Phys109-MECHANICS

Altuğ Özpineci

METU

PHYS109

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Your Lecturer

- **Title and Name:** Prof. Dr. Altuğ Özpineci
- **Administrative Duty:** Advisor to Dept. Chair
- **Specialization:** High Energy Physics, Hadron Physics (properties of quarks and gluons)
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- **Prefered method to contact:** through e-mail
- **Web Page:** `http://www.metu.edu.tr/~ozpineci`
  Course Web Page: Course Material -> Phys109

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Syllabus

- Text Book: “Principles of Physics,” 10^{th} Edition, J. Walker, D. Halliday, R. Resnick
- Subjects to be covered:
  - Physics as a Science
  - Describing Motion-Kinematics
  - Causes for Changes in Motion - Dynamics
  - Applications of Newton’s Laws
  - (?)Probability in Physics - Thermodynamics

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Applications of Newton's Laws:

- Friction
- Circular Motion
- Gravitation
- Work and Energy
- Systems of Particles
- Collisions
- Rotation
- Statics
- Fluids
- Oscillations and Waves
- ...

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Supplamentary Material and References

- H. C. Ohanian, “Physics”
- R. P. Feynman “Lectures on Physics, Vol. 1”
  http://www.feynmanlectures.info
- Open Courseware Project (OCW)
  - MIT OCW (http://ocw.mit.edu) (in particular see Physics I: Classical Mechanics, Prof. Walter Lewin, Turkish translation is also available)
  - TÜBA OCW (in Turkish) (http://www.acikders.org.tr)
  - METU OCW (http://ocw.metu.edu.tr)
- “Feynman’s Lost Lecture: The Motion of Planets Around the Sun ”
- Any other related book in the **library**

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Grading: Phys109

Grades are **NOT** the aim of studying! They are only means of measuring how much you have learned!

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- 5% Pop Quizes
- 25% each midterm (two midterms in total)
- 25% Final Exam
- 20% Homework
- 0% Prereports
- Bonus: Can increase your grade upto half a letter
  - Translate 5 items from English wikipedia to Turkish wikipedia(or your mother tongue if different). You have to let us know in a month (deadline: November 6, 2015) which items you are planning to translate
  - Ask good questions on discussion in ODTUClass

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Pop Quizes

- The purpose is to encourage you
  - to attend the lectures
  - to study regularly,
- Will be simple question that can be easily done if you have payed attention to the lecture
- Their time and number will not be announced! They can be any day, and any time during the lecture

Prereports

- The purpose is to make attend the lecture/lab already having an idea about the material to be covered/experiment to be done.
- One page report: half page a summary of what you understood from the reading assignments, half a page questions that you have.
- No Questions, No Summary, Too much overlap with other resources: your prereport will not be accepted!

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HOMEWORK

- Their purpose is to encourage you to study regularly and to practice
- You can discuss the solutions in groups
- The homework you hand in should be what **YOU** understand

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Final Exam

Conditions under which you will **NOT** be allowed to take the final exam and receive an NA:

- Being absent in more than 30% of Pop Quizes
- Not taking any of the midterms without any excuse
- Not preparing more than two pre reports
- Not completing the term project

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10 ways to annoy a college professor (and lose a reference)

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2. Arrive late for class and then ask questions about topics discussed before you arrived. *Please anticipate traffic or parking issues, and build in extra time to get to class. If you had a plane to catch, a red carpet event to attend, or a business deadline, would you be late? If yes, then PR is not the industry for you.*

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10 ways to annoy a college professor (and lose a reference)

3. Skip class and arrive the following session asking, "Did I miss anything?" *No, of course you didn't. We stared at the wall and contemplated why you decided not to show up.*

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10 ways to annoy a college professor (and lose a reference)

9. Complain about the workload. *Do you think we enjoy taking 200 or more pages of students' work home to edit, grade, and assess? We give you work to help you learn and to prepare you for future success. Professors spend countless hours outside of class creating meaningful assignments, grading, and assessing your work to help you learn.*

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10 ways to annoy a college professor (and lose a reference)

10. Don't do your homework and then make excuses. *Please take responsibility for your own life and academic choices and success. In the job market, your employer will not care about your really great excuse regarding why you missed a deadline.*

