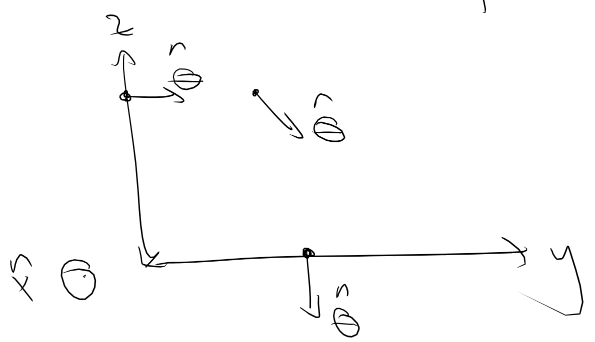
HW

$$\vec{E} = \vec{k}$$

$$\vec{\nabla}\cdot\vec{E} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 k_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, k_\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}(k_\phi) \overset{?}{\neq} 0$$

$$k_r = \vec{k}\cdot\hat{r} \qquad \underline{k_\theta = \vec{k}\cdot\hat{\theta}} \qquad k_\theta(\theta,\phi)$$

$$\vec{\nabla}\cdot\vec{E} = \frac{\partial k_x}{\partial x} + \frac{\partial k_y}{\partial y} + \frac{\partial k_z}{\partial z} = 0$$



$\hat{\theta}(\theta,\phi)$
$\hat{r}(\theta,\phi)$
$\hat{\phi}(\phi)$

$$\vec{r} \equiv r\,\hat{r} + 0\,\hat{\phi} + 0\,\hat{\theta}$$

$$\vec{k} \qquad \hat{r} = \frac{\vec{r}}{r}$$

$$E = \frac{1}{2}\int \rho(\vec{r})\phi(\vec{r})\,d^3r \qquad \rho(\vec{r}),\ \phi(\vec{r})$$

Ex

$$\rho(\vec{r}) = q_1\,\delta^{(3)}(\vec{r}-\vec{r}_1) + q_2\,\delta^{(3)}(\vec{r}-\vec{r}_2)$$

$$E = q_1\,\phi(\vec{r}_1) + q_2\,\phi(\vec{r}_2) = q_1\frac{q_2}{|\vec{r}_1-\vec{r}_2|} + q_2\frac{q_1}{|\vec{r}_1-\vec{r}_2|}$$

$$E = 2\,\frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|}$$

?

$$E = \frac{1}{2}\int \rho(\vec{r})\,\phi(\vec{r})\,d^3r \Leftarrow$$

$$\vec{\nabla}\cdot\vec{E} = 4\pi\rho \Rightarrow \rho(\vec{r}) = \frac{1}{4\pi}\vec{\nabla}\cdot\vec{E}$$

$$E = \frac{1}{8\pi}\int(\vec{\nabla}\cdot\vec{E})\,\phi(\vec{r})\,d^3r$$

$$= \frac{1}{8\pi}\int_V \partial_i E_i\,\phi\,d^3r$$

$$= \frac{1}{8\pi}\int_V \left[\partial_i(E_i\phi) - E_i\partial_i\phi\right]d^3r$$

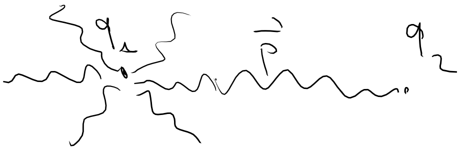
$$= \frac{1}{8\pi}\int_V \left[\partial_i(\underbrace{E_i\phi}) + E_i^2\right]d^3r$$

$$= \frac{1}{8\pi}\int_V \left[\vec{\nabla}\cdot(\vec{E}\phi) + \vec{E}^2\right]d^3r$$

$$E = \frac{1}{8\pi}\int_V \vec{E}^2\,d^3r + \frac{1}{8\pi}\int_{\partial V}\phi\,\vec{E}\cdot d\vec{S}$$

take $V \to \infty$

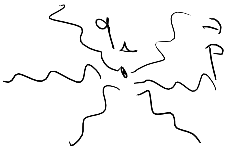
$$E = \frac{1}{8\pi}\int \vec{E}^2\,d^3r$$



