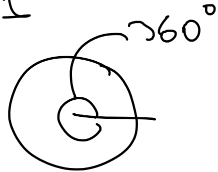
8 October 2015

$\cos(0.5) = 0.88$

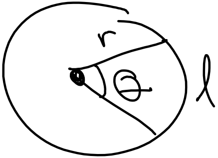
$\cos(0.5) = 0.999961$



Angles: degree

radians

conventions



$$\theta(\text{rad}) \equiv \frac{l}{r}$$

$$0.5\ \text{rad} = 0.5\ \text{rad}\ \frac{360^\circ}{2\pi\ \text{rad}}$$

$$0.5\ \text{rad} \simeq 30^\circ$$

$$\frac{r}{r+1.5\ \text{m}}$$

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Kinematics

reference point $x_u = +(3.2 \mp 0.1)\,m$

different ref. point : $x'_u = 0\ m$

$x''_u = (-3.2 \mp 0.1)\,m$

Displacement

$x_i = (3.2 \pm 0.1)\,m$

$x_f = (5.3 \pm 0.2)\,m$

$\Delta x \equiv x_f - x_i = 2.1 \pm 0.1\ m$

$\pm 0.2\ m$

$\pm 0.5\ m$

$\pm 0.3\ m$ ✓

$\pm 0.4\ m$

↳ Delta

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Ex

$x_i = (3.2 \pm 0.2)\,m$ ←

$x_f = (2.1 \pm 0.2)\,m$

$\boxed{\Delta x = x_f - x_i} = -0.9 \pm 0.4$

$-1.1 \pm 0.4\ m$ ✓

-1.1

$+1.1 \pm 0.4$

$-0.9\ m$

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Velocity : displacement per unit time

$$V_{av} = \frac{\Delta X}{\Delta t} = \frac{X_f - X_i}{t_f - t_i}$$

velocity vs speed

$$x_i = (4.5 \pm 0.1)\,m \qquad t_i = 0\,s$$

$$x_f = 0\,m \qquad t_f = (3.0 \pm 0.2)\,s$$

$$V_{av} = ? \simeq \frac{\Delta x}{\Delta t} \simeq \frac{-4.5m}{3s} \simeq -1.5 m/s$$

$$|V_{av}| \simeq 1.5\, m/s$$

$$x_{ff} = (4.5 \pm 0.1)\,m \qquad t_{ff} = (13.0 \pm 0.5)\,s$$

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$$v_{av}(t_i \to t_{ff}) = \frac{\Delta x}{\Delta t} \sim 0 \ m/s$$

$$v_{av}(t_f \to t_{ff}) = \frac{\Delta x}{\Delta t} \sim \frac{4.5 m}{10 s} \sim 0.45 \ m/s$$

$x_{ff} \sim 4.5 m$ $\quad$ $t_{ff} \sim 10 s$

$x_f \sim 0 m$ $\quad$ $t_f \sim 0 s$

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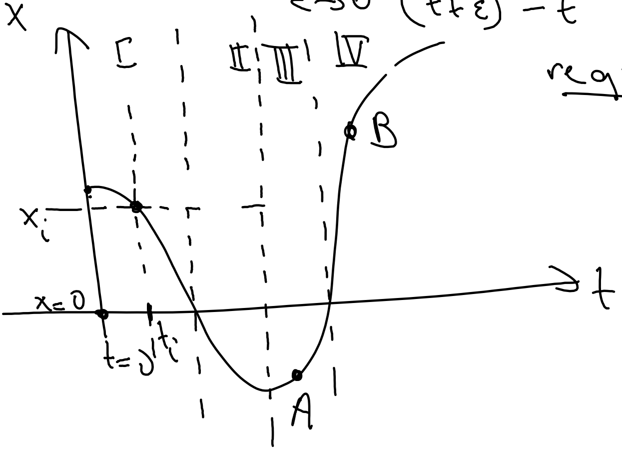
Instantaneous Velocity

$$V_{inst}(t) = \lim_{\varepsilon \to 0} \frac{X(t+\varepsilon) - X(t)}{(t+\varepsilon) - t}$$



region	X	V
I	>0	<0
II	<0	<0
III	<0	>0
IV	>0	>0

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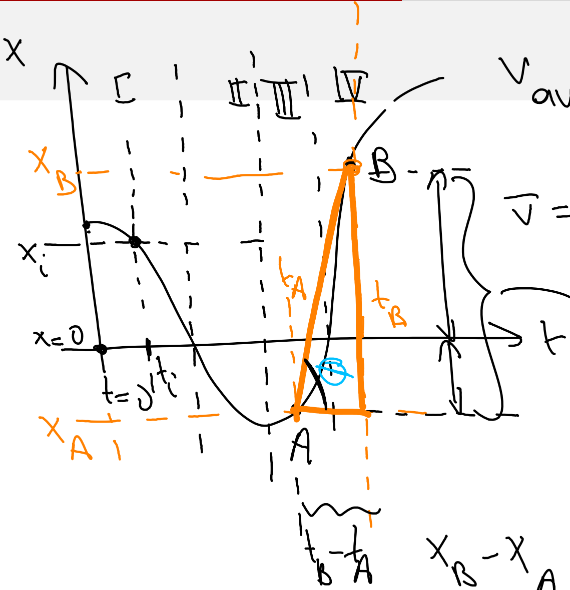

$$v_{av} \equiv \bar{v} = \frac{\Delta x}{\Delta t}$$

$$\bar{v} = \frac{x_B - x_A}{t_B - t_A} = \tan\theta$$

$x_A + x_B$

$-x_A + x_B$

$$x_B - x_A = |x_A| + |x_B|$$

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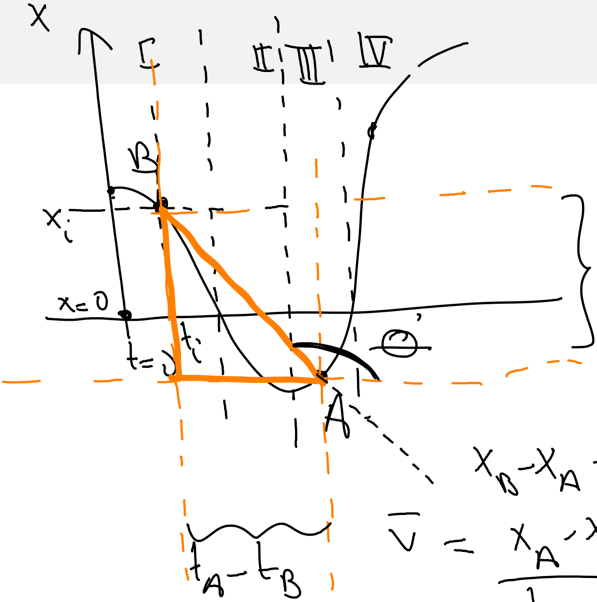


$x_A - x_B$

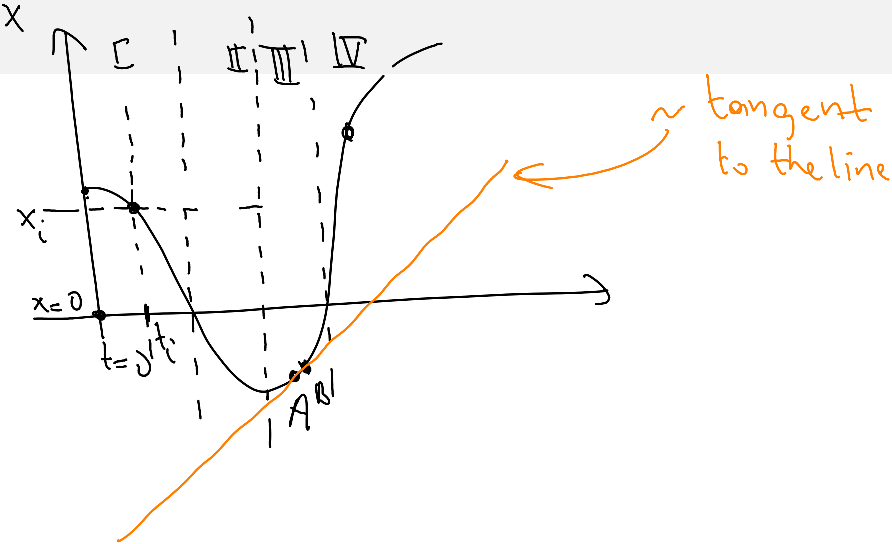
$|x_B - x_A|$ ✓

$x_A - x_B$

$x_B - x_A$ ✓

$$x_B - x_A = |x_B| + |x_A|$$

$$\bar{v} = \frac{x_A - x_B}{t_A - t_B} = \tan\theta'$$


X
I
II
III
IV
x_i
x=0
t=0
t_i
A
B
~ tangent
to the line

Obtaining Position from Velocity

Motion with constant velocity: v_0

$$\bar{v} = v_0 = \frac{x(t) - x_0}{t - t_0} \Rightarrow x(t) - x_0 = v_0(t - t_0)$$

$$x(t) = x_0 + v_0 \cdot (t - t_0)$$

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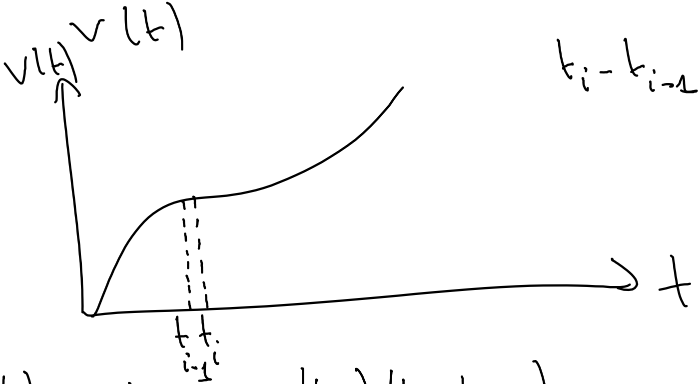
Motion with variable velocity.



$t_i - t_{i-1} \equiv \varepsilon$

↳ epsilon

$$x(t_i) = x(t_{i-1}) + v(t_{i-1})(t_i - t_{i-1})$$

$$x(t_i) = x(t_{i-1}) + v(t_{i-1})\,\varepsilon$$

$$x(t_1) = x(t_0) + v(t_0)\,\varepsilon$$

$$x(t_2) = x(t_1) + v(t_1)\,\varepsilon$$

$$x(t_3) = x(t_2) + v(t_2)\,\varepsilon$$

$$\cdots$$

$$+\quad x(t) = x(t-\varepsilon) + v(t-\varepsilon)\,\varepsilon$$

$$x(t) = x(t_0) + \sum_{t_0}^{t-\varepsilon} v(t_i)\,\varepsilon$$

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$$x(t) = x(t_0) + \lim_{\varepsilon \to 0} \sum_{t_i = t_0}^{t-\varepsilon} v(t_i)\,\varepsilon \quad (\varepsilon = \delta t)$$

$$x(t) \equiv x(t_0) + \int_{t_0}^{t} v(t')\,dt'$$

δ : minuscle delta

Δ : capital delta

$S \to \int$

position $\rightleftarrows$ change in position: velocity $\longrightarrow$ change in velocity: acceleration

$$v_{av} = \frac{x_f - x_i}{t_f - t_i} \qquad a_{av} = \frac{v_f - v_i}{t_f - t_i}$$

Motion with Constant Acceleration

$$\bar{a} = a_0$$

$$\bar{a}(t_0, \Delta t) = \frac{v(t) - v(t_0)}{t - t_0} = a_0$$

$$\Rightarrow v(t) = v(t_0) + a_0 (t - t_0)$$

compare with

$$x(t) = x(t_0) + v_0 (t - t_0)$$

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$$a_{inst} = a(t) = \lim_{\varepsilon \to 0} \frac{v(t+\varepsilon) - v(t)}{(t+\varepsilon) - t}$$

October 13, 2015

$$v_{av} = \frac{\Delta x}{\Delta t}$$

Δx: displacement

$$s_{av} = \frac{\ell}{\Delta t}$$

ℓ: covered distance

$$v_{inst} = v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx(t)}{dt}$$

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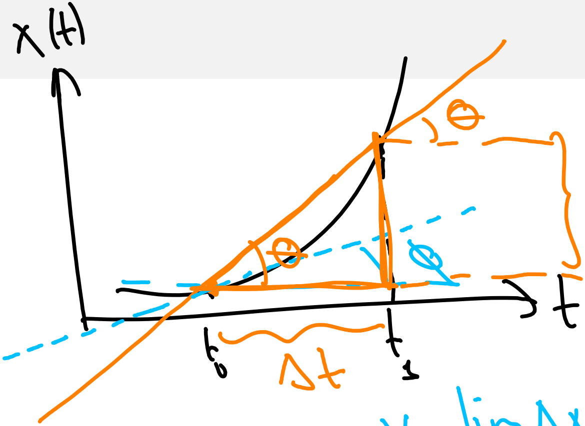


$V_{av} = \tan\theta$

$V = \tan\phi$

$V_{av} = \frac{\Delta x}{\Delta t} = \tan\theta$

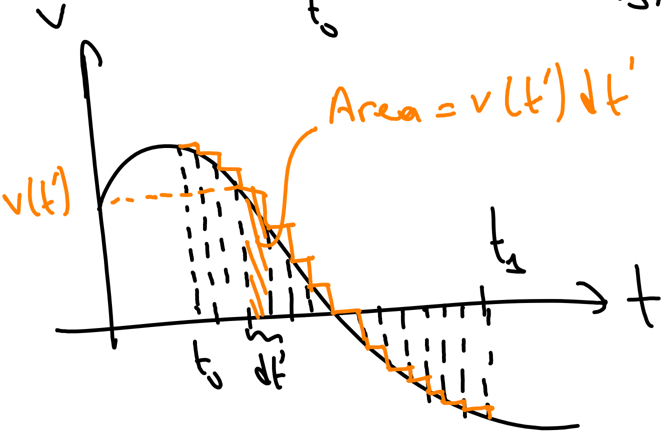
$V = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$

θ : theta

ϕ : phi

$$\Delta x = v_0 \Delta t$$

$$x(t_1) - x(t_0) = \int_{t_0}^{t_1} v(t')\, dt'$$



Δx : area between velocity curve and time access

Acceleration: change in velocity per unit time

$$v_{av} = \frac{\Delta x}{\Delta t}$$

$$a_{av} = \frac{\Delta v}{\Delta t}$$

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

$$a_{inst} \equiv a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

$$v = \frac{dx}{dt}$$

$$a = \frac{dv}{dt}$$

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$$a_{av} = \frac{v_1 - v_0}{t_1 - t_0} = - \frac{v_0 - v_1}{t_1 - t_0} = -\tan\theta = \tan(\pi - \theta)$$

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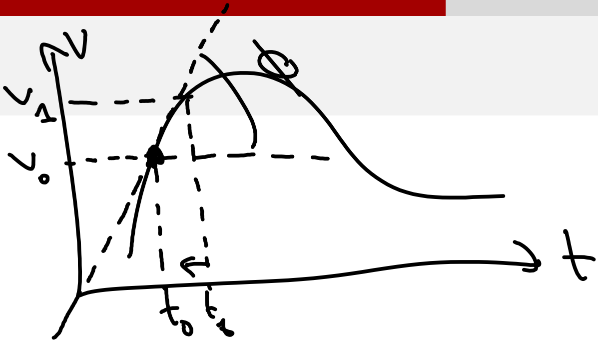

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \tan\phi$$

$t_f - t_i = 2s$

1) $v_i = -3\ m/s$ $\qquad s_i = 3m/s \equiv |v_i|$

$v_f = -5\ m/s$ $\qquad s_f = 5m/s \equiv |v_f|$

$$a_{av} = \frac{(-5m/s) - (-3m/s)}{2s} = -1m/s^2$$

2) $v_i = -5m/s$ $\qquad s_i = 5m/s$

$v_f = -3m/s$ $\qquad s_f = 3m/s$

$$a_{av} = \frac{(-3m/s) - (-5m/s)}{2s} = 1m/s^2$$

3) $v_i = 5m/s;\ v_f = 3\ m/s$ $\qquad a_{av} = -1m/s^2$

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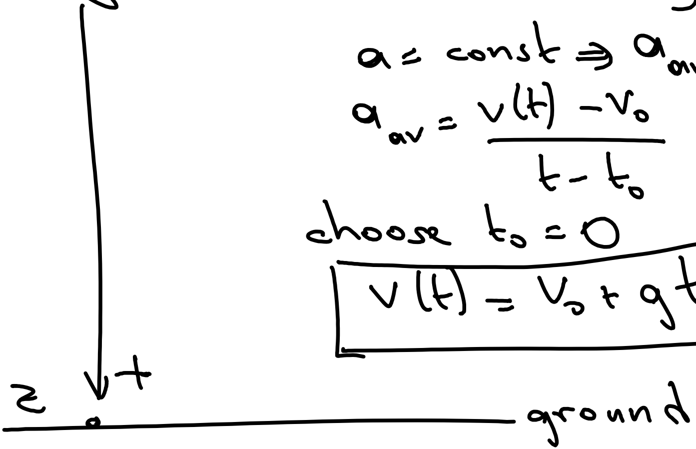
Motion with Constant Acceleration

$g \simeq 9.8\ m/s^2 \qquad a = g$



$a = \text{const} \Rightarrow a_{av} = a$

$$a_{av} = \frac{v(t) - v_0}{t - t_0} = g$$

choose $t_0 = 0$

$$\boxed{v(t) = v_0 + g t}$$

$s_0 = 5\,m/s$, upwards

$v_0 = -5\,m/s$

$v(t) = (-5\,m/s) + (9.8\,m/s^2)\,t$

$v(t=0.1s) = (-5\,m/s) + (9.8\,m/s^2)(0.1s)$

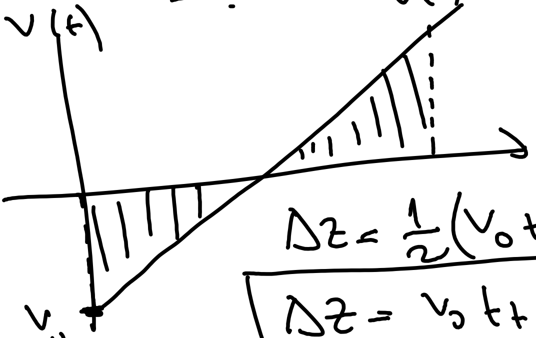
$= -5\,m/s + 0.98\,m/s$

$v(t=0.1s) \simeq -4\,m/s$

$v(t_1) = 0 \Rightarrow t_1 = ?$

$$v(t_1) = v_0 + g t_1 = 0 \Rightarrow t_1 = -\frac{v_0}{g}$$

$$z(t) = ?$$



area = Δz

$$\Delta z = v_{av} \Delta t$$

$$= \frac{1}{2}(v_0 + v(t))(t - t_0), \quad t_0 = 0$$

$$\Delta z = \frac{1}{2}(v_0 + v_0 + gt)\, t$$

$$\boxed{\Delta z = v_0 t + \frac{1}{2} g t^2}$$

$$z(t) = z_0 + v_0 t + \frac{1}{2} g t^2$$

$$\Delta z = \int_{t_0=0}^{t} v(t')\,dt'$$

$$= \int_0^t (v_0 + g t')\,dt' = v_0 t + g \frac{1}{2} t^2$$

$$\boxed{z = z_0 + v_0 t + \frac{1}{2} g t^2}$$

$$z(t_1) - z_0 = v_0 \left(-\frac{v_0}{g}\right) + \frac{1}{2} g \left(-\frac{v_0}{g}\right)^2$$

$$z(t_1) - z_0 = -\frac{1}{2} \frac{v_0^2}{g} \Rightarrow h = \frac{v_0^2}{2g}$$

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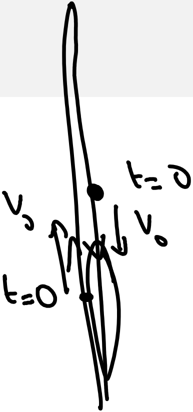
t=0
v_0
v_0
t=0

Dimensional analysis

$[a] = m/s^2$

$[h] = m$

$t = ?$

$[m] = kg$

$t = A\, a^n h^k m^l$

A: dimensionless constant.

$[A]$: dimension of A

$$s = \frac{m^n}{s^{2n}} m^k (kg)^l$$

$l = 0$

$1 = -2n \Rightarrow n = -\frac{1}{2}$

$0 = n + k \Rightarrow k = \frac{1}{2}$

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$$t = A\, a^{n} h^{k} m^{l}$$

A: dimensionless constant.

$l = 0$

$1 = -2n \Rightarrow n = -\frac{1}{2}$

$0 = n + k \Rightarrow k = \frac{1}{2}$

$$t = A\, a^{-\frac{1}{2}} h^{1/2}$$

$$t = A\sqrt{\frac{h}{a}} \Rightarrow \boxed{h = \frac{1}{A^2} a t^2}$$

explicit calculation $\Rightarrow A^2 = 2$

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$$v(t) = v_0 + gt$$

$$z(t) = z_0 + v_0 t + \frac{1}{2} g t^2$$

~~$t = \dfrac{d}{\sqrt{v_i + v_f}}$~~

Vectors

direction + magnitude ≡ vectors

1D

Δx: displacement

$$\Delta x = x_f - x_i$$

$$= x_f + (-1)x_i$$

3D (2D)

$\Delta \vec{r}$: displacement vector.

$$\Delta \vec{r} = \vec{r}_f - \vec{r}_i$$

$$\Delta \vec{r} = \vec{r}_f + (-1)\vec{r}_i$$

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Multiplying Vectors By a Number

$\vec{r}$: vector

r : (magnitude of vector $\vec{r}$) $= |\vec{r}|$

$a\vec{r}$: vector $\quad |a\vec{r}| = |a|\, r$

↳ number

direction of $a\vec{r}$ = $\begin{cases} \text{direction of } \vec{r} \text{ if } a>0 \\ \text{opposite direction if } a<0 \end{cases}$

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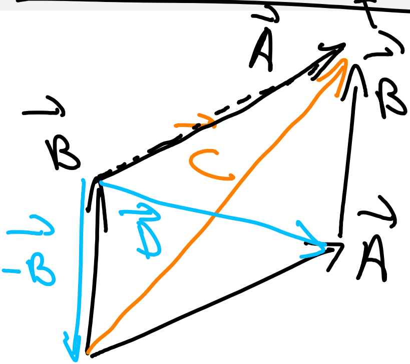
Addition of Vectors



$\vec{C} = \vec{A} + \vec{B}$

$\vec{A} + \vec{B} = \vec{B} + \vec{A}$

$\vec{D} = \vec{A} - \vec{B} = ?$

$= \vec{A} + (-\vec{B})$

October 15, 2015

PHED students that are taking Phys109, come to see me in the 10 minute break

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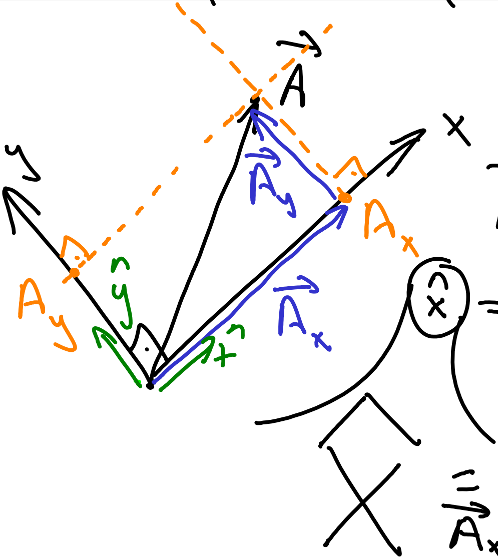
Components of Vectors



A_x, A_y: components of vector A.

$\vec{A} = \vec{A}_x + \vec{A}_y$

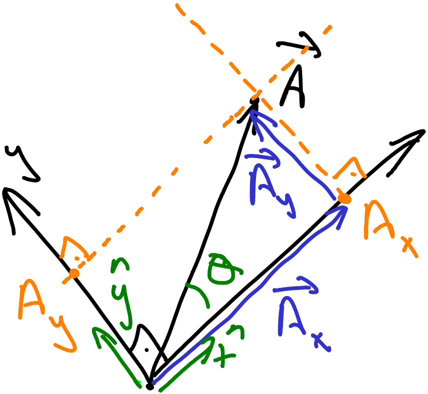
$\hat{x} = \hat{i}$ = x hat (i hat)

= a unit vector in the direction of x.

$\vec{A}_x = A_x \hat{x}$

$$\vec{A} = A_x \hat{x} + A_y \hat{y}$$

$\vec{A}$ vector in terms of its components.

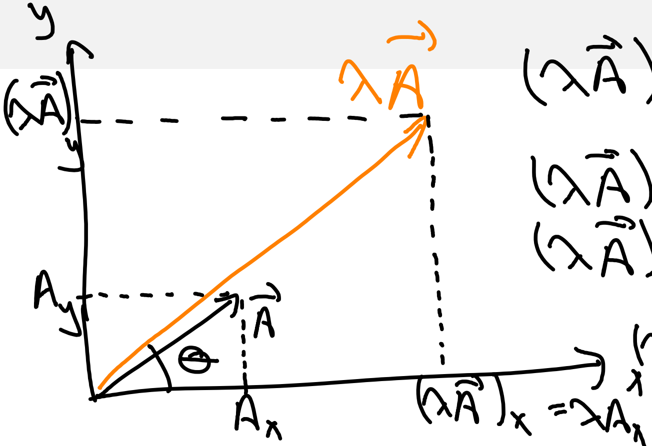


$$A_x = A\cos\theta$$

$$A_y = A\sin\theta$$

$$A_x^2 + A_y^2 = A^2$$

$$\frac{A_y}{A_x} = \tan\theta$$



$$(\lambda\vec{A})_x = |\lambda\vec{A}|\cos\theta = \lambda A\cos\theta$$

$$(\lambda\vec{A})_x = \lambda A_x$$

$$(\lambda\vec{A})_y = |\lambda\vec{A}|\sin\theta = \lambda A\sin\theta$$

$$(\lambda\vec{A})_y = \lambda A_y$$

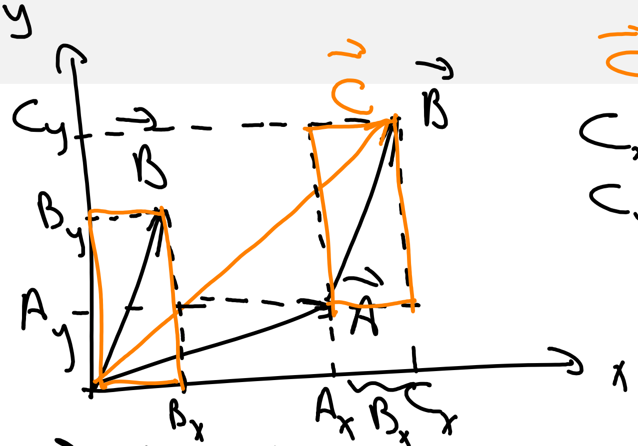
λ: small greek letter lambda

Λ: capital λ

$$\lambda\vec{A} = \lambda A_x\hat{x} + \lambda A_y\hat{y}$$
$$= \lambda(A_x\hat{x} + A_y\hat{y})$$

<u>Circular Coordinates</u>

$$\vec{A} = (A, \theta)$$
$$\lambda\vec{A} = (\lambda A, \theta) \quad (\lambda > 0)$$



$$\vec{C} = \vec{A} + \vec{B}$$

$$C_x = A_x + B_x$$

$$C_y = A_y + B_y$$

$$\vec{A} + \vec{B} = (A_x\hat{x} + A_y\hat{y}) + (B_x\hat{x} + B_y\hat{y})$$

$$= \underbrace{(A_x + B_x)}_{C_x}\hat{x} + \underbrace{(A_y + B_y)}_{C_y}\hat{y}$$

$\vec{A}$: a vector

$|\vec{A}| \equiv A$: magnitude of vector $\vec{A}$

$\hat{A}$: a unit vector in the direction of $\vec{A}$

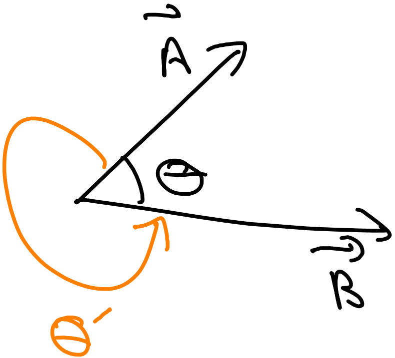
Products of Vectors

$\vec{A}, \vec{B}$ —scalar product→ $\vec{A} \cdot \vec{B}$ (number)

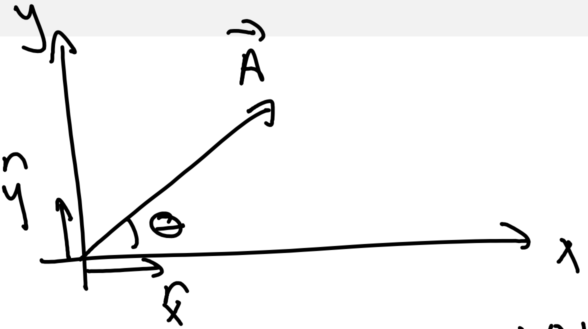
$\vec{A}, \vec{B}$ —vector product→ $\vec{A} \times \vec{B}$ (vector)

Scalar Product ≡ Dot Product

$$\vec{A}\cdot\vec{B} = AB\cos\theta = AB\cos\theta'$$

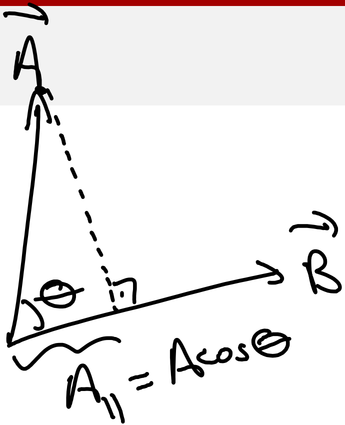


$$|\vec{A}| \equiv A$$



$$A_x = A\cos\theta = A|\hat{x}|\cos\theta = \vec{A}\cdot\hat{x}$$

$$A_y = A\sin\theta = A|\hat{y}|\cos\left(\frac{\pi}{2}-\theta\right) = \vec{A}\cdot\hat{y}$$



$$\vec{A}\cdot\vec{B} = AB\cos\theta$$
$$= B A_{\parallel}$$
$$= A B_{\parallel}$$

$$\vec{A} = A_x \hat{x} + A_y \hat{y}$$
$$\vec{B} = B_x \hat{x} + B_y \hat{y}$$

$$\hat{x} \cdot \hat{x} \equiv \hat{x}^2 = 1$$
$$\hat{y} \cdot \hat{y} \equiv \hat{y}^2 = 1$$
$$\hat{x} \cdot \hat{y} = 0$$

$$\vec{A} \cdot \vec{B} = (A_x \hat{x} + A_y \hat{y}) \cdot (B_x \hat{x} + B_y \hat{y})$$
$$= A_x B_x + A_x B_y \, 0$$
$$+ A_y B_x \, 0 + A_y B_y \, 1$$

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y$$

Example

$$\vec{A} = 3\hat{x} + \hat{y} - 2\hat{z}$$
$$\vec{B} = \hat{x} - \hat{y} + \hat{z}$$

If $\vec{A}$ & $\vec{B}$ are perpendicular

$$\vec{A}\cdot\vec{B} = AB\cos\frac{\pi}{2} = 0$$

$$\vec{A}\cdot\vec{B} = 3 + (-1) + (-2) = 0 \quad \checkmark$$
$$= AB\cos\theta$$

$$\Rightarrow \cos\theta = 0 \Rightarrow \theta = \frac{\pi}{2}$$

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$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{AB}$$

$$\vec{A}\cdot\vec{A} = \vec{A}^2 = A^2$$

$$A = \sqrt{\vec{A}^2}$$

Examples Cosine Thm

1)

$$c^2 = a^2 + b^2 - 2ab\cos\theta$$

$$\vec{C} = \vec{A} + \vec{B}$$

$$c^2 = \vec{C}^2 = (\vec{A} + \vec{B})^2 = \vec{A}^2 + \vec{B}^2 + 2\vec{A}\cdot\vec{B}$$

$$c^2 = a^2 + b^2 + 2ab\cos(\pi - \theta)$$

$$\boxed{c^2 = a^2 + b^2 - 2ab\cos\theta}$$

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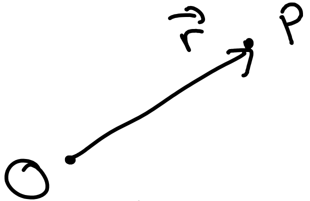
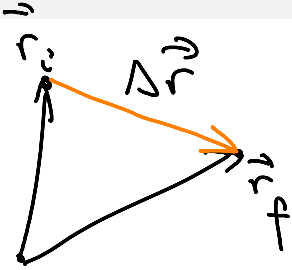
Motion in 2D & 3D

Position Vector : $\vec{r}$



Displacement Vector :

$$\vec{\delta} \equiv \Delta\vec{r} = \vec{r}_f - \vec{r}_i$$



$$\vec{r}_i = x_i \hat{x} + y_i \hat{y} + z_i \hat{z}$$

$$\vec{r}_f = x_f \hat{x} + y_f \hat{y} + z_f \hat{z}$$

$$\Delta \vec{r} = (x_f - x_i)\hat{x} + (y_f - y_i)\hat{y} + (z_f - z_i)\hat{z}$$

$$\vec{V}_{av} \equiv \frac{\Delta \vec{r}}{\Delta t} = \frac{x_f - x_i}{\Delta t}\hat{x} + \frac{y_f - y_i}{\Delta t}\hat{y} + \frac{z_f - z_i}{\Delta t}\hat{z}$$

$$= (V_{av})_x \hat{x} + (V_{av})_y \hat{y} + (V_{av})_z \hat{z}$$

$$(v_{av})_x = \frac{\Delta x}{\Delta t}\, ; \quad (v_{av})_y = \frac{\Delta y}{\Delta t}\, ;$$

$$(v_{av})_z = \frac{\Delta z}{\Delta t}$$

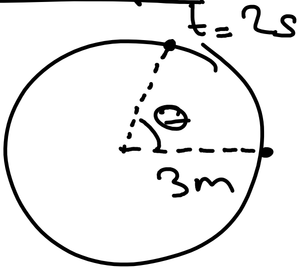
$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

$$\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t} = \left(\frac{\Delta v_x}{\Delta t}\right)\hat{x} + \left(\frac{\Delta v_y}{\Delta t}\right)\hat{y} + \left(\frac{\Delta v_z}{\Delta t}\right)\hat{z}$$

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

Example

$s = 2\,m/s$ (const)

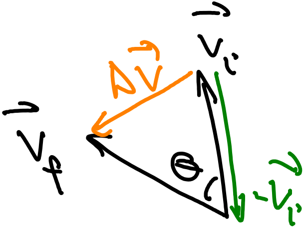


$\vec{a}_{av} = 0$

towards the centre $\frac{4}{3}\,m/s^2$

$1\,m/s^2$

$8 - 8\cos\left(\frac{8}{9}m\right)$



$|\vec{v}_i| = s$

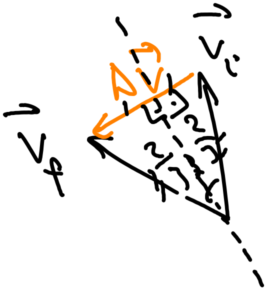
$|\vec{v}_f| = s$

$\Delta\vec{v} = \vec{v}_f - \vec{v}_i$

$90^\circ \sim \frac{\pi}{2} \sim 1.6$

$\frac{4}{3} \sim 1.3$

$$\theta = \frac{s \cdot \Delta t}{3m} = \frac{(2\,m/s)(2s)}{3m} = \frac{4m}{3m} = \frac{4}{3}$$



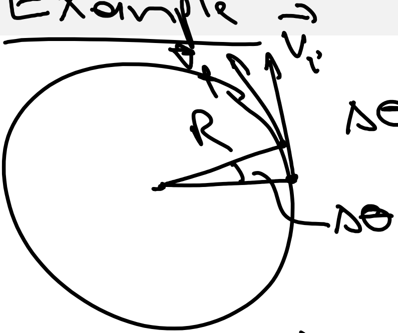
$$|\Delta\vec{v}| = 2\cdot\left(2\,m/s\ \sin\frac{2}{3}\right)$$

$$= 4\sin\frac{2}{3}\ m/s$$

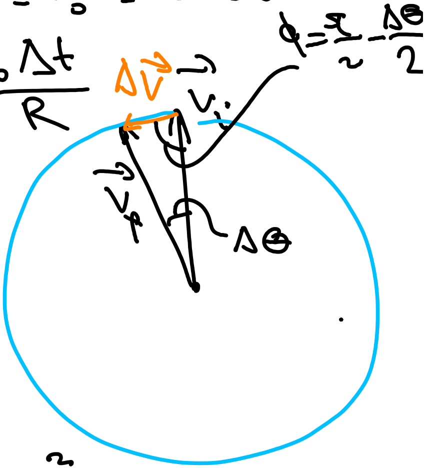
$$\boxed{|\vec{a}_{av}| = 2\sin\frac{2}{3}\ m/s^2}$$

Example

$|\vec{v}| = v_0 = \text{const}$



$\Delta\theta = \frac{v_0 \Delta t}{R}$



$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$$

$$|\Delta \vec{v}| = v_0 \Delta\theta$$

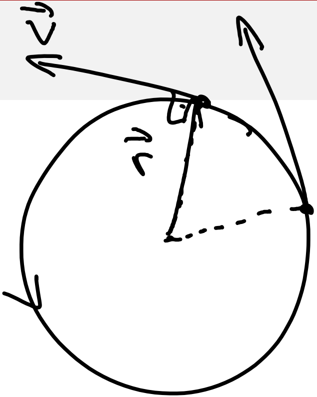
$$|\vec{a}| = v_0 \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{v_0^2}{R}$$

October 20, 2015

Uniform Circular Motion

circular: going around a circle
uniform: speed is constant

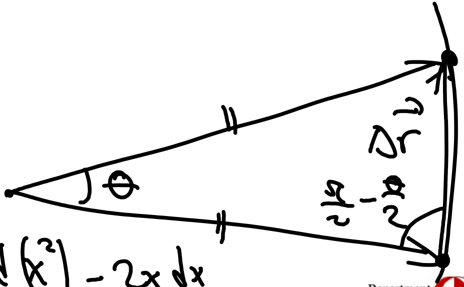
$$\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\text{change in velocity}}{\text{time it takes}}$$



"trajectory"

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t}$$

$$\frac{d}{dt}\left(\vec{r}^{\,2} = R^2\right) = 0$$



$$\frac{d}{dt}(x^2) = 2x\frac{dx}{dt}$$

$$\vec{r}^2 = x^2 + y^2 + z^2$$

$$\frac{d\vec{r}^2}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt} + 2z\frac{dz}{dt}$$

$$\boxed{\vec{r} = x(t)\hat{x} + y(t)\hat{y} + z(t)\hat{z}}$$

$$\frac{d\vec{r}^2}{dt} = 2(x v_x + y v_y + z v_z)$$

$$= 2\vec{r}\cdot\vec{v} = 0$$

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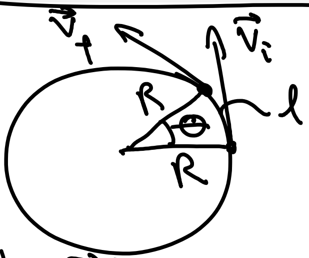
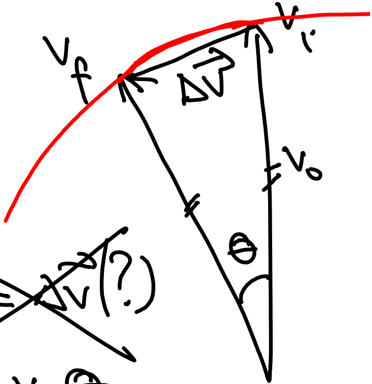
Acceleration

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$$



$$|\vec{v}_i| = |\vec{v}_f| = v_0$$



$$\Delta \vec{r} = \Delta \vec{v} \ (?)$$

$$R\theta = l \Leftarrow \theta = \frac{l}{R}$$

$$l_v = v_0 \theta$$

$$\Delta t = \frac{l}{v_0}$$

$$\lim_{\theta \to 0} \left(\frac{|\Delta \vec{v}| - l_v}{\theta} \right) = 0$$

space (mathematics) collection of vectors

$$|\Delta\vec{v}| \sim l_v$$

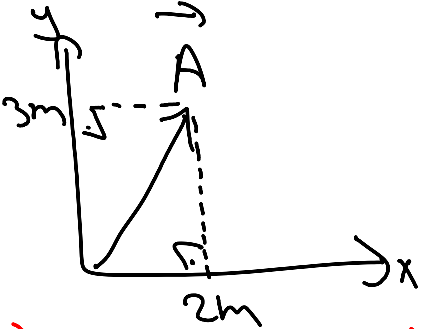
$$\frac{|\Delta\vec{v}|}{\Delta t} \sim \frac{l_v}{\Delta t} = \frac{l_v}{l/v_0} = \frac{v_0\,\theta}{R\theta/v_0} = \frac{v_0^2}{R}$$

$$|\vec{a}| = \lim_{\Delta t \to 0} \frac{|\Delta\vec{v}|}{\Delta t} = \frac{v_0^2}{R} = \text{const}$$

$\vec{a}$ points towards the center!

Quiz 1

- One full page!
- everybody hands in their own quiz!



Q: Write $\vec{A}$ in terms of its components.

$\vec{A} = 2m\,\hat{x} + 3m\,\hat{y}$

$\vec{A} = 2m\,\hat{i} + 3m\,\hat{j}$

$\vec{A} \neq \vec{2m}\,\hat{x} + \vec{3m}\,\hat{y}$

$A \neq 2m\,\hat{x} + 3m\,\hat{y}$

$\vec{A} \neq 2\hat{x} + 3\hat{y}$

$\vec{A} = (2\hat{x} + 3\hat{y})m$

$$\vec{v}^2 = v_0^2 = \text{const}$$

$$\frac{d}{dt}(\vec{v}^2) = 0$$

$$2\vec{v}\cdot\underbrace{\frac{d\vec{v}}{dt}}_{\vec{a}} = 0$$

$$\boxed{\vec{v}\cdot\vec{a} = 0} \Longrightarrow \vec{v}\perp\vec{a}$$

: they are perpendicular to each other

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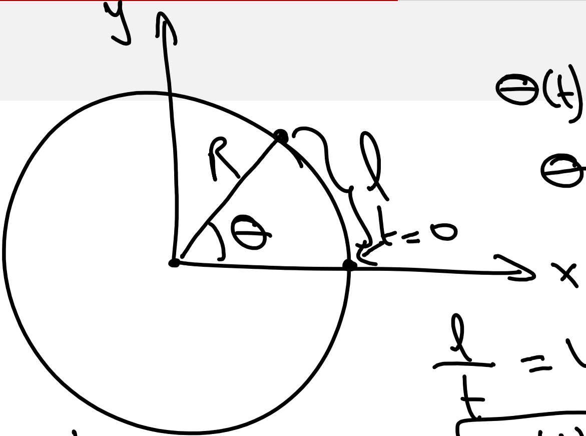


$\theta(t) = ?$

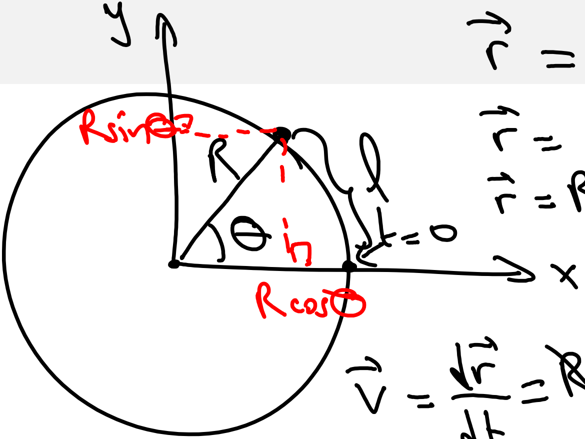
$\theta = \frac{l}{R}$

$\frac{l}{t} = v_0 \Rightarrow l = v_0 t$

$$\boxed{\theta(t) = \frac{v_0}{R} t}$$

How long does one full revolution take?

