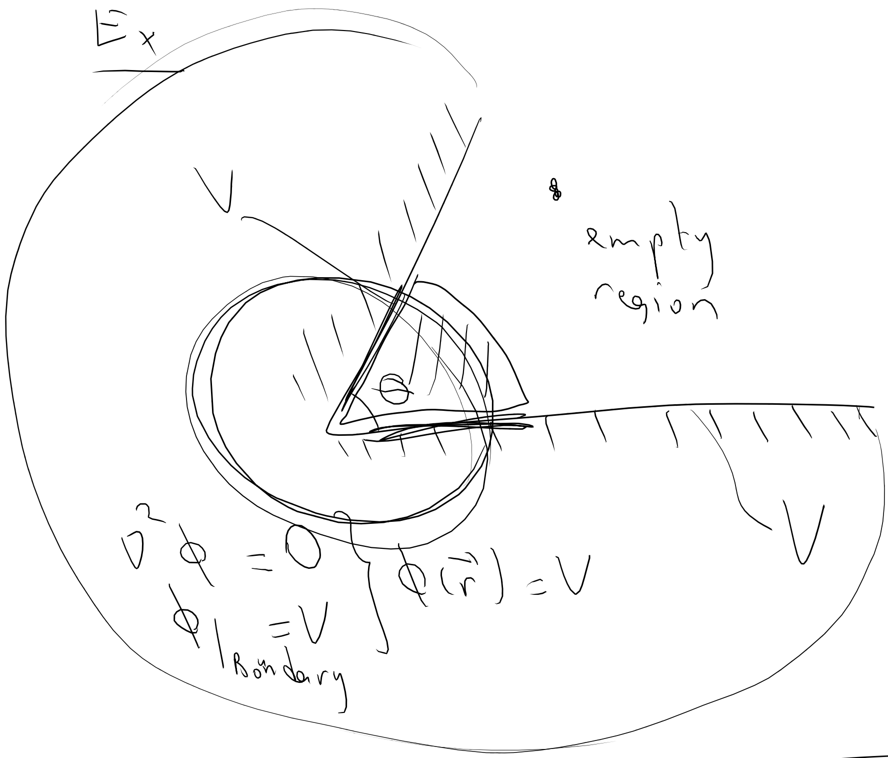
$$\phi(\vec{r}) = \int_V dV' \frac{\tilde{\rho}(\vec{r}')}{|\vec{r}-\vec{r}'|} + \int_{\text{all space}\setminus V} dV' \frac{\tilde{\rho}(\vec{r}')}{|\vec{r}-\vec{r}'|}$$

$$\phi(\vec{r}) = \int_V dV' \frac{\rho(\vec{r}')}{|\vec{r}-\vec{r}'|} + \int_{\text{all space}\setminus V} dV' \frac{\tilde{\rho}(\vec{r}')}{|\vec{r}-\vec{r}'|}$$

$$G(\vec{r},\vec{r}') = \frac{1}{|\vec{r}-\vec{r}'|} + F(\vec{r},\vec{r}') \qquad \nabla^2 F = 0$$

$$\phi(\vec{r}) = \int_V dV' \rho(\vec{r}') G(\vec{r},\vec{r}') + \int ds \left(\phi \frac{\partial G}{\partial n} - \frac{\partial \phi}{\partial n} G \right)$$

