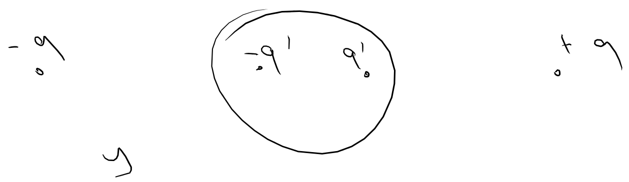
$$\boxed{\vec{p} = \sum q_i \vec{r_i}}$$

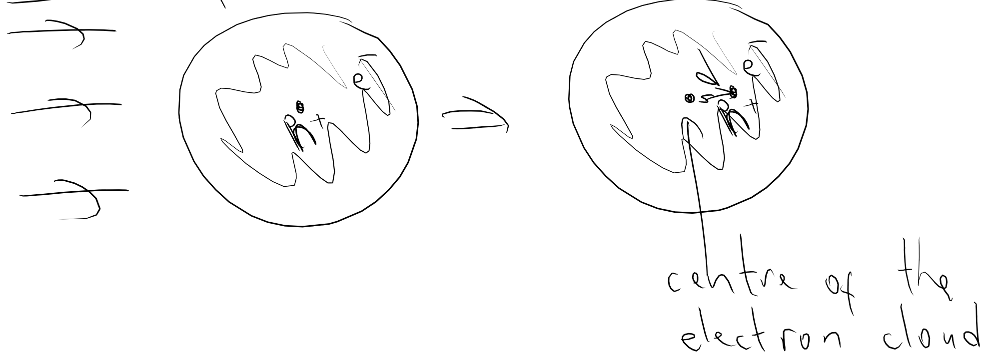


$$\vec{p} = (q)\vec{0} + (2q)\, d\hat{x}$$

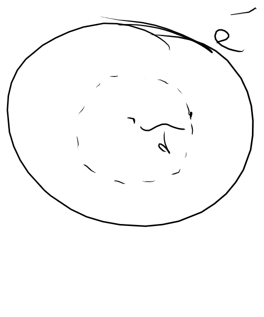
$$\boxed{\vec{p} = 2qd\hat{x}}$$

Polarization

Example



$$|\vec{p}| = qd$$



$$\frac{1}{4\pi\varepsilon_0}\left(\frac{q}{\cancel{\frac{4}{3}\pi R^3}}\right)\cancel{\frac{4}{3}\pi d^3}\,\frac{1}{d^2} = E$$

$$\boxed{\frac{(qd)}{4\pi\varepsilon_0 R^3} = E}$$

$$\boxed{\vec{p} = (4\pi\varepsilon_0 R^3)\vec{E}}$$

$$\vec{p} = 3\varepsilon_0 v \vec{E}$$

$\boxed{v : \text{volume of the atom}}$

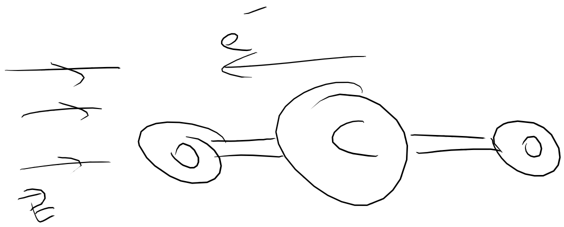
$$v = \frac{4}{3}\pi R^3$$

$$4\pi R^3 = 3v$$

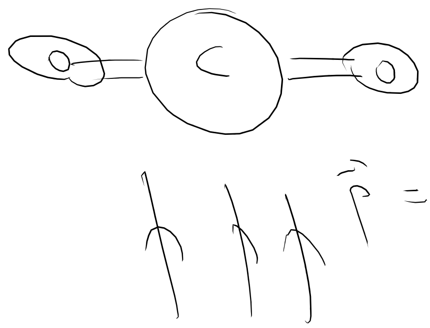
$\vec{p}$: dipole moment created by displacement

$$\vec{p}(\vec{E}) = \alpha \vec{E} + O(\vec{E}^2)$$

susceptibility



$$\vec{p} = \alpha_{\parallel} \vec{E}_{\parallel}$$



$$\vec{p} = \alpha_{\perp} \vec{E}_{\perp}$$

$$\vec{E} = \vec{E}_{\parallel} + \vec{E}_{\perp}$$

$$\Downarrow$$

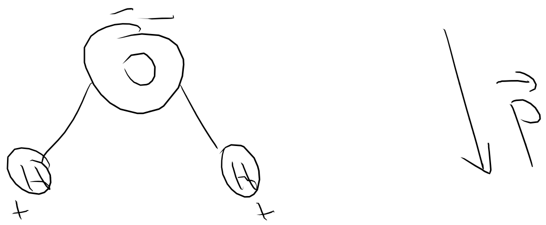
$$\vec{p} = \alpha_{\parallel} \vec{E}_{\parallel} + \alpha_{\perp} \vec{E}_{\perp} \neq \alpha \vec{E}$$