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Ethics

It is your responsability to

- report your findings as they are even if you do not like them
- be honest about the consequences of your claims

Ethics

- You should never claim the work to be yours if you have not done it
  - Handing in somebody else's (from your friends, from internet, or from some other source) solutions in homework/midterms (cheating/plagiarism)
  - Claiming that your data/solution is correct even if you know that they are not (falsification/data fabrication)
- If you quote somebody else's work, make sure that you cite him/her so that the reader understands that you do not claim to be the owner of your work
- According to discipline regulation of committee of higher education, cheating is punished by sending the student away for at least one semester (YOK Öğrenci Disiplin Yönetmeliği)

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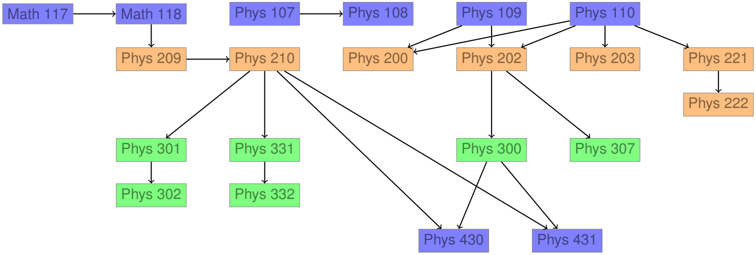
Prerequisite Courses

- Phys109/110 and Math117/118 are prerequisites to the higher level courses
- If you fail them, most probably, you can not graduate in four years.



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Learning/Lecturing Physics

- Watch: *Confessions of a Converted Lecturer, by Eric Mazur*
- Main lessons:
  - *"Traditional lecturing is nothing but the transfer of lecture notes from the notes of the lecturer to the notes of the student without passing through the brains of neither" by XXX YYY*
  - In a traditional lecture, students only learn 22% of what they don't know no matter what the lecturer does/doesn't do
  - It can increase to 44% on average if the students participate in the class

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“LECTURING”

- **Lecture:** late 14c., "action of reading, that which is read," from M.L. lectura "a reading, lecture," from L. lectus, $\cdots$
- **lesson:** l early 13c., "a reading aloud from the Bible," $\cdots$

Source: http://www.etymonline.com

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The way we lecture did not change since 14^{th} century.

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Learning

- When do you think that you have *LEARNED* a subject?
- I think that I have learned a subject if I can
  - carry out derivations without looking at any other reference
  - repeat the reasonings

  starting from the first principles
- Learning physics is **NOT** about memorizing formulas and applying them (you will be given formulas in all the exams)

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Pay Attention To Answers That are Not Answers

- Not all answers are really answers.
- Why the leaves are green?
- Because they reflect green light.
- Rephrase the question: Why do leaves reflect green light?
- Because they contain chlorophyll.
- Rephrase the question: Why does chlorophyll reflect green light?
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- What? and When? questions get you high grades
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- A common conversation:

  Q: What is ...?
  A: I calculate it by ... ?
  Q: Why do you do this calculation?
  A: We solved similar questions like this in the class!
  Q: Why?
  A: Is it wrong?
  Q: It is not wrong, but what is the meaning of our steps? Why did we take those steps?
  A: ??!??

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Best Teachers:

Your best teachers are (in order of importance):

1. Yourself: learn to learn on your own. (use the library!) You are the only person that will know when you have really learned a subject. Use your free time to review, learn future subjects, learn a new subject, etc.
2. Your friends: Collaborate. Your professors will not have a clue why you do not understand things that are obvious for them.
3. Your professor. Even if you think that a professor *does not know anything*, s/he still knows more physics than you, and has more experience.

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What is Physics

- Student participation is crucial for students to learn.
- Not so easy in large classrooms with $\approx$ 150 student
- Optimum number of students in a class $\approx$ 17 (PHED graduate students)
- Try the SMS system: send your answers to 4660 (Turkcell, Avea, or Vodafone free of charge)

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Physics

- Physics is about everything around you!
- Physics tries to find relationship between observations.
- Measurement is a crucial part of physics

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MEASUREMENT

- Has to be repeatable by anybody (that has the necessary equipment)
- Units and errors are a crucial part of the measurement!

