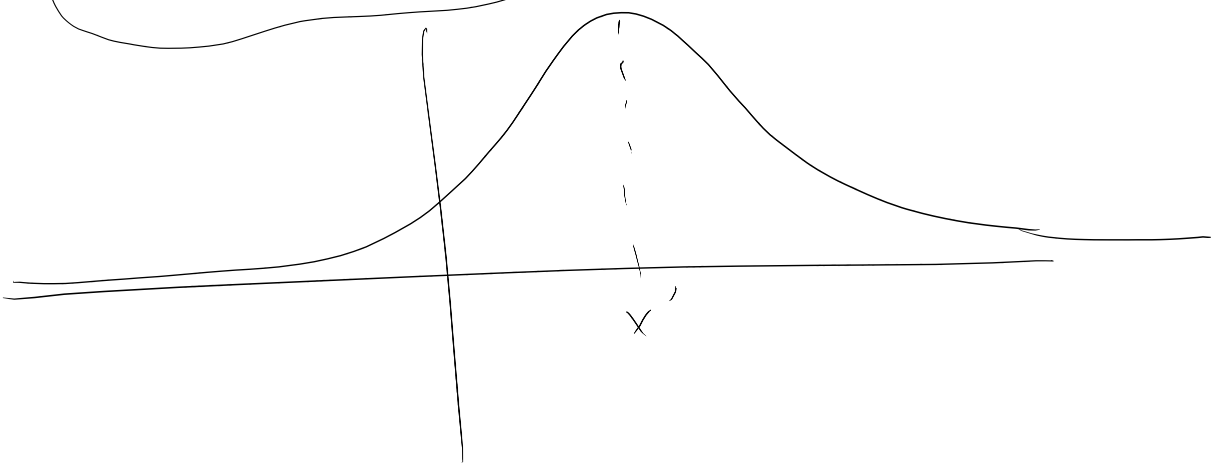
$$\int_{\text{all space}} d^3r'' \int_{\text{all space}} d^3r' \; f(\vec{r}\,' - \vec{r}\,'') \dots$$

$$\vec{\delta} = \vec{r}\,' - \vec{r}\,''$$

$$\Rightarrow \int d^3r' \; f(\vec{r}\,' - \vec{r}\,'') = \int d^3\delta \; f(\vec{\delta})$$

$$\int d^3r'' \int d^3\delta' \, \rho(\vec{\delta}) \dots = \int d^3\delta \; f(\vec{\delta}) \int d^3r'' \dots$$

$$\int_{-\infty}^{\infty} dx' \int_{-\infty}^{\infty} dx \; \frac{1}{(x-x')^2 + \alpha^2} \; f(x')$$



$$\phi(\vec{r}) = \int d^3r' \, \frac{\rho(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|} + \int d^3r' \, \frac{\vec{P}(\vec{r}\,') \cdot (\vec{r} - \vec{r}\,')}{|\vec{r} - \vec{r}\,'|^3}$$

$$\vec{P}(\vec{r}) = \frac{1}{V'} \int_{V'} d^3r' \, \rho(\vec{r}\,') (\vec{r}\,' - \vec{r}) \; d^3r'$$

$$\rho_b = -\vec{\nabla}\cdot\vec{P}(\vec{r})$$
$$\sigma_b = \vec{P}(\vec{r})\cdot\hat{n}$$