$$\boxed{\vec{F} = \frac{\Delta \vec{p}}{\Delta t}}$$

$$\Delta P_{q_1} = -\vec{p}$$

$$\Delta P_{q_2} = \vec{p}$$



$$\vec{P} \longrightarrow \vec{P} - \vec{p}$$

$$\vec{P} = (\vec{P} - \vec{p}) + \vec{p}$$

$$E_{q_1}(\vec{P}) \neq E_{q_1}(\vec{P} - \vec{p}) + E_{\gamma}(\vec{p})$$

$$\Delta E \, \Delta t \gtrsim \frac{\hbar}{2}$$

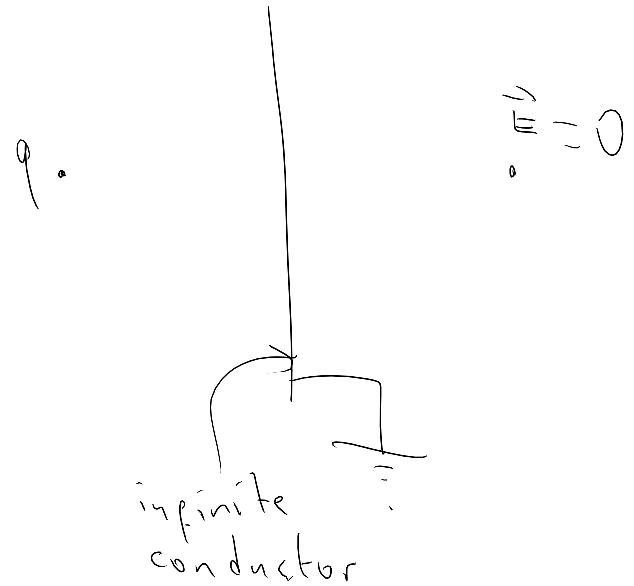
$$E_{q_1}(\vec{P}_1) + E_{q_2}(\vec{P}_2)$$

$$= E_{q_1}(\vec{P}_1 - \vec{p}) + E_{q_2}(\vec{P}_2 + \vec{p})$$

$$E = \sqrt{p^2 + m^2} \qquad (c = 1)$$

$$V \propto \frac{e^{-mr}}{r} \qquad (\hbar = c = 1)$$

$$\text{range} \sim \frac{\hbar}{mc}$$



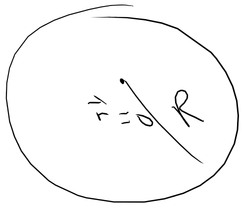
$$\vec{\nabla} \cdot \left(\frac{\hat{r}}{r^2} \right) = 4\pi \delta^{(3)}(\vec{r})$$

$$\vec{\nabla} \cdot \left(\frac{\hat{r}}{r^2} \right) = 0 \quad \text{if} \quad r \neq 0$$

$$\vec{\nabla} \cdot \vec{E} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 E_r) + \ldots$$

$$\vec{\nabla} \cdot \left(\frac{\hat{r}}{r^2} \right) = \frac{1}{r^2} \frac{\partial}{\partial r} \left[r^2 \left(\frac{1}{r^2} \right) \right] = 0$$

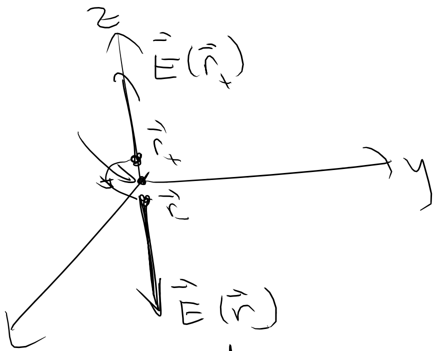
$$\int_V \vec{\nabla} \cdot \left(\frac{\hat{r}}{r^2} \right) d^3 r = \int_{\partial V} \frac{\hat{r}}{r^2} \cdot d\vec{S} = 4\pi$$



$$\vec{\nabla} \cdot \left(\frac{\hat{r}}{r^2} \right) = 0 \quad \text{if} \quad \vec{r} \neq 0$$

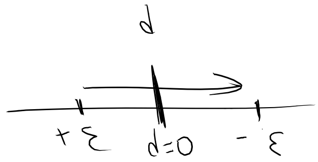
integral of $\vec{\nabla} \cdot \left(\frac{\hat{r}}{r^2} \right)$ over a volume containing $r = 0$ is finite ($= 4\pi$)

$$\boxed{\vec{\nabla} \cdot \left(\frac{\hat{r}}{r^2} \right) = 4\pi \delta^{(3)}(\vec{r})}$$