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- Precision: how well repeated measurements yield similar results
- Accuracy: how close the measurement is to the real value

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- Accuracy: how close the measurement is to the real value **How can one know the real value without measurement? How is it possible to be sure that a given measurement is accurate?**

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Certain quantities are **defined**, not measured!

- 1 m: Length that light travels in 1/299,792,458 seconds
- 1 s: Time required for 9,192,631,770 periods of radiation emitted by cesium atoms

Speed of light is **exactly** 299,792,458 m/s!

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Speed of light is **exactly** 299,792,458 m/s!

- Platinum cylinder in International Bureau of Weights and Measures, Paris

These are the **fundamental units** (in SI system). Everything else is measured relative to these units.

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Q: How to measure learning?

Learn $\LaTeX$

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nature

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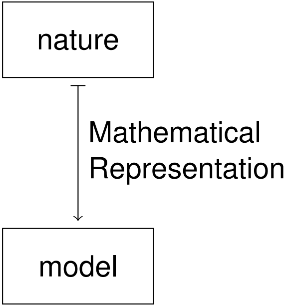
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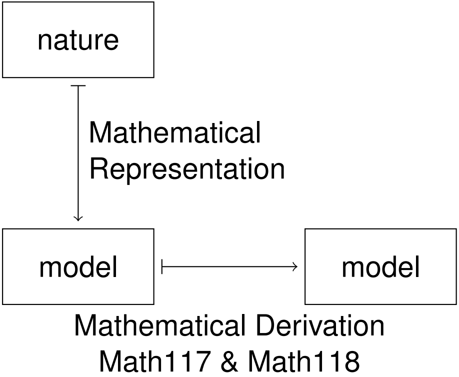
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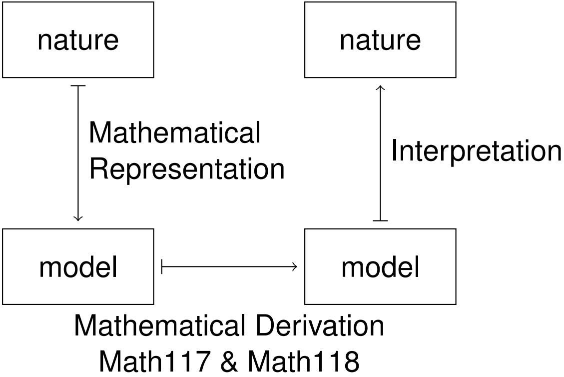
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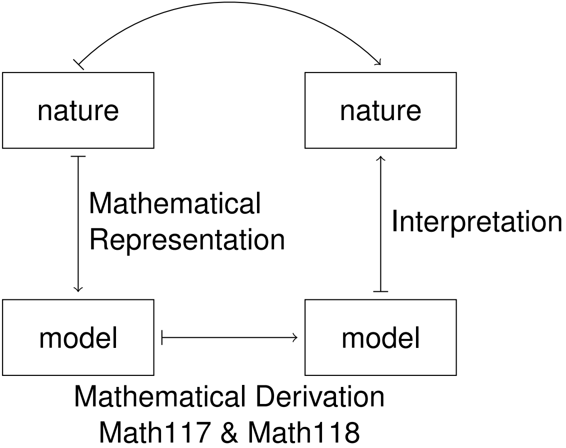
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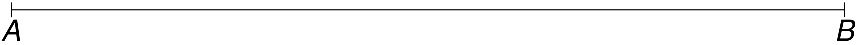
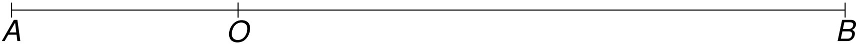
Math117 & Math118

- Assume all the motion is along a given line.



- Assume all the motion is along a given line.
- The position can be specified by a unique number: distance from origin O.