$\theta(T) = 2\pi = \frac{v_0}{R} T \Rightarrow T = \frac{2\pi R}{v_0}$ ← period of motion



$$\vec{r} = ?$$

$$\vec{r} = R\cos\theta\,\hat{x} + R\sin\theta\,\hat{y}$$

$$\vec{r} = R\left[\cos\left(\frac{v_0}{R}t\right)\hat{x} + \sin\left(\frac{v_0}{R}t\right)\hat{y}\right]$$

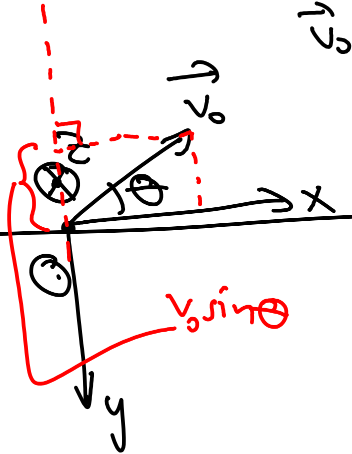
$$\vec{v} = \frac{d\vec{r}}{dt} = R\left[-\sin\left(\frac{v_0}{R}t\right)\frac{v_0}{R}\hat{x} + \cos\left(\frac{v_0}{R}t\right)\frac{v_0}{R}\hat{y}\right]$$

$$\vec{v} = v_0\left[-\sin\left(\frac{v_0}{R}t\right)\hat{x} + \cos\left(\frac{v_0}{R}t\right)\hat{y}\right]$$

Projectile Motion (in 2D & 3D)

$\vec{v}_0 = v_{0y}\hat{y} + \vec{v}_\perp$

$\downarrow \vec{a}$ $|\vec{a}| = g$



$\vec{a} = g\hat{y}$

$\vec{v}_0 = -v_0 \sin\theta\, \hat{y} + v_0 \cos\theta\, \hat{x}$

$\vec{r}_0 = 0$

$$\vec{a} = \vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}(t) - \vec{v}(t=0)}{t-0}$$

$$\vec{v}(t) = \vec{v}(t=0) + \vec{a}\, t = \vec{v}_0 + \vec{a}\, t$$

$$= (v_0 \cos\theta\, \hat{x} - v_0 \sin\theta\, \hat{y}) + (g\, \hat{y})\, t$$

$$\boxed{\vec{v}(t) = (v_0 \cos\theta)\hat{x} + (-v_0 \sin\theta + g t)\hat{y}}$$

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- horizontal component is constant
- vertical component changes downwards
- acceleration is never zero
- different components are indep. of each other

$$v_y(t) = (-v_0 \sin\theta + gt)$$

$$v_0 \sin\theta > 0$$

$$v_y(t_m) = 0 \Rightarrow t_m = \frac{v_0 \sin\theta}{g}$$

$v_y(t) < 0$ if $t < t_m$

$v_y(t) > 0$ if $t > t_m$

At $t = t_m$, the object reaches maximum height

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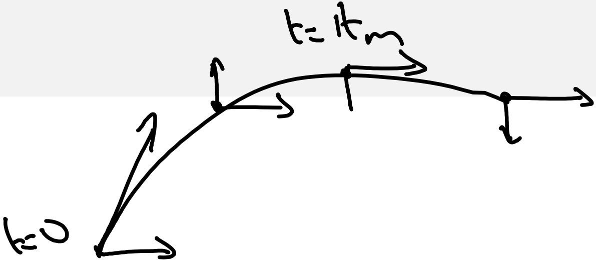

$\vec{v}(t) = \vec{v}_0 + \vec{a}t$ $\qquad \vec{a}$: constant vector

$$\vec{v}_{av}(t_1 \to t_2) = \frac{\vec{v}(t_2) + \vec{v}(t_1)}{2}$$

← valid only for constant acceleration!

$$\vec{v}_{av}(0 \to t) = \frac{1}{2}\left(\vec{v}_0 + \vec{a}t + \vec{v}_0\right)$$

$$\vec{v}_{av}(0 \to t) = \vec{v}_0 + \frac{1}{2}\vec{a}t$$

$$\vec{v}_{av}(0 \to t) = \frac{\vec{r}(t) - \vec{r}(t=0)}{t - 0} \quad ; 0$$

$$= \frac{\vec{r}(t)}{t}$$

$$\vec{r}(t) = t\left(\vec{v}_0 + \frac{1}{2}\vec{a}t\right)$$

$$\boxed{\vec{r}(t) = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2}$$

$$x(t) = v_0 \cos\theta\, t$$

$$y(t) = -v_0 \sin\theta\, t + \frac{1}{2}gt^2$$

$$z(t) = 0$$

$$x(t) = v_0 \cos\theta\, t \qquad t_m = \frac{v_0 \sin\theta}{g}$$

$$y(t) = -v_0 \sin\theta\, t + \frac{1}{2} g t^2$$

$$z(t) = 0$$

$$x(t_m) = v_0 \cos\theta \frac{v_0 \sin\theta}{g} = \frac{v_0^2}{g} \sin\theta \cos\theta$$

$$\left[\frac{v_0^2}{g}\right] = \frac{(m/s)^2}{m/s^2} = m$$

$$y(t_m) = -v_0 \sin\theta \frac{v_0 \sin\theta}{g} + \frac{1}{2} g \left(\frac{v_0 \sin\theta}{g}\right)^2$$

$$\boxed{y(t_m) = -\frac{1}{2g} v_0^2 \sin^2\theta}$$

$$h_m = \frac{1}{2g} v_0^2 \sin^2\theta$$

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What is the range of the projectile?

$v_0 \cos\theta \, t_f$

$t_f = 2t_m$?

t_f : time of flight

$y(t_f) = 0 = -v_0 \sin\theta \, t_f + \frac{1}{2} g t_f^2 = 0$

~~$t_f = 0$~~ $\quad t_f = \frac{2v_0 \sin\theta}{g} = 2t_m$

$R = x(t_f) = v_0 \cos\theta \frac{2 v_0 \sin\theta}{g} = \frac{v_0^2 \sin 2\theta}{g}$

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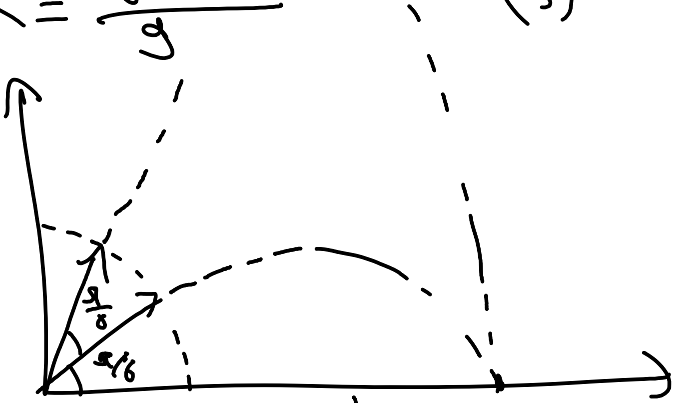
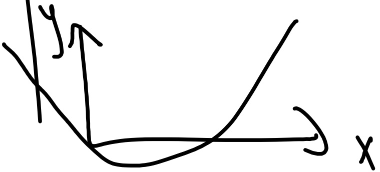
$$R = \frac{v_0^2 \sin 2\theta}{g} \qquad \sin\left(\frac{2\pi}{3}\right) = \sin\left(\frac{\pi}{3}\right)$$



same initial speed.

$$x(t) = v_0 \cos\theta \, t$$
$$y(t) = -v_0 \sin\theta \, t + \frac{1}{2} g t^2$$
$$z(t) = 0$$

$$y(x) = ?$$



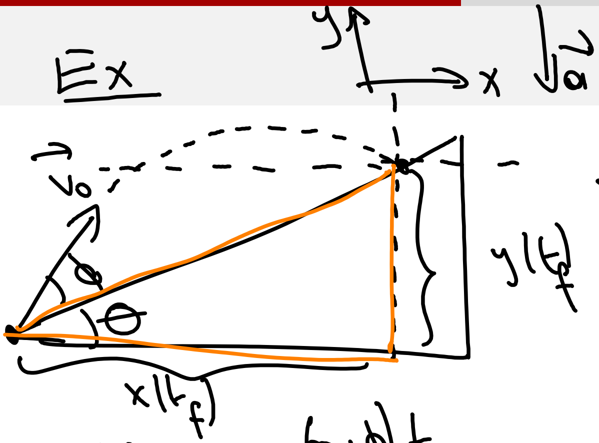
$$t = \frac{x}{v_0 \cos\theta}$$

$$y = -v_0 \sin\theta \frac{x}{v_0 \cos\theta} + \frac{1}{2} g \frac{x^2}{v_0^2 \cos^2\theta}$$

$$y = -Ax + Bx^2$$

Are curved balls possible?

Ex



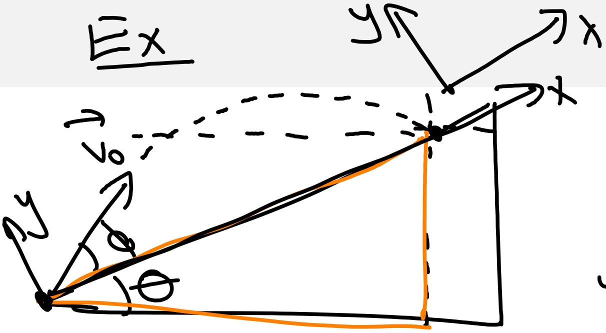
$$\frac{y(t_f)}{x(t_f)} = \tan\theta$$

$$x(t) = v_0 \cos(\theta+\phi)\, t$$

$$y(t) = v_0 \sin(\theta+\phi)\, t - \frac{1}{2} g t^2$$

$$z(t) = 0$$

Ex



$\vec{a} = a_x\hat{x} + a_y\hat{y}$

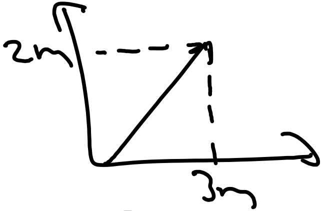
$x(t) = ?$

$y(t) = ?$

$y(t_f) = 0$

October 22, 2015

$$\vec{A} = 3m\,\hat{x} + 2m\,\hat{y}$$

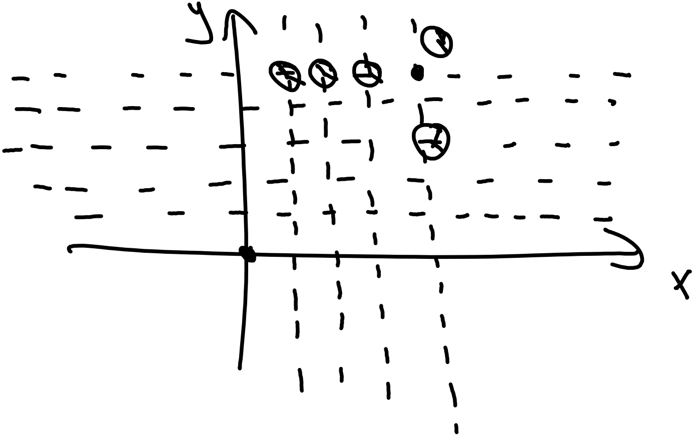


$$A = \sqrt{2^2 + 3^2} = \sqrt{13}$$

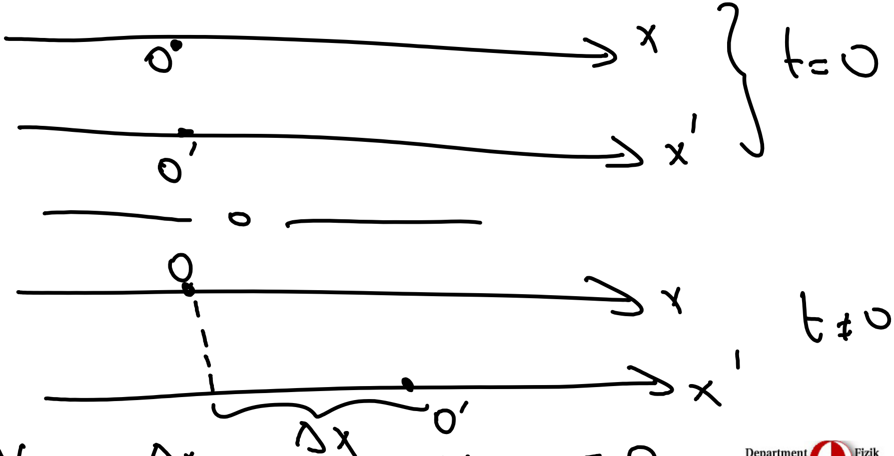
$$\vec{A} = \sqrt{6\,\hat{x}\,\hat{y}}$$

$\hat{x}\hat{y}$: Dyadics

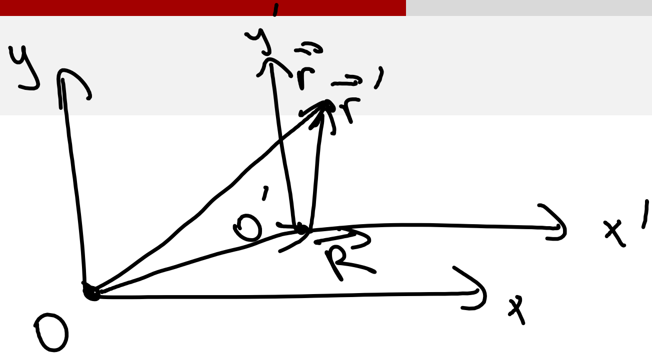
Reference Frames



Relative Motion



$$v_{O':O} = \frac{\Delta x}{t} \qquad v_{O':O'} = 0$$



$$\boxed{\vec{r}_0 = \vec{R}_0 + \vec{r}\,'_0} \qquad t = t_0$$

$$\vec{r}_1 = \vec{R}_1 + \vec{r}\,'_1 \qquad t = t_1$$

$$\frac{\Delta \vec{r}}{\Delta t} = \frac{\Delta \vec{R}}{\Delta t} + \frac{\Delta \vec{r}\,'}{\Delta t} \Rightarrow \boxed{\vec{v} = \vec{V} + \vec{v}\,'}$$

$$\vec{v} = \vec{V} + \vec{v}\,' \Longrightarrow \frac{\Delta \vec{v}}{\Delta t} = \frac{\Delta \vec{V}}{\Delta t} + \frac{\Delta \vec{v}\,'}{\Delta t}$$

$$\boxed{\vec{a} = \vec{A} + \vec{a}\,'}$$

if $\vec{A} = 0 \Longrightarrow \vec{a} = \vec{a}\,'$

$(\vec{V} = \text{const})$

Inertial Reference Frame

Newton's First Law:

In an inertial reference frame, an isolated object (an object that feels zero net force) keeps its state of motion.

$$\boxed{\vec{a} = 0}$$

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Newton's Second Law

The acceleration of an object is proportional to the force acting on it and inversely proportional to its mass.

$$\vec{a} \propto \frac{\vec{F}}{m} \Rightarrow \vec{F} = m\vec{a}$$

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Newton's Third Law

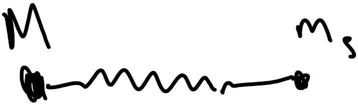
If A exerts a force, $\vec{F}_A$, on B, then B exerts a force, $\vec{F}_B$, on A and

$$\vec{F}_A = -\vec{F}_B$$

action-reaction pairs

$M, \; m_s$

m_s: standard mass.



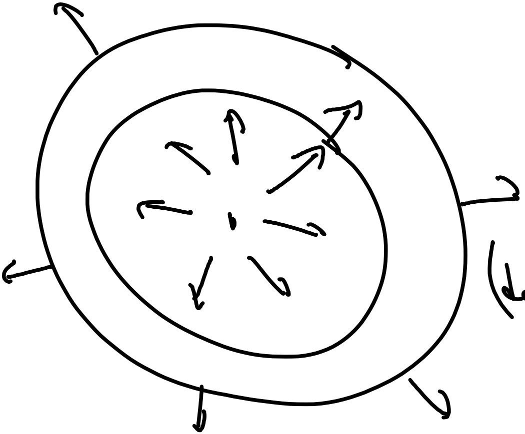
$$\vec{F}_M = -\vec{F}_{m_s}$$

$$F_M = F_{m_s}$$

$$M a_M = m_s a_s \Rightarrow \boxed{M = m_s \frac{a_s}{a_M}}$$

Modern understanding of force

$F_e \propto \frac{1}{r^2}$; $F_G \propto \frac{1}{r^2}$



$N = \text{const}$

$= (\text{surface density})\, 4\pi r^2$

$(\text{density}) \propto \frac{1}{r^2}$

- Gravity
- resistance
- push
- pull

resistance, push, pull → Electromagnetic force

HAND IN YOUR HOMEWORK!

Now!

1) inertial reference frames

2) $\frac{\vec{F}}{m} = \vec{a}$

3) Action-Reaction forces.

$\frac{\vec{F}}{m} = \vec{a} \not\Rightarrow m = \frac{\vec{F}}{\vec{a}}$ no vector division!

$\frac{F}{m} = a \Rightarrow m = \frac{F}{a}$

$\frac{F}{m} = a \Rightarrow m = \frac{F}{a}$ if $a \neq 0$

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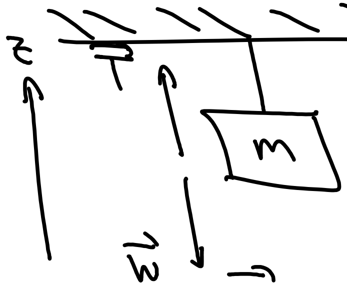
$$[\vec F] = [m][\vec a] = kg\, m/s^2 \equiv N$$

Example



$\vec a = 0$

N. 2nd law $\vec F = m\vec a = 0$

$$\vec W = mg(-\hat z)$$

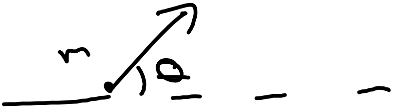
$$\vec T = T\hat z$$

$$\vec F_{net} = \vec W + \vec T = -mg\hat z + T\hat z$$

$$\vec F_{net} = (T - mg)\hat z = 0$$

$$\boxed{T = mg}$$

Weight



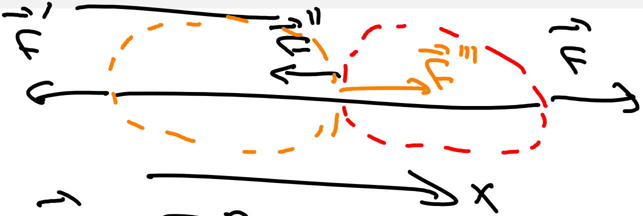
$\vec{a}\downarrow$

$|\vec{a}| = g$

$|\vec{F}| = |m\vec{a}| = w$

$w = mg$

Tension



string is massless.

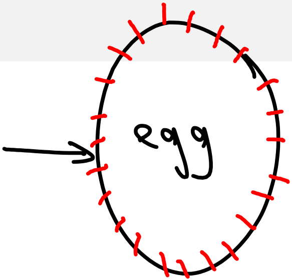
$\vec{F} = m\vec{a} = 0$

$\vec{F} = F\hat{x}$

$\vec{F}' = F'(-\hat{x})$

$\vec{F} + \vec{F}' = (F - F')\hat{x} = 0 \Rightarrow |\vec{F}| = |\vec{F}'|$

$\vec{F}'' = -F''\hat{x}$ $\qquad F'' = F = F' \equiv T$: tension

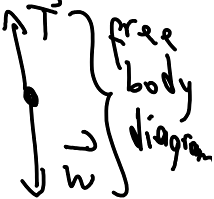

egg

Ex

massive string, lineer mass density is ρ

$$\rho = \frac{\text{mass}}{\text{length}}$$

what is the tension at any point on the string?



~~$T = \text{weight} = mg = \rho L g$~~

$\vec{r} = h\hat{x}$

$\vec{T} = T(-\hat{x})$

$\vec{w} = m_h g \hat{x}$

$$\vec{F}_{tot.} = \vec{w} + \vec{T} = (m_h g - T)\hat{x} = 0$$

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$$T = m_h g$$

$$m_h = ?$$

$$m_h = \rho (L-h)$$

$$T(h) = \rho g (L-h)$$

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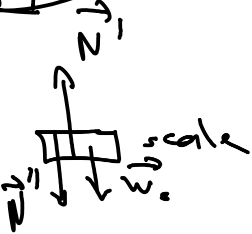
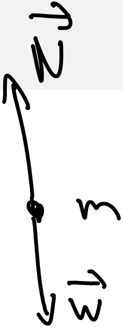
Apparent Weight

$$\vec{a} = a\hat{z}$$





$$\vec{N} = N\hat{z}$$

$$\vec{W} = mg(-\hat{z})$$

$$\vec{F}_T = \vec{N} + \vec{W} = (N - mg)\hat{z} = ma\hat{z}$$

$$N - mg = ma$$

$$\boxed{N = m(a+g)}$$

$$N'' = N$$

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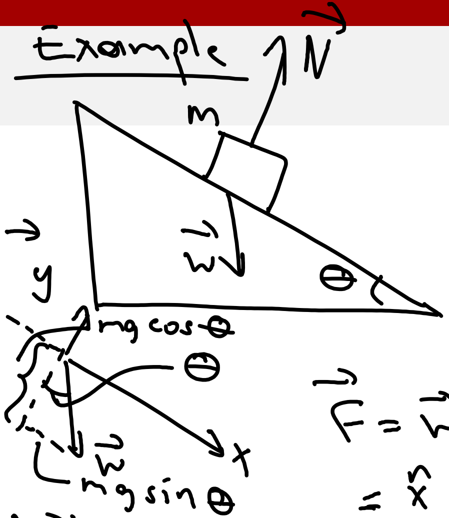
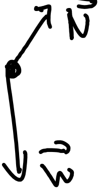
Example



$|\vec{w}| = mg$

$\vec{a} = a\hat{x}$

$\vec{a} = ?$ (no friction)



$\vec{N} = N\hat{y}$

$\vec{w} = mg\cos\theta(-\hat{y}) + mg\sin\theta(\hat{x})$

$\vec{F} = \vec{w} + \vec{N}$

$= \hat{x}(mg\sin\theta) + \hat{y}(-mg\cos\theta + N)$

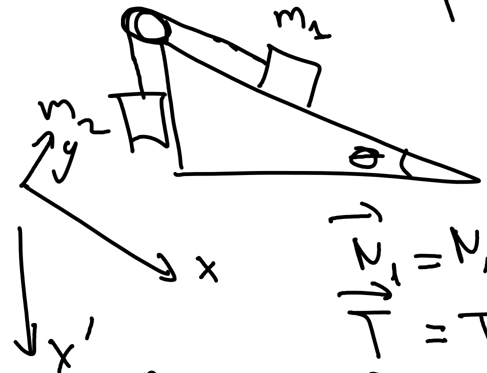
$= ma\hat{x}$

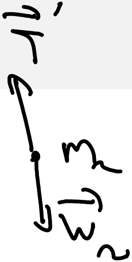
$-mg\cos\theta + N = 0 \Rightarrow N = mg\cos\theta$

$mg\sin\theta = ma \Rightarrow a = g\sin\theta$

$\vec{a} = g\sin\theta\,\hat{x}$

Example (no friction)





$$\vec{N}_1 = N_1 \hat{y}$$

$$\vec{T} = T(-\hat{x})$$

$$\vec{w}_1 = m_1 g \cos\theta(-\hat{y}) + m_1 g \sin\theta(\hat{x})$$

$$\vec{a}_1 = a_1 \hat{x}$$

$$\vec{T}' = T(-\hat{x}')$$

$$\vec{w}_2 = m_2 g \hat{x}'$$

$$\vec{a}_2 = a_2 \hat{x}'$$

$$\left.\begin{array}{l} \Delta\vec{r}_1 = d\hat{x} \\ \Delta\vec{r}_2 = -d\hat{x}' \\ |\vec{v}_1| = |\vec{v}_2| \end{array}\right\} \quad \boxed{a_1 = -a_2}$$

unknowns: a_1, a_2, N_1, T

$$\vec{F}_{t1} = (N_1 - m_1 g\cos\theta)\hat{y} + (m_1 g\sin\theta - T)\hat{x}$$
$$= m_1 a_1 \hat{x}$$

$$m_1 a_1 = m_1 g\sin\theta - T \qquad (1)$$
$$0 = N_1 - m_1 g\cos\theta \qquad (2)$$

$$\boxed{N_1 = m_1 g\cos\theta}$$

$$\vec{F}_{t2} = (m_2 g - T)\hat{x}' = m_2 a_2 \hat{x}'$$
$$m_2 g - T = m_2 a_2 \qquad (3)$$

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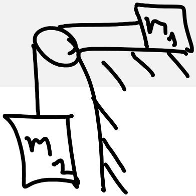
$\theta = 0$

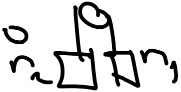


$$m_1 a_1 = m_1 g \sin\theta - T$$

$$m_2 g - T = m_2 a_2$$

$$a_1 = -a_2$$

$\theta = 90$

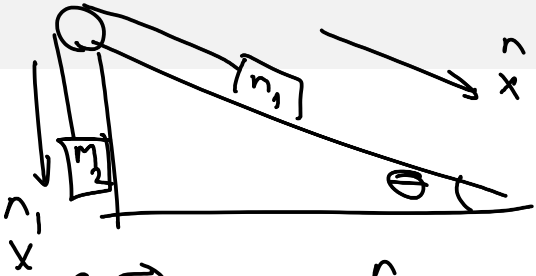


$$T = m_1 g \sin\theta - m_1 a_1 = m_2 g + m_2 a_1$$

$$\Longrightarrow m_1 g \sin\theta - m_2 g = (m_1 + m_2) a_1$$

$$\boxed{a_1 = \frac{m_1 \sin\theta - m_2}{m_1 + m_2} g}$$

$$\vec{a}_1 = \frac{m_1 \sin\theta - m_2}{m_1 + m_2} g \hat{x}$$



$$\Delta \vec{r}_1 = d\,\hat{x}$$

$$\Delta \vec{r}_2 = -d\,\hat{x}'$$

$$\vec{v}_1 = v_{1x}\hat{x}$$

$$\vec{v}_2 = -v_{1x}\hat{x}'$$

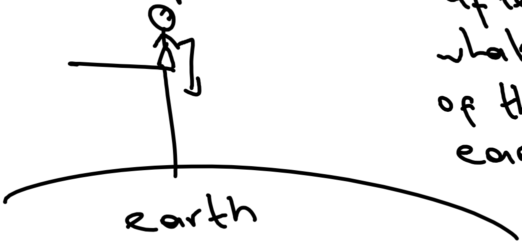
$$\frac{1}{\Delta t}\begin{cases} \Delta \vec{v}_1 = \Delta v_{1x}\hat{x} \\ \Delta \vec{v}_2 = -\Delta v_{1x}\hat{x}' \end{cases}$$

$$a_1 = \frac{\Delta v_1}{\Delta t}$$

$$\vec{a}_1 = a_{1x}\hat{x}$$

$$\vec{a}_2 = -a_{1x}\hat{x}$$

Example



after you jump what is the magnitude of the acceleration of earth (order of magnitude)

$$g = 9.8\,m/s^2$$

$$\frac{w}{M_E} \approx \frac{800\,N}{10^{24}\,kg} \sim 10^{-21}\,m/s^2 \simeq 10^{-22}\,g$$

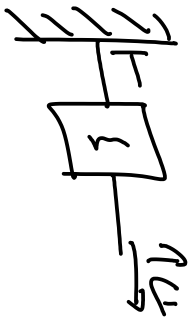
$$100\,m \sim \frac{1}{2}\cdot(10\,m/s^2)\,t^2$$

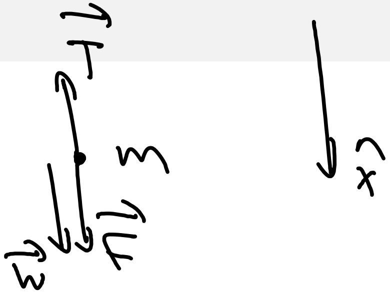
$$t^2 \sim 20\,s^2 \Rightarrow t \sim 4\,s$$

$$d_\varepsilon \sim \frac{1}{2}(10^{-21}\,m/s^2)\,10^1\,s^2 \sim 10^{-20}\,m$$

$$r_{atom} \sim 10^{-10}\,m$$

Example





$$\vec{W} = mg\,\hat{x}$$
$$\vec{F} = F\,\hat{x}$$
$$\vec{T} = -T\,\hat{x}$$
$$\vec{F}_t = (mg + F - T)\,\hat{x} = 0$$
$$\boxed{T = mg + F}$$

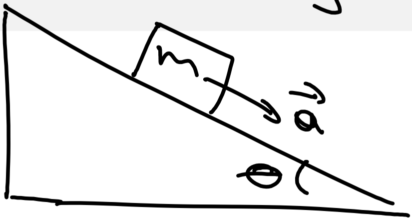
November 3, 2015

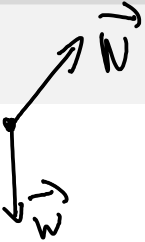
1st Midterm: Saturday 13^{30}

Chapter 6: Force & Motion

NOT Gravity

$|\vec{a}| = g \sin\theta$





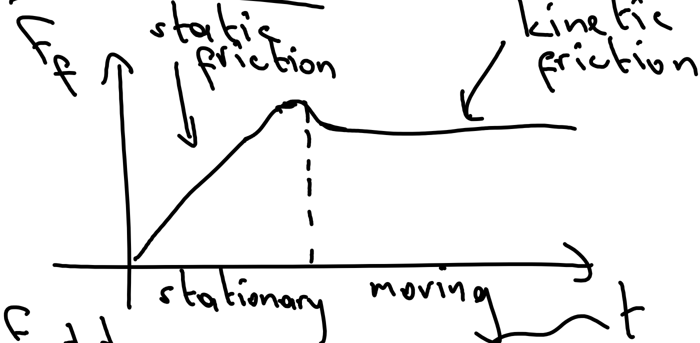
free body diagram

$|\vec{w}| = mg$

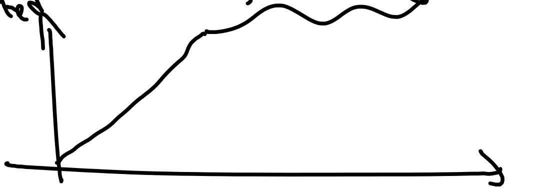
$g = 9.8\, m/s^2$

$\vec{w}$ does <u>NOT</u> act on the surface

Friction



F_f
static friction
kinetic friction
stationary
moving
t
$F_{exerted}$



Direction of the friction force is opposite to the motion of the surfaces!

$\vec{F}_f$: force <u>acting</u> on our feet <u>acted by</u> our legs

$|\vec{F}_s|$: any value sufficient to prevent the relative motion of surfaces.

$$|\vec{F}_s| \leqslant F_{s,max} = \mu_s N$$

$$|\vec{F}_k| = \mu_k N$$

μ_k : coefficient of kinetic friction

μ_s : " " static "

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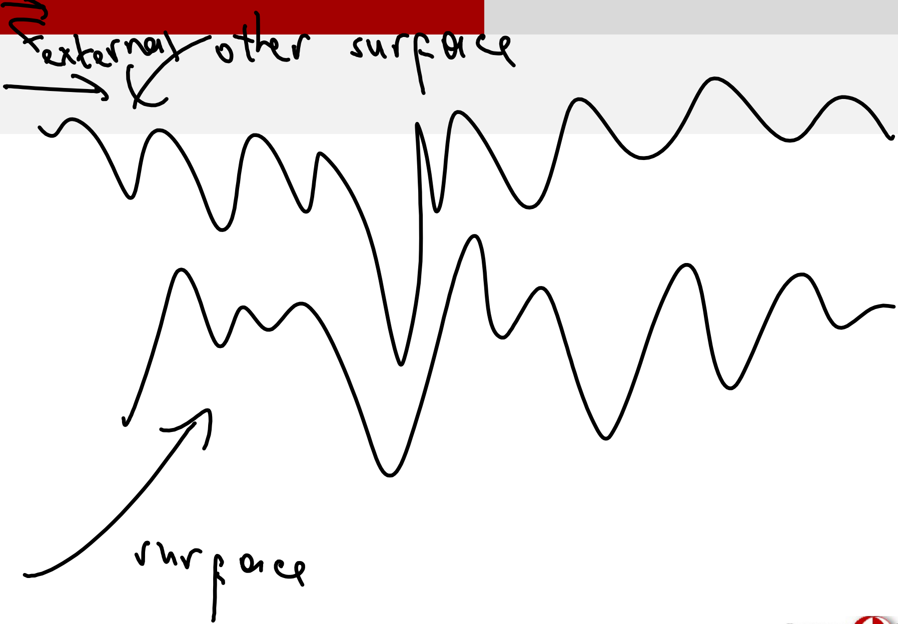
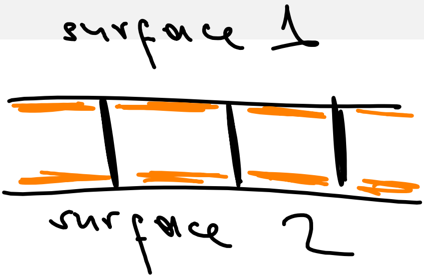
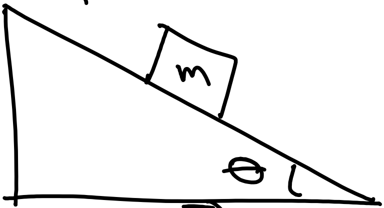
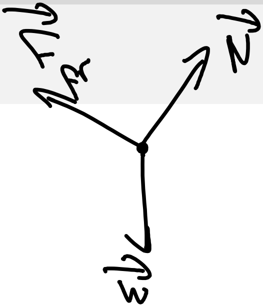

external
other surface
surface


surface 1
surface 2

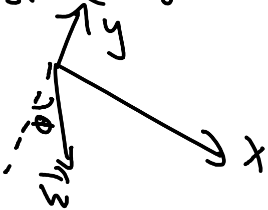
Example

$\vec{v}_0 = 0$





Since $\vec{v}_0 = 0$; $\vec{F}_{fr}$ is static



$$\vec{N} = N\hat{y}$$

$$\vec{F}_{fr} = (-\hat{x})\,F_{fr}$$

$$\vec{w} = mg\cos\theta\,(-\hat{y}) + mg\sin\theta\,(\hat{x})$$

$$\vec{N} = N\hat{y}$$
$$\vec{F}_{fr} = (-\hat{x}) F_{fr}$$
$$\vec{w} = mg\cos\theta(-\hat{y}) + mg\sin\theta(\hat{x})$$

$$\vec{F}_{tot} = \hat{x}(mg\sin\theta - f_{fr}) + \hat{y}(N - mg\cos\theta)$$

$$F_{tot,y} = 0 \Rightarrow N - mg\cos\theta = 0$$
$$\boxed{N = mg\cos\theta}$$

If $\vec{a} = 0 \Rightarrow \vec{F} = 0 \Rightarrow mg\sin\theta - F_{fr} = 0$
$$\boxed{F_{fr} = mg\sin\theta}$$

$$F_{fr} \leq F_{s,max} = \mu_s N = \mu_s mg\cos\theta$$

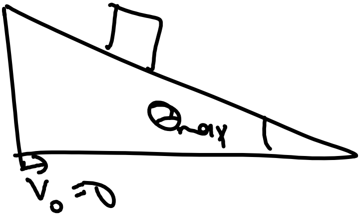
$$\cancel{mg}\sin\theta \leq \mu_s \cancel{mg}\cos\theta$$

$$\boxed{\tan\theta \leq \mu_s}$$

⟸ criteria for not starting to slide.

Assume $\theta = \theta_{max}$, $\tan\theta_{max} = \mu_s$

How long does it take for the mass to slide a distance L?



$$\vec{F}_{tot} = \hat{x}\left(mg\sin\theta - f_{fr}\right) + \hat{y}\left(N - mg\cos\theta\right)$$

$$F_{fr} = \mu_k N = \mu_k mg\cos\theta$$

$$\vec{F}_{tot} = \hat{x}\left(mg\sin\theta - \mu_k mg\cos\theta\right)$$

$$= mg\cos\theta\left(\tan\theta - \mu_k\right)\hat{x}$$

$$\vec{F}_{tot} = mg\cos\theta\left(\mu_s - \mu_k\right)\hat{x} = m\vec{a}$$

$$\vec{a} = g\cos\theta_{max}\left(\mu_s - \mu_k\right)\hat{x}$$

$$\tan\theta = \mu_s$$

$$\Delta x = v_{0x} t + \frac{1}{2} a_x t^2$$

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$$\Delta x = L$$

$$L = \frac{1}{2} g \cos\theta (\mu_s - \mu_k) t^2$$

$$t = \sqrt{\frac{2L}{g \cos\theta (\mu_s - \mu_k)}}$$

$$\mu_s = \tan\theta$$

for $\theta_1 < \theta < \theta_2 (= \theta_{max})$

s.t. if $\vec{v}_0 = 0$; $\vec{a} = 0$,

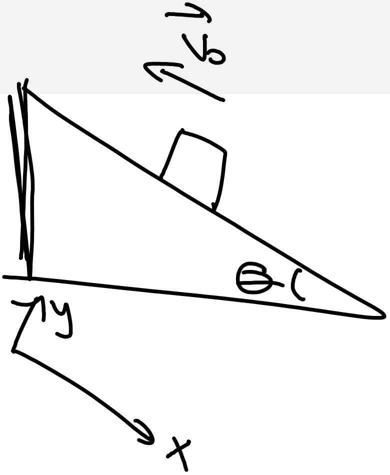
if $\vec{v}_0 \neq 0$, if the is moving down the incline, it will keep accelerating

$\theta_1 = ?$

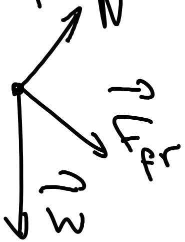
$$\vec{F}_{tot} = \hat{x}(mg\sin\theta - \mu_k mg\cos\theta)$$

$$\cancel{mg}\sin\theta_1 - \mu_k \cancel{mg}\cos\theta_1 = 0$$

$$\boxed{\tan\theta_1 = \mu_k}$$



$\tan\theta = \mu_s$



– weight
– Normal force
– friction force
– drag force

Drag Force "friction" force acting on objects moving in gases and liquids.

velocity dependent!

v is large $F_D \propto v^2$

v is smaller $F_D \propto v$

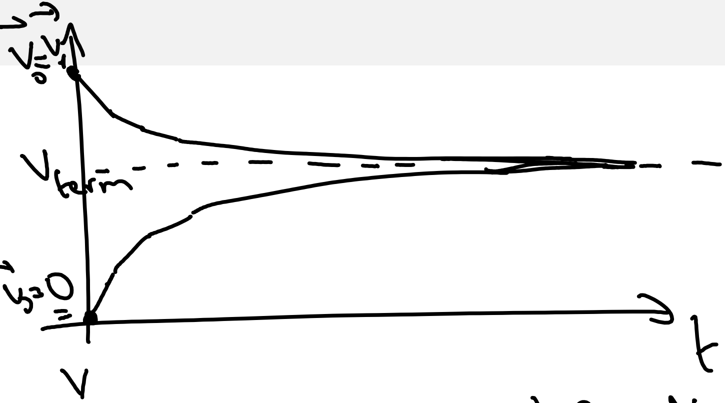
# of collisions: $\rho A v \Delta t$

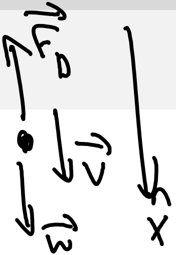
$\underset{\text{per collision}}{F} \propto v$

$F \propto A \rho v^2$

Δt

$$F_D = \frac{1}{2} C A \rho v^2$$





$$\vec F_x = \left(mg - \frac{1}{2} C \rho A v^2\right)\hat x$$

$$V_{term} \equiv \sqrt{\frac{2mg}{C\rho A}}$$

terminal velocity

if $v = 0 \Rightarrow \vec F_x \propto \hat x$

if $v > \sqrt{\frac{2mg}{C\rho A}} \Rightarrow \vec F_x \propto (-\hat x)$

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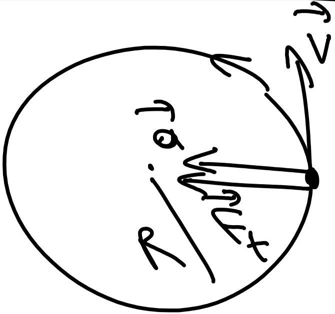
$$F_D = (\ )v + (\ )v^2 + (\ )v^3 + \cdots$$

Circular Motion



$$a = \frac{v^2}{R}$$

$$\vec{F}_t \neq 0$$

$$F_t = \frac{mv^2}{R}$$

November 5, 2015

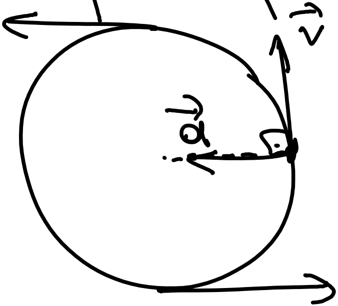
Hand in your HW! (now!)