$$\vec{E} = -\vec{\nabla}\Phi$$

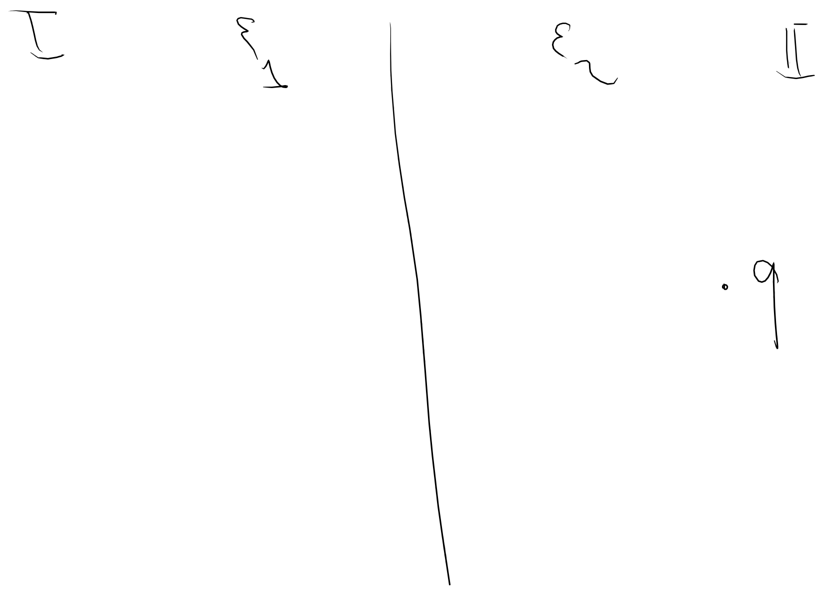
$$\vec{\nabla}\cdot\vec{E} = 4\pi\left(\rho(\vec{r}) + \rho_b(\vec{r})\right)$$
$$= 4\pi\left(\rho(\vec{r}) - \vec{\nabla}\vec{P}\right)$$

$$\vec{\nabla}\cdot\underbrace{\left(\vec{E} + 4\pi\vec{P}\right)}_{\vec{D}(\vec{r})\text{: displacement field.}} = 4\pi\rho(\vec{r})$$

$$\boxed{\vec{P}(\vec{r}) = \chi_e\vec{E}(\vec{r})}$$

χ_e : susceptibility

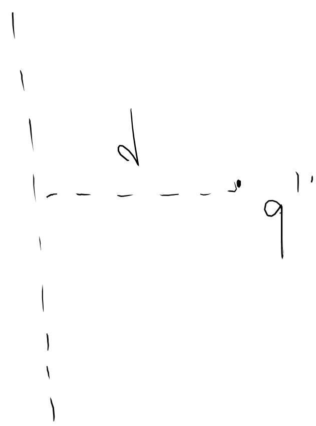
$$\vec{D} = \underbrace{\left(1 + 4\pi\chi_e\right)}_{\varepsilon\text{ : permittivity}}\vec{E}$$



In region II

$$q' = \frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_2 + \varepsilon_1} q$$

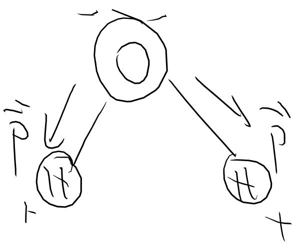
In region I



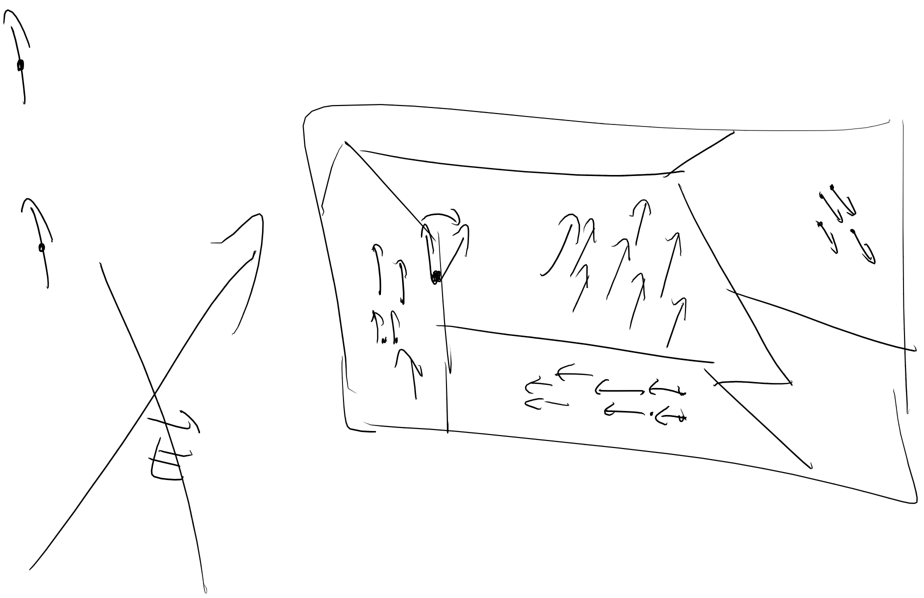
$$q'' = \frac{2\varepsilon_1}{\varepsilon_1 + \varepsilon_2} q$$