$$p_x = \alpha_{xx} E_x + \alpha_{xy} E_y + \alpha_{xz} E_z$$

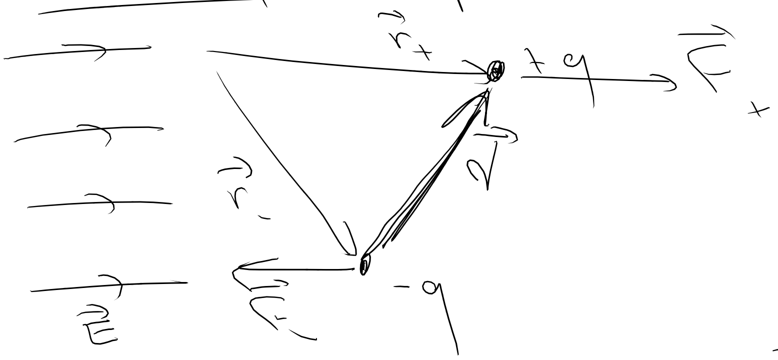
$$p_y = \ldots$$

α_{ij} : susceptibility tensor.

Polarization by Rotation



Simple Dipole in an $\vec{E}$ field



If $\vec{E}$ is uniform, $\vec{F}_+ = -\vec{F}_-$

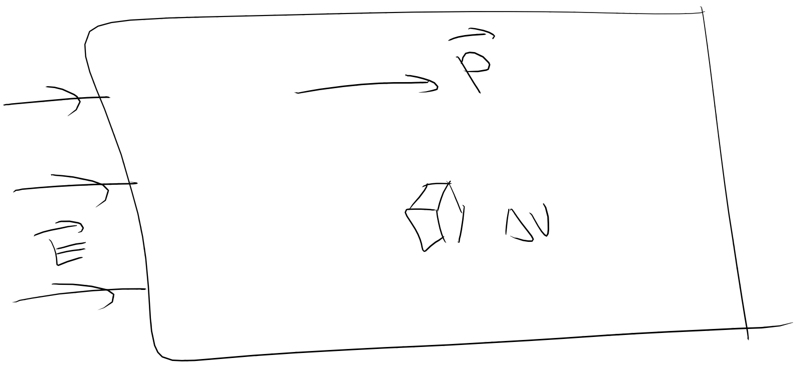
$$\vec{F}_{tot} = 0$$

$$\vec{\tau} = \vec{r}_+ \times \vec{F}_+ + \vec{r}_- \times \vec{F}_-$$

$$= (\vec{r}_+ - \vec{r}_-) \times q\vec{E}$$

$$= (q\vec{d}) \times \vec{E}$$

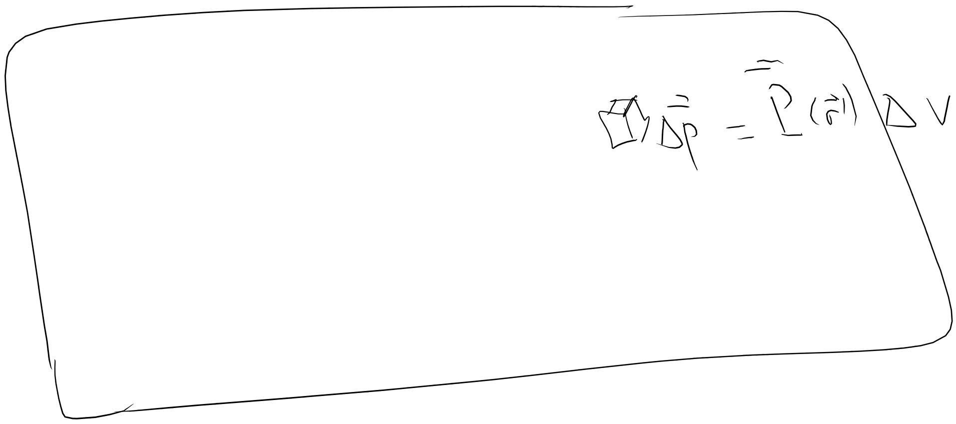
$$\boxed{\vec{\tau} = \vec{p} \times \vec{E}}$$



$\vec{P}(\vec{r})$ polarization = dipole moment density
= dipole moment per unit volume.