Midterm Places: U1, U2, U3

Applications of Newton's Dynamics

ii. Circular Motion

- circular: trajectory is a circle.
- uniform: speed is constant

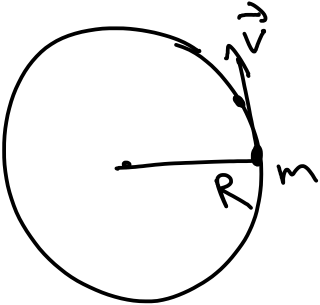


$$a = \frac{v^2}{R}$$

$$\vec{F} = m\vec{a}$$

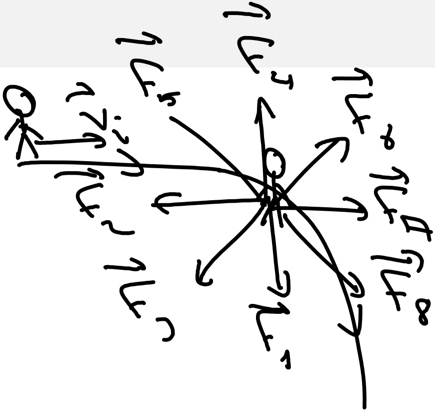
$$F = \frac{mv^2}{R}$$

Example

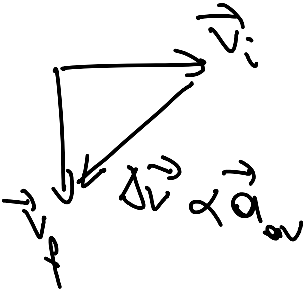


$$\vec{T} = m\vec{a}$$

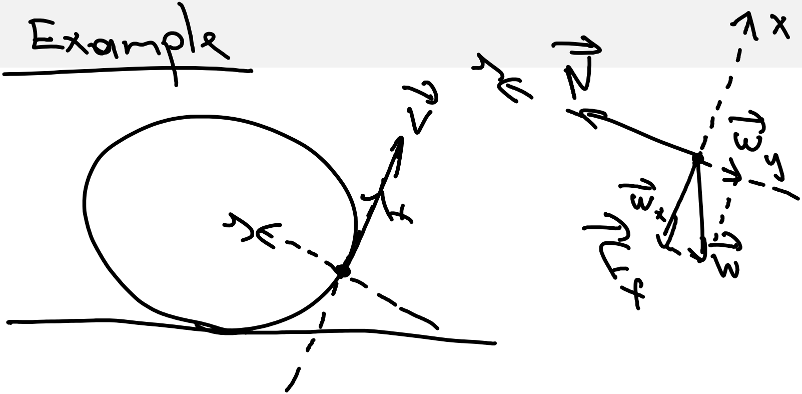
$$T = \frac{mv^2}{R}$$



a) No force (just inertia)



Example

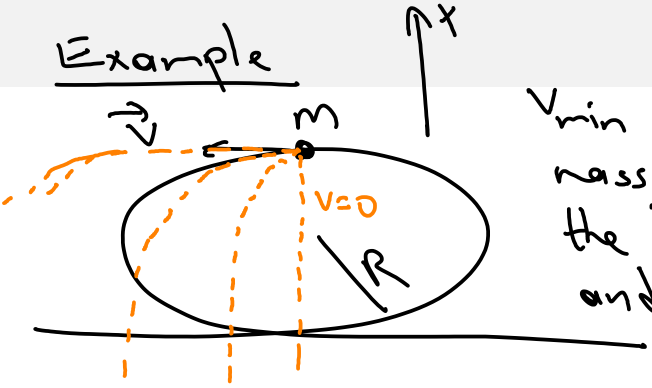


$$\vec{w} = \vec{w}_x + \vec{w}_y$$

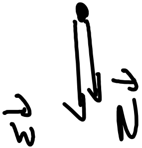
$$\vec{F}_T = F_{Tx}\hat{x} + F_{Ty}\hat{y} \; ; \; F_{Tx} < 0$$

$$a_{tan} = \frac{F_{tan}}{m}, \quad a_c = \frac{F_{cen}}{m} = \frac{v^2}{R}$$

Example



V_{min} s.t. the mass follows the trajectory and doesn't fall down?



$$N \geq 0 \iff a_c \geq \frac{v^2}{R}$$ ← you said

$$\vec{F}_r = (mg+N)(-\hat{x}) = \frac{mv^2}{R}(-\hat{x})$$

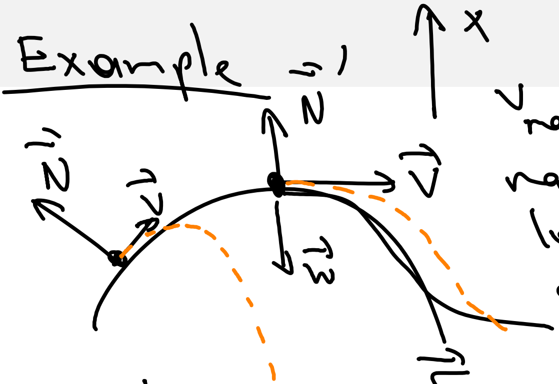
$$N + mg = \frac{mv^2}{R} \Rightarrow \boxed{N = \frac{mv^2}{R} - mg}$$

$$N \geq 0 \Rightarrow \frac{mv^2}{R} - mg \geq 0$$

$$\boxed{v^2 \geq gR}$$

$$\boxed{v \geq \sqrt{gR}}$$

Example - 1



v_{max} at the mass doesn't loose contact at the top?

$$\vec{N}' = N\hat{x}$$

$$\vec{w} = mg(-\hat{x})$$

$$\vec{F}_r = (N - mg)(\hat{x}) = m\frac{v^2}{R}(-\hat{x})$$

$$N - mg = -m\frac{v^2}{R}$$

$$\boxed{N = mg - m\frac{v^2}{R}} \geq 0$$

Example Conical Pendulum

$$\vec{a} = \frac{v^2}{R}(-\hat{x})$$

$$\vec{T} = T\sin\theta(-\hat{x}) + T\cos\theta(\hat{y})$$

$$\vec{w} = mg(-\hat{y})$$

$$\vec{F}_T = T\sin\theta(-\hat{x}) + (T\cos\theta - mg)\hat{y}$$

$$\vec{F}_T = T\sin\theta(-\hat{x}) + (T\cos\theta - mg)\hat{y}$$

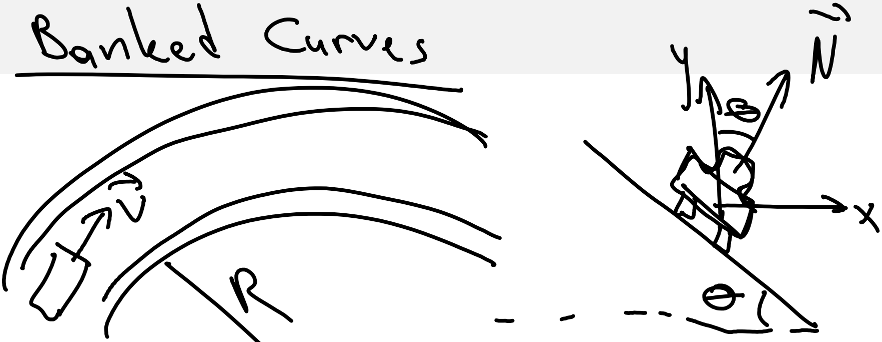
$$= \frac{mv^2}{R}(-\hat{x})$$

$$T\cos\theta - mg = 0 \Rightarrow \boxed{T = \frac{mg}{\cos\theta}}$$

$$-T\sin\theta = -\frac{mv^2}{R}$$

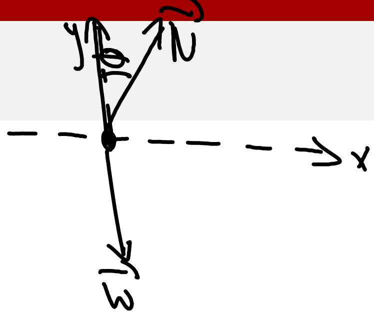
$$mg\tan\theta = \frac{mv^2}{R} \Rightarrow \boxed{v^2 = gR\tan\theta}$$

Banked Curves



θ, v, R ?

$a_c = \frac{v^2}{R} \Rightarrow F_c = \frac{mv^2}{R}$



$$\vec{a}_c = \frac{v^2}{R}\hat{x}$$

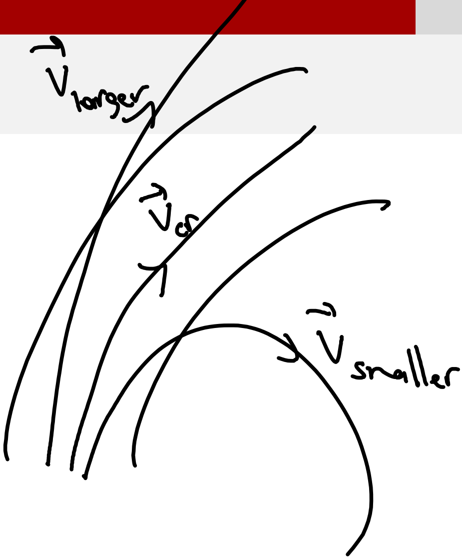
$$\vec{N} = N\sin\theta\,\hat{x} + N\cos\theta\,\hat{y}$$

$$\vec{W} = mg(-\hat{y})$$

$$\vec{F}_n = N\sin\theta\,\hat{x} + (N\cos\theta - mg)\hat{y} = \frac{mv^2}{R}\hat{x}$$

$$N\cos\theta - mg = 0 \qquad N = \frac{mg}{\cos\theta}$$

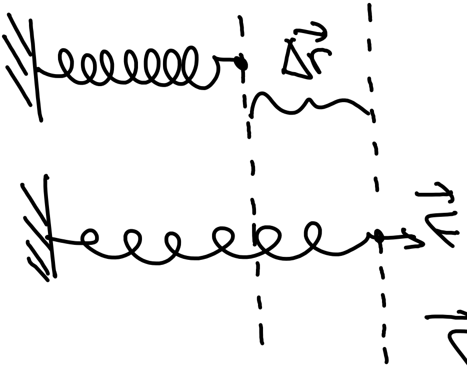
$$N\sin\theta = \frac{mv^2}{R}$$



$\vec{V}_{larger}$
$\vec{V}_{cr}$
$\vec{V}_{smaller}$

November 10, 2015

Spring Force



$$\vec{F} = +k\Delta\vec{r}$$

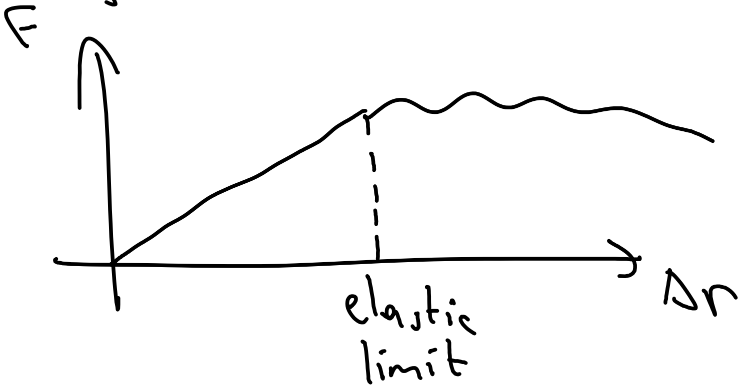
k: spring constant

$$\vec{F}_s = -k\Delta\vec{r}$$

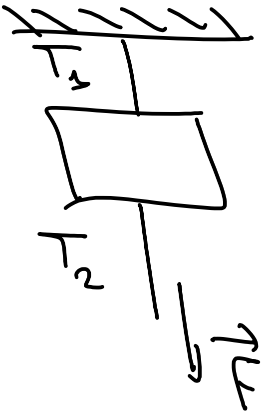
$\vec{F}_s$: force that spring exerts.

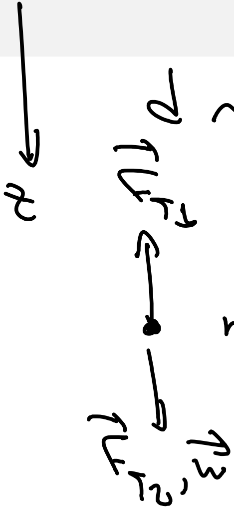
spring force ~ restoring force

$\vec{F}_s = -k\Delta\vec{r}$: Hooke's Law



Example





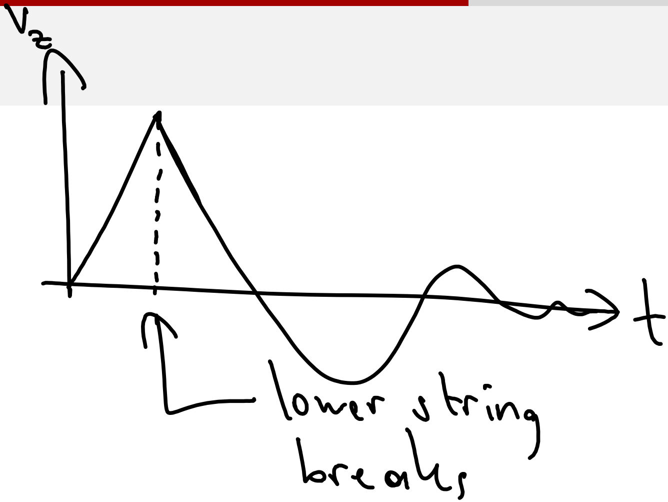
$$d \sim \frac{1}{2} a t^2$$

$$\vec{F}_{T_1} + \vec{F}_{T_2} + \vec{w} = 0$$

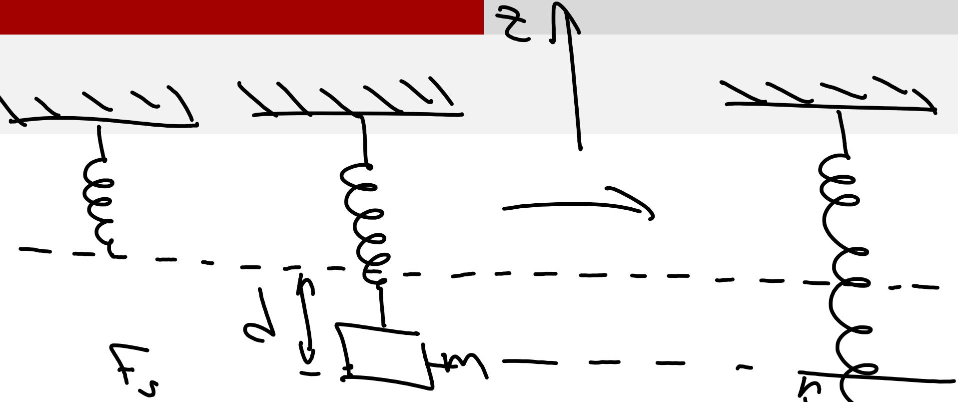
m (if mass at rest)

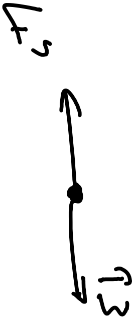
$$\boxed{T_1 = T_2 + mg}$$

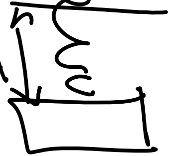
$$T_2 = F$$



V_z
t
lower string breaks





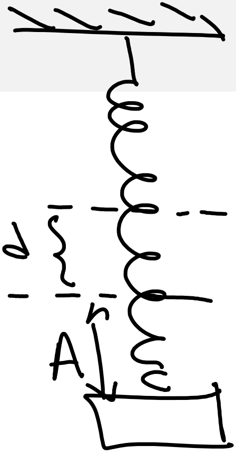


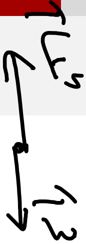
$$\vec{F}_s = k d \hat{z} = -k(-d\hat{z})$$

$$\vec{W} = -mg\hat{z}$$

$$\vec{F}_s = (kd - mg)\hat{z} = 0$$

$$\boxed{d = mg/k}$$



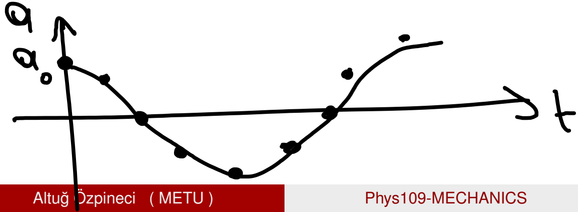


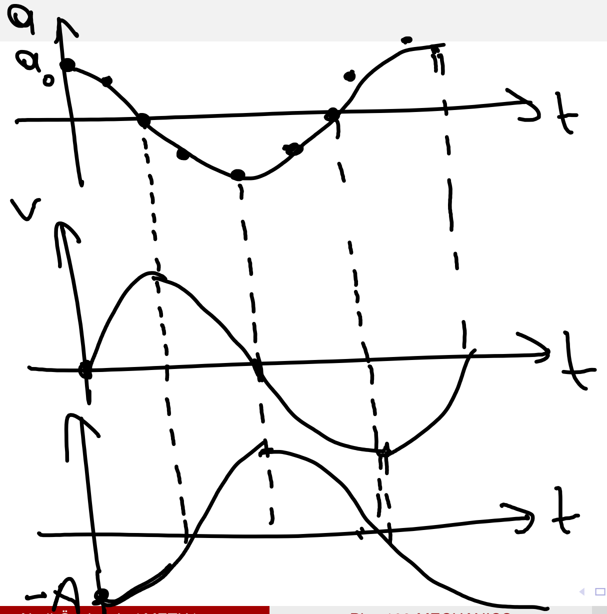
$$\vec{F}_s = k(d+A)\hat{z}$$

$$\vec{w} = -mg\hat{z}$$

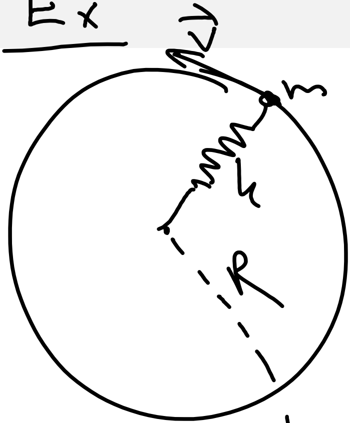
$$\vec{F}_T = (\cancel{kd - mg} + Ak)\hat{z}$$

$$\boxed{\vec{F}_T = Ak\hat{z}}$$



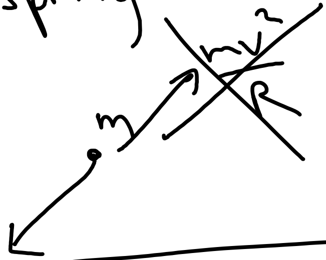

a
a_0
t
v
t
t
-A

Ex



m carries out uniform circular motion.

L: equilibrium length of the spring



$$\vec{F}_s$$

$$F_s = k(R-L) = \frac{mv^2}{R}$$

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Example One dimensional motion with constant acceleration

$$\Delta x = \frac{(v_i + v_f)}{2} \Delta t$$

$$\Delta v \equiv v_f - v_i = a \Delta t$$

$$\Delta x = \frac{1}{2} (v_i + v_f) \frac{(v_f - v_i)}{a}$$

$$\boxed{a \Delta x = \frac{1}{2} (v_f^2 - v_i^2)}$$

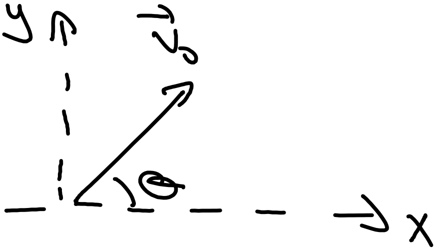
$$a \Delta x = \frac{1}{2}(v_f^2 - v_i^2)$$

$$a = \frac{F}{m}$$

$$F \Delta x = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

$$F \Delta x = \Delta \left(\frac{1}{2} m v^2 \right)$$

Example



$$\Delta v_x = 0$$

$$F_y \Delta y = \Delta\left(\frac{1}{2} m v_y^2\right)$$

$$F_x \Delta x = \Delta\left(\frac{1}{2} m v_x^2\right)$$

$$F_x \Delta x + F_y \Delta y = \Delta\left(\frac{1}{2} m \left(v_x^2 + v_y^2\right)\right)$$

$$= \Delta\left(\frac{1}{2} m v^2\right)$$

$$\vec{F} = F_x \hat{x} + F_y \hat{y}$$

$$\Delta \vec{r} = \Delta x \hat{x} + \Delta y \hat{y}$$

$$\vec{F} \cdot \Delta\vec{r} = \Delta\left(\frac{1}{2}mv^2\right)$$

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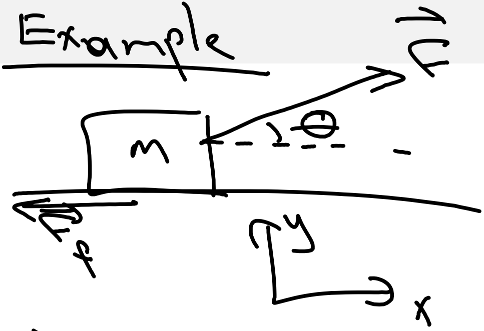
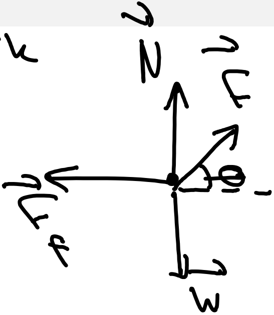
Example



μ_k



$$\vec F = F\cos\theta\,\hat x + F\sin\theta\,\hat y$$

$$\vec F_{tot} = \hat x\left(F\cos\theta - F_f\right) + \hat y\left(F\sin\theta + N - mg\right)$$

$$\vec{F}_{tot} = \hat{x}\left(F\cos\theta - F_f\right) + \hat{y}\left(F\sin\theta + N - mg\right)$$

$$a_y = 0 \Rightarrow F_y = 0 \Rightarrow F\sin\theta + N - mg = 0$$

$$\boxed{N = mg - F\sin\theta}$$

$$\boxed{\vec{F}_{tot} = \hat{x}\left(F\cos\theta - F_f\right) = ma\,\hat{x}}$$

$$a = \frac{1}{m}\left(F\cos\theta - F_f\right)$$

$$\vec{F}_{tot} = \vec{F} + \vec{F}_{fr} + \vec{N} + \vec{w} \; ; \; \vec{w} \cdot \Delta\vec{r} = 0 \; ; \; \vec{N} \cdot \Delta\vec{r} = 0$$

$$\frac{1}{m}\left(F\cos\theta - F_{fr}\right)\Delta x = \Delta\left(\frac{1}{2}v_x^2\right)$$

$$= \Delta\left(\frac{1}{2}v_x^2 + \frac{1}{2}v_y^2\right) = \Delta\left(\frac{1}{2}\;v^2\right)$$

$$\left(F\cos\theta - F_{fr}\right)\Delta x + \left(F\sin\theta + N - mg\right)\Delta y$$

$$\vec{F}_{tot} \cdot \Delta\vec{r} = \Delta\left(\frac{1}{2}mv^2\right)$$

$$\left(\vec{F} + \vec{F}_{fr}\right) \cdot \Delta\vec{r} = \Delta\left(\frac{1}{2}mv^2\right)$$

$\frac{1}{2}mv^2$: Kinetic energy

$\vec{F} \cdot \Delta\vec{r} = W$ work done by a constant force $\vec{F}$

$\Delta(KE) = W_{tot}$

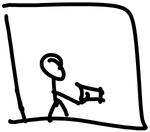
$$W = \vec{F} \cdot \Delta \vec{r} = F \Delta r \cos\theta$$

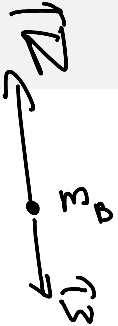
$$\Delta\left(\frac{1}{2}mv^2\right) = W_T$$

$$W > 0 \quad \text{if} \quad \cos\theta > 0 \Rightarrow 0 \leq \theta < \frac{\pi}{2}$$

$$W < 0 \quad \text{if} \quad \cos\theta < 0 \Rightarrow \frac{\pi}{2} < \theta \leq \pi$$

Example

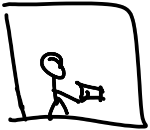




$$\vec{a} = 0$$

$$\vec{F}_T = 0$$

$$\vec{N} + \vec{w} = 0$$



$$\vec{v} = \text{const} \qquad \vec{N} = -\vec{w}$$

$$\Delta(KE) = W_{tot} = \vec{F}_{tot} \cdot \Delta\vec{r}$$

$$= (\vec{N} \cdot \Delta\vec{r}) + (\vec{w} \cdot \Delta\vec{r})$$

Work is NOT a vector!

Work can be positive or negative.

Work done by a force on an object

He holds the ball and stops the ball instantaneously so that the ball does not move after he holds it.

$$a_{av} = \frac{\Delta v}{\Delta t = 0} = \infty \Rightarrow F = \infty$$

Δt small but non zero

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$$W = \vec{F} \cdot \Delta\vec{r}$$

Example

$$\Delta(KE) = W_{tot}$$

$$v \geq \sqrt{gR}$$

$$W_{tot} = W_{gravity} = \vec{w} \cdot \vec{\Delta r} =$$

$$W_{gravity} = \vec{w} \cdot \Delta\vec{r}$$

$$= mg\,\Delta r \underbrace{\cos\left(\theta + \frac{\pi}{2}\right)}_{-\sin\theta}$$

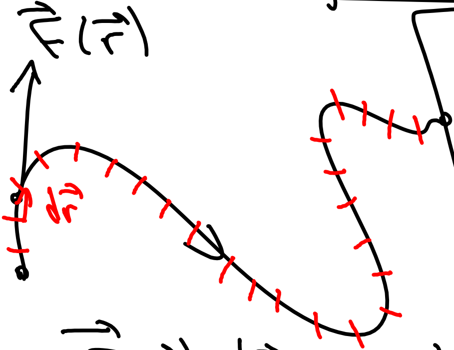
$$= -mg\,\Delta r \sin\theta$$

$$W_{tot} = W_{gravity} = -mgh$$

$$\frac{1}{2} m v^2 - \frac{1}{2} m v_0^2 = -mgh$$

$$v_0^2 = v^2 + 2gh \geq gR + 2gh$$

Work Done By a Variable Force



$W = \vec{F} \cdot \Delta\vec{r}$ only if $\vec{F}$ is constant

$dW = \vec{F}(\vec{r}) \cdot d\vec{r}$: work done by the force $\vec{F}$ as the objects moves through $d\vec{r}$

$W = \sum dW$

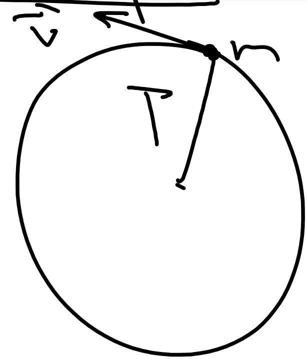
$W = \sum \vec{F}(\vec{r}) \cdot d\vec{r}$

Work done by normal force

$$dW = \vec{N} \cdot d\vec{r} = 0$$

Example

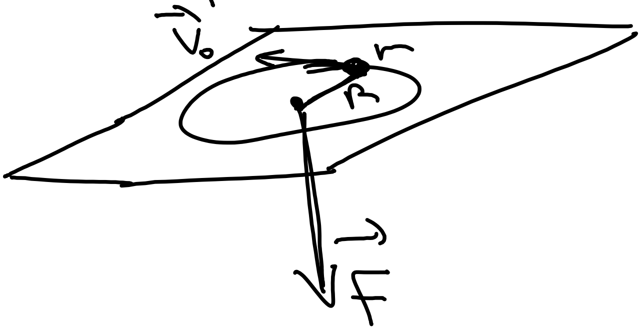
uniform circular motion



$$\vec{F}_c \cdot d\vec{r} = 0$$

$$W_{tension} = 0$$

Example



If you pull the string down by l

initially v_0, R, F_0 — stuck

finally $v, R-l, F$

$v = ?$

$$F(r) = \frac{m v^2(r)}{r}$$

$F(r)$: magnitude of the force when the circle has radius r.

$v(r)$: speed of the mass when the circle has radius r!

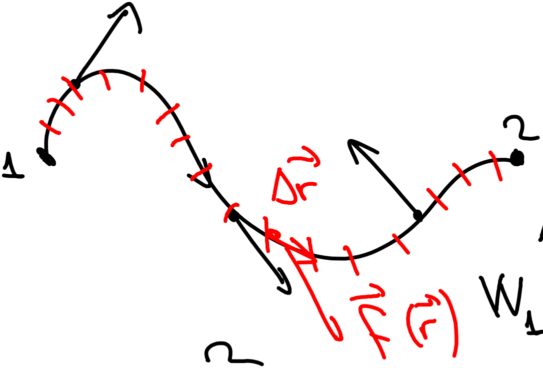
November 17, 2015

$W = \vec{F} \cdot \Delta\vec{r}$ work done by a constant force

$$\Delta(KE) = W_{tot}$$

$$KE = \frac{1}{2}mv^2$$

Work Done by a Variable Force

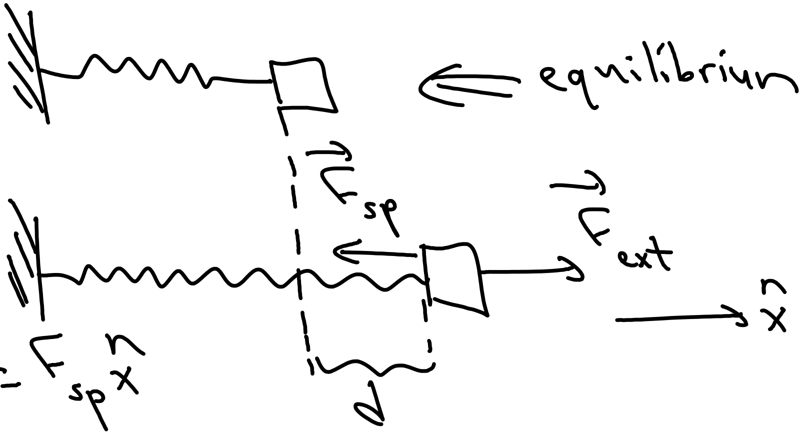


$$\Delta W = \vec{F}(\vec{r}) \cdot \Delta\vec{r}$$

$$W_{1\to2} = \sum \Delta W$$

$$W_{1\to2} = \int_1^2 \vec{F} \cdot d\vec{r}$$: line integral

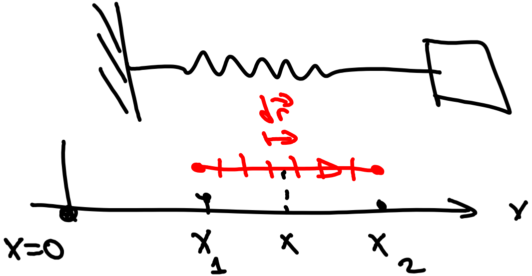
Work Done By a Spring



$$\vec{F}_{sp} = -F_{sp}\hat{x}$$

$$\boxed{F_{sp} = -kd}$$ Hooke's Law

Work Done By the Spring



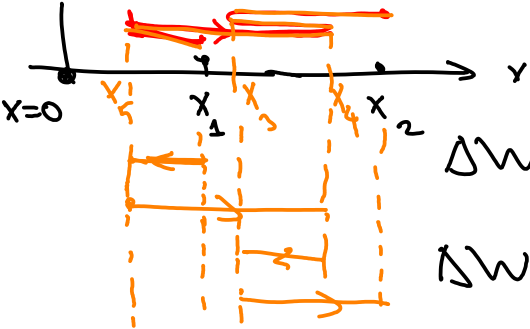
$$d\vec{r} = dx\,\hat{x}$$

$$\vec{F} = -kx\,\hat{x}$$

$$dW = \vec{F}\cdot d\vec{r} = -kx\,dx$$

$$W = \sum dW = \int_{x_1}^{x_2} (-kx\,dx) = -\frac{1}{2}kx^2\Big|_{x=x_1}^{x_2}$$

$$\boxed{W = -\frac{1}{2}kx_2^2 + \frac{1}{2}kx_1^2}$$



$$\Delta W_{x_1 \to x_5} = -\frac{1}{2}kx_5^2 + \frac{1}{2}kx_1^2$$

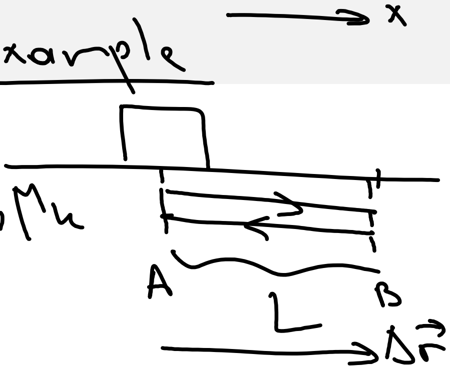
$$\Delta W_{x_5 \to x_4} = -\frac{1}{2}kx_4^2 + \frac{1}{2}kx_5^2$$

$$\Delta W_{x_4 \to x_3} = -\frac{1}{2}kx_3^2 + \frac{1}{2}kx_4^2$$

$$\Delta W_{x_3 \to x_2} = -\frac{1}{2}kx_2^2 + \frac{1}{2}kx_3^2$$

$$W_{tot} = -\frac{1}{2}kx_2^2 + \frac{1}{2}kx_1^2$$

Example



$W_f = ?$

μ_s, μ_k

$$W_f = W_{A\to B} + W_{B\to A} = -2\mu_k m g L$$

$$F_{fr} = \mu_k m g$$

$$W_{A\to B} = \vec{F}_{fr}\cdot\Delta\vec{r} = \mu_k m g L \cos\theta$$

$$W_{A\to B} = -\mu_k m g L$$

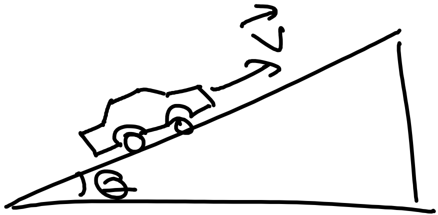
$$W_{B\to A} = \vec{F}_{fr}'\cdot\Delta\vec{r}' = (+\vec{F}_{fr})\cdot(+\Delta\vec{r}) = W_{A\to B}$$

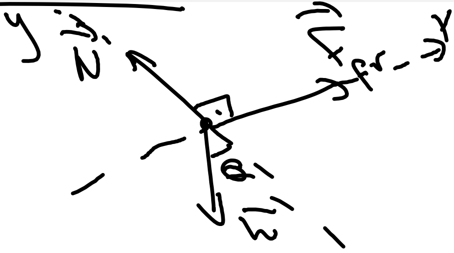
Power

$$P = \lim_{\Delta t \to 0} \frac{\Delta \text{Work done}}{\Delta \text{time}}$$

$$P = \frac{\vec{F} \cdot d\vec{r}}{dt} = \vec{F} \cdot \vec{v} = P$$

Car on Inclined Plane





$$\vec N = N\hat y$$

$$\vec F_{fr} = F_{fr}\hat x$$

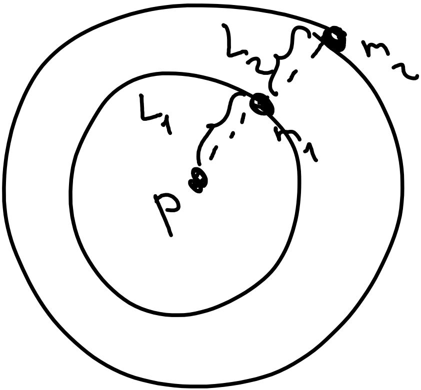
$$F_{fr} = mg\sin\theta$$

$$\boxed{P = mg\sin\theta\, v}$$

$$\vec w = mg\cos\theta(-\hat y) + mg\sin\theta(-\hat x)$$

$$(\vec F_{tot})_x = -mg\sin\theta + F_{fr} = 0$$

Homework Q2



$L_1 = R_1$

$L_2 = R_2$

$$\Delta(KE) = W_{tot}$$

spring $\quad W_{x_1 \to x_2} = -\frac{1}{2}kx_2^2 + \frac{1}{2}kx_1^2$

constant force $\quad W_{\vec{r}_1 \to \vec{r}_2} = \vec{F} \cdot (\vec{r}_2 - \vec{r}_1) = \vec{F} \cdot \vec{r}_2 - \vec{F} \cdot \vec{r}_1$

friction force $\quad W_{\vec{r}_1 \to \vec{r}_2} \neq f(\vec{r}_2) - f(\vec{r}_1)$

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Conservative force: work done $\vec{r}_1 \to \vec{r}_2$ is independent of how you go from $\vec{r}_1 \to \vec{r}_2$

e.g.: constant force, spring

Nonconservative force: work done $\vec{r}_1 \to \vec{r}_2$ depends on how you go from $\vec{r}_1 \to \vec{r}_2$

e.g.: friction

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Conservative force:

$$W_{\vec{r}_1 \to \vec{r}_2} = -U(\vec{r}_2) + U(\vec{r}_1)$$

spring $\quad W_{x_1 \to x_2} = -\frac{1}{2}kx_2^2 + \frac{1}{2}kx_1^2$

constant force $\quad W_{\vec{r}_1 \to \vec{r}_2} = \vec{F}\cdot(\vec{r}_2 - \vec{r}_1) = \vec{F}\cdot\vec{r}_2 - \vec{F}\cdot\vec{r}_1$

spring $\quad U(\vec{r}) = \frac{1}{2}kx^2 + \text{const}$

const. force $\quad U(\vec{r}) = -\vec{F}\cdot\vec{r} + \text{const}$

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Consider object of mass m,

$\vec{F}_{tot} = \vec{F}$ $\quad \vec{F}$ is a conservative force.

$$\Delta(KE) = W_{tot} = W_F = -\Delta U$$

$$KE(\vec{v}_2) - KE(\vec{v}_1) = -U(\vec{r}_2) + U(\vec{r}_1)$$

$$KE(\vec{v}_2) + U(\vec{r}_2) = KE(\vec{v}_1) + U(\vec{r}_1)$$

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KE + U has the same value at any point on the trajectory.

KE + U ≡ Mechanical Energy is conserved

U: potential energy corresponding to $\vec{F}$.

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$$\vec{F}_{tot} = \vec{F}_{conservative} + \vec{F}_{non\text{-}conservative}$$

forces   forces.

$$W_{tot} = W_{cf} + W_{ncf}$$

$$W_{tot}(\vec{r}_1 \to \vec{r}_2) = -U(\vec{r}_2) + U(\vec{r}_1) + W_{ncf}$$

$$W_{tot} = \Delta(KE)$$

$$\Delta(KE) = -\Delta U + W_{ncf}$$

$$\Delta(KE + U) = W_{ncf}$$

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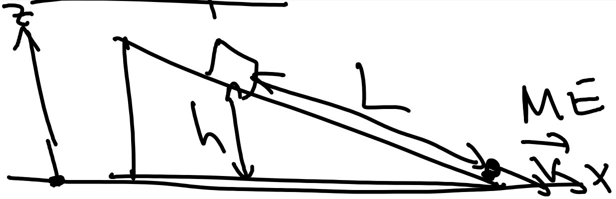
Example



M_s , M_k

$ME = KE + U_w$

$$U = -\vec{F}\cdot\vec{r} = -(-mg\hat{z})\cdot(x\hat{x}+y\hat{y}+z\hat{z})$$

$$\boxed{U = mgz}$$

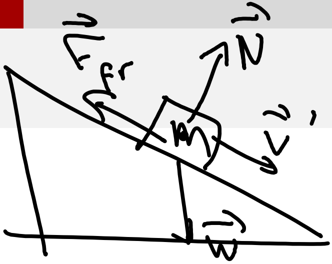
$$(ME)_{initial} = \frac{1}{2}m0^2 + mgh = mgh$$

$$(ME)_{pinal} = \frac{1}{2}mv^2 + mg0 = \frac{1}{2}mv^2$$

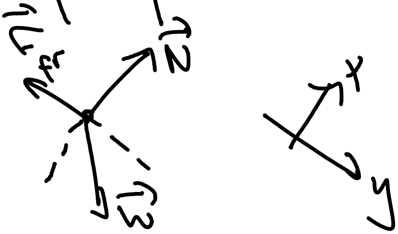
If $\mu_k = 0$, $W_{ncf} = 0$

$$\Delta(ME) = \frac{1}{2}mv^2 - mgh = 0$$

$$\Rightarrow \boxed{v = \sqrt{2gh}}$$



$\mu_k \neq 0$



$$\vec{N} = N\hat{x}$$

$$\vec{F}_{fr} = F_{fr}(-\hat{y})$$

$$\vec{w} = mg\cos\theta(-\hat{x}) + mg\sin\theta(\hat{y})$$

$$F_{fr} = \mu_k N$$

$$a_x = \frac{F_x}{m} = 0 = (-mg\cos\theta + N)\frac{1}{m}$$

$$\boxed{N = mg\cos\theta}$$

$$F_{fr} = \mu_k mg\cos\theta$$

$$W_{ncf} =$$

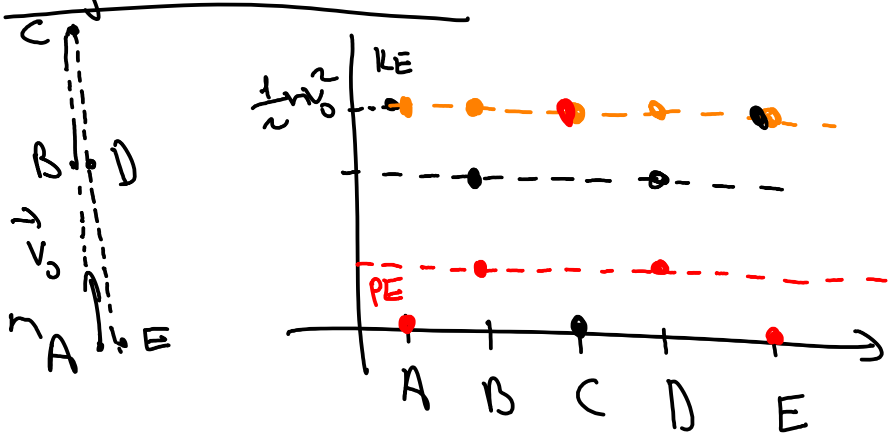
$$\Delta(ME) = \frac{1}{2}mv^2 - mgh = W_{ncf}$$

$$W = \vec{F}\cdot\Delta\vec{r} = (\mu_k mg\cos\theta)\,L\cos\pi$$

$$W_{ncf} = -\mu_k mgL\cos\theta$$

$$\boxed{\frac{1}{2}mv^2 = mgh - \mu_k mgL\cos\theta}$$

Projectile Motion



Constant Force

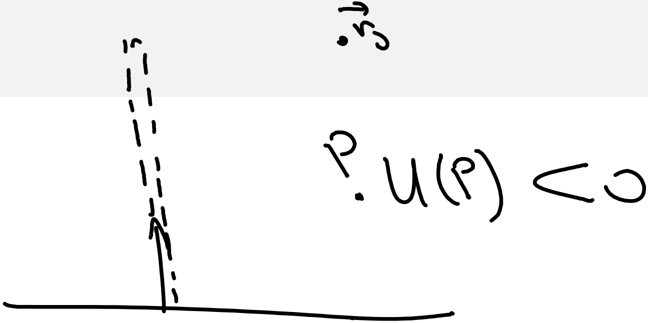
$$U(\vec{r}) = \vec{F}\cdot\vec{r} + \text{const}$$

$$U(\vec{r}=0) = \text{const}$$

$$\boxed{U(\vec{r}) = \vec{F}\cdot\vec{r} + U(\vec{r}=0)}$$

$$U(\vec{r}=0) = -\vec{F}\cdot\vec{r}_0$$

$$U(\vec{r}) = \vec{F}\cdot(\vec{r}-\vec{r}_0)$$


U(P) < 0

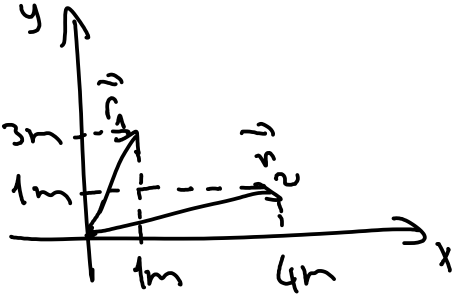
Conservation of Energy

Energy is **ALWAYS** conserved!

$E = ME + E_{internal}$

internal energy

Quiz



$$\vec{F} = (1N)(\hat{x} + \hat{y})$$

$$W(\vec{r}_1 \to \vec{r}_2) = ?$$

Everybody will get a 5 from this quiz!

~~One full page!~~

Don't tear the page!