CO_2

H_2O



$U = -\vec{E}\vec{p}$

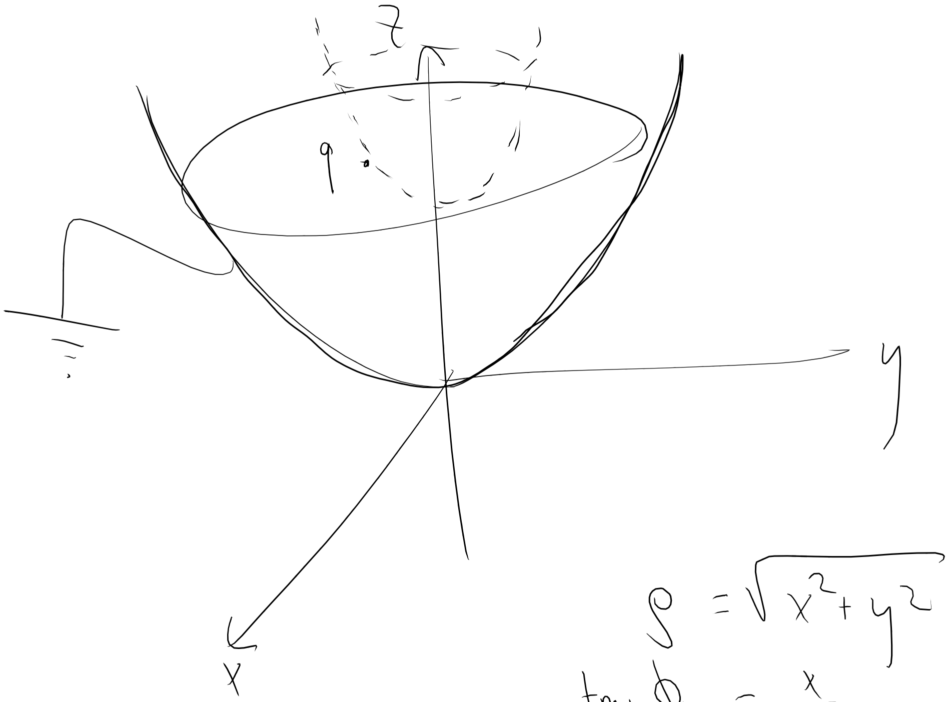


piezoelectric } materials
pyroelectric }

$\boxed{n \simeq \sqrt{\varepsilon}}$

$n = \sqrt{\varepsilon \mu}$

$\mu \simeq 1$



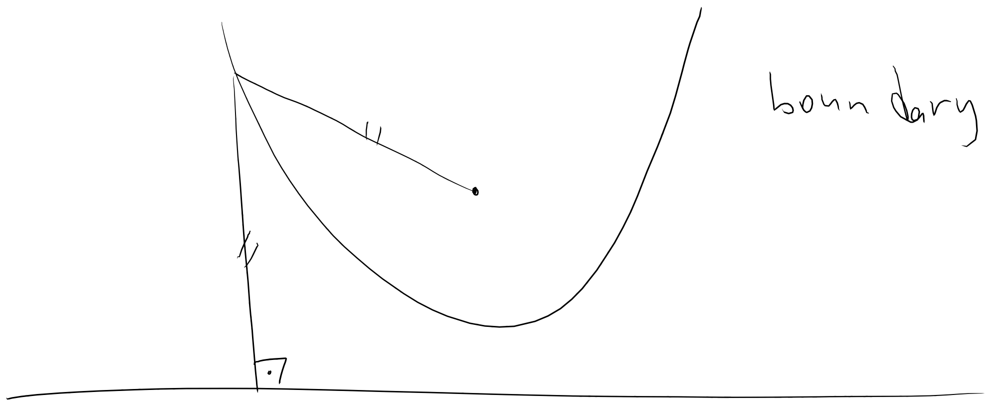
$$\rho = \sqrt{x^2 + y^2}$$

$$\tan\phi = \frac{x}{y}$$

$$z = (x^2 + y^2)$$

$$u = z - (x^2 + y^2)$$

boundary $u = 0$



$$\sigma = \frac{Q}{4\pi abc}\left(\frac{x^2}{a^4}+\frac{y^2}{b^4}+\frac{z^2}{c^4}\right)^{-1/2}$$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

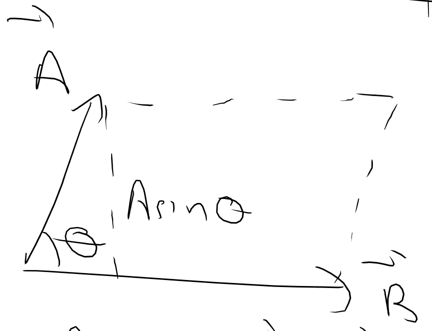
$$\boxed{\vec{E} = 4\pi\sigma\,\hat{n}}$$