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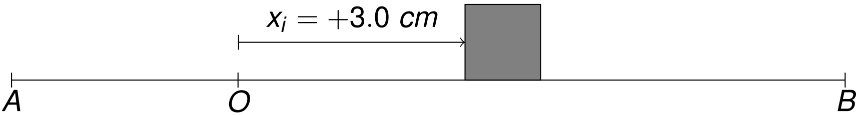
- The position can be specified by a unique number: distance from origin O.
- One side is denoted as "+"



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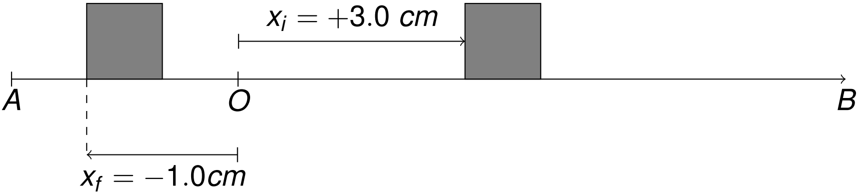

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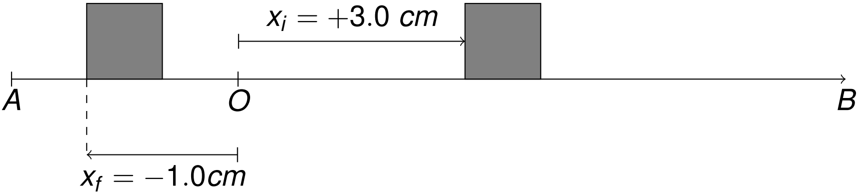
- Assume all the motion is along a given line.
- The position can be specified by a unique number: distance from origin O.
- One side is denoted as "+" , the other side "-"
- The choice of O and the "+" side is completely arbitrary



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Definitions:

- Displacement: the change in the position of an object Δx.

$$\begin{aligned} \Delta x &= \text{(final position)} - \text{(initial position)} \\ &= (3.0\ cm) - (-1.0\ cm) = 4.0\ cm \end{aligned} \quad (1)$$

- Average velocity: If Δt is the time that an object moves by Δx, average velocity is

$$\bar{v} = \frac{\Delta x}{\Delta t} \quad (2)$$

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- Average ve ves by Δx, average vel

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Greek Letters

Δ: Finite differences of any size
δ: Finite differences of small size
d: Infinitesimal difference (smaller than anything else)

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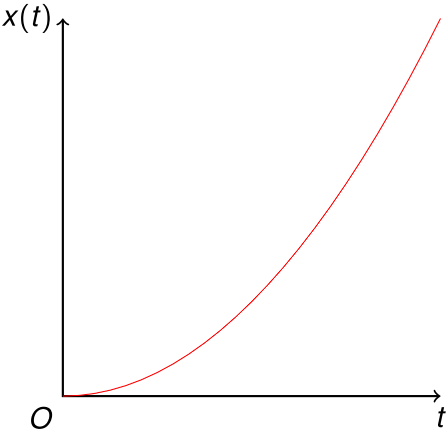

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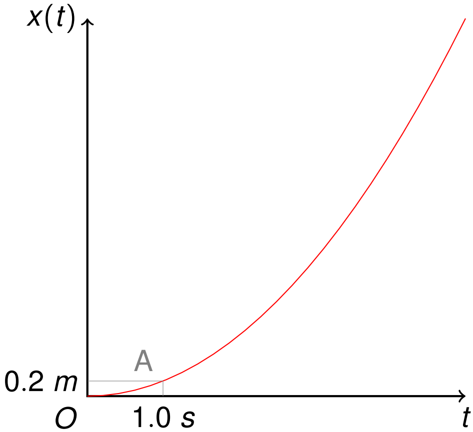

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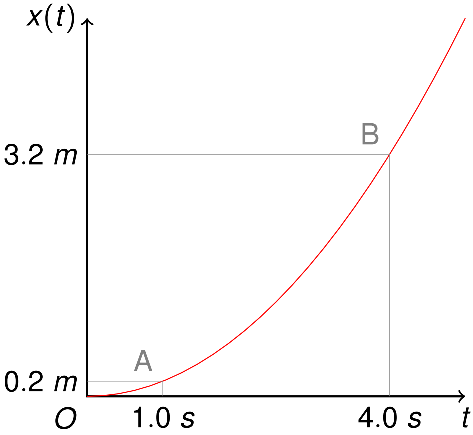

