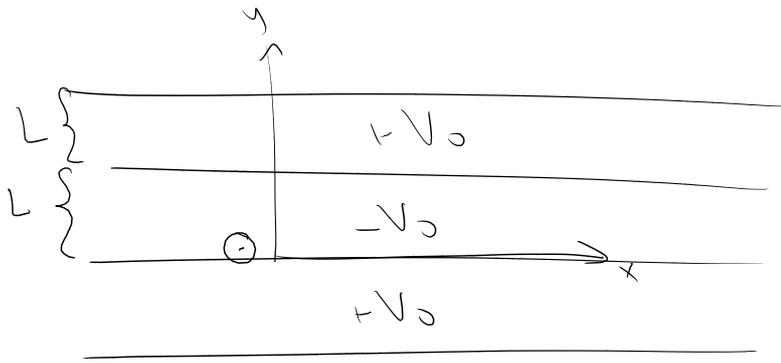


3)



$$V(x, y+2L, z) = V(x, y, z) \quad V(x, -y, z) = -V(x, y, z)$$

$$V(x, y, z) = V(y, z) \quad V(x, y, -z) = V(x, y, z)$$

$$V(x, y, z \rightarrow \pm\infty) = 0$$

a) $V(y, z) = Y(y)Z(z)$

$$\nabla^2 V = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) V(y, z)$$

$$= \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

$$= \left(\frac{\partial^2 Y}{\partial y^2} \right) Z(z) + Y(y) \left(\frac{\partial^2 Z}{\partial z^2} \right)$$

$$\frac{0}{YZ} = \frac{Y'' Z + Y Z''}{YZ}$$

$$\frac{Y''}{Y} + \frac{Z''}{Z} = 0$$

$$\frac{Y''}{Y} = -\frac{Z''}{Z} = C_1$$

$$Y'' = C_{\Delta} Y$$

$$Z'' = -C_{\Delta} Z$$

b) $V(x, y+2L, z) = V(x, y, z)$

$$V(x, y, z) = V(y, z) = Y(y)Z(z)$$

$$V(x, y, z \rightarrow \pm\infty) = 0$$

$Y(y+2L) = Y(y)$ $Z(z \rightarrow \pm\infty) = 0$	$\Rightarrow C_{\Delta} < 0$ $C_{\Delta} = -k^2$
$Y(y \rightarrow -y) = -Y(y)$	

$$Y'' = -k^2 Y \Rightarrow Y = A_k \sin(ky + \delta_k)$$
$$= a_k \sin(ky) + b_k \cos(ky)$$

$$Y(-y) = -Y(y) \Rightarrow b_k = 0$$

$$Y(y) = a_k \sin(ky)$$

$$Y(y+2L) = Y(y)$$

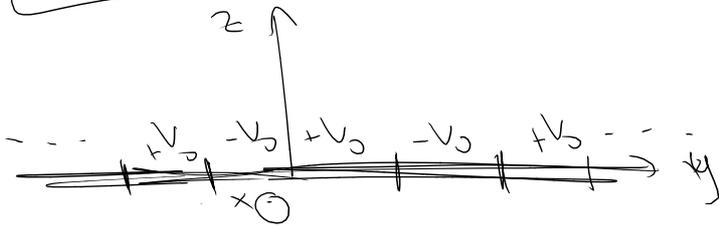
$$a_k \sin(ky + k2L) = a_k \sin(ky)$$

$$2Lk = n2\pi \Rightarrow k_n = n \frac{\pi}{L}$$

$$Y_n(y) = a_n \sin\left(n \frac{\pi}{L} y\right)$$

$$z'' = k^2 z = \left(\frac{ns}{L}\right)^2 z$$

$$z = c_n e^{\frac{ns}{L}z} + d_n e^{-\frac{ns}{L}z}$$



$$z^>(z < 0) = c_n e^{\frac{ns}{L}z} + d_n e^{-\frac{ns}{L}z}$$

$$z^<(z > 0) = c_n e^{\frac{ns}{L}z} + d_n e^{-\frac{ns}{L}z}$$

$$z(z \rightarrow +\infty) = \lim_{z \rightarrow +\infty} (c_n e^{\frac{ns}{L}z} + d_n e^{-\frac{ns}{L}z}) = 0$$

$$c_n = 0$$

$$z(z \rightarrow -\infty) = \lim_{z \rightarrow -\infty} (c_n e^{\frac{ns}{L}z} + d_n e^{-\frac{ns}{L}z}) = 0$$

$$d_n = 0$$

$$z(z) = \begin{cases} d_n e^{-\frac{ns}{L}z} & z < 0 \\ c_n e^{\frac{ns}{L}z} & z > 0 \end{cases}$$