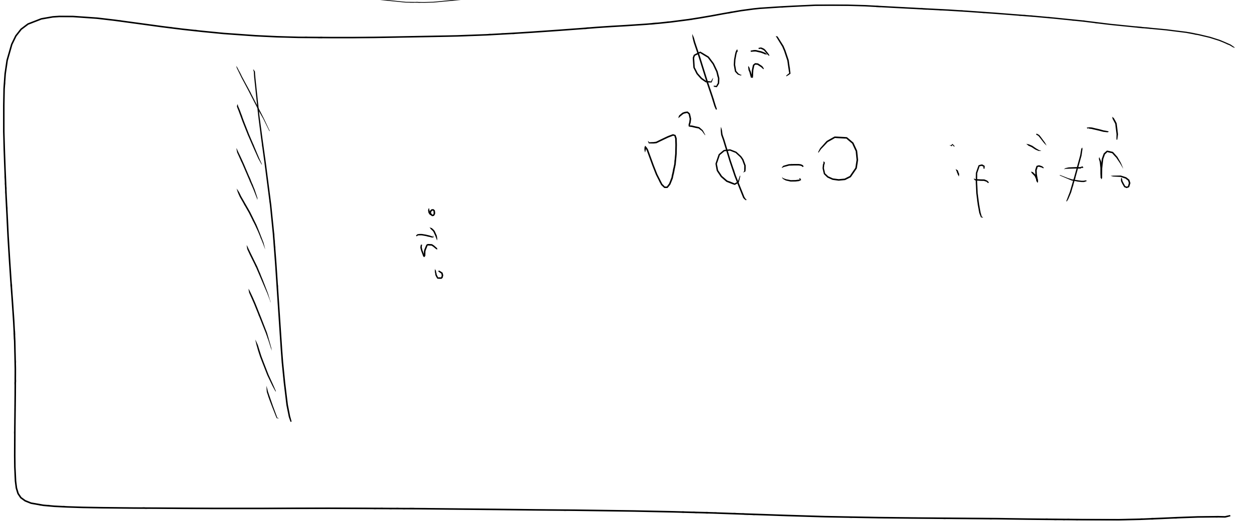
$$\nabla^2 \cancel{G} = -4\pi\delta(\vec{r}-\vec{r}') \qquad G|_{\partial V} = 0$$



$\Theta = \frac{\pi}{4}, \frac{\pi}{2}, \frac{\pi}{3}, \ldots$

charge density at the surface

$$\nabla^2 \phi = 0 \quad \left.\phi\right|_{\text{Boundary}} = V \Bigg\} \quad \phi(\vec{r}) = V$$



$\phi(\vec{r})$

$$\nabla^2 \phi = 0 \quad \text{if } \vec{r} \neq \vec{r}_0$$

$$\phi = V + \sum_m a_m \rho^{m\frac{\pi}{\beta}} \sin\left(m \pi \frac{\phi}{\beta}\right)$$

$a_m = ?$

$$\vec{\nabla}\phi = \frac{\partial \phi}{\partial \rho}\hat{\rho} + \ldots$$

assume $\frac{\pi}{\beta} < 1$

$$\frac{\partial}{\partial \rho}\rho^{\frac{\pi}{\beta}} = \frac{\pi}{\beta}\rho^{\frac{\pi}{\beta}-1}$$

$\frac{\pi}{\beta} - 1 < 0$

$$(\phi \vec{\nabla} \psi) \cdot \hat{n} = \phi \frac{\partial \psi}{\partial n}$$

$$\phi = \frac{q}{|\vec{r} - \vec{r}'|}$$

$$\boxed{\phi(\vec{r}) = \int_V dV' \rho(\vec{r}') G(\vec{r}', \vec{r}) + \oint_{\partial V} ds \left(\phi(\vec{r}) \frac{\partial G}{\partial n} - \frac{\partial \phi}{\partial n} G \right)}$$

$\frac{1}{r}$ $\frac{1}{r^2}$ $\frac{1}{r^2}$ $\frac{1}{r}$

$$\boxed{\phi(\vec{r}) = \int \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} d^3 r'}$$

all of space

$$G \neq \frac{1}{|\vec{r} - \vec{r}'|}$$

$$G = \frac{1}{|\vec{r} - \vec{r}'|} + F(\vec{r}, \vec{r}')$$

$$\rho(\vec{r}) = 0 \quad \text{if} \quad \vec{r} \notin V$$

$$\boxed{\begin{gathered} \nabla^2 G = -4\pi \delta^{(3)}(\vec{r} - \vec{r}') \\ G|_{\partial V} = 0 \Leftarrow \end{gathered}}$$