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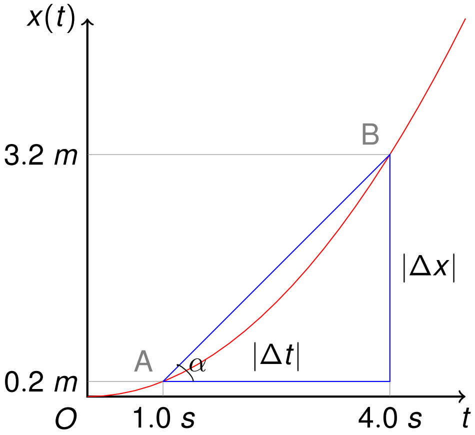

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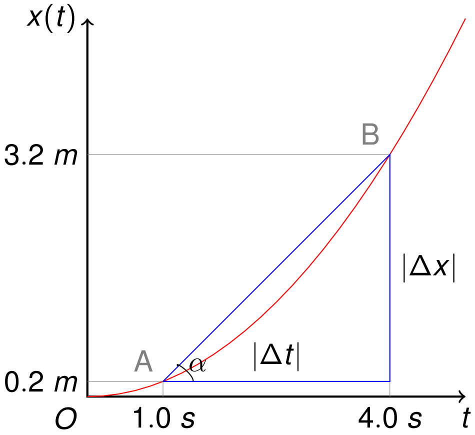

$$\bar{v}_{AB} = \frac{(3.2\ m) - (0.2\ m)}{(4.0\ s) - (1.0\ s)} = 1.0\ m/s \tag{3}$$

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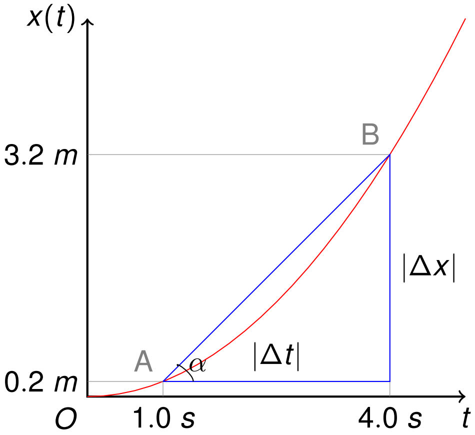

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- As the final time moves closer to the initial time, i.e. the point B moves towards point A, we obtain the instantaneous velocity:

$$v_{inst} = \lim_{B \to A} \bar{v}_{AB} = \lim_{t_f \to t_i} \frac{\Delta x}{\Delta t} = \lim_{t_f \to t_i} \frac{x_f - x_i}{t_f - t_i} = \frac{dx}{dt} \tag{4}$$

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- If δt is a sufficiently small amount of time, the displacement during this time is $\delta x = v \delta t$

$$x_f = x_i + v \delta t \tag{5}$$

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Question

If $v(t)$ is know for all $t \in (t_i, t_f)$, and a particle is at the position $x(t_i) = x_0$ initially, how can we find $x(t)$ for any $t \in (t_i, t_f)$?

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A: Assume δt is sufficiently small and $t_f = t_i + N\delta t$.

$$\begin{aligned} x(t_i + \delta t) - x(t_i) &= v(t_i)\delta t \\ x(t_i + 2\delta t) - x(t_i + \delta t) &= v(t_i + \delta t)\delta t \\ x(t_i + 3\delta t) - x(t_i + 2\delta t) &= v(t_i + 2\delta t)\delta t \\ &\cdots \\ x(t_i + N\delta t = t_f) - x(t_i + (N-1)\delta t) &= v(t_2 + (N-1)\delta t) \end{aligned} \tag{6}$$

$+$

$$x(t_f) - x_0 = \sum_{k=0}^{N-1} v(t_i + k\delta t)\delta t \tag{7}$$

Read Zeno's paradox!

odtu

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$$x(t_f) - x_0 = \sum_{k=0}^{N-1} v(t_i + k\delta t)\delta t \xrightarrow{\delta t \to 0} \int_{t_i}^{t_f} v(t)dt \qquad (7)$$

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Read Zeno's paradox!