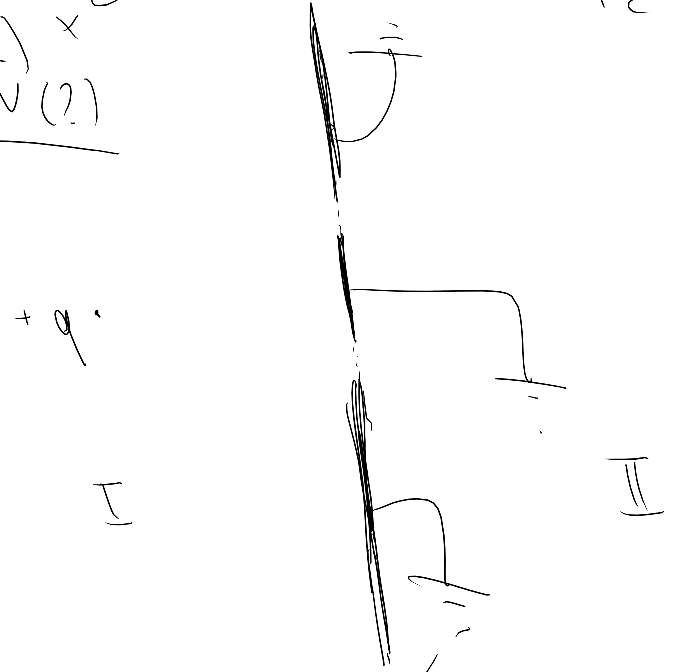


$\vec{r}_\pm = \pm \varepsilon \hat{z}$ $\quad$ ε very small

$\vec{r} = d\hat{z}$



(future) HW (?)

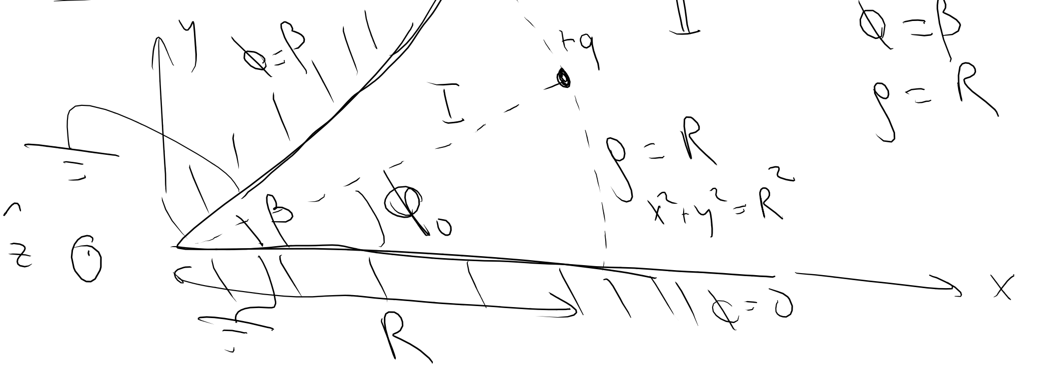


$\vec{E}_I = \ldots$

$\vec{E}_{II} = \ldots$

impose B.C.

Ex



Boundaries:

$\phi = 0$

$\phi = \beta$

$\rho = R$

except than on the Boundaries $\nabla^2 \Phi = 0$

$$\nabla^2 \Phi = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial \Phi}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 \Phi}{\partial \phi^2} + \frac{\partial^2 \Phi}{\partial z^2} = 0$$

Trial

$$\Phi(\rho, \phi, z) = R(\rho)\,\varphi(\phi)\,Z(z)$$

$$\frac{\nabla^2 \Phi}{\Phi} = \underbrace{\frac{1}{R}\frac{1}{\rho}\frac{d}{d\rho}\left(\rho \frac{dR}{d\rho}\right) + \frac{1}{\rho^2}\frac{1}{\varphi}\frac{d^2\varphi}{d\phi^2}}_{\rho,\ \phi} \underbrace{+\frac{1}{Z}\frac{d^2 Z}{dz^2}}_{z} = 0$$

$$\boxed{\frac{1}{Z}\frac{d^2 Z}{dz^2} = K^2}$$

$$0 = \frac{\nabla^2 \Phi}{\Phi} = \frac{1}{R}\frac{1}{\rho}\frac{d}{d\rho}\left(\rho\frac{dR}{d\rho}\right) + K^2 + \frac{1}{\rho^2}\underbrace{\left(\frac{1}{\varphi}\frac{d^2\varphi}{d\phi^2}\right)}_{\phi}$$

$$\boxed{\frac{1}{\varphi}\frac{d^2\varphi}{d\phi^2} = -m^2}$$

$$\varphi(\phi + 2\pi) = \varphi(\phi)$$

$$\boxed{\frac{1}{R}\frac{1}{\rho}\frac{d}{d\rho}\left(\rho\frac{dR}{d\rho}\right) + K^2 - \frac{m^2}{\rho^2} = 0}$$