$z(z)$ is continuous at $z = 0$.

$$\lim_{z \rightarrow 0^-} z(z) = \lim_{z \rightarrow 0^+} z(z)$$

$$c_n = d_n$$

$$= d_n$$

$$\Rightarrow z(z) = c_n e^{-\frac{ns}{L}|z|}$$

$$= c_n e^{-\frac{ns}{L}|z|}$$

$$V(y, z) = \sum_{n=1}^{\infty} a_n e^{-\frac{n\pi}{L}|z|} \sin\left(\frac{n\pi}{L}y\right)$$

$$V(y, z=0) = \begin{cases} +V_0 & 0 < y < L \\ -V_0 & -L < y < 0 \end{cases}$$

$$V_0 = \sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi}{L}y\right) \quad \text{for } 0 < y < L$$

$$\int_0^L dy \sin\left(\frac{n\pi}{L}y\right) \sin\left(\frac{m\pi}{L}y\right) = \frac{L}{2} \delta_{n,m}$$

$$\int_0^L dy \sin\left(\frac{m\pi}{L}y\right) V_0 = \sum_{n=1}^{\infty} a_n \int_0^L dy \sin\left(\frac{n\pi}{L}y\right) \sin\left(\frac{m\pi}{L}y\right)$$

$$\frac{2V_0}{m\pi} \left[-\cos\left(\frac{m\pi}{L}y\right) \right]_{y=0}^L = \sum_{n=1}^{\infty} a_n \frac{L}{2} \delta_{n,m}$$

$$\frac{2V_0}{m\pi} \left[-\cos(m\pi) + 1 \right] = a_m$$

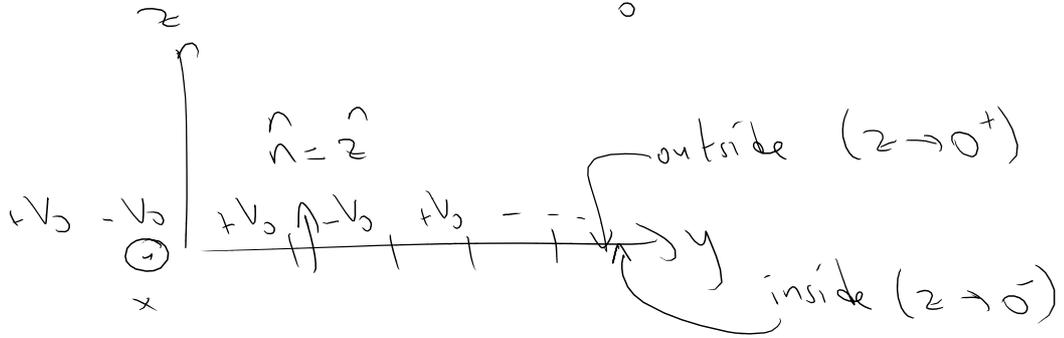
$$a_m = \frac{2V_0}{m\pi} \left[1 - (-1)^m \right]$$

$$a_m = \frac{2V_0}{m\pi} \begin{cases} 2 & \text{if } m \text{ is odd} \\ 0 & \text{if } m \text{ is even} \end{cases}$$

$$V(y, z) = \sum_{\substack{n \text{ odd} \\ n\pi}} \frac{4V_0}{n\pi} e^{-\frac{n\pi}{L}|z|} \sin\left(\frac{n\pi}{L}y\right)$$

$$V(y, z) = \frac{4V_0}{\pi} \sum_{n=0}^{\infty} \frac{1}{(2n+1)} e^{-\frac{(2n+1)\pi}{L}|z|} \sin\left(\frac{(2n+1)\pi}{L}y\right)$$

$$\vec{E}_{\text{outside}} - \vec{E}_{\text{inside}} = \frac{\sigma}{\epsilon_0} \hat{n}$$



$$E_z = -\frac{\partial V}{\partial z}$$

$$E_z^{\text{outside}} = -\frac{\partial}{\partial z} \left[\frac{4V_0}{\pi} \sum_{n=0}^{\infty} \frac{1}{(2n+1)} e^{-\frac{(2n+1)\pi}{L}z} \sin\left(\frac{(2n+1)\pi}{L}y\right) \right]$$