$$Q = \int \sigma\, dS$$

$$dS = ?$$

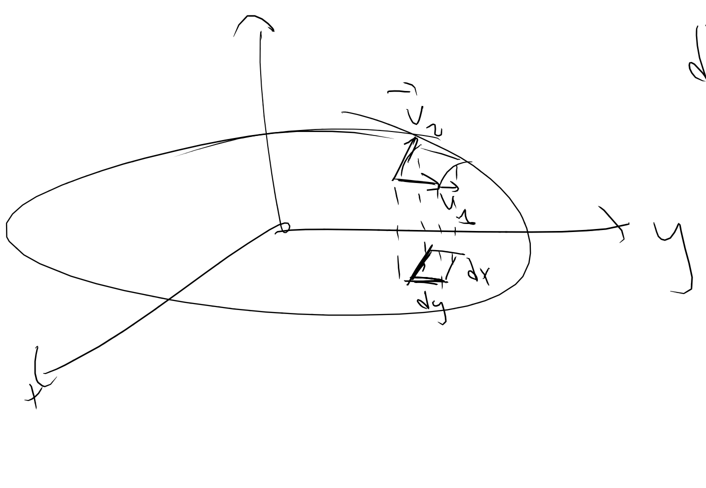
$$\boxed{dS = R^2\, d\Omega}$$

$$|\vec{A}\times\vec{B}| = AB\sin\theta$$



$$\text{Area} = |\vec{A}\times\vec{B}|$$

$$d\vec{S} = \vec{v}_1\times\vec{v}_2$$



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

$$\to (x, y, z) \nearrow (x+dx,\ y+dy,\ z+dz)$$

$$d\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}\right) = 0$$

$$\frac{2x\,dx}{a^2} + \frac{2y\,dy}{b^2} + \frac{2z\,dz}{c^2} = 0$$

$$\boxed{\frac{x\,dx}{a^2} + \frac{y\,dy}{b^2} + \frac{z\,dz}{c^2} = 0}$$

por $\vec{v}_1$ $\quad dx = 0$

$$\vec{v}_1 = 0\,\hat{x} + (dy)_1\,\hat{y} + (dz)_1\,\hat{z}$$

$$\frac{y\,(dy)_1}{b^2} + \frac{z\,(dz)_1}{c^2} = 0$$

$$(dz)_1 = -\frac{c^2}{b^2}\frac{y}{z}(dy)_1$$

$$\boxed{\vec{v}_1 = dy\,\hat{y} - \frac{c^2}{b^2}\frac{y}{z}\,dy\,\hat{z}}$$

por $\vec{v}_2$ $\quad (dy)_2 = 0$

$$\vec{v}_2 = (dx)_2\,\hat{x} + (dz)_2\,\hat{z}$$

$$\frac{x\,dx}{a^2} + \frac{y\,dy}{b^2} + \frac{z\,dz}{c^2} = 0$$

$$\frac{x\,(dx)_2}{a^2} + \frac{z\,(dz)_2}{c^2} = 0$$