$$\boxed{\frac{1}{Z}\frac{d^2 Z}{dz^2} = +K^2}$$

$$\Rightarrow \varphi_m = A_m \sin(m\phi + \delta_m)$$

$$Z(z) = A e^{Kz} + B e^{-Kz}$$

$$Z_K(z) = A e^{-K|z|}$$

$$\boxed{\sum_{Km} C^{I,II}_{Km} R^{I,II}_{Km}(\rho)\, \varphi_m(\phi)\, Z_K(z) = \Phi^{I,II}}$$

region I : $\rho < R$ region II $\rho > R$

$$\boxed{C^{I}_{Km} R^{I}_{Kn}(R) = R^{II}_{Kn}(R)\, C^{II}_{Km}}$$

$$\Delta \left.\frac{\partial \Phi}{\partial \rho}\right|_{\substack{\text{boundary}\\ \rho = R}} = 4\pi q\, \delta(\phi - \phi_0)\, \frac{1}{R}(\ )\, \delta(z)$$

$$\Phi(\phi = 0) = 0 \Rightarrow \sin \delta_m = 0 \Rightarrow \delta_m = 0$$

$$\Phi(\phi = \beta) = 0 \Rightarrow \sin(m\beta) = 0 \Rightarrow m = \frac{\pi}{\beta} n$$

$$\Phi^{I,II}(\rho, \phi, z) = \sum C^{I,II}_{nK} R^{I,II}_{nK}(\rho)\, \sin\left(\frac{\pi}{\beta} n \phi\right) e^{-K|z|}$$

$$m = \frac{\pi}{\beta} n$$

$$R^{II}_{nK}(\rho) \xrightarrow{\rho \to \infty} 0$$

$$R^{I}_{nK}(\rho) \xrightarrow{\rho \to 0} \text{finite}$$

$$\frac{1}{R} \frac{1}{\rho} \frac{d}{d\rho}\left(\rho \frac{dR}{d\rho}\right) + K^2 - \frac{m^2}{\rho^2} = 0$$

$$\frac{1}{\rho}\left[\rho \frac{d^2R}{d\rho^2} + \frac{dR}{d\rho}\right] + \left(K^2 - \frac{m^2}{\rho^2}\right) R = 0$$

$$\frac{d^2R}{d\rho^2} + \frac{1}{\rho} \frac{dR}{d\rho} + \left(K^2 - \frac{m^2}{\rho^2}\right) R = 0$$

Close to $\rho = 0$

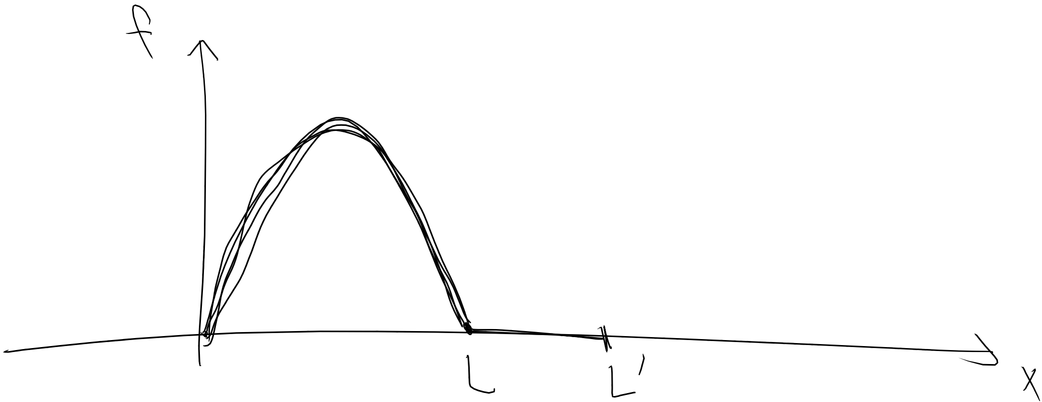
$$\rho^2 \frac{d^2R}{d\rho^2} + \rho \frac{dR}{d\rho} - m^2 R = 0$$

$$R = \rho^{\nu} \qquad \nu(\nu-1)\cancel{\rho^{\nu}} + \nu\,\cancel{\rho^{\nu}} - m^2 \cancel{\rho^{\nu}} = 0$$

$$\nu^2 - m^2 = 0$$

$$\nu = \pm m$$

Finite soln is $\boxed{R = \rho^{m}}$



$0 < x < L \qquad f(x) = \sum_n a_n \sin\left(\frac{n\pi}{L} x\right) \Leftarrow$

$f'(x) = \sum b_n \sin\left(\frac{n\pi}{L'} x\right) \Leftarrow$

$f(x) = f'(x)$ if $0 < x < L$

$f(x) \neq f'(x)$ if $L < x\ L'$