$$= \frac{4V_0}{\pi} \sum_{n=0}^{\infty} \frac{1}{2n+1} \left(+\frac{(2n+1)\pi}{L} \right) \sin\left(\frac{(2n+1)\pi}{L}y\right)$$

$$E_z^{\text{outside}} = \frac{4V_0}{L} \sum_{n=0}^{\infty} \sin\left(\frac{(2n+1)\pi}{L}y\right)$$

$$E_z^{\text{inside}} = -\frac{\partial}{\partial z} \left[\frac{4V_0}{\pi} \sum_{n=0}^{\infty} \frac{1}{(2n+1)} e^{-\frac{(2n+1)\pi}{L}(L-z)} \sin\left(\frac{(2n+1)\pi}{L}y\right) \right]$$

$$E_z^{\text{inside}} = -\frac{4V_0}{L} \sum_{n=0}^{\infty} \sin\left(\frac{(2n+1)\pi}{L}y\right)$$

$$\sigma = \epsilon_0 (E_z^{\text{outside}} - E_z^{\text{inside}})$$

$$\sigma = 8\epsilon_0 \frac{V_0}{L} \sum_{n=0}^{\infty} \sin\left(\frac{(2n+1)\pi}{L}y\right)$$

$$V(y, z) = \frac{4V_0}{\pi} \sum_{n=0}^{\infty} \frac{1}{(2n+1)} e^{-\frac{(2n+1)\pi}{L}|z|} \sin\left(\frac{(2n+1)\pi}{L}y\right)$$

$$\nabla^2 V = -\frac{\rho}{\epsilon_0} \Rightarrow \rho = -\epsilon_0 \nabla^2 V$$

$$= -\epsilon_0 \left(\frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} \right)$$

$$\frac{\partial^2 V}{\partial y^2} = \frac{4V_0}{\pi} \sum_{n=0}^{\infty} \frac{1}{(2n+1)} \left[-\frac{(2n+1)^2 \pi^2}{L^2} e^{-\frac{(2n+1)\pi}{L}|z|} \sin\left(\frac{(2n+1)\pi}{L}y\right) \right]$$

$$\frac{\partial^2 V}{\partial z^2} = \frac{4V_0}{\pi} \sum_{n=0}^{\infty} \frac{1}{(2n+1)} \left[\frac{(2n+1)\pi}{L} e^{-\frac{(2n+1)\pi}{L}|z|} \sin\left(\frac{(2n+1)\pi}{L}y\right) \right]$$

$$= -\frac{(2n+1)\pi}{L} \left\{ \frac{(2n+1)\pi}{L} e^{-\frac{(2n+1)\pi}{L}|z|} \sin\left(\frac{(2n+1)\pi}{L}y\right) \right\}$$

$$\frac{\partial^2 V}{\partial z^2} = \frac{4V_0}{\pi} \begin{cases} 2 & 0 < z < L \\ -2 & z > L \end{cases} = \begin{cases} 1 & 0 < z < L \\ -1 & z > L \end{cases}$$

$$\int \frac{\partial^2 V}{\partial z^2} dz = 1$$

$$\frac{\partial V}{\partial z} = \Theta(z) - \Theta(-z)$$

$$\frac{\partial^2 V}{\partial z^2} = \delta(z) - \delta(-z)(-1) = 2\delta(z)$$

$$\begin{aligned}
 &= - (2n+1) \frac{\epsilon_0}{L} \left\{ - (2n+1) \frac{\epsilon_0}{L} e^{- (2n+1) \frac{\pi}{L} |z|} \left(\frac{\epsilon_0 |z|}{2\epsilon} \right)^2 \right. \\
 &\quad \left. - (2n+1) \frac{\epsilon_0}{L} |z| \frac{\epsilon_0}{2\epsilon} \right\} \\
 &= (2n+1)^2 \frac{\epsilon_0^2}{L^2} e^{- (2n+1) \frac{\pi}{L} |z|} - (2n+1) \frac{\epsilon_0}{L} e^{- (2n+1) \frac{\pi}{L} |z|} \quad 2\delta(z)
 \end{aligned}$$

$$\nabla_{\rho}^2 e^{\frac{z}{L}} = \frac{\epsilon_0 V_0}{\epsilon} \sum_{n=0}^{\infty} \frac{1}{(2n+1)} \left[- (2n+1)^2 \frac{\epsilon_0^2}{L^2} e^{- (2n+1) \frac{\pi}{L} |z|} \sin\left(\frac{(2n+1)\pi y}{L}\right) \right]$$