November 19, 2015

Hand in your HW NOW!
I will not accept your HW
later during the class!

$$W = \int_{P_0}^{P_f} \vec{F} \cdot d\vec{l}$$

$$W_{tot} = \Delta(KE)$$

$$KE = \frac{1}{2} m v^2$$

conservative forces:

$$\int_{P_0}^{P_f} \vec{F} \cdot d\vec{l} = -U(P_f) + U(P_i)$$

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Only conservative forces acting

$$\Delta(KE) = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

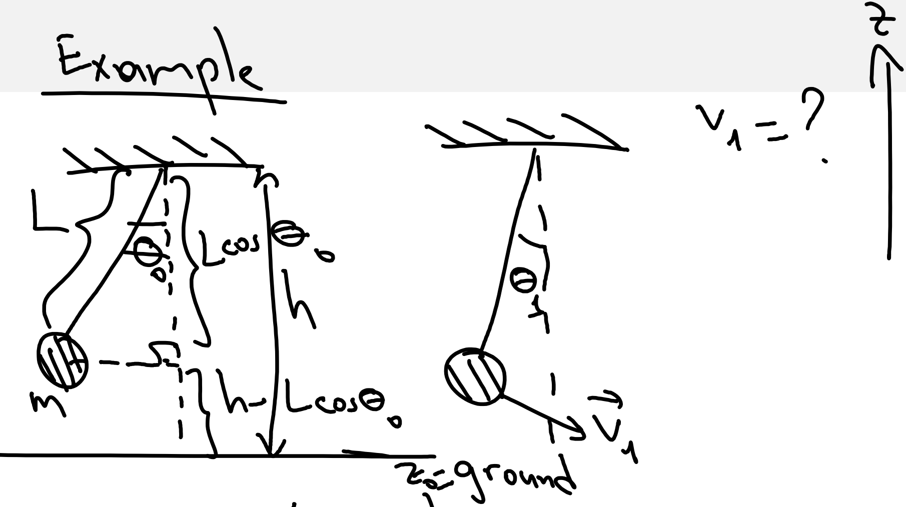
$$= W_{tot} = -U(P_f) + U(P_i)$$

$$\boxed{\frac{1}{2}mv_f^2 + U(P_f) = \frac{1}{2}mv_i^2 + U(P_i)}$$

$$ME = \frac{1}{2}mv^2 + U(P)$$

$$\Delta(ME) = W_{ncf}$$

Example



$$U = mg(z - z_0)$$

$$U_i = mg(h - L\cos\theta_0)$$

$$U_i = mg(h - L\cos\theta_0)$$

$$U_f = mg(h - L\cos\theta_1)$$

$$(ME)_i = \frac{1}{2}m0^2 + mg(h - L\cos\theta_0)$$

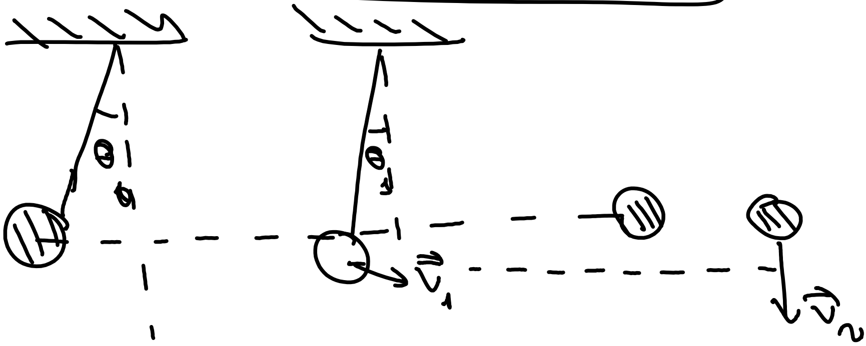
$$(ME)_f = \frac{1}{2}mv_1^2 + mg(h - L\cos\theta_1)$$

$$(ME)_i = (ME)_f \quad \text{bc. } W_{ncf} = 0$$

$$\frac{1}{2}\cancel{m}v_1^2 + \cancel{mgh} - \cancel{m}gL\cos\theta_1$$

$$= \cancel{mgh} - \cancel{m}gL\cos\theta_0$$

$$v_1^2 = 2gL(\cos\theta_1 - \cos\theta_0)$$

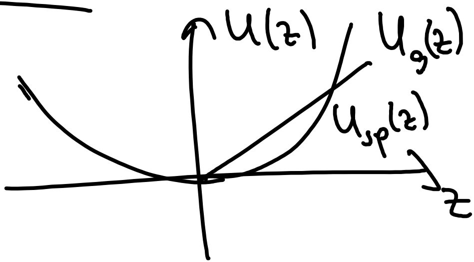


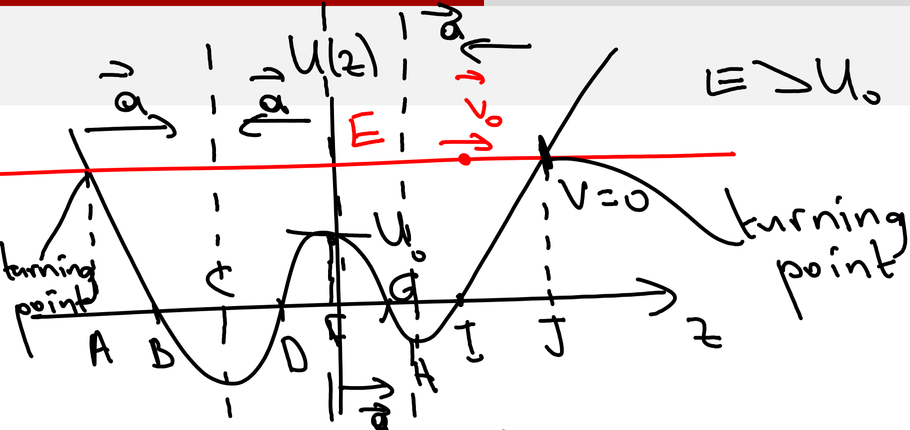
$$|\vec{v}_1| = |\vec{v}_2|$$

Potential Graphs (1D case)

$U_g(z) = mgz$

$U_{sp}(z) = \frac{1}{2}kz^2$



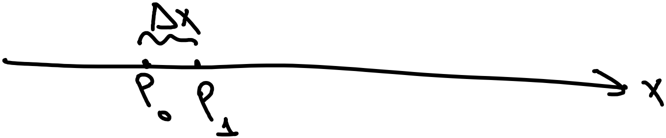


$$E \equiv ME = \frac{1}{2} m V^2 + U(z)$$

$$F = -\frac{dU}{dz}$$

Force from the Potential Energy

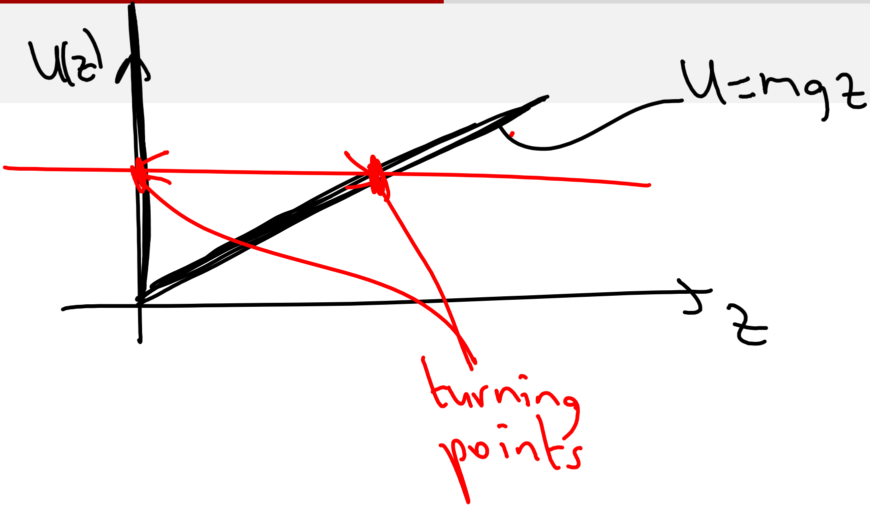
(1D case)



$$W = F\Delta x = -U(P_1) + U(P_0)$$

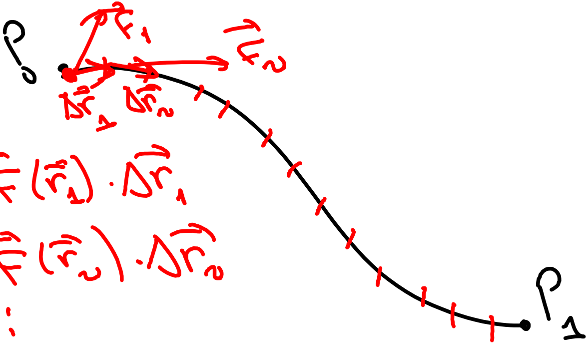
$$= -U(x_0+\Delta x) + U(x_0)$$

$$F = -\frac{U(x_0+\Delta x) - U(x_0)}{\Delta x} = -\frac{dU}{dx}$$


U(z)
U=mgz
z
turning points

Constant Force

$$W = \vec{F}\cdot\Delta\vec{r}$$



$$W_1 = \vec{F}(\vec{r}_1)\cdot\Delta\vec{r}_1$$

$$W_2 = \vec{F}(\vec{r}_2)\cdot\Delta\vec{r}_2$$

$\vdots$

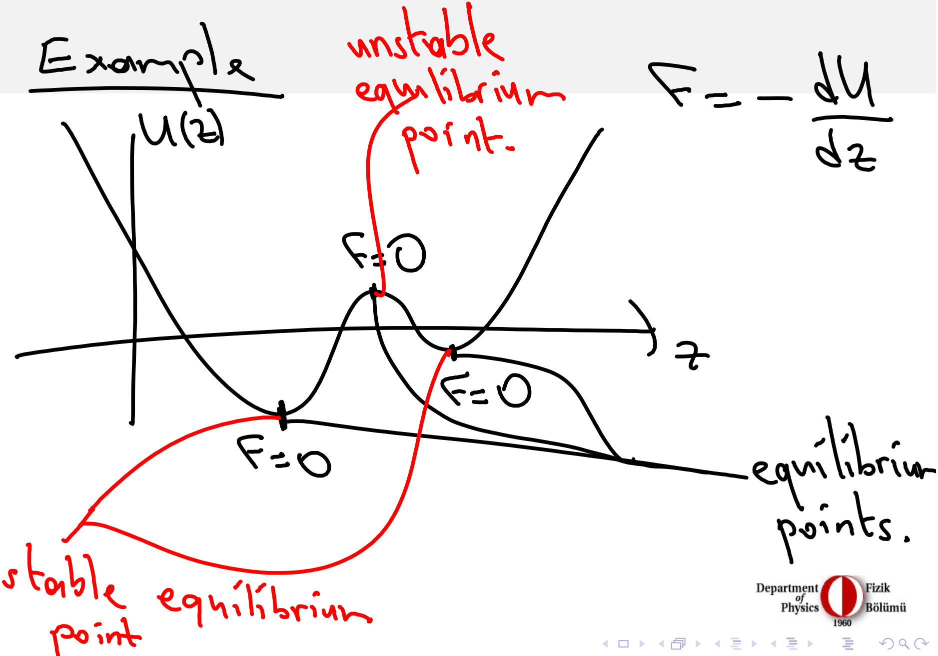
$+$

$$W = \sum W_i = \sum_i \vec{F}(\vec{r}_i)\cdot\Delta\vec{r}_i \to \int_{P_0}^{P_1} \vec{F}\cdot d\vec{r}$$

$$W_{cf}(P_0 \to P_1) = -U(P_1) + U(P_0)$$

Example

$$F = -\frac{dU}{dz}$$



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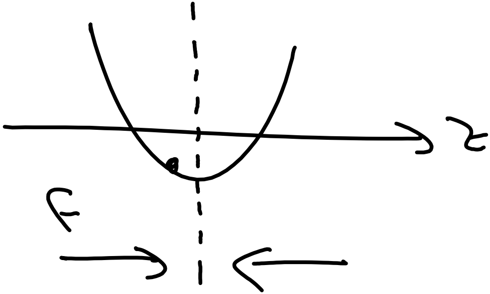
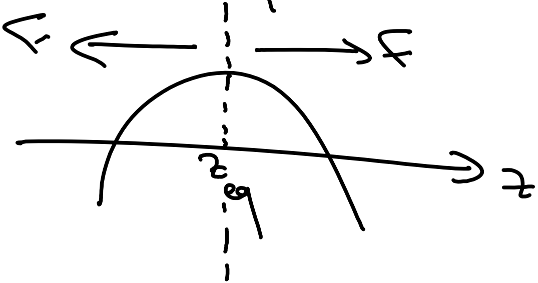
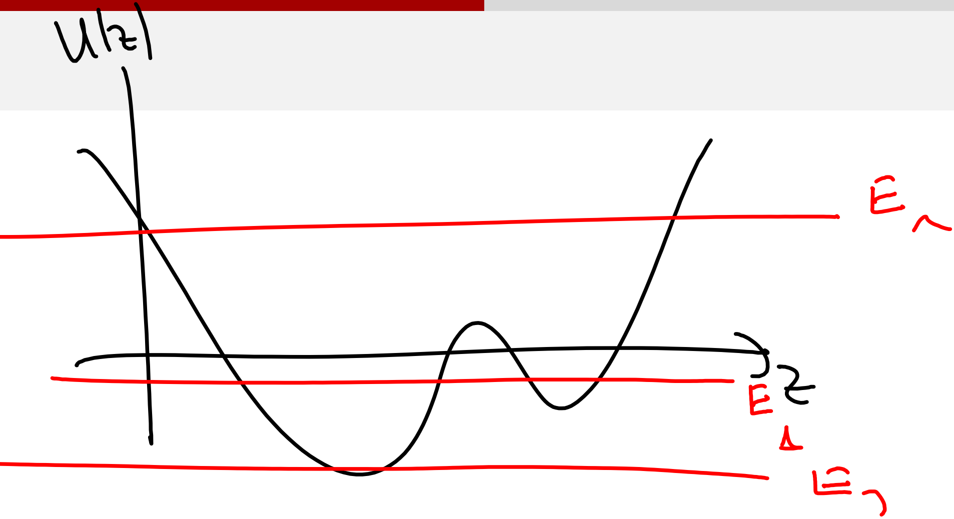
stable equilibrium point:

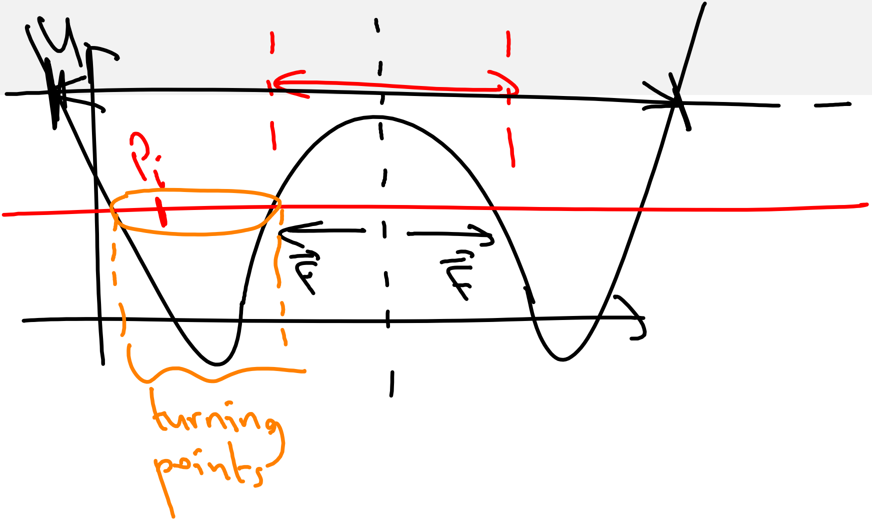


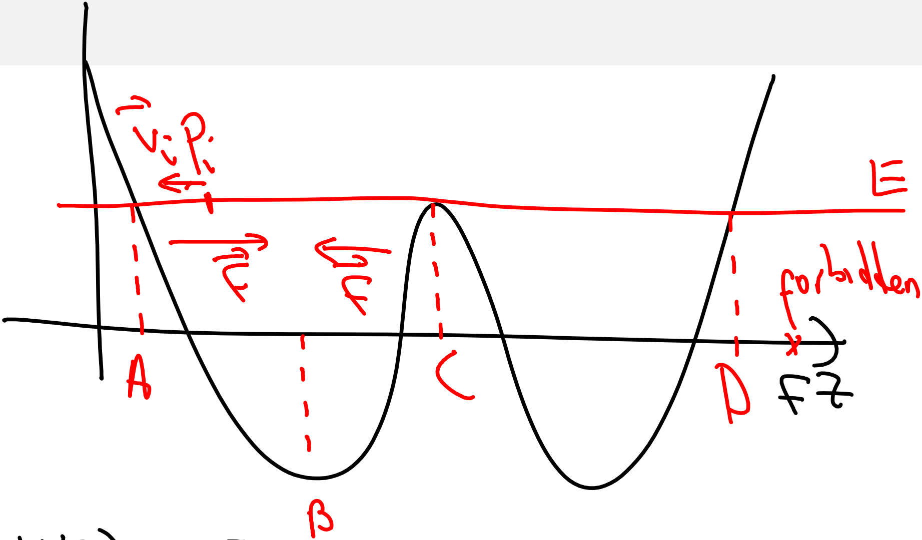
$$F = -\frac{dU}{dz}$$

unstable equilibrium Point




U(z)
z
E
E
E


U
pi
F
F
turning points



$$U(r) > E = U(r) + \frac{1}{2}mv^2$$

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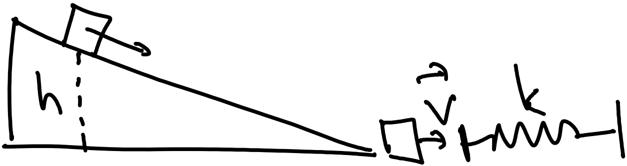
Example



$$(ME)_1 = mgh$$

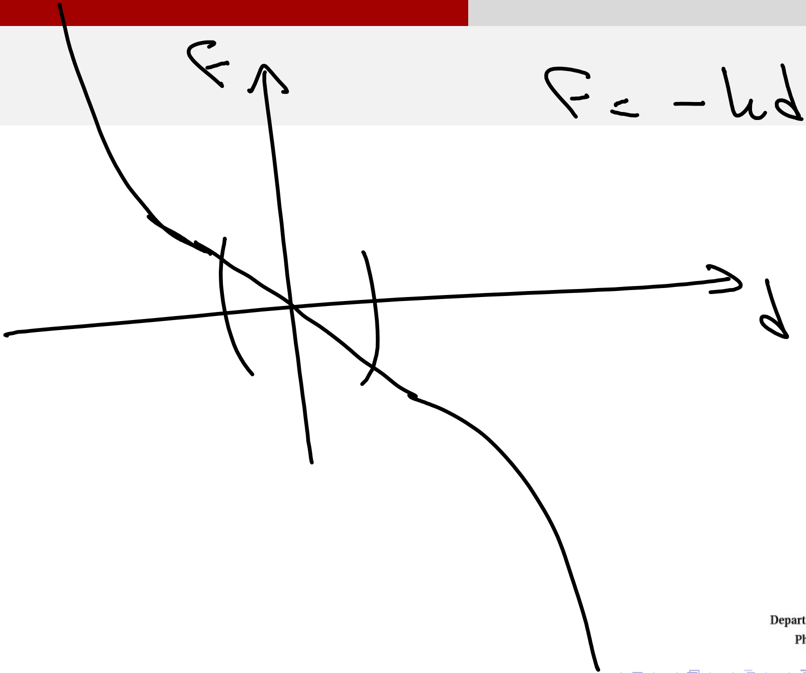
$$(ME)_2 = \frac{1}{2} m v^2$$

$$(ME)_3 = \frac{1}{2} k d^2$$

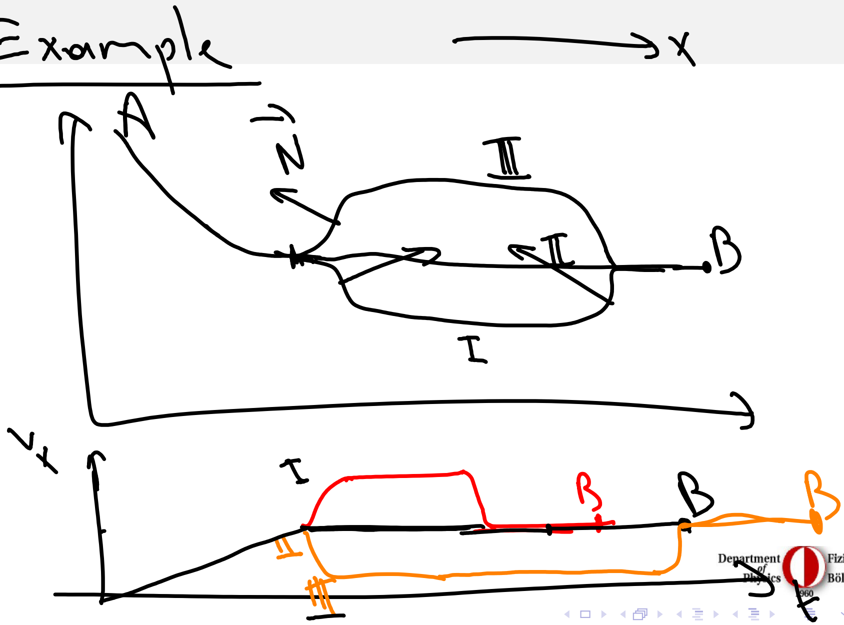
$$(ME)_1 = (ME)_2 = (ME)_3$$

if $\exists$ friction

$$(ME)_1 > (ME)_2 > (ME)_3$$


F
F = -kd
d

Example



November 24, 2015

Newton's 2nd Law:

$$\vec{a} = \frac{\vec{F}}{m} \Leftarrow$$

Newton's 3rd Law:

Forces always form action reaction pairs

$$\vec{F}_{AB} = -\vec{F}_{BA}$$

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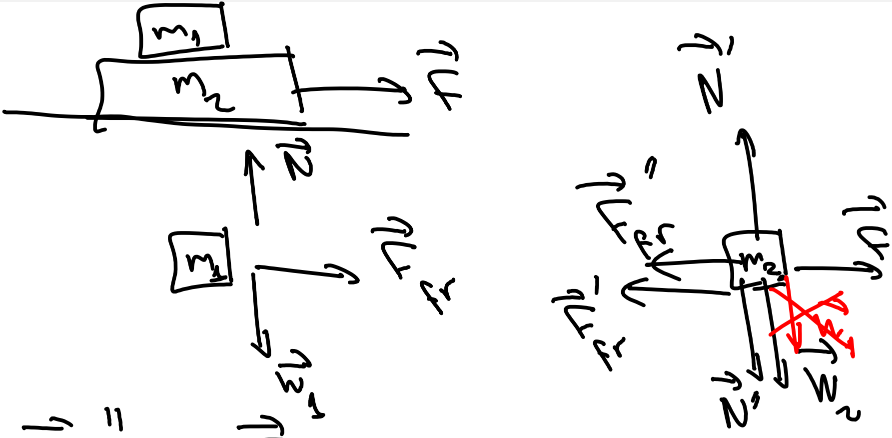
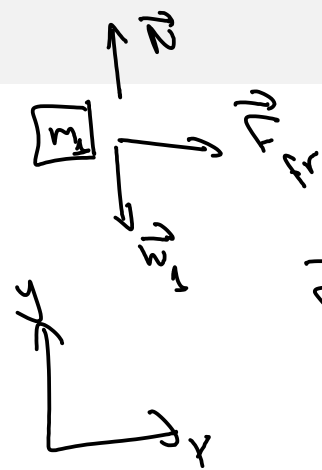


$$\vec{F}''_{fr} = -\vec{F}_{fr}$$

$$\vec{N} = -\vec{N}''$$



$$\vec{F}_{fr} = F_{fr}\hat{x}$$

$$\vec{N} = N\hat{y}$$

$$\vec{w}_1 = -mg\hat{y}$$

$$\vec{F}_{tot} = (N - mg)\hat{y} + F_{fr}\hat{x} = m_1 a_1 \hat{x}$$

$$N - mg = 0$$
$$F_{fr} = m_1 a_1$$

$F_{fr} \leq \mu_s N$ (if static)

$F_{fr} = \mu_k N$ (if kinetic)

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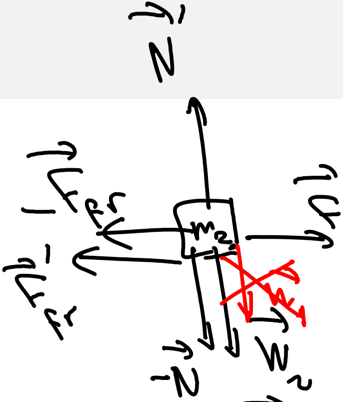


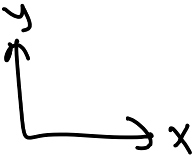
$\vec{N}' = N'\hat{y}$ $\quad$ $\vec{N} = N\hat{y}$

$\vec{F} = F\hat{x}$

$\vec{F}_{fr} = F_{fr}\hat{x}$

$\vec{F}'_{fr} = -F'_{fr}\hat{x}$

$\vec{W}_2 = -m_2 g\hat{y}$



$$\vec{F}_{tot} = \hat{y}(N' - N - m_2 g) + \hat{x}(F - F_{fr} - F'_{fr}) = m_2 a_2 \hat{x}$$

$$\boxed{N' = N + m_2 g} \qquad \boxed{F - F_{fr} - F'_{fr} = m_2 a_2}$$

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$$N - m_1 g = 0$$
$$F_{fr} = m_1 a_1$$
$$N' = N + m_2 g$$
$$F - F_{fr} - F'_{fr} = m_2 a_2$$

$$a_2 \neq 0$$
$$a_1 = a_2$$
$$F_{fr} = \text{static}$$
$$F'_{fr} = \mu_k N' \text{ (kinetic)}$$

$$\vec{F}'_{fr} \neq \mu_k \vec{N}'$$

$$F_{fr} = m_1 a_1$$
$$F - F_{fr} - \mu_k (m_1 + m_2) g = m_2 a_1$$

$$N = m_1 g$$
$$N' = (m_1 + m_2) g$$

$$a_1 = \frac{F_{fr}}{m_1} = \frac{F - F_{fr} - \mu_k (m_1 + m_2) g}{m_2}$$

$$\frac{F_{fr}}{m_1} = \frac{F - F_{fr} - \mu_k (m_1+m_2) g}{m_2}$$

$$m_2 F_{fr} = m_1 F - m_1 F_{fr} - \mu_k (m_1+m_2) g\, m_1$$

$$F_{fr} = \frac{m_1 F - m_1 \mu_k (m_1+m_2) g}{m_1+m_2}$$

$$\boxed{F_{fr} = \frac{m_1}{m_1+m_2} F - m_1 \mu_k g < \mu_s m_1 g}$$

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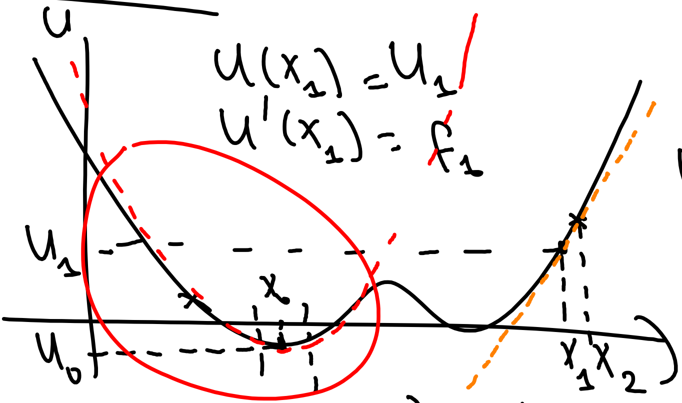
$$a_1 = a_2 = \frac{F_{fr}}{m_1} > 0$$

"Elastic" Potential

$U(x_2) \cong U(x_1) + f_1(x_2 - x_1)$

$U(x_1) = U_1$

$U'(x_1) = f_1$



$U(x_0) = U_0$

$U'(x_0) = f_0 = 0$

$U''(x_0) = k_0$

$x \simeq x_0$ $\qquad U(x) \cong U_0 \qquad U(x) \simeq U_0 + f_0(x - x_0) + \frac{1}{2}k_0(x - x_0)^2$

$x \simeq x_1$ $\qquad U(x) \simeq U_1$

$$U(x) = U(x_0) + U'(x_0)(x - x_0) + U^{(2)}(x_0)\frac{(x-x_0)^2}{2!} + \cdots$$

$$U^{(n)}(x_0) = \frac{d^n}{dx^n} U(x) \Big|_{x = x_0}$$

$$U(x) = \sum_{n=0}^{\infty} U^{(n)}(x_0) \frac{(x-x_0)^n}{n!}$$

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close to a minimum at $x = x_0$

$$U(x) = U_0 + \frac{1}{2}k(x - x_0)^2$$

shift origin so that $x_0 = 0$

shift the pot. s.t $U_0 = 0$

$$U(x) \simeq \frac{1}{2}kx^2$$

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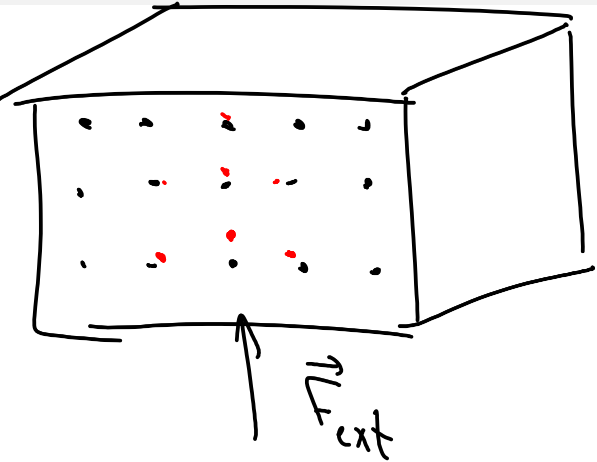

$\vec{F}_{ext}$

1) Come prepared - prereports
2) ask questions
3) solve questions (chapter question)
4) discuss — forums
5) work regularly

Momentum

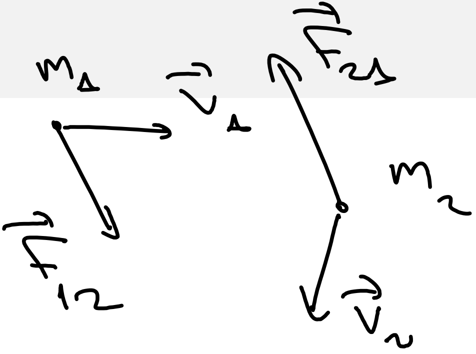
$$\vec{F} = m\vec{a} = m\frac{\Delta\vec{v}}{\Delta t}$$

$$\vec{F}\Delta t = m\Delta\vec{v} = m(\vec{v}_f - \vec{v}_i)$$

$$= (m\vec{v}_f) - (m\vec{v}_i)$$

$$\vec{F}\Delta t = \Delta(m\vec{v})$$

$$\vec{p} \equiv m\vec{v} \text{ : momentum}$$



3rd Law:

$$\vec{F}_{12} = -\vec{F}_{21}$$

$$\vec{F}_{12}\Delta t = -\vec{F}_{21}\Delta t$$

$$\Delta(\vec{p}_1) = -\Delta(\vec{p}_2)$$

$$\Delta(\vec{p}_1) + \Delta(\vec{p}_2) = 0$$

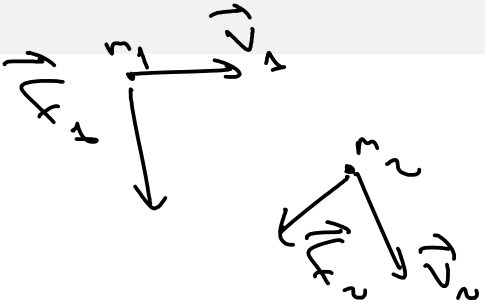
$$\Delta(\vec{p}_1) + \Delta(\vec{p}_2) = 0$$

$$\boxed{\Delta(\vec{p}_1 + \vec{p}_2) = 0}$$

$$\vec{p}_1 + \vec{p}_2 = \text{conserved}$$

$$\Updownarrow$$

Newton's 3rd Law is Valid



$$\vec{F}_1 = \vec{F}_1^{ext} + \vec{F}_{12}$$

$$\vec{F}_2 = \vec{F}_2^{ext} + \vec{F}_{21}$$

$$\Delta(\vec{p}_1 + \vec{p}_2) = \Delta\vec{p}_1 + \Delta\vec{p}_2$$

$$= \vec{F}_1 \Delta t + \vec{F}_2 \Delta t$$

$$= (\vec{F}_1 + \vec{F}_2)\Delta t$$

$$= (\vec{F}_1^{ext} + \vec{F}_2^{ext})\Delta t$$

$$\Delta(\vec{p}_1+\vec{p}_2) = \vec{F}^{ext}_{tot}\,\Delta t$$

$$\vec{P} = \vec{p}_1+\vec{p}_2$$

$$\boxed{\Delta\vec{P} = \vec{F}^{ext}_{tot}\,\Delta t} \iff \vec{F}^{ext}_{tot} = \frac{d\vec{P}}{dt}$$

$$\vec{P} = \sum \vec{p}_i$$

$$\vec{F}^{ext}_{tot} = \sum \vec{F}^{ext}_i \left(= \sum \vec{F}_i\right)$$

compare $\vec{F} = \frac{d\vec{p}}{dt}$

$$\vec{F}^{ext}_{tot} = \frac{d\vec{P}}{dt} \Longleftrightarrow \vec{F} = \frac{d\vec{p}}{dt}$$

$$\vec{P} \equiv M\vec{V}_{cm} \qquad \vec{p} = m\vec{v}$$

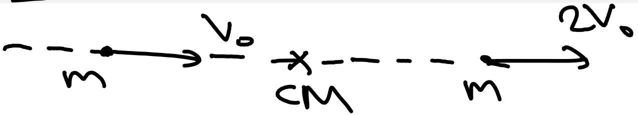
↳ center of mass

M: total mass of the system

$$M = \sum_i m_i \quad (= m_1 + m_2)$$

$$\vec{V}_{cm} = \frac{1}{M}\vec{P} = \frac{1}{M}\sum \vec{p}_i$$

Ex 1D motion



$$\underline{P} = m v_0 + m(2v_0) = 3 m v_0$$

$$M = m + m = 2m$$

$$v_{cm} = \frac{\underline{P}}{M} = \frac{3 m v_0}{2m} = \frac{3}{2} v_0$$

$$\vec{a}_{cm} \equiv \frac{d\vec{v}_{cm}}{dt} \qquad \vec{F}^{ext}_{tot} = M\vec{a}_{cm}$$

$$\vec{v}_{cm} \equiv \frac{d\vec{r}_{cm}}{dt}$$

$$\vec{v}_{cm} = \frac{1}{M}\sum_i m_i\vec{v}_i = \frac{1}{M}\sum_i m_i\frac{d\vec{r}_i}{dt}$$

$$\vec{v}_{cm} = \frac{1}{M}\sum \frac{d}{dt}(m_i\vec{r}_i) = \frac{d}{dt}\left(\frac{1}{M}\sum_i m_i\vec{r}_i\right)$$

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$$\vec{r}_{cm} = \frac{1}{M}\sum m_i \vec{r}_i$$

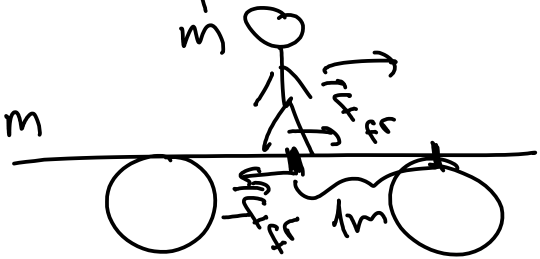
Ex $m_1 = m_2 = m$; $M = 2m$

$$\vec{r}_{cm} = \frac{1}{2m}\left[(m\vec{r}_1) + (m\vec{r}_2)\right]$$

$$\vec{r}_{cm} = \frac{1}{2}\left[\vec{r}_1 + \vec{r}_2\right]$$

Example

no friction between the cart and ground, no drag force.



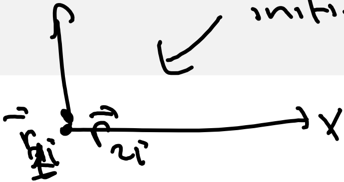
$$\vec{r}_{cm} = \frac{1}{2}(\vec{r}_1+\vec{r}_2) = \frac{m_1\vec{r}_1+m_2\vec{r}_2}{m_1+m_2}$$

$$0 = \Delta\vec{r}_{cm} = \frac{1}{2}(\Delta\vec{r}_1+\Delta\vec{r}_2)$$

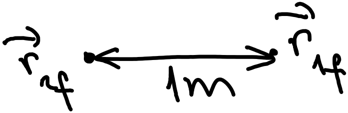
$$\boxed{\Delta\vec{r}_1 = -\Delta\vec{r}_2}$$

$$m_1\Delta\vec{r}_1 + m_2\Delta\vec{r}_2 = 0$$

initial state



$$\vec r_{1i} = \vec r_{2i} = 0$$



$$|\vec r_{1f} - \vec r_{2f}| = 1m$$

$$r_{1f} - r_{2f} = 1m$$

$$\Delta r_1 + \Delta r_2 = r_{1f} + r_{2f} = 0$$

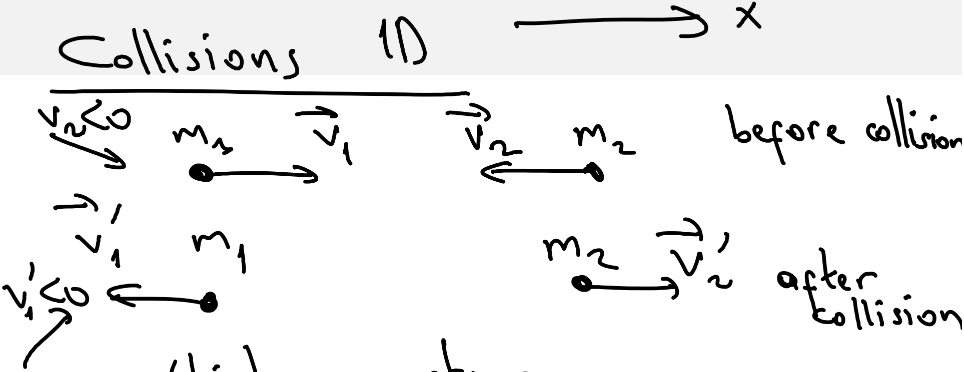
$$\Delta r_1 = r_{1f} - r_{1i} = 0.5m \qquad 2r_{1f} = 1m$$

$$\boxed{r_{1f} = \frac{1}{2}m}$$

November 26, 2015

Hand in your HW now!

Collisions 1D



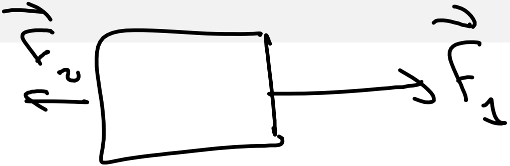
P_i : initial momentum

$$P_i = m_1 v_1 + m_2 v_2$$

$$P_f = m_1 v_1' + m_2 v_2'$$

$$\vec{v}_1 = v_1 \hat{x}$$

$$\vec{v}_2 = v_2 \hat{x}$$



$$\vec{F}_{tot} = \vec{F}_1 + \vec{F}_2 = (|\vec{F}_1| - |\vec{F}_2|)\hat{x}$$

Conservation of momentum

$$p_i = p_f$$

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

Unknowns: v_1' & v_2'

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

one possible (extreme) case: conserved kinetic energy (elastic collision.

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2$$

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$$m_1(v_1 - v_1') = m_2(v_2' - v_2)$$

$$m_1(v_1^2 - v_1'^2) = m_2(v_2'^2 - v_2^2)$$

$$m_1(v_1 - v_1')(v_1 + v_1') = m_2(v_2' - v_2)(v_2' + v_2)$$

$$m_1(v_1 - v_1') = m_2(v_2' - v_2)$$

$$v_1 + v_1' = v_2 + v_2'$$

$$m_1(v_1 - v_1') = m_2(v_2' - v_2)$$

$$v_1 + v_1' = v_2 + v_2' \Rightarrow v_1 - v_2 = -(v_1' - v_2')$$

$$-\frac{m_1}{m_2}(v_1 - v_1') = -v_2' + v_2$$

$$\left(1 - \frac{m_1}{m_2}\right)v_1 + \left(1 + \frac{m_1}{m_2}\right)v_1' = 2v_2$$

$$\boxed{v_1' = \frac{m_2 - m_1}{m_2 + m_1} v_1 + \frac{2m_2}{m_1 + m_2} v_2}$$

$$v_1' = -\frac{m_2 - m_1}{m_2 + m_1} v_1 + \frac{2m_2}{m_1 + m_2} v_2$$

$$v_1 + v_1' = v_2 + v_2' \Rightarrow v_2' = v_1 + v_1' - v_2$$

$$v_2' = \frac{2m_1}{m_1 + m_2} v_1 + \frac{m_2 - m_1}{m_2 + m_1} v_2$$

Limiting case

$m_1 \to 0$ (a fly hits you on the back)

$$v_2' = \frac{2m_1}{m_1+m_2} v_1 + \frac{m_2-m_1}{m_2+m_1} v_2$$

$$\overset{m_1 \to 0}{\simeq} \frac{0}{m_2} v_1 + \frac{m_2}{m_2} v_2$$

$$\boxed{v_2' = v_2}$$

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Completely Inelastic Collisions

$$KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

$$v_{cm} = \frac{1}{m_1 + m_2} \left(m_1 v_1 + m_2 v_2 \right)$$

$$v = v_1 - v_2$$

$$\left. \begin{aligned} v_1 &= v_{cm} + \frac{m_2}{m_1 + m_2} v \\ v_2 &= v_{cm} - \frac{m_1}{m_1 + m_2} v \end{aligned} \right\} \quad KE = \frac{1}{2}(m_1 + m_2) v_{cm}^2 + \frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} v^2$$

completely inelastic collision:

$$v_1' = v_2'$$

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2' = (m_1 + m_2) v_1'$$

$$v_1' = \frac{1}{m_1 + m_2}(m_1 v_1 + m_2 v_2) = v_{cm}$$

$$\boxed{\frac{v_2' - v_1'}{v_1 - v_2} \equiv \eta}$$

Example

mass A

$v_x^i = 0.65 \pm 0.05$ m/s

$v_x^f = 0.2 \pm 0.1$ m/s

$m = 48g = 0.048$ kg

$p_x^i = 0.03$ kgm/s

$p_x^f = 0.01$ kgm/s

mass B

$v_x^i = (-0.05 \pm 0.05)$ m/s

$v_x^f = (0.4 \pm 0.05)$ m/s

$p_x^i = -0.002$ kgm/s

$p_x^f = 0.02$ kgm/s

$P_x^i = 0.03$ kgm/s ; $P_x^f = 0.03$ kgm/s

$$v_A^i = (0.65 \pm 0.05)\, m/s$$

$$v_B^i = (0.5 \pm 0.05)\, m/s$$

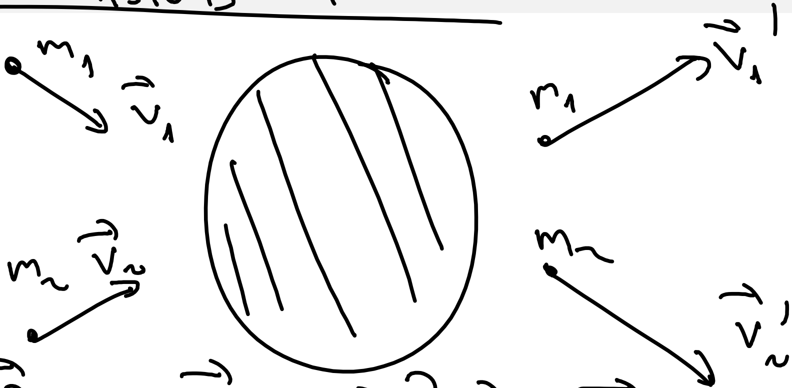
$$v_B^f = (0.4 \pm 0.05)\, m/s$$

$$v_A^f = (0.2 \pm 0.05)\, m/s$$

$$(v_A^i)^2 + (v_B^i)^2 = (v_A^f)^2 + (v_B^f)^2 \ ?$$

$$0.7\, m^2/s^2 \stackrel{?}{=} 0.2\, m^2/s^2$$

Collisions in 2D



$$\left.\begin{aligned}\vec{P}_i &= m_1\vec{v}_1 + m_2\vec{v}_2 \\ \vec{P}_f &= m_1\vec{v}_1' + m_2\vec{v}_2'\end{aligned}\right\} \quad \vec{P}_i = \vec{P}_f$$

$$(P_i)_x = (P_f)_x \Rightarrow m_1 v_{1x} + m_2 v_{2x} = m_1 v'_{1x} + m_2 v'_{2x}$$

$$(P_i)_y = (P_f)_y \Rightarrow m_1 v_{1y} + m_2 v_{2y} = m_1 v'_{1y} + m_2 v'_{2y}$$

Completely inelastic collision: $\vec{v}_1' - \vec{v}_2' = 0$

$$v'_{1x} = v'_{2x}$$

$$v'_{1y} = v'_{2y}$$

Completely elastic collision:

$$(KE)_i = (KE)_f$$

Example mass A & mass B

$$\Delta P_{Ax} = -0.02 \ kgm/s$$

$$\Delta \vec{p} = \vec{F}_{av} \Delta t$$

$$(F^A_{av})_x = \frac{(\Delta P_A)_x}{\Delta t}$$

$$\Delta t = 0.1 \ s$$

$$(F^A_{av})_x = -0.2 \ kgm/s^2 = -0.2N$$

$$\left(F^{B}_{av}\right)_x = \frac{\Delta p^{B}_{x}}{\Delta t} = \frac{0.02\ \text{kg m/s}}{0.1\ \text{s}}$$

$$= 0.2\ \text{kg m/s}^2 = 0.2\ \text{N}$$

$$\Delta\vec{p} = \vec{F}\Delta t \quad (\text{if } \vec{F} \text{ is constant})$$

$$= \int_{t_i}^{t_f} \vec{F}(t)\,dt \quad (\text{if } \vec{F} \text{ is } \underline{\text{NOT}} \text{ constant})$$

$$= \vec{J} \equiv \vec{I} : \text{impulse}$$

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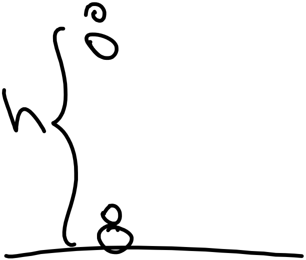
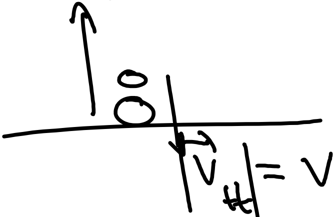