Special Case: Motion with constant velocity v_0:
In this case

$$x(t_f) - x_0 = \sum_{k=0}^{N-1} v(t_i + k\delta t)\delta t = \sum_{k=0}^{N-1} v_0 \delta t = v_0 N \delta t = v_0(t_f - t_i) \qquad (8)$$

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$$x(t) = v_0(t - t_i) + x_0 \qquad (9)$$

Note that for motion with constant velocity $\bar{v} = v_0$. Hence $\Delta x = v_0 \Delta t$

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- The same steps can be repeated for the change of velocity.
  - $\bar{a} = \frac{\Delta v}{\Delta t}$. The unit of acceleration is m/s^2
  - $a_{inst} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} \equiv a$
  - $v(t) = v(t_0) + \int_{t_0}^{t_f} a(t')dt'$

odtu

- The same steps can be repeated for the change of velocity.
  - $\bar{a} = \frac{\Delta v}{\Delta t}$. The unit of acceleration is m/s^2
  - $a_{inst} =$ l
  - $v(t) =$

Acceleration is in the direction of Δv, **NOT** in the direction of v.

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Example:

Motion with Constant Acceleration. Initial conditions: $x(0) = 0$, $v(0) = 0$. Realistic case: You stand at the top of a building. You are holding a mass m in your and release it from rest outside a window.

- Let a be the constant acceleration.

$$v(t) = v(0) + \int_0^t a dt' = at \tag{10}$$

- The position:

$$\begin{aligned} x(t) &= x(0) + \int_0^t v(t') dt' \\ &= \int_0^t (at') dt' = \frac{1}{2} a t'^2 \Big|_0^t = \frac{1}{2} a t^2 \end{aligned} \tag{11}$$

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Dimensional Analysis

Most of the time, the final formula can be estimated unto overall coefficients using dimensions only. Denote the dimension of any quantity O by $[O]$

- Dimension of $x(t)$ is $[x(t)] = m$
- The dimensionful parameters in the problem are the acceleration a and the time t.
- Assume $x(t) = Aa^k t^l$ where A, k and l are numbers.

$$[Aa^m t^l] = [A][a]^k[t]^l = 1\left(\frac{m}{s^2}\right)^k s^l = m^k s^{l-2k} \qquad (12)$$

- $x = Aa^k t^l \implies k = 1$ and $l - 2k = 0 \implies x(t) = Aat^2$
- Explicit calculation shows $A = \frac{1}{2}$.

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- In principle these steps can be done for the change in acceleration, change in the change in the acceleration, etc.
- Newton's Laws tell us that this is not necessary
- The acceleration of an object is determined by external effects.

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Compare

$$v(t) = \frac{dx}{dt} \Longleftrightarrow x(t) = x(0) + \int_0^t v(t')dt' \qquad (13)$$

$$a(t) = \frac{dv}{dt} \Longleftrightarrow v(t) = v(0) + \int_0^t a(t')dt' \qquad (14)$$

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Integration is the inverse of differentiation

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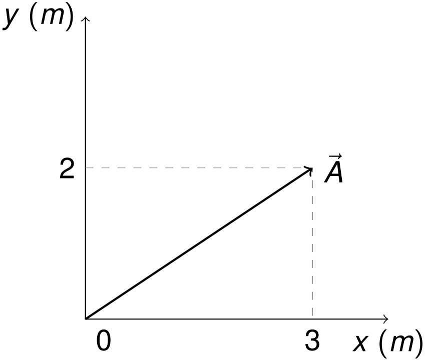
Vectors