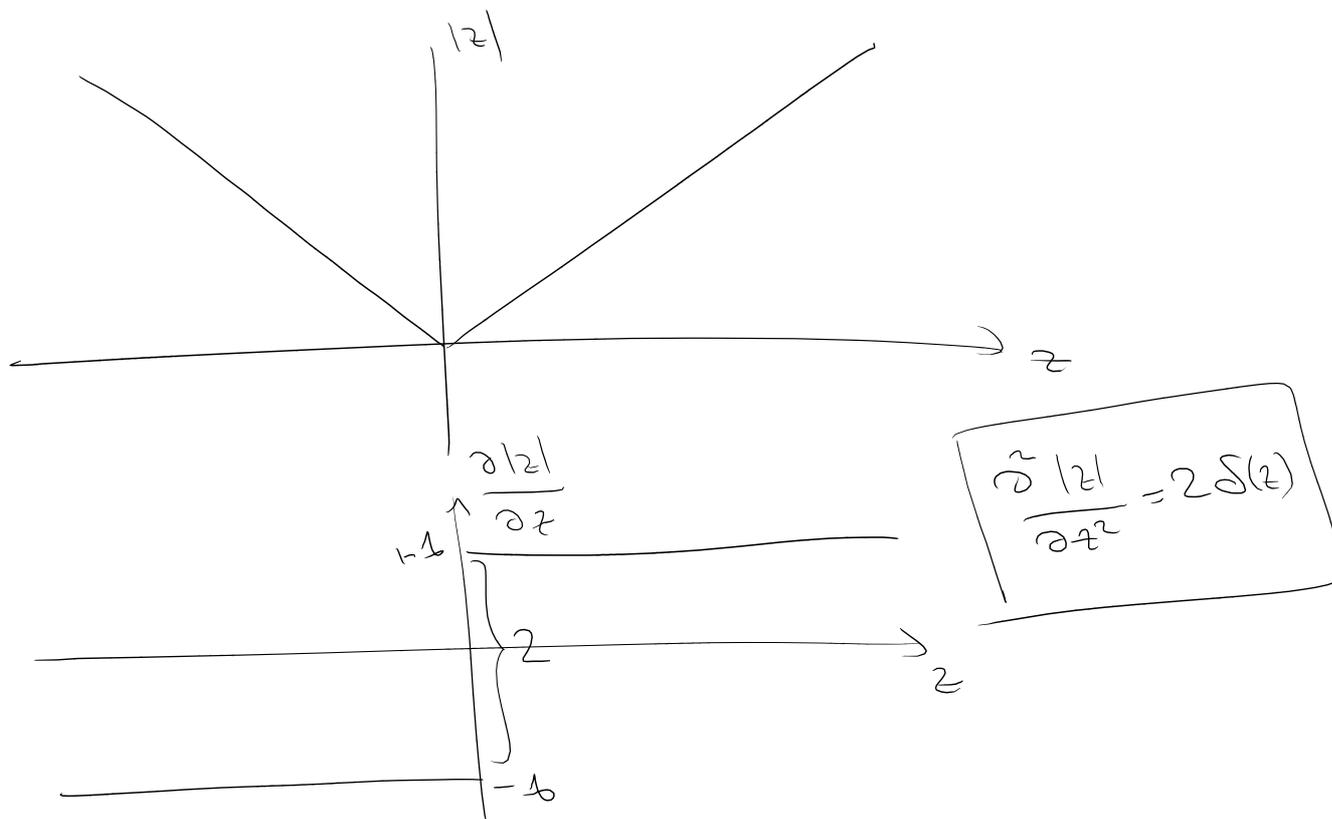
$$\nabla_{\rho}^2 e^{-\frac{z}{L}} = \frac{\epsilon_0 V_0}{\epsilon} \sum_{n=0}^{\infty} \frac{1}{(2n+1)} \left[(2n+1)^2 \frac{\epsilon_0^2}{L^2} e^{- (2n+1) \frac{\pi}{L} |z|} \sin\left(\frac{(2n+1)\pi y}{L}\right) \right]$$