$$(dz)_2 = -\frac{c^2}{a^2}\frac{x}{z}(dx)_2$$

$$\boxed{\vec{v}_2 = dx\,\hat{x} - \frac{c^2}{a^2}\frac{x}{z}\,dx\,\hat{z}}$$

$$\vec{v}_1 = dy\,\hat{y} - \frac{c^2}{b^2}\frac{y}{z}\,dy\,\hat{z}$$

$$d\vec{S} = \vec{v}_1 \times \vec{v}_2$$

$$= dx\,dy\left[-\hat{z} - \frac{c^2}{a^2}\frac{x}{z}\hat{x} - \frac{c^2}{b^2}\frac{y}{z}\hat{y}\right]$$

$$dS = dx\,dy\left[1 + \frac{c^4}{a^4}\frac{x^2}{z^2} + \frac{c^4}{b^4}\frac{y^2}{z^2}\right]^{1/2}$$

$$= \frac{dx\,dy}{z^2}c^2\left[\frac{x^2}{a^4} + \frac{y^2}{b^2} + \frac{z^2}{c^4}\right]^{1/2}$$

$$\sigma\,dS = \frac{Q}{4\pi abc}\frac{dx\,dy\,c^2}{z\,c}$$

$$\begin{cases} z = ct \\ x = au \\ y = bv \end{cases} \qquad u^2+v^2 = \left(1-\frac{z^2}{c^2}\right)(\,)$$

$$\sigma \, dS = \frac{Q}{4\pi ab} \cancel{c} \frac{dx\,dy}{\cancel{c}\sqrt{1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}}}$$

$$= \frac{Q}{4\pi} \frac{du\,dv}{\sqrt{1-u^2-v^2}}$$

$$\rho = \sqrt{u^2+v^2}$$

$$\tan\phi = \frac{u}{v}$$

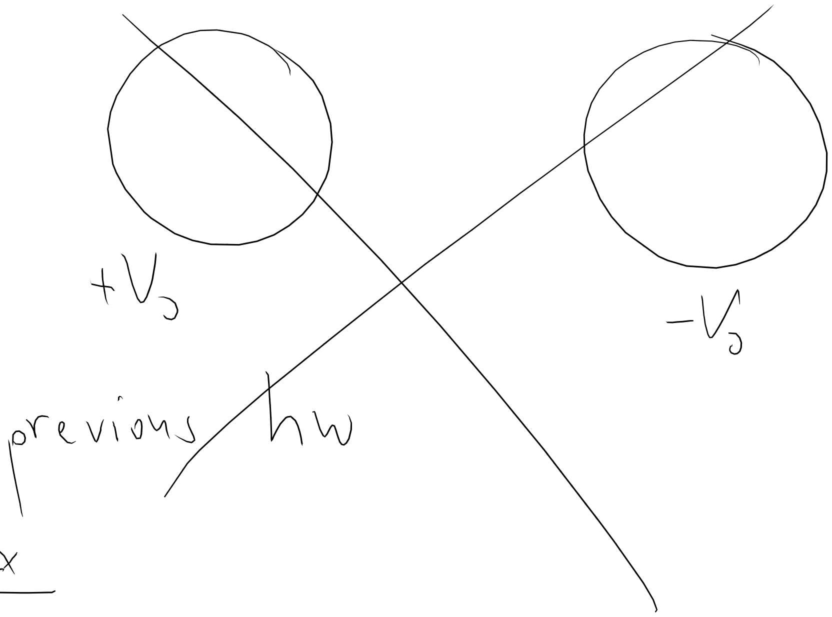
$$\sigma \, dS = \frac{Q}{4\pi} \frac{\rho \, d\rho \, d\phi}{\sqrt{1-\rho^2}}$$