$h \gg$ size of the balls

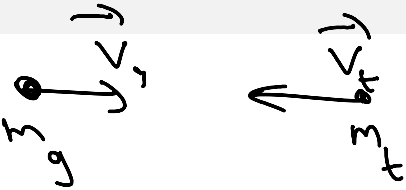
$$mgh = \frac{1}{2}mv^2$$

$$\boxed{h = \frac{v^2}{2g}}$$

$|\vec{V}_{gb}| = V$



$|\vec{V}_{tt}| = V$

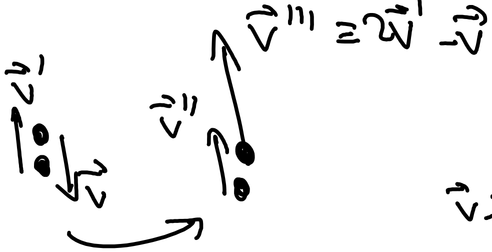


$m_g \gg m_t$

$$\vec{V}_t' = \frac{m_t \vec{V}_t + m_g \vec{V}_g}{m_t} \approx \frac{m_g}{m_t}\vec{V}_g$$

add $\vec{v}'$ to all velocities

initial ref frame

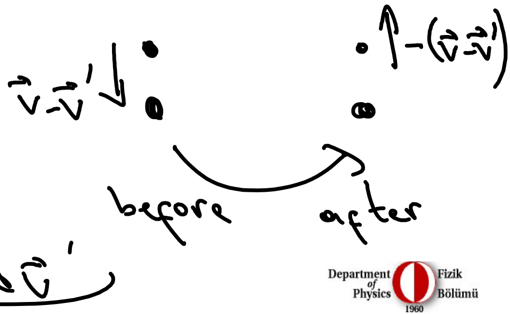


$|\vec{v}|=|\vec{v}'|=v$

$|\vec{v}''|\simeq v$

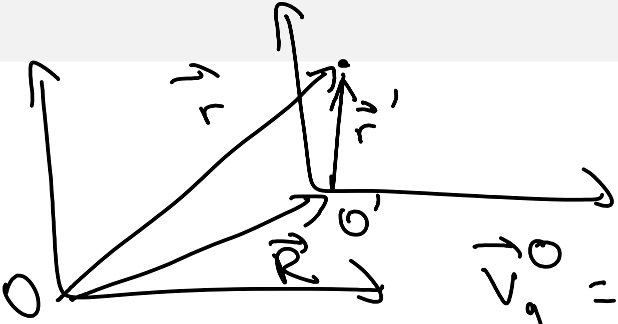
$|\vec{v}'''|=3v$

ref frame in which the golf ball is at rest during collision



$\Delta\vec{p} = \vec{F} \cdot \Delta t$

if Δt is very small, $\Delta\vec{p} \cong 0$



$$\vec{r} = \vec{r}\,' + \vec{R}$$

$$\boxed{\vec{v} = \vec{v}\,' + \vec{V}}$$

$$\boxed{\vec{v}\,' = \vec{v} + (-\vec{V})}$$

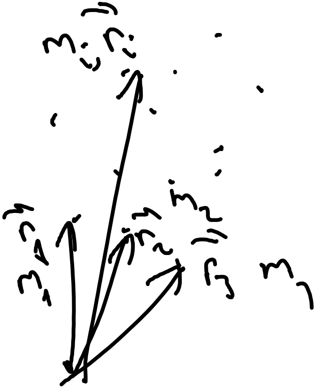
$$\vec{v}_g^{\,O} = \vec{v}\,'$$

$$\vec{v}_g^{\,O'} = 0$$

$$\vec{v}_g^{\,O} = \vec{v}_g^{\,O'} + \vec{V}$$

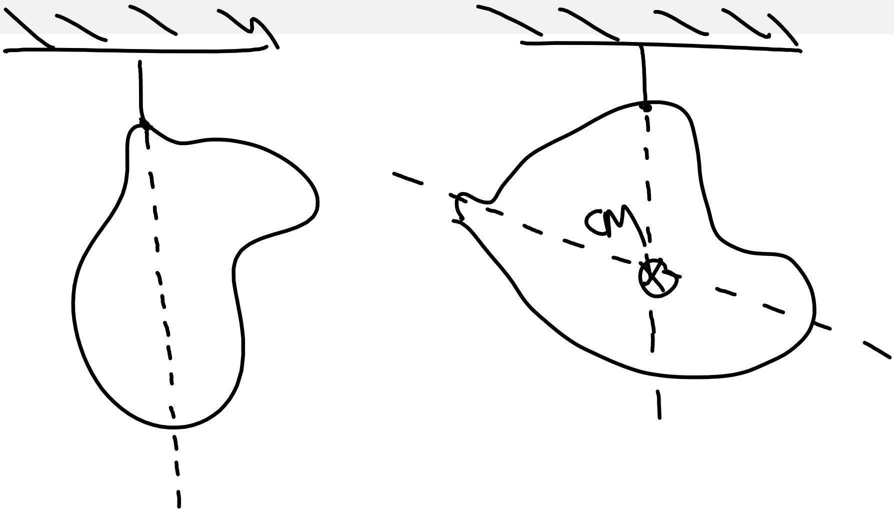
$$\boxed{\vec{v}_g^{\,O'} = \vec{v}_g^{\,O} - \vec{V} \neq 0}$$

Center of Mass

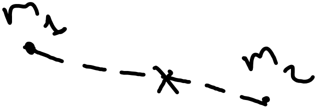


$$\vec{r}_{cm} = \left(\sum m_i \vec{r}_i\right)\frac{1}{M}$$

$$M = \sum m_i : \text{total mass}$$

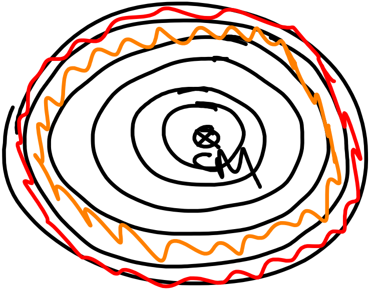

CM

Example

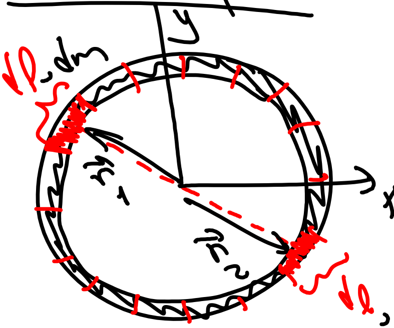


$$\vec{r}_{cm} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2}$$

Example where is the CM?



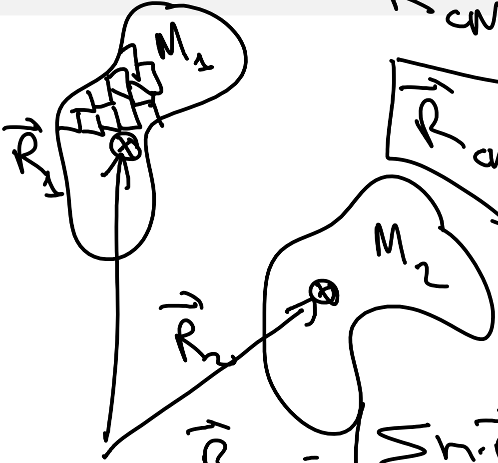
Example CM = ?



assume: mass is homogeneously distributed

$$\vec r_{cm} = \frac{\sum dm\, \vec r_i}{M} = \frac{\int dm\, \vec r_i}{M}$$

$$\boxed{\vec r_{cm} = 0}$$

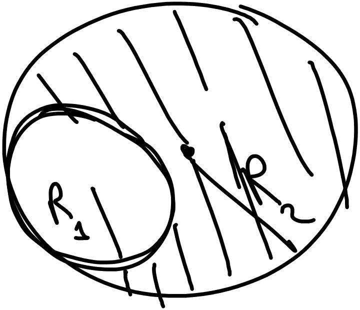


$$\vec R_{cm} = \frac{M_1 \vec R_1 + M_2 \vec R_2}{M_1 + M_2}$$

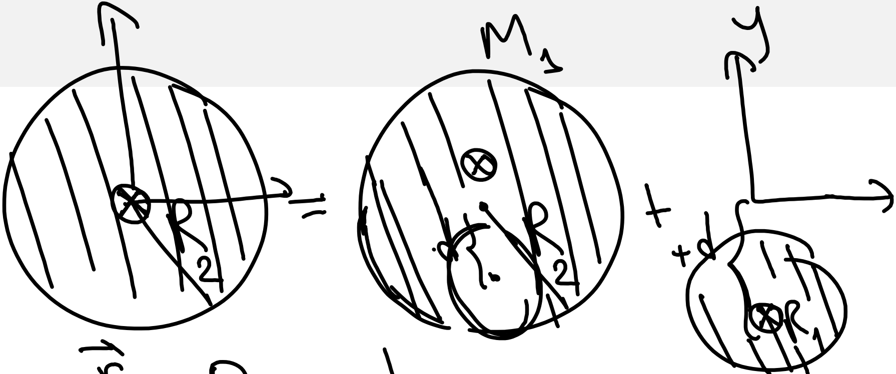
$$\vec R_{cm} = \sum m_i \vec r_i \, \frac{1}{M}$$

$$\vec R_{cm} = \Big(\underbrace{\sum_{m_i \in M_1} m_i \vec r_i}_{M_1 \vec R_1} + \underbrace{\sum_{m_i \in M_2} m_i \vec r_i}_{M_2 \vec R_2} \Big) \frac{1}{M_1 + M_2}$$

Example ρ : surface mass density



$$\vec{R}_{cm} = \frac{\sum m_i \vec{r}_i}{\sum m_i}$$



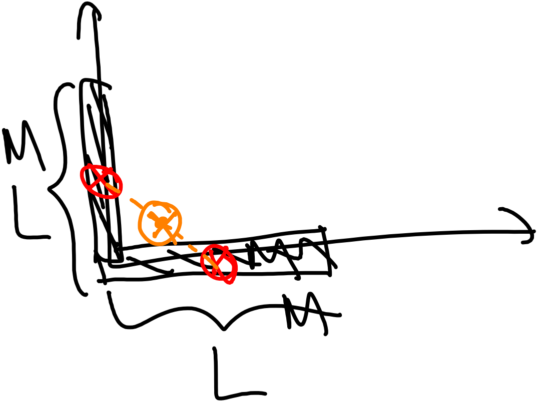
$\vec{r}_{cm} = 0$ unknown

$$\vec{0} = M_1 \vec{R}_{cm} + M_2 \left(d(-\hat{y}) \right)$$

$$\boxed{\vec{R}_{cm} = \frac{M_2}{M_1} d \hat{y}}$$


CM

Example



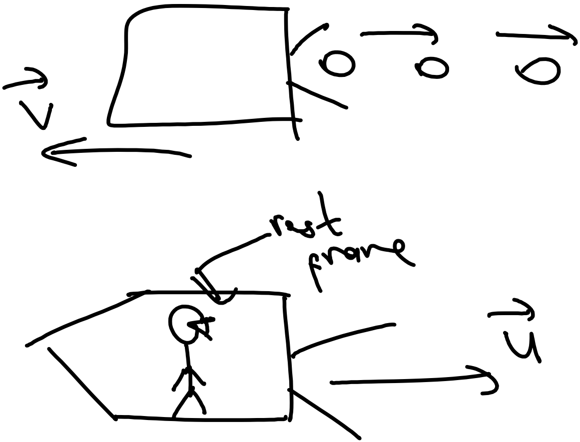
$$\boxed{\vec{a}_{cm} = \frac{\vec{F}^{ext}}{M}}$$

$$\vec{F}^{ext} = \sum_i (m_i \vec{g}) = \left(\sum m_i\right)\vec{g} = M\vec{g}$$

$$\boxed{\vec{a}_{cm} = \vec{g}}$$

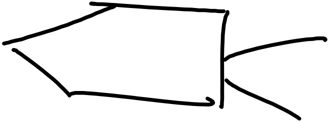
$$\Delta \vec{P} = 0 \quad \text{if} \quad \vec{F}^{ext} = 0$$

Example



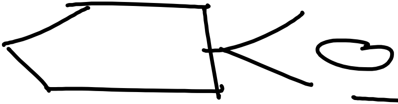
$\hat{z} \leftarrow$

$\underline{t}$ $\quad \vec{v}$, $M(t)$ $\quad \vec{P} = P\hat{z}$



$P_i = M(t)\,v$

$\underline{t+dt}$ $\quad \vec{v} + d\vec{v}$, $M - dM$ $\quad (dM > 0)$



$dM,\ \vec{v} + \vec{u} = (v-u)\hat{z}$

$$P_f = (M - dM)(v + dv) + dM(v - u)$$

$$P_i = Mv$$

negligible

$$\cancel{Mv} = \cancel{Mv} + M\,dv - \cancel{(dM)\,v} - \cancel{dM\,dv} + \cancel{(dM)\,v} - u\,dM$$

$$\boxed{M\,dv = u\,dM}$$

$$M\,dv = u\,dM + dM\,dv$$

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$$M\frac{dv}{dt} = u\frac{dM}{dt}$$

$$\boxed{M(t)\,a(t) = u\,\frac{dM}{dt}}$$

$$\frac{dM}{dt} > 0$$

$$\frac{dM}{dt} = -\frac{dM_R}{dt}$$

dM = change in mass of the exhaust

$$M\,dv = u\,dM$$

dM_R : change in the mass of the rocket

$$dM_R = -dM$$

$$M_R\,dv = -u\,dM_R$$

$$\int_{v_0}^{v} dv = -u\int_{M_i}^{M_f} \frac{dM_R}{M_R}$$

$$v - v_0 = u \ln \frac{M_f}{M_i}$$

$$v = v_0 - u \ln \frac{M_f}{M_i}$$

$$v = v_0 + u \ln \frac{M_f}{M_i}$$

$$v = v_0 + u \ln \frac{M_i}{M_f}$$

$$\lim_{dt\to 0}\left[M\frac{dv}{dt} = u\frac{dM}{dt} + \frac{dM\,dv}{dt} \right]$$

$$M(t)\,a(t) = u\frac{dM}{dt} + \frac{dM}{dt}\underbrace{\lim_{dt\to 0}(dv)}_{=0}$$

$$\boxed{M\,dv = u\,dM + dM\,dv}$$

$$\boxed{a(t) = \frac{1}{M(t)}\,u\,\frac{dM}{dt}}$$

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Example

n : # of hits per unit time.

$F_{av} = ?$ $\quad |\vec{F}_{av}| = \left|\frac{\Delta \vec{p}}{\Delta t}\right| = 2mnu$

$\Delta p_{1ball} = 2mu$

$n\Delta t$ = # of collision in Δt

$\Delta P = 2mun\Delta t$: momentum change in Δt

December 3, 2015

Hand in your HW!

Now!

Kinematics : <u>angular</u> displacement

<u>angular</u> velocity

<u>angular</u> acceleration

" kinetic energy

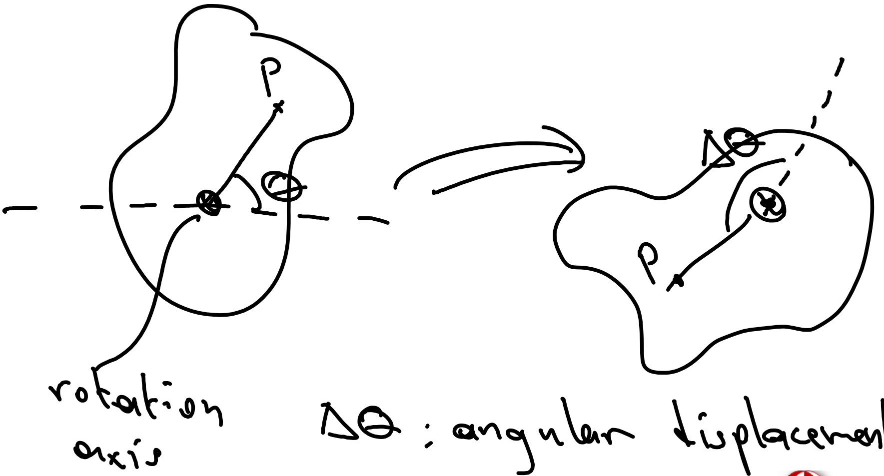
angular momentum

" dynamics

mass $\propto$ moment of inertia

rigid body: shape is constant

rotations around a fixed axis: orientation is (almost) constant



Δθ : angular displacement

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$$\omega_{av} = \frac{\Delta\theta}{\Delta t}$$: average angular velocity

$$\omega = \lim_{t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

$\Delta\omega$: change in angular velocity

$$\alpha_{av} = \frac{\Delta\omega}{\Delta t}$$: average angular accel.

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$$

1D case

Δx

$v_{av} = \frac{\Delta x}{\Delta t}$

$v = \frac{dx}{dt}$

$a_{av} = \frac{\Delta v}{\Delta t}$

angular

$\Delta\theta$

$\omega_{av} = \frac{\Delta\theta}{\Delta t}$

$\omega = \frac{d\theta}{dt}$

$\alpha_{av} = \frac{\Delta\omega}{\Delta t}$

1D case

$a(t)$

$$v(t) = v_0 + \int_{t_0}^{t} a(t')\,dt'$$

const acc.

$$v(t) = v_0 + a_0 (t - t_0)$$

$$x(t) = x_0 + \int_{t_0}^{t} v(t')\,dt'$$

$$x(t) = x_0 + v_0(t-t_0) + \frac{1}{2} a (t-t_0)^2$$

angular

$\alpha(t)$

$$\omega(t) = \omega_0 + \int_{t_0}^{t} \alpha(t')\,dt'$$

$$\omega(t) = \omega_0 + \alpha_0 (t - t_0)$$

$$\theta(t) = \theta_0 + \int_{t_0}^{t} \omega(t')\,dt'$$

$$\theta(t) = \theta_0 + \omega_0 (t - t_0) + \frac{1}{2} \alpha (t-t_0)^2$$

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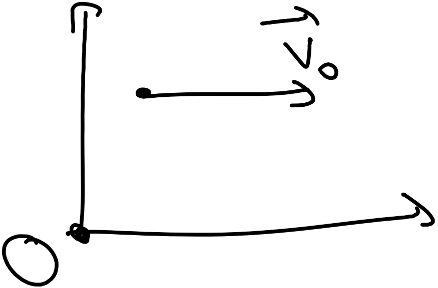
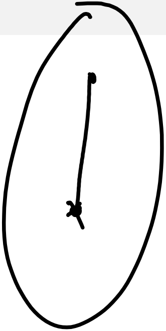
Example



$$\omega = \frac{\Delta\theta}{\Delta t} \qquad v = \frac{\Delta x}{\Delta t}$$

$$[\omega] = \frac{rad}{s} = \frac{1}{s} \qquad [v] = \frac{m}{s}$$

$$\Delta\theta = \frac{\Delta l}{R}$$



$t = t_0$

$$\frac{\Delta l}{\Delta t} = R\frac{\Delta\theta}{\Delta t}$$

$$\boxed{v_t = R\omega}$$

$$v = R\omega \longleftarrow \omega = \frac{v}{R}$$

$$[v] = [R][\omega]$$

$$m/s = m \, \frac{rad}{s} = \frac{m}{s} \quad [rad = 1]$$

$$a_t = \frac{dv_t}{dt} = R\frac{d\omega}{dt} = \boxed{R\alpha = a_t}$$

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Quiz

An object of mass 1 kg moves with a speed of 2 m/s in the +x direction. Write $\vec{p}$ in a vector form.

$\vec{p}$: linear momentum!

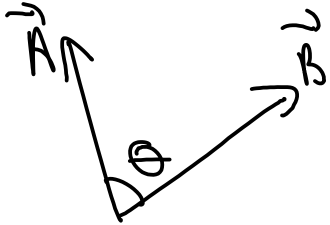
$\vec{\omega}$: magnitude & direction
direction is determined by
the right hand!

$$\vec{\alpha} \doteq \frac{d\vec{\omega}}{dt}$$

Vector Multiplication

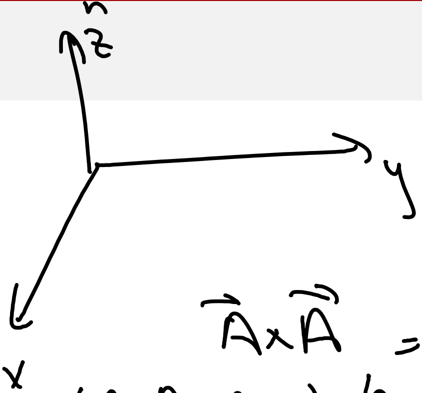
$$\vec{A} \times \vec{B} = \vec{C}$$

$$|\vec{C}| = |\vec{A}||\vec{B}| \sin\theta$$



direction is determined by the right hand rule.

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$



$$\hat{x}\times\hat{y} = \hat{z} = -\hat{y}\times\hat{x}$$

$$\hat{y}\times\hat{z} = \hat{x} = -\hat{z}\times\hat{y}$$

$$\hat{z}\times\hat{x} = \hat{y} = -\hat{x}\times\hat{z}$$

$$\vec{A}\times\vec{A} = \hat{x}\times\hat{x} = 0 = \hat{y}\times\hat{y} = \hat{z}\times\hat{z}$$

e.g. $(A_x\hat{x} + A_y\hat{y})\times(B_z\hat{z})$

$$= A_x B_z(\hat{x}\times\hat{z}) + A_y B_z(\hat{y}\times\hat{z})$$

$$= A_x B_z(-\hat{y}) + B_y A_z(\hat{x})$$

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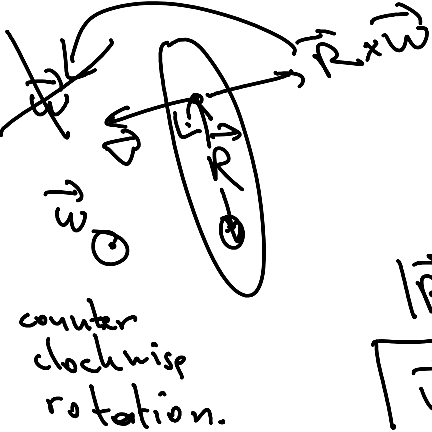
Ex



⊙ : vector pointing out of the screen

⊗ : vector pointing into the screen.

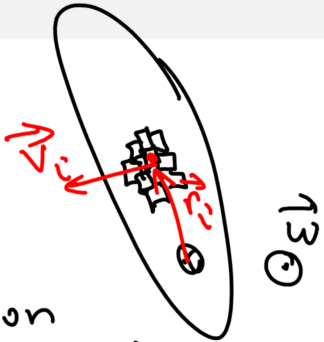
$v = Rw$

$|\vec{R}\times\vec{w}| = Rw = |\vec{v}|$

$$\boxed{\vec{w}\times\vec{R} = \vec{v}}$$

Kinetic Energy

$$KE = \sum \frac{1}{2} m_i v_i^2$$



$$v_i = \omega r_i$$

r_i : distance from rotation axis (not from the origin)

$$KE = \sum_i \frac{1}{2} m_i \omega^2 r_i^2 = \frac{1}{2}\left(\sum m_i r_i^2\right)\omega^2$$

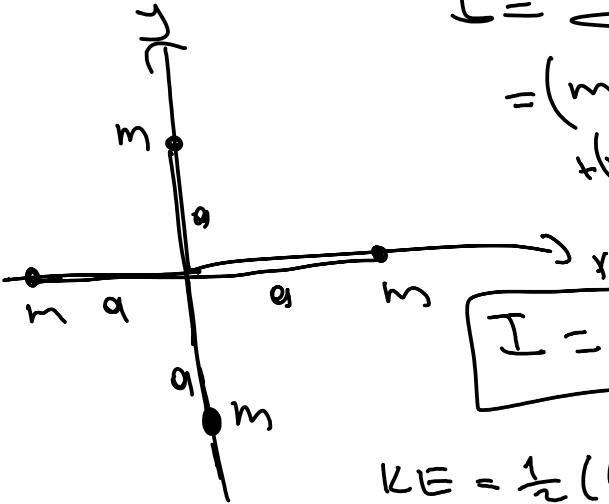
$$KE_{rot} = \frac{1}{2}\underbrace{\left(\sum m_i r_i^2\right)}_{I} w^2$$

compare $KE = \frac{1}{2}MV^2$

$$\boxed{I = \sum m_i r_i^2}$$

: moment of inertia of an object.

Ex



$$I = \sum m_i r_i^2$$

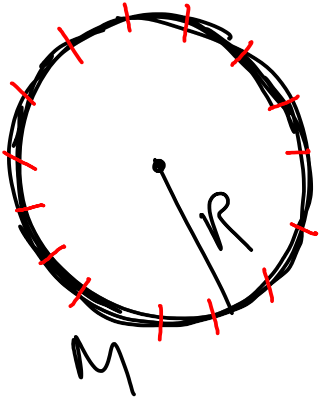
$$= (ma^2) + (ma^2) + (ma^2) + (ma^2)$$

$$I = 4ma^2$$

$$KE = \frac{1}{2}(4ma^2)\omega^2$$

Ex.

$$I = \sum m_i r_i^2$$



$$I = \sum (m_i R^2)$$

$$= \left(\sum m_i\right) R^2$$

$$\boxed{I = MR^2}$$

for rotation around an axis passing through the center & perpendicular

December 8, 2015

ω, α

$KE = \frac{1}{2} I \omega^2$

$I = \sum m_i r_i^2$

$\vec{\omega} \longleftrightarrow \vec{v}$

$\alpha \longleftrightarrow a$

$\Delta\theta \longleftrightarrow \Delta x$

cross-product

$|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}| \sin\theta$

direction is determined by the right hand rule

$\vec{v} = \vec{\omega} \times \vec{r}$

$\vec{a}_{tan} = \vec{\alpha} \times \vec{r}$

$$\boxed{a_{tan} = \alpha r}$$

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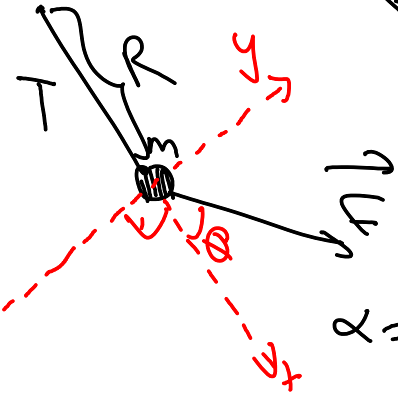


$$\vec{F} = F\cos\theta\,\hat{x} + F\sin\theta\,\hat{y}$$

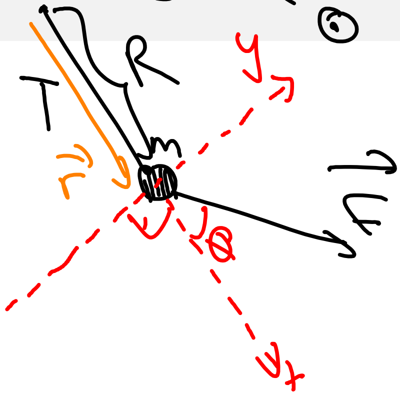
$$\vec{T} = T(-\hat{x})$$

$$\vec{F}_{tot} = -(T - F\cos\theta)\,\hat{x} + F\sin\theta\,\hat{y}$$

$$a_{tan} = \frac{F\sin\theta}{m}$$

$$\alpha = \frac{a_{tan}}{R} = \frac{F\sin\theta}{mR}$$

$\odot \ \vec{\alpha} \ (\vec{r} \times \vec{F}) \ \odot$



$$\boxed{\alpha = \frac{F \sin\theta}{mR}}$$

$$\alpha = \frac{|\vec{F} \times \vec{r}|}{mR^2}$$

$$|\vec{F} \times \vec{r}| = F R \sin\theta$$

$$F \sin\theta = \frac{|\vec{F} \times \vec{r}|}{R}$$

$$\boxed{(mR^2)\alpha = |\vec{F} \times \vec{r}|}$$

$$I = mR^2$$

$$I\alpha = |\vec{F} \times \vec{r}| = |\vec{r} \times \vec{F}|$$

$$|I\vec{\alpha}| = |\vec{r} \times \vec{F}| \quad \odot \quad \odot$$

$$I\vec{\alpha} = \vec{r} \times \vec{F} \equiv \vec{\tau}$$

$$\boxed{I\vec{\alpha} = \vec{\tau}}$$

$$m\vec{a} = \vec{F}$$

$$|\vec{\tau}| = RF\sin\theta$$

$$\boxed{I\vec{\alpha} = \vec{\tau}}$$

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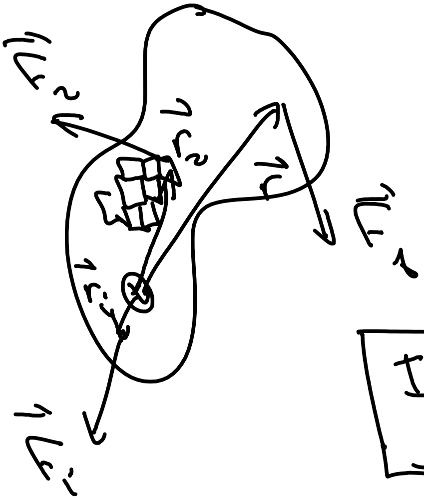


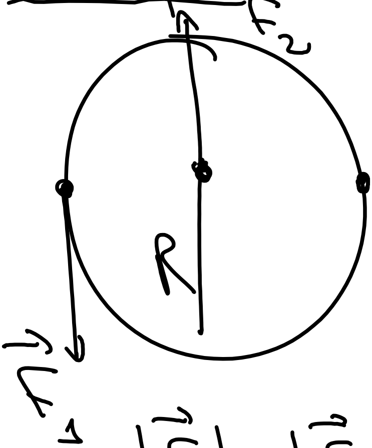
$$\vec\tau_{tot} = \sum \vec\tau_i = \sum \vec r_i \times \vec F_i$$

$$I = \sum m_j d_j^2$$

d_j: distance of mass m_j from rotation axis.

$$\boxed{I\vec\alpha = \vec\tau_{tot}}$$

Example → wheel of mass M



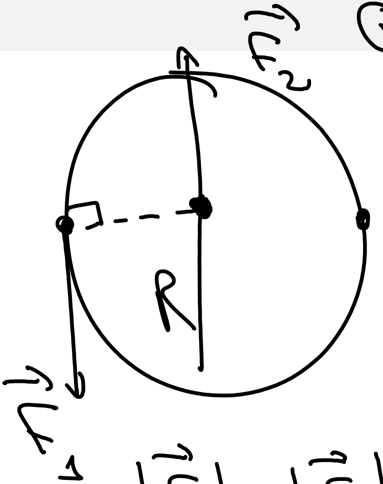
$$I = \sum m_i d_i^2 = \sum m_i R^2$$

$$\boxed{I = MR^2}$$

$$\vec F_{tot} = \vec F_1 + \vec F_2 = 0$$

$$\vec a_{cm} = \frac{\vec F_{tot}}{m} = 0$$

$$|\vec F_1| = |\vec F_2|$$



$$\vec{\tau}_2 = \vec{r}_2 \times \vec{F}_2 = 0$$

$$\vec{\tau}_1 = F_1 R \hat{z}$$

$$\boxed{\vec{\tau} = F_1 R \hat{z}}$$

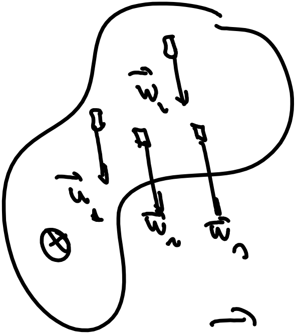
$$\vec{\alpha} = \frac{F_1 R}{I} \hat{z}$$

$$|\vec{F}_1| = |\vec{F}_2|$$

$$\boxed{\vec{\alpha} = \frac{F_1}{MR} \hat{z}}$$

- torque created by gravity
- work done by a torque

Torque Due to the Weight



$$\vec{\tau} = \sum \vec{r}_i \times \vec{F}_i$$

$$= \sum \vec{r}_i \times \vec{W}_i$$

$$= \sum \vec{r}_i \times (m_i \vec{g})$$

$$= \sum m_i (\vec{r}_i \times \vec{g})$$

$$= \left(\sum m_i \vec{r}_i\right) \times \vec{g}$$

$$= M R_{cm} \times \vec{g}$$

$$= \vec{R}_{cm} \times \vec{W}$$

$$\frac{1}{M} \sum m_i \vec{r}_i \equiv \vec{R}_{cm}$$

$$\vec{W} = M\vec{g}$$

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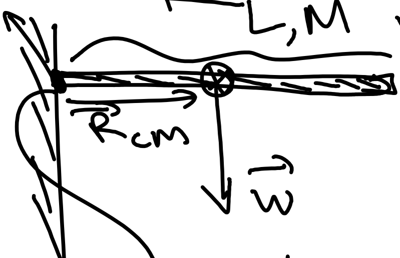
$\vec F$ Example

$\odot \hat z$ $\quad \rightarrow \hat x$

L, M uniporm.



free to rotate

I

$\vec\tau = \vec r \times \vec F$

$\vec\tau_{\vec F} = 0$

$\vec\tau_{\vec W} = \vec R_{cm} \times \vec W = \frac{L}{2} Mg(-\hat z)$

$$\vec\alpha = \frac{\vec\tau}{I} = \frac{MgL}{2I}(-\hat z)$$

$$\vec a^{cm}_{tan} = \vec\alpha \times \vec r^{\,cm} = \frac{MgL}{2I}\left(\frac{L}{2}\right)\hat x$$

$$\vec{a}^{CM}_{tan} = \frac{MgL^2}{4I}\hat{x}$$

$$I = \frac{ML^2}{3}$$

$$\vec{a}^{CM}_{tan} = \vec{a}^{CM} = \frac{MgL^2}{4} \cdot \frac{3}{ML^2}\hat{x}$$

$$\vec{a}^{CM} = \frac{3}{4}g\hat{x}$$

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$$\vec{F}_{tot} = \vec{F} + \vec{W} = \vec{F} + Mg(+\hat{x})$$

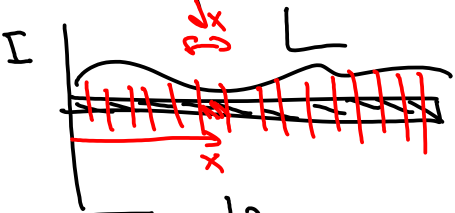
$$\vec{F}_{tot} = M\vec{a}^{cm} = Mg\frac{3}{4}\hat{x}$$

$$\Rightarrow \boxed{\vec{F} = -\frac{Mg}{4}\hat{x}}$$

$$\vec{W} = Mg\hat{x}$$

Example

$I = ?$, ρ : mass per unit length



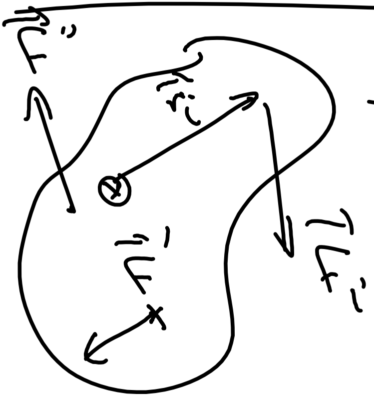
$$\rho = \frac{M}{L}$$

$$I = \sum m_i d_i^2$$

$$I = \sum_{L} \left(\frac{M}{L} dx\right) x^2$$

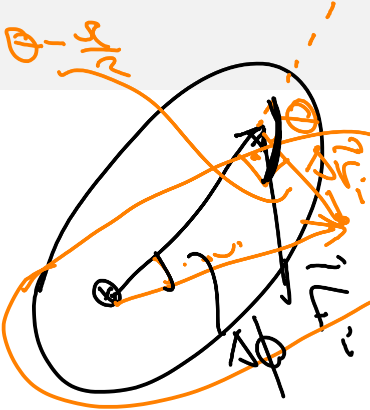
$$= \int_0^L \frac{M}{L} x^2 dx = \frac{M}{L} \frac{x^3}{3} \Big|_{x=0}^{L} = \frac{ML^2}{3}$$

Work Done on a Rigid Object



$$\Delta W = \sum \vec{F}_i \cdot \Delta\vec{r}_i$$

$\Delta\vec{r}_i$: displacement of the point that the force is exerted



$$\Delta W_i = \vec{F}_i \cdot \Delta\vec{r}_i$$
$$= |\vec{F}_i||\Delta\vec{r}_i| \cos\theta'$$
$$= |\vec{F}_i||\Delta\vec{r}_i| \cos\left(\theta - \frac{\pi}{2}\right)$$
$$= |\vec{F}_i||\Delta\vec{r}_i| \sin\theta$$

$$\boxed{\Delta W_i = |\vec{F}_i \times \Delta\vec{r}_i|}$$

$$|\Delta\vec{r}_i| = r_i \Delta\phi$$

$$\boxed{\Delta W_i = |\vec{F}_i| r_i \Delta\phi \sin\theta}$$

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$$P_i = \frac{\Delta W_i}{\Delta t} = |\vec{F}_i||\vec{r}_i| \sin\theta \frac{\Delta\phi}{\Delta t}$$

$$P_i = \tau_i \, \omega = \vec{\tau}_i \cdot \vec{\omega}$$

$$P = \sum P_i = \sum \vec{\tau}_i \cdot \vec{\omega}$$

$$\boxed{P = \vec{\tau}_{tot} \cdot \vec{\omega}}$$

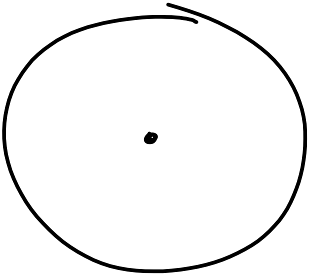
$$W = \vec{\tau}_{tot} \cdot \Delta\vec{\theta} \Leftrightarrow W = \vec{F} \cdot \Delta\vec{x}$$

$$W = \Delta\left(KE_{rot}\right)$$

Example

$\vec{\omega} = \omega(t)\hat{z}$

$\vec{\tau}_{tot} = \tau_0 \hat{z}$



$P = \vec{\omega} \cdot \vec{\tau}_{tot}$

$P = \omega(t)\tau_0$

W: ω : omega

$$\int_0^W \frac{dW}{dt} dt = \int_{\omega=0}^{\omega} \omega(t) I \frac{d\omega(t)}{dt} dt$$

$$W = \frac{1}{2} I \omega^2 \Big|_{\omega=0}^{\omega} = \frac{1}{2} I \omega^2 = W$$

$$\frac{dW}{dt} = \omega(t) I \frac{d\omega(t)}{dt} \Rightarrow dW = \omega I d\omega$$

$$W_{tot} = \int_0^{W_{tot}} dW = \int_{\omega_i}^{\omega_f} I \omega d\omega$$

$$W_{tot} = \frac{1}{2} I \omega^2 \Big|_{\omega=\omega_i}^{\omega_f} = \frac{1}{2} I \omega_f^2 - \frac{1}{2} I \omega_i^2$$

$$W_{tot} = \int^{W_{tot}} dW = \int \omega \tau_0 dt$$

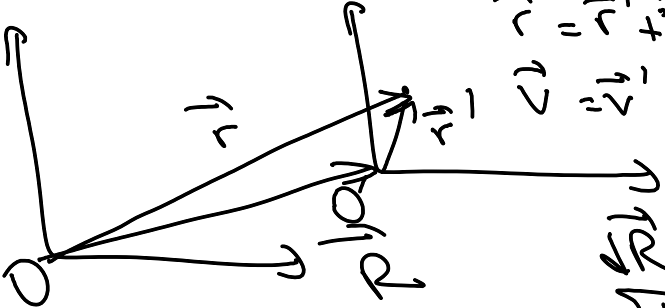
$$P = \frac{dW}{dt} = \omega \tau_0 \Rightarrow dW = \omega \tau_0 \, dt$$

$$W_{tot} = \int_{t_i}^{t_f} \omega(t) \tau_0 \, dt = \tau_0 \left[\theta(t_f) - \theta(t_i) \right]$$

Rotation and Translation

Axis of rotation is moving with constant velocity.

Change of reference frames



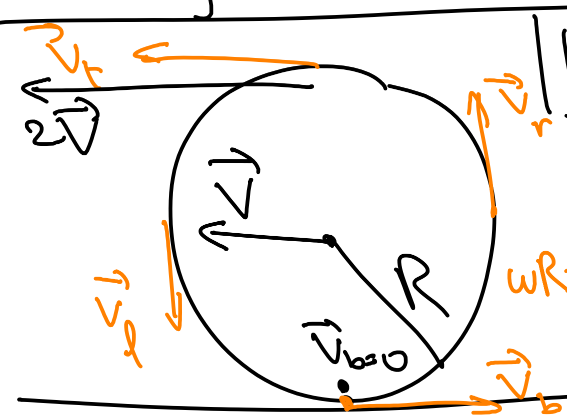
$$\vec{r} = \vec{r}\,' + \vec{R}$$

$$\vec{V} = \vec{v}\,' + \vec{V}$$

$$\frac{d\vec{R}}{dt} = \vec{V}$$

O' moves with the axis of rotation.

Rolling Without Slipping



$$|\vec{V}| = \omega R$$

$\odot \vec{\omega}$

$$\omega R = |\vec{v}_l| = |\vec{v}_r| = |\vec{v}_t| = |\vec{v}_b|$$

$$|\vec{V}| = |\vec{v}_b'| = \omega R$$

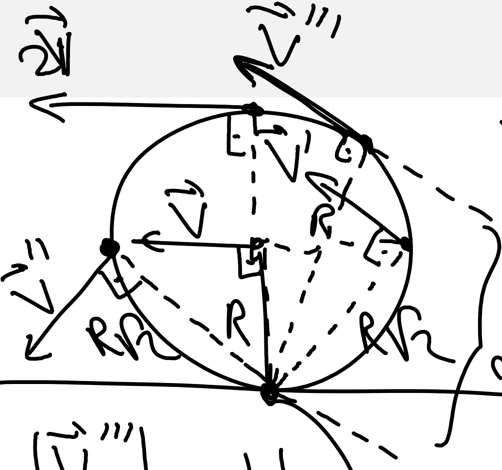
ground

- velocities relative to the reference frame of the axis of rotation

$$\vec{v}_b = \vec{v}_b + \vec{V} = 0 \Rightarrow \vec{v}_b = \vec{v}_b' = -\vec{V}$$

$$\boxed{\vec{V} = -\vec{v}_b'}$$

$$\boxed{\vec{v} = \vec{v}' + (-\vec{v}_b')} \Rightarrow \boxed{\vec{v} = \vec{v}' + \vec{V}}$$



$$|2\vec{V}| = \omega(2R)$$

$$|\vec{V}| = \omega R$$

$$|\vec{V}'| = \omega R\sqrt{2}$$

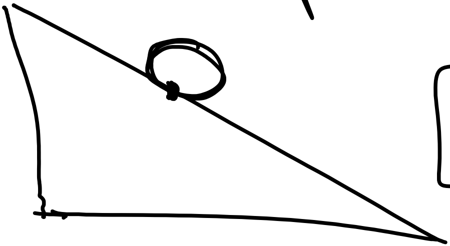
$$|\vec{V}''| = \omega R\sqrt{2}$$

$$|\vec{V}'''| = \omega d$$

instantaneous axis of rotation.

angular velocity is a property of rotating object, not the rotating axis!

Example Rolling (without slipping) down an inclined plane.