$$+ \frac{\epsilon_0 V_0}{\epsilon} \sum_{n=0}^{\infty} \frac{1}{(2n+1)} \left[- (2n+1) \frac{\epsilon_0}{L} 2\delta(z) e^{- (2n+1) \frac{\pi}{L} |z|} \sin\left(\frac{(2n+1)\pi y}{L}\right) \right]$$

$$\nabla_{\rho}^2 e^{\frac{z}{L}} + \nabla_{\rho}^2 e^{-\frac{z}{L}} = - \frac{\epsilon_0 V_0}{L} \sum_{n=0}^{\infty} \sin\left(\frac{(2n+1)\pi y}{L}\right) \delta(z) = - \frac{\epsilon_0 V_0}{L}$$

$$\delta(z) = \left[\frac{\epsilon_0 V_0}{L} \sum_{n=0}^{\infty} \sin\left(\frac{(2n+1)\pi y}{L}\right) \right] \delta(z)$$

$$\delta = \frac{\epsilon_0 V_0}{L} \sum_{n=0}^{\infty} \sin\left(\frac{(2n+1)\pi y}{L}\right) \delta(z)$$



$$V(y, z) = \frac{4V_0}{\pi} \sum_{n=0}^{\infty} \frac{1}{(2n+1)} e^{-\frac{(2n+1)\pi|z|}{L}} \sin\left(\frac{(2n+1)\pi y}{L}\right)$$

$$\begin{aligned} V(y, z=0) &= \frac{4V_0}{\pi} \sum_{n=0}^{\infty} \frac{1}{(2n+1)} \sin\left(\frac{(2n+1)\pi y}{L}\right) \\ &= \frac{4V_0}{\pi} \sum_{n=0}^{\infty} \operatorname{Im} \left(\frac{e^{i \frac{(2n+1)\pi y}{L}}}{(2n+1)} \right) \\ &= \frac{4V_0}{\pi} \operatorname{Im} \left(\sum_{n=0}^{\infty} \frac{e^{i \frac{\pi y}{L}}}{(2n+1)} \right)^{(2n+1)} \end{aligned}$$

$$f(z) = \sum_{n=0}^{\infty} \frac{z^{2n+1}}{2n+1} = z + \frac{z^3}{3} + \dots \quad f(z=0) = 0$$

$$\frac{df}{dz} = \sum_{n=0}^{\infty} z^{2n} = \frac{1}{1-z^2}$$

$$\frac{df}{dz} = \left(\frac{1}{1-z} + \frac{1}{1+z} \right) \frac{1}{2}$$

$$= \frac{1}{2} \frac{d}{dz} (-\ln|1-z|) + \frac{1}{2} \frac{d}{dz} \ln(1+z)$$

$$= \frac{d}{dz} \frac{1}{2} \ln \left(\frac{1+z}{1-z} \right)$$

$$f(z) = \frac{1}{2} \ln \left(\frac{1+z}{1-z} \right) + C$$

$$0 = f(z=0) = \frac{1}{2} \ln(1) + C = C$$

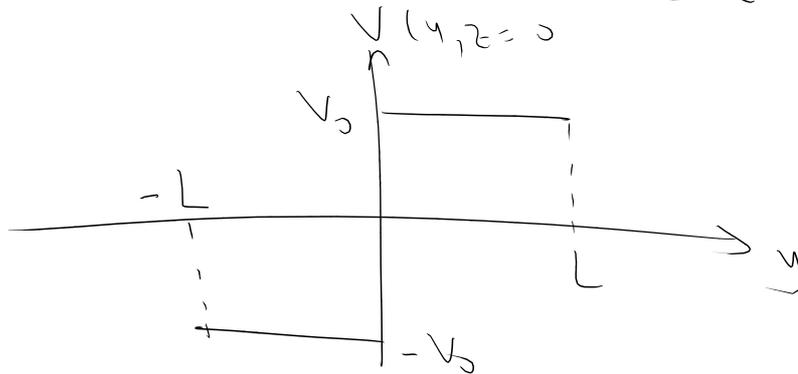
$$f(z) = \frac{1}{2} \ln \left(\frac{1+z}{1-z} \right) = \frac{1}{2} \ln(1+z) - \frac{1}{2} \ln(1-z)$$

$$\ln(1+z) = \sum_{n=1}^{\infty} \frac{z^n}{n}$$

$$V(y, z=0) = \frac{4V_0}{\pi} \operatorname{Im} \ln \frac{\sum_{n=0}^{\infty} \left(e^{i \frac{\pi}{L} y} \right)^{(2n+1)}}{(2n+1)}$$