- For motion that is not confined to a line, more than a number is necessary to describe the direction.
- A vector is *a recipe* for how to go to the point *A* from the origin.
- A vector is a number and a direction
- Origin is arbitrarily chosen



$$\vec{A} = (3,2)\ m \tag{15}$$

$$\vec{A} = (3\ m)\hat{x} + (2\ m)\hat{y} \tag{16}$$

$$\vec{A} = (3\ m)\hat{i} + (2\ m)\hat{j} \tag{17}$$

$$\vec{A} = (\sqrt{13}\ m, \arctan\frac{2}{3}) \tag{18}$$

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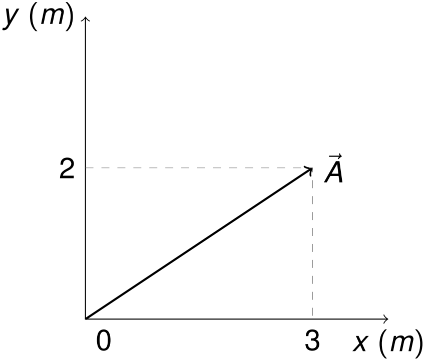

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$$\vec{A} = (2,3)\ m \tag{19}$$
$$\vec{A} = (\sqrt{13}\ m, \arctan\frac{3}{2})$$

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Vector Operations-Multiplication by a number

- A vector $\vec{A}$ is a number (the length of the vector, $|\vec{A}|$) and a direction.
- The vector $\lambda\vec{A}$ is another vector
  - The length of $\lambda\vec{A}$ is $|\lambda\vec{A}| = |\lambda||\vec{A}|$
  - The direction of $\lambda\vec{A}$ is the same as the direction of $\vec{A}$ if $\lambda > 0$, and opposite to $\vec{A}$ if $\lambda < 0$

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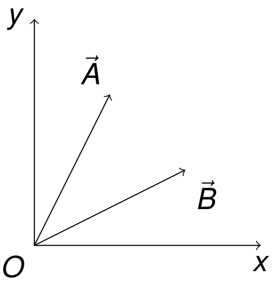
Vector Operations-Addition of Vectors

Geometrical Addition



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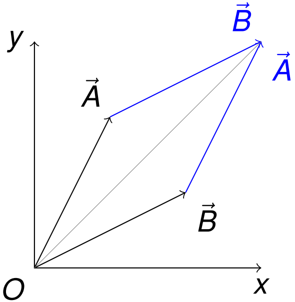

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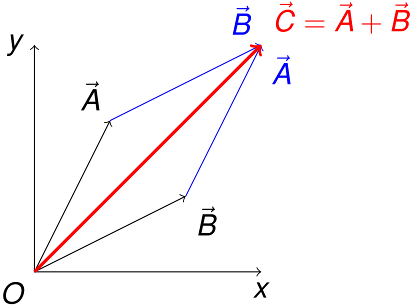

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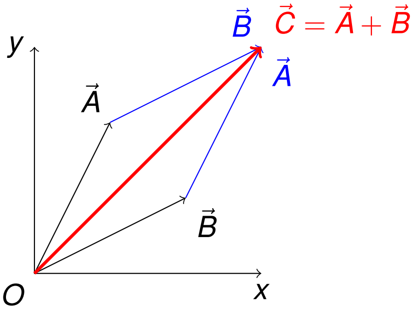
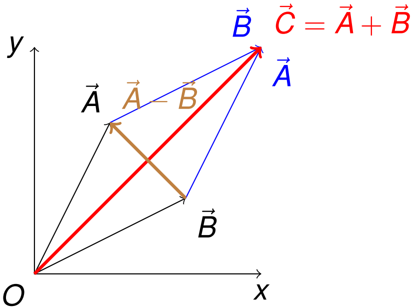


Componentwise Addition

- $\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}$
- $\vec{B} = B_x\hat{x} + B_y\hat{y} + B_z\hat{z}$
- $\vec{C} = C_x\hat{x} + C_y\hat{y} + C_z\hat{z}$
- $C_x = A_x + B_x$, $C_y = A_y + B_y$ $C_z = A_z + B_z$
- $C_i = A_i + B_i$, $i = x,\ y$ or z

Vector Operations-Addition of Vectors

Geometrical Addition



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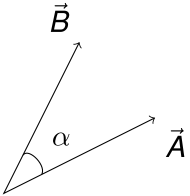
Subtraction

$$\vec{A} - \vec{B} = \vec{A} + ((-1)\vec{B})$$

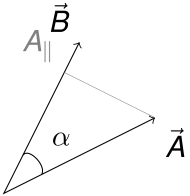
Vector Operations: Scalar Product



- Scalar product gives a number from two vectors
- $\vec{A} \cdot \vec{B} \equiv |\vec{A}||\vec{B}| \cos\alpha$

odtu

Vector Operations: Scalar Product



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odtu

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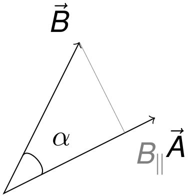