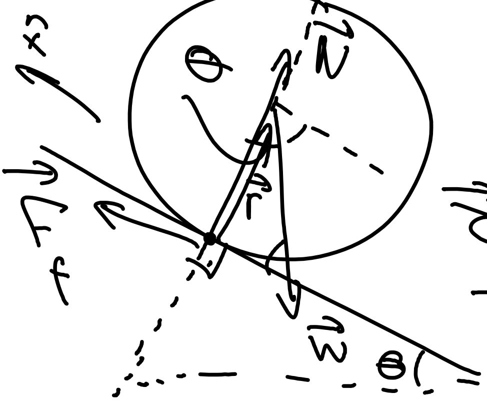


M, R, I

$$I = 2MR^2$$

$$I = I_{cm} + MR^2$$

$\odot \hat{z}$ $\vec{\tau}_f = \vec{r}_f \times \vec{F}$



$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\vec{\tau}_{tot} = \vec{\tau}_w = Rmg\sin(\pi-\theta)(-\hat{z})$$

$$\vec{\tau}_{tot} = mgR\sin\theta(-\hat{z})$$

$$\vec{\alpha} = \frac{\vec{\tau}_{tot}}{I} = \frac{mgR\sin\theta(-\hat{z})}{I}$$

$$\vec{\alpha} = \frac{mgR\sin\theta}{I}(-\hat{z}) = \frac{g}{2R}\sin\theta(-\hat{z})$$

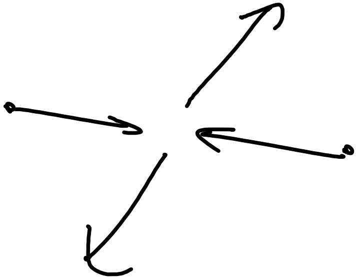
$$a_t^{cm} = \frac{g\sin\theta}{2}$$

December 10, 2015

HAND IN YOUR HOMEWORK!

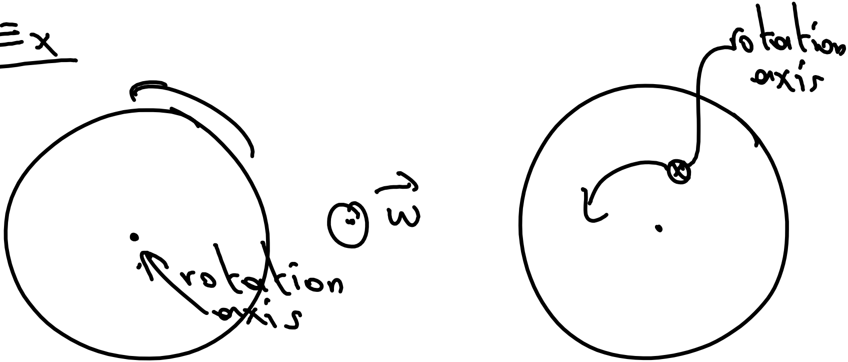
$$\Delta(KE) = W_{tot}$$

$$KE = \frac{1}{2} M V_{cm}^2 + \underbrace{\frac{1}{2}\frac{m_1 m_2}{m_1+m_2} v^2}_{E_{int}}$$



$\frac{1}{2} I \omega^2$: Kinetic energy of rotation

Ex



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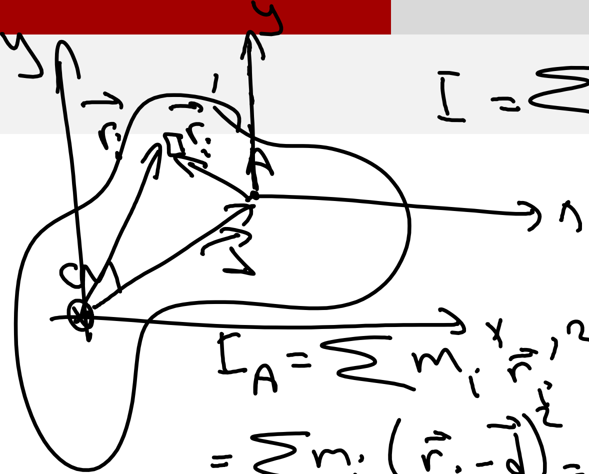
Parallel Axis Thm

I_{CM}: moment of inertia for an axis passing through the CM.

I_A: moment of inertia for axis passing through A parallel to the axis passing through the CM.

$$I_A = I_{CM} + Md^2$$



$$I = \sum m_i d_i^2$$

$$\vec{R}_{cm} = \frac{1}{M}\sum m_i \vec{r}_i$$

$$I_A = \sum m_i \vec{r}_i{}'^2$$

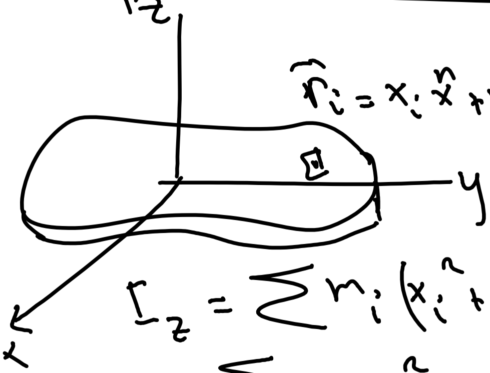
$$= \sum m_i (\vec{r}_i - \vec{d})^2 =$$

$$= \sum m_i \vec{r}_i^2 + \sum m_i d^2 - 2\sum m_i \vec{r}_i \cdot \vec{d}$$

$$I_A = I_{cm} + M d^2 - 2\left(\sum m_i \vec{r}_i\right)\cdot\vec{d}$$

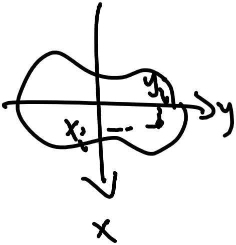
$$\sum m_i \vec{r}_i = 0$$

Perpendicular Axis Thm



$z_i \cong 0$

$\vec{r}_i = x_i \hat{x} + y_i \hat{y}$



$$I_z = \sum m_i (x_i^2 + y_i^2) =$$

$$= \sum m_i x_i^2 + \sum m_i y_i^2$$

$$I_z = I_y + I_x$$

$$I_y = \sum m_i (x_i^2 + z_i^2)$$

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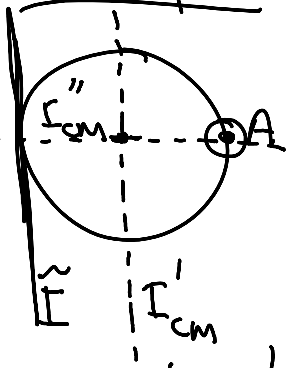
Example



$$I_{cm} = MR^2$$

$$I_A = MR^2 + MR^2 = 2MR^2$$

$$I'_{cm} = ?$$

$$I_{cm} = I'_{cm} + I''_{cm}$$

$$I''_{cm} \overset{?}{=} I'_{cm} \Rightarrow I'_{cm} = I''_{cm} = \frac{I_{cm}}{2} = \frac{MR^2}{2}$$

$$\tilde{I} = I'_{cm} + MR^2 = \frac{3}{2}MR^2 = \tilde{I}$$

KE of an object translating & Rotating

$\vec{r}_i = \vec{R} + \vec{d}_i$

$KE = \sum \frac{1}{2} m_i v_i^2$

$\vec{R}$: position vector of the rotation axis.

$\vec{r}_i$: position of the mass m_i relative to the inertial reference frame.

$\vec{d}_i$: position of the mass m_i relative to the rotation axis.

$$\vec{r}_i = \vec{R} + \vec{d}_i \Rightarrow \vec{v}_i = \vec{V} + \vec{u}_i$$

$\vec{u}_i$: velocity relative to the axis.

$$KE = \sum \frac{1}{2} m_i v_i^2 = \sum \frac{1}{2} m_i V^2 + \sum \frac{1}{2} m_i u_i^2$$

$$u_i = \omega d_i \qquad + \sum \frac{1}{2} m_i 2\vec{V} \cdot \vec{u}_i$$

$$KE = \frac{1}{2} M V^2 + \frac{1}{2} \omega^2 \underbrace{\left(\sum m_i d_i^2 \right)}_{I}$$

$$+ \vec{V} \cdot \left(\sum m_i \vec{u}_i \right)$$

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$$KE = \frac{1}{2}MV^2 + \frac{1}{2}I\omega^2 + M\vec{V}\cdot\vec{u}_{cm}$$

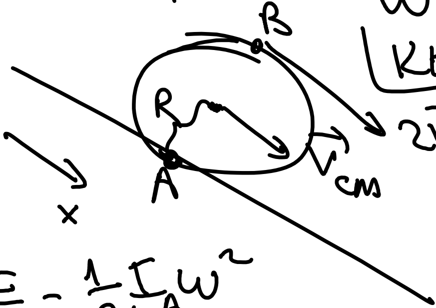
$\vec{u}_{cm}$: velocity of the CM relative to the rotation axis.

Choose rotation axis to pass through the CM! $\vec{u}_{cm} = 0$

$$KE = \frac{1}{2}MV_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$$

Example

rolling without slipping



$$\omega$$

$$KE = \frac{1}{2} M V_{cm}^2 + \frac{1}{2} I_{cm} \omega^2$$

$$KE = \frac{1}{2} I_A \omega^2$$

$$V_{cm} = R\omega$$

$$= \frac{1}{2} M (\omega R)^2 + \frac{1}{2} I_{cm} \omega^2$$

$$\frac{1}{2} I_A \omega^2 = \frac{1}{2} (I_{cm} + MR^2) \omega^2$$

$$\boxed{I_A = I_{cm} + MR^2}$$

$$(KE)_B = \frac{1}{2} M V_B^2 + \frac{1}{2} I_B \omega^2 + M \vec{V}_B \cdot \vec{u}_{cm}$$

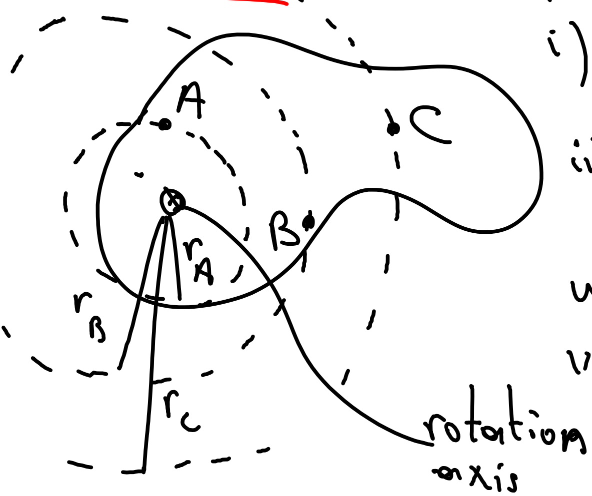
$$I_B = I_{cm} + MR^2 = 2MR^2 = I_A$$

$$\vec{V}_B = 2\vec{V}_{cm} = (2\omega R)\hat{x}$$

$$\vec{u}_{cm} = -\vec{V}_{cm} = (-\omega R)\hat{x}$$

$$(KE)_B = \frac{1}{2} M (4(\omega R)^2) + \frac{1}{2} 2M (\omega R)^2 + M(-2)(\omega R)^2 = \frac{1}{2} \underbrace{(2MR^2)}_{I_A} \omega^2$$

Quiz Order from large to small:



i) $\omega_A, \omega_B, \omega_C$

ii) v_A, v_B, v_C

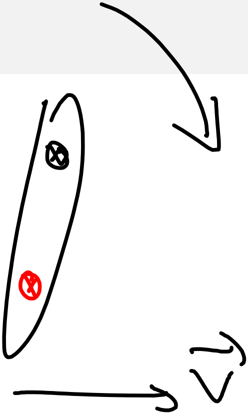
$$\omega_A = \omega_B = \omega_C$$

$$v = \omega d$$

$$v_C > v_B > v_A$$

December 15, 2015

- parallel axis thm
- yo-yo



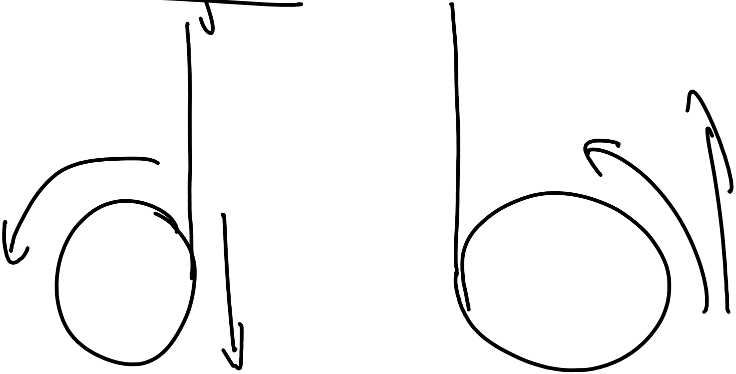
$$I = \sum m_i d_i^2$$

$$I_A = I_{cm} + M d^2$$

M: total mass of object

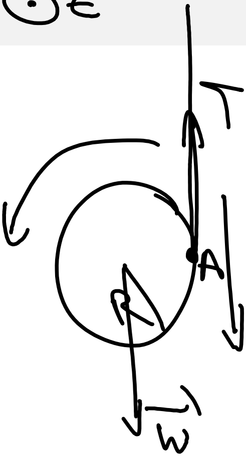
d: distance of A to the axis passing through the CM

Yo-yo



$\odot \hat{z}$

$I_{cm} + MR^2 = I_A$



$\vec{\tau}_A = 0 + RMg\hat{z}$

$I_A \alpha = MgR$

$$\boxed{\alpha = \frac{MgR}{I_A}}$$

$$a^{cm}_{tan} = \alpha d = \frac{Mg}{I_A} R^2 = \frac{MR^2}{I_{cm} + MR^2} g$$

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$$a^{cm}_{tan} = \frac{MR^2}{I_A} g = \frac{MR^2}{I_{cm} + MR^2} g$$

$$a^{cm}_{tan} = \frac{1}{1 + I_{cm}/MR^2} g < g$$

$$\vec{F}_{net} = M\vec{a}_{cm}$$

$$F_{tot} = \frac{Mg}{1 + I_{cm}/MR^2} = Mg - T$$

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$$F_{tot} = \frac{Mg}{1 + I_{cm}/MR^2} = Mg - T$$

$$T = Mg \frac{I_{cm}/MR^2}{1 + I_{cm}/MR^2} = Mg \frac{I_{cm}}{I_{cm} + MR^2}$$

$$\boxed{T = Mg \frac{I_{cm}}{I_{cm} + MR^2}} \qquad \alpha = \frac{MgR}{I_A}$$

$$\frac{TR}{I_{cm}} = \frac{MR}{I_{cm} + MR^2} g = \alpha$$

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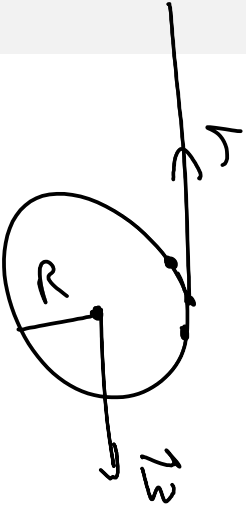
$$TR = I_{cm}\alpha$$

$$TR = \tau_{cm} = I_{cm}\alpha$$



$\tau = I\alpha$ is valid if

i) Rotation axis is fixed (or moving at constant velocity)

ii) Rotation axis goes through the CM.

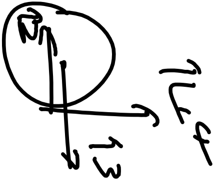
Rolling with Slipping

$\omega_i \neq 0$

$v_i = 0$

$\vec{\tau} = \vec{r} \times \vec{F}$

$|\vec{\tau}| = rF\sin\theta$



kinetic friction

$$\vec{F}_{tot} = \mu_k N \hat{x} + N\hat{y} + mg(-\hat{y}) = ma\hat{x}$$

$$N - mg = 0 \Rightarrow N = mg$$

$$ma = \mu_k N = \mu_k mg$$

$$\boxed{a = \mu_k g} \qquad \vec{v} = \vec{a}t = \mu_k g t\,\hat{x}$$

$$\vec{\alpha} = \frac{\vec{\tau}_{cm}}{I_{cm}} = \frac{R\,\mu_k m g\,\hat{z}}{I_{cm}}$$

$$\vec{\omega}(t) = \vec{\omega}_0 + \vec{\alpha}t = \omega_i(-\hat{z}) + t\frac{mg\mu_k R}{I_{cm}}\hat{z}$$

$$\vec{\omega}(t) = \left(\frac{mg\mu_k R}{I_{cm}}t - \omega_i\right)\hat{z}$$

$$|\vec{\omega}| = \left(\omega_i - \frac{mg\mu_k R}{I_{cm}}t\right)$$

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$$v_{cm} = \omega R$$

condition for rolling without slipping.

$$\mu_u g t_0 = \left(\omega_i - \frac{mg\mu_u R}{I_{cm}} t_0\right) R$$

t_0: time at which the ball starts rolling without slipping.

$$v_{cm} = \mu_k g t$$

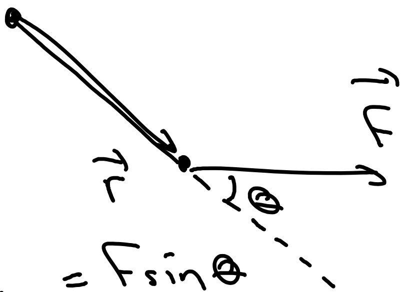
$$w = \left(w_i - \frac{mg\mu_k R}{I_{cm}} t\right)$$

$$I_{cm} w(t) + MRv_{cm}(t) = I_{cm}\left(w_i - \frac{mg\mu_k R}{I_{cm}} t\right) + MR\mu_k g t$$

$$I_{cm} w(t) + MRV_{cm}(t) = I_{cm} w(t=0) + MRV_{cm}(t=0)$$

$$I_{cm} w + RMV_{cm} = \text{const} \Leftarrow \text{conservation law}$$

Angular Momentum



$$a_{tan} = \frac{F\sin\theta}{m}$$

$$a_r = \ldots$$

$$\alpha = \frac{a_{tan}}{R}$$

$$\alpha = \frac{F\sin\theta}{mR}$$

$$mR^2\alpha = RF\sin\theta$$

$$I\vec{\alpha} = \vec{R}\times\vec{F}$$

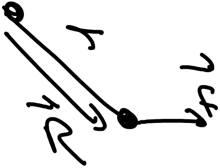
$$\vec{R} \times \vec{F} = \vec{R} \times (\vec{F} + \vec{T})$$

$$= \vec{R} \times \vec{F}_{tot}$$

$$= \vec{R} \times \frac{d\vec{p}}{dt}$$

$$= \frac{d}{dt}(\vec{R} \times \vec{p}) - \underbrace{\frac{d\vec{R}}{dt} \times \vec{p}}_{= 0}$$

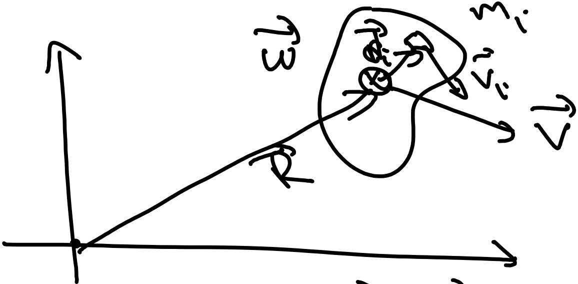
$$\frac{d}{dt}(\vec{R} \times \vec{p}) = \frac{d\vec{R}}{dt} \times \vec{p} + \vec{R} \times \frac{d\vec{p}}{dt}$$



$$\vec{\tau} = \vec{R} \times \vec{F} = \frac{d}{dt}(\vec{R} \times \vec{p}) \Leftrightarrow \vec{F} = \frac{d\vec{p}}{dt}$$

$\vec{L} = \vec{R} \times \vec{p}$: angular momentum of a point object.

Angular momentum of a rigid body



$$\vec{L} = \vec{r} \times \vec{p}$$

$$\vec{v}_i = \vec{V} + \vec{\omega} \times \vec{d}_i$$

$$\vec{r}_i = \vec{R} + \vec{d}_i$$

$$\vec{L} = \sum m_i \vec{r}_i \times \vec{v}_i$$

$$\vec{L} = \sum m_i (\vec{R} + \vec{d}_i) \times (\vec{V} + \vec{\omega} \times \vec{d}_i)$$

$$\vec{L} = \sum m_i (\vec{R} + \vec{d}_i) \times (\vec{V} + \vec{\omega} \times \vec{d}_i)$$

$$= \sum m_i \vec{R} \times \vec{V} + \sum m_i \vec{R} \times (\vec{\omega} \times \vec{d}_i)$$

$$+ \sum m_i \vec{d}_i \times \vec{V} + \sum m_i \vec{d}_i \times (\vec{\omega} \times \vec{d}_i)$$

$$= \vec{R} \times (M\vec{V}) + \vec{R} \times \left(\vec{\omega} \times \left(\sum m_i \vec{d}_i\right)\right)$$

$$+ \left(\sum m_i \vec{d}_i\right) \times \vec{V} + \sum m_i \vec{d}_i \times (\vec{\omega} \times \vec{d}_i)$$

Choose an axis that goes through the CM:

$$\sum m_i \vec{d}_i = 0 \; ; \; \vec{V} = \vec{V}_{cm}$$

$$\vec{L} = \vec{R}_{cm} \times \vec{P}_{cm} + \sum m_i \vec{d}_i \times (\vec{\omega} \times \vec{d}_i)$$

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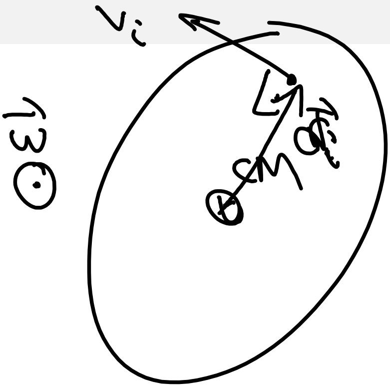
$\vec{V}_{cm} = 0$



$$\vec{L} = \sum m_i \vec{d}_i \times (\vec{\omega} \times \vec{d}_i)$$

$$= \sum m_i \vec{d}_i \times \vec{v}_i$$

$$= \sum m_i v_i d_i \hat{\omega}$$

$$= \sum m_i (\omega d_i) d_i \hat{\omega}$$

$$= \left(\sum m_i d_i^2\right)(\omega \hat{\omega})$$

$$\boxed{\vec{L} = I \vec{\omega}}$$

true even if the rotation axis does not go through the CM as long as it is fixed!

$$\vec{L} = I\vec{\omega} \quad \text{(if axis fixed)}$$

$$= \vec{R}_{cm} \times \vec{P}_{cm} + I_{cm}\vec{\omega}$$

(if axis goes through the CM)

BUT!

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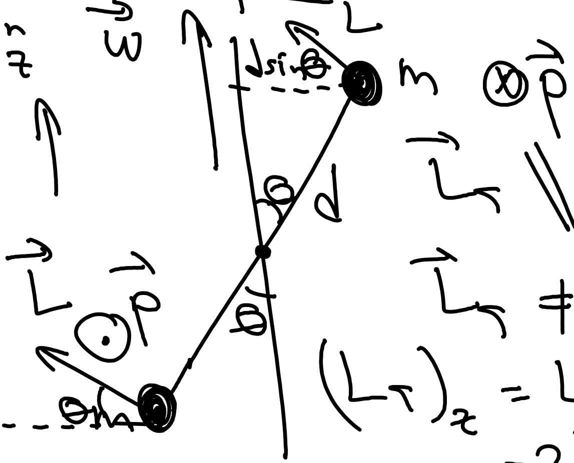
Example

$$\vec{L} = \vec{r} \times \vec{p}$$



$$\vec{L}_T \parallel \vec{\omega} \;!!$$

$$\vec{L}_T \neq I\vec{\omega}$$

$$(L_T)_z = L_T \sin\theta$$
$$= 2L\sin\theta$$

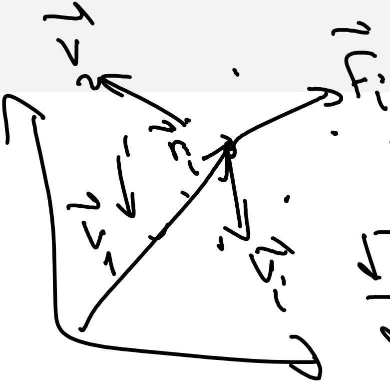
$$(L_T)_z = 2 d m \omega d \sin\theta \sin\theta = 2m(d\sin\theta)^2\omega$$

$$L_z = I\omega$$

always!

z : direction of $\vec{\omega}$

$$\vec{\tau} = \frac{d\vec{L}}{dt}$$

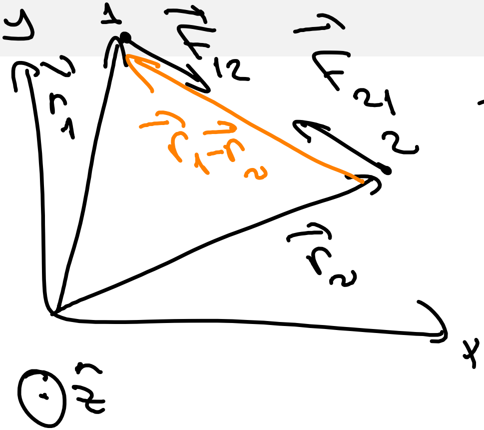


$$\vec{F}_i = \vec{F}_i^{ext} + \sum_i \vec{F}_{ij}^{int}$$

$$\vec{L} = \sum_i \vec{r}_i \times \vec{p}_i$$

$$\frac{d\vec{L}}{dt} = \sum_i \vec{r}_i \times \vec{F}_i^{ext} + \sum_{ij} \vec{r}_i \times \vec{F}_{ij}^{int}$$

$$\frac{d\vec{L}}{dt} = \vec{\tau}^{ext}$$



$$\vec{F}_{12} = -\vec{F}_{21}$$

$$\vec{\tau}_1 + \vec{\tau}_2$$

$$= \vec{r}_1 \times \vec{F}_{12} + \vec{r}_2 \times \vec{F}_{21}$$

$$= \vec{r}_1 \times \vec{F}_{12} - \vec{r}_2 \times \vec{F}_{12}$$

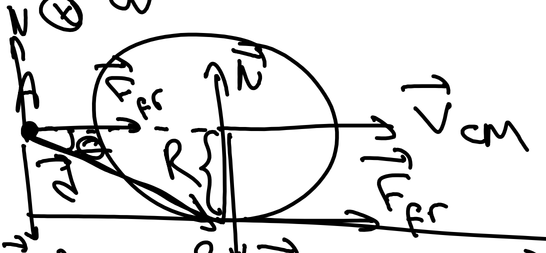
$$= (\vec{r}_1 - \vec{r}_2) \times \vec{F}_{12}$$

$$= 0$$

If $\vec{\tau}^{ext} = 0$, $\vec{L}$ is conserved!

Example

$\odot \; \hat{z}$



$$\frac{d\vec{L}}{dt} = \vec{\tau}$$

$$\vec{L} = \vec{r} \times \vec{p}$$

$$\vec{\tau} = \vec{r} \times \vec{F}$$

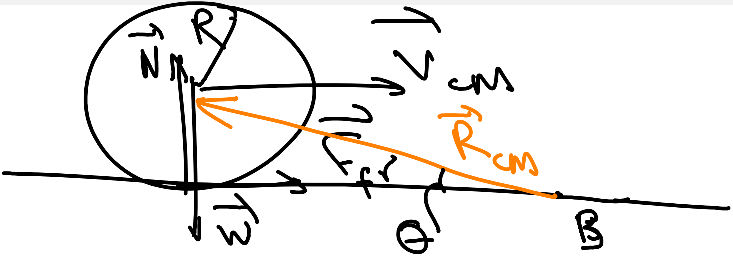
$$L_z = ?$$

$$\vec{L} = \vec{R}_{cm} \times \vec{P}_{cm} + I_{cm}\vec{\omega}$$

$$L_A^z = 0 + I_{cm} \omega (-1)$$

$$\vec{\tau}_A = d N \sin\left(\frac{\pi}{2} + \theta\right) \hat{z} + \left(d \cos\theta\right) mg(-\hat{z}) + d F_{fr} \sin\theta \, \hat{z} \neq 0$$

$\otimes \vec{w}$ $\odot \hat{z}$



$$\vec{\tau}_B = \vec{\tau}_B^{F_{fr}} = 0$$

$$\vec{L}^B = \vec{r}_{cm} \times \vec{P}_{cm} + I_{cm}\vec{w}$$

$$L_z^B = R_{cm} M V_{cm} \sin(\pi - \theta)(-\hat{z}) + I_{cm}(-w)\hat{z}$$

$$L_z^B = R_{cm} M V_{cm} \sin(\pi - \theta)(-1) + I_{cm}(-\omega)$$

$$L_z^B = -MV_{cm}(R_{cm}\sin\theta) - I_{cm}\omega$$

$$\boxed{L_z^B = -\left[MV_{cm}R + I_{cm}\omega\right]}$$ is conserved!

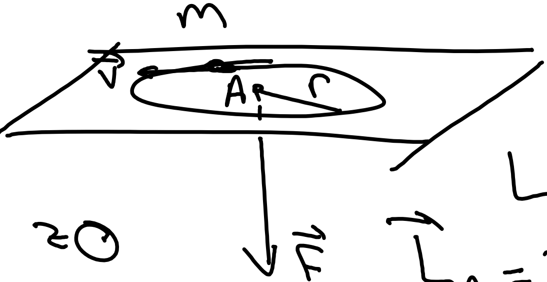
Example $\omega_i = \omega_0$; $V_{cm}^i = 0$

rolling without slipping $\omega_f = \omega_1$, $V_f = \omega_1 R$

$$I_{cm}\omega_0 = I_{cm}\omega_1 + MR\omega_1 = (I_{cm} + MR^2)\omega_1$$

$L = I\omega = mvr$

Example



$F = \frac{mv^2}{r} = \frac{m(vr)^2}{r^3}$

$\vec{\tau}_A = 0$

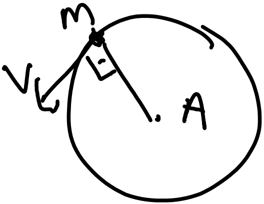
$L_A =$ conserved.

$\vec{L}_A = \vec{r} \times \vec{p}$

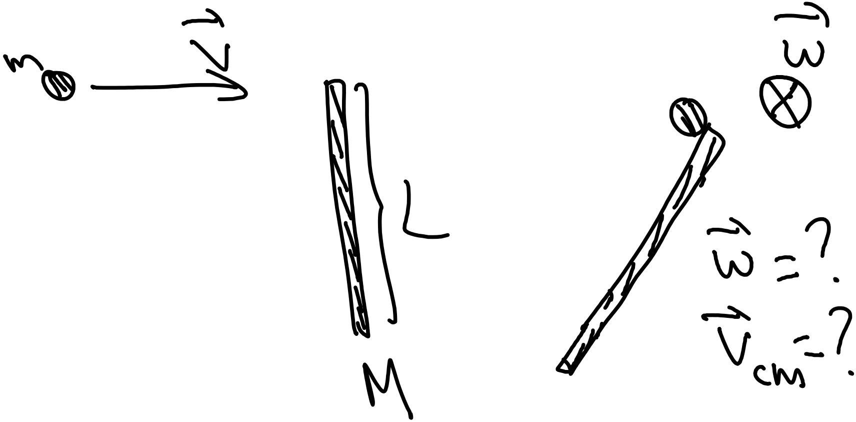
$= rmv\hat{z}$

$\Rightarrow$ $vr = \text{const}$

$v = \frac{v_0 r_0}{r}$



Example (for Thursday)



December 17, 2015

Hand in Your Homework! Now!

$$\vec{\tau} = \frac{d\vec{L}}{dt}$$ always

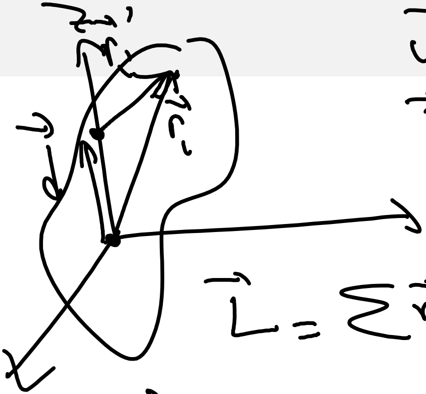
$$\vec{L} = I\vec{\omega}$$ almost always

$$\vec{v} = \vec{\omega} \times \vec{R} + \vec{V}_0$$

$\vec{V}_0$: velocity of the axis

$\vec{R}$: distance to the axis

$\vec{v}$: velocity of a point on the rigid body



$$\vec{\omega} = \omega \hat{z}$$

$\vec{L}$: $\vec{L}$ relative to O

$$\vec{r}_i{}' + \vec{d} = \vec{r}_i$$

$$\vec{L} = \sum \vec{r}_i \times \vec{p}_i$$

$\vec{L}_A$: angular momentum relative to A

$$\vec{L} = \sum (\vec{d} + \vec{r}_i{}') \times \vec{p}_i = \vec{d} \times \left(\sum \vec{p}_i\right) + \underbrace{\sum \vec{r}_i{}' \times \vec{p}_i}_{\vec{L}_A}$$

$$\vec{L} = \vec{L}_A + \vec{d} \times \left(\sum \vec{p}_i\right)$$

$$\boxed{\vec{L} = \vec{L}_A + \vec{d} \times \vec{P}_{cm}}$$

$\vec{d}$: vector connecting the two reference points.

is it possible that $\vec{L}_A$ is parallel to $\vec{w}$

if possible $\vec{w} \times \vec{L}_A = 0$

$$\vec{w} \times \vec{L} = \vec{w} \times (\vec{d} \times \vec{P}_{cm})$$

$$\vec{\omega} \times \vec{L} = \vec{\omega} \times (\vec{d} \times \vec{P}_{cm})$$

↑ if this eqn has a soln, then

∃ a point A st $\boxed{\vec{L}_A = I_A \vec{\omega}}$

$$\vec{L} \equiv \vec{r} \times \vec{p} \qquad \vec{v} = \vec{\omega} \times \vec{r}$$

$$\vec{\tau} \equiv \vec{r} \times \vec{F}$$

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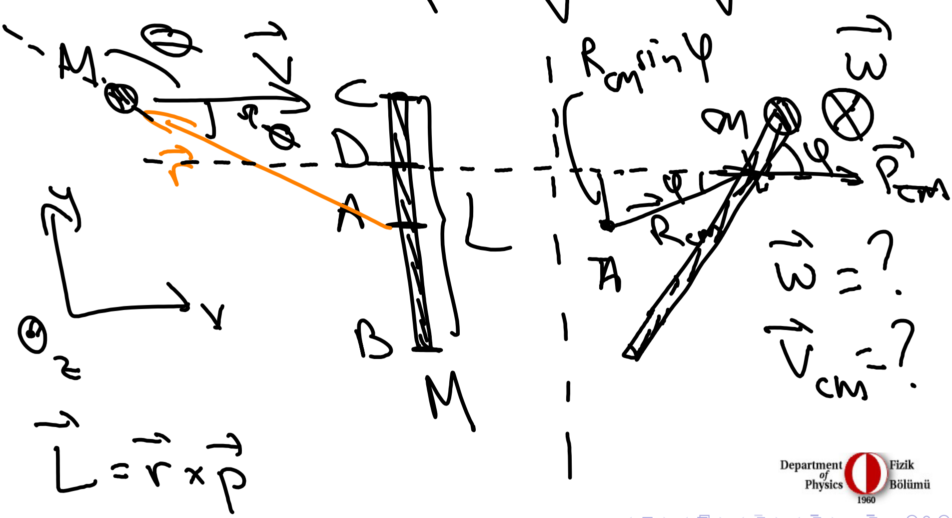
Example


E ✗
$\vec{P}$ ✓
$\vec{L}$ ✓
M
$\vec{v}$
$\vec{r}$
C
D
A
B
L
M
$R_{cm}\sin\varphi$
m
$\vec{\omega}$
φ
$\vec{P}_{cm}$
R_{cm}
A
$\vec{\omega} = ?$
$\vec{V}_{cm} = ?$
y
x
z
$\vec{L} = \vec{r} \times \vec{p}$

$$\vec{P} = (2M)\vec{V}_{cm} = M\vec{V} + 0 \Rightarrow \boxed{\vec{V}_{cm} = \frac{\vec{V}}{2}}$$

$$\vec{L}^{i}_{A} = (-\hat{z})\, r m v \sin\theta$$

$$= (-\hat{z})\, m v r \sin(\pi - \theta)$$

$$\vec{L}^{i}_{A} = (-\hat{z})\, m v \frac{L}{2}$$

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$\vec{P}_{cm} = m\vec{v} = (2m)\vec{V}_{cm}$

$\vec{\omega} = \omega\hat{z}$

$\vec{L}^f_A = \vec{R}_{cm} \times \vec{P}_{cm} + \vec{L}^{rot}_{cm}$

$= (-\hat{z}) R_{cm} (mv) \sin\varphi + I_{cm}\vec{\omega}$

$\vec{L}^f_A = (-\hat{z}) mv \frac{L}{4} + I_{cm}\omega_f \hat{z}$

$-\hat{z}\, mv \frac{L}{2} = (-\hat{z}) mv \frac{L}{4} + I_{cm}\omega_f \hat{z}$

$$\boxed{\omega_f = -\frac{1}{4}\frac{mvL}{I_{cm}}} = -\frac{1}{4} mvL \frac{12}{10}\frac{1}{mL^2}$$

$= \boxed{-\frac{6}{5}\frac{v}{L} = \omega_f}$

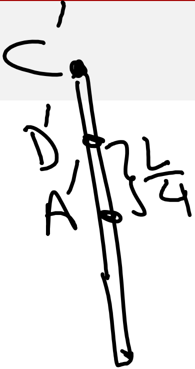
$$I^{rod}_{tip} = \frac{1}{3} M L^2$$

$$I_{cm} = I^{rod} + I^{point\ mass}$$

$$I_{cm} = I^{rod} + M\left(\frac{L}{4}\right)^2$$

$$I^{rod} \overset{?}{\neq} \frac{1}{3} M L^2 + M\left(\frac{L}{4}\right)^2$$

$$I_{cm} = \frac{7}{48} M L^2 + \frac{1}{16} M L^2 = \frac{10}{48} M L^2$$



$$I^{C}_{rod} = \frac{1}{3}ML^2$$

$$I^{D}_{rod} = I_A + M\left(\frac{L}{4}\right)^2$$

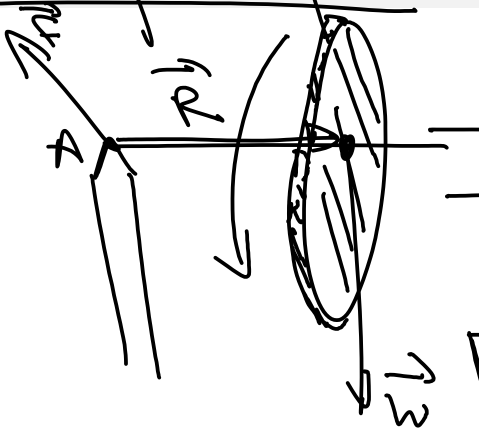
$$I^{C}_{rod} = I_A + M\left(\frac{L}{2}\right)^2$$

$$\frac{1}{12}ML^2 = \frac{1}{3}ML^2 - \frac{1}{4}ML^2 = I_A$$

$$I^{D}_{rod} = \frac{1}{12}ML^2 + M\frac{L^2}{16} = \frac{7}{48}ML^2$$

Gyroscope

$z \odot$



$\vec{L} = I\vec{\omega}$

$\vec{\tau}_A = \vec{R} \times \vec{w}$

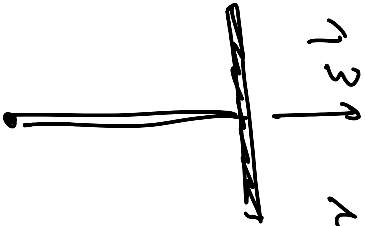
$$\boxed{\vec{\tau}_A = (-\hat{z})\, R m g}$$

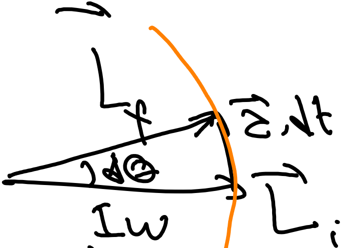
$$\vec{\tau}_A = \frac{d\vec{L}}{dt}$$

$$\vec{L} = \vec{L}_0 + \vec{\tau}\, dt$$

ω_p: precession angular velocity!

top view





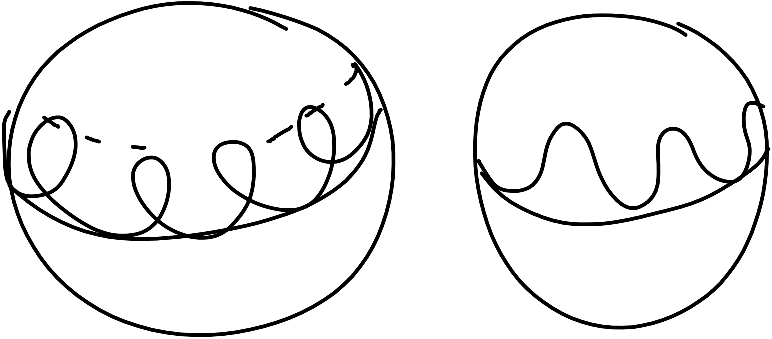
$$\tau\, dt = mgR\, dt = L\, d\theta = I\omega\, d\theta$$

$$L_i = I\omega$$

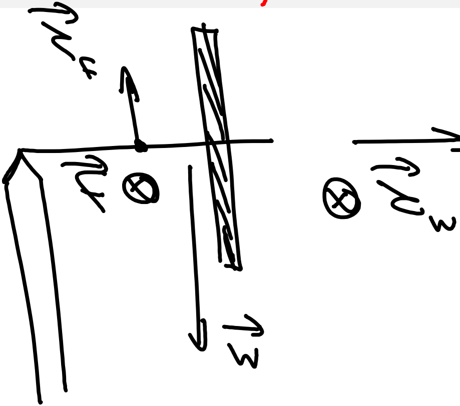
$$L_f = I\omega$$

$$\frac{d\theta}{dt} = \omega_p = \frac{mgR}{I\omega}$$

nutation



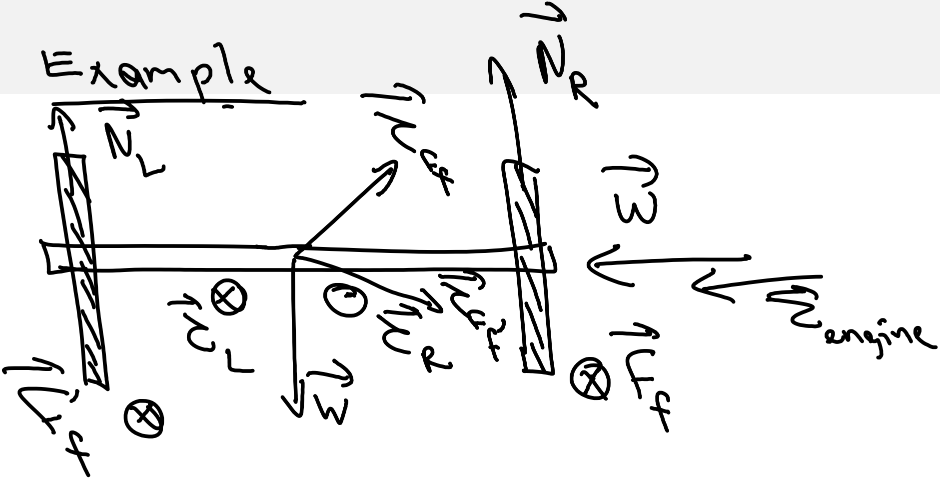
December 22, 2015



$$\Delta \vec{L} = \vec{\tau} \Delta t$$

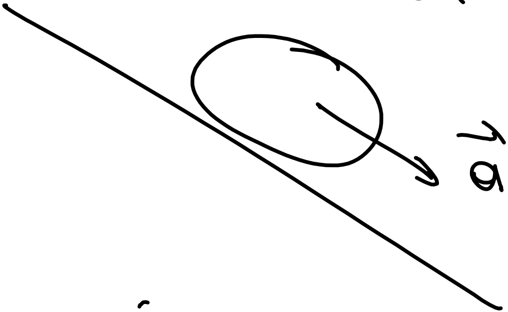
$$\vec{\tau} = \vec{r} \times \vec{F}$$

Example



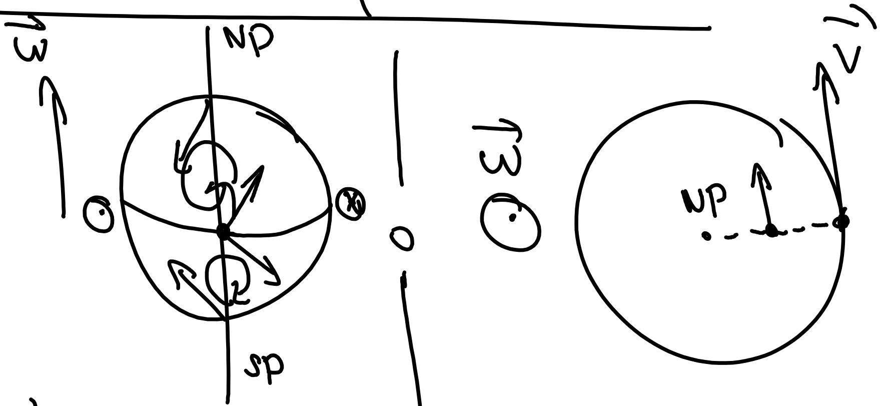
$\vec{N}_L$
$\vec{N}_R$
$\vec{\tau}_{f_f}$
$\vec{W}$
$\vec{\tau}_L$
$\vec{\tau}_R$
$\vec{\tau}_{f_f}$
$\vec{\tau}_{engine}$
$\vec{F}_f$
$\vec{W}$
$\vec{F}_f$