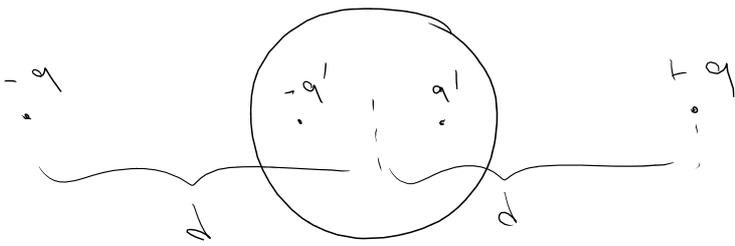
$$V(y, z=0) = \frac{4V_0}{\pi} \operatorname{Im} \ln \left(\frac{1 + e^{i \frac{\pi}{L} y}}{1 - e^{i \frac{\pi}{L} y}} \right)$$

$$V(y, z=0) = \frac{4V_0}{\pi} \operatorname{Im} \ln \left(\frac{1 + e^{\frac{i\pi}{2L}y}}{1 - e^{\frac{i\pi}{2L}y}} \right)$$



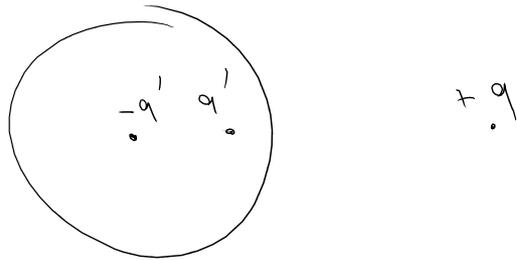
$$\begin{aligned} V(y, z=0) &= \frac{4V_0}{\pi} \operatorname{Im} \ln \left(\frac{1 + e^{\frac{i\pi}{2L}y}}{1 - e^{\frac{i\pi}{2L}y}} \right) \\ &= \frac{4V_0}{\pi} \operatorname{Im} \ln \left(\frac{e^{-\frac{i\pi}{2L}y} + e^{\frac{i\pi}{2L}y}}{e^{-\frac{i\pi}{2L}y} - e^{\frac{i\pi}{2L}y}} \right) \\ &= \frac{4V_0}{\pi} \operatorname{Im} \ln \left(\frac{2 \cos\left(\frac{\pi}{2L}y\right)}{-2i \sin\left(\frac{\pi}{2L}y\right)} \right) \\ &= \frac{4V_0}{\pi} \operatorname{Im} \ln \left(\frac{i \cos\left(\frac{\pi}{2L}y\right)}{\sin\left(\frac{\pi}{2L}y\right)} \right) \end{aligned}$$

$$\ln z = \ln|z| + i \arg(z)$$



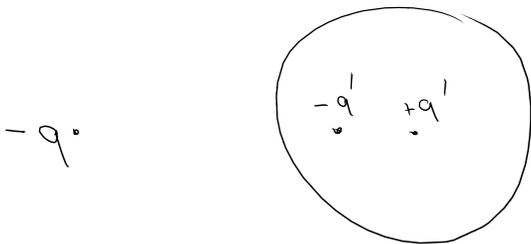
$$q' = -q \frac{R}{d}$$

||

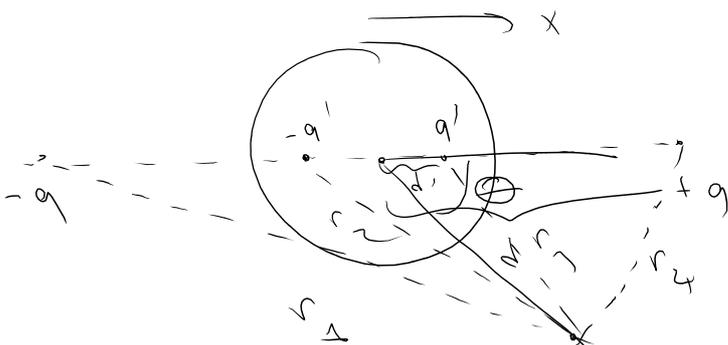


+

$$q' = -q \frac{R}{d}$$



$$V = \frac{1}{4\pi\epsilon_0} \left[\frac{-q}{r_1} + \frac{q}{r_2} - \frac{q'}{r_3} + \frac{q'}{r_4} \right]$$