$$\boxed{\vec{P}(\vec{r}) = \frac{\Delta \vec{p}}{\Delta V}}$$

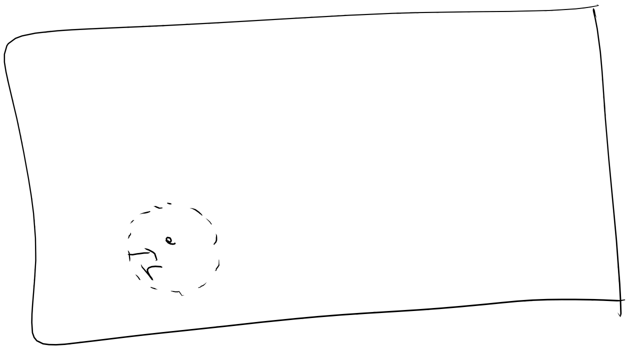
Electric Field Created by Polarization



$\cdot\vec{r}$

$$\Delta V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{(\Delta\vec{p})\cdot(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3} = \frac{1}{4\pi\varepsilon_0}\frac{\vec{P}(\vec{r}')\cdot(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}\Delta V$$

$$\boxed{V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\vec{P}(\vec{r}')\cdot(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}\, d^3r'}$$



$$\boxed{V(\vec r) = \frac{1}{4\pi\varepsilon_0}\int \vec P(\vec r')\cdot\frac{(\vec r-\vec r')}{|\vec r-\vec r'|^3}\,d^3r'}$$

whether $\vec r$ is inside or outside the material.

$$\frac{\vec r-\vec r'}{|\vec r-\vec r'|^3} = \vec\nabla_{\vec r'}\frac{1}{|\vec r-\vec r'|}$$

$$V(\vec r) = \frac{1}{4\pi\varepsilon_0}\int \vec P(\vec r')\cdot\vec\nabla_{\vec r'}\left(\frac{1}{|\vec r-\vec r'|}\right)d^3r'$$

$$= \frac{1}{4\pi\varepsilon_0}\int\left[\vec\nabla_{\vec r'}\cdot\left[\frac{\vec P(\vec r')}{|\vec r-\vec r'|}\right] - \frac{1}{|\vec r-\vec r'|}\vec\nabla_{\vec r'}\cdot\vec P(\vec r')\right]d^3r'$$

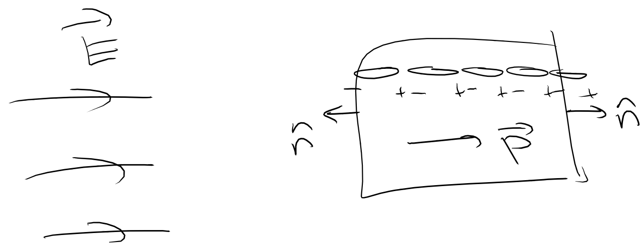
$$= \frac{1}{4\pi\varepsilon_0}\oint\frac{\vec P\cdot d\vec S}{|\vec r-\vec r'|} + \frac{1}{4\pi\varepsilon_0}\int\frac{\left(-\vec\nabla_{\vec r'}\cdot\vec P(\vec r')\right)}{|\vec r-\vec r'|}d^3r'$$

$d\vec S = dS\,\hat n$

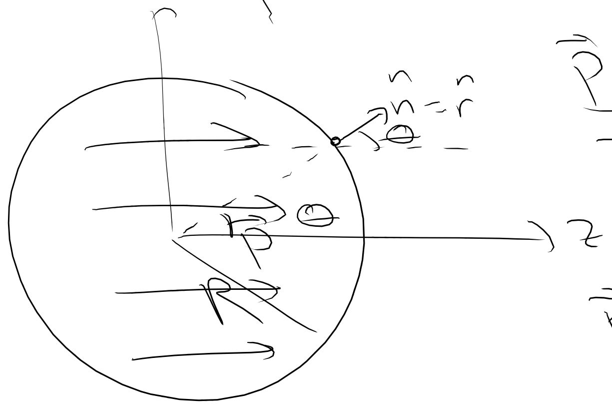
$$V(\vec r) = \frac{1}{4\pi\varepsilon_0}\oint\frac{\sigma_b\,dS}{|\vec r-\vec r'|} + \frac{1}{4\pi\varepsilon_0}\int\frac{\rho_b(\vec r')}{|\vec r-\vec r'|}d^3r'$$

$$\boxed{\sigma_b = \vec P\cdot\hat n} \qquad \boxed{\rho_b = -\vec\nabla\cdot\vec P}$$

$$\vec{P} \Longleftrightarrow \sigma_b = \vec{P}\cdot\hat{n} \quad \rho_b = -\vec{\nabla}\cdot\vec{P}$$