Exercise ring rolling down an incline

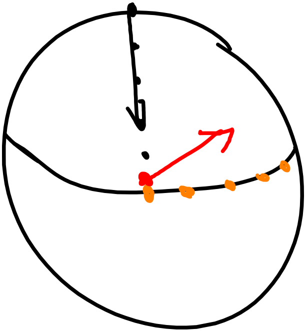


$$a = \frac{g\sin\theta}{2}$$

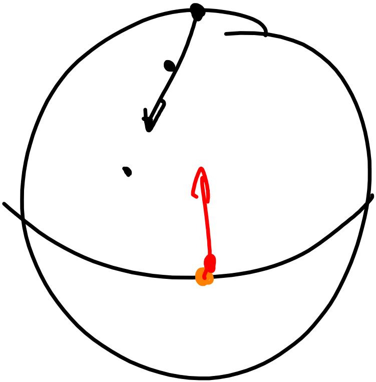
Rotation of Hurricanes



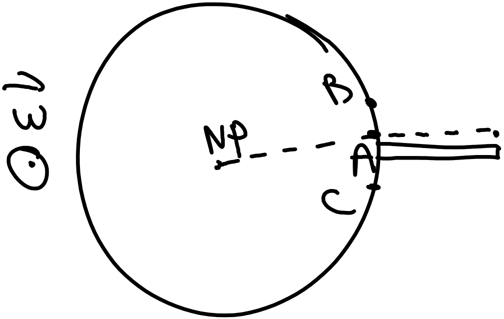
observer is rotating with earth!



inertial observer.



observer on the orange dot



inertial observer

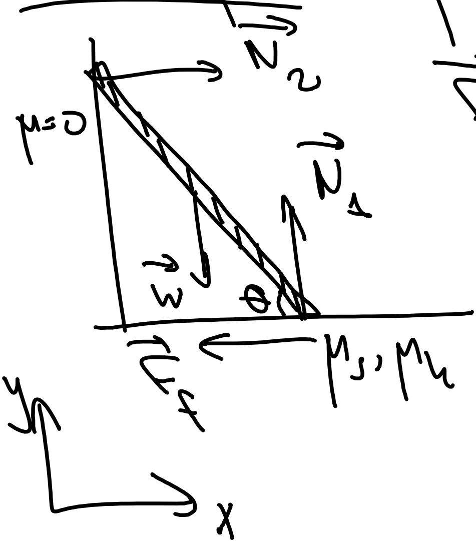
- Statics
- Elasticity
- Oscillations (Ch 15)
- Gravity
- Fluids

Statics

$\vec{a}_{cm} = 0 \Rightarrow \vec{F}_{net} = 0$

$\vec{\alpha}_{cm} = 0 \Rightarrow \vec{\tau}_{net} = 0$

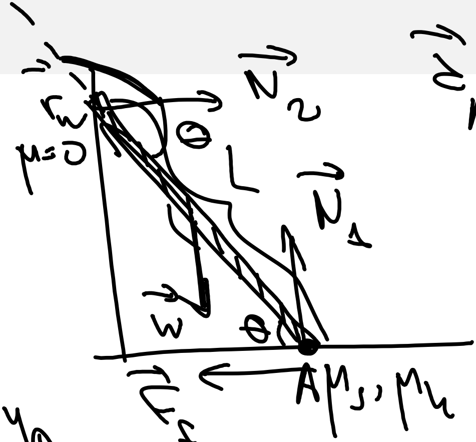
Example



$$|\vec{F}_f| \le \mu_s N_1$$

$$\vec{F}_{tot} = \hat{y}(N_1 - mg) + \hat{x}(N_2 - F_f) = 0$$

$$\boxed{\begin{array}{l} N_1 = mg \\ N_2 = F_f \end{array}}$$



$$\vec{\tau}_A = \hat{z}\left[mg\frac{L}{2}\sin\left(\frac{\pi}{2}+\theta\right) - N_2 L\sin(\pi-\theta)\right]$$

$$= \hat{z}\left[mg\frac{L}{2}\cos\theta - N_2 L\sin\theta\right] = 0$$

$$N_2 L\sin\theta = mg\frac{L}{2}\cos\theta$$

$$\boxed{N_2 = \frac{mg}{2}\frac{\cos\theta}{\sin\theta}}$$

$$F_f = N_2 = \frac{mg}{2}\frac{\cos\theta}{\sin\theta} \leq \mu_s N_1 = \mu_s mg$$

$$\tan\theta > \frac{mg}{2\mu_s mg} = \frac{1}{2\mu_s}$$

$$\boxed{\tan\theta \geq \frac{1}{2\mu_s}}$$

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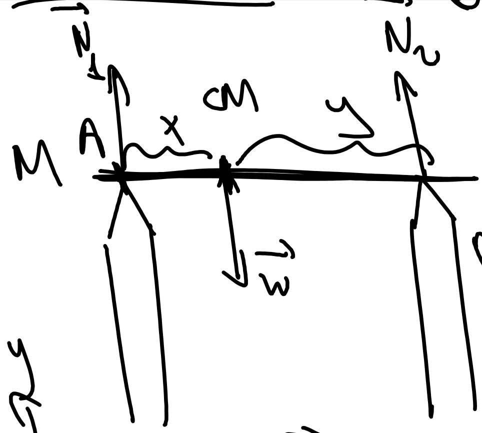
Exercise



$$0 = \vec{F}_{tot} = \hat{y}\left(N_1 + N_2 - mg\right)$$

$$\boxed{N_1 + N_2 = mg}$$

$$0 = \vec{\tau}^{cm}_{tot} = \hat{z}\left(-N_1 x + N_2 y\right)$$

$$\boxed{N_1 x = N_2 y}$$

$$0 = \vec{\tau}_A = \hat{z}\left(-xmg + N_2(x+y)\right)$$

$$\boxed{N_2 = \frac{x}{x+y} mg}$$

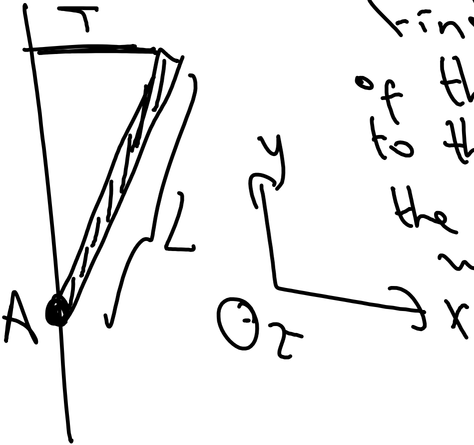
$$N_1 x = N_2 y \Rightarrow N_1 = N_2 \frac{y}{x}$$

$$N_1 + N_2 = mg \Rightarrow N_2\left(\frac{y}{x} + 1\right) = mg$$

$$N_2 = \frac{x}{x+y} mg$$

$$N_1 = \frac{y}{x+y} mg$$

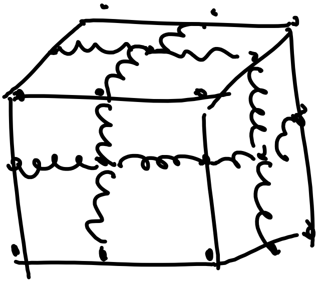
Quiz 5 one full page! hand in your own Quiz!

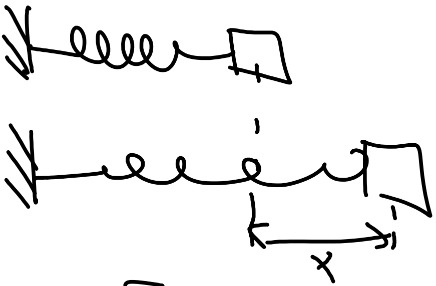


Find the **directions** of the torque due to the tension and the weight of the rod with respect to the point A.

Elasticity

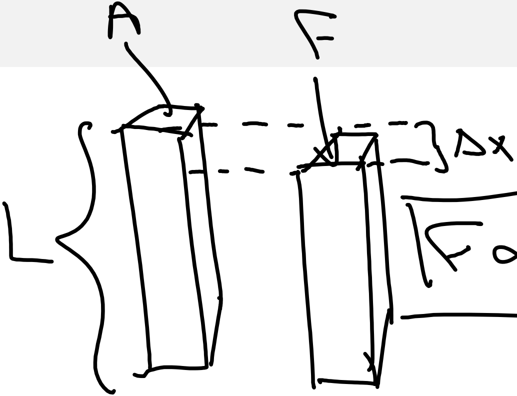
Hooke's law





$$F = -kx$$

↳ spring constant



$$\Delta x = \frac{F}{A} L \frac{1}{Y}$$

$$F \propto \Delta x$$

Y: Young's Modulus

Shear

$$\Delta x = \frac{hF}{A}\frac{1}{S}$$

S : shear modulus

$$\boxed{S\frac{\Delta x}{h} = \frac{F}{A}}$$

$\frac{F}{A}$: stress

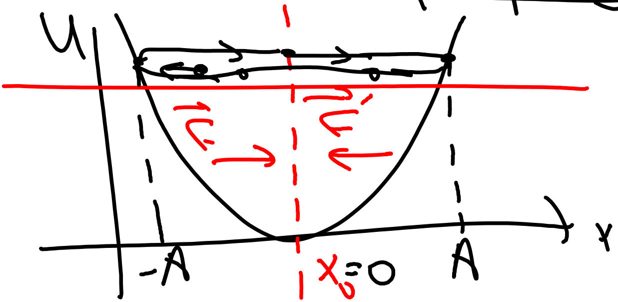
$\frac{\Delta x}{h}$: strain

Spring

$F = -kx$

Commercial spring

Potential energy of spring



Periodic Motion

$$U = \frac{1}{2}kx^2 \; ; \; m\frac{d^2x}{dt} = -kx$$

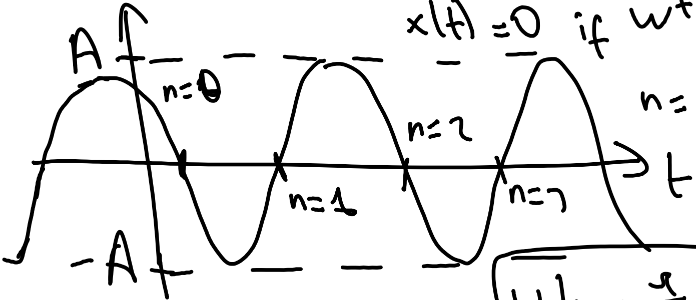
Harmonic Motion

A motion is said to be harmonic if

$$x(t) = A\cos(\omega t + \phi)$$

$x(t) = 0$ if $\omega t + \phi = \frac{\pi}{2} + n\pi$

$n = 0, \mp 1, \mp 2, \ldots$



$$\omega t = \frac{\pi}{2} - \phi + n\pi$$

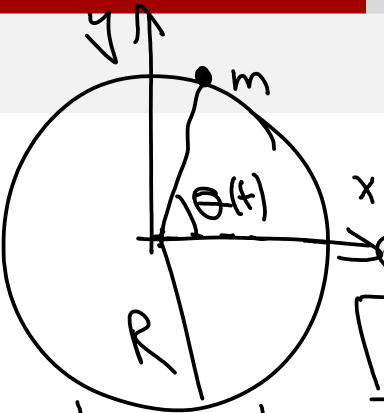
ϕ: phase shift

$$x(t) = A\cos(\omega t + \phi)$$

ϕ: phase shift

A: amplitude

ω: angular velocity



m carries out uniform circular motion

$$\theta(t) = \omega t + \phi$$

$$\boxed{x(t) = R\cos(\omega t + \phi)}$$

ω: angular speed

$$\theta(t=0) = \phi$$

$$T = \frac{2\pi}{\omega}$$: period of oscillation

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$$x(t) = A\cos(\omega t + \phi)$$

$$F = ma$$

$$v(t) = \frac{dx(t)}{dt} = -A\omega\sin(\omega t + \phi)$$

$$a(t) = \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = -A\omega^2\cos(\omega t + \phi)$$

$$= -\omega^2 x$$

$$\boxed{a(t) = -\omega^2 x(t)}$$

$$x(t) = A\cos(\omega t + \phi)$$
$$a(t) = -\omega^2 x(t)$$

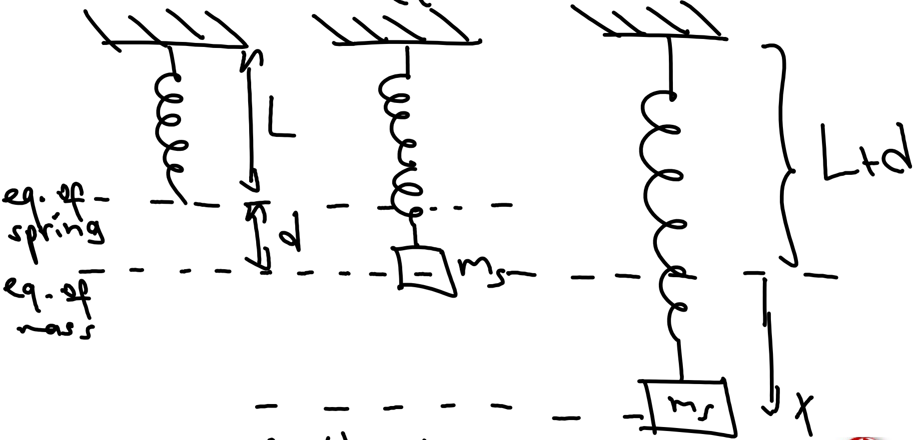
$$F = ma = -(m\omega^2)\, x(t) = -kx$$

$$m\omega^2 = k \Rightarrow \boxed{\omega = \sqrt{\frac{k}{m}}}$$

Example

$x(t=0) = A$

$\frac{dx}{dt}(t=0) = 0$



$F(x) = mg - k(d+x)$

$$F(x=0) = 0 \Rightarrow mg - kd = 0$$

$$d = \frac{mg}{k}$$

$$F = -kx = m\frac{d^2x}{dt^2}$$

$$x(t) = C\cos(wt+\phi) \quad ; \quad w = \sqrt{\frac{k}{m}}$$

$$x(t=0) = C\cos\phi = A$$

$$\frac{dx}{dt}(t=0) = -Cw\sin(wt+\phi)\Big|_{t=0}$$
$$= -Cw\sin\phi = 0$$

$$\Bigg\} \quad x(t) = A\cos(wt)$$

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$$x(t) = A\cos(\omega t + \phi)$$

$$x(t) = A\cos\phi\cos(\omega t) - A\sin\phi\sin(\omega t)$$

$$x_0 = x(0) = A\cos\phi$$

$$v(t) = -A\omega\cos\phi\sin(\omega t) + A\omega\sin\phi\cos\omega t$$

$$v_0 = v(0) = A\omega\sin\phi \Rightarrow A\sin\phi = \frac{v_0}{\omega}$$

$$\boxed{x(t) = x_0\cos\omega t - \frac{v_0}{\omega}\sin\omega t}$$

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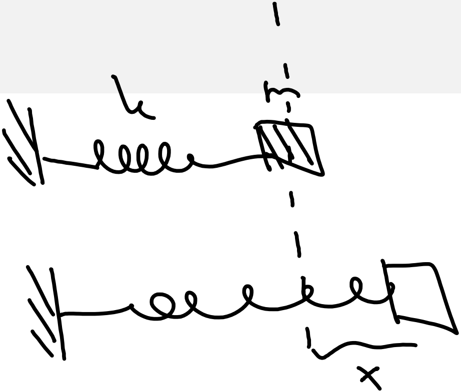
December 24, 2015

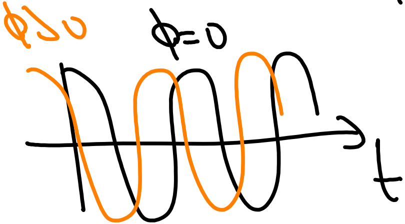
OKAY TÜZEL

Go and see dept. secretary urgently!

..



$$x = A\cos(\omega t + \phi)$$



$$ma = -kx$$

$$m\frac{d^2x}{dt^2} = -kx$$

$$\phi = \omega t_0$$

$$x = A\cos[\omega(t - t_0)]$$

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Physical Pendulum

$$\omega = \frac{d\theta}{dt}$$

$$\alpha = \frac{d^2\theta}{dt^2}$$

$$\vec{\omega} = \frac{d\theta}{dt}\hat{z}$$

$$\vec{\alpha} = \frac{d^2\theta}{dt^2}\hat{z} = \frac{\vec{\tau}}{I} = 0 + \frac{(-\hat{z})\, r m g \sin\theta}{I}$$

$$\frac{dr}{dt} = 0$$

$$\boxed{\frac{d^2\theta}{dt^2} = -\frac{mgr\sin\theta}{I}}$$

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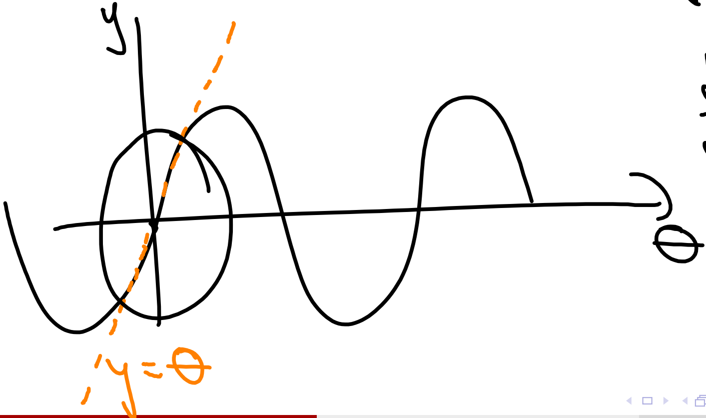
$$\frac{d^2\theta}{dt^2} = -\frac{mgr}{I}\sin\theta$$

harmonic motion

$$\frac{d^2\theta}{dt^2} = -\omega^2\theta$$

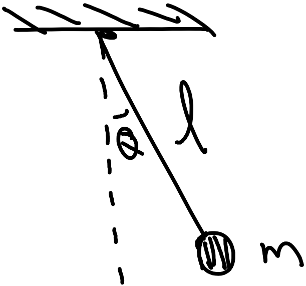


$$\left.\frac{d}{d\theta}\sin\theta\right|_{\theta=0} = 1$$

$$\frac{d^2\theta}{dt^2} = -\frac{mgr}{I}\sin\theta \simeq -\frac{mgr}{I}\theta \qquad \theta \ll 1$$

$$\frac{d^2\theta}{dt^2} = -w^2\theta \quad ; \quad w = \sqrt{\frac{mgr}{I}}$$

Simple Pendulum



$$I = m\ell^2$$

$$\omega = \sqrt{\frac{mgr}{I}} = \sqrt{\frac{mg\ell}{m\ell^2}}$$

$$\boxed{\omega = \sqrt{\frac{g}{\ell}}}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{\ell}{g}}$$

$9T = 5\,s \qquad T_1 = \frac{5}{9}\,s \qquad l = 9.91\ \text{cm}$

$6.5T = 5\,s \qquad T_2 = \frac{5}{6.5}\,s \qquad l \simeq 20\ \text{cm}$

$$\frac{T_1}{T_2} = \frac{6.5}{9} = 0.7 = \frac{1.4}{2} = \frac{\sqrt{2}}{2} \simeq \frac{1}{\sqrt{2}}$$

$$T \propto \sqrt{l} \qquad T = \frac{2\pi\sqrt{l}}{\sqrt{g}} \Rightarrow g = 4\pi^2 l / T^2$$

$$g = 4\pi^2 l / T^2 \simeq \frac{4\pi^2 \, 10\,cm}{(5/9\ s)^2} = \frac{400\,cm}{(0.5)^2 s^2}$$

$$\simeq \frac{4\ m/s^2}{0.5 \cdot 0.5} = 20\,m/s^2$$

$$\boxed{g = 4\pi^2 \frac{l}{T^2} \simeq 12.6\ m/s^2}$$

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$$x = A\cos(\omega t + \phi) \qquad \omega^2 = \frac{k}{m} \Rightarrow k = m\omega^2$$

$$v = \frac{dx}{dt} = -A\omega\sin(\omega t + \phi)$$

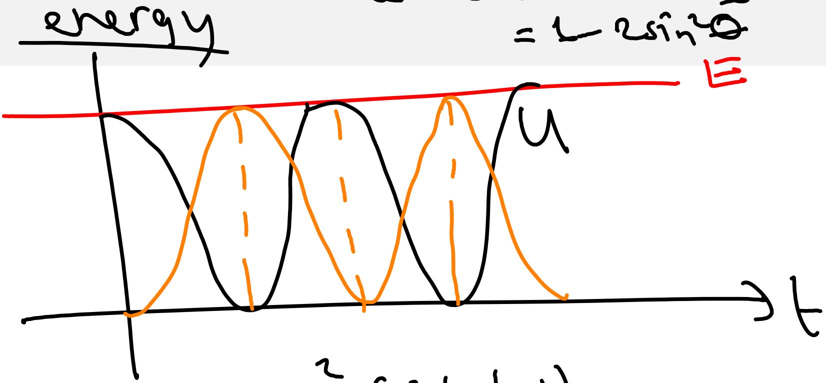
$$KE = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2 A^2 \sin^2(\omega t + \phi)$$

$$PE = \frac{1}{2}kx^2 = \frac{1}{2}kA^2\cos^2(\omega t + \phi)$$

$$PE = \frac{1}{2}m\omega^2 A^2 \cos^2(\omega t + \phi)$$

$$\boxed{E = \frac{1}{2}m\omega^2 A^2}$$

$$\cos 2\theta = 2\cos^2\theta - 1$$
$$= 1 - 2\sin^2\theta$$



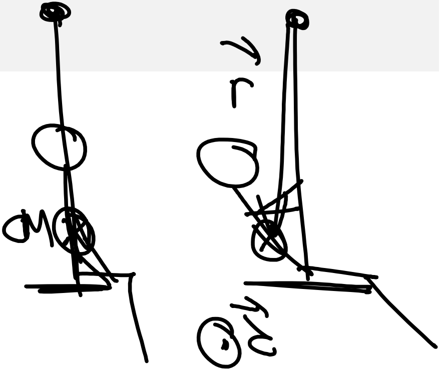
$$KE = \frac{1}{2} m \omega^2 A^2 \sin^2(\omega t + \phi)$$
$$= \frac{1}{2} m \omega^2 A^2 \frac{1}{2}\left(1 + \cos(2\omega t + 2\phi)\right)$$

$$I\vec{\alpha} = \vec{\tau}$$

$$\omega = \sqrt{\frac{g}{l}}$$

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\boxed{T = 2\pi\sqrt{\frac{l}{g}}}$$



"resonance"

$$m\ddot{x} = -kx - \gamma\dot{x}$$

$$\dot{x} \equiv \frac{dx}{dt}$$

$$\ddot{x} \equiv \frac{d^2x}{dt^2}$$

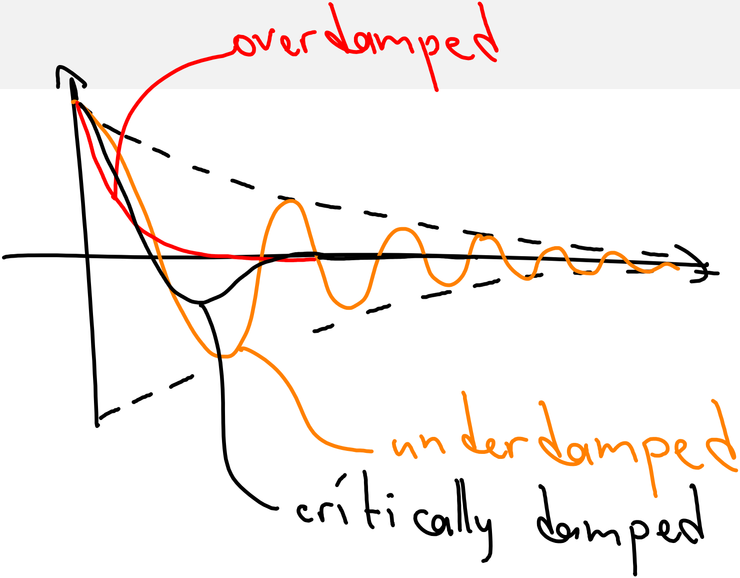
$$\boxed{m\ddot{x} + \gamma\dot{x} + kx = 0}$$

$$x = Ae^{-Bt}\cos(\omega' t + \phi)$$

γ is sufficiently small

$$\omega' = \sqrt{\frac{g}{l} - \frac{\gamma^2}{4m^2}}$$

$$B \propto \gamma$$



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Driven Oscillator

$$m\ddot{x} = -kx + f_{ext}(t)$$

$$m\ddot{x} + kx = f_{ext}(t) = f_0 \cos(\omega t)$$

$$\omega_0^2 = \frac{k}{m}$$

$$x(t) = A\cos(\omega_0 t + \phi) + \frac{f_0}{k - m\omega^2}\cos(\omega t)$$

$$x(t) = A\cos(\omega_0 t + \phi) + \frac{f_0}{m}\frac{1}{\omega_0^2 - \omega^2}\cos(\omega t)$$

$$\omega \neq \omega_0$$

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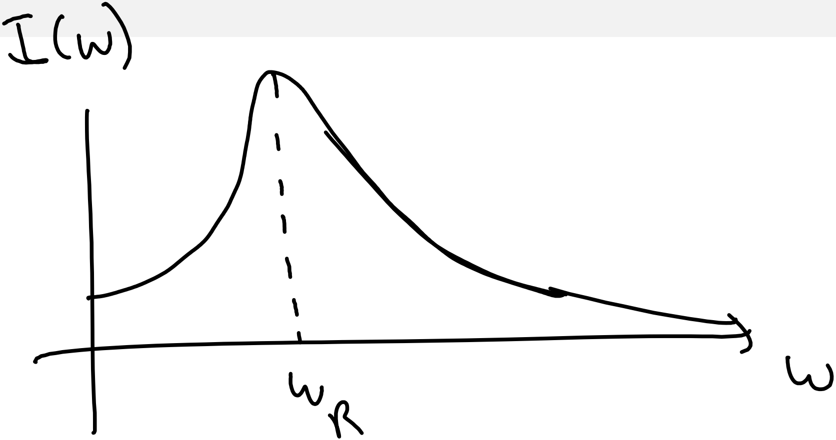
$x(t) = (A + Bt)\cos(\omega_0 t + \phi)$ if $\omega = \omega_0$

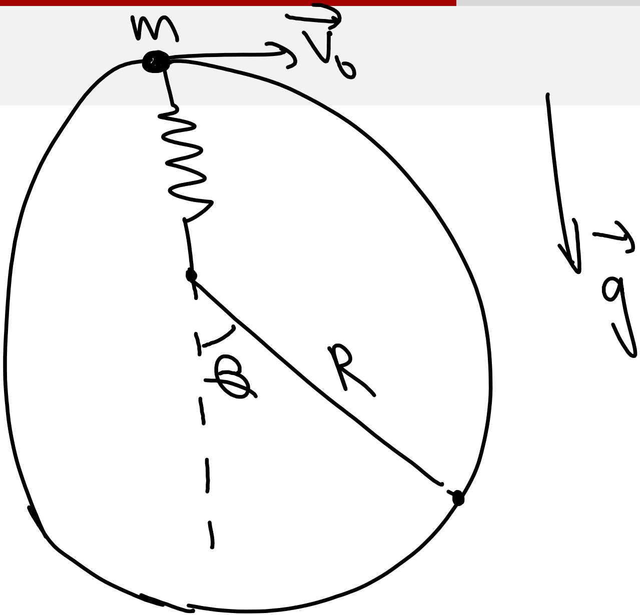
$$m\ddot{x} + \gamma\dot{x} + kx = f_0\cos(\omega t)$$

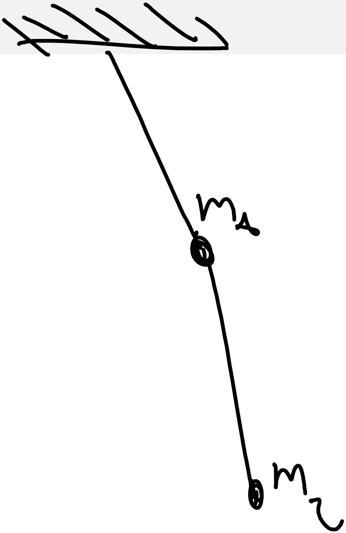
$$x(t) = Ae^{-\beta t}\cos(\omega' t + \phi)$$

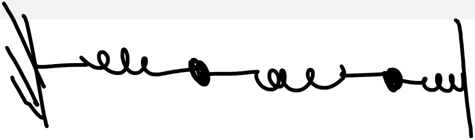
$$+ \underbrace{\frac{f_0}{\sqrt{(k-m\omega^2)^2 + \gamma^2\omega^2}}}_{I(\omega)}\cos(\omega t + \delta)$$

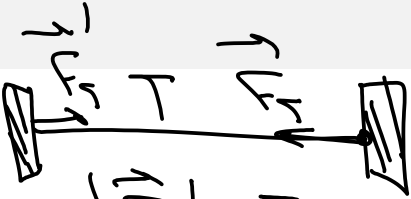
$$I(\omega) = \frac{f_0}{\sqrt{m^2(\omega^2-\omega_0^2)^2 + \gamma^2\omega^2}}$$


I(ω)
ω
ω_R


m
$\vec{V}_0$
θ
R
$\vec{g}$

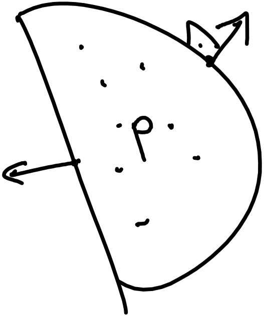

m1
m2



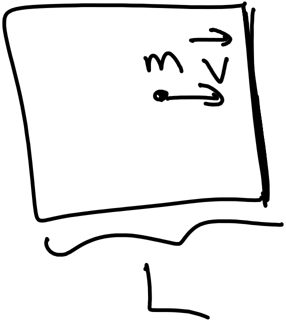


$$|\vec{F}_T| = T$$

$$|\vec{F}_T'| = T$$



Example



Δt : time between two collisions with the wall on the right

$$\Delta t = \frac{2L}{v}$$

$$\Delta p = 2mv$$

$$F_{av} = \frac{\Delta p}{\Delta t} = \frac{2mv}{2L/v} = \frac{mv^2}{L}$$

$$F_{av}^{(1)} = \frac{mv^2}{L}$$

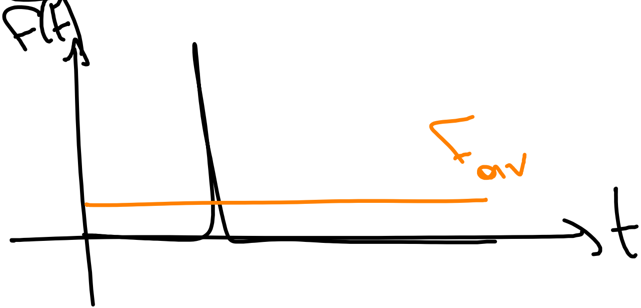
N point masses

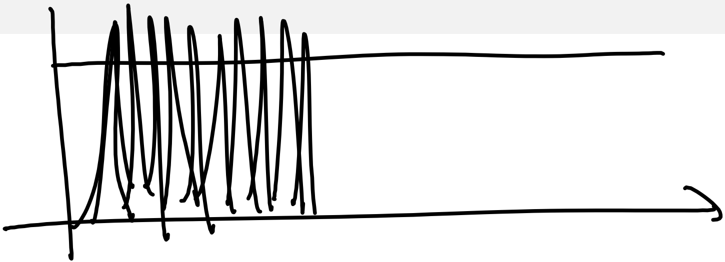
$$F_{av}^{(N)} = N \frac{mv^2}{L}$$

$$P = \frac{F^{(N)}}{A} = \frac{N\, mv^2}{\underbrace{(LA)}_{V}} \Rightarrow PV = N\left(mv_x^2\right)$$

$$\frac{PV}{N} = \langle m v_x^2 \rangle_{av} = \text{const} = k_B T$$

(1)



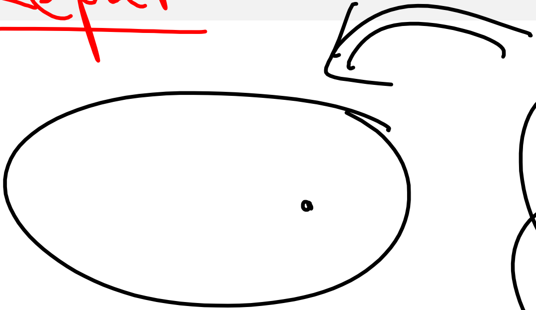


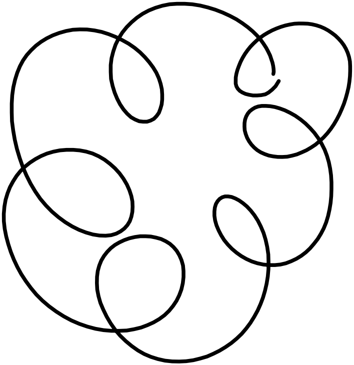
December 29, 2016

$$f(x) = a_0 + a_1 x + a_2 x^2 + \ldots$$

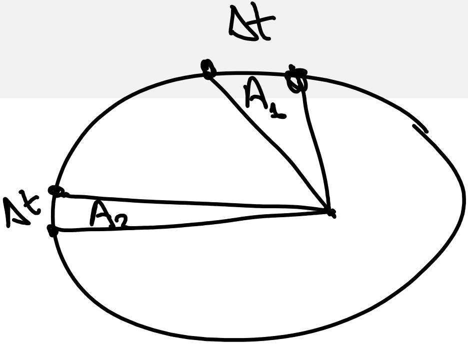
$$g(x) = a_0 + \sin\_\_ + \cos \ldots$$

Kepler





"Feynman's Lost Lecture"

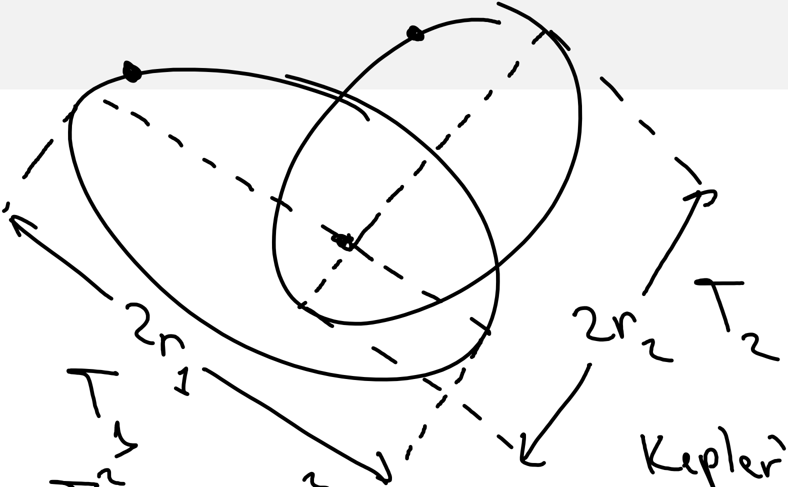


$$A_1 = A_2$$

$$\frac{A_1}{\Delta t} = \frac{A_2}{\Delta t} = \text{rate of sweeping of area}$$

Kepler's Second Law

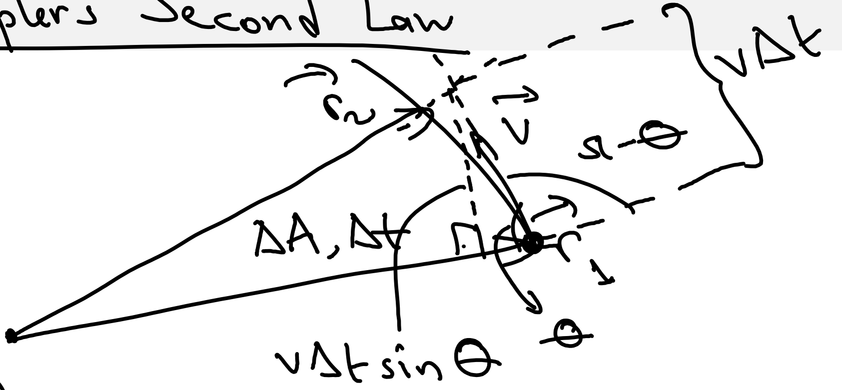
rate of sweeping area is conserved!



$$\frac{T_1^2}{r_1^3} = \frac{T_2^2}{r_2^3}$$

Kepler's Third Law

Kepler's Second Law



$$\frac{\Delta A}{\Delta t} = \frac{1}{2} \frac{v \Delta t \sin\theta \, r_1}{\Delta t} = \frac{1}{2} v \sin\theta \, r_1$$

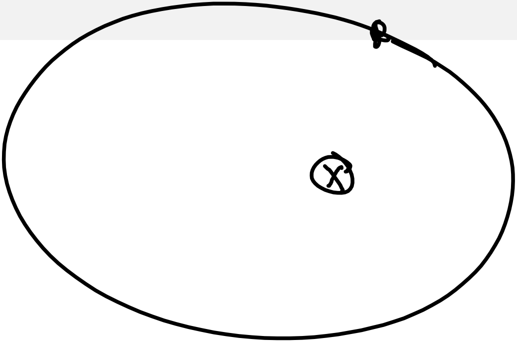
$$\boxed{v \sin\theta \, r = \text{const}}$$

$$v r \sin\theta = v r \sin(\pi - \theta)$$

$$= |\vec{r} \times \vec{v}| = \frac{1}{m} |\vec{r} \times \vec{p}|$$

Kepler's Second Law

$\Longleftrightarrow$ conservation of angular momentum



Kepler's Second Law

$$\Rightarrow \vec{F} = f(r)\,\hat{r}$$

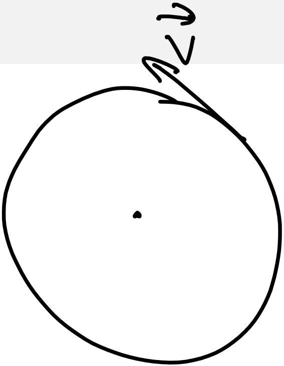
Kepler's Third Law

$$\frac{T^2}{r^3} = \text{const}$$

$$r v \sin\theta = \text{const}$$

$$T = \frac{2\pi r}{v} \Rightarrow v = \frac{2\pi r}{T}$$

$$a = \frac{v^2}{r} = (2\pi)^2 \frac{r}{T^2}$$



$$a = (2\pi)^2 \frac{r}{T^2}$$

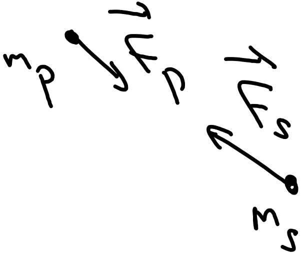
$$F = ma = (2\pi)^2 m \frac{r}{T^2}$$

$$\frac{F r^2}{m} = (2\pi)^2 \frac{r^3}{T^2} = \text{const}$$ (for all planets in the solar system)

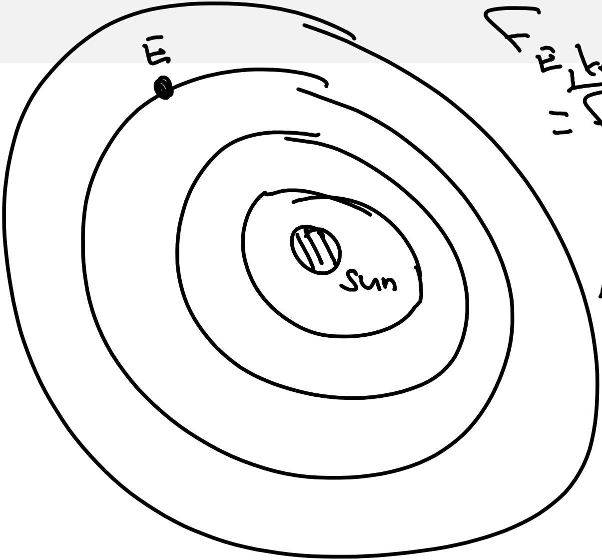
$$\boxed{F \propto m \frac{1}{r^2}}$$

Newton's Law of Gravity

$$\vec{F} = G_N \frac{m_p M_s}{r^2}\hat{r}$$

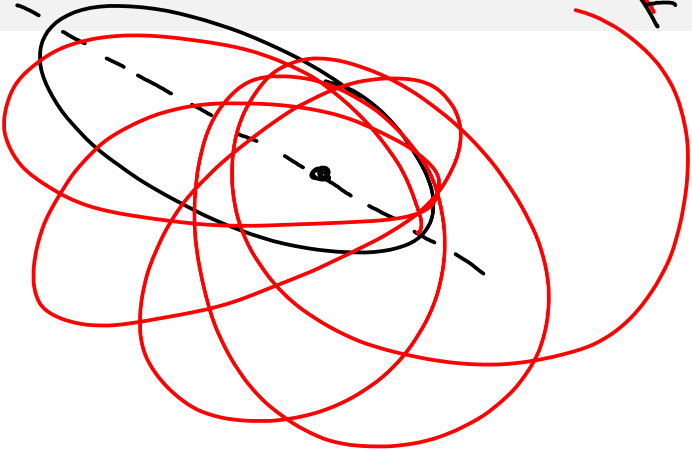


$$\vec{F}_p = -\vec{F}_s$$



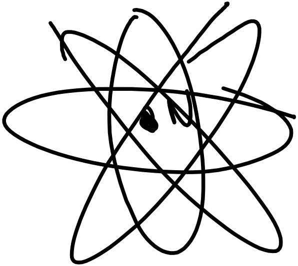
$$\vec{F}_{E,tot} = \vec{F}_{E,S} + \vec{F}_{E,M} + \vec{F}_{E,V} + \ldots$$

"Principle of Superposition"

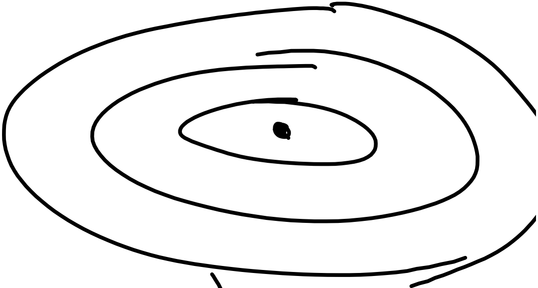


$$F = \frac{(\ )}{r^2} + \frac{(\ )}{r^3} + O\left(\frac{1}{r^4}\right)$$

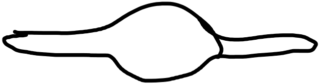
Model of Atom (False model)