$\partial V \to \infty$ $\quad G = \frac{1}{|\vec{r} - \vec{r}'|}$

on the surface

$G \sim \frac{1}{r}$

$$\nabla_r^2 G = \nabla_{r'}^2 G = -4\pi \delta^{(3)}(\vec{r} - \vec{r}') \quad (?)$$

$$G|_{r \in \partial V} = G|_{r' \in \partial V} = 0$$

$$\nabla_r^2 G(\vec{r}, \vec{r}') = -4\pi \delta^{(3)}(\vec{r} - \vec{r}')$$

$$\nabla_{r'}^2 G(\vec{r}', \vec{r}) = -4\pi \delta^{(3)}(\vec{r}' - \vec{r}) = -4\pi \delta^{(3)}(\vec{r} - \vec{r}')$$

Dirichlet

$$G_D(y,x) = 0 \qquad y \in \partial V$$

$$G_D(y,x) \stackrel{?}{=} 0 \qquad x \in \partial V$$

$$\phi(\vec{r}) = \int_V dV' \rho(\vec{r}') G(\vec{r}', \vec{r})$$

$$+ () \oint ds' \left(\phi(\vec{r}') \underset{\frac{1}{r}}{} \frac{\partial G}{\partial n'} \underset{\frac{1}{r^2}}{} - \frac{\partial \phi}{\partial n'} \underset{\frac{1}{r^2}}{} \overset{\frac{1}{r}}{G} \right)$$

$$\nabla_r^2 G = -4\pi \delta^{(3)}(\vec{r}' - \vec{r}') \qquad -4\pi \delta^{(3)}(\vec{r} - \vec{r}')$$

$$\nabla_r^2 \phi(\vec{r}) = \int_V dV' \rho(\vec{r}') \nabla_r^2 G$$

$$+ () \int ds \left(\phi(\vec{r}') \frac{\partial}{\partial n} \underbrace{(\nabla_r^2 G)}_{-4\pi \delta^{(3)}(\vec{r} - \vec{r}') = 0} - \frac{\partial \phi}{\partial n'} \underbrace{\nabla_r^2 G}_{= 0} \right)$$

$$\boxed{\nabla_r^2 \phi(\vec{r}) = -4\pi \rho(\vec{r})}$$

$$\underset{\vec{r} \in \partial V}{\phi(\vec{r})} \stackrel{?}{=} \int dV' \rho(\vec{r}') \underbrace{G(\vec{r}, \vec{r}')}_{= 0} + () \oint ds' \left(\phi(\vec{r}') \frac{\partial G}{\partial n'} \right)$$

$$\boxed{\underset{\vec{r} \to \vec{0}^+ \in \partial V}{\phi(\vec{r})} \stackrel{?}{=} () \int ds' \left(\phi(\vec{r}') \frac{\partial G}{\partial n} \right)}$$

? HW!

$$\nabla^2 G = -4\pi \delta^{(3)}(\vec{r} - \vec{r}')$$

$$G = 0 \text{ if } \vec{r} \in \partial V$$

Simple example: inside A sphere with a uniform surface charge density,

$$\phi\big|_{\text{Boundary}} = \frac{Q}{R}$$

$$G = \frac{1}{|\vec{r} - \vec{r}_0|} - q'\frac{1}{|\vec{r} - \vec{r}_0'|}$$

$$\frac{\partial G}{\partial n} = \frac{\partial G}{\partial r} = \frac{\partial}{\partial r}\left(\frac{1}{\sqrt{r^2 + r_0^2 - 2rr_0\cos\theta}} - q'\frac{1}{\sqrt{r^2 + r_0'^2 - 2rr_0'\cos\theta}}\right)$$

$$\underset{\vec{r} \in \partial V}{\phi(\vec{r})} = (\;)\int ds'\, \underbrace{\phi(\vec{r}')}\, \frac{\partial G}{\partial r}$$

$$\frac{Q}{R} = \left(-\frac{1}{4\pi}\right)\frac{Q}{R}\underbrace{\oint ds'\, \frac{\partial G}{\partial r}}_{-4\pi}$$