$$q' = -q \frac{R}{d}$$

$$d' = \frac{R^2}{d}$$

$$r_1 = |\vec{r} - (-d\hat{x})| = \sqrt{(\vec{r} + d\hat{x})^2} = \sqrt{r^2 + d^2 + 2rd\cos\theta}$$

$$r_2 = |\vec{r} - (d'\hat{x})| = \sqrt{r^2 + d'^2 + 2rd'\cos\theta}$$

$$r_3 = |\vec{r} - (d'\hat{x})| = \sqrt{r^2 + d'^2 - 2rd'\cos\theta}$$

$$r_4 = |\vec{r} - (d\hat{x})| = \sqrt{r^2 + d^2 - 2rd\cos\theta}$$

$\downarrow \rightarrow \delta$

$$\frac{1}{r_4} = (r^2 + d'^2 - 2rd' \cos \theta)^{-1/2}$$
$$= \frac{1}{d'} \left(1 + \left(\frac{r}{d'}\right)^2 - 2\frac{r}{d'} \cos \theta \right)^{-1/2}$$

$$= \frac{1}{d'} \left[1 + \left(-\frac{1}{2}\right) \left(\left(\frac{r}{d'}\right)^2 - 2\frac{r}{d'} \cos \theta\right) + \mathcal{O}\left(\left(\frac{r}{d'}\right)^2\right) \right]$$

$$\frac{1}{r_4} \approx \frac{1}{d'} \left[1 + \frac{r}{d'} \cos \theta \right] + \mathcal{O}\left(\left(\frac{r}{d'}\right)^2\right)$$

$$\frac{1}{r_2} \approx \frac{1}{d'} \left[1 - \frac{r}{d'} \cos \theta \right] + \mathcal{O}\left(\left(\frac{r}{d'}\right)^2\right)$$

$$\frac{1}{r_2} = (r^2 + d'^2 + 2rd' \cos \theta)^{-1/2}$$

$$= \left[r^2 + \left(\frac{R^2}{d'}\right)^2 + 2r \left(\frac{R^2}{d'}\right) \cos \theta \right]^{-1/2}$$

$$= \frac{1}{r} \left[1 + \underbrace{\frac{R^2}{d'^2 r^2} + 2\frac{R^2}{d'r} \cos \theta}_{K_1} \right]^{-1/2}$$

$$= \frac{1}{r} \left[1 - \frac{1}{2} \left(2\frac{R^2}{d'r} \cos \theta \right) + \mathcal{O}\left(\left(\frac{R^2}{d'r}\right)^2\right) \right]$$

$$\frac{1}{r_2} = \frac{1}{r} \left[1 - \frac{R^2}{d'r} \cos \theta \right]$$

$$\frac{1}{r_1} = \frac{1}{r} \left[1 + \frac{R^2}{d'r} \cos \theta \right]$$

$$V = \frac{1}{4\pi\epsilon_0} \left[\frac{-q}{r_1} + \frac{q}{r_4} - \frac{q'}{r_2} + \frac{q'}{r_2} \right]$$

$$= \frac{1}{4\pi\epsilon_0} \left\{ -q \frac{1}{d} \left[1 - \frac{r}{d} \cos\theta \right] + q \frac{1}{d} \left[1 + \frac{r}{d} \cos\theta \right] + \left(-\frac{qR}{d} \right) \frac{1}{r} \left[1 - \frac{R^2}{r^2} \cos\theta \right] + \left(-\frac{qR}{d} \right) \frac{1}{r} \left[1 + \frac{R^2}{r^2} \cos\theta \right] \right\} + Q \left(\frac{1}{r} \right)$$

$$V = \frac{1}{4\pi\epsilon_0} \left[2 \frac{q}{d^2} r \cos\theta - 2 \frac{q}{d^2} \frac{R^3}{r^3} r \cos\theta \right]$$

$1 \rightarrow 8$
 $9 \rightarrow 8$

$\frac{q}{d^2} \rightarrow 2\epsilon_0 E_0$

$$\frac{1}{2\epsilon_0} (2\epsilon_0 E_0) z = \left(1 - \frac{R^3}{r^3} \right)$$

$$V = E_0 z \left(1 - \frac{R^3}{r^3} \right)$$

$$z = r \cos\theta$$