$$\int \sigma \, dS = \frac{Q}{4\pi} 2\pi \int_0^1 \frac{\rho \, d\rho}{\sqrt{1-\rho^2}}$$

$$= \frac{Q}{2} \left(-\sqrt{1-\rho^2} \right|_{\rho=0}^{1}$$

$$= \frac{Q}{2}$$

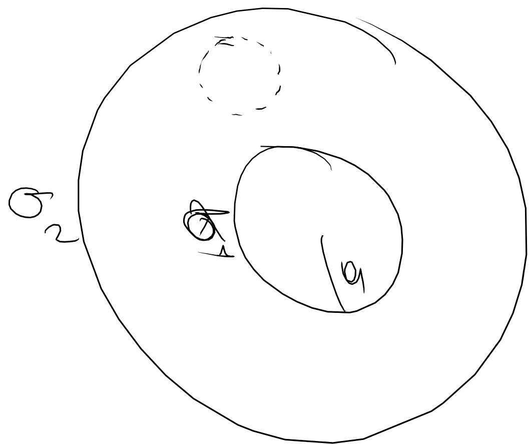
total charge in half of the elipsoid (above the xy plane)



<u>Ex</u>

$+\lambda$ $\quad\quad$ $-\lambda$

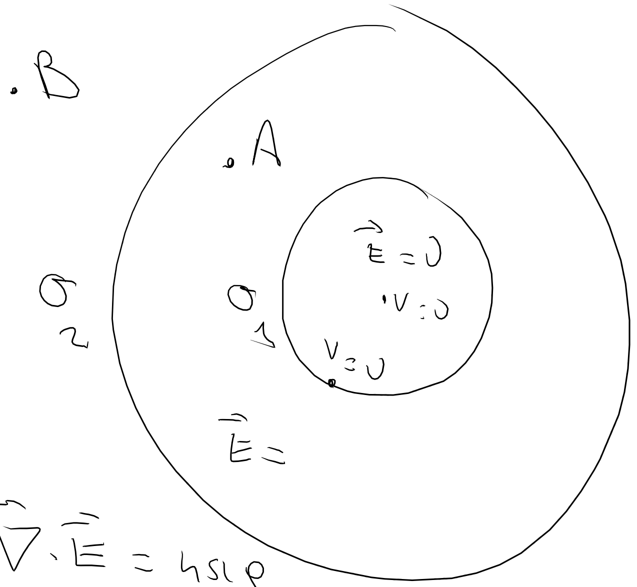
$V =$

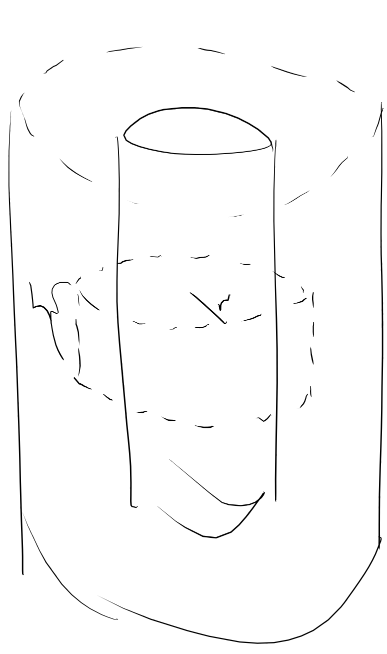


$$\frac{1}{A}\int V(\vec{r}')\,dS = V(r)$$

$$V(\vec{r}') = \sigma_1 2\pi a \ln\left(\frac{r'}{a}\right)$$

$$r' = \sqrt{x^2+y^2}$$





$$\vec{\nabla}\cdot\vec{E} = 4\pi\rho$$

$$\oint \vec{E}\cdot d\vec{S} = E\,\cancel{2\pi} r \cancel{h} = 4\pi(\cancel{2\pi} a \cancel{h})\,\sigma_1$$