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odtu

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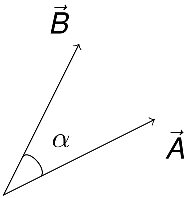
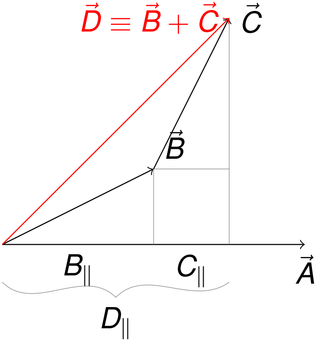


- Scalar product gives a number from two vectors
- $\vec{A} \cdot \vec{B} \equiv |\vec{A}||\vec{B}| \cos\alpha$
- Scalar product is linear: $\vec{A} \cdot (a\vec{B} + b\vec{C}) = a(\vec{A} \cdot \vec{B}) + b(\vec{A} \cdot \vec{C})$
- $\hat{x} \cdot \hat{x} = \hat{y} \cdot \hat{y} = \hat{z} \cdot \hat{z} = 1$, $\hat{x} \cdot \hat{y} = \hat{x} \cdot \hat{z} = \hat{y} \cdot \hat{z} = 0$

odtu

Vector Operations: Scalar Product



$$\vec{A}\cdot\vec{D} = AD_{||} = A(B_{||}+C_{||}) = \vec{A}\cdot\vec{B}+\vec{A}\cdot\vec{C}$$

duct gives a number from

;

$\vec{B}| \cos\alpha$

duct is linear:

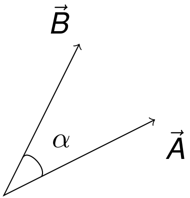
$\vec{C}) = a(\vec{A}\cdot\vec{B}) + b(\vec{A}\cdot\vec{C})$

$= \hat{z}\cdot\hat{z} = 1,$

$= \hat{y}\cdot\hat{z} = 0$

odtu

Vector Operations: Scalar Product



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- $\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$
- $A_x = \vec{A}\cdot\hat{x}$, $A_y = \vec{A}\cdot\hat{y}$, and $A_z = \vec{A}\cdot\hat{z}$

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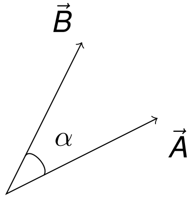
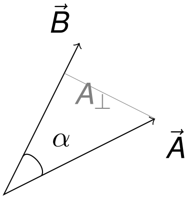
Vector Operations: Vector Product



- Vector product gives a vector from two vectors
- $|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}| \sin \alpha$
- Direction of $\vec{A} \times \vec{B}$ is given by the right hand rule. ($\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$)

Vector Operations: Vector Product



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odtu

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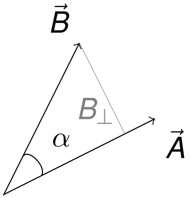

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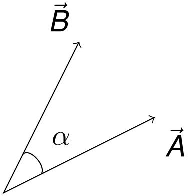

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- $\hat{x} \times \hat{x} = \hat{y} \times \hat{y} = \hat{z} \times \hat{z} = 0$, $\hat{x} \times \hat{y} = \hat{z}$, $\hat{x} \times \hat{z} = -\hat{y}$, $\hat{y} \times \hat{z} = \hat{x}$

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Vector Operations- Vector Division

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Vector Operations- Vector Division

Division by a vector DOES NOT exist!

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Equality of Vectors

Two vectors are equal only if all their components are equal:

$$\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z} \quad (20)$$

$$\vec{B} = B_x\hat{x} + B_y\hat{y} + B_z\hat{z} \quad (21)$$

if $\vec{A} = \vec{B}$, then

$$A_x = B_x, \quad A_y = B_y, \quad A_z = B_z \quad (22)$$

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