<u>Ex</u> Electric field of a uniformly polarized sphere



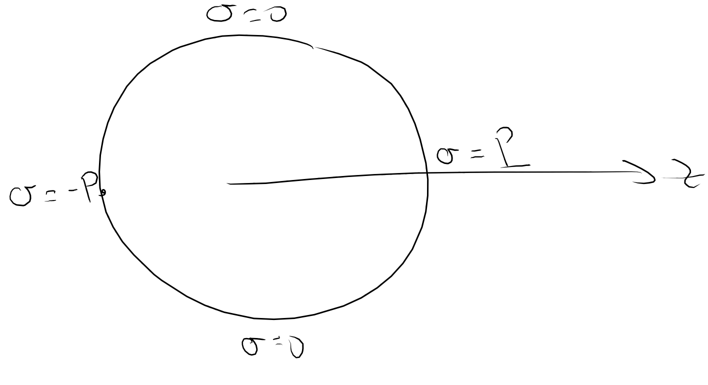
$\vec{P} = \text{constant}$

$\vec{P} = P\hat{z}$

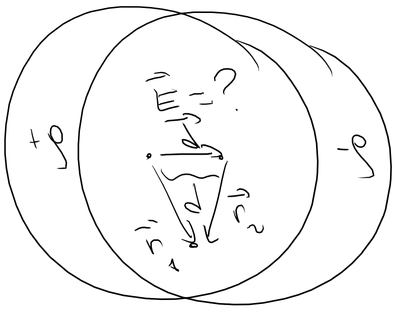
$\vec{p} = \vec{P}\,\frac{4}{3}\pi R^3$

$$\rho_b = -\vec{\nabla}\cdot\vec{P} = 0$$

$$\sigma_b = \vec{P}\cdot\hat{n} = P\cos\Theta$$

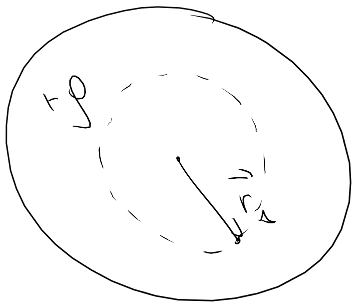


Ex



$$\vec{E} = \frac{1}{3\varepsilon_0}(+\rho)\vec{r}_1 + \frac{1}{3\varepsilon_0}(-\rho)\vec{r}_2$$

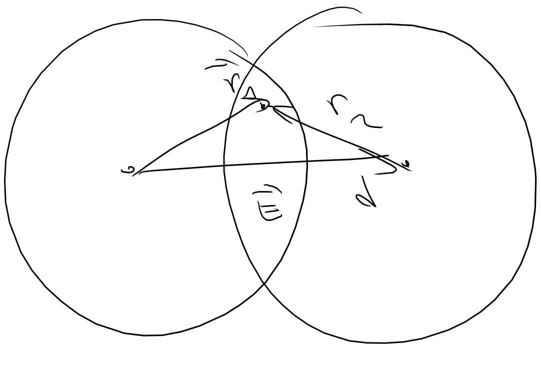
$$\vec{E} = \frac{1}{3\varepsilon_0}\rho(\vec{r}_1 - \vec{r}_2) = \frac{1}{3\varepsilon_0}\rho\vec{d}$$



$$\vec{E}_+ = \frac{1}{\cancel{4\pi}\varepsilon_0}\frac{\left(\frac{\cancel{4}}{3}\cancel{\pi} r_1^{\cancel{3}}\rho\right)}{\cancel{r_1^2}}\hat{r}_1$$

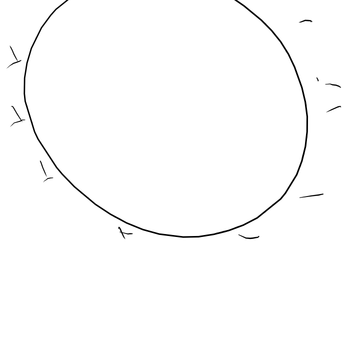
$$= \frac{1}{3\varepsilon_0}\rho\, r_1 \hat{r}_1$$

$$\boxed{\vec{E}_+ = \frac{1}{3\varepsilon_0}\rho\vec{r}_1}$$



$$\vec{E} = \frac{1}{3\varepsilon_0}\rho\vec{d}$$

Take the limit $\left.\begin{matrix}\rho \to \infty \\ d \to 0\end{matrix}\right\}$ $\rho d = \text{const.}$



$\sigma \propto \cos\theta$