$$\boxed{E = \frac{4\pi a\,\sigma_1}{r}}$$

$$V(\vec{r}_A) - V(\vec{r}=0) = -\int_{\vec{0}}^{\vec{r}_A} \vec{E}\cdot d\vec{l} = -\int_a^{r_A} \frac{4\pi a\,\sigma_1}{r}\,dr$$

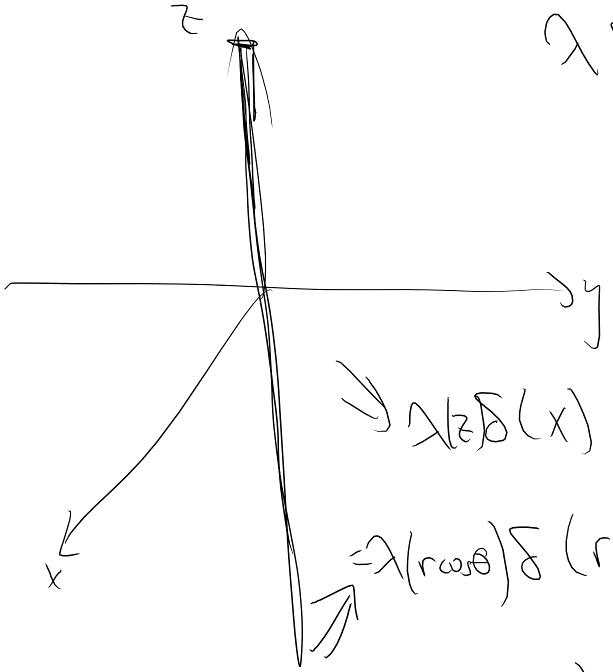
$$\boxed{V(\vec{r}_A) = -4\pi a\,\sigma_1 \ln\left(\frac{r_A}{a}\right)}$$

$$V(\vec{r}_B) - V(|\vec{r}|=b) = -\int_{|\vec{r}|=b}^{\vec{r}_B} \frac{4\pi}{r}\left[a\sigma_1 + b\sigma_2\right]\cdot dr$$

$$V(|\vec{r}|=b) = -4\pi a\,\sigma_1 \ln\left(\frac{b}{a}\right)$$

$$V(\vec{r}_B) = -4\pi a\,\sigma_1 \ln\left(\frac{b}{a}\right) - 4\pi\left[a\sigma_1 + b\sigma_2\right]\ln\left(\frac{r_B}{b}\right)$$

$$= -4\pi a\,\sigma_1 \ln\left(\frac{r_B}{a}\right) - 4\pi b\,\sigma_2 \ln\left(\frac{r_B}{b}\right)$$



$$\lambda\delta(x)\delta(y)$$

$$x = r\sin\theta\cos\phi$$
$$y = r\sin\theta\sin\phi$$

$$\Rightarrow \lambda(z)\delta(x)\delta(y)$$

$$= \lambda(r\cos\theta)\,\delta(r\sin\theta\cos\phi)\,\delta(r\sin\theta\sin\phi)$$

$$= \lambda(r\cos\theta)\quad\delta(\sin\theta)\,A$$

$$= \lambda(r\cos\theta)\left(\delta(\theta)+\delta(\theta-\pi)\right)A'$$

$$\delta(f(x)) = \sum \frac{\delta(x-x_0)}{|f'(x_0)|}$$

$$f(x_0) = 0$$