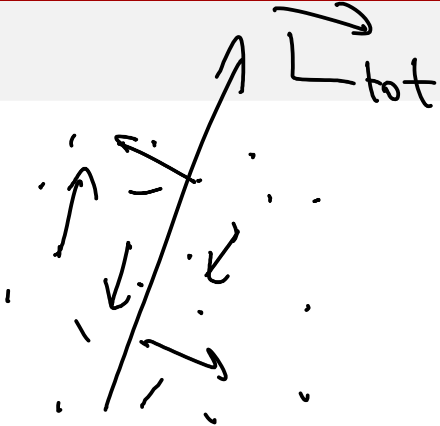


Solar System



Galaxy (From the Side)

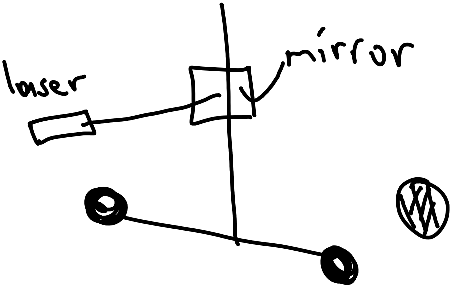




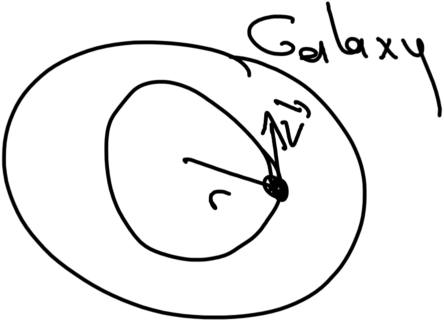
$$KE = \frac{L^2}{2I}$$

$$\vec{F} = G_N \frac{m_1 m_2}{r^2} \hat{r}$$

Cavendish Experiment



Rotation Curves



$$F = \frac{mv^2}{r} = G_N \frac{mM(r)}{r^2}$$

$$v^2 = \frac{mM(r)}{r}$$

$$v \propto \sqrt{\frac{M(r)}{r}}$$

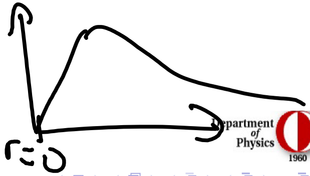
$$v \propto \sqrt{\frac{M(r)}{r}}$$

inside the galaxy $M(r) \sim r^3$

$$V_{cir} \sim r$$

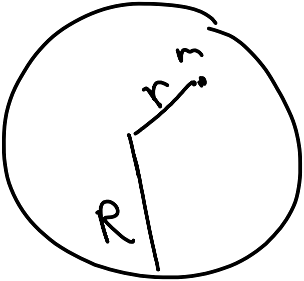
out of the galaxy $M(r) = \text{const}$

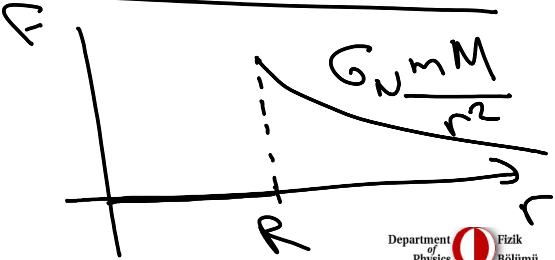
$$V_{cir} \sim \frac{1}{\sqrt{r}}$$



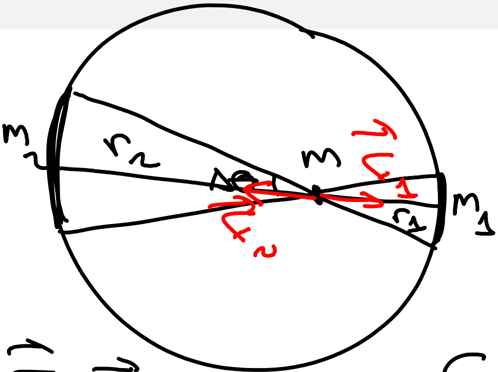
$$\vec{F} = G_N \frac{m_1 m_2}{r^2}\hat{r}$$ valid for point objects

Gravitational Field of a Shell





inside the shell



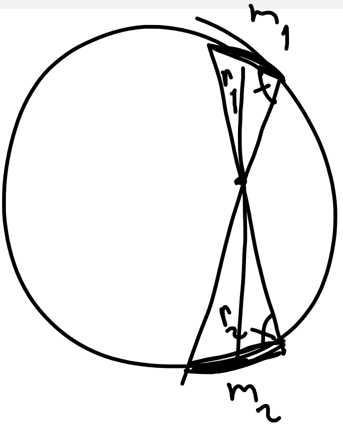
$\vec{F}_1 + \vec{F}_2 = 0$

$\Delta\theta \to 0$

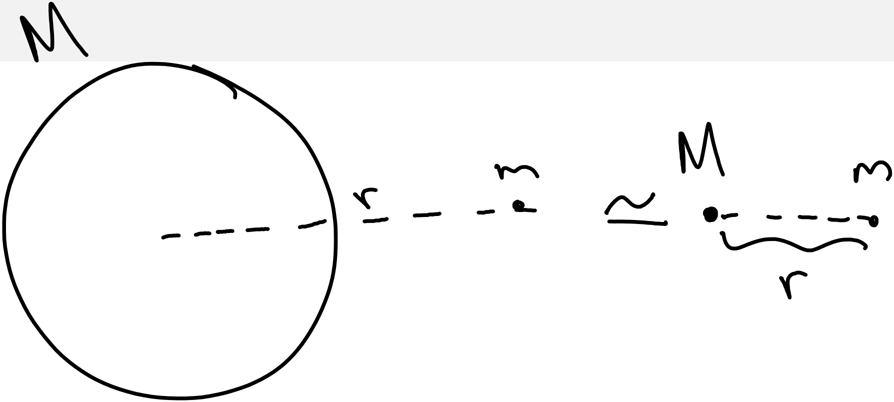
$$\frac{m_1}{m_2} = \frac{r_1^2}{r_2^2}$$

$$\frac{m_1}{r_1^2} = \frac{m_2}{r_2^2}$$

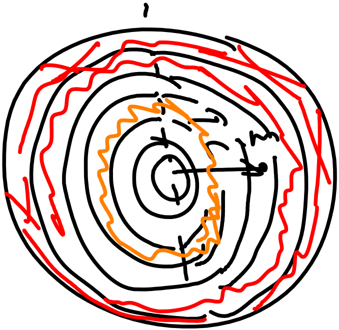
$$\underbrace{G_N m \frac{m_1}{r_1^2}}_{|\vec{F}_1|} = \underbrace{G_N \frac{m m_2}{r_2^2}}_{|\vec{F}_2|}$$



$$\frac{m_1}{m_2} = \frac{A_1}{A_2} = \frac{r_1^2}{r_2^2}$$


M
r
m
≃
M
m
r

Example



$$\vec{F} = 0$$

$$\vec{F} = G_N \frac{m M_{shell}}{r^2}$$

$$\vec{F} = -\sum_{\text{shells for which } m \text{ is outside}} G_N \frac{m M_{shell}}{r^2} \hat{r}$$

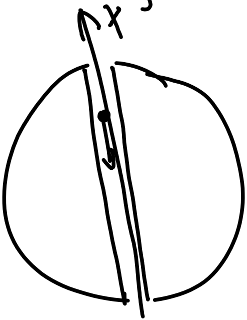
$F = G_N \frac{m M(r)}{r^2}$; $M(r)$ is the mass of the sphere of radius r.

$$F = G_N m \frac{M(r)}{r^2}$$

If sphere is uniform, $M(r) = \frac{4}{3}\pi r^3 \rho$

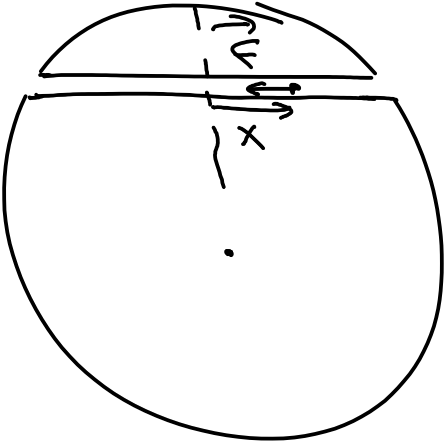
$$F = \left(G_N m \frac{4}{3}\pi\rho\right) r$$

$$F = -\left(G_N m \frac{4}{3}\pi\rho\right) x$$



Example

$|\vec{F}| \propto x$



December 31, 2015

Hand in your HW (NOW!)

- Potential energy for gravity
- Gravitational acceleration on the surface of the Earth
- Equivalence Principle (General Th. of Relativity)

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Gravitational Potential Energy

A force (field) is conservative if $\int_A^B \vec{F}\cdot d\vec{\ell}$ is independent of the path.

$$U(P) = U(P_0) - \int_{P_0}^{P_1} \vec{F}\cdot d\vec{\ell}$$

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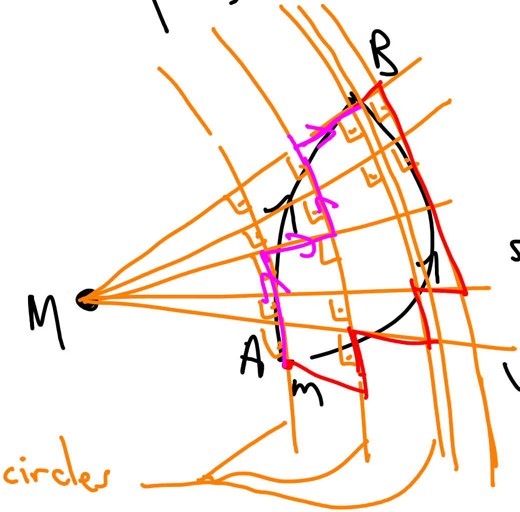
$\vec{F} = \hat{r} f(r)$: is conservative



$W_{arc} = \vec{F} \cdot \Delta\vec{r}$

$= f(r)\hat{r} \cdot \Delta\vec{r}$

$= 0$

since along the arc $\hat{r} \cdot \Delta\vec{r} = 0$

$W_{rad. seg} = \vec{F} \cdot \Delta\vec{r}$

$= f(r) \Delta r$

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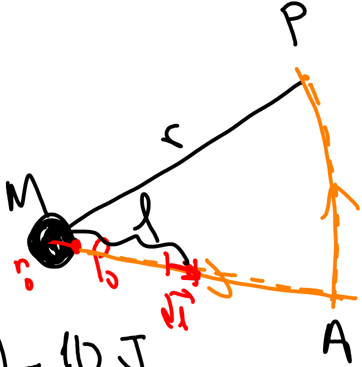
$$U(P) - U(P_0) = -\int_{P_0}^{P} \vec{F}\cdot d\vec{l}$$

$$= \left(-\int_{P_0}^{A} \vec{F}\cdot d\vec{l}\right) + \left(-\int_{A}^{P} \vec{F}\cdot d\vec{l}\right), \qquad -\int_{A}^{P} \vec{F}\cdot d\vec{l} = 0$$

$$= +\int_{P_0}^{A} \left(+G_N \frac{mM}{l^2}\right) dl$$

$$= -G_N \frac{mM}{l}\Big|_{l=r_0}^{r}$$



$U(P_0) = 10\ \text{J}$

$d\vec{l} = (dl)\hat{r}$; $\vec{F} = -G_N \frac{mM}{l^2}\hat{r}$

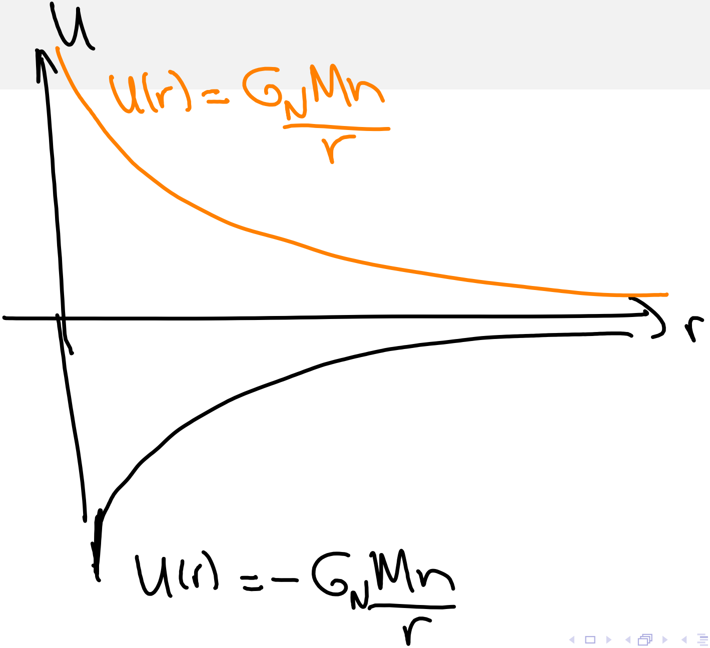
$$U(P) = U(P_0) + \left(-\frac{G_N m M}{\ell} \right) \Big|_{\ell = r_0}^{r}$$

$$U(P) = \left(U(P_0) + \frac{G_N m M}{r_0} \right) - \frac{G_N m M}{r}$$

Conventional choice

$$U(P_0) = -\frac{G_N m M}{r_0}$$

$$\boxed{U(P) = -\frac{G_N m M}{r}}$$


U
$U(r) = \frac{G_N M m}{r}$
r
$U(r) = -\frac{G_N M m}{r}$

$$U(P) = U(P_0) - G_N M m \left(\frac{1}{R+h} - \frac{1}{R} \right)$$

R: radius of Earth

M: mass of Earth

m: mass of the object close to the surface of earth.

$$U(P) = mgh \;; \quad U(h=0) = 0$$

$$\Downarrow$$

$$U(P_0) = 0$$

$$U = -G_N M m \left(\frac{1}{R+h} - \frac{1}{R} \right) = \frac{G_N M m h}{R(R+h)}$$

$$\cong mgh$$

$$U \cong m \left(\frac{G_N M}{R^2} \right) h \qquad \boxed{h \ll R}$$

$$\boxed{g = \frac{M G_N}{R^2}}$$

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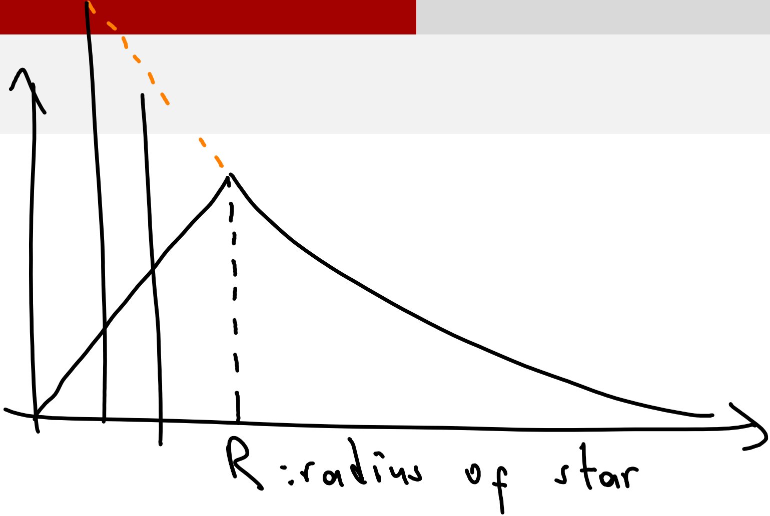
Circular Orbits

$$v = \frac{2\pi r}{T}$$

$$\frac{m v^2}{r} = G_N \frac{M m}{r^2}$$

$$\frac{m (2\pi)^2 r^2}{T^2} = G_N \frac{M m}{r}$$

$$\boxed{\frac{r^3}{T^2} = \frac{G_N}{(2\pi)^2} M}$$


R : radius of star

Escape Velocity

$$\frac{1}{2}\cancel{m}v_i^2 = \frac{1}{2}\cancel{m}v_f^2 + \cancel{m}gh_{max}$$

($v_f = 0$)

$$\boxed{h_{max} = \frac{v_i^2}{2g}}$$

$$\frac{1}{2}mv_i^2 - G_N\frac{mM}{R} = 0 - G_N\frac{mM}{R_{max}}$$

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$$\frac{1}{2}mv_i^2 - \frac{G_N mM}{R} = 0 - \frac{G_N mM}{R_{max}}$$

$$\boxed{\frac{G_N mM}{R_{max}} = \frac{G_N mM}{R} - \frac{1}{2}mv_i^2}$$

$$\frac{1}{2}mv_i^2 = \frac{G_N mM}{R} \Rightarrow R_{max} = \infty$$

$$\boxed{v_{esc} = \sqrt{\frac{2G_N M}{R}}}$$

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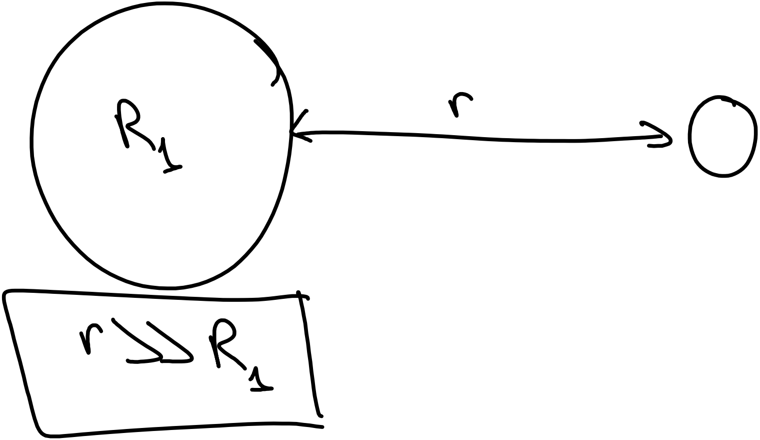

R1
r
r >> R1

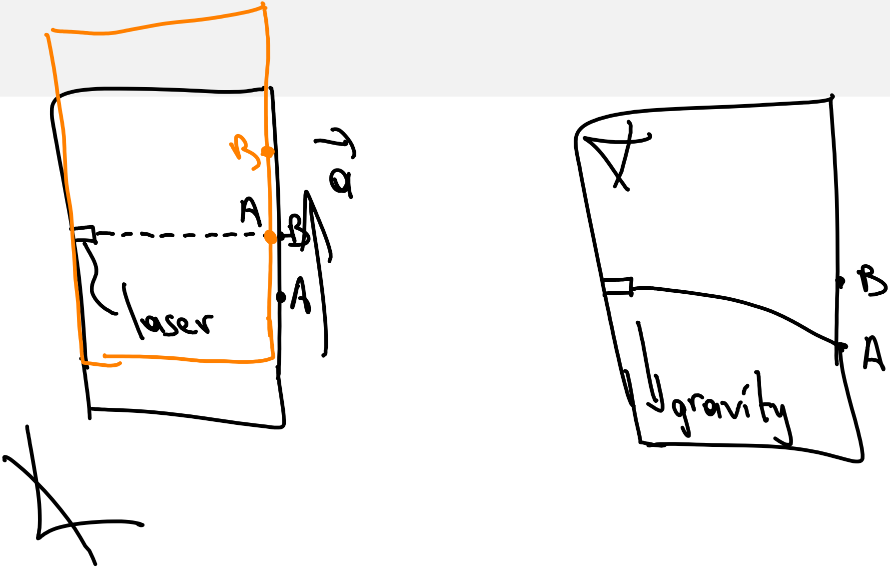
Equivalence Principle

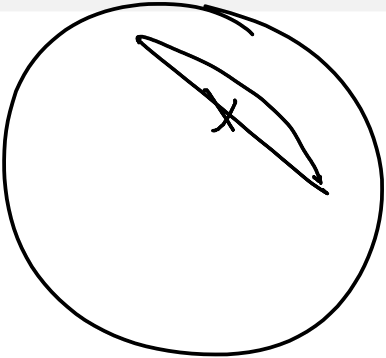
$$\cancel{m} a = G_N \frac{\cancel{m} M}{r^2}$$

$$\boxed{a = G_N \frac{M}{r^2}}$$

inertial mass

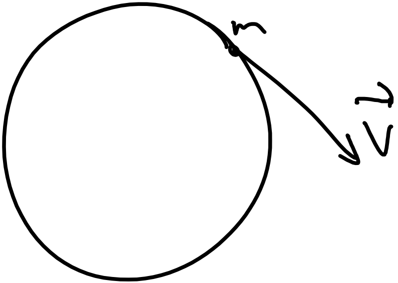
gravitational mass


B
A
B
A
a
laser
B
A
gravity



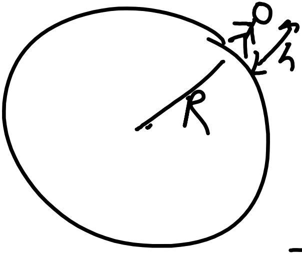
January 5, 2016

$$|\vec{v}| = v_{esc}$$



$$-G_N \frac{m M_E}{R_E} + \frac{1}{2} m v_{esc}^2 + \frac{1}{2} M v_E^2 = 0$$

Tides

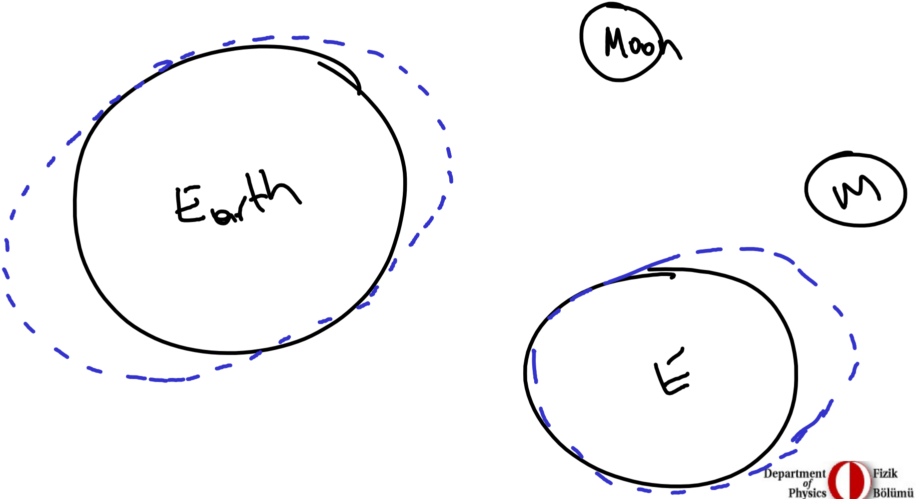


$$g = \frac{G_N M_E}{r^2}$$

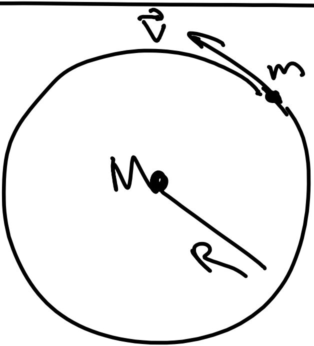
$$|\Delta g| = \frac{2 G_N M_E}{r^3} \Delta r$$

$$= \frac{2 G_N M_E}{R_E^3} h$$

$$\frac{\Delta g}{g} = \frac{2h}{R_E} \simeq \frac{4 m}{6\ 10^6 m} \sim 10^{-6}$$


Earth
Moon
m
E

Circular Orbits



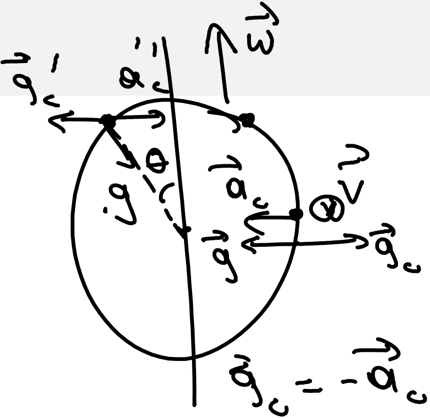
$$\frac{mv^2}{r} = \frac{G_N M m}{r^2}$$

$$v^2 = \frac{G_N M}{r}$$

$$KE = \frac{1}{2} m v^2 \qquad ME = KE + PE = -(KE)$$

$$PE = -\frac{G_N M m}{r} = -mv^2 = -2(KE)$$


M
M
moon



$$\vec{g}_c = -\vec{a}_c$$

$$|\vec{g}| = |\vec{a}|$$

$$g_t = \frac{G_N M}{R_E^2} - w^2 R_E$$

$$\vec{g}_c' = -\vec{a}_c' \; ; \; |\vec{g}_c'| = w^2 R_E \sin\theta$$

Fluids

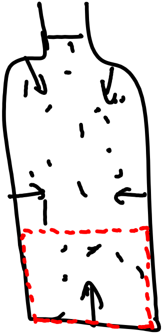
- Solids
- liquids – fluids
- gases

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

$$\rho_w \equiv d_w \cong \frac{1\ \text{gr}}{1\ \text{cm}^3} = 1\ \text{gr}/\text{cm}^3$$

ρ : rho

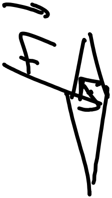
Pressure



$\vec{a}_{cm} = 0$

weigh = $Mg = \rho V g$

$P = \frac{F}{A}$ P is a scalar



$P = \frac{F}{A}$ F perpendicular to the area

$\vec{F}_{net} = 0$

$\Rightarrow \vec{F}_1 + \vec{F}_2 + \vec{w} = 0$ vertical forces

$\vec{F}_3 + \vec{F}_4 = 0$ horizontal forces

$\frac{F_3}{A_3} = \frac{F_4}{A_3} \Rightarrow P_3 = P_4$

$\Rightarrow$ P can depend only at the height of the point.

$$\vec{F}_1 = \hat{z}\, P(z)\, A_1$$

$$\vec{F}_2 = -\hat{z}\, P(z+h)\, A_1$$

$$\vec{w} = \hat{z}\, mg = \hat{z}\, \rho A_1 h g$$

$$\vec{F}_1 + \vec{F}_2 + \vec{w} = A_1 \hat{z} \left[P(z) - P(z+h) + \rho h g \right] = 0$$

$$\frac{P(z+h) - P(z)}{h} = \frac{\rho h g}{h} \Rightarrow \frac{dP(z)}{dz} = \rho g$$

$$\boxed{P(z) = P_0 + \rho g z}$$

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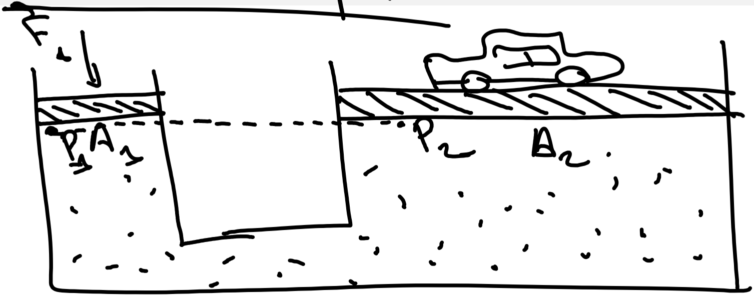
Pascal Principle



$$P_1 = \frac{F_1}{A_1}$$

$$P_2 = \frac{w}{A_2} = \frac{m_c g}{A_2}$$

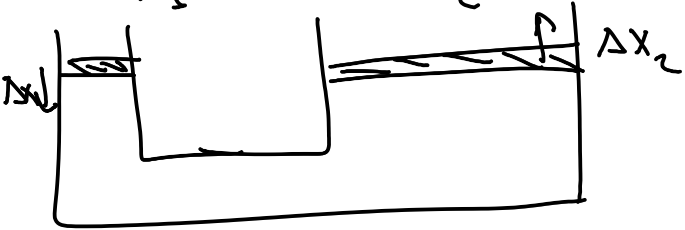
$$P_1 = P_2 \Rightarrow \boxed{\frac{F_1}{A_1} = \frac{w}{A_2}}$$

$$\frac{F_1}{w} = \frac{A_1}{A_2}$$

$$\rho_1 A_1 \Delta x = \rho_2 A_2 \Delta x_2$$

incompressible fluid: $\rho_1 = \rho_2$

$$A_1 \Delta x = A_2 \Delta x_2$$



$$\underbrace{P_1 A_1 \Delta x}_{|W_{F_1}|} = \underbrace{P_2 A_2 \Delta x_2}_{|W_{F_2}|}$$

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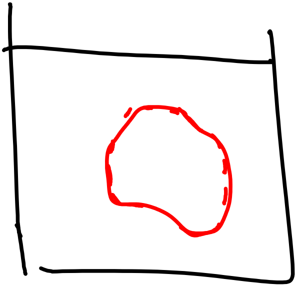
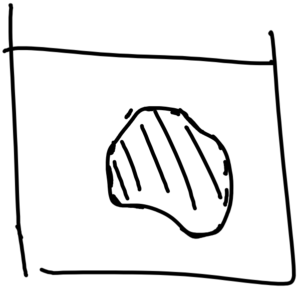
Buoyoncy - The lifting force of the liquid



$\vec{F}_{net} = 0$

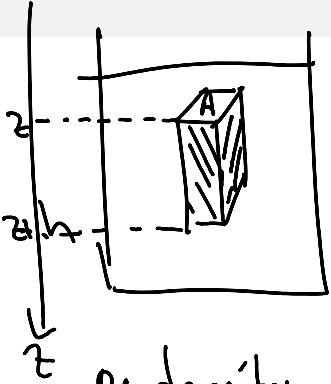
$\vec{w} + \vec{F}_B = 0$

$\vec{F}_B = -\vec{w}$



$$\vec{F}_t = \vec{W}_{solid} + \vec{F}_B$$

$|\vec{F}_B|$ = weight of the liquid that the object displaces.



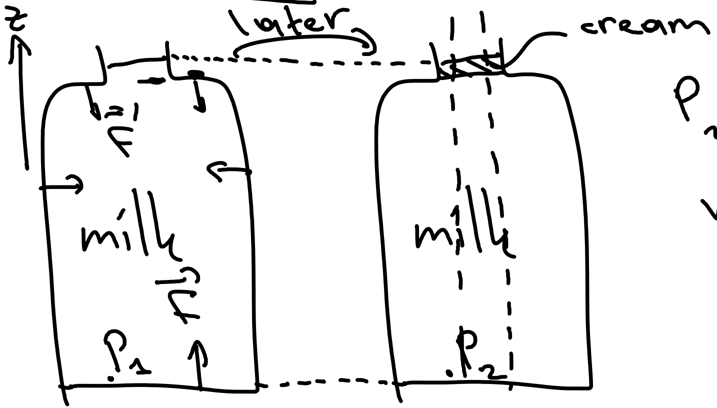
ρ: density of the liquid

$$\vec{F}_B = \hat{z}\left(P(z)A\right) - \hat{z}\left(P(z+h)A\right)$$

$$= \hat{z}\,A\left(\underbrace{P(z) - P(z+h)}_{-\rho g h}\right)$$

$$= \hat{z}\,(-\rho g)\underbrace{(Ah)}_{V}$$

Question



$P_2 < P_1$

why?

$P_1 - P_2 = ?$

> 0

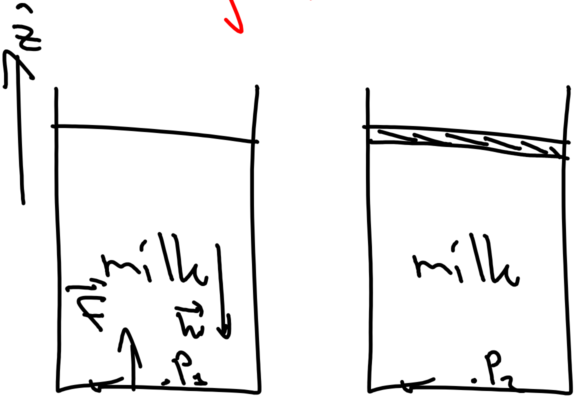
< 0

$= 0$

$\hat{z} P_1 A_1 + \vec{F}' - mg\hat{z} = 0$

$$\boxed{P_1 A_1 \hat{z} = mg\hat{z} - \vec{F}'}$$

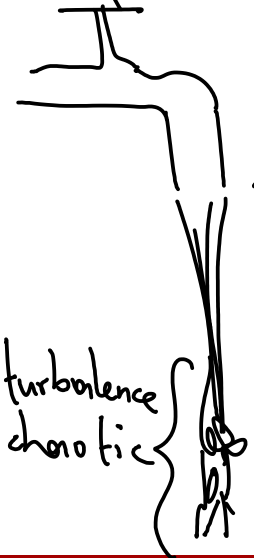
January 7, 2016



$P_1 = P_2$

$$\vec{F} = (P_1 A_1 - mg)\hat{z} = 0 \Rightarrow P_1 = \frac{mg}{A_1}$$

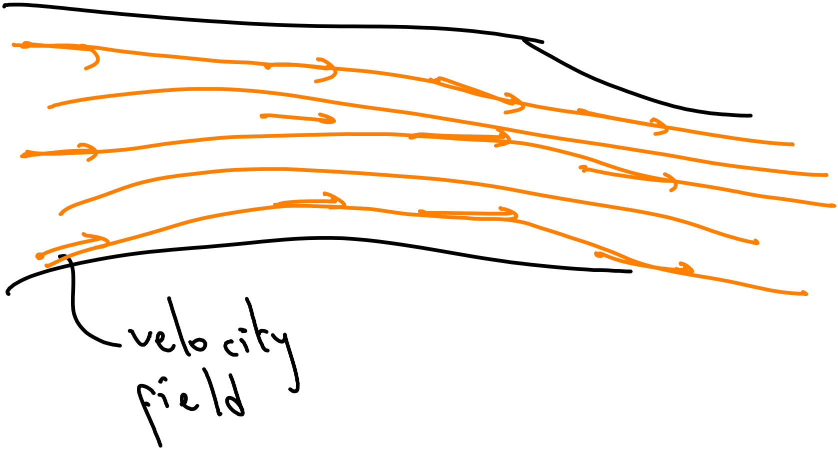
Liquid Flow



incompressible flow

streamlines

steady
non-chaotic
laminar



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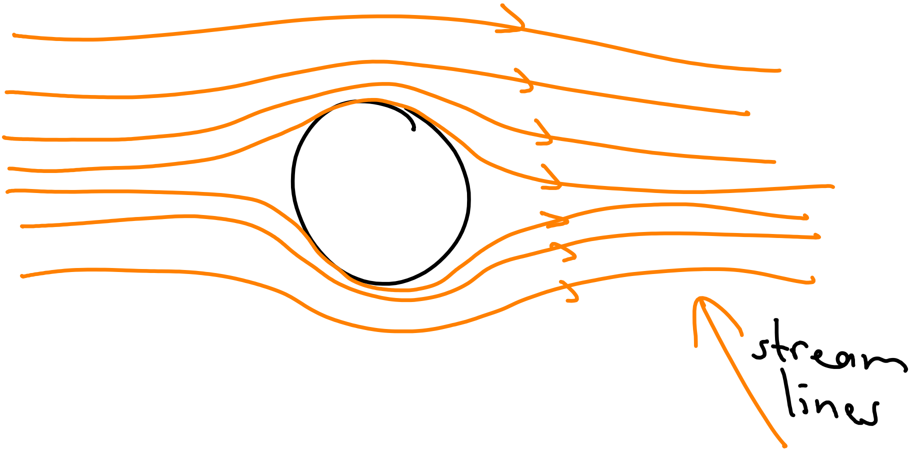
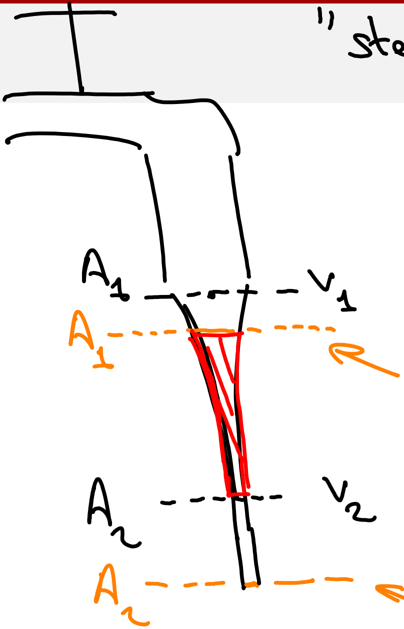

stream
lines

"steady flow"



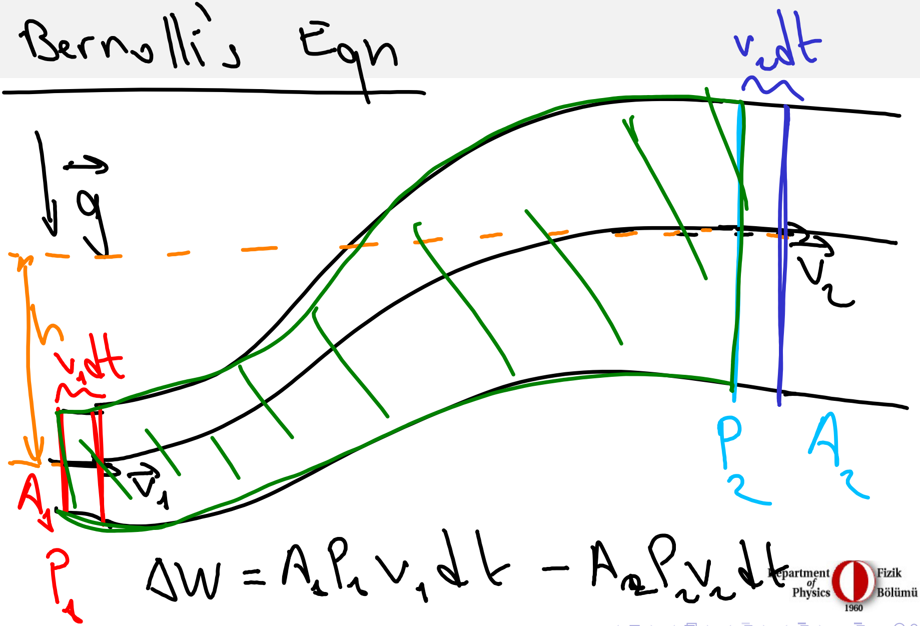
$$\rho_1 A_1 v_1 dt = A_2 v_2 dt \rho_2$$

$$\boxed{A_1 v_1 = A_2 v_2}$$

$\rho_1 = \rho_2$ incompressibility

$$\Delta m = \rho_1 A_1 v_1 dt - \rho_2 A_2 v_2 dt = 0$$

Bernolli's Eqn



$$\Delta W = A_1 P_1 v_1 dt - A_2 P_2 v_2 dt$$

Example

$$1\,atm = 101325\,Pa \simeq 10^5\,Pa$$

$$1\,Pa = 1\,N/m^2$$

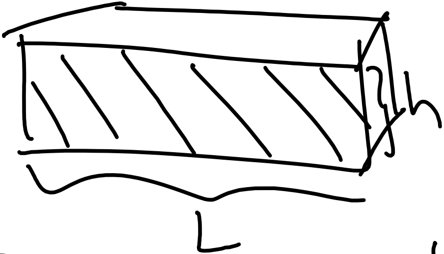
$$A = 0.1 \times 0.2\,m^2 = 2\times10^{-2}\,m^2$$

$$F = PA = 10^5\,\frac{N}{m^2}\,2\times10^{-2}\,m^2 \simeq 10^3\,N$$

$$F \simeq (100\,kg)\,g$$

Exercise

$$F_T \neq PA$$



$$P_{av} = \frac{P_0 + P_h}{2} = \frac{P_0 + (P_0 + \rho g h)}{2} = P_0 + \frac{1}{2}\rho g h$$

$$F_T = \left(P_0 + \frac{1}{2}\rho g h\right) L h - P_0 L h$$

$$F_T = \frac{1}{2}\rho g L h^2 \simeq \frac{1}{2} 10^3 \frac{kg}{m^3} \, 10 \frac{m}{s^2} \, \frac{1}{5} m \cdot 1 m^2 \sim 500 N$$

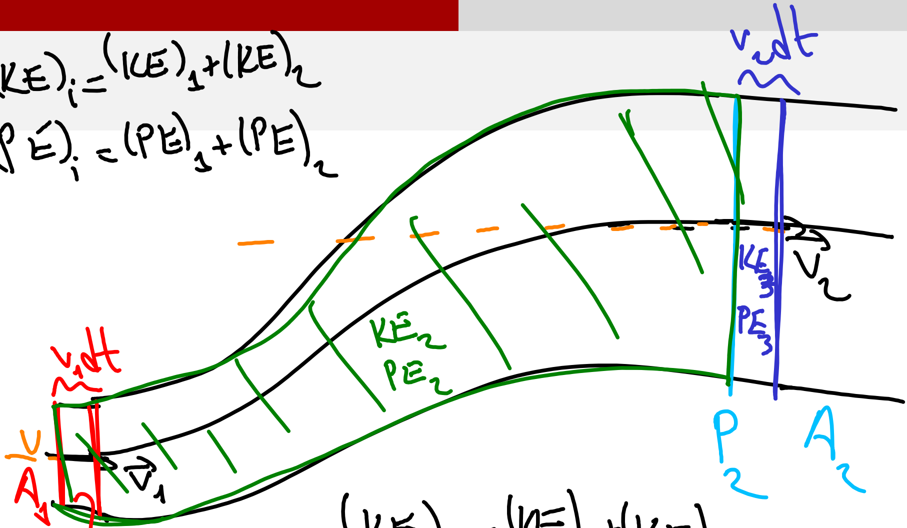
$$\Delta W = A_1 P_1 v_1 \, dt - A_2 P_2 v_2 \, dt = \Delta(ME)$$

$$= \frac{1}{2}\left[(A_2 v_2 \, dt)\rho\right] v_2^2 - \frac{1}{2}\left[(A_1 v_1 \, dt)\rho\right] v_1^2$$

$$+ \left[(A_2 v_2 \, dt)\rho\right] g h - 0$$

$(KE)_i = (KE)_1 + (KE)_2$

$(PE)_i = (PE)_1 + (PE)_2$



$(KE)_f = (KE)_2 + (KE)_3$

$(PE)_f = (PE)_2 + (PE)_3$

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$$A_1 P_1 v_1 dt - A_2 P_2 v_2 dt$$

$$= \frac{1}{2}[(A_2 v_2 dt)\rho] v_2^2 - \frac{1}{2}[(A_1 v_1 dt)\rho] v_1^2$$

$$+ [(A_2 v_2 dt)\rho] g h$$

$$A_1 v_1 = A_2 v_2$$

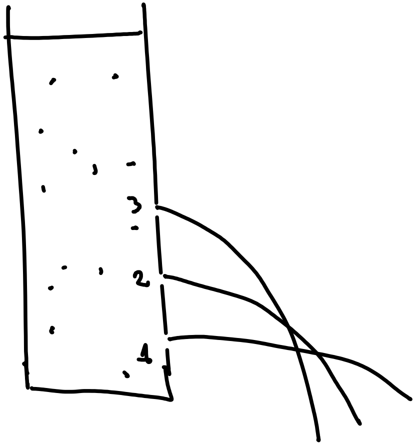
$$P_1 - P_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 + \rho g (h_2 - h_1)$$

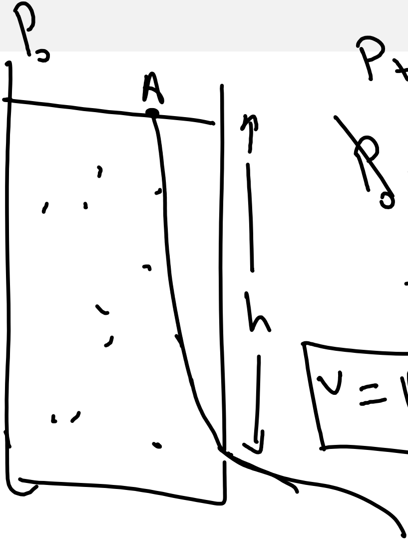
$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$$P + \frac{1}{2}\rho v^2 + \rho g h = \text{const. along a streamline.}$$

Bernolli's Eqn.

Exercise

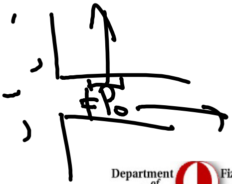




$$P + \frac{1}{2}\rho v^2 + \rho g h = \text{const.}$$

$$P_0 + \frac{1}{2}\rho v_A^2 + \rho g h = P_0 + \frac{1}{2}\rho v^2$$

$$v = \sqrt{2gh}$$